Minimum Quality Standard and Protectionism

Olivier BONROY*†

GREEN, University Laval

CATT, University of Pau

January 2003

Abstract

We studied the protectionist character of a miminum quality standard (MQS). We show that in the fixed cost model where two firms, one local and one foreign, compete in a local market, the implementation of a MQS on the local market is a protectionist political only if the local firm supplies the lower quality, whatever the type of competition (Cournot or Bertrand) and the technology differential between the firms.

Keywords: Minimum Quality Standard, Protecionism, Endogenous Quality.

JEL classification: F12, F13.

* *E-mail address:* olivier.bonroy@ecn.ulaval.ca

† I would like to thank Antoine Bouët, Lionel Fontagné, Jacques Le Cacheux, Jean-Marc Tallon, and participants at the 50^{st} annual conference of the French Association of Economics (AFSE) and the 12^{st} conference of GDR International Economics and Finance for hepful comments and suggestions.

1 Introduction

The success of GATT in reducing the trade barriers such as tariffs, quotas and voluntary exportations restrictions has been accompanied by an increase in the using of others trade restrictions specially the standards. The Marrakech agreements established that local standards can differ from international standards only when the objective is legitimate and when there is a scientific evidence to the use of these instruments.

So the standard may be considered like legitimate. Its objective may be the protection of human health or safety, or the live or health of plant or the environment. But this instrument may be used in a protectionist aim, they may alter the market outcome in favor of local industry. In this case, it is very difficult to see if the goal of the standard is legitimate or protectionist.

A series of articles have studied the protectionist character of standards. So Barrett (1994) shows that when two firms, one domestic and the other foreign, compete in quantities in a third market, then the domestic government has an incentive a weak restrictive standard for his firm. This standard increases the competitiveness of the local firm. The environmental objective is inferior to commercial objective. Fisher and Serra (2000) analyse the implementation of a standard when a local firm and a foreign firm compete in quantities in a domestic market. They show that the local social planner always implements a protectionist standard. Indeed the level of the standard chosen by the local social planner is always superior to the level chosen by the social planner when both firms are domestic. Moreover, Mattoo (2001) shows that even when a standard is not discriminatory (i.e. its implementation entails an identical incremental cost to all producers), it can alter the market outcome in favor of the domestic producers. The feature of these models is to concern minimum standards and not minimum quality standards (MQS). Conversely to MQS, the minimum standards have not direct effect on the utility of consumer, indeed the quality of products is the same after and before the standard. In these models the products are not differentiated, and the standards do not increase the quality of the products but the marginal cost of production of these products.

If the protectionist character of minimum standards is studied by the literature, in my knowledge only Das and Donnenfeld (1989) are studied the protectionist character of MQS. These authors propose a duopolistic model comprising a local and a foreign firm. These firms compete in qualities and in quantities. The authors show that the MQS alter the market outcome in favor of local industry only in the case where the local firm supplies the lower quality, this instrument is then protectionist.

This paper expands the approach of Das and Donnenfeld to a "fixed cost of quality model". The literature which analyses the effects of the MQS on the market proposes two ways. A part of authors assumes that the MQS increases only the marginal production costs, the models are then referred to as a "variable cost of quality model". It is the case for Das and Donnenfeld (1989), Ecchia and Lambertini (1997) and Crampes and Hollander (1995). The other part of authors assumes that the MQS increases only the fixed costs, the models are then referred to as a "fixed cost of quality model". It is the case for Ronnen (1991), Boom (1995.) or Constantatos and Perrakis (1998).

In order to analyze the protectionist character of SQM, we must define when the MQS is an instrument protectionist. Baldwin (1970) defines a protectionist measure such as the real global income decrease after its implementation. Das and Donnenfeld (1989) characterize a MQS as protectionist if its implementation improves the situation of the local firm to the detriment of the foreign firm. Fisher and Serra (2001) propose the following definition: "*A minimum standard is said to be non-protectionist when it corresponds to the standard the local social planner would use if both firms were domesti*c". In this paper, we decide to use the Das and Donnenfeld's criteria, so we assume that the MQS is protectionist if it increases the market shares and the profit of the local firm to the detriment to the market shares and the profit of foreign firm. We have two criteria, one on the quantities and one on the profit. We study exclusively when both firms stay in the market after the implementation of the MQS. Indeed, it is easy to conclude that the SQM is protectionist when the foreign firm goes out the market , but it is not easy when both firms stay in the market.

The contribution of this paper is to show that in the fixed cost model where two firms, one local and one foreign, compete in a local market, the MQS is protectionist only if the local firm supplies the lower quality whatever the type of competition (Cournot or Bertrand) and the technology differential between the firms.

The next section describes the model. In the section 3 and 4 we present successively the Cournot competition and the Bertrand competition. In the last section we discuss the results and we conclude.

2 The basic model

There are two firms, one located in an home country and other, in foreign country. Each firms produces a quality differentiated product, all of which is sold to home country market. The firm h produces a good h with high quality q_h and the firm l produces a good l with low quality q_l. There are two possible cases : the firm h is the local firm and the firm l is the foreign firm and conversely. The games between firms involves a sub-game perfect equilibrium with two stages of decision. In stage 1: the firms chose the quality level at a Nash equilibrium. In stage 2: the producers decide simultaneously whether how many customers to supply if quantity is the decision variable (Cournot competition) or the prices (Bertrand competition). As Das and Donnenfeld (1989) and Zhou and *al.* (2001) we assume that the costs of firms are asymmetric. This assumption generalizes the results to the trade north south (high technology differential) and the trade north north (low technology differential). So, the firm h requires an investment $\gamma_h F(q_h)$ to produce a product h and the firm l requires an investment $\gamma_l F(q_l)$ to produce a product l, with $\gamma_l > \gamma_h > 0$. $\gamma_l - \gamma_h$ represents the technology differential between the foreign firm and the home firm. $F(q)$ and $F'(q)$ are assumed increasing functions of q for all feasible qualities $q \in [0, \infty)$. We assume that $F(0) = F'(0) = 0$, $\lim_{q \to \infty} F'(q) = \infty$ and $F'''(q) = 0$ i.e. $F''(q_h) = F''(q_l)$. And we assume that there are no variable production costs. This model is a "fixed cost of quality model".

The basic features of consumer demand used are standard in studies of quality differentiation, as Shaked and Sutton (1982) and Ronnen (1991). There is a continuum of consumers, uniformly distributed on $[0,1]$ according to their taste parameter θ. Consumers purchase at most one unit of either firm h's product or firme l's product. Otheir things being equal, consumers prefer a higher quality product. Consumer θ maximizes the following indirect utility function:

$u_{\theta,i} = \theta q_i - p_i$ if the consumer purchases the quality q_i at price p_i, 0 if the consumer does not purchases, $\forall i \in l, h$

To determine the demand faced by the high quality and low quality firm, divide the interval $[0,1]$ into three segments. Let $\theta_1 = \frac{p_h - p_l}{q_h - q_l}$ and $\theta_0 = \frac{p_l}{q_l}$ (see figure 1). Consumers with $\theta = \theta_0$ will be indifferent between purchasing the low quality product and no purchasing at all. Consumers with $\theta = \theta_1$ will be indifferent between purchasing the high quality product or the low quality product.

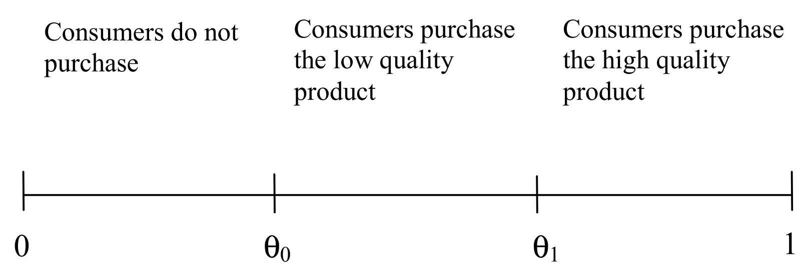

Figure 1: Market shares of firms

The demands are then given by:

$x_h = 1 - \theta_1 = 1 - \frac{p_h - p_l}{q_h - q_l}$ and $x_l = \theta_1 - \theta_0 = \frac{p_h - p_l}{q_h - q_l} - \frac{p_l}{q_l}$

3 Duopoly equilibrium under Cournot competition

From the demand functions we obtain the inverse demand functions:

$p_l = (1 - x_h)q_l - x_l q_l$ et $p_h = (1 - x_h)q_h - x_l q_l$.

We examine the second-stage first. The firms compete in quantities and maximize their respective profit[1] $\pi_i = x_i p_i - \gamma_i F(q_i)$ with $i = h, l$. We obtain the best reponse functions: $x_h = \frac{q_h - x_l q_l}{2q_h}$ and $x_l = \frac{1}{2}(1 - x_h)$. From the best reponse functions and the demand functions, we detremine the prices $p_h = \frac{2q_h^2 - q_h q_l}{4q_h - q_l}$ and $p_l = \frac{q_h q_l}{4q_h - q_l}$, the equilibrium quantities $x_h = \frac{2q_h - q_l}{4q_h - q_l}$ and $x_l = \frac{q_h}{4q_h - q_l}$, as the marginal consumers: $\theta_0 = \frac{q_h}{4q_h - q_l}$ and $\theta_1 = \frac{2q_h}{4q_h - q_l}$.

Notice that the quality-adjusted price $P_i = \frac{p_i}{q_i}$ with $(i \in h, l)$ equals the equilibrium quantities x_i. So :$P_h = x_h = \frac{2r-1}{4r-1}$ and $P_l = x_l = \frac{r}{4r-1}$ with $r = \frac{q_h}{q_l} \geq 1$, $\frac{dP_h}{dr} = \frac{dx_h}{dr} > 0$ and $\frac{dP_l}{dr} = \frac{dx_l}{dr} < 0$.

We determine the properties of R_h and R_l with $R_h = x_h p_h$ and $R_l = x_l p_l$ (Appendix 1).

In the first-stage the firms chose the quality level. The firm h chose q_h, the firm l chose q_l, with $q_h \geq q_l$. Consider first the high quality producer's best reponse to q_l. For a given q_l the high quality seller maximizes its profit subject to $q_l \leq q_h$. This problem has a unique solution satisfysing $MR_h(q_h, q_l) = \gamma_h F'(q_h)$ with $q_h \in [q_l, \infty[$, and the second order condition : $\frac{\partial^2 \pi_h}{\partial q_h^2} < 0$ (Appendix 2). Using the first order condition, we determine the high quality producer's best reponse noted $b_h(q_l)$. The same way, we determine the low quality producer's best reponse to q_h noted $b_l(q_h)$. For a given q_h the low quality seller maximizes its profit subject to $q_l \leq q_h$. This problem has a unique solution satisfysing $MR_l(q_h, q_l) = \gamma_l F'(q_l)$ with $q_l \in [0, q_h]$, as the second order condition : $\frac{\partial^2 \pi_l}{\partial q_l^2} < 0$ (Appendix 2). The crosed condition is respected :

$$\frac{\partial^2 \pi_h}{\partial q_h^2} \frac{\partial^2 \pi_l}{\partial q_l^2} - \frac{\partial^2 \pi_h}{\partial q_l \partial q_h} \frac{\partial^2 \pi_l}{\partial q_h \partial q_l} = \frac{\partial^2 \pi_h}{\partial q_h^2} \frac{\partial^2 \pi_l}{\partial q_l^2} - \frac{\partial MR_h}{\partial q_l} \frac{\partial MR_l}{\partial q_h} > 0.$$

By fully differentiating the first order conditions with respect to q_l, and by using the Euler's theorem we determine a relation between the best reponse functions and the differenciation degree r (Appendix 4).

$$r > \frac{\partial b_h(q_l)}{\partial q_l} = \frac{\frac{\partial MR_h}{\partial q_l}}{\gamma_h F'' - \frac{\partial MR_h}{\partial qh}} > 0 \text{ et } \frac{1}{r} > 0 > \frac{\partial b_l(q_h)}{\partial q_h} = \frac{\frac{\partial MR_l}{\partial q_h}}{\gamma_l F'' - \frac{\partial MR_l}{\partial q_l}} \tag{1}$$

[1] The first order and crosed conditions are respected. :
$\frac{\partial^2 \pi_h}{\partial x_h^2} = -2q_h < 0$ et $\frac{\partial^2 \pi_l}{\partial x_l^2} = -2q_l < 0$; $\frac{\partial^2 \pi_h}{\partial x_h^2} \frac{\partial^2 \pi_l}{\partial x_l^2} - \frac{\partial^2 \pi_h}{\partial x_l \partial x_h} \frac{\partial^2 \pi_l}{\partial x_h \partial x_l} = 4q_h q_l - q_l^2 > 0$

The best reponse function $b_h(q_l)$ is restricted, because if q_l is relatively high, the best reponse of the firm h is to enter the market as the quality q_h seller with $q_h \leq q_l$. Just as for the best reponse function $b_l(q_h)$, so if q_h is relatively low, the best reponse of the firm l is to enter the market as the quality q_l seller with $q_l \geq q_h$.

When the technology differential is important, the best reponse functions are not restricted. There is only one equilibrium. The equilibrium where the firm h supplies the low quality and the firm l supplies the high quality does not exist.

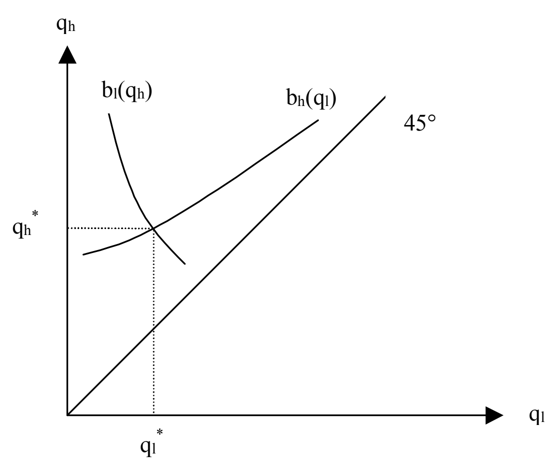

Figure 2: Best reponse functions under Cournot competition.

The best reponse function $b_h(q_l)$ is increasing with the level of q_l, conversely of best reponse function $b_l(q_h)$ which is decreasing with the level of q_h (see figure 2). As $r = \frac{q_h}{q_l} > 1$, the best reponse functions are on top of the 45° straight line.

The qualities equilibrium are $q_l^* = b_l(q_h^*)$ et $q_h^* = b_h(q_l^*)$, they are stable. And the profit are positive for $q_h > q_l$ (Annexe 5).

Consequences of a minimum quality standard.

The local social planner introduces a MQS q^{sqm} such as $q^{sqm} > q_l^*$. We assume that $q^{sqm} - q_l^*$ is weak to assure that two firms stay in the market. Indeed, since the profits are positive before the introduction of the SQM and since $R_h(b_h(q_l), q_l) - \gamma_h F(b_h(q_l))$ and $R_l(b_h(q_l), q_l) - \gamma_l F(q_l)$

are continous in q_l, there exists q_l noted $\overline{q}^c$ and superior to q_l^* such that if $q^{sqm} \in (q_l^*, \overline{q}^c]$ the profits are positive when $(b_h(q^{sqm}), q^{sqm})$ is played. Hence $(b_h(q^{sqm}), q^{sqm})$ is an equilibrium when $q^{sqm} \in (q_l^*, \overline{q}^c]$. The firm l sells a product of standard quality q^{sqm} and the firm h a product of quality $q_h = b_h(q^{sqm})$ with $q_h > q_h^*$ and $q_h > q^{sqm}$.

Proposition 1 *The introduction of a MQS, close to q_l^*, improves the level of high quality and decreases the differentiation degree.*

Proof. $\frac{dq_h}{dq_l} = \frac{\frac{\partial MR_h}{\partial q_l}}{\gamma_h F'' - \frac{\partial MR_h}{\partial q_h}} > 0$ and $\frac{dr}{dq_l} = \frac{q_h^{'} - r}{q_l} < 0$ because $r > q_h^{'}$ with $q_h' = \frac{dq_h}{dq_l} > 0$ ■

This result is in accordance with the result of Ronnen (1991). The introduction of the MQS increases the level of low quality. The revenue of the firm h decreases $\left(\frac{\partial R_h}{\partial q_l} < 0\right)$, so this firm improves its quality: q_h increases. But increasing the quality is more costly to the high quality seller than to the low quality seller. The proportional increase in q_h is less than the proportional increase in q_l, r decreases. This result can appear in contradiction to the assumption that $\gamma_l > \gamma_h > 0$, but lower is γ_h higher is q_h^* (Appendix 6), so it is impossible to the firm h to improve its quality in the same proportion that the rise of the low quality. The proportional increase in q_h is equal to the proportional increase in q_l only if $r = q_h'$ i.e. $\gamma_h = 0$.

Proposition 2 *The introduction of a MQS, close to q_l^*, decreases the high quality product demand, increases the low quality product demand and consumers which are no active (see figure 3).*

If the local firm produces the high quality, the MQS decreases its market shares and increases the market shares of the foreign firm ; if the local firm produces the low quality, the MQS increases its market shares and decreases the market shares of the foreign firm

Proof. $\frac{dx_h}{dq_l} = \frac{dP_h}{dq_l} = \frac{2q_h^{'}q_l - 2q_h}{(4q_h - q_l)^2} < 0$ because $r > q_h^{'}$; and $\frac{dx_l}{dq_l} = \frac{dP_l}{dq_l} = \frac{q_h - q_h^{'}q_l}{(4q_h - q_l)^2} > 0$ because $r > q_h^{'}$

$\frac{d\theta_0}{dq_l} = \frac{q_h - q_h^{'}q_l}{(4q_h - q_l)^2} > 0$ because $r > q_h^{'}$; and $\frac{d\theta_1}{dq_l} = \frac{2(q_h - q_h^{'}q_l)}{(4q_h - q_l)^2} > 0$ because $r > q_h^{'}$ ■

For understand this result, note that the introduction of a SQM decreases r (Proposition 1), and for x_h and x_l fixed, a decreases of r shifts down the demand curve for product h, decreasing the willingness of consumers to pay for the high quality product, but the willingness of consumers to pay for the low quality product is the same[2]. The high quality product demand is lower so the firm h decreases its supply. The firm l reacts and increases its production. Notice that the quality-adjusted price of the high quality product decreases while the quality-adjusted price of the low quality product increases. The marginal consumer of the firm l becomes no active, while the marginal consumer of the firm h alters his behaviour and purchases the low quality product.

Consumers' interval	Before MQS	After MQS
$[0, \theta_0]$	no purchase	no purchase
$[\theta_0, \theta_0^{sqm}]$	purchase q_l^*	no purchase
$[\theta_0^{sqm}, \theta_1]$	purchase q_l^*	purchase q_l^{sqm}
$[\theta_1, \theta_1^{sqm}]$	purchase q_h^*	purchase q_l^{sqm}
$[\theta_1^{sqm}, 1]$	purchase q_h^*	purchase q_h^{sqm}

Figure 3: SQM impacts on the market shares of firms

Conversely to Das and Donnenfeld (1989), the number of consumers who switch from the firm h to the firm l is sufficient to offset the number of customers that exit the market.

The surplus of consumers increases:

$$SC = \int_{\theta_0}^{\theta_1} (\theta q_l - p_l) d\theta + \int_{\theta_1}^{1} (\theta q_h - p_h) d\theta$$

$$\frac{dSC}{dq_l} = \frac{\partial SC}{\partial q_l} + \frac{\partial SC}{\partial q_h} \frac{dq_h}{dq_l} = \frac{q_h^2(12q_h - 7q_l)}{2(4q_h - q_l)^3} + \frac{16q_h^3 - 12q_h^2 q_l + 2q_h q_l^2 + q_l^3}{2(4q_h - q_l)^3} \frac{dq_h}{dq_l} > 0$$

We may note that the surplus increases for all consumers (Appendix 7)

While the profits of both firms decreases. Using the envelope theorem, we observe that:

$$\frac{d\pi_h(q_h, q_l)}{dq_l} = \frac{\partial R_h}{\partial q_l} < 0 \text{ et } \frac{d\pi_l(q_h, q_l)}{dq_l} = \frac{\partial R_l}{\partial q_h} \frac{dq_h}{dq_l} < 0$$

The rise of low quality decreases the revenue of the firm h, this firm improves its quality, but it is insufficient, its profit decreases. The rise of high quality decreases the revenue of the firm l.

[2] v_h the willingness of consumers to pay for the high quality product and v_l the willingness of consumers to pay for the low quality product, as $v_h = \frac{p_h}{q_h} = (1 - x_h) - \frac{x_l}{r}$ and $v_l = \frac{p_l}{q_l} = (1 - x_h - x_l)$. We have for x_h and x_l constant: $\frac{\partial v_h}{\partial r} = \frac{x_l}{r^2} > 0$ and $\frac{\partial v_l}{\partial r} = 0$

Both firms are worse off compared to the equilibrium of the unregulated market. But the firm h cannot drive the firm l to play the quality q_l of the initial Cournot Nash game, because this one cannot supply a quality inferior to the standard quality.

Now, we examine the effects of the MQS on the social welfare[3].

The MQS has ambigous effects on the social welfare, these effects depend to origins of the firms.

Proposition 3 *When the home firm produces the high quality good, the introduction of the MQS, close to q_l^*, decreases the social welfare w_h.*

Proof. $w_h = \pi_h + sc_h + sc_l = \int_{\theta_1}^{1} (\theta q_h) d\theta - \gamma_h F(q_h) + \int_{\theta_0}^{\theta_1} (\theta q_l - p_l) d\theta$. By fully differentiating w_h with respect to q_l, substituting $\gamma_h F'(q_h)$ by MR_h and $\gamma_h F''$ by $\frac{MR_h}{q_h}$, we obtain :

$\frac{dw_h}{dq_l} = -\frac{1}{2} q_h^7 \frac{256 - 448\sigma + 432\sigma^2 - 212\sigma^3 + 28\sigma^4 + 7\sigma^5}{(4q_h - q_l)^3 \left(64q_h^4 - 64q_h^3 q_l + 36q_h^2 q_l^2 - 16q_h q_l^3 + q_l^4\right)} < 0$ with $\sigma = \frac{q_l}{q_h}$ and $\sigma \in]0,1[$ (Appendix 8) ■

The rise of the surplus of consumers is insufficient to compensate the reduction of the profit of the local firm. Das and Donnenfeld (1989) obtain the same results in a "variable cost of quality model". This result is ambigous, indeed the MQS improves the qualities but decreases the local social welfare.

Proposition 4 *When the home firm produces the low quality good, the introduction of the MQS, close to q_l^*, increases the social welfare w_l.*

Proof. $w_l = \pi_l + sc_l + sc_h = \int_{\theta_0}^{\theta_1} (\theta q_l) d\theta - \gamma_l F(q_l) + \int_{\theta_1}^{1} (\theta q_h - p_h) d\theta$. By fully differentiating w_l with respect to q_l, substituting $\gamma_l F'(q_l)$ by MR_l, we obtain :

$\frac{dw_l}{dq_l} = \frac{1}{2} \frac{16q_h' q_h^3 - 12q_h' q_h^2 q_l - 2q_h' q_l^2 q_h + q_h' q_l^3 + 12q_h^3 - 7q_h^2 q_l}{(4q_h - q_l)^3} > 0$ with $q_h' = \frac{dq_h}{dq_l} < 1$ ■

To understand these both results, note that the MQS has a negative effect more important on the firm h than the firm l. Indeed, as a result of the MQS, both firms increase theirs costs, but

[3] The social welfare is the sum between the surplus of consumers and the profit of the local firm.

only the firm h must decrease its output, this benefits to the firm l (Proposition 2). So, the rise of surplus is sufficient to compensate the reduction of the profit of the local firm, if only this one supplies the lower quality.

Finally, we show that the protectionist character also depends to origins of the firms.

Proposition 5 *When the home firm produces the high-quality product, the introduction of the MQS, close to q_l^*, may not be considered like a protectionist instrument, the quantity of the foreign products does not decrease, and there is not monetary transfer of the foreign firm profit to the home firm profit. Whereas when the home firm produces the low-quality product, the introduction of the MQS, close to q_l^*, decreases the quantity of the foreign products in favor of the home firm, the MQS is a protectionist instrument, the criterion on the quantity is broken.*

We note that if the objective of the government is to improve the qualities and increase the home social welfare, a MQS (equal to $\overline{q}^c$) is implement only if the local firm supplies the lower quality. Then this instrument may be consider like protectionist because one of both criteria is broken.

These results and the results of Das and Donnenfeld (1989) are similar. The MQS may be consider like a protectionist instrument only when the home firm produces the low quality level. We note that in the model of Das et Donnenfeld, only the criteria on the profit is broken. In our model only the criterion on the quantity is broken.

4 Duopoly equilibrium under Bertrand competition

We examine the second-stage first. The firms compete in prices and maximize their respective profit[4]. We obtain the best reponse functions: $p_h = \frac{1}{2}q_h - \frac{1}{2}q_l + \frac{1}{2}p_l$ and $p_l = \frac{1}{2}\frac{p_h}{q_h}q_l$. From the best

[4]The first order and crosed conditions are respected. :
$\frac{\partial^2 \pi_h}{\partial x_h^2} = \frac{2}{-q_h+q_l} < 0$ et $\frac{\partial^2 \pi_l}{\partial x_l^2} = 2\frac{q_h}{(-q_h+q_l)q_l} < 0$; $\frac{\partial^2 \pi_h}{\partial x_h^2}\frac{\partial^2 \pi_l}{\partial x_l^2} - \frac{\partial^2 \pi_h}{\partial x_l \partial x_h}\frac{\partial^2 \pi_l}{\partial x_h \partial x_l} = \frac{4}{(-q_h+q_l)^2}\frac{q_h}{q_l} > 0$

reponse functions and the demand functions, we determine the prices, the equilibrium quantities and the marginal consumers:

$p_h = \frac{2(q_h - q_l)q_h}{4q_h - q_l}$ and $p_l = \frac{(q_h - q_l)q_l}{4q_h - q_l}$

$x_h = 2\frac{q_h}{4q_h - q_l}$ and $x_l = \frac{q_h}{4q_h - q_l}$

$\theta_0 = \frac{q_h - q_l}{4q_h - q_l}$ and $\theta_1 = \frac{2q_h - q_l}{4q_h - q_l}$

We may notice that an increase of r decreases the consummation but increases the quality-adjusted prices:

$P_h = \frac{p_h}{q_h} = \frac{2(q_h - q_l)}{4q_h - q_l} = \frac{2(r-1)}{4r-1}$ and $P_l = \frac{p_l}{q_l} = \frac{(q_h - q_l)}{4q_h - q_l} = \frac{(r-1)}{4r-1}$

$\frac{dP_h}{dr} = \frac{d\frac{2(r-1)}{4r-1}}{dr} = \frac{6}{(4r-1)^2} > 0$ and $\frac{dP_l}{dr} = \frac{d\frac{(r-1)}{4r-1}}{dr} = \frac{3}{(4r-1)^2} > 0$

$\frac{dx_h}{dr} = \frac{d2\frac{r}{4r-1}}{dr} = -\frac{2}{(4r-1)^2} < 0$ et $\frac{dx_l}{dr} = \frac{d\frac{r}{4r-1}}{dr} = -\frac{1}{(4r-1)^2} < 0$

In a Cournot competition, an increase of r increases the consummation and the quality-adjusted price of the high quality product but decreases the consummation and the quality-adjusted price of the low quality product.

We determine the properties of R_h and R_l with $R_h = x_h p_h$ and $R_l = x_l p_l$ (Appendix 1).

Like in the environment of Cournot competition, the firms chose the quality level. The firm h chose q_h, the firm l chose q_l, with $q_h \geq q_l$. Consider first the high quality producer's best reponse to q_l. For a given q_l the high quality seller maximizes its profit subject to $q_l \leq q_h$. This problem has a unique solution satisfysing $MR_h(q_h, q_l) = \gamma_h F'(q_h)$ with $q_h \in [q_l, \infty[$, and the second order condition : $\frac{\partial^2 \pi_h}{\partial q_h^2} < 0$ (Appendix 2). Using the first order condition, we determine the high quality producer's best reponse noted $b_h(q_l)$. The same way, we determine the low quality producer's best reponse to q_h noted $b_l(q_h)$. For a given q_h the low quality seller maximizes its profit subject to $q_l \leq q_h$. This problem has a unique solution satisfysing $MR_l(q_h, q_l) = \gamma_l F'(q_l)$ with $q_l \in [0, q_h]$, as the second order condition : $\frac{\partial^2 \pi_l}{\partial q_l^2} < 0$ (Appendix 2). The crosed condition is respected :

$\frac{\partial^2 \pi_h}{\partial q_h^2}\frac{\partial^2 \pi_l}{\partial q_l^2} - \frac{\partial^2 \pi_h}{\partial q_l \partial q_h}\frac{\partial^2 \pi_l}{\partial q_h \partial q_l} = \frac{\partial^2 \pi_h}{\partial q_h^2}\frac{\partial^2 \pi_l}{\partial q_l^2} - \frac{\partial MR_h}{\partial q_l}\frac{\partial MR_l}{\partial q_h} > 0$ (Appendix 3).

By fully differentiating the first order conditions with respect to q_l, and by using the Euler's theorem we determine a relation between the best reponse functions and the differenciation degree r (Appendix 4).

$$r > \frac{\partial b_h(q_l)}{\partial q_l} = \frac{\frac{\partial MR_h}{\partial q_l}}{\gamma_h F'' - \frac{\partial MR_h}{\partial qh}} > 0 \text{ et } \frac{1}{r} > \frac{\partial b_l(q_h)}{\partial q_h} = \frac{\frac{\partial MR_l}{\partial q_h}}{\gamma_l F'' - \frac{\partial MR_l}{\partial q_l}} > 0 \tag{2}$$

The best reponse function $b_h(q_l)$ is restricted, because if q_l is relatively high, the best reponse of the firm h is to enter the market as the quality q_h seller with $q_h \leq q_l$. Just as for the best reponse function $b_l(q_h)$, so if q_h is relatively low, the best reponse of the firm l is to enter the market as the quality q_l seller with $q_l \geq q_h$.

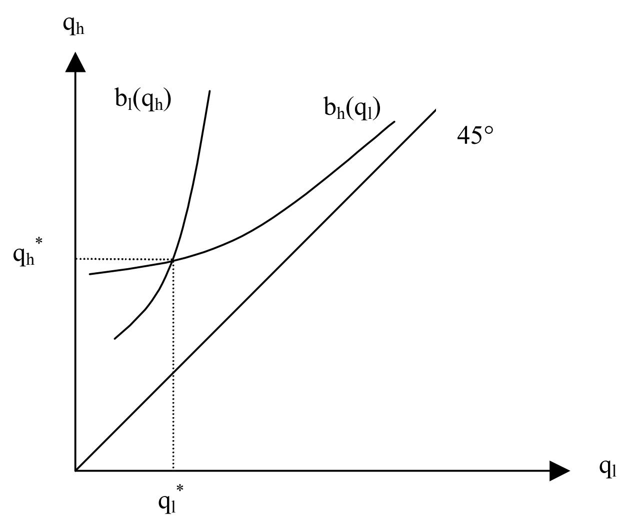

Figure 4: Best reponse functions under Bertrand competition.

The best reponse function $b_h(q_l)$ is increasing with the level of q_l, like the best reponse function $b_l(q_h)$ which is increasing with the level of q_h (see figure 4). As $r = \frac{q_h}{q_l} > 1$, the best reponse functions are on top of the 45° straight line.

The qualities equilibrium are $q_l^* = b_l(q_h^*)$ et $q_h^* = b_h(q_l^*)$, it are stable. And the profit are positive for $q_h > q_l$ (Appendix 5).

Consequences of a minimum quality standard.

Like in the environment of Cournot competition, thelocal social planner introduces a MQS q^{sqm} such as $q^{sqm} > q_l^*$. We assume that $q^{sqm} - q_l^*$ is weak to assure that two firms stay in the market, with $q^{sqm} \in \left(q_l^*, \overline{q}^b\right]$.

This instrument increases the high quality level and decreases the differentiation degree.

Proposition 6 *The introduction of a MQS, close to q_l^*, increases the competition. The demand of both products increases and consumers which are no active decreases (see figure 5).*

Whatever the quality produced by the local firm, the MQS increases its market shares and the market shares of the foreign firm.

Proof. $\frac{dx_h}{dq_l} = \frac{2\left(q_h - q_h^{'} q_l\right)}{(4q_h - q_l)^2} > 0$ because $r > q_h^{'}$; and $\frac{dx_l}{dq_l} = \frac{q_h - q_h^{'} q_l}{(4q_h - q_l)^2} > 0$ because $r > q_h^{'}$ $\frac{d\theta_0}{dq_l} = \frac{2\left(q_h^{'} q_l - q_h\right)}{(4q_h - q_l)^2} < 0$ because $r > q_h^{'}$; and $\frac{d\theta_1}{dq_l} = \frac{2\left(q_h^{'} q_l - q_h\right)}{(4q_h - q_l)^2} < 0$ because $r > q_h^{'}$ ■

Consumers' interval	Before MQS	After MQS
$[0, \theta_0^{sqm}]$	no purchase	no purchase
$[\theta_0^{sqm}, \theta_0]$	no purchase	purchase q_l^{sqm}
$[\theta_0, \theta_1^{sqm}]$	purchase q_l^*	purchase q_l^{sqm}
$[\theta_1^{sqm}, \theta_1]$	purchase q_l^*	purchase q_h^{sqm}
$[\theta_1, 1]$	purchase q_h^*	purchase q_h^{sqm}

Figure 5: SQM impacts on the market shares of firms

These results are similar with Ronnen results but not with the results of the last section. All consumers, who are active in the market before the implementation of the MQS, purchase a product of higher quality. And more consumers are active in the market.

The effets of the implementation of the label are different depending on the competition. The decrease of differentiation degree r decreases the wilingness to pay for the high quality product (the willingness of consumers to pay for the low quality product is the same) whatever the competition. In a Bertrand competition, the firm h decreases its price, so the firm l reacts and decreases its price. In a Cournot competition, the firm h decreases its supply, so the firm l reacts and increases its production. In Bertrand, all quality-adjusted prices décrease. In Cournot, only the quality-adjusted price of the high quality product decreases.

Like in a Cournot Competition, the surplus of consumers increases (Appendix 7) :

$\frac{dSC}{dq_l} = \frac{\partial SC}{\partial q_l} + \frac{\partial SC}{\partial q_h}\frac{dq_h}{dq_l} = \frac{1}{2}\frac{\left(112q_h^3-24q_h^2q_l+99q_hq_l^2+20q_l^3\right)q_h}{\left(16q_h^2-16q_hq_l+21q_l^2\right)(4q_h-q_l)^2} > 0.$

But only the profit of the firm h decreases:

$\frac{d\pi_h(q_h,q_l)}{dq_l} = \frac{\partial R_h}{\partial q_l} < 0$ et $\frac{d\pi_l(q_h,q_l)}{dq_l} = \frac{\partial R_l}{\partial q_h}\frac{dq_h}{dq_l} > 0.$

The increase of q_l decreases the R_h, the firm h increases its quality, but it is insufficient for prevent the decrease of its profit. The firm h cannot improve its position because it cannot force the firm l to play the initial quality, indeed the firm l cannot supply a quality inferior to the MQS. Conversely to the case of Cournot competition, this situation benefits to the firm l. Indeed, the increase of q_h increases R_l.

Now, we analyze the effects of the MQS on the social welfare.

Proposition 7 *The introduction of the MQS, close to q_l^*, increases the social welfare, whatever the quality produced by the local firm.*

Proof. $w_h = \pi_h + sc_h + sc_l = \int_{\theta_1}^{1}(\theta q_h)d\theta - \lambda_h F(q_h) + \int_{\theta_0}^{\theta_1}(\theta q_l - p_l)d\theta$. By fully differentiating w_h with respect to q_l, substituting $\gamma_h F'(q_h)$ by MR_h and $\gamma_h F''$ by $\frac{MR_h}{q_h}$, we obtain :

$\frac{dw_h}{dq_l} = q_h\frac{-192q_h^4+80q_h^3q_l-212q_h^2q_l^2+187q_hq_l^3+20q_l^4}{(-4q_h+q_l)^3\left(16q_h^2-16q_hq_l+21q_l^2\right)} > 0$ (Appendix 8)

$w_l = \pi_l + sc_l + sc_h = \int_{\theta_0}^{\theta_1}(\theta q_l)d\theta - \gamma_l F(q_l) + \int_{\theta_1}^{1}(\theta q_h - p_h)d\theta$. By fully differentiating w_l with respect to q_l, we obtain :

$\frac{dw_l}{dq_l} = \frac{\partial w_l}{\partial q_l} + \frac{\partial w_l}{\partial q_h}q_h' = \frac{1}{2}q_h^2\frac{28q_h+5q_l}{(4q_h-q_l)^3} + \frac{2q_h^2-q_hq_l-q_l^2}{(4q_h-q_l)^2}q_h' > 0$ ■

Like in the case of Cournot competition, the protectionist character of the SQM depends to origins of the firms.

Proposition 8 *When the home firm produces the high-quality product, the introduction of the MQS, close to q_l^*, may not be considered like a protectionist instrument, the quantity of the foreign products does not decrease, and there is not monetary transfer of the foreign firm profit to the home firm profit. Whereas when the home firm produces the low-quality product, the introduction of the MQS, close to q_l^*, decreases the profit of the foreign firm and increases the profit of the home firm, the MQS is a protectionist instrument, the criterion on the profit is broken.*

We note that if the objective of the government is to improve the qualities and increase the home social welfare, a MQS (equal to $\overline{q}^b$) is implement whatever the firm which supplies the lower quality. When the home firm produces the low quality product, then the MQS may be consider like protectionist because one of both critera is broken.

5 Conclusion and discussion

In this paper we demonstrate that, in a duopolistic model, the MQS is protectionist only if the lower quality seller is local.

We note that the SQM benefits to the low-quality sellers. This result contradicts the critics of Southern countries which think that the MQS penalizes theirs exports (generally low-quality products) to Northern countries. Indeed, if the Southern countries can supply a quality superior to the Northern countries SQM (i.e. the SQM is not over restrictive), this instrument benefits them whatever the competition (Cournot or Bertrand) and the technology differential between the southern industry and the northern industry.

In Cournot competition, when the home firm produces the high-quality product, the introduction of a weakly restrictive MQS may not be considered like a protectionist instrument, the quantity of the foreign products (x^*) does not decrease, and there is not monetary transfer of the foreign firm profit (π^*) to the home firm profit (π). Whereas when the home firm produces the low-quality product, the introduction of a weakly restrictive MQS decreases the quantity of the foreign products in favor of the home firm, the MQS is a protectionist instrument, the criterion on the quantity is broken (see figure 6).

	COURNOT *h* the local firm	**BERTRAND** *h* the local firm	**COURNOT** *l* the local firm	**BERTRAND** *l* the local firm
x	-	+	+	+
x*	+	+	-	+
π	-	-	-	+
π*	-	+	-	-
sc	+		+	
w	-	+	+	+

Figure 6: MQS effects on the payoff of the players

In Bertrand competition, when the home firm produces the high-quality product, the introduction of a weakly restrictive MQS may not be considered like a protectionist instrument, the quantity of the foreign products does not decrease, and there is not monetary transfer of the foreign firm profit to the home firm profit. Whereas when the home firm produces the low-quality product, the introduction of a weakly restrictive MQS decreases the profit of the foreign firm and increases the profit of the home firm, the MQS is a protectionist instrument, the criterion on the profit is broken (see figure 6).

This paper has focused on the environment perfect information, so it is interesting to wonder whether the results would be different under imperfect information.

Appendix

Appendix 1

Cournot competition

$R_h = \frac{q_h(2q_h-q_l)^2}{(4q_h-q_l)^2}$ et $R_l = \frac{q_h^2 q_l}{(4q_h-q_l)^2}$.

$\frac{\partial R_h}{\partial q_h} = MR_h = \frac{(2q_h-q_l)(8q_h^2-2q_hq_l+q_l^2)}{(4q_h-q_l)^3}$ et $\frac{\partial R_l}{\partial q_l} = MR_l = \frac{q_h^2(4q_h+q_l)}{(4q_h-q_l)^3}$.

$\frac{\partial MR_h}{\partial q_h} = \frac{8q_l^2(q_l-q_h)}{(4q_h-q_l)^4} < 0$ et $\frac{\partial MR_l}{\partial q_l} = \frac{2q_h^2(8q_h+q_l)}{(4q_h-q_l)^4} > 0$.

$\frac{\partial R_h}{\partial q_l} = \frac{4q_h^2(q_l-2q_h)}{(4q_h-q_l)^3} < 0$ et $\frac{\partial R_l}{\partial q_h} = \frac{-2q_hq_l^2}{(4q_h-q_l)^3} < 0$.

$\frac{\partial MR_h}{\partial q_l} = \frac{8q_hq_l(q_h-q_l)}{(4q_h-q_l)^4} > 0$ et $\frac{\partial MR_l}{\partial q_h} = \frac{-2q_hq_l(8q_h+q_l)}{(4q_h-q_l)^4} < 0$.

Bertrand competition

$R_h = 4q_h^2 \frac{q_h-q_l}{(4q_h-q_l)^2}$ et $R_l = \frac{q_lq_h(q_h-q_l)}{(4q_h-q_l)^2}$.

$\frac{\partial R_h}{\partial q_h} = MR_h = -4q_h \frac{4q_h^2-3q_hq_l+2q_l^2}{(-4q_h+q_l)^3}$ et $\frac{\partial R_l}{\partial q_l} = MR_l = q_h^2 \frac{7q_l-4q_h}{(-4q_h+q_l)^3}$.

$\frac{\partial MR_h}{\partial q_h} = -8q_l^2 \frac{5q_h+q_l}{(4q_h-q_l)^4} < 0$ et $\frac{\partial MR_l}{\partial q_l} = -2q_h^2 \frac{8q_h+7q_l}{(4q_h-q_l)^4} < 0$.

$\frac{\partial R_h}{\partial q_l} = 4q_h^2 \frac{2q_h+q_l}{(-4q_h+q_l)^3} < 0$ et $\frac{\partial R_l}{\partial q_h} = -q_l^2 \frac{2q_h+q_l}{(-4q_h+q_l)^3} > 0$.

$\frac{\partial MR_h}{\partial q_l} = 8q_hq_l \frac{5q_h+q_l}{(4q_h-q_l)^4} > 0$ et $\frac{\partial MR_l}{\partial q_h} = 2q_hq_l \frac{8q_h+7q_l}{(4q_h-q_l)^4} > 0$.

Appendix 2

Cournot competition

We verify the second order condition i.e.:

$\frac{\partial^2 \pi_h}{\partial q_h^2} < 0$ and $\frac{\partial^2 \pi_l}{\partial q_l^2} < 0$.

- $\frac{\partial^2 \pi_h}{\partial q_h^2} < 0$ because $\frac{\partial MR_h}{\partial q_h} < 0$ and $\gamma_h F''(q_h) > 0$.

- alike, $\frac{\partial^2 \pi_l}{\partial q_l^2} < 0$.

  Proof. $\frac{\partial^2 \pi_l}{\partial q_l^2} = \frac{\partial MR_l}{\partial q_l} - \gamma_l F''(q_l)$.

  As $F''(q_l) = F''(q_h) = \frac{F'(q_h)}{q_h} = \frac{F'(q_l)}{q_l}$, we obtain: $\frac{F'(q_l)}{F'(q_h)} = \frac{1}{r}$.

  Then we can write:

  $\frac{MR_l}{MR_h} = \frac{\gamma_l}{\gamma_h} \frac{F'(q_l)}{F'(q_h)} = \frac{\gamma_l}{\gamma_h} \frac{1}{r}$.

  So $\gamma_l F'(q_l) = MR_l = \frac{\gamma_l}{\gamma_h} \frac{MR_h}{r}$. Now we can determine the sign of $\frac{\partial^2 \pi_l}{\partial q_l^2}$

  $\frac{\partial^2 \pi_l}{\partial q_l^2} = \frac{\partial MR_l}{\partial q_l} - \frac{\gamma_l}{\gamma_h} \frac{MR_h}{q_h} < \frac{\partial MR_l}{\partial q_l} - \frac{MR_h}{q_h} = \frac{-\left[48q_h^4 - 66q_h^3 q_l + 28q_h^2 q_l^2 - 8q_h q_l^3 + q_l^4\right]}{q_h (4q_h - q_l)^4} < 0,$

  with $\frac{\gamma_l}{\gamma_h} > 1$. ■

Bertrand competition

- $\frac{\partial^2 \pi_h}{\partial q_h^2} < 0$ indeed, $\frac{\partial MR_h}{\partial q_h} < 0$ and $\gamma_h F''(q_h) > 0$.

- $\frac{\partial^2 \pi_l}{\partial q_l^2} < 0$ indeed, $\frac{\partial MR_l}{\partial q_l} < 0$ and $\gamma_l F''(q_l) > 0$.

Appendix 3

As in the Bertrand competition $\frac{\partial MR_h}{\partial q_l} \frac{\partial MR_l}{\partial q_h} > 0$, then we must verify the crosed condition.

As $\frac{\partial MR_h}{\partial q_h} = -\frac{\partial MR_h}{\partial q_l} \frac{q_l}{q_h}$ and $\frac{\partial MR_l}{\partial q_l} = -\frac{\partial MR_l}{\partial q_h} \frac{q_h}{q_l}$ (annexe 1) then:

$\frac{\partial^2 \pi_h}{\partial q_h^2} \frac{\partial^2 \pi_l}{\partial q_l^2} - \frac{\partial^2 \pi_h}{\partial q_l \partial q_h} \frac{\partial^2 \pi_l}{\partial q_h \partial q_l} = \left(-\frac{\partial MR_h}{\partial q_l} \frac{q_l}{q_h} - \gamma_h F''(q_h)\right) \left(-\frac{\partial MR_l}{\partial q_h} \frac{q_h}{q_l} - \gamma_l F''(q_l)\right) - \frac{\partial MR_h}{\partial q_l} \frac{\partial MR_l}{\partial q_h}$

$\frac{\partial^2 \pi_h}{\partial q_h^2} \frac{\partial^2 \pi_l}{\partial q_l^2} - \frac{\partial^2 \pi_h}{\partial q_l \partial q_h} \frac{\partial^2 \pi_l}{\partial q_h \partial q_l} = \frac{\partial MR_h}{\partial q_l} \frac{q_l}{q_h} \frac{\partial MR_l}{\partial q_h} \frac{q_h}{q_l} + \frac{\partial MR_h}{\partial q_l} \frac{q_l}{q_h} \gamma_l F''(q_l) + \frac{\partial MR_l}{\partial q_h} \frac{q_h}{q_l} \gamma_h F''(q_h)$

$+\gamma_h F''(q_h) \gamma_l F''(q_l) - \frac{\partial MR_h}{\partial q_l} \frac{\partial MR_l}{\partial q_h}$.

So:

$$\frac{\partial^2 \pi_h}{\partial q_h^2} \frac{\partial^2 \pi_l}{\partial q_l^2} - \frac{\partial^2 \pi_h}{\partial q_l \partial q_h} \frac{\partial^2 \pi_l}{\partial q_h \partial q_l} = \frac{\partial MR_h}{\partial q_l} \frac{q_l}{q_h} \gamma_l F''(q_l) + \frac{\partial MR_l}{\partial q_h} \frac{q_h}{q_l} \gamma_h F''(q_h) + \gamma_h F''(q_h) \gamma_l F''(q_l) > 0.$$

Appendix 4

Cournot competition

Using Ronnen methodology (1991), we determine a connection between the differentiation degree and $\frac{\partial b_h(q_l)}{\partial q_l}$.

$MR_h(q_h, q_l) = \gamma_h F'(q_h)$.

By fully differentiating this equation with respect to q_l, we obtain :

$\frac{\partial b_h(q_l)}{\partial q_l} = \frac{\frac{\partial MR_h}{\partial q_l}}{\gamma_h F'' - \frac{\partial MR_h}{\partial q_h}} > 0.$

Following the Euler's theorem we obtain:

$q_h(\frac{\partial MR_h}{\partial q_h}) + q_l(\frac{\partial MR_h}{\partial q_l}) = 0$, because $MR_h(q_h, q_l)$ is homogeneous of degree zero, $MR_h(kq_h, kq_l) = k^0 MR_h(q_h, q_l)$.

$\Leftrightarrow r = \frac{q_h}{q_l} = \frac{-\frac{\partial MR_h}{\partial q_l}}{\frac{\partial MR_h}{\partial q_h}} > 0.$

So:

$\frac{-\frac{\partial MR_h}{\partial q_l}}{\frac{\partial MR_h}{\partial q_h}} > \frac{\frac{\partial MR_h}{\partial q_l}}{\gamma_h F'' - \frac{\partial MR_h}{\partial q_h}} \Leftrightarrow r > \frac{\partial b_h(q_l)}{\partial q_l} > 0.$

A like:

$\frac{1}{r} > 0 > \frac{\partial b_l(q_h)}{\partial q_h} = \frac{\frac{\partial MR_l}{\partial q_h}}{\gamma_l F'' - \frac{\partial MR_l}{\partial q_l}}$ with $\gamma_l F'' - \frac{\partial MR_l}{\partial q_l} > 0$ because $\frac{\partial^2 \pi_l}{\partial q_l^2} < 0$.

Bertrand competition

A like:

$r > \frac{\partial b_h(q_l)}{\partial q_l} = \frac{\frac{\partial MR_h}{\partial q_l}}{\gamma_h F'' - \frac{\partial MR_h}{\partial q_h}} > 0$ and $\frac{1}{r} > \frac{\partial b_l(q_h)}{\partial q_h} = \frac{\frac{\partial MR_l}{\partial q_h}}{\gamma_l F'' - \frac{\partial MR_l}{\partial q_l}} > 0.$

Appendix 5

Cournot Competition

- $\pi_l > 0$.

  Proof. $\pi_l = R_l - \gamma_l F(q_l)$

  MR_l is continous and increasing in q_l ($\frac{1}{16}$ for $q_l = 0$ to $\frac{5}{27}$ for $q_l = q_h$). F' is continous and increasing in q_l (0 for $q_l = 0$ to > 0 for $q_l = q_h$). The condition $MR_l(q_h, q_l) = \gamma_l F'(q_l)$ has a unique solution $q_l \in [0, q_h]$.

  We can deduce : $MR_l(q_h, 0) > \gamma_l F'(0)$ i.e $\lim_{q_l \to 0} \frac{\partial \pi_l}{\partial q_l} > 0$ so if $q_h > 0$ then $\pi_l > 0$ for q_l low.

  ■

- $\pi_h > 0$ with $q_h > q_l$.

  Proof. As $\pi_l > 0$, we show that $\pi_h - \pi_l > 0$.

  $R_h - R_l = \frac{q_h(q_h - q_l)}{4q_h - q_l} > 0$.

  As $\gamma F'(q)q > \gamma F(q)$ we obtain:

  $F(q_h) - F(q_l) < F'(q_h)(q_h - q_l)$.

  As $F'(q_h) = \frac{MR_h}{\gamma_h}$ et $F'(q_l) = \frac{MR_l}{\gamma_l}$, we obtain:

  $F(q_h) - F(q_l) < \frac{MR_h}{\gamma_h}(q_h - q_l)$ i.e.:

  $\gamma_h (F(q_h) - F(q_l)) < MR_h(q_h - q_l)$.

  We deduce:

  $\pi_h - \pi_l = R_h - R_l - (\gamma_h F(q_h) - \gamma_l F(q_l)) > R_h - R_l - (\gamma_h F(q_h) - \gamma_h F(q_l))$

  with $\gamma_l > \gamma_h$.

  $\pi_h - \pi_l > R_h - R_l - (\gamma_h F(q_h) - \gamma_h F(q_l)) > R_h - R_l - MR_h(q_h - q_l)$

  $\pi_h - \pi_l > (q_h - q_l)\, q_l \frac{4q_h^2 - 3q_h q_l + q_l^2}{(4q_h - q_l)^3} > 0$. ■

We obtain $\pi_h > \pi_l > 0$.

Bertrand competition

- $\pi_l > 0$

  Proof. $\pi_l = R_l - \gamma_l F(q_l)$

  MR_l is continous and decreasing in q_l ($\frac{1}{16}$ for $q_l = 0$ to $-\frac{1}{9}$ for $q_l = q_h$). F' is continous and increasing in q_l (0 for $q_l = 0$ to > 0 for $q_l = q_h$). The condition $MR_l(q_h, q_l) = \gamma_l F'(q_l)$ has a unique solution $q_l \in [0, q_h]$.

  We can deduce : $MR_l(q_h, 0) > \gamma_l F'(0)$ i.e $\lim\limits_{q_l \to 0} \frac{\partial \pi_l}{\partial q_l} > 0$ so if $q_h > 0$ then $\pi_l > 0$ for q_l low.

  ■

- $\pi_h > 0$ pour $q_h > q_l$

  Proof. As $\pi_l > 0$, we show that $\pi_h - \pi_l > 0$.

  $R_h - R_l = \frac{q_h(q_h - q_l)}{4q_h - q_l} > 0$.

  As $\gamma F'(q)q > \gamma F(q)$ we obtain:

  $F(q_h) - F(q_l) < F'(q_h)\,(q_h - q_l)$.

  As $F'(q_h) = \frac{MR_h}{\gamma_h}$ et $F'(q_l) = \frac{MR_l}{\gamma_l}$, we obtain:

  $F(q_h) - F(q_l) < \frac{MR_h}{\gamma_h}(q_h - q_l)$ i.e.:

  $\gamma_h\,(F(q_h) - F(q_l)) < MR_h(q_h - q_l)$.

  We deduce:

  $\pi_h - \pi_l = R_h - R_l - (\gamma_h F(q_h) - \gamma_l F(q_l))$

  $\pi_h - \pi_l = R_h - R_l - (\gamma_h F(q_h) - \gamma_h F(q_l)) + (\gamma_l - \gamma_h)F(q_l)$

  $\pi_h - \pi_l > \frac{q_h(q_h - q_l)}{4q_h - q_l} - \left(4q_h \frac{4q_h^2 - 3q_h q_l + 2q_l^2}{(4q_h - q_l)^3}(q_h - q_l)\right) + (\gamma_l - \gamma_h)F(q_l)$

$\pi_h - \pi_l > q_h\left(q_h - q_l\right) q_l \frac{4q_h - 7q_l}{(4q_h - q_l)^3} + (\gamma_l - \gamma_h)F(q_l)$ with $\gamma_l > \gamma_h$

$\pi_h - \pi_l > 0$ for $4q_h - 7q_l > 0$ i.e.: $\frac{q_h}{q_l} > \frac{7}{4}$.

Choi et Shin (1992) show that in a duopolistic model with products differantiated and Bertrand competition, the differentiation degree is $\frac{7}{4}$ when the cots are equal to zero. Motta (1993) show that if the costs are positive and symmetrical, the differentiation degree is 5.2512 (with $F(q_i) = \frac{q_i^2}{2}$). As, the more important the technology differential is, the more respected the condition $\pi_h - \pi_l > 0$ is, we may assume that $\frac{q_h}{q_l} > \frac{7}{4}$ is weakly restrictive. ■

We obtain $\pi_h > \pi_l > 0$.

Appendix 6

Cournot competition

By fully differentiating $MR_h(q_h, q_l) = \gamma_h F'(q_h)$ with respect to γ_h, we obtain:

$$\frac{dq_h}{d\gamma_h} = \frac{F'(q_h) - \frac{\partial MR_h}{\partial q_l}\frac{\partial q_l}{\partial \gamma_h}}{\frac{\partial MR_h}{\partial q_h} - \gamma_h F''(q_h)}.$$

To determine $\frac{\partial q_l}{\partial \gamma_h}$, we fully differentiate $MR_l(q_h, q_l) = \gamma_l F'(q_l)$ with respect to γ_h.

We obtain:

$$\frac{dq_l}{d\gamma_h} = \frac{-\frac{\partial MR_l}{\partial q_h}\frac{\partial q_h}{\partial \gamma_h}}{\frac{\partial MR_l}{\partial q_l} - \gamma_l F''(q_l)}.$$

Then:

$$\frac{dq_h}{d\gamma_h} = \frac{F'(q_h)}{\frac{\partial MR_h}{\partial q_h} - \gamma_h F''(q_h) - \frac{\frac{\partial MR_h}{\partial q_l}\frac{\partial MR_l}{\partial q_h}}{\frac{\partial MR_l}{\partial q_l} - \gamma_l F''(q_l)}} < 0$$

with $\frac{\partial MR_h}{\partial q_h} - \gamma_h F''(q_h) < 0$ and $\frac{\partial MR_h}{\partial q_l} - \gamma_l F''(q_l) < 0$ (annexe 2).

A like we obtain:

$$\frac{dq_l}{d\gamma_l} = \frac{F'(q_l)}{\frac{\partial MR_l}{\partial q_l} - \gamma_l F''(q_l) - \frac{\frac{\partial MR_l}{\partial q_h}\frac{\partial MR_h}{\partial q_l}}{\frac{\partial MR_h}{\partial q_h} - \gamma_h F''(q_h)}} < 0.$$

Bertrand competition

$$\frac{dq_h}{d\gamma_h} = \frac{F'(q_h)}{\frac{\partial MR_h}{\partial q_h} - \gamma_h F''(q_h) - \frac{\frac{\partial MR_h}{\partial q_l}\frac{\partial MR_l}{\partial q_h}}{\frac{\partial MR_l}{\partial q_l} - \gamma_l F''(q_l)}} = \frac{F'(q_h)\left(\frac{\partial MR_l}{\partial q_l} - \gamma_l F''(q_l)\right)}{\left(\frac{\partial MR_h}{\partial q_h} - \gamma_h F''(q_h)\right)\left(\frac{\partial MR_l}{\partial q_l} - \gamma_l F''(q_l)\right) - \frac{\partial MR_h}{\partial q_l}\frac{\partial MR_l}{\partial q_h}} < 0$$

because $\left(\frac{\partial MR_h}{\partial q_h} - \gamma_h F''(q_h)\right)\left(\frac{\partial MR_l}{\partial q_l} - \gamma_l F''(q_l)\right) - \frac{\partial MR_h}{\partial q_l}\frac{\partial MR_l}{\partial q_h} = \frac{\partial^2 \pi_h}{\partial q_h^2}\frac{\partial^2 \pi_l}{\partial q_l^2} - \frac{\partial^2 \pi_h}{\partial q_l \partial q_h}\frac{\partial^2 \pi_l}{\partial q_h \partial q_l} > 0.$

A like:

$$\frac{dq_l}{d\gamma_l} = \frac{F'(q_l)}{\frac{\partial MR_l}{\partial q_l} - \gamma_l F''(q_l) - \frac{\frac{\partial MR_l}{\partial q_h}\frac{\partial MR_h}{\partial q_l}}{\frac{\partial MR_h}{\partial q_h} - \gamma_h F''(q_h)}} = \frac{F'(q_l)\left(\frac{\partial MR_h}{\partial q_h} - \gamma_h F''(q_h)\right)}{\left(\frac{\partial MR_l}{\partial q_l} - \gamma_l F''(q_l)\right)\left(\frac{\partial MR_h}{\partial q_h} - \gamma_h F''(q_h)\right) - \frac{\partial MR_l}{\partial q_h}\frac{\partial MR_h}{\partial q_l}} < 0.$$

Appendix 7

Cournot competition

$$\frac{dq_h}{dq_l} = \frac{\frac{\partial MR_h}{\partial q_l}}{\gamma_h F'' - \frac{\partial MR_h}{\partial q_h}} = \left(\frac{\frac{8q_h q_l (q_h - q_l)}{(4q_h - q_l)^4}}{\gamma_h F'' - \frac{8q_l^2 (q_l - q_h)}{(4q_h - q_l)^4}}\right),$$

but as $\gamma_h F'' = \frac{\gamma_h F'(q_h)}{q_h} = \frac{MR_h}{q_h} = \frac{(2q_h - q_l)(8q_h^2 - 2q_h q_l + q_l^2)}{q_h (4q_h - q_l)^3}$,

we obtain $\frac{dq_h}{dq_l} = \frac{8(q_h - q_l) q_h^2 q_l}{64q_h^4 - 64q_h^3 q_l + 36q_h^2 q_l^2 - 16q_h q_l^3 + q_l^4}$.

- By fully differentiating sc_l with respect to q_l, we obtain:

  $sc_l = \int_{\frac{q_h}{4q_h - q_l}}^{2\frac{q_h}{4q_h - q_l}} (\theta q_l - \frac{q_h q_l}{4q_h - q_l}) d\theta = \frac{1}{2} q_l \frac{q_h^2}{(-4q_h + q_l)^2}$

  $\frac{\partial sc_l}{\partial q_l} = -\frac{1}{2} q_h^2 \frac{4q_h + q_l}{(-4q_h + q_l)^3}$

  $\frac{\partial sc_l}{\partial q_h} = q_l^2 \frac{q_h}{(-4q_h + q_l)^3}$

  $\frac{dsc_l}{dq_l} = \frac{-\frac{1}{2}\left(q_l^4 + 8q_h q_l^3 - 12q_h^2 q_l^2 + 32q_h^3 q_l - 64q_h^4\right) q_h^2}{\left(64q_h^4 - 64q_h^3 q_l + 36q_h^2 q_l^2 - 16q_h q_l^3 + q_l^4\right)(-4q_h + q_l)^2}.$

  We obtain:

  $\frac{dsc_l}{dq_l} = \frac{-\frac{1}{2} q_h^6 \left(\sigma^4 + 8\sigma^3 - 12\sigma^2 + 32\sigma - 64\right)}{\left(64q_h^4 - 64q_h^3 q_l + 36q_h^2 q_l^2 - 16q_h q_l^3 + q_l^4\right)(-4q_h + q_l)^2} > 0$ with $\sigma = \frac{q_l}{q_h}$ and $\sigma \in]0,1[$

  because $\sigma^4 + 8\sigma^3 - 12\sigma^2 + 32\sigma - 64 < 0$ with $\sigma \in]0,1[$.

- By fully differentiating sc_h with respect to q_l, we obtain:

  $sc_h = \int_{2\frac{q_h}{4q_h - q_l}}^{1} (\theta q_h - \frac{2q_h - q_l}{4q_h - q_l} q_h) d\theta = -\frac{1}{2} q_h \frac{-4q_h^2 + q_l^2}{(-4q_h + q_l)^2}$

$$\frac{\partial sc_h}{\partial q_l} = 4q_h^2 \frac{q_l - q_h}{(-4q_h + q_l)^3}$$

$$\frac{\partial sc_h}{\partial q_h} = -\frac{1}{2} \frac{16q_h^3 - 12q_h^2 q_l + 4q_l^2 q_h + q_l^3}{(-4q_h + q_l)^3}$$

$$\frac{dsc_h}{dq_l} = \frac{8\left(q_l^3 - 2q_l^2 q_h + 4q_h^2 q_l - 8q_h^3\right) q_h^2 (q_l - q_h)}{\left(64q_h^4 - 64q_h^3 q_l + 36q_h^2 q_l^2 - 16q_h q_l^3 + q_l^4\right)(-4q_h + q_l)^2} > 0.$$

Bertrand competition

$$\frac{dq_h}{dq_l} = \frac{\frac{\partial MR_h}{\partial q_l}}{\gamma_h F'' - \frac{\partial MR_h}{\partial q_h}} = \left(\frac{8q_h q_l \frac{5q_h + q_l}{(4q_h - q_l)^4}}{\gamma_h F'' + 8q_l^2 \frac{5q_h + q_l}{(4q_h - q_l)^4}} \right),$$

but as $\gamma_h F'' = \frac{\gamma_h F'(q_h)}{q_h} = 4\frac{4q_h^2 - 3q_h q_l + 2q_l^2}{(4q_h - q_l)^3}$,

we obtain: $\frac{dq_h}{dq_l} = \frac{2(5q_h + q_l) q_l}{16q_h^2 - 16q_h q_l + 21q_l^2}$.

- By fully differentiating SC with respect to q_l, we obtain:

$$\frac{\partial SC}{\partial q_l} = -\frac{1}{2} q_h^2 \frac{28q_h + 5q_l}{(-4q_h + q_l)^3}$$

$$\frac{\partial SC}{\partial q_h} = q_h \frac{6q_h q_l + 5q_l^2 - 8q_h^2}{(-4q_h + q_l)^3}$$

$$\frac{dSC}{dq_l} = \frac{\frac{1}{2} q_h \left(112q_h^3 - 24q_h^2 q_l + 99q_h q_l^2 + 20q_l^3\right)}{\left(16q_h^2 - 16q_h q_l + 21q_l^2\right)(4q_h - q_l)^2} > 0.$$

- By fully differentiating sc_l with respect to q_l, we obtain:

$$sc_l = \int_{\frac{q_h - q_l}{4q_h - q_l}}^{\frac{2q_h - q_l}{4q_h - q_l}} \left(\theta q_l - \frac{q_h - q_l}{4q_h - q_l} q_l\right) d\theta = \frac{1}{2} q_h^2 \frac{q_l}{(4q_h - q_l)^2}$$

$$\frac{\partial sc_l}{\partial q_l} = \frac{1}{2} q_h^2 \frac{4q_h + q_l}{(4q_h - q_l)^3}$$

$$\frac{\partial sc_l}{\partial q_h} = -q_h \frac{q_l^2}{(4q_h - q_l)^3}$$

$$\frac{dsc_l}{dq_l} = \frac{\frac{1}{2}\left(16q_h^3 - 8q_h^2 q_l + 15q_h q_l^2 + 4q_l^3\right) q_h}{\left(16q_h^2 - 16q_h q_l + 21q_l^2\right)(4q_h - q_l)^2} > 0.$$

- By fully differentiating sc_h with respect to q_l, we obtain:

$$sc_h = \int_{\frac{2q_h - q_l}{4q_h - q_l}}^{1} \left(\theta q_h - \frac{2q_h(q_h - q_l)}{4q_h - q_l}\right) d\theta = 2q_h^2 \frac{q_h + q_l}{(4q_h - q_l)^2}$$

$$\frac{\partial sc_h}{\partial q_l} = 2q_h^2 \frac{6q_h + q_l}{(4q_h - q_l)^3}$$

$$\frac{\partial sc_h}{\partial q_h} = 2q_h \frac{4q_h^2 - 3q_h q_l - 2q_l^2}{(4q_h - q_l)^3}$$

$$\frac{dsc_h}{dq_l} = \frac{2\left(24q_h^3 - 4q_h^2 q_l + 21q_h q_l^2 + 4q_l^3\right) q_h}{\left(16q_h^2 - 16q_h q_l + 21q_l^2\right)(4q_h - q_l)^2} > 0.$$

Appendix 8

Cournot competition

$$w_h = \pi_h + sc_h + sc_l = \int_{\theta_z}^{1} (\theta q_h) d\theta - \gamma_h F(q_h) + \int_{\theta_l}^{\theta_z} (\theta q_l - p_l) d\theta$$

$$\int_{\frac{q_h}{4q_h - q_l}}^{2\frac{q_h}{4q_h - q_l}} (\theta q_l - \frac{1}{4q_h - q_l} q_h q_l) d\theta + \int_{2\frac{q_h}{4q_h - q_l}}^{1} (\theta q_h) d\theta - \gamma_h F(q_h) = \frac{1}{2} q_h \frac{3q_h - q_l}{4q_h - q_l} - \gamma_h F(q_h).$$

$$\frac{\partial w_h}{\partial q_l} = \frac{\partial \left(\frac{1}{2} q_h \frac{3q_h - q_l}{4q_h - q_l} \right)}{\partial q_l} = -\frac{1}{2} \frac{q_h^2}{(4q_h - q_l)^2}$$

$$\frac{\partial w_h}{\partial q_h} = \frac{\partial \left(\frac{1}{2} q_h \frac{3q_h - q_l}{4q_h - q_l} - \lambda_h C(q_h) \right)}{\partial q_h} = \frac{1}{2} \frac{12q_h^2 - 6q_h q_l + q_l^2}{(4q_h - q_l)^2} - \gamma_h F'(q_h).$$

Substituting $\gamma_h F'(q_h)$ by MR_h, we obtain:

$$\frac{\partial w_h}{\partial q_h} = \frac{1}{2} \frac{16q_h^3 - 12q_h^2 q_l + 2q_h q_l^2 + q_l^3}{(4q_h - q_l)^3}.$$

As $\frac{dq_h}{dq_l} = \frac{8(q_h - q_l) q_h^2 q_l}{64q_h^4 - 64q_h^3 q_l + 36q_h^2 q_l^2 - 16q_h q_l^3 + q_l^4}$.

By fully differentiating w_h with respect to q_l, we obtain:

$$\frac{dw_h}{dq_l} = -\frac{1}{2} q_h^2 \frac{256q_h^5 - 448q_h^4 q_l + 432q_h^3 q_l^2 - 212q_h^2 q_l^3 + 28q_h q_l^4 + 7q_l^5}{(4q_h - q_l)^3 \left(64q_h^4 - 64q_h^3 q_l + 36q_h^2 q_l^2 - 16q_h q_l^3 + q_l^4 \right)}.$$

Substituting $\frac{q_l}{q_h}$ by σ, we can write:

$$\frac{dw_h}{dq_l} = -\frac{1}{2} q_h^7 \frac{256 - 448\sigma + 432\sigma^2 - 212\sigma^3 + 28\sigma^4 + 7\sigma^5}{(4q_h - q_l)^3 \left(64q_h^4 - 64q_h^3 q_l + 36q_h^2 q_l^2 - 16q_h q_l^3 + q_l^4 \right)} < 0 \text{ with } \sigma = \frac{q_l}{q_h} \text{ and } \sigma \in]0, 1[.$$

Bertrand competition

$$w_h = \pi_h + sc_h + sc_l = \int_{\theta_z}^{1} (\theta q_h) d\theta - \gamma_h F(q_h) + \int_{\theta_l}^{\theta_z} (\theta q_l - p_l) d\theta$$

$$w_h = \int_{\frac{2q_h - q_l}{4q_h - q_l}}^{1} (\theta q_h) d\theta - \gamma_h F(q_h) + \int_{\frac{q_h - q_l}{4q_h - q_l}}^{\frac{2q_h - q_l}{4q_h - q_l}} (\theta q_l - \frac{q_h - q_l}{4q_h - q_l} q_l) d\theta = \frac{3}{2} \frac{q_h^2}{4q_h - q_l} - \gamma_h F(q_h).$$

$$\frac{\partial w_h}{\partial q_l} = \frac{3}{2} \frac{q_h^2}{(4q_h - q_l)^2}$$

$$\frac{\partial w_h}{\partial q_h} = 3q_h \frac{2q_h - q_l}{(4q_h - q_l)^2} - \gamma_h F'(q_h).$$

Substituting $\gamma_h F'(q_h)$ by MR_h, we obtain:

$$\frac{\partial w_h}{\partial q_h} = q_h \frac{8q_h^2 - 6q_h q_l - 5q_l^2}{(4q_h - q_l)^3}.$$

As $\frac{dq_h}{dq_l} = \frac{2(5q_h+q_l)q_l}{16q_h^2-16q_hq_l+21q_l^2}$.

By fully differentiating w_h with respect to q_l, we obtain:

$\frac{dw_h}{dq_l} = q_h \frac{-192q_h^4+80q_h^3q_l-212q_h^2q_l^2+187q_hq_l^3+20q_l^4}{(-4q_h+q_l)^3\left(16q_h^2-16q_hq_l+21q_l^2\right)} > 0.$

References

BALDWIN R.E. (1970), *Nontariff Distortions of International Trade*, The Brookings Institution, Washington.

BARRETT S. (1994), “Strategic environmental policy and international trade”, *Journal of Publics Economics*, 54, p. 325-338.

BOOM, A. (1995), “Asymetric International Minimum Quality Standards and Vertical Differentition”, *The Journal of Industrial Economics*, 43, p.101-119.

CHOI J.C. and SHIN H.S. (1992), “A Comment on a Model of Vertical Poduct Differnciation”, *The journal of Industrial Economics*, 40, p. 229-231.

COSTANTATOS C and PERRAKIS S. (1998), “Minimum Quality Standards, Entry, and the Timing of Quality Decision”, *Journal of Regulatory Economics*, 13, p. 47-58.

CRAMPES C. and HOLLANDER A. (1995), “Duopoly and Quality Standards”, *European Economic Review*, 39, p. 71-82.

DAS S.P. and DONNENFELD S. (1989), “Oligopolistic Competition and International Trade”, *Journal of International Economics*, 27, p. 299-318.

ECCHIA G. et LAMBERTINI L. (1997), “Minimum Quality Standards and Collusion”, *The journal of Industrial Economics*, 45, p. 101-113.

FISCHER R. and SERRA P. (2000), “Standards and protection”, *Journal of International Economics*, 52, p. 377-400.

LAMBERTINI L. and SCARPA C. (1999), “Minimum Quality Standards and Predatory Behaviour”, Nota di Lavoro 86.99, Fondazione ENI Enrico Mattei, Milan.

MATTOO A. (2001), "Discriminatory Consequences of Non-discriminatory Standards", *Journal of Economic Integration*, 16 p. 78-105.

MOTTA M. (1993), "Endogenous Quality Choice Price vs Quantity Competition", *The journal of Industrial Economics*, 41, p. 113-131.

RONNEN U. (1991), "Minimum Quality Standard, fixed Costs, and Competition" *Rand Journal of Economics,* 22, p.490-504.

SALANT D.J. (1986), "Equilibrium in a spacial model of imperfect competition with sequential choice of locations and quantities", *Canadian Journal of Economics*, 19, p. 685-715.

SHAKED A. and SUTTON J. (1982),"Relaxing Price Competition through Product Differenciation", *Review of Economic Studies*, 49, p. 3-14.

TANAKA Y. (2001), "Profitability of Price and Quantity strategies in an Oligopoly", *Journal of Mathematical Economics*, 35, p. 409-418.

ZHOU D, SPENCER B.J. and VERTINSKY I. (2002), "Strategic trade policy whith endogenous choice of quality and asymmetric costs", *Journal of International Economics*, 56, p. 205-232.