International Competition between Public or Mixed Enterprises

Jean-Thomas Bernard, Marc Dupéré and Michel Roland*

Département d'économique

Université Laval

Québec, QC

Canada, G1K 7P4

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Abstract

We develop a model in which two firms from different countries compete on each other domestic market. Each firm is jointly owned by the residents and the government of its country. The extent of the government's stake in the public enterprise is endogenous and it determines the weight given to domestic consumers' surplus in the firm's payoff function. We show that the choice of each government's stake depends on a trade-off between allocative efficiency on the domestic market and profitability of foreign markets. We also highlight the fact that the government's stake in one country has an impact on firms' behavior in both countries.

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*Corresponding author: Phone: +1 (418) 656-2337, Fax: +1 (418) 656-7412, email: Michel.Roland@ecn.ulaval.ca

1 Introduction

In recent years, several countries have implemented regulatory reforms in their public utility sectors such as telecommunications, electricity and postal services. A common feature of these reforms is the move away from franchised monopolies toward more open markets by removing some or all existing barriers to entry.[1]

In many cases, most notably in countries of the European Union, the franchised monopoly which was providing the service before the introduction of regulatory reforms was a state-owned enterprise. Governments have often, but not always, combined their regulatory reforms with partial or total privatization of the incumbent public firm. Since the most likely entrants in one market are often incumbents operating in neighbouring markets, competition in newly liberalized markets is likely to involve firms which display different ownership patterns. For instance, state-owned *Électricité de France* competes with *Électrabel*, a Belgian private firm. In the same way, 55% of *France Telecom* shares are owned by the French government; this firm could eventually compete with *Telecom Italia* (3% of shares owned by the Italian government).[2] In North America, publicly-owned *Hydro-Québec (HQ)* competes in the US wholesale electricity markets against producers that are mainly privately-owned. Even though the Quebec provincial government has shown no intent to privatize *HQ* in the aftermath of the opening of the US electricity market, the neighbouring province of Ontario has taken a different stand by adopting a plan to privatize parts of the generation assets of its publicly-owned utility, *Ontario Hydro*.

The choices made by governments with respect to the ownership structure of their public utilities and the resulting competition among firms with mixed (private and state) ownership,

[1] In electricity, franchised monopoly is generally maintained for transmission and distribution, which are still considered to be a natural monopoly. Franchising is also likely to be maintained for specific postal services, such as rural distribution, that are deemed to be essential and to be natural monopolies under current technologies.

[2] Data on shareholding structure are for 2001.

raise two questions. First, what are the relevant trade-offs made by governments when they choose their stake in a public utility which faces competition at home and abroad? Second, what is the impact of state ownership on market operations?

The second question is puzzling since, in the face of the recognized inefficiencies of the former regulatory framework, we would expect that governments would aim at improving market efficiency. However, little is known on the efficiency properties of an industry structure which involves competition between mixed enterprises. On the one hand, economic models in industrial organization are generally based on the assumption that firms maximize profit. Such an assumption seems ill suited to describe the behavior of firms which are partly or totally controlled by governments. On the other hand, models that focus explicitly on state-owned enterprises generally assume that the market is monopolistic. At this stage, the expected efficiency improvement from the substitution of regulated monopolies by competition as currently implemented rests more on hunches than on appropriate theoretical results.

This paper analyzes the decisions with respect to output by two firms that compete on two markets under different jurisdictions (country, state, etc.); for ease of presentation, the latter are taken to be countries. Firms are partly or wholly owned by the governments of their respective domestic markets. The government's stake in a firm determines the weight given to domestic consumer's surplus in the firm's objective function. Each firm maximizes profit flowing from foreign market operations. Allocation of output between the two firms is then the result of a two-stage game. In the first stage, each government chooses its stake in the domestic firm in order to maximize domestic welfare. In the second stage, each firm determines output on domestic and foreign markets in order to maximize the weighted sum of domestic consumers' surplus and firm's profit, where the weights are determined by the first stage.

Because the occurence of multi-market competition among public enterprises is rather new, to our knowledge, little attention has been paid to this topic thus far. We nevertheless draw on two strands of literature. The first is the literature on "mixed oligopolies", where a public firm competes with one or several private firms on the same market. De Fraja and Delbono [7] provides a survey of this literature. Our model is related to this literature in the sense that, within a single market (domestic or foreign), a publicly-owned firm competes with a profit-maximizing one. We add to this setting the interaction between domestic and foreign markets. In this respect, our model extends the work of Matsumara's [12]. It is also akin to the idea of White [15] that public ownership allows the government to assign "an objective function to the public firm administrator, strategically designed to maximize the governing body's true objective function" (p. 488). Although White uses this fact to let the government pursue a hidden agenda instead of welfare maximization, here the discrepency between the firm's objective function and both welfare or profit maximization will be strategically chosen by government in order to reduce market inefficiencies of the oligopoly structure.

A second strand of literature comes from the much larger field of international economics. Brander and Spencer [4], [5], Dixit [9], Brander and Krugman [3], Krugman [11] and Eaton and Grossman [10] have studied the impact of government policies, such as export subsidies or import tariffs, on exchanges between countries in oligopoly structures. In all those papers, government policies are used to increase the domestic firms profit at equilibrium. In Eaton and Grossman[10], they are also used to reduce the difference between price and marginal cost at equilibrium. In our paper, public ownership will also be used strategically with the aim of increasing domestic welfare, and this will involve a trade-off between domestic firms' profitability on foreign markets and the price-marginal cost discrepency. In a context where subsidies and tariffs can be contested as unfair practices (e.g. at the WTO), public ownership can then be seen as a substitute for subsidies and tariffs to increase domestic welfare.

In the next section, we develop the model. Sections 3 and 4 present the Nash equilibria of the second and first stages, respectively. We conclude by discussing possible extensions.

2 Model

An homogeneous good is sold on two separate markets located in two countries, labelled 1 and 2. The good is not storable, and thus arbitrage opportunities from one market to the other are precluded. Market i's inverse demand function for this good is $p_i(X_i)$, where X_i is total consumption in country i and where $p_i(\cdot)$ is a strictly decreasing and twice differentiable function.

Each country i is served by the same two firms, a domestic firm, which is jointly owned by the private sector and country i's government, and a foreign firm, which is also jointly owned by the private sector and the foreign government $j \neq i$. We denote by $\alpha_i \in [0,1]$ the portion of shares of firm i ($i \in \{1,2\}$) owned by government i.[3]

We assume that output delivered to the foreign market is a perfect substitute to output delivered to the domestic market. For instance, this would be the case when firms use the same equipment to supply both markets. Then, letting x_{ij} be the output of firm i sold on market j and C_i be its (twice differentiable) cost function, we can write the firm's profit function $\pi_i(\cdot)$ as follows:

$$\pi_i(x_{ii}, x_{ij}; x_{ji}, x_{jj}) = p_i\left(x_{ii} + x_{ji}\right) x_{ii} + p_j\left(x_{ij} + x_{jj}\right) x_{ij} - C_i\left(x_{ii} + x_{ij}\right), \quad j \neq i, \quad i, j \in \{1,2\} \tag{1}$$

Assuming that the private shareholders of firm i are residents of country i, welfare of country i is given by:

$$W_i(x_{ii}, x_{ij}, x_{ji}, x_{jj}) = CS_i(x_{ii}, x_{ji}) + \pi_i(x_{ii}, x_{ij}; x_{ji}, x_{jj}) \tag{2}$$

where $CS_i(x_{ii}, x_{ji}) \equiv \left[\int_0^{x_{ii}+x_{ji}} p_i(x)dx - p_i(x_{ii} + x_{ji})(x_{ii} + x_{ji})\right]$ is country i consumers' surplus.

Private shareholders seek to maximize profit while government aims at aggregate consumers' and producer's surplus maximization.

[3] We exclude the case where a government would also own part of the foreign firm.

The government can exercise control over its domestic firm through its shareholding of the firm. The aim of this control is to make the domestic firm's managers to take also into account welfare in their objective function. Government thus chooses a share of ownership α_i that allows it to impose its preferred welfare weight γ_i in the firm's objective function. The latter is thus:

$$\begin{aligned} U_i(x_{ii}, x_{ij}; x_{ji}, x_{jj}) &= \gamma_i W_i(x_{ii}, x_{ij}, x_{ji}, x_{jj}) + (1-\gamma_i)\pi_i(x_{ii}, x_{ij}; x_{ji}, x_{jj}) \\ &= \gamma_i CS_i(x_{ii}, x_{ji}) + \pi_i(x_{ii}, x_{ij}, x_{ji}, x_{jj}) \end{aligned} \tag{3}$$

We do not explicitly model the relationship between α_i and γ_i as this can vary with the institutional context of each country. As a result, we consider that government i chooses directly γ_i and buys shares accordingly. We assume that $\gamma_i(\alpha_i)$ is non-decreasing in α_i.[4] This encompasses almost all situations. For instance, Matsumara [12] and Bös [1] use a continuous, non-decreasing function with $\gamma_i(0) = 0$ and $\gamma_i(1) = 1$. However we can also consider a case where the majority owner obtains total control over firms' decisions. In such a case, this means that the firm's objective function is standard profit maximization as long as government remains a minority shareholder. On the other hand, government could assign any weight to welfare whenever it is a majority holder, i.e. it can modulate its effective control as it sees fit whenever it gets 50% of the shares. In such a case, we have $\gamma_i(\alpha_i) = 0, \forall\gamma_i = [0, 0.5)$ and $\gamma_i(\alpha_i) \in [0, 1], \forall\alpha_i = [0.5, 1]$. Hereafter, the decision variable is taken to be γ_i in order to avoid this indeterminancy between shareholding and effective control.[5]As γ_i represents the weight given by the domestic firm's managers to government's objective, we will call it the government's stake in the firm (as opposed to its shareholding).

[4]Note that $\gamma_i(\alpha_i)$ is not necessarily a function but can be a relation as the exemple on majority shareholding will show below.

[5]We assume that there is no direct cost associated with public ownership, so a transfer of shares from domestic private shareholders to the public sector is a transfer of money within the country and does not affect welfare. Including a shadow cost of public funds would not affect results qualitatively.

In a framework where governments and firms have perfect knowledge of both demand functions as well as both cost functions, we consider the following two-stage game. In the first stage, governments determine independently their stakes in order to maximize (2). Given these stakes, firms determine their output for both markets in order to maximize (3). We search for Nash equilibria at both stages. As usual, we begin the analysis with the second stage.

3 Firms' Choices of Output

3.1 First Order Conditions

We search for a Cournot-Nash equilibrium where firm i maximizes its payoff U_i given output (x_{ji}, x_{jj}) of firm j:

$$\max_{x_{ii},x_{ij}} U_i(x_{ii}, x_{ij}; x_{ji}, x_{jj}) \qquad i = 1, 2 \tag{4}$$

The first order conditions for firm i are:

$$\begin{aligned}\frac{\partial U_i}{\partial x_{ii}} &= -\gamma_i p_i'(x_{ii} + x_{ji}) \cdot (x_{ii} + x_{ji}) + p_i(x_{ii} + x_{ji}) + p_i'(x_{ii} + x_{ji}) \cdot x_{ii} - C_i'(x_{ii} + x_{ij}) \leq 0 \\ &\frac{\partial U_i}{\partial x_{ii}} x_{ii} = 0 \qquad x_{ii} \geq 0\end{aligned} \tag{5}$$

$$\begin{aligned}\frac{\partial U_i}{\partial x_{ij}} &= p_j(x_{ij} + x_{jj}) + p_j'(x_{ij} + x_{jj}) \cdot x_{ij} - C_i'(x_{ii} + x_{ij}) \leq 0 \\ &\frac{\partial U_i}{\partial x_{ij}} x_{ij} = 0 \qquad x_{ij} \geq 0\end{aligned} \tag{6}$$

The term $-\gamma_i p_i' \cdot (x_{ii} + x_{ji}) + p_i + p_i' \cdot x_{ii}$ in condition (5) represents the marginal benefit of domestic production to firm i. Since the government is a stakeholder, this benefit is not restricted to the marginal revenue $(p_i + p_i' \cdot x_{ii})$: it involves also the gain in consumer surplus from increased production $(-p_i' \cdot (x_{ii} + x_{ji}))$. This gain is weighted by the government's stake in the firm. Condition (5) thus compares the firm's marginal benefit of domestic sales to their marginal cost. As the firm maximizes profit on the foreign market, condition (6) compares marginal revenue of foreign sales to their marginal cost.

When solving (5) and (6), several cases arise as each of the four variables can take a positive or zero value at equilibrium. However, as mentionned in the introduction, we are particularly interested in multi-market competition of mixed enterprises. This allows us to restrict the number of cases, as shown in the following lemmas.

Lemma 1 *If $\gamma_i > 0$ for $i = 1$ or $i = 2$, then at least one of the market is supplied by the domestic firm at equilibrium, i.e either $x_{11} > 0$ or $x_{22} > 0$ (or both).*

Proof. Without loss of generality, suppose that $0 < \gamma_1 \leq 1$ (while $0 \leq \gamma_2 \leq 1$) and that the equilibrium is such that $x_{11} = x_{22} = 0$. Then $x_{21} = X_1$ and $x_{12} = X_2$ and the first order conditions become:

$$
\begin{aligned}
(1-\gamma_1)p_1'X_1 + p_1 - C_1' &\leq 0 \quad (7)\\
p_2 + p_2'X_2 - C_1' &= 0 \quad (8)\\
p_1 + p_1'X_1 - C_2' &= 0 \quad (9)\\
(1-\gamma_2)p_2'X_2 + p_2 - C_2' &\leq 0 \quad (10)
\end{aligned}
$$

From (8), (7) and the fact that $\gamma_1 > 0$, we get that $p_2 + p_2'X_2 \geq (1-\gamma_1)p_1'X_1 + p_1 > p_1 + p_1'X_1$. From the fact that $\gamma_2 \geq 0$, and from (10) and (9), we get $p_2 + p_2'X_2 \leq (1-\gamma_2)p_2'X_2 + p_2 \leq p_1 + p_1'X_1$. We thus have a contradiction. ■

Lemma 1 can be explained as follows. The government's stake in firm 1 adds a positive contribution to domestic output which is above domestic market marginal revenue. If firm 1 nevertheless decides to sell only on market 2, this is because the market 2 marginal revenue is greater than marginal benefit in market 1. As the firm 2 marginal benefit in market 2 is at least as great as market 2 marginal revenue, firm 2 will be a situation where marginal benefit in market 2 is greater than marginal revenue in market 1 and thus, should produce a positive amount for market 2.

Note that even if the two firms were profit maximizers ($\gamma_1 = \gamma_2 = 0$), the case where each firm would sell only on the foreign market ($x_{11} = x_{22} = 0$) would be rather unusual, as it would require that $C_1' = C_2'$ and $p_1 + p_1'X_1 = p_2 + p_2'X_2$ simultaneously. Throughout the analysis below, we will thus assume that either x_{11} or x_{22} is positive at equilibrium.

Lemma 2 *If $\gamma_i < 1$ for $i = 1$ or $i = 2$, then there exists international trade at equilibrium, i.e. either $x_{12} > 0$ or $x_{21} > 0$ (or both).*

Proof. Without loss of generality, suppose that $0 \leq \gamma_1 < 1$ (while $0 \leq \gamma_2 \leq 1$) and that the equilibrium is such that $x_{12} = x_{21} = 0$. Then $x_{11} = X_1$ and $x_{22} = X_2$ and first order conditions become:

$$\begin{aligned}(1-\gamma_1)p_1'X_1 + p_1 - C_1' &= 0 && (11)\\ p_2 - C_1' &\leq 0 && (12)\\ p_1 - C_2' &\leq 0 && (13)\\ (1-\gamma_2)p_2'X_2 + p_2 - C_2' &= 0 && (14)\end{aligned}$$

From (12) and (11) and the fact that $\gamma_1 < 1$, we have that $p_2 \leq (1-\gamma_1)p_1'X_1 + p_1 < p_1$. From the fact that $\gamma_2 \leq 1$ and from (14) and (13), we have $p_2 \geq (1-\gamma_2)p_2'X_2 + p_2 \geq p_1$. We thus have a contradiction. ■

Lemma 2 is explained in a similar manner as Lemma 1. If firm 1 decides to sell only on market 1, its marginal benefit on market 1 is greater or equal to its marginal revenue on market 2, which, at $x_{12} = 0$, is the price on market 2: $(1-\gamma_1)p_1'X_1 + p_1 \geq p_2$. As the price on one market is necessarily greater than the domestic firm's marginal benefit on the same market, we then have $p_1 > (1-\gamma_1)p_1'X_1 + p_1 \geq p_2 > (1-\gamma_2)p_2'X_2 + p_2$. This implies that firm 2's marginal benefit on market 1, which, at $x_{12} = 0$, is the price on market 1, is greater than its marginal benefit on market 2. This should bring firm 2 to sell on market 1.

Note that even if the two firms were welfare maximizers ($\gamma_1 = \gamma_2 = 1$), the case where there would be no international trade ($x_{12} = x_{21} = 0$) would be unusual as it would require that $p_1 = p_2$ and $C_1' = C_2'$. Throughout the analysis below, we will assume that either x_{12} or x_{21} is positive at equilibrium.

Let ε_i be the price elasticity of demand on market i and s_{ij} be the market share of firm i in country j; this allows to rewrite (5) in the following way:

$$\begin{aligned}\frac{p_i(x_{ii}+x_{ji}) - C_i'(x_{ii}+x_{ij})}{p_i(x_{ii}+x_{ji})} &\leq -\frac{p_i'(x_{ii}+x_{ji}) \cdot [(1-\gamma_i)x_{ii} - \gamma_i x_{ji}]}{p_i(x_{ii}+x_{ji})} \\ &= -\frac{s_{ii}-\gamma_i}{\varepsilon_i(p_i)} \qquad i \neq j \end{aligned} \tag{15}$$

where both sides become equal whenever $x_{ii} > 0$. The second equality was obtained by multiplying both the numerator and the denominator of the first line RHS by $(x_{ii} + x_{ji})$. In a similar manner, condition (6) can be rewritten as:

$$\frac{p_j(x_{ij}+x_{jj}) - C_i'(x_{ii}+x_{ij})}{p_j(x_{ij}+x_{jj})} \leq -\frac{s_{ij}}{\varepsilon_j(p_j)} \qquad i \neq j \tag{16}$$

where both sides are equal whenever $x_{ij} > 0$.

Condition (16) is the usual condition on the Lerner index in a Cournot equilibrium. This reflects profit maximizing behavior on the foreign market. The impact of public control is seen in equation (15): for given price elasticity and market share,[6] it decreases the Lerner index to the extent of the government's stake. Intuitively, the weight attached by governement to consumer surplus increases the marginal benefit of domestic consumption and thus, leads the domestic firm to increase output delivered to the domestic market, relative to the profit maximizing output.

Whenever we have an interior solution ($x_{ij} > 0, i = 1,2; j = 1,2$), conditions (5) and (6) yield together:

$$\frac{p_i(x_{ii}+x_{ji}) - \left(\frac{C_i'(x_{ii}+x_{ij}) + C_j'(x_{ji}+x_{jj})}{2}\right)}{p_i(x_{ii}+x_{ji})} = -\left(\frac{1-\gamma_i}{2}\right)\frac{1}{\varepsilon_i(p_i)}, \qquad i = 1,2; j \neq i \tag{17}$$

[6] Obviously, the price elasticity and the market share are endogenous. In section 4, we perform the comparative statics following a change in public ownership.

Since the price mark-up is computed from the same marginal cost benchmark in both markets, (17) can be written as

$$p_1\left[1-\left(\frac{1-\gamma_1}{2}\right)\frac{1}{|\varepsilon_1|}\right]=p_2\left[1-\left(\frac{1-\gamma_2}{2}\right)\frac{1}{|\varepsilon_2|}\right]=\frac{C_1'(x_{11}+x_{12})+C_2'(x_{21}+x_{22})}{2} \tag{18}$$

Thus, if governments of both countries have exactly the same stake in their home enterprise, we obtain the well-known result that the price will the lowest in the market which has the most elastic demand. However, public ownership qualifies this result : for given elasticities, price will be lower in the country where government has a greater stake in the national enterprise.

3.2 Second Order Conditions

In order to insure global uniqueness of the solution , we make the following assumption (see Nikaido [13], chap. VII).

Assumption 1 *The matrix*

$$H\equiv\begin{pmatrix}\partial^2U_1/\partial x_{11}^2 & -C_1'' & \partial^2U_1/\partial x_{11}\partial x_{21} & 0\\ -C_1'' & \partial^2U_1/\partial x_{12}^2 & 0 & \partial^2U_1/\partial x_{12}\partial x_{22}\\ \partial^2U_2/\partial x_{21}\partial x_{11} & 0 & \partial^2U_2/\partial x_{21}^2 & -C_2''\\ 0 & \partial^2U_2/\partial x_{22}\partial x_{12} & -C_2'' & \partial^2U_2/\partial x_{22}^2\end{pmatrix} \tag{19}$$

is an N-P matrix, i.e. all the principal minors of odd orders are negative and those of even orders are positive.

Elements of H are the partial derivatives with respect to x_{11} (1st column), x_{12} (2nd column), x_{21} (3rd column) and x_{22} (4th column) of $\partial U_1/\partial x_{11}$ (1st row), $\partial U_1/\partial x_{12}$ (2nd row), $\partial U_2/\partial x_{21}$ (3rd row) and $\partial U_2/\partial x_{22}$ (4th row), respectively. As these first order derivatives must vanish for interior solutions, this matrix of second-order derivatives lies behind the comparative statics analysis.

Among other things, assumption 1 implies that:[7]

$$\frac{\partial^2 U_i}{\partial x_{ii}^2} = -\gamma_i p_i'' \cdot (x_{ii} + x_{ji}) - \gamma_i p_i' + p_i'' x_{ii} + 2p_i' - C_i'' < 0 \qquad i \neq j,\ i \in \{1,2\} \tag{20}$$

$$\frac{\partial^2 U_i}{\partial x_{ij}^2} = p_j'' x_{ij} + 2p_j' - C_i'' < 0 \qquad i \neq j,\ i \in \{1,2\} \tag{21}$$

and

$$\frac{\partial^2 U_i}{\partial x_{ii}^2}\frac{\partial^2 U_i}{\partial x_{ij}^2} - C_i'' > 0 \qquad i \neq j,\ i \in \{1,2\} \tag{22}$$

which are the second-order conditions for firm i's maximization problem.

Moreover, in order to have well-behaved best-response functions, we assume the following.

Assumption 2 *For any* $(x_{ii}, x_{ij}, x_{ji}, x_{jj})$,

(i) $p_i' < C_i''$

(ii) $p_j' < C_i''$

(iii) $p_j' + p_j'' x_{ij} < 0$

Assumptions 2 (i) and 2 (ii) are always satisfied if inverse demand functions are decreasing and cost functions are convex. Assumption 2 (iii) means that firm i's marginal revenue on the foreign market falls as its rival's output increases.

3.3 Comparative Statics

In this section, we analyze what happens to outputs when government of country i changes γ_i, i.e. the stake that it has in its own enterprise. We assume an interior equilibrium at initial stakes.[8]

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[7] Arguments of functions are omitted for ease of presentation. We will draw upon other implications of Assumption 1 when we perform comparative statics.

[8] If $x_{i1} = x_{i2} = 0$, we would have that firm $j \neq i$ acts as a monopoly on both markets, which is the classic textbook case on first degree price discrimination. If $x_{ii} = 0$, firm $j \neq i$ is a monopoly on market i. If

Proposition 1 *Let us assume that, for given* γ_1 *and* γ_2, $(x_{11}, x_{12}, x_{21}, x_{22}) > 0$ *at equilibrium. Suppose there is a change in government 1 stake, so that* $d\gamma_1 \neq 0$ *while* $d\gamma_2 = 0$. *Then*

$$\left.\frac{dx_{11}}{d\gamma_1}\right|_{\gamma_2} > 0; \qquad \left.\frac{dx_{12}}{d\gamma_1}\right|_{\gamma_2} < 0; \qquad \left.\frac{dx_{21}}{d\gamma_1}\right|_{\gamma_2} < 0; \qquad \left.\frac{dx_{22}}{d\gamma_1}\right|_{\gamma_2} > 0;$$

$$\left.\frac{dX_1}{d\gamma_1}\right|_{\gamma_2} > 0; \qquad \left.\frac{dX_2}{d\gamma_1}\right|_{\gamma_2} > 0$$

Proof. See Appendix A ■

Here is the intuition behind Proposition 1.[9] An increase in government 1's stake in firm 1 means that the marginal benefit of domestic production has increased for this firm. This leads firm 1 to increase its domestic sales. As a result, its marginal cost of production is initially higher. Since the marginal benefit function on market 2 has not changed, firm 1 is forced to lower foreign sales. Thus firm 2 marginal benefit of its sales on its own market increases and this leads firm 2 to increase its sales on market 2. As this increases its marginal cost while its marginal benefit function on market 1 remains the same, firm 2 then reduces production on market 1. We then enter a second round where firm 1 sees its marginal benefit increase as firm 2 retreats from its market. The assumption made on matrix H insures that this process converges.

The upshot is that an increase in government's stake in one market increases production in *both* markets. Nevertheless, the impact of public ownership is to "isolate" markets in the sense that the share of the foreign firm in a given market is reduced: sales of the foreign firm are lowered while those of the domestic firm are increased in an overall bigger market.

$x_{ij} = 0$ for $j \neq i$, then firm j is a monopoly on market j. The last two cases of a monopoly in one market and a duopoly in another has been thouroughly analyzed by Bulow et al. [6]. It turns out that results of Proposition 1 carry over to these cases in the sense that the signs of derivatives remain the same for non-zero variables.

[9]For ease of interpretation, we consider in this paragraph that cost functions are convex, although this is not necessary. Proposition 1 holds under more general cost functions provided that Assumption 1 still holds.

4 Choice of Governments' Stakes

4.1 First Order Conditions

We now turn to the choice of governments' stakes in their respective domestic firm. Each government wants to maximise social welfare function W_i $(i = 1, 2)$ given the stake of the other government in its national firm. Labeling $x_{ij}(\gamma_i, \gamma_k)$, $i, j, k = 1, 2$, $k \neq i$, the stage 2 output equilibrium for firm i on market j, the problem of government i is thus:

$$\begin{aligned} \max_{\gamma_i} \quad & W_i(x_{ii}(\gamma_i; \gamma_j), x_{ij}(\gamma_i; \gamma_j), x_{ji}(\gamma_i, \gamma_j), x_{jj}(\gamma_i, \gamma_j)) \\ \text{s.t. } 0 \quad \leq & \quad \gamma_i \leq 1 \end{aligned}$$

From definition (2), the total derivative of W_i with respect to γ_i is:

$$\left.\frac{dW_i}{d\gamma_i}\right|_{\gamma_j} = (p_i - p_i' x_{ji} - C_i')\frac{dx_{ii}}{d\gamma_i} - p_i' x_{ji}\frac{dx_{ji}}{d\gamma_i} + p_j' x_{ij}\frac{dx_{jj}}{d\gamma_i} \tag{23}$$

Let us assume that this total derivative is equal to zero for $0 < \gamma_i^* < 1$ and that $x_{ii}(\gamma_i^*; \gamma_j) > 0$. Then, from first order condition (5) with respect to the optimal choice of output by the firm, we have:

$$p_i - p_i' x_{ji} - C_i' = -(1 - \gamma_i^*) p_i' (x_{ii} + x_{ji}) \tag{24}$$

Substituting (24) into (23) yields:

$$\begin{aligned} \gamma_i^*(\gamma_j) &= 1 + \frac{p_i' x_{ji} \frac{dx_{ji}}{d\gamma_i} - p_j' x_{ij} \frac{dx_{jj}}{d\gamma_i}}{p_i' X_i \frac{dx_{ii}}{d\gamma_i}} \\ &= 1 + \frac{p_i s_{ji} \varepsilon_i^{-1} \frac{dx_{ji}}{d\gamma_i} - p_j s_{ij} \varepsilon_j^{-1} \frac{dx_{jj}}{d\gamma_i}}{p_i \varepsilon_i^{-1} \frac{dx_{ii}}{d\gamma_i}} \\ &= 1 + s_{ji}\frac{dx_{ji}}{dx_{ii}} - s_{ij}\frac{p_j \varepsilon_i}{p_i \varepsilon_j}\frac{dx_{jj}}{dx_{ii}} \end{aligned} \tag{25}$$

The second term of the last line shows that the optimal stake is the lower the higher is the foreign enterprise share of the domestic market (s_{ji}) and the higher is the crowding-out effect (dx_{ji}/dx_{ii}), i.e. the more the foreign firm reduces sales as the domestic firm increases prouduction for its own market. This comes from the fact that if it is profitable for the foreign firm to enter the domestic market (so that $s_{ji} > 0$), it is welfare-enhancing to accommodate entry since this brings a reduction in marginal cost. Such accomodation is done through a decrease of public ownership, which leads the domestic firm to decrease domestic production. The more responsive is the foreign firm to a decrease of domestic output (i.e the greater is $|dx_{ji}/dx_{ii}|$), the lesser is the optimal government stake in the domestic firm. In other words, the greater is the crowding-out of foreign production made by domestic output (again, the greater is $|dx_{ji}/dx_{ii}|$), the lesser is the optimal government stake in the domestic firm.

The third term of the RHS of (25) shows the relationship between the domestic and foreign markets. This term weighs the importance of the foreign market in the profitability of the domestic firm : the greater are the opportunities on the foreign market, the more interesting it becomes to have the firm to act as a profit maximizer. As a result, the government's stake is the lower *(i)* the greater is the share of the domestic firm in the foreign market consumption; *(ii)* the greater is the relative price of foreign sales to domestic sales; *(iii)* the more elastic is the domestic demand, since market power of the firm is then lower, so that private ownership is less costly in terms of welfare; *(iv)* the less elastic is the foreign demand, because of the greater price-marginal cost mark-up that it can result; and *(v)* the greater is the increase of the foreign firm production following an increase of public ownership in the foreign market, since such an increase lowers the marginal revenue of the domestic firm on the foreign market.

Condition (25) is also useful to analyze under which conditions corner solutions could arise. In order for firm i to be entirely controlled by government, we should have that $x_{12} = x_{21} = 0$. In turn, this would mean that government j would also choose to have a 100% stake. We have observed from Lemma 2 that this could occur only if $p_1 = p_2$ simultaneously with $C_1' = C_2'$, as there is no possible gain from trade possible and each government then chooses to price at the (same) marginal cost. As this situation would rather be exceptional, the model predicts that opening of markets will generally be followed by the privatization of (some) shares of the state-owned enterprises.

It is however possible that one or both firms will be profit maximizer at optimum, i.e. that $\gamma_1 = 0$ or $\gamma_2 = 0$ (or both). A plausible case is one where both markets are of very different sizes : the small country could then see that the profit opportunity of the foreign market is overwhelming compared to social welfare increases that are possible on the domestic market.

4.2 Existence

Existence of an equilibrium stems simply from the fact that reaction functions in the first stage are continuous. To see this, we totally differentiate the welfare function W_i

$$dW_i = \sum_{i=1}^{2} \left[(p_i - p_i' x_{ji} - C_i') \left.\frac{dx_{ii}}{d\gamma_i}\right|_{\gamma_j} - p_i' x_{ji} \left.\frac{dx_{ji}}{d\gamma_i}\right|_{\gamma_j} + p_j' x_{ij} \left.\frac{dx_{jj}}{d\gamma_i}\right|_{\gamma_j} \right] d\gamma_i \qquad j \neq i \tag{26}$$

to obtain

$$\frac{d\gamma_i}{d\gamma_j} = \frac{(p_j - p_j' x_{ij} - C_j') \left.\frac{dx_{jj}}{d\gamma_j}\right|_{\gamma_i} - p_j' x_{ij} \left.\frac{dx_{ij}}{d\gamma_j}\right|_{\gamma_i} + p_i' x_{ji} \left.\frac{dx_{ii}}{d\gamma_j}\right|_{\gamma_i}}{(p_i - p_i' x_{ji} - C_i') \left.\frac{dx_{ii}}{d\gamma_i}\right|_{\gamma_j} - p_i' x_{ji} \left.\frac{dx_{ji}}{d\gamma_i}\right|_{\gamma_j} + p_j' x_{ij} \left.\frac{dx_{jj}}{d\gamma_i}\right|_{\gamma_j}} \qquad j \neq i, \quad j = \{1, 2\} \tag{27}$$

All functions in the RHS, including the derivatives, are continuous. As a result, the best-response functions are continuous. Since each player's strategy set ($[0, 1]$) is compact and convex, there exists at least one Nash equilibrium. However, this equilibrium need not be unique.

5 Conclusion

The choice of a government's stake in a public utility implies a trade-off between the allocative efficiency on the domestic market and potential profits on external markets. In this paper, we have built a model that make this trade-off explicit and that highlight the fact that the ownership structure of a firm in one country has an impact on the behavior and the ownership of firms in all countries.

Some of the factors that we have identified as correlated with state ownership, such as firms' output shares, are observable. Some empirical tests could then be performed on markets where exchanges are mostly bilateral (e.g. Canada and U.S.) in order to make predictions on state ownership in certain industries (e.g. electricity). However, we can already notice that recent privatizations which accompanied regulatory reforms are well explained by our model. First, by going from a monopolistic structure to an oligopolistic one, market power is reduced. Second, with market liberalization, the former incumbent national monopoly can obtain profits on foreign markets, which were previously protected by barriers to entry. According to our model, both changes favor privatization.

On the theorical side, an immediate extension of the model would be to have more than two firms/countries. This should tend to reduce governments' stakes as each firm's market power would be reduced and thus, domestic allocative efficiency would be increased.

Also, the model is presently biased towards public ownership because government maximizes welfare while private shareholders want to restrain output to maximize profit. As there is often a presumption that productive efficiency is impaired by public ownership,[10] a more realistic model would make the cost function to be dependent upon γ_i, with $\partial C_i / \partial \gamma_i > 0$. This is left for future research.

[10] There is pervasive evidence that state-owned enterprises have higher costs than private companies. Boardman and Vining [2], Vining and Boardman [14] and Dewenter and Malatesta [8] find that the profitability performance of state-owned enterprises is worse than that of private companies on competitive markets.

A Proof of Proposition 1

Total differentiation of conditions (5) and (6) for $i = 1, 2$ results in the following equation system:

$$\begin{pmatrix} h^1_{11} & -C''_1 & h^1_{21} & 0 \\ -C''_1 & h^1_{12} & 0 & h^1_{22} \\ h^2_{11} & 0 & h^2_{21} & -C''_2 \\ 0 & h^2_{12} & -C''_2 & h^2_{22} \end{pmatrix} \cdot \begin{pmatrix} dx_{11} \\ dx_{12} \\ dx_{21} \\ dx_{22} \end{pmatrix} = \begin{pmatrix} p'_1(x_{11}+x_{21})d\gamma_1 \\ 0 \\ 0 \\ p'_2(x_{12}+x_{22})d\gamma_2 \end{pmatrix} \tag{28}$$

where $h^i_{ii} \equiv \partial^2 U_i/\partial x^2_{ii}$, $h^i_{ji} \equiv \partial^2 U_i/\partial x_{ii}\partial x_{ji}$, $h^i_{ij} = \partial^2 U_i/\partial x^2_{ij}$ and $h^i_{jj} = \partial^2 U_i/\partial x_{ij}\partial x_{jj}$, $i = 1, 2$, $i \neq j$. The square matrix on the LHS is matrix H defined in (19). From assumption 1, we have that $\det H > 0$.

Let $d\gamma_2 = 0$ and $d\gamma_1 \neq 0$. From Cramer's rule, we have:

$$\left.\frac{dx_{11}}{d\gamma_1}\right|_{\gamma_2} = \frac{\left|\begin{pmatrix} p'_1(x_{11}+x_{21}) & -C''_1 & h^1_{21} & 0 \\ 0 & h^1_{12} & 0 & h^1_{22} \\ 0 & 0 & h^2_{21} & -C''_2 \\ 0 & h^2_{12} & -C''_2 & h^2_{22} \end{pmatrix}\right|}{\det H} \tag{29}$$

The numerator is then $p'_1(x_{11}+x_{21}) \cdot \det H_{11}$ where H_{11} is the cofactor of element h_{11}. The determinant of this cofactor is negative by assumption 1, so that $dx_{11}/d\gamma_1 > 0$.

Similarly,

$$\left.\frac{dx_{22}}{d\gamma_1}\right|_{\gamma_2} = \frac{\left|\begin{pmatrix} h^1_{11} & -C''_1 & h^1_{21} & p'_1(x_{11}+x_{21}) \\ -C''_1 & h^1_{12} & 0 & 0 \\ h^2_{11} & 0 & h^2_{21} & 0 \\ 0 & h^2_{12} & -C''_2 & 0 \end{pmatrix}\right|}{\det H} \tag{30}$$

The numerator of this expression is $-p_1' X_1 (C_1'' h_{21}^2 h_{12}^2 + h_{12}^1 h_{11}^2 C_2'')$. This is positive as $C_1'' > 0$, $h_{21}^2 < 0$ and $h_{12}^2 < 0$ by assumption 1, and $h_{12}^1 < 0$, $h_{11}^2 < 0$ and $C_2'' > 0$ in virtue of assumptions 1 and 2(iii), respectively. As the denominator is also positive, we have $dx_{22}/d\gamma_1 > 0$.

From (28.2) and (28.3), we obtain:

$$\left.\frac{dx_{ij}}{d\gamma_1}\right|_{\gamma_2} = \frac{C_i''}{h_{ij}^i} \left.\frac{dx_{ii}}{d\gamma_1}\right|_{\gamma_2} - \frac{h_{jj}^i}{h_{ij}^i} \left.\frac{dx_{jj}}{d\gamma_1}\right|_{\gamma_2} \qquad i = 1, 2 \quad i \neq j \tag{31}$$

which is negative since $C_i'' > 0$, $h_{ij}^i < 0$ by assumption 1 and $h_{jj}^i < 0$ by assumption assumption 2(iii).

Turning to total consumption in a given market, we compare the absolute value of $dx_{ij}/d\gamma_1$ with $dx_{ii}/d\gamma_1$. From (31), we have:

$$\begin{aligned} \left.\frac{dx_{ij}}{d\gamma_1}\right|_{\gamma_2} &= \frac{C_i''}{h_{ij}^i} \left.\frac{dx_{ii}}{d\gamma_1}\right|_{\gamma_2} - \frac{h_{jj}^i}{h_{ij}^i} \left.\frac{dx_{jj}}{d\gamma_1}\right|_{\gamma_2} \\ &< -\frac{h_{jj}^i}{h_{ij}^i} \left.\frac{dx_{jj}}{d\gamma_1}\right|_{\gamma_2} \\ &= -\frac{p_j'' x_{ij} + p_j'}{p_j'' x_{ij} + 2p_j' - C_i''} \left.\frac{dx_{jj}}{d\gamma_1}\right|_{\gamma_2} \end{aligned} \tag{32}$$

But $\left|p_j'' x_{ij} + p_j'\right| < \left|p_j'' x_{ij} + 2p_j' - C_i''\right|$ as $p_j' - C_i'' < 0$ from assumption 2(ii). We thus have

$$\left|\frac{dx_{ij}}{d\gamma_1}\right| < \left|\frac{dx_{ii}}{d\gamma_1}\right| \tag{33}$$

which implies that $\left|\frac{dX_i}{d\gamma_1}\right| > 0$.

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