Investment and screening under asymmetric endogenous information

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This paper provides an analysis of screening contracts in a complete but imperfect information environment as opposed to the usual incomplete information (Bayesian) environment. An agent faces a hold-up situation while making a cost-reducing specific investment that is not observed by the principal. To prevent the hold-up, the agent randomizes his investment strategy and the principal offers a screening contract. The informational rents provided by the equilibrium contract finance the investment. Because uncertainty is endogenous, the equilibrium contract depends only on tastes, technology and on the strategic opportunities of both players.

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1. Introduction

■ This paper recasts the problem of screening an agent with respect to his production costs in a complete but imperfect information framework instead of the incomplete information Bayesian framework analyzed in the literature (Baron and Myerson 1982). Consider a firm (the agent) that sinks an investment to increase its production capacity of a good desired by its client (the principal). The principal is limited to short-term contracts: she cannot sign a binding contract before the investment is undertaken. For instance, at the time of investment, the firm may know that a demand exists for its product although a client has yet to be identified. When exchange finally takes place, a price-setting principal agrees to pay the agent no more than his reservation price. Since the agent's reservation price does not incorporate the sunk cost of investment, there is a hold-up problem: the agent has less incentive to invest and the benefits of investment may be lost.

When complete binding contracts are available, the hold-up problem can be solved under various information structures (Rogerson, 1992).[1] In this paper, a hold-up occurs because the parties are unable to commit themselves to *any* contract at the time the investment is made. However, the assumption of symmetric information is relaxed: all costs are the private information of the agent. Tirole (1986) was the first to point out that since the *ex post* sharing rule that results from a given bargaining process is generally sensitive to the information structure, the privacy of the investment decision provides a strategic advantage to protect the return on investment from a hold-up. This idea is further developed as the basis of a theory of screening contracts in which the asymmetric information is *endogenous*.

The standard analysis of screening contracts under asymmetric information starts with the Bayesian notion of a *type* that resumes the private information of the agent. In incomplete information models, the distribution of types is exogenous. Any inference drawn with these models about economic structures depends on the distribution of types. But in many instances of screening, the "type" of an agent refers to instrumental factors that he controls and that have a well-defined economic value. For instance, a "low-cost type" results from past investment.

In the framework presented here, the distribution of "types" emerges as the (Nash) equilibrium randomization of the agent's investment strategy. The model works like a classical principal-agent model to which an initial investment stage is added. The agent has the opportunity of choosing his "type" at that stage[2], at a price (*i.e.* the cost of investment). An unobserved randomized strategy allows the agent to hide his investment behind a veil of noise to prevent a hold-up. In equilibrium, this randomization induces a common-knowledge endogenous distribution of "types". Naturally, the principal will offer a screening contract that provides *ex post* production incentives. Surprisingly, the equilibrium contract turns out to be a cost-plus contract.

There is a long tradition of models with mixed strategy equilibria in complete but imperfect information settings, but few studies consider the screening problem in this context.[3] Fudenberg and Tirole (1990) analyze the moral hazard problem in a way that involves screening. A principal wants her risk averse agent to exert an effort that increases the probability of a good outcome in a stochastic environment. Towards this end, the principal offers an incentive contract that links the agent's compensation to the random outcome, thereby exposing the agent to some risk. Yet, once the effort is undertaken, there is room to renegotiate the contract in order to provide insurance to the agent. This renegotiation leads to screening, since the agent has private information about the amount of effort that was provided and therefore about the likelihood of a good outcome. Fudenberg and Tirole's (1990) model is set in an environment with partial commitment: there is no hold-up problem since the principal can commit to a compensation scheme prior to investment (effort). Consequently, the contract they study is a *maximizer* in the set of renegotiation-proof contracts and the agent's randomization is directly induced by the contract. In this paper, the contract is an *equilibrium best response* to the agent's randomization.

Gul (2000) analyzes a model of bargaining between a seller and a buyer in an environment similar to the one presented here. In Gul's (2000), the buyer has the *ex ante* opportunity to make an investment that increases the gains from trade *ex post*. By allowing the investment to be made privately by the buyer, and by considering a sequential bargaining subgame of offers and counter-offers, the hold-up problem vanishes as the length of time between successive offers goes to zero. Gul (2000) puts the emphasis on the bargaining subgame, which is more complex than the one used here. Information about the buyer's *ex post* willingness to pay is revealed through a process of offers and counter-offers while this study relies on a screening contract offered on a take-it-or-leave-it basis.

Linking the hold-up and the screening problems clarifies the role of asymmetric information in *explaining* screening (nonlinear pricing). The need and the means of screening come from the heterogeneity of the agent's characteristics and the monopoly power of the principal (Wilson 1993). Asymmetric information plays no role in that story; if anything, it reduces the ability of the principal to discriminate agents. On the other hand, the monopoly power of the principal creates a hold-up problem. Combined with asymmetric information, the hold-up problem generates the heterogeneity of the agent's *ex post* characteristics that justifies the need for *ex post* screening. According to this view, asymmetric information is an important ancillary condition that rationalizes screening.

The model is presented and solved in the next section. It is illustrated with three analytical examples. Comparative statics are pursued in Section 3. The paper concludes with a discussion about normative and positive issues that favor the use of complete but imperfect information principal-agent models. The Appendix contains the proofs.

2. The model

■ Consider an exchange between a buyer (hereafter the principal) and a producer (the agent). The agent produces a quantity $q \geq 0$ of a good for the principal. The principal pays the agent a sum of money t or a price $p = t/q$ per unit produced (when $q > 0$). Both players have quasi-linear preferences and the principal values each unit produced at 1. The principal's and agent's payoffs are then respectively

$$(1-p)q,$$
$$\pi(q,k,p) \equiv pq - c(q,k),$$

where $c(q,k)$ is the expected *opportunity cost* of dealing with the principal. This cost includes production costs as well as any relevant loss of profits (related to outside opportunities) induced by this exchange. If no exchange takes place, then $q = 0$ and $-c(0,k)$ is the agent's payoff function. The variable $k \geq 0$ represents an investment in capital. Investment determines the *ex post* production costs, and can also determine the profitability of other ventures pursued by the agent affected by this exchange. Let c be a thrice continuously differentiable and strictly convex function that exhibits an unbounded long-run marginal cost as q increases in order to ensure bounded returns.

The short-run and long-run profit functions are defined by

$$\pi^*(k,p) \equiv \max_{q \geq 0} \pi(q,k,p),$$
$$\pi^{**}(p) \equiv \max_{q \geq 0, k \geq 0} \pi(q,k,p).$$

Let $q^*(k,p)$ and $(q^{**}(p), k^{**}(p))$ be the solutions to the first and second programs. These expressions define the *conditional ex post supply* function q^* and the *long-run supply* and *capital demand* functions q^{**} and k^{**}. To simplify the notation, denote $k_0 \equiv k^{**}(0)$, $k_1 \equiv k^{**}(1)$, $q_1 \equiv q^{**}(1)$ and $\pi_0 \equiv \pi^{**}(0)$. In Figure 1, the NE panel contains three short-run marginal cost curves (the supply curves with investment levels k_0, x and k_1), two short-run average cost curves (with investment levels x and k_1) and the long-run marginal and average cost curves. Convexity ensures that the marginal cost curves are upward slopping. The functions q^* and q^{**} are given by the inverse of the marginal cost curves. The value q_0 is equal to zero.[4]

The following assumptions are made about the cost function. They will be explained in detail later.

- Stability (Stab.): $c_k(0,k_0) = 0$. The solution k_0 is interior.
- Single crossing (S.C.): $c_{qk} < 0$. Investment increases capacity.
- Regularity (Reg.): $c_{qqk} \leq 0$. The marginal benefit of capital increases with q at a non-decreasing rate.
- Interest (Int.): $c_q(0,k_0) < 1$. Gains from trade always exist.

Total surplus is given by $(1-p)q + \pi(q,k,p) = \pi(q,k,1)$. The ex ante *efficient* levels of investment and production are thus k_1 and q_1, generating a total surplus of $\pi^{**}(1)$. Given k, the ex post *efficient* level of production is $q^*(k,1)$ which yields a total surplus of $\pi^*(k,1)$. The *ex post* efficient allocations $(k, q^*(k,1))$ yield a downward slopping curve in the SE panel of Figure 1. When $k = k_1$, the *ex ante* and *ex post* efficient allocations coincide.

The game has three stages; Figure 2 provides a sketch of its extensive form. It begins with the *investment stage* at the *initial node* where an amount k is invested. The shaded triangle to the right of the initial node represents the possible investment moves that may be realized. In the standard incomplete information game, the initial node belongs to Nature and k is a random exogenous type. An alternate approach is to let this node belong to the agent so that the choice of k becomes part of his strategy.[5]

The second stage is the *contracting stage* in which the principal offers a contract which is labeled here $(\tilde{q}, \tilde{t})$. That stage begins at some node n^k that follows the investment move k. If the principal observes the investment, then her information set at that stage is the singleton $\{n^k\}$. Otherwise, her information set is N. The second shaded triangle stands for the possible contracts that may be offered at that stage.

The final stage is the *acceptance stage*. It begins after some k has been invested and some contract $(\tilde{q}, \tilde{t})$ has been proposed by the principal leading to some node n'. The agent then either *accepts* the contract or he *refuses* it. The payoffs attached to the nodes following these moves are the sequential values for both players of the subsequent subgames (not shown in Figure 2). A refusal by the agent of the principal's offer puts an end to the relationship: zero unit are sold, leaving the principal with a payoff of zero and the agent with a payoff of $\pi^*(k,0)$. The payoffs obtained when a deal is reached will be detailed later.

The analysis proceeds in three steps. In Step 1, I analyze the *perfect information game* where the initial node belongs to the agent and the investment move is observed by the principal. In Step 2, I analyze the *incomplete information game* where the initial node belongs to Nature and the investment move is not observed by the principal. In Step 3, I analyze the *complete but imperfect information game* where the initial node belongs to the agent and where the investment move is not observed by the principal. The main result of the paper is Proposition 2 of Step 3 which characterizes the equilibrium of this latter game.

□ **Step 1: The perfect information game.** Suppose that the initial node belongs to the agent and that the principal observes the investment move. She will then offer to pay a transfer t for a quantity q where the pair (q,t) solves

$$\text{Program 1:} \quad \max_{q,t} \quad q - t$$

$$\text{subject to} \quad t - c(q,k) \geq \pi^*(k,0). \qquad \text{(IR)}$$

Inequality (IR) is an *individual rationality* constraint, stating that the agent is no worse off by accepting the contract than by refusing it. Define the total *ex post* surplus of contracting as

$$s(q,k) \equiv \pi(q,k,1) - \pi^*(k,0), \tag{1}$$

so that (IR) may be rewritten

$$t \geq q - s(q,k). \tag{IR$'$}$$

Program 1 is solved when $q = q^*(k,1)$ and (IR$'$) binds. Since contracting takes place with perfect information, it must yield an *ex post* efficient allocation and since the principal has all the bargaining power, she will pay no more than what is required to satisfy (IR$'$).

Hence, in a game with perfect information, the principal captures the total *ex post* surplus $s(q^*(k,1),k)$ and the agent receives $\pi^*(k,0)$. In a subgame perfect equilibrium, the agent maximizes $\pi^*(k,0)$ to π_0 by investing k_0 at the investment stage. The principal offers her agent to produce $q^*(k_0,1)$ for a transfer $q^*(k_0,1) - s(q^*(k_0,1),k_0)$ and she realizes a payoff $s(q^*(k_0,1),k_0)$. The equilibrium allocation pair $(q^*(k_0,1),k_0)$ is identified in the SE panel of Figure 1. Although *ex post* efficient, this allocation is *ex ante* inefficient since total *ex ante* surplus peaks at $\pi^{**}(1) > \pi_0$ with an investment $k_1 > k_0$ and a production level $q_1 > q^*(k_0,1)$. Because of the hold-up, there is under-investment and reduced production.

□ **Step 2: The incomplete information game.** Let the initial node belong to Nature. Investment k is a randomly chosen type that is the private information of the agent. The solution of that game is well known: the principal offers a screening contract that equalizes the expected marginal benefit of production to the expected marginal informational rent conceded to the agent. This contract is characterized in Proposition 1 and Corollary 1 below. To the extent that the contract depends on the distribution of types, the incomplete information approach provides a *family* of contracts as a solution to the screening problem.

Let F represent the beliefs of the principal with respect to the distribution of k over $K = [\underline{k}, \overline{k}]$, where $\underline{k} \geq k_0$. To maximize her expected payoff, the principal will offer a screening contract: a pair of real bounded functions $\tilde{q}$ and $\tilde{t}$ over K that specify a production level $\tilde{q}(m)$ and a payment $\tilde{t}(m)$ that depend on the information m reported by the agent about his type k. By the Revelation Principle, there is no loss of generality in considering a Bayesian equilibrium where the agent reports truthfully his type.

Define the informational rent of a type k agent as total *ex post* surplus minus the principal's share

$$\tilde{r}(k) \equiv s(\tilde{q}(k),k) - \left[\tilde{q}(k) - \tilde{t}(k)\right]. \tag{2}$$

By (2), a contract may be equivalently represented by a pair $(\tilde{q},\tilde{t})$ or by a pair $(\tilde{q},\tilde{r})$.

Assumption S.C. ensures that investment always increases the agent's *ex post* capacity, playing the role of a single-crossing condition. Assumption Reg. ensures that the solution to Program 2 below is a singleton.[6] The following representation then holds:

Proposition 1 Let F be a twice-continuously differentiable distribution function defined on $K = [\underline{k}, \overline{k}]$ with density f and a hazard rate $h(k) = f(k)/(1 - F(k))$. The incomplete information game has a Bayesian-Nash equilibrium in which the principal offers a contract $(\tilde{q}, \tilde{r})$ defined over K by

$$\tilde{r}(k) = \tilde{r}(\underline{k}) + \int_{\underline{k}}^{k} s_k(q(\kappa), \kappa) d\kappa, \tag{3}$$

where the function $\tilde{q}$ and the quantity $\tilde{r}(\underline{k})$ solve

$$\text{Program 2:} \quad \max_{\tilde{q}, \tilde{r}(\underline{k})} \quad -\tilde{r}(\underline{k}) + \int_{\underline{k}}^{\overline{k}} \left[s(\tilde{q}(k), k) - \frac{s_k(\tilde{q}(k), k)}{h(k)} \right] f(k) dk,$$

$$\text{subject to} \quad \tilde{r}(\underline{k}) \geq 0,$$

$$\tilde{q}_k(k) \geq 0 \quad \forall k \in K. \tag{4}$$

The agent accepts the contract and truthfully reveals his type. □

When (4) does not bind, the solution to Program 2 is easy to characterize:

Corollary 1 Define $\tilde{q} : K \to \mathbb{R}$ by letting $\tilde{q}(k)$ solves

$$s_q(\tilde{q}(k), k) = \frac{s_{qk}(\tilde{q}(k), k)}{h(k)}, \quad \text{for all } k \in K. \tag{5}$$

If $\tilde{q}$ is a function that satisfies (4), then $\tilde{q}$ and $\tilde{r}(\underline{k}) = 0$ solve Program 2. □

Given an arbitrary distribution of types, Program 2 yields an arbitrary downward sloping dotted curve $\tilde{q}$ in the SE panel of Figure 1. In Program 2, the bracketed term $[s - s_k/h]$ is the *ex post* virtual surplus (Myerson 1981). Corollary 1 characterizes a solution to Program 2 where $\tilde{q}(k)$ maximizes the virtual surplus for all k. As $k \to \overline{k}$, the hazard rate diverges $h(k) \to \infty$, and the terms s_k/h in Program 2 and s_{qk}/h in Corollary 1 vanish. Hence, the real and the maximized virtual surplus coincide at $\overline{k}$ so that there is "no distortion at the top".

When k is strategically chosen by the agent, this solution has a normative content: if the principal holds beliefs F, she should propose the screening contract $(\tilde{q}, \tilde{r})$ specified in Proposition 1. From a positive point of view, that is, if we are trying to explain the structure of actual screening contracts, this proposition is rather incomplete. The problem is that this contract yields an *ex ante* payoff of $\pi^*(k, 0) + \tilde{r}(k)$ to an agent that has invested k. Through r, that payoff *depends* on F; given an arbitrary F, the function $\pi^*(\cdot, 0) + \tilde{r}(\cdot)$ is generally not constant. Hence, if the agent expects the contract $(\tilde{q}, \tilde{r})$ to be offered and if $\pi^*(\cdot, 0) + \tilde{r}(\cdot)$ reaches a strict maximum at k_F, the agent has an incentive to invest k_F. But then, there is little rationale for the principal to screen the agent in the first place: if the principal expects her agent to play k_F in pure strategy on the equilibrium path, she should propose the contract derived in Step 1.

In a setting where the "type" of an agent refers to characteristics that have a clear market value (here, the opportunity cost of investment) and that are under the agent's control, it is inappropriate to assume that the distribution of "types" is an exogenous variable with a predictive content. In such a setting, technology, preferences, and the strategic opportunities (the fundamentals) alone should explain the structure of screening contracts. I now depart from the incomplete information model by focusing on a game of *complete* but *imperfect* information where the agent "chooses his type" by choosing k.

□ **Step 3: The complete but imperfect information game.** The agent not only has private information about k, but he also decides its value. The investment k is still a random variable although the randomization is endogenous. Hence, the results obtained in Step 2 should apply. On the other hand, Proposition 1 is not applicable if k is not distributed on a bounded set or if the distribution is degenerate. These issues are addressed in lemmas 1 and 3 below. The equilibrium of this game is presented in Proposition 2. In equilibrium, the *ex post* moves of both players are of the form described in Corollary 1, but the distribution of k is no longer arbitrary and is associated with the agent's best response strategy. While the analysis in Step 2 yields a *family* of contracts as a solution to the screening problem, the complete information approach selects a *single* contract in that family, namely the contract associated with the equilibrium distribution of k as chosen by the agent.

Once the investment opportunity is reintegrated into the model, we have a game of complete information, since the principal can evaluate the agent's payoff of playing any of his strategies. The principal can ensure her payoff realized in Step 1 by offering the equilibrium contract of Step 1 which is always accepted. Yet, there is more surplus to capture with a screening contract if the agent has invested more than k_0. Hence, as in Step 2, a strategy for the principal is a contract $(\tilde{q}, \tilde{r})$. A strategy for the agent must specify k at the investment stage and a decision function (acceptance or refusal) at each of the possible nodes of the acceptance stage. In a subgame perfect equilibrium, the agent accepts any contract that satisfies individual rationality at that stage. In that case, the sequential values of both players at each of these nodes can be characterized as a function of the contract and the investment level. Hence, a strategy for the agent is resumed by the choice of k. Lemma 1 establishes that the agent's choice is bounded:

Lemma 1 There exists an investment level $\hat{k}$ such that any strategy $k > \hat{k}$ does not survive iterative elimination of weakly dominated strategies. □

To construct a Nash equilibrium for this game, I focus on a partial characterization of the best responses of both players. If a pure strategy equilibrium is played, the principal anticipates the agent's strategy. Observability does not matter and the game is to be played like the perfect information game in Step 1. In an equilibrium where the agent randomizes his investment move, the principal will hold some equilibrium beliefs F about this randomization and she will accordingly propose a screening contract based on those beliefs as in Step 2. If the agent's equilibrium randomization is well-behaved, then her best response $(\tilde{q}, \tilde{t})$ follows readily from Corollary 1.

It is shown in Proposition 2 below that the equilibrium allocation $\tilde{q}$ is given by the inverse of the *conditional capital demand*:

$$k^*(q) \equiv \operatorname*{argmin}_{k \geq 0} c(q, k). \tag{6}$$

The conditional capital demand function k^* yields the level of investment that minimizes the total cost of production. Assumptions Stab. and S.C. ensure that (6) has an interior solution for all $q \geq 0$. The function k^* may then be derived directly by applying the implicit function theorem to the first-order condition

$$c_k(q, k) = 0. \tag{7}$$

Equation (7) defines a curve in the SE panel of Figure 1 that represents the relationship between q and k at the point of tangency between any straight vertical line passing through some q and some isocost curve. Assumption S.C. ensures that this relationship is positive, which implies that k^{**} is a strictly increasing function as well. Let p^* be the inverse of k^{**} on $[k_0, \infty)$. Given k, the value $p^*(k)$ is the price for which investing k maximizes profits. Both k^{**} and p^* are drawn in the SW panel of Figure 1 where $p_0 \equiv p^*(k_0) = 0$ and $p^*(k_1) \equiv 1$. Notice that $C(q) \equiv c(q, k^*(q))$ is the long-run cost curve of the firm. Given q^* and p^*, one may define $\hat{q}$ on $[k_0, \infty)$ such that $\hat{q}(k) \equiv q^*(k, p^*(k))$. The following lemma relates the function $\hat{q}$ to the long-run cost curve.

Lemma 2 $\hat{q}$ is the inverse of k^*. □

Lemma 2 is illustrated in Figure 1. Starting from an investment level x, we obtain a price through p^* at point $y = p^*(x)$. The long-run supply at price y is $z = q^{**}(y)$ where the long-run marginal cost $C_q(z)$ equals y. At that point, $z = q^*(x, p^*(x)) = \hat{q}(x)$. The cost of producing at point z is then minimized by investing $k^*(z) = x$.

Assumption Int. and S.C. ensure that $c_q(0,k) < 1$, for all $k \geq k_0$. Hence, there are *always* strict gains from trade to be realized at the contracting stage. Because each player is trying to secure these gains for themselves, we get the following simple but important result.

Lemma 3 The complete information game does not have an equilibrium in pure strategy. □

In an equilibrium in pure strategies, all moves are anticipated along the equilibrium path. Observability of the investment move is irrelevant and the game is played like in Step 1. But if the principal proposes her agent to produce $q^*(k_0, 1)$ like in Step 1, the agent should invest strictly more than k_0 to minimize costs. Hence, the pure strategy profile identified in Step 1 does not hold as an equilibrium when investment is not observed. Figure 1 illustrates the cobweb-like strategic structure of this game. If the principal thinks that her agent invested x, she identifies the relevant short-run cost structure through price y at point b. Going up $c_q(\cdot, x)$, she identifies point z' that maximizes the short-run surplus and pays her agent his average opportunity cost $y' = c(z', x)/z'$. If the agent expects to produce in point z', he will invest in point $x' > x$ and no equilibrium obtains except in the *ex ante* efficient allocation point (q_1, k_1). But the principal is paying a price $c(q_1, k_1)/q_1$ at that point, which is lower than the price $p^*(k_1) = 1$ that justifies investing k_1. However, an equilibrium does exist when the agent randomizes his investment strategy.

Proposition 2 Let $K = [k_0, k_1]$. The complete but imperfect information game has a Nash equilibrium in which the principal offers a contract $(\tilde{q}, \tilde{r})$ defined over K by

$$\tilde{q}(k) = \hat{q}(k), \tag{8}$$

$$\tilde{r}(k) = \pi_0 - \pi^*(k, 0), \tag{9}$$

and where the agent randomizes over K with F given by

$$F(k) = 1 - \exp\left(-\int_{k_0}^{k} h(\kappa)d\kappa\right), \tag{10}$$

$$\text{and} \qquad h(k) = \frac{s_{qk}(\hat{q}(k), k)}{s_q(\hat{q}(k), k)}. \tag{11}$$

The agent accepts the contract and truthfully reveals his investment move. □

In Proposition 2, the key equation is (8). In equilibrium, the agent is ready to randomize his investment move only if he is indifferent about its value. But since the principal plays a pure strategy, the agent anticipates his production level q and, given q, he is never indifferent about k. He wants to invest $k^*(q)$ to strictly minimize costs. A contract $(\tilde{q}, \tilde{r})$ is incentive compatible if type k wants to produce $\tilde{q}(k)$. Hence, using Lemma 2, $\tilde{q} \equiv [k^*]^{-1} \equiv \hat{q}$ is a necessary condition to have both indifference and incentive compatibility.[7]

Once (8) is established, the rest of the proposition follows easily. For (5) to characterize $\hat{q}$ as the best response of the principal, h must be of the form given by (11). In (10), recovering the distribution F from its associated hazard rate h involves solving an ordinary differential equation.[8] Given (8) and (11), h is completely specified; hence (10) is obtained. Applying (3) yields (9).

Given the equilibrium strategy F for the agent, the contract offered by the principal is the screening contract described in Proposition 1. Since all pure strategies in the support of the mixed strategy played by the agent must yield the same payoff, his ex post informational rent must match the investment cost of having a more or less *ex post* efficient type. When the agent invests $k > k_0$, he does so with the intent of producing for the principal but he is also reducing his ex post reservation payoff by $\pi_0 - \pi^*(k, 0)$. The *ex post* rent $\tilde{r}(k)$ of the equilibrium contract conceded to a type k agent compensates exactly for that amount. These are *quasi-rents*, in the Marshallian sense, since these rents are nothing more than a minimum fair return on past investment in capital (Hart 1995).

The incomplete information approach of Step 2 states that the agent should produce an amount that maximizes the *ex post* virtual surplus. The complete information approach of Step 3 states that the agent should produce an amount that turns his rent into a quasi-rent. It is rational to invest as long as the opportunity cost of investment is no greater than the *ex post* informational rent associated to a higher investment level. The informational rent thus includes the cost of investment as a quasi-rent. From (1), (2) and (9),

$$\tilde{t}(k) = \pi_0 + c(\hat{q}(k), k). \tag{12}$$

Substitute k by $k^*(q)$ in (12) and (2) to get expressions for the transfer and the rent as functions of q,

$$\begin{aligned} \tilde{t} \circ k^*(q) &= \pi_0 + C(q), \\ \tilde{r} \circ k^*(q) &= \pi_0 - \pi^*(k^*(q), 0), \end{aligned} \tag{13}$$

where "$\circ$" is the composition operator (see the proof of Lemma 2). The transfer combines a fixed payment π_0 and a conditional payment that evolves with the long-run cost. Having received π_0, a "type" k agent must then choose q in order to minimize $c(q, k) - C(q)$. By the envelope property of the long-run cost function, the agent will minimize this loss to zero by choosing $\hat{q}(k)$. Doing so, his informational rent is reduced to the quasi-rent $\pi_0 - \pi^*(k, 0)$.

From (12) or (13), the equilibrium contract is a *cost-plus contract*, generally considered to be at the lowest end of the spectrum of incentive contracts. However, the contract does provide incentives to produce efficiently, since an agent who produces q does so at the lowest possible cost on his long-run cost curve. It is only to the extent that he does not invest k_1 with certainty that the allocation remains inefficient. Investment is nevertheless greater than in the perfect information case of Step 1.

As is apparent from (10) and (11), the shape of the distribution of investment depends on the opportunity cost function alone. The following analytical examples show that the technological assumptions allow many shapes of the density function: In the first example, the distribution is skewed to the right, in the second it is uniform, while in the third, it is skewed to the left.

□ **Example 1.** Let the cost function be

$$c(q,k) = \alpha\Big[\exp(q-k) + \exp(k)\Big], \quad 0 < \alpha < 1.$$

With this specification, $k_0 = 0$ and $k_1 = -\ln(\alpha)$. The efficient allocation is to invest k_1 and to produce $q_1 = -2\ln(\alpha)$. If investment is observable, the agent invests k_0, produces $q^*(k_0, 1) = -\ln(\alpha)$ and gets a zero payoff. If investment is not observable, he gets the same payoff over K while producing $\hat{q}(k) = 2k$. Using (10) and (11), the equilibrium distribution is given by $F(k) = \alpha\exp(k)$. The density increases on K and is skewed toward k_1.

□ **Example 2.** The cost function is

$$c(q,k) = (q-k)^2 + k^2.$$

The first best solution is to invest $k_1 = 1/2$ and to produce $q_1 = 1$. If investment is observable, the agent invests $k_0 = 0$ and produces $q^*(k_0, 1) = 1/2$. If investment is not observable, the equilibrium allocation is implemented by having a type k agent produce $\hat{q}(k) = 2k$ for a transfer $\tilde{t}(k) = 2k^2$. The agent randomizes on $K = [0, 1/2]$ with a uniform distribution given by $f(k) = 2$.

□ **Example 3.** The distribution is now skewed toward k_0. Let

$$c(q,k) = \frac{q^2}{2k} + \frac{k^2}{4}.$$

The first best solution is to invest $k_1 = 1$ and to produce $q_1 = 1$. If investment is observable, the agent invests $k_0 = 0$ and produces $q^*(k_0, 1) = 0$. If investment is not observable, the equilibrium allocation is implemented by having a type k agent produce $\hat{q}(k) = k^{\frac{3}{2}}$. The hazard rate function is $h(k) = \left(\sqrt{k} - k\right)^{-1}$ and the the distribution on $[0, 1]$ is given by $F(k) = 1 - \left(1 - \sqrt{k}\right)^2$. It has a decreasing density and is skewed to the left.

3. Comparative Statics

■ Consider a family of cost functions parameterized by η. As η changes, the equilibrium contract and the equilibrium distribution F also change. Comparative statics for the contract are easy because once $c(q,k;\eta)$ is defined, the contract $\hat{q}$ is obtained directly from (7). What is less obvious though is how this change affects the equilibrium distribution of investment. Suppose that, as η changes[9], the new agent's randomization first-order stochastically dominates the old one; allowing a slight abuse of terminology, I will say that investment becomes *stochastically larger*. [10]

Proposition 3 Suppose that k_0 weakly increases with η and that h weakly decreases[11] with η. Then investment becomes stochastically larger as η is increased. □

Corollary 2 Suppose that c has a separable form $c(q,k) = \psi(q,k) + \phi(k;\eta)$, where $\phi_{k\eta}(k;\eta) \leq 0$ for all $k \in K$. Then investment becomes stochastically larger as η is increased. □

I present two variations of the base model that permit comparative statics using these results. In the first, Proposition 3 is used to show that an increase in the reservation price of the agent leads to more investment. In the second, Corollary 2 is used to show that an increase in the bargaining power of the agent or in the privacy of his investment move leads to more investment.

□ **Market opportunity.** Suppose that the good may be produced either in a generic or in a specific variety. The generic variety can be bought or sold at a price $\mu < 1$ on the market. The specific variety can also be sold at price μ on the market but can only be bought from the agent at a price to be agreed upon. The principal values the generic variety at zero. The market does not attribute a different value to the specific variety but the principal does. There are no economies of scope in using the agent's installed capacity for a joint production of both varieties so that the cost of producing any bundle of the two varieties is a function of the total amount produced alone. The agent is free to sell any quantity he wishes on the market.

Let $\tilde{c}(q+\theta,k)$ be the *production* cost of a total quantity $q+\theta$ where q is sold to the principal and θ is sold on the market. Given q, the agent will choose θ to maximize his profits. Let $\tilde{\pi}^{**}$, $\tilde{q}^{**}$ and $\tilde{k}^{**}$ be defined like π^{**}, q^{**} and k^{**} but from $\tilde{c}$ instead of c. Define the region $Y(\mu) \equiv \{(q,k) | \tilde{c}_q(q,k) \geq \mu\}$ where it is not strictly profitable to produce for the market. Taking opportunity costs as the converse of profits, define

$$c(q,k) \equiv -\max_{\theta\geq 0} \mu\theta - \tilde{c}(q+\theta,k) \equiv \begin{cases} \tilde{c}(q,k) & \text{if } (q,k) \in Y(\mu), \\ \mu q - \tilde{\pi}^*(k,\mu) & \text{otherwise.} \end{cases}$$

The agent either uses all his capacity to supply the principal (by raising his short-run marginal cost above the market price) and his opportunity cost equals the production cost, or he retains some capacity and the opportunity cost of contracting equals the actual loss in market sales μq net of the total profit he could have made on the market. Notice that $k_0 = \tilde{k}^{**}(\mu)$, $p_0 = \mu$ and $\pi_0 = \tilde{\pi}^{**}(\mu)$.

Consider applying the technological assumptions presented in section 2 to $\tilde{c}$ with assumption Int. simply stating that a profit can be realized at the current market price. Within $Y(\mu)$, these assumptions translate directly to c. It is easy to verify that $Y(\mu)$ includes the graph of all the equilibrium points $\{(q,k)|k \in K, q = \hat{q}(k)\}$. The contract of Proposition 2 yields $q_0 \equiv \hat{q}(k_0) = \tilde{q}^{**}(\mu)$ at k_0 which is the maximum amount the agent is ready to produce for the principal if he expects to be paid no more than the market price.[12] From (13), the principal pays an agent who produces q units (where $q > q_0$) a transfer $t \circ k^*(q) = \pi_0 + C(q) > \mu q$. Since the principal pays more than the market, the strategy profile described in Proposition 2 remains an equilibrium here.[13]

A rise in the market price μ shifts the point (q_0, p_0) in the NE panel of Figure 1 along the long-run marginal cost curve and the point (k_0, q_0) in the SE panel downward the $\hat{q}$ curve. The $\hat{q}$ curve is not affected by this change since the cost function is independent of μ within $Y(\mu)$. As a result, the effect of a rise in the market price is characterized by a shrinking of K and the image $\hat{q}(K)$. Since $\hat{q}$ does not change, h does not change. Since k_0 strictly increases with the market price μ, Proposition 3 implies that a rise in the market price leads to a stochastically larger investment.

□ **Bargaining and Observability.** Consider a generalized version of the model where both the concepts of bargaining power and observability are parameterized. Add exogenous uncertainty by considering the two following exogenous events:

event B: the principal has the bargaining power at the contracting stage;

event U: the agent's investment is not observed.

In event $\sim B$ (the complement of B), it is the agent that holds the bargaining power while in event $\sim U$, the investment is observed by the principal. Assume that both players commonly learn which combination of events is realized after the investment stage but prior contracting take place (in point E in Figure 2). When the agent has all the bargaining power, the allocation (then decided by the agent) is efficient and independent of whether his investment was observed or not. Partition B into two other events $B \cap U$ and $B \cap \sim U$. Let β be the marginal probability that the principal has the bargaining power and υ be the conditional (on B) probability that the agent's investment is not observed. With this notation, we get three exclusive events:

event	$\sim B$	$B \cap U$	$B \cap \sim U$
probability	$1-\beta$	$\beta\upsilon$	$\beta(1-\upsilon)$

The parameters β and υ are to be interpreted as measures of the the expected bargaining power of the principal and of the privacy of the agent's investment move. The cases $(\beta, \upsilon) = (1, 0)$ and $(\beta, \upsilon) = (1, 1)$ were studied in Steps 1 and 3. The cases $(\beta, 0)$ are a simple variation of Step 1. In the cases $(0, \upsilon)$, the agent realizes and captures the total *ex ante* surplus. I consider now the remaining cases where $\beta\upsilon > 0$.

Let $\tilde{c}$ be the "real" opportunity cost function and $\tilde{\pi}^*$ its associated short-run profit function. In the event $\sim B$, the agent will propose to produce efficiently in order to realize and capture the full surplus $\tilde{\pi}^*(k, 1)$. In the event $B \cap \sim U$, the principal (now fully informed) will also propose that the agent should produce efficiently, but for a reduced transfer that allows the agent to realize only his *ex post* reservation payoff $\tilde{\pi}^*(k, 0)$ like in Step 1. The problem is to characterize the *ex ante* investment move of the agent and the screening contract offered by the principal in the event $B \cap U$.

Define the opportunity cost function $c(q, k)$ of producing q with k in the event $B \cap U$ from

$$-\beta\upsilon \cdot c(q, k) \equiv (1 - \beta)\tilde{\pi}^*(k, 1) + \beta\Big[(1 - \upsilon)\tilde{\pi}^*(k, 0) - \upsilon\tilde{c}(q, k)\Big]. \tag{14}$$

Up to a multiplicative constant $(\beta\upsilon)$, the agent's expected profit if the principal buys q units at price p in the event $B \cap U$ is then $\pi(q, k, p)$. If the function $\tilde{c}$ satisfies the technological assumptions, so does c. The equilibrium distribution of investment and the equilibrium contract proposed by the principal in the event $B \cap U$ are then given by Proposition 2. Notice that c is separable with respect to β and υ with:

$$\phi(k; -\beta, \upsilon) = -\left[\frac{1 - \beta}{\beta\upsilon}\tilde{\pi}^*(k, 1) + \frac{1 - \upsilon}{\upsilon}\tilde{\pi}^*(k, 0)\right].$$

Lemma 4 below ensures that Corollary 2 applies, so that a decrease in β or an increase in υ leads to a stochastically larger investment.

Lemma 4 We have $\phi_{k,-\beta}(k; -\beta, \upsilon) < 0$ and $\phi_{k,\upsilon}(k; -\beta, \upsilon) \leq 0$, for all $k \in K$. □

4. Conclusion

■ When investment is unobserved by the principal, its expected level rises as the agent is given more incentives to invest, thus increasing the incidence of *ex post* inefficiencies in production. Under perfect information, the agent invests too little but always produces efficiently. Under imperfect information, the agent invests more but generally under-produces. Since he receives a payoff π_0 whether or no the game is played under asymmetric or symmetric information, and since the principal can always ensure herself the equilibrium payoff of the perfect information game, unobservability increases social welfare. This is in sharp contrast with Bayesian games of incomplete information where the unobservability of types diminishes social welfare as players engage in rent-seeking behavior.

This important difference can be explained by a classical second-best argument: when dealing with one market imperfection (non commitment), introducing another (imperfect information) may improve efficiency. This effect does not appear in traditional Bayesian models, because it is assumed that the distribution of types is exogenous, and therefore unaffected by the observability issue. It follows that going from unobservable to observable types increases welfare as all inefficiencies associated with bargaining under asymmetric information are resolved. When "types" are endogenous, observability causes the type distribution to collapse to k_0 at a great cost in social welfare. Unobservability allows more "types" to be played and the presence of more efficient types dominates the fact that most types now produce inefficiently.

The approach taken here can be extended to most adverse selection models to improve our understanding of contracts both on normative and positive grounds. Incomplete information models are routinely used, for instance in the regulatory literature (see Laffont and Tirole, 1993) to rationalize the use of high-powered incentives schemes like price caps in lieu of rate-of-return regulation. In such a setting, the distribution of types is assumed exogenous and the return to a "good" type is considered a "rent". Any conclusion about the relative merits of various compensation schemes will depend heavily on the assumptions made with respect to the distribution of types.

For example, if a principal believes that the proportion of high-cost types in a population is high, she should construct an incentive scheme designed mainly for these types; that is, an incentive scheme that allows the few low-cost types to extract a lot of informational rent. But if the distribution of "types" is endogenous, then these high rents motivate high-cost agents to improve their "type". In the end, because of the distribution shift, the principal may end up paying a larger rent than she intended.

This study internalizes all of these effects. The equilibrium contract shares the same qualitative features as the incomplete information model, but it is robust to the feedback effect of a given contract on the distribution of "types". Factors that affect the incentives to invest determine the equilibrium distribution of types and thus the nature of the contract.

From a positive point of view, the aim of contract and organization theory is to explain economic structures as a systemic endogenous response to address transaction costs. In particular, opportunity costs generated by opportunistic behavior and asymmetrical information are assumed paramount. Ultimately, the links between pure technological factors (or preferences) and economic structures should be explicit. Models that rely on an exogenously specified distribution of (economic) types are an important but transitional step toward the development of such a theory. The screening contract constructed here does not depend on an exogenous distribution of types but it takes as given the commitment capabilities of the players. A natural step forward for future research would be to endogenize these capabilities.

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A Appendix

■ The proofs of all the lemmas, propositions and of corollaries follow.

Proof of Proposition 1. Given F, an optimal contract solves

$$\text{Program 2:} \qquad \max_{\tilde{q},\tilde{t}} \int_{\underline{k}}^{\overline{k}} (\tilde{q}(k) - \tilde{t}(k)) f(k) dk,$$

$$\text{subject to,} \quad k \in \underset{m\in[\underline{k},\overline{k}]}{\operatorname{argmax}}\ \tilde{t}(m) - c(\tilde{q}(m), k), \quad \forall k \in K, \tag{IC}$$

$$\tilde{t}(k) - c(\tilde{q}(k), k) \geq \pi^*(k, 0) \quad \forall k \in K. \tag{IR$''$}$$

Equation (IC) expresses the incentive compatibility constraints. Given that the agent reports information m about k that maximizes his *ex post* payoff, they state that reporting k should be a best response for a type k agent.

Program 2 is a classical principal-agent problem à la Guesnerie and Laffont (1984) that can be solved using the differentiable approach that resumes the constraints (IC) via their associated first-order conditions when they are necessary and sufficient. Assuming a differentiable solution $(\tilde{q}, \tilde{r})$, the first-order condition of (IC) yields an expression for the marginal rent:

$$\tilde{r}_k(m) = s_k(\tilde{q}(m), m) + \Big[s_q(\tilde{q}(m), m) - s_q(\tilde{q}(m), k)\Big]\tilde{q}_k(m). \tag{A1}$$

In a truth-telling equilibrium, $m = k$, so that the term in the square brackets vanishes. If q is an increasing function, (A1) will also be sufficient to solve (IC).[14] The constraints (IC) are then replaced by (4) and

$$\tilde{r}_k(k) = s_k(\tilde{q}(k), k). \tag{A2}$$

Integrating (A2) over $[\underline{k}, k]$ yields (3). Using (2), the (IR$''$) constraints now read $\tilde{r}(k) \geq 0$ for all k. S.C. and (A2) imply that $\tilde{r}_k(k) \geq 0$ for all k so that the (IR$''$) constraint of type $\underline{k}$ subsumes all the others.

Substitute (3) into (2) and substitute the resulting expression for $t(k)$ into the maximand of Program 2. Express that maximand in terms of *ex post* virtual surplus by integrating by parts and replace the (IC) and (IR$''$) constraints by (4) and $\tilde{r}(\underline{k}) \geq 0$ to get the reduced form given in the proposition. *Q.E.D.*

Proof of Corollary 1. $\tilde{r}(\underline{k}) = 0$ is a necessary condition. Consider the relaxed program where constraint (4) is not imposed. Reg. then ensures that the maximand of Program 2 is strictly concave in $\tilde{q}(k)$ so that the first-order condition (5) associated to $\tilde{q}(k)$ characterizes a global maximum. If the solution of the relaxed program is monotonous, then it also solves Program 2. *Q.E.D.*

Proof of Lemma 1. Once k is invested, a maximal expected social surplus $\pi^*(k,1)$ may be realized. The principal's payoff is bounded below by zero: any contract that would result in a negative payoff against some type k is weakly dominated by one that stipulates $\tilde{q}(k) = \tilde{t}(k) = 0$ in that event. Consequently, the agent's payoff is bounded above by $\pi^*(k,1)$. Because returns are bounded, this bound eventually decreases below π_0 as k is increased beyond some $\hat{k}$ where $\pi^*(\hat{k},1) = \pi_0$. Hence, any $k > \hat{k}$ is now dominated by k_0. *Q.E.D.*

Proof of Lemma 2. Let "$\circ$" be the composition operator: $f \circ g(x) \equiv f(g(x))$. Stab. and S.C. ensure that the demand and supply functions are interior solutions. By definition $c_q(\hat{q}(k),k) \equiv p^*(k)$. Replace k by $k^{**}(p)$ to get $c_q(\hat{q} \circ k^{**}(p), k^{**}(p)) \equiv p$ and conclude from S.C. that $\hat{q} \circ k^{**} \equiv q^{**}$. Apply k^* to both sides: $k^* \circ \hat{q} \circ k^{**} \equiv k^* \circ q^{**} \equiv k^{**}$. Hence $k^* \circ \hat{q}$ is the identity function. *Q.E.D.*

Proof of Lemma 3. Suppose that such an equilibrium exists. The agent invests a level $k \in [k_0, \hat{k}]$ that is anticipated by the principal. Since strict gains from trade can be realized, her best response is to offer the contract (q,t) derived in Step 1 where $q = q^*(k,1)$. Similarly, the best response of the agent to any q proposed by the principal is to invest $k^*(q)$ in order to minimize costs. These two best response functions (see Figure 1) intersect only when $q^*(k,1) = [k^*]^{-1}(k) \equiv \hat{q}(k)$; that is when $k = k_1$. The agent's payoff $\pi^*(k_1, 0)$ is then strictly less than the feasible payoff π_0. Since the agent is not maximizing at the investment stage, we have a contradiction. *Q.E.D.*

Proof of Proposition 2. Suppose that the agent's equilibrium strategy F has support K and is given by (10) and (11). Notice that $\lim_{k \downarrow k^{**}(p)} F(k) = 0$ and that $\lim_{k \uparrow k^{**}(1)} F(k) = 1$, so that there are no atoms at the ends of the support. Given F, compute $(\tilde{q}, \tilde{r})$ from (5) and (3). Equation (5) is solved for all $k \in K$ if and only if $\tilde{q} = \hat{q}$. Since $\hat{q}$ is a strictly increasing function, Corollary 1 applies and $(\tilde{q}, \tilde{r})$ maximizes the principal's payoff. To show that (9) obtains, notice that if $\tilde{q} = \hat{q}$ then

$$s_k(\tilde{q}(k), k) \equiv -c_k(\hat{q}(k), k) - \pi_k(k, p).$$

By (7), $-c_k(\hat{q}(k), k) \equiv 0$. Applying (3) then yields (9).

I now verify that randomizing on K is optimal. Consider the proposed equilibrium contract in its form $(\hat{q}, \tilde{t})$. Using (1), (2) and (9),

$$\tilde{t}(k) = \pi_0 + c(\hat{q}(k), k). \tag{A3}$$

Substitute $k^*(q)$ for k in (A3)

$$\tilde{t} \circ k^*(q) \equiv \pi_0 + c(\hat{q} \circ k^*(q), k^*(q)) \equiv \pi_0 + C(q).$$

The marginal revenue of producing an additional unit is thus given by the long-run marginal cost up to $\hat{q}(k_1) = q^*(k_1, 1) = q_1$. It is discontinuous in q_1 and zero afterward. Since marginal cost is increasing, the lowest value that marginal revenue may take (beside zero) is $C_q(0) \equiv c_q(0, k_0)$. S.C. implies that $c_q(0, k_0) \geq c_q(0, k)$ for all $k \geq k_0$. In this case, increasing production as long as marginal revenue is no lesser than marginal cost $c_q(q, k)$ is a necessary condition for profit maximization. An agent that has invested $k \in K$ produces a quantity q that solves $C_q(q) = c_q(q, k)$, that is $q = \hat{q}(k)$, and realizes a profit $\tilde{t}(k) - c(\hat{q}(k), k) = \pi_0$. The short-run marginal cost curve of an agent that has invested $k < k_0$ lies everywhere above the marginal revenue curve. He produces nothing and realizes a profit $-c(0, k)$ that is no greater than π_0. The short-run marginal curve of an agent that has invested $k > k_1$ lies everywhere below the marginal revenue curve. He produces q_1 and realizes a profit $\tilde{t}(k_1) - c(q_1, k) = \pi_0 - [c(q_1, k) - C(q_1)]$ that is strictly less than π_0. It follows that K is a subset of the set of the agent's best responses to $(\hat{q}, \tilde{t})$. The agent's payoff on K is independent of k so that he is ready to randomize. The randomization may then be set arbitrarily to F. We have an equilibrium. *Q.E.D.*

Proof of Proposition 3 and of Corollary 2. Using Leibniz's rule, differentiate (10),

$$F_\eta(k) = (1 - F(k)) \left(-h(k_0) \frac{\partial k_0}{\partial \eta} + \int_{k_0}^{k} h_\eta(\kappa) d\kappa \right).$$

If k_0 increases with η and $h(k)$ decreases with η, then $F_\eta(k) \leq 0$ for all k, which is equivalent to first-order stochastic dominance (see footnote 10).

If $\phi_{k\eta} \leq 0$, then $c_{k\eta} \leq 0$. Apply the implicit function theorem on (7) to show that

$$\frac{\partial k_0}{\partial \eta} = -\frac{c_{k\eta}}{c_{kk}} \geq 0; \qquad \frac{\partial \hat{q}(k)}{\partial \eta} = -\frac{c_{k\eta}}{c_{qk}} \leq 0, \quad \forall k \in K.$$

Differentiate (11) with respect to η to obtain

$$\left[1 - c_q\right] \frac{\partial h}{\partial \eta} = \left[h c_{qq} - c_{qqk}\right] \frac{\partial \hat{q}}{\partial \eta} + \left[h c_{q\eta} - c_{qk\eta}\right], \quad \forall k \in K.$$

The first two bracketed terms are positive. When c is separable with respect to q and η, the last bracketed term on the r.h.s. vanishes, and h is positively related to $\hat{q}$, hence negatively related to η. Proposition 3 can then be applied. *Q.E.D.*

Proof of Lemma 4. Let $\tilde{\pi}^{**}$ and $\tilde{k}^{**}$ be defined from $\tilde{c}$. An agent who invests k_1 at the top of the equilibrium support gets to produce efficiently $\hat{q}(k_1) = q^*(k_1, 1)$. Differentiate (14) with respect to k, evaluate the result at $(\hat{q}(k_1), k_1)$ and use S.C. to get

$$0 \equiv -\beta\upsilon c_k(\hat{q}(k_1), k_1) = (1 - \beta + \upsilon\beta)\tilde{\pi}^*_k(k_1, 1) + \beta(1 - \upsilon)\tilde{\pi}^*_k(k_1, 0) < \tilde{\pi}^*_k(k_1, 1).$$

Since $\tilde{\pi}^*$ is strictly concave in k, the previous result $\tilde{\pi}^*_k(k_1, 1) > 0$ implies that $k_1 < \tilde{k}^{**}(1)$. For all $k < \tilde{k}^{**}(1)$, hence for all $k \in K$, we have $\tilde{\pi}^{**}_k(k, 1) > 0$. It follows that

$$\phi_{k,-\beta}(k; -\beta, \upsilon) = -\frac{\tilde{\pi}^*_k(k, 1)}{\beta^2\upsilon} < 0, \quad \forall k \in K.$$

Differentiate (14) and use S.C. to get

$$-\beta\upsilon c_k(q, k) \equiv (1 - \beta)\tilde{\pi}^*_k(k, 1) + \beta\Big[(1 - \upsilon)\tilde{\pi}^*_k(k, 0) - \upsilon\tilde{c}_k(q, k)\Big] \geq (1 - \beta)\tilde{\pi}^*_k(k, 1) + \beta\tilde{\pi}^*_k(k, 0).$$

For any $k \in K$ and any equilibrium pair $(\hat{q}(k), k)$, we have $c_k(\hat{q}(k), k) = 0$ so that

$$\phi_{k,\upsilon}(k; -\beta, \upsilon) = \frac{(1 - \beta)\tilde{\pi}^*_k(k, 1) + \beta\tilde{\pi}^*_k(k, 0)}{\beta\upsilon^2} \leq 0, \quad \forall k \in K.$$

Q.E.D.

Notes

[1] Hart and Moore (1988) have initiated a string of models where a hold-up occurs because *ex post* renegotiation is unavoidable and contracts are assumed to be incomplete. Information is symmetric between both contracting parties while a third enforcing party (the courts) stays uninformed. An appropriate design of the renegotiation process (Aghion, Dewatripont and Rey, 1994; Noldeke and Schmidt, 1995) or of the litigation process (Edlin and Reichelstein, 1996) can solve the hold-up problem in this context.

[2]In game theoretic words, this is not a "type" but a "move". Nevertheless, the (quoted) word "type" will be used for an investment level since it characterizes *ex post* the agent's payoff function like a true type does in the agency literature.

[3]There are some older examples in non-strategic competitive settings in the "quality-guaranteeing prices" literature. See Shapiro's (1986) model of a lemon market where sellers have the choice, through investment in human capital, to either sell lemons or quality services. Daughety and Reinganum (1995) have proposed a model where an endogenous "type" distribution emerges when firms play a pure strategy with respect to an R&D investment decision that involves a random outcome. The distinction should be made between models in which the informed parties play *pure* strategies that exogenously involve a random outcome and those, such as in this paper, where *different* pure strategies are played that belong to the support of a same mixed strategy. Adapted to a strategic setting, Shapiro's model would fall into the latter class while the other belongs to the former. The important difference between the two classes of models is that observability of the strategy played yields no strategic effect in the pure strategy case – only the outcome matters.

For instance, Laffont and Tirole's (1993) version of the hold-up problem under asymmetric information does not result in a mixed strategy for investment because they make the implicit assumption that it is not possible to contract after investment has taken place but prior the agent knows precisely his random production set. Furthermore, investment does not affect the support of that random set. One can show in that context that it is does not pay for the principal to induce separation of agents with respect to their investment level (which is not related to the feasible *ex post* gains to trade). The agent's payoff function then becomes strictly concave in investment and has a unique maximizer that is played in pure strategy.

[4]The value q_0 refers to Figure 1. It equals $q^{**}(0) \equiv 0$ in this section but it takes a different value in the first application of section 3. See footnote 12.

[5]The marker E in Figure 2 is used in section 3.

[6]Consider the slopes of any two short-run marginal cost curves at q such as points a and b of Figure 1. When assumption Reg. holds, the slope at point a is no lesser than the slope at point b.

[7]Fudenberg and Tirole (1990) make a similar observation.

[8]Let $y = F(k)$ and $y' = f(k)$. Then, $y' + h(k)y = h(k)$ which is a first-order linear differential equation solved by (10).

[9]Differentiability with respect to η is implicitly assumed throughout this section.

[10]Let x and y be random variables with distributions F and G. The variable x is said to *first-order stochastically dominates* y, denoted $x \succeq_{\text{FSD}} y$, if $F(z) \leq G(z)$ for all z. If $F(\cdot;\eta)$ is a family of differentiable distributions parameterized by η, then $F_\eta \leq 0$ is a sufficient condition to order these distributions. When $x \succeq_{\text{FSD}} y$, x is said to be *stochastically larger* than y in the sense that we can always find two random variables x' and y' that have the same distributions as x and y and that are such that $\text{Prob}(x' \geq y') = 1$. See Wolfstetter (1999).

[11]This monotonicity condition is not related to the usual one which specifies how h changes with k. In the standard incomplete information model, the condition $h_k \geq 0$ is sufficient (when $c_{qkk} \geq 0$) to ensure that $\tilde{q}$ satisfies the monotonicity condition (4). No such condition was required in Step 3 because S.C. ensures that the *equilibrium* $\tilde{q}$, namely $\hat{q}$, is monotonous anyway.

[12]The value q_0 in Figure 1 is not zero as in the base model because the opportunity cost function c does not satisfy the technological assumptions everywhere. Notice that c is convex but not strictly so outside $Y(\mu)$ and is continuously differentiable but not three times at the frontier of $Y(\mu)$. Nevertheless, the structure of the equilibrium is not affected in this case.

[13]Notice that $\mu = \tilde{c}_q(q_0, k_0) = C_q(q_0)$. For $q \geq q_0$, we have

$$\begin{aligned}\pi_0 + C(q) = \tilde{\pi}^{**}(\mu) + C(q) = \mu q_0 - C(q_0) + C(q) &= \mu q_0 + \int_{q_0}^{q} C_q(\theta) d\theta, \\ &\geq \mu q_0 + \int_{q_0}^{q} C_q(q_0) d\theta = \mu q_0 + \mu(q - q_0) = \mu q,\end{aligned}$$

where the equality holds only in q_0. Hence, a "type" k_0 agent is paid a price μ by the principal. This agent is indifferent between producing for the market or the principal. Proposition 2 selects an equilibrium where this agent produces only for the principal. For any other k, the principal pays more than the market and the agent produces only for the principal.

[14]See Laffont and Tirole (1993), § A1.4, for a proof.

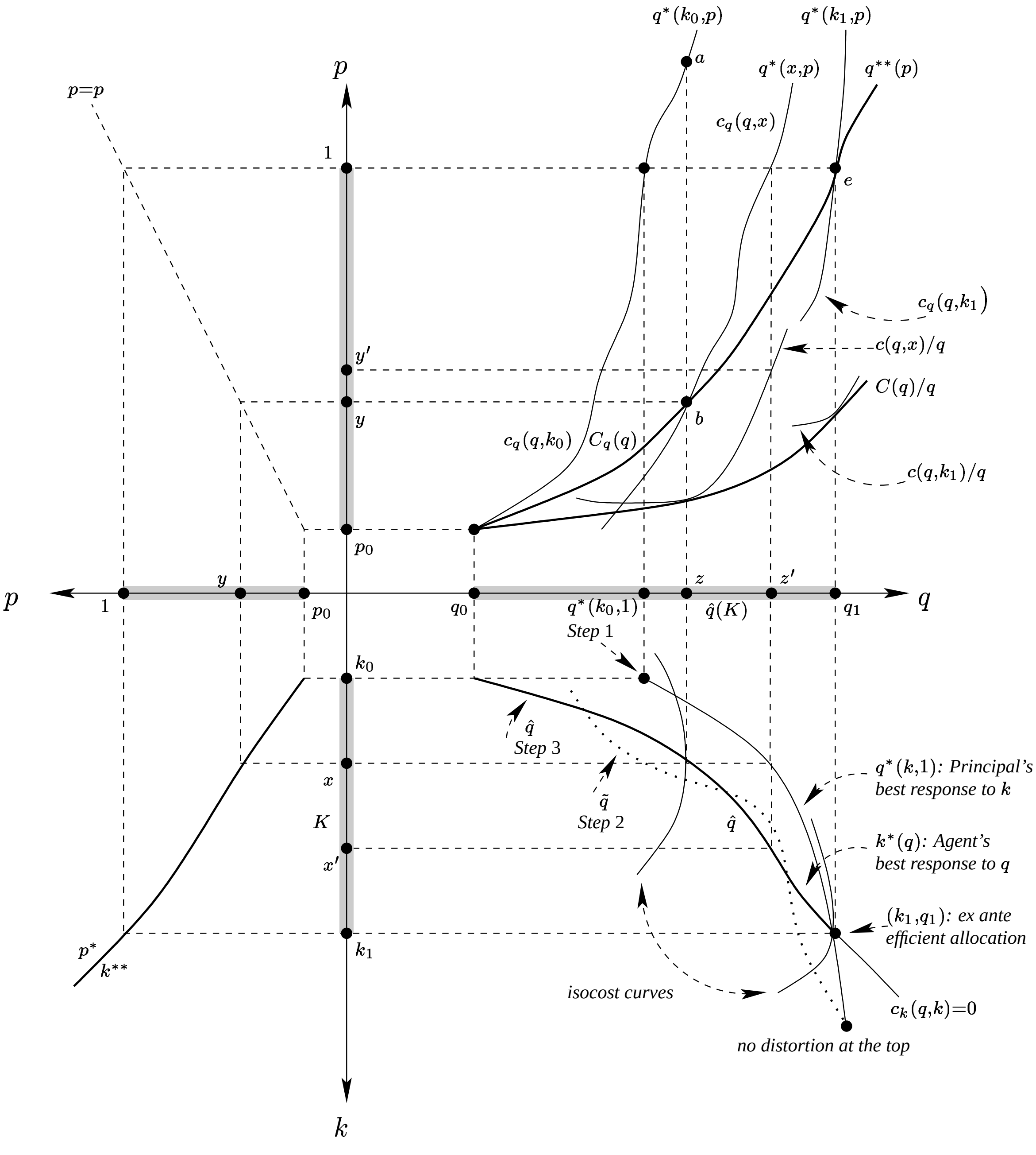

Figure 1: Costs Structure.

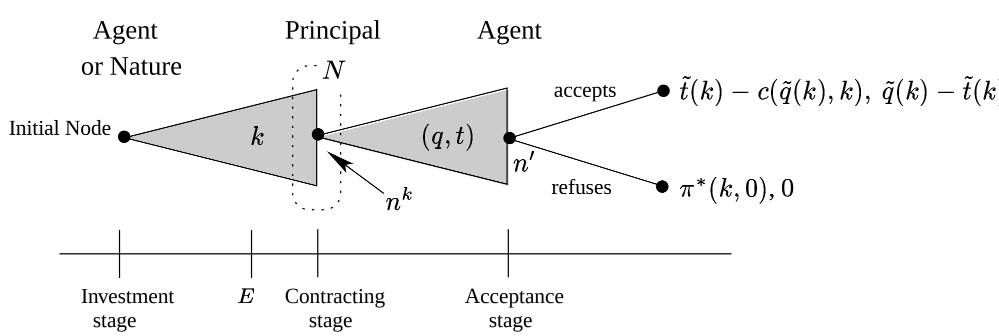

Figure 2: A sketch of the game in extensive form.