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FOR IN-KIND REDISTRIBUTION**

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DIFFERENTIAL TIME AND MONEY PRICING AS A MECHANISM FOR IN-KIND REDISTRIBUTION

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ABSTRACT. We propose a mechanism to implement the distributional goal of “specific egalitarianism”, or that allocation of a good be independent of income, but increasing in relative strength of preference or need. Governments could offer the good at multiple “outlets” that charge different money and time prices. Individuals would self-select between outlets based on time opportunity cost. We show conditions under which differential pricing achieves specific egalitarianism more efficiently than uniform public provision funded from income tax, with or without optional private purchase. Differential pricing becomes more efficient than uniform provision as 1) the relative importance of the good rises, 2) the elasticity of substitution between goods falls, 3) variation in preferences increases and 4) income inequality rises or the proportion of the poor falls.

Keywords: in-kind provision, specific egalitarianism, differential pricing

JEL Classifications: D30, D45, H31, I18

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1. Introduction

Economists have traditionally been wary of distributional concerns over private goods, or of publicly providing private goods for redistributive purposes. It has commonly been argued that distributional concerns grounded in utilitarian social welfare could be met at least cost by transferring *income* from rich to poor, and then allowing market prices to allocate resources to their most valued uses (the Second Welfare Theorem). Nevertheless, two factors have increased the attention economists have paid to in-kind provision over the past thirty years.

First, good-specific distributional concerns, particularly over items “essential to life and citizenship,” have proven remarkably robust over time (Tobin, 1970). For example, compulsory public health insurance in Canada, implemented federally in 1968, was reviewed in 2002 and justified in part on the basis that Canadians want the poor to have the same access to health care as the rich (Romanow Commission, 2002, p. xvi). The enduring popularity of the Food Stamp program in the United States has ensured its survival through welfare reform, prompting some economists to suggest that good-specific distributional concerns be taken seriously as a public policy objective (Tobin, (1970), Weitzman (1977), and Rosen (2002a, p. 175)).

Second, the incorporation of imperfect information into public economics has shown that egalitarian in-kind provision of some types of goods can actually increase utilitarian social welfare. Goods such as health insurance may not be available to all in private markets if providers cannot distinguish high and low risk individuals (Rothschild and Stiglitz (1976)). Similarly, governments may be unable to distinguish high from low ability workers, and thus face restrictions on the degree of income redistribution possible through optimally designed tax systems. For individuals with high ability may mimic those with low ability in order to avoid taxes or qualify for cash

transfers (Blackorby and Donaldson (1988), Boadway and Marchand (1995), and Blomquist and Christiansen (1995)). These authors have used unknown risk or ability to explore conditions where social welfare would be higher if certain private goods were publicly provided at a uniform level to all.

In this paper, we are agnostic as to the basis for good-specific egalitarianism, or the domain of goods to which it might apply. Instead we ask how such a distributional concern could be achieved at least cost in efficiency. The good in question could be as major as health care, or minor as access to popular national park sites, though we restrict our attention to goods for which resale is infeasible. We propose a differential pricing mechanism, where the government could make the target good available at “outlets” charging different time and money price combinations. By appropriately choosing the money price of the good across outlets, the government can ensure that individuals self-select outlets by their earnings ability. Individuals of each ability level will purchase the same quantity of the target good on average, while those who value the target good more relative to other goods will purchase more of it than those who value it less.

Of course, any use of time as an allocation device involves the waste of an otherwise valuable resource. There is then inescapably an efficiency cost to our proposed form of differential pricing. We show, however, that the cost of achieving specific egalitarianism may be less under our proposal than under more conventional instruments, such as uniform public provision funded by proportional income tax, with or without private purchase. In particular, differential pricing is likely to be more efficient than tax-funded uniform provision as 1) the relative importance of the target good in people’s utilities rises, 2) the elasticity of substitution between the target good

and other goods falls, 3) heterogeneity of preference for the target good increases and 4) income inequality rises or the proportion of the poor falls.

The layout of the paper is as follows. Section 2 provides a brief review of the literature on the distributional and efficiency aspects of queuing as an allocation mechanism. Section 3 provides a formal model of our allocation mechanism. Section 4 compares social welfare under this mechanism with tax funded uniform provision. We conclude in Section 5.

2. Equitably Inefficient Queues¹

Allocating goods that are “essential to life and citizenship” using queues is often seen as fairer than using price because time is more evenly distributed than income (Nichols et al. (1971), Barzel (1974), O’Shaughnessy (2000), and Alexeev and Leitzel (2001)). Yet allocating goods by time rather than price creates two major costs in efficiency. First, buyers who wait in line are surrendering a valuable resource, time, that unlike money does not get transferred to the seller. The opportunity cost of that lost time may be leisure, but also forgone production. Thus, widespread queuing for goods in an economy would ultimately make fewer of these goods available. Second, the time price of queuing penalizes those with a higher opportunity cost of time. When compared to pricing, queuing will thus transfer goods from some who value them more to others who value them less (Tobin (1970), Suen (1989), and O’Shaughnessy (2000)). This is why economists have generally recommended meeting distributional concerns at a general level with a tax and transfer system, and then allocating private goods by price, or congestible public services with user fees set at marginal social cost (Rosen (2002b)).

On the other hand, tax and transfer systems carry their own distortions in work disincentives (Tobin (1970) and Bucovetsky (1984)) and imperfect targeting of the truly needy vs. the opportunistic (Alexeev and Leitzel (2001)). Similarly, user fees for congestible public services may have regressive distributional effects (Nichols et al. (1971)). In response, a number of studies have compared the efficiency of alternative re-distributional instruments, such as tax/transfers, in-kind transfers, queuing, or rationing with resale (Bucovetsky (1984), Sah (1987), Blackorby and Donaldson (1988), Polterovich (1993), O'Shaughnessy (2000) and Alexeev and Leitzel (2001)). In general, when re-sale is not practical, the inefficiency of queuing must be traded-off against the inefficiency of allocating uniform quantities of a good to heterogeneous people.

Our approach begins with a key insight by Nichols et al. (1971) that if people could *choose* whether to pay by money or by time, much of the re-distributional potential of queuing could be preserved, and its inefficiency reduced. Indeed, private firms with a degree of monopoly power commonly offer goods with a variety of price / wait combinations as a form of second degree price discrimination to increase profits (Tirole (1988)). Governments could do the same with a target good, but to pursue distributional ends such as specific egalitarianism. Low wage individuals would self-select to pay partly by (low) price and partly by time, while those with a high wage would self-select to pay only by money. If wage captures the opportunity cost of time, and differences in wages reflect differences in marginal product, then the time lost in queues would have low foregone cost in wages and production. The costly and error-prone apparatus of means testing individuals would be unnecessary. Nichols et al. provided no formal model of differential pricing, but O'Shaughnessy (2000) and Alexeev and Leitzel (2001)) do when comparing social welfare under such a system

with that under conventional tax and transfer systems. Both of the latter studies assume, however, that preferences are identical across the population. They also model economies in which only a single good is produced, and thus eligible for redistribution.

Though independently derived, we formalize the differential pricing mechanism proposed by Nichols et al. and show that it can make consumption of any particular good independent of income, but dependent on relative strength of preference or need. We then illustrate conditions under which it can do this with greater efficiency than public uniform provision, with or without allowance for private purchase.

3. A Model of Redistribution through Differential Pricing

Consider an economy of N people who have constant elasticity of substitution (CES) preference orderings over leisure l , a composite commodity y , and a “target” good g whose distribution is of concern to a policy maker.² While each individual values g , some have a stronger preference (or need) for it relative to l and y than others.

$$U(1, y, g) = \left(1^\rho + y^\rho + \theta_i g^\rho\right)^{\frac{1}{\rho}} \quad i = R \text{ or } S \quad (1)$$

θ_i represents this relative strength of preference for the target good (“strong” or “regular”), where $\theta_S > \theta_R$. ρ (< 1) represents the individual’s elasticity of substitution between the target good and other goods, and could range from almost perfect flexibility ($\rho \rightarrow 1$), to Cobb Douglas ($\rho = 0$), to Leontief ($\rho \rightarrow -\infty$).

Individuals earn income from their choice of labour hours L , which they spend at competitive firms producing either Y or G .³ Workers are paid a wage equal to the value of their marginal product. We assume for simplicity that production technology is identical in the Y and G sectors, and that an individual’s marginal product is the same at both. Income inequality arises in part because of differences in taste (θ_i), but mostly

because of differences in ability. N_H of N workers have a high ability and marginal product, and so receive a high wage w_H . The remaining N_L workers ($= N - N_H$) have a low ability, marginal product, and wage w_L . Workers of high and low ability divide their labour hours between production sectors according to $L_{i,H} = L_{i,H}^G + L_{i,H}^Y$ and $L_{i,L} = L_{i,L}^G + L_{i,L}^Y$ respectively, where $i = R$ or S .

The prices individuals face are as follows. The price of leisure is a person's wage, w_j ($j = L$ or H), while the price of y is normalized to 1. Under the mechanism we propose, the *full* money and time price of the target good g at a given outlet is $P_{g,j} = w_j h + p$, where p is the money price per unit, and h is the hours of waiting time required per unit.⁴ With these prices and income, each person faces a budget constraint $w_j L = pg + y$. Individuals also face a time constraint, as they have an (identical) time endowment T that can be spent working L , in leisure ℓ , or in line (hg). As is easily shown in Appendix 1, individuals will choose the g outlet offering the lowest full price given their opportunity cost of time w_j . Conditional on this choice of outlet, an individual's problem is:

$$\begin{aligned} \text{Max}_{1,y,g} U &= \left(1^\rho + y^\rho + \theta_i g^\rho\right)^{1/\rho} \\ \text{s.t.} \quad & pg + y = w_j L, \\ & 1 + L + hg = T \quad \text{where } i=R \text{ or } S \text{ and } j = L \text{ or } H. \end{aligned} \tag{2}$$

With interior solutions for any $\rho \in (-\infty, 1)$, the corresponding demand functions are:

$$\begin{aligned} l_{i,j}^* &= \left(\frac{w_j^{\frac{\rho}{\rho-1}} T}{D_{i,j}} \right), \\ y_{i,j}^* &= \left(\frac{w_j T}{D_{i,j}} \right), \text{ and} \end{aligned} \tag{3}$$

$$g_{i,j}^* = \left(\frac{w_j P_{g,j}^{\frac{\rho}{\rho-1}} \theta_i^{\frac{1}{1-\rho}} T}{D_{i,j}} \right),$$

where $D_{i,j} = w_j^{\frac{\rho}{\rho-1}} + P_{g,j}^{\frac{\rho}{\rho-1}} \theta_i^{\frac{1}{1-\rho}} + 1$, $i=R$ or S , and $j = L$ or H .

Substituting demands into utility yields an individual's indirect utility function:

$$V_{i,j} = (l_{i,j}^{*\rho} + y_{i,j}^{*\rho} + \theta_i g_{i,j}^{*\rho})^{1/\rho} = (1 + P_{g,j}^{\frac{\rho}{\rho-1}} \theta_i^{\frac{1}{1-\rho}} w_j^{\frac{\rho}{\rho-1}} + w_j^{\frac{\rho}{\rho-1}})^{\frac{1-\rho}{\rho}} T$$

$$i=R \text{ or } S, \text{ and } j = L \text{ or } H \quad (4)$$

Note from (3) or (4) that individuals with a given wage are indifferent as to the composition of g 's full price, $P_{g,j}$, between the time (h) and money (p) components.

Firms operate in perfectly competitive markets, producing either Y or G by employing workers with both ability levels and tastes using constant returns technology. With constant (exogenous) marginal products and price of y set to one, wages adjust to equal marginal product, and so the price of both leisure and y are determined. With identical technology in the G sector as in Y , the competitive price of g without re-distribution would equal that of y , 1.

Under our mechanism, however, the policy maker would purchase all G produced by firms at cost, and sell it at a higher money price $p_H > 1$ at one outlet, and at a lower money price p_L at a second outlet.⁵ Note that the low price p_L could even be set negative, functioning as a unit subsidy funded from the tax at the high price outlet. Separation of buyers by wage will occur if members of each wage group find the full price at their own outlet lower than at the alternative, given their opportunity cost of time. By choosing the money price at each outlet, the policy maker could seek to maximize social welfare subject to "specific egalitarianism," or that

1. consumption of g be equalized across the average low and high wage person, yet
2. individuals with a strong preference or need for g receive as much or more of it than individuals with a relatively weak preference, regardless of income.

We have set the requirement that consumption of g be weakly rather than strictly increasing in strength of preference, in order to permit comparison of our mechanism with more conventional redistributive instruments such as uniform provision. As we shall see, differential pricing can satisfy the stricter requirement.

To formally model the policy maker's problem, we describe finally the distribution of strong and regular tastes for g among ability groups. We denote by $0 \leq s \leq 1$ the proportion of individuals with a strong preference θ_s for g , and assume initially that it is the same among low and high wage individuals. We discuss the relaxation of this assumption in the final section of the paper.

The policy maker's formal problem is:

$$\underset{p_H, h_H, p_L, h_L}{Max} \quad SW = (1-s)N_L V_{R,L} + sN_L V_{S,L} + (1-s)N_H V_{R,H} + sN_H V_{S,H} \quad (5)$$

subject to

$$(1-s)g_{R,L}^* + sg_{S,L}^* = (1-s)g_{R,H}^* + sg_{S,H}^* \quad (6a)$$

$$g_{S,j}^* \geq g_{R,j}^*, \quad g_{S,L}^* \geq g_{R,H}^*, \quad g_{S,H}^* \geq g_{R,L}^* \quad (6b)$$

$$(p_H - 1)N_H = (1 - p_L)N_L \quad (7)$$

$$P_{g,H} \leq w_H h_L + p_L \quad (8a)$$

$$P_{g,L} \leq w_L h_H + p_H \quad (8b)$$

$$(1-s)N_L g_{R,L}^* + sN_L g_{S,L}^* + (1-s)N_H g_{R,H}^* + sN_H g_{S,H}^* =$$

$$(1-s)N_L w_L L_{R,L}^G + sN_L w_L L_{S,L}^G + (1-s)N_H w_H L_{R,H}^G + sN_H w_H L_{S,H}^G \quad (9)$$

Constraints (6a) and (6b) define specific egalitarianism as described above. Constraint (7) is a reduced form of the policy maker's balanced budget condition, where the right- and left hand side of (6a) have cancelled. Equations (8a) and (8b) are incentive compatibility constraints for low and high ability individuals to remain at their own outlets. Finally, (9) is the economy's resource constraint for the supply and demand for g .⁶

We characterize the solution to this problem in steps. First, the expression for p_H in the balanced budget condition (7) can be substituted into the specific egalitarianism constraint (6a). The money price p_L that satisfies (6a) can be expressed as an implicit function of h_L and h_H , as then from (7) can p_H . $P_{g,H}$ and $P_{g,L}$ in social welfare (5) can then be expressed using these implicit functions of h_L and h_H . Compacting the notation yields

$$SW = \sum_{h_H, h_L} \sum_{i=R,S} \sum_{j=L,H} (1 + (P_{g,j}(h_L, h_H, s, w_j, T, \theta_i, \rho, N_j))^{\frac{\rho}{\rho-1}} \theta_i^{\frac{1}{1-\rho}} w_j^{\frac{\rho}{\rho-1}} + w_j^{\frac{\rho}{1-\rho}})^{\frac{1-\rho}{\rho}} T \quad (10)$$

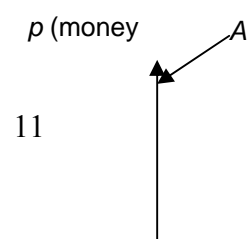
Next, we show that social welfare as expressed in (10) is unambiguously falling in both high and low wage queuing times. So a planner will maximize (10) by setting both h_H and h_L as low as possible, subject to meeting separation constraints (8a) and (8b). First, given some optimal queuing time at the high ability outlet, h_H^* , we claim SW will fall in h_L . This is because as h_L rises, the full price at the low ability outlet must also rise. Why? $P_{g,L}$ could remain constant or fall only if the money price p_L fell, which from budget balance (7) would require the policy maker to raise p_H . From (6a), a

higher p_H given h_H^* would reduce demand for g among those with high ability. To satisfy egalitarianism, this would have to be matched with a drop in demand by those with low ability, which cannot occur if $P_{g,L}$ has fallen or remained constant. Thus, a rise in h_L must raise $P_{g,L}$, lower the indirect utility of low ability individuals, and so lower social welfare. h_L^* will thus be set at the minimum level consistent with keeping the low wage outlet unattractive to those with a high wage, or from (8a), $h_L = (P_{g,H} - p_L)/w_H$. Unfortunately, we can show from the numerator that this time price will have to be positive. This follows because $P_{g,H}$ must exceed $P_{g,L}$ for (6a) to be satisfied, since it can be shown that $\partial g_{i,j} / \partial w_j > 0$ and $\partial g_{i,j} / \partial P_{g,j} < 0$. Yet $P_{g,H}$ cannot exceed $P_{g,L}$ if $h_L \leq 0$, since setting it minimally implies $P_{g,H} = w_H h_L + p_L \leq w_L h_L + p_L = P_{g,L}$.

Conversely, given an optimal h_L^* , social welfare will also be falling in h_H , for the analogous reason that when h_H rises, the full price $P_{g,H}$ must also rise, lowering the indirect utility of high wage individuals. But unlike for low wage individuals, there need be no queuing at the high wage outlet. Setting $h_H = 0$ will still satisfy (8b), or that $p_H \geq P_{g,L}$, because (8a) becomes $w_H h_L^* + p_L = P_{g,H} = p_H$, which exceeds $w_L h_L^* + p_L = P_{g,L}$. In short, $h_H^* = 0$, and h_L^* can be re-expressed as $(p_H - p_L)/w_H$.

Substituting these requirements for optimal queuing times into the resource constraint (9), the technical conditions are satisfied to ensure there exist unique money prices p_H and p_L that the policy maker can choose to ensure a feasible allocation.⁷

The reader may gain intuition concerning the planner's optimal pricing policy $\{p_H^*, h_H^*, p_L^*, h_L^*\}$ from Figure 1. The figure illustrates the time/money price pairs for g that would yield the same full price for high wage individuals, including the socially optimal pair at point A , where $h_H^* = 0$ and p_H^* . It also shows the time/money price pairs that would yield low wage individuals the same full price, including the optimal pair at



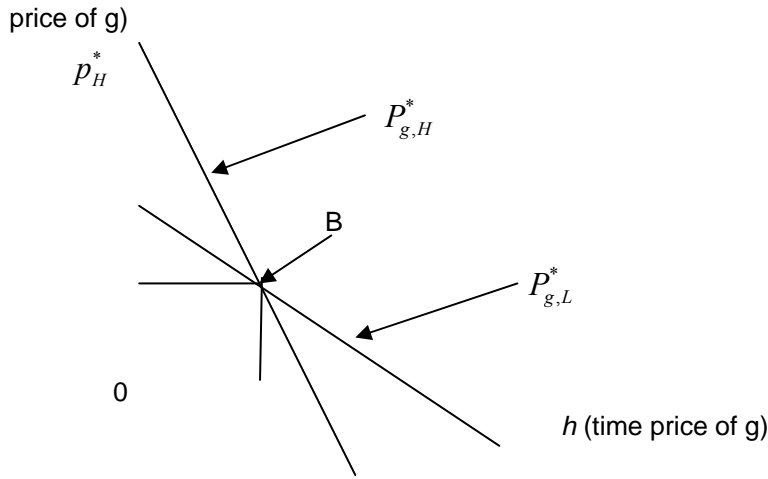


Figure 1. Illustration of efficient separating pricing for two income levels

point B , h_L^* and p_L^* .⁸ The optimal time/money prices at points A and B are incentive compatible, in that no individual from either wage group would be better off by going to the outlet targeted to the other. Yet inefficiency is minimised by making the least use of time pricing possible.

While we have constructed our optimal policy to make consumption of g equalized between wage groups (6a), we have not yet shown that it will be increasing in relative strength of preference, or (6b). For a given wage group at a given outlet, it can easily be shown that g will be increasing in θ . Differentiating demand at an outlet,

$$\frac{\partial g_{i,j}}{\partial \theta_i} = \frac{1}{1-\rho} \frac{\theta_i^{\frac{\rho}{1-\rho}} w_j^{\frac{1}{1-\rho}} P_{g,j}^{\rho-1} T(1+w_j^{\rho-1})}{D_{i,j}^2} > 0 \quad (11)$$

Thus, within an outlet, $g_{R,j}^* < g_{S,j}^*$. Furthermore, since the pricing solution must satisfy (6a), average demand must be equalized across outlets, say at g^* . It follows that at the low wage outlet, for any distribution of s , $g_{R,L}^* < g^* < g_{S,L}^*$. Similarly, for any distribution of s at the high wage outlet, $g_{R,H}^* < g^* < g_{S,H}^*$. Combining the two, an individual with a high need for g will always purchase an above average amount at his

outlet, while an individual with a low need for g will always purchase a below average amount. Or formally, $g_{S,L}^* > g_{R,H}^*$ and $g_{S,H}^* > g_{R,L}^*$, so that (6b) is strictly and therefore weakly satisfied .

Once the policy maker has determined the optimal prices, the total quantity of G that will be sold at each outlet follows easily, and thus the total quantity that must be purchased from the G production sector. G will need to be allocated to each outlet in proportion to the distribution of ability types in the population. Our assumption of identical, constant returns technology in the G and Y production sectors means that the precise allocation of individuals' labour hours across the two sectors is under-identified.⁹ See Appendix 2 for details.

Our characterization of the differential pricing mechanism for in-kind redistribution is complete. By commanding exclusive purchase rights over a target good and setting a high money price at one outlet, and a low money price with queue at another, a policy maker can ensure that consumption of the good is equalized across wage groups, while increasing in relative need. This is done at least cost to efficiency by setting the money price at the low wage outlet high enough that the resulting queue is the minimum necessary to keep high wage earners out of the outlet. The money price at the high wage outlet is set high enough to clear that outlet without queuing.

To evaluate the cost in efficiency of achieving specific egalitarianism under differential pricing, we consider next two plausible alternative mechanisms. Each mechanism can (just) achieve specific egalitarianism as we have defined it, without requiring the government to identify the ability status of a given individual. The first is uniform public provision of the target good, funded through a proportional income tax, with additional private purchase of g allowed. The second mechanism is uniform provision without private purchase, again financed through proportional taxation. We

then identify the conditions under which our mechanism can achieve the policy maker's distributional goal more efficiently than the better of the alternatives.

4. Uniform Provision with Proportional Income Tax

Suppose that instead of differential pricing, the policy maker were to provide every individual with a uniform quantity of g , funded by a proportional income tax t on labour income. This mechanism would additionally require the policy maker to know each individual's total labour income, but not their individual ability or wage. Along with public provision, individuals could be free to purchase additional units of g privately (as with school vouchers), or not (as with public health insurance in Canada). As we shall see, uniform provision funded by income tax replaces the inefficiency of queuing time and g price distortions with labour/leisure price distortions and under- or over-provision of the target good. It will turn out that specific egalitarianism comes at a greater cost in efficiency when private purchase *is* allowed, because the government must provide sufficient g to crowd out all private purchases. Our final comparison will be therefore between no-purchase uniform provision and differential pricing.

4.1 Uniform Provision with Private Purchase

Returning to our N -person economy, assume that the distribution of preferences and technology are the same as before. The government will provide \hat{g} units of the target good to everyone. An individual with θ_i relative strength of preference for g and with wage w_j faces the problem:

$$\begin{aligned} \underset{1, y, \hat{g}}{\text{Max}} U &= \left(1^\rho + y^\rho + \theta_i (\hat{g} + \hat{g})^\rho \right)^{1/\rho} \\ \text{s.t.} \quad y + \hat{g} &= (1-t)w_j L, \\ 1 + L &= T \quad \text{where } i=R \text{ or } S \text{ and } j = L \text{ or } H. \end{aligned} \tag{12}$$

Of relevance here, an individual's supplementary demand function for \hat{g} on top of g is

$$\hat{g}_{i,j}^* = \left(\frac{(1-t)w_j T - g \theta_i^{\frac{1}{\rho-1}} (1 + ((1-t)w_j)^{\frac{\rho}{\rho-1}})}{\theta_i^{\frac{1}{\rho-1}} (1 + ((1-t)w_j)^{\frac{\rho}{\rho-1}}) + 1} \right) \quad i=R \text{ or } S \text{ and } j = L \text{ or } H. \quad (13)$$

Note from (13) that there is a quantity of g that would just crowd out the individual's private purchase of \hat{g} . It can easily be shown that this quantity is rising in both w and θ , or that $\partial \hat{g} / \partial w_j > 0$ and $\partial \hat{g} / \partial \theta_i > 0$. As we show in Appendix 3, a policy maker wishing to ensure specific egalitarianism in g will find it necessary to set the level of uniform provision at the "highest common denominator," or at the level desired by an individual with the highest wage and strength of preference for the target good. Otherwise, private purchases by those with higher incomes will prevent the equalization of average consumption across wage groups. From the numerator of (13), uniform provision must then satisfy

$$g \geq \left(\frac{(1-t)w_H \theta_S^{\frac{1}{1-\rho}} T}{(1 + ((1-t)w_H)^{\frac{\rho}{\rho-1}})} \right) \quad (14)$$

Note that by providing g at a uniform level and crowding out private purchase, the policy maker only *weakly* satisfies specific egalitarianism. Consumption across income groups will be equalized, but individuals with a high need for the good will receive the same amount as those with a low need, rather than more.

With no private purchase of g , an individual's remaining demand functions are

$$l_{i,j}^* = \left(\frac{((1-t)w_j)^{\frac{\rho}{\rho-1}} T}{1 + ((1-t)w_j)^{\frac{\rho}{\rho-1}}} \right),$$

$$y_{i,j}^* = \left(\frac{(1-t)w_j T}{1 + ((1-t)w_j)^{\frac{\rho}{\rho-1}}} \right). \quad (15)$$

With crowding out, the leisure and consumption choices of individuals become independent of their strength of preference for g . Substituting these demands functions into utility yields corresponding indirect utilities, $V_{i,j} = (l_{i,j}^{*\rho} + y_{i,j}^{*\rho} + \theta_i g_{i,j}^{\rho})^{1/\rho}$.

The policy maker must choose a tax rate and uniform provision of g to solve:

$$\underset{t, g}{\text{Max}} SW = (1-s)N_L V_{R,L} + sN_L V_{S,L} + (1-s)N_H V_{R,H} + sN_H V_{S,H} \quad (16)$$

subject to

$$g \geq \left(\frac{(1-t)w_H \theta_S^{\frac{1}{1-\rho}} T}{(1 + ((1-t)w_H)^{\frac{\rho}{\rho-1}})} \right) \quad (17)$$

$$N_L t w_L L_L + N_H t w_H L_H = (N_L + N_H) g \quad (18)$$

(17) is necessary for specific egalitarianism and (18) is the government budget balance. With private markets operating only for y and all prices but t given, a resource constraint is redundant, and the exact allocation of labour across G and Y production is under-identified as before. Intuitively, the solution to the policy maker's problem will be an optimal tax rate t^* that supports the minimum level of uniform provision g^* that just crowds out all private purchase, or the (t^*, g^*) that satisfy (17) and (18) with equality.

4.2 Uniform Provision without Private Purchase

Achieving specific egalitarianism comes at a high cost in efficiency under the above mechanism, because the government must impose on everyone the tax/provision trade-off that would be chosen by the keenest, wealthiest individual. This inefficiency could be reduced if the policy maker could restrict private purchase of g , and provide

instead an “average” amount \bar{g} to all at a lower tax rate. With no possible purchase of g , an individual with θ_i and w_j would face the problem:

$$\begin{aligned} \text{Max}_{1,y} U &= (1^\rho + y^\rho + \theta_i \bar{g}^\rho)^{1/\rho} \\ \text{s.t.} \quad y &= (1-t)w_j L, \\ 1 + L &= T \quad \text{where } i=R \text{ or } S \text{ and } j = L \text{ or } H. \end{aligned} \quad (19)$$

The individual’s demand functions for leisure and the composite commodity would be identical to those in (15), leading again to indirect utility $V_{i,j} = (1_{i,j}^{\rho} + y_{i,j}^{\rho} + \theta_i \bar{g}_{i,j}^{\rho})^{1/\rho}$. Now, since everyone must consume an identical g , the policy maker will automatically (weakly) satisfy specific egalitarianism, whatever the level of provision. The policy maker will choose t and \bar{g} to solve:

$$\text{Max}_{t,\bar{g}} SW = (1-s)N_L V_{R,L} + sN_L V_{S,L} + (1-s)N_H V_{R,H} + sN_H V_{S,H} \quad (20)$$

subject to

$$N_L t w_L L_L + N_H t w_H L_H = (N_L + N_H) \bar{g} \quad (21)$$

The technical conditions are satisfied to ensure that one or more (\bar{g}^*, t^*) pairs exist that solve this problem, and can be compared to find a global maximum.¹⁰ With no binding distributional constraint corresponding to (17), the social welfare resulting under this pair will be at least as high as that where private purchase was allowed. Intuitively, the policy maker will choose the g/t tradeoff that would be chosen by the average person, weighted by the distribution of wage and preference strengths in the population. Because social welfare will be at least as high without private purchase, we shall concentrate on this case for the purpose of comparison with differential pricing.

4.3 Comparing Policies

Ideally, we would like to make a global comparison of social welfare when specific egalitarianism is achieved using differential pricing vs. uniform provision

without private purchase. That is, we would like to compare

$$\sum_{i=R,S} \sum_{j=L,H} N_{i,j} V_{i,j}(P_{g,j}^*) \quad \text{and} \quad \sum_{i=R,S} \sum_{j=L,H} N_{i,j} V_{i,j}(\bar{g}^*, t^*). \quad (22)$$

Such a comparison is complicated by the fact that we cannot in general derive closed form solutions for policy variables. We can, however, derive closed form solutions and make comparisons when the elasticity of substitution between goods, ρ , is zero (Cobb-Douglas). We then rely on simulations to compare policies for other values of ρ .

As ρ approaches zero, an individual's preferences converge to

$$U(1, y, g) = 1^{\frac{1}{2+\theta_i}} y^{\frac{1}{2+\theta_i}} g^{\frac{\theta_i}{2+\theta_i}} \quad \text{for } i = R \text{ or } S \quad (23)$$

Under differential pricing, the individual's demand functions from problem (2) become:

$$l_{i,j}^* = \left(\frac{T}{2+\theta_i} \right), \quad y_{i,j}^* = \left(\frac{w_j T}{2+\theta_i} \right), \quad \text{and} \quad g_{i,j}^* = \left(\frac{\theta_i w_j T}{P_{g,j} (2+\theta_i)} \right). \quad (24)$$

There are closed form solutions for the $P_{g,j}$ in (24), which take the simple form $P_{g,H} = p_H = N/N_H$, and $P_{g,L} = w_L h_L + 0 = w_L(p_H - 0)/w_H = w_L N/w_H N_H$. Note that with a Cobb-Douglas degree of substitution, the low wage outlet will charge a zero price and rely completely on queuing to deter high wage individuals.¹¹

In contrast, under uniform provision the policy maker's optimal choice of \bar{g} and t become:

$$\bar{g}^* = \left(\frac{\bar{\theta} \bar{w} T}{2(1+\bar{\theta})} \right), \quad t^* = \frac{\bar{\theta}}{1+\bar{\theta}}. \quad (25)$$

where \bar{w} and $\bar{\theta}$ are the weighted averages $[(N_L/N)w_L + (N_H/N)w_H]$ and $(1-s)\theta_R + s\theta_S$, respectively. The individual's after-tax demand functions from (19) become

$$l_{i,j}^* = \left(\frac{T}{2} \right), \quad y_{i,j}^* = \left(\frac{w_j T}{2(1+\bar{\theta})} \right) \quad (26)$$

We can now identify the variables that determine which policy achieves higher social welfare.

4.3.1 The Importance of the Target Good

The first variable of interest is θ , the relative weight that individuals place on the target good relative to other goods. To simplify the comparison, we assume initially that preferences for g are homogeneous at θ , and later consider the effect of a mean preserving spread. As we show in Appendix 4, differential pricing will achieve specific egalitarianism more efficiently than uniform provision if and only if θ and wage disparity are related as follows:

$$\sum_{j=L,H} N_j V_j(P_{g,j}^*) \geq \sum_{j=L,H} N_j V_j(\bar{g}^*, t^*) \Leftrightarrow \frac{w_H}{w_L} \geq \frac{N_L / N_H}{\left(\frac{2}{2+\theta}\right)^{\frac{2+\theta}{\theta}} (1+\theta)^{\frac{1+\theta}{\theta}} - 1} \quad (27)$$

As proven in the Appendix, for a given ratio of high to low wages, the right hand side of the inequality in (27) is falling in θ . Thus, for a given wage disparity, differential pricing will generate higher social welfare than uniform provision if θ is sufficiently high. That is, there will exist a θ^* at which the policies yield equal welfare, and above which differential pricing will dominate. Intuitively, this is because the tax rate needed to support uniform provision rises in θ more rapidly than do changes in the relative price of g across outlets. As a result, as θ rises, uniform provision creates more substantial distortions to labour supply and consumption decisions than differential pricing when compared against a benchmark of no redistribution.

4.3.2 Disparities in Wage and the Proportion of Low Ability Individuals

The second two variables of interest are the degree of wage disparity and the proportion of low ability individuals in the population. Returning to (26), it is clear that differential pricing becomes more efficient than uniform provision as the ratio of high

to low wage increases, or as the proportion of low ability individuals falls, all else constant. Intuitively, less queuing time and target good price distortions are required to keep high wage individuals out of the low wage outlet as relative wage differentials grow, or as fewer individuals need to be subsidized, thus raising the relative efficiency of differential pricing to uniform provision.

4.3.3 Disparities in Preference

The next variable of interest is the degree of variation in taste for g relative to other goods. As we show in Appendix 5, when $\rho = 0$ any mean preserving spread in θ_R and θ_S around θ^* makes differential pricing more efficient than uniform provision. Intuitively, preference heterogeneity favours differential pricing over uniform provision because the importance of allowing unequal consumption of g grows. Under uniform provision heterogeneous individuals must pay the same tax, receive the same g , and so choose identical labour/leisure and composite consumption.

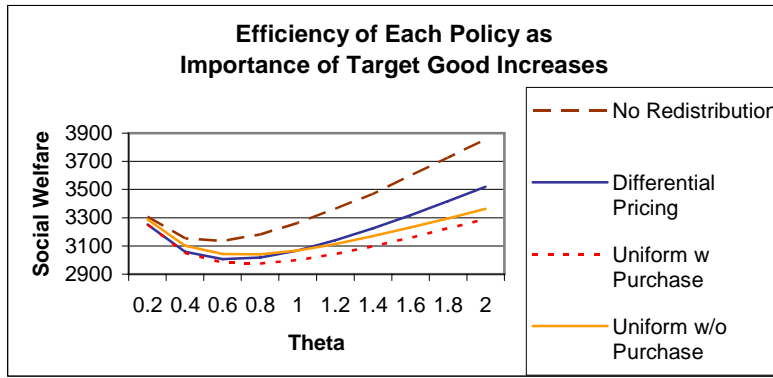
4.3.4 The Elasticity of Substitution Between Goods

Our final variable of interest is the elasticity of substitution that individuals hold between goods. We cannot make general welfare comparisons between policies for values of ρ other than zero, because we cannot derive closed form solutions for the optimal policy instruments. However, simulations under diverse parameters show that as individuals grow less willing to substitute other goods for the target good, differential pricing yields higher social welfare than uniform provision. Our partial intuition for this is as follows. Under a high elasticity of substitution, low wage individuals take far more leisure under tax-funded uniform provision than differential pricing, because income tax discourages work effort, and because they need not spend time queuing for g . With little difference in g or y consumption between mechanisms, the poor are thus better off under uniform provision. As the elasticity of substitution

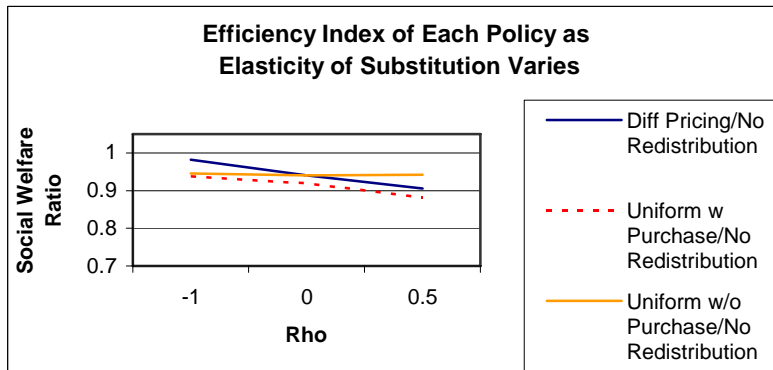
drops, however, the poor lose the “leisure premium” under uniform provision, and receive noticeably less g . This is because the policy maker must offer a negative price (subsidy) at the low income outlet as $\rho \rightarrow -\infty$ to compensate low income individuals for the long waiting time required to keep high income individuals out of the outlet. These subsidies encourage the poor to consume more g and leisure, while still purchasing y . The poor thus become better off under differential pricing.

High wage individuals face a reverse welfare ranking. Under a high elasticity of substitution, they receive a “ y premium” under differential pricing, with little difference in the other goods. This is because differential pricing raises the price of g , providing substitution incentives towards y , and because the income tax under uniform provision discourages labour that makes y affordable. The rich thus prefer differential pricing when ρ is high. Once ρ drops, however, the rich lose the “ y premium” under differential pricing, and purchase more g . It appears that the large price differential for g set by the policy maker to ensure separation as ρ falls is not sufficient to turn the rich from purchasing g to purchasing y . Thus the rich become better off under uniform provision. Despite the asymmetry in welfare rankings for the rich and poor, overall social welfare rankings of the policies track those for the poor, because of a diminishing marginal utility of consumption. Thus differential pricing becomes socially preferable as the elasticity of substitution falls.

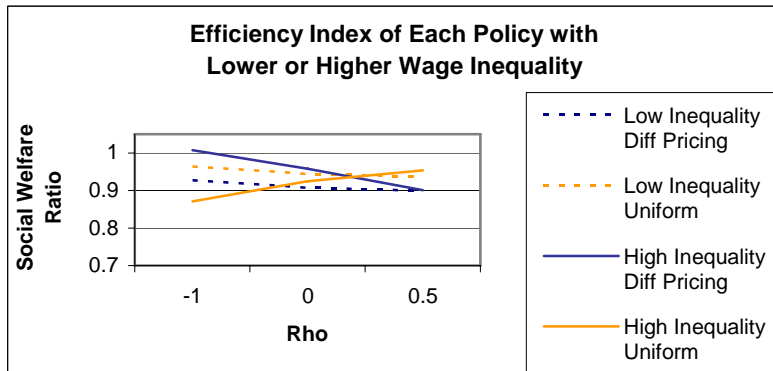
We illustrate the preceding results in simulations in Figure 2. In window (a), we show that a high value of (uniform) θ favours differential pricing. In the simulation we have selected a wage ratio (5.4 to 1) that makes the two policies equivalent in welfare at $\theta^* = 1$. In window (b), we illustrate how changing people’s elasticity of substitution between goods away from $\rho = 0$ affects the relative efficiency of the two policies. The results are illustrated with homogenous preferences, a wage ratio of 5.4



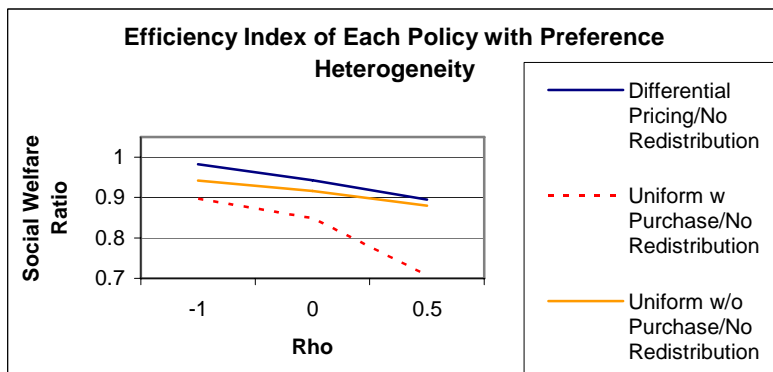
(a)



(b)



(c)



(d)

Figure 2: Simulations for $N_H = N_L = 100$, $s_L = s_H = .5$, $T = 24$, $w_L = 1$, $w_H = 5.4$, $\theta^* = 1$.

to 1, and value of θ that makes the policies welfare equivalent when $\rho = 0$. A low value of ρ favours differential pricing.

As window (c) illustrates, an increase in the wage disparity ratio from 5.4 to 15 increases the range in ρ over which differential pricing dominates uniform provision, while a decrease from 5.4 to 3 reduces it.¹² Finally, window (d) illustrates that the introduction of mean-preserving heterogeneity in relative taste for g , from $\theta^* = 1$ to $\theta_R = .5$ and $\theta_S = 1.5$, also increases the range in ρ over which differential pricing dominates uniform provision.

5. Discussion and Conclusion

In this paper, we have considered how a policy maker could achieve specific egalitarianism, making the consumption of a good “essential to life or citizenship” independent of income, but increasing in relative strength of preference or need. We have assumed that good in question cannot feasibly be on-sold, and that the policy maker’s information is limited to the distribution of wages and preference strengths in the population, and not the earnings ability of any individual.¹³ The policy maker would be exclusive purchaser of the good of interest from competitive producers, and then make it available at outlets charging different money and time prices. A below-cost money price at one outlet would be accompanied by a positive time price, which would be set just high enough to make high wage individuals better off purchasing the good at the higher price outlet with no wait. These prices could be set to ensure that consumption of the target good was equalized across wage groups, while within wage groups, those who valued the good more would purchase more of it.

Redistribution by differential pricing carries the efficiency cost of alterations to relative prices, and time lost in queues that is not transferred to sellers. However, we

show that this efficiency cost may be less than that under more conventional policies, such as uniform provision financed by proportional income taxation, with or without private purchase. For income tax also distorts relative prices, and uniform provision ignores differences in relative preferences for the target good. Allowing optional private purchase to accompany uniform provision actually makes specific egalitarianism more costly to achieve, because the policy maker must tax and spend enough to crowd out all private purchases of the target good.

Even without private purchase, we find that uniform provision is likely to be a more costly way to achieve specific egalitarianism than differential pricing as 1) the relative importance of the target good rises, 2) the elasticity of substitution between the target good and other goods falls, 3) heterogeneity of preference for the target good rises, and 4) wage inequality increases or the proportion of the poor falls. Furthermore, uniform provision satisfies specific egalitarianism in letter but not in spirit, as consumption of the target good does not strictly increase in strength of need.

We note that real world examples of differential pricing, whether in health care, highway and ferry tolls, hiking permits, postal services, or immigration processing offer at most a few price/time combinations. This despite the fact that incomes (and abilities) follow a wide distribution. Nonetheless, with judicious use, even a few money/time price combinations will greatly diminish the disparity of income of individuals per outlet, and thus the inequality of consumption caused by such disparity.

Our proposal suffers from several limitations. First, as Nichols et al. (1971) observed, the existence of non-labour income uncorrelated with wage raises the possibility that, e.g., wealthy retirees might choose outlets targeted to the poor. Second, the static nature of our model precludes its application to goods whose relative value to an individual would depreciate during the delay of optimal queuing time, such as acute

surgery. Perhaps most importantly, as mentioned earlier, our mechanism was modelled with the restrictive assumption that the distribution of preferences for the target good was identical across income groups. Our mechanism still functions when the distribution of tastes diverges by income, but two problems emerge as this divergence grows. First, the distributional requirement that consumption be equalized across income groups (6a) will begin to penalize individuals in income groups with a higher proportion of strong preference for the good. Adjusting the equalization requirement to account for each income group's relative s could address this problem, but may jeopardise the second distributional requirement (6b) that consumption of g be non-decreasing in relative strength of preference, regardless of income. A second problem as s diverges across income groups is that the existence of feasible prices with a non-negative queuing time for low wage individuals can no longer be guaranteed. In particular, as the disparity in s across wage groups increases, our mechanism's tolerance for extreme disparities between strong and weak preference or relative size of wage groups is reduced.¹⁴

With these caveats in mind, we have provided a mechanism that can achieve specific egalitarianism without compulsory queues or income or ability tests, while respecting differences in people's relative preferences or needs.

Appendix 1

Proof that Individuals are Best Off Choosing the Outlet with the Lowest Full Price.

We claim that an individual will choose the target good outlet that offers the lowest full price given his wage. (He adjusts his time allocation between work and queuing accordingly).

Proof: from (4), an individual's indirect utility function is

$$V_{i,j} = (1^{*\rho} + y^{*\rho} + \theta_i g^{*\rho})^{1/\rho} = \left(1 + P_{g,j} \frac{\rho}{\rho-1} \theta_i^{1-\rho} w_j \frac{\rho}{\rho-1} + w_j \frac{\rho}{\rho-1} \frac{1-\rho}{\rho} T\right)^{\rho} T$$

where $i = R$ or S , and $j = L$ or H (A.1)

It follows that for any $\rho \in (-\infty, 1)$

$$V_{i,j} \Big|_{\hat{P}_{g,j}} > V_{i,j} \Big|_{\hat{P}_{g,j}^0} \text{ if and only if } \hat{P}_{g,j} < \hat{P}_{g,j}^0 \text{ and vice versa.}$$

Appendix 2

Proof that the allocation of labour hours across production sectors is under-identified.

Define by $\alpha_{i,j}$ the proportion of individual i,j 's total chosen labour hours devoted to production in the Y sector. For any arbitrary set of $\alpha_{i,j}$, total production of G and Y is given by

$$\sum_{i=R,S} \sum_{j=L,H} N_{i,j} w_j (1 - \alpha_{i,j}) L_{i,j} + \sum_{i=R,S} \sum_{j=L,H} N_{i,j} w_j \alpha_{i,j} L_{i,j} = \sum_{i=R,S} \sum_{j=L,H} N_{i,j} w_j L_{i,j} \quad (\text{A.2})$$

$$\begin{aligned} &= \sum_{i=R,S} \sum_{j=L,H} N_{i,j} w_j (T - 1_{i,j} - h_{i,j} g_{i,j}) \\ &= \sum_{i=R,S} \sum_{j=L,H} N_{i,j} w_j \left(\frac{(P_{g,j} \frac{\rho}{\rho-1} \theta_i^{1-\rho} + 1 - P_{g,j} \frac{\rho}{\rho-1} \theta_i^{1-\rho} h_{i,j} w_j) T}{w_j \frac{\rho}{\rho-1} + P_{g,j} \frac{\rho}{\rho-1} \theta_i^{1-\rho} + 1} \right) \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned}
&= \sum_{i=R,S} \sum_{j=L,H} N_{i,j} [g_{i,j} p_j + y_{i,j}] = N_H p_H [(1-s)g_{R,H} + s g_{S,H}] + N_L p_L [(1-s)g_{R,L} + s g_{S,L}] \\
&\quad + N_H [(1-s)y_{R,H} + s y_{S,H}] + N_L [(1-s)y_{R,L} + s y_{S,L}] \tag{A.4}
\end{aligned}$$

With specific egalitarianism achieved (6a), this can be written as

$$[N_H p_H + N_L p_L] g^* (\equiv (1-s)g_{R,j}^* + s g_{S,j}^*) + \sum_{i=R,S} \sum_{j=L,H} N_{i,j} y_{i,j}.$$

Substituting in $p_H = \frac{N}{N_H} - \frac{N_L}{N_H} p_L$ from (7),

$$= [N_H + N_L] g^* + \sum_{i=R,S} \sum_{j=L,H} N_{i,j} y_{i,j} = \sum_{i=R,S} \sum_{j=L,H} N_{i,j} g_{i,j} + \sum_{i=R,S} \sum_{j=L,H} N_{i,j} y_{i,j}. \tag{A.5}$$

Appendix 3

Proof that for the policy maker to achieve specific egalitarianism, it is both necessary and sufficient to crowd out all private purchase of g .

Sufficiency:

Define as $g_{i,j}^0$ the level of uniform provision that would crowd out private purchase by an individual of preference type i and wage j . From the rhs of (14) it can be shown that $\partial g_{i,j}^0 / \partial w_j > 0$ and $\partial g_{i,j}^0 / \partial \theta_i > 0$. It follows that if g^0 is set high enough to crowd out the maximum $g_{i,j}^0$, namely $g^0 \geq g_{S,H}^0$, every individual will consume only g^0 (or set $\hat{g}_{i,j} = 0$) and both (6a) and (6b) will be weakly but trivially satisfied.

Necessity:

We show that under all possible cases, g^0 must be set $\geq g_{S,H}^0$ or else (6a) or (6b) will be violated. From $\partial g_{i,j}^0 / \partial w_j > 0$ and $\partial g_{i,j}^0 / \partial \theta_i > 0$ it follows that $g_{R,L}^0 < g_{R,H}^0$ and

$\vartheta'_{S,L} < \vartheta'_{S,H}$. As well, from (13) it can be shown that $\hat{g}_{R,L} < \hat{g}_{R,H}$ and $\hat{g}_{S,L} < \hat{g}_{S,H}$ for any positive $\hat{g}_{i,j}$.

Case I: $\vartheta'_{R,L} < \vartheta'_{R,H} \leq \vartheta'_{S,L} < \vartheta'_{S,H}$.

If $\vartheta' \leq \vartheta'_{R,L}$ then the lhs of (6a) will be $(1-s)(\vartheta' + \hat{g}_{R,L}) + s(\vartheta' + \hat{g}_{S,L})$, and the rhs will be

$$(1-s)(\vartheta' + \hat{g}_{R,H}) + s(\vartheta' + \hat{g}_{S,H}).$$

Since $\hat{g}_{R,H} > \hat{g}_{R,L}$ and $\hat{g}_{S,H} > \hat{g}_{S,L}$, average

consumption across the two wage groups will not be equal.

If $\vartheta'_{R,L} < \vartheta' \leq \vartheta'_{R,H}$ then the lhs of (6a) will be $(1-s)\vartheta' + s(\vartheta' + \hat{g}_{S,L})$, and the rhs will be

$$(1-s)(\vartheta' + \hat{g}_{R,H}) + s(\vartheta' + \hat{g}_{S,H}).$$

Again, since $\hat{g}_{R,H} \geq 0$ and $\hat{g}_{S,H} > \hat{g}_{S,L}$, equality

will not hold.

If $\vartheta'_{R,H} < \vartheta' \leq \vartheta'_{S,L}$ then the lhs of (6a) will be $(1-s)\vartheta' + s(\vartheta' + \hat{g}_{S,L})$ and the rhs will be

$$(1-s)\vartheta' + s(\vartheta' + \hat{g}_{S,H}).$$

Equality will not hold.

If $\vartheta'_{S,L} < \vartheta' < \vartheta'_{S,H}$ then (6a) reduces to $\vartheta' < (1-s)\vartheta' + s(\vartheta' + \hat{g}_{S,H})$, or inequality.

If $\vartheta'_{S,H} \leq \vartheta'$, then (6a) reduces to $\vartheta' = \vartheta'$. Only here is equality satisfied for any s and

(6b) is (weakly) satisfied.

Case II: $\vartheta'_{R,L} < \vartheta'_{S,L} < \vartheta'_{R,H} < \vartheta'_{S,H}$.

If $\vartheta' \leq \vartheta'_{R,L}$ then the lhs of (6a) will be $(1-s)(\vartheta' + \hat{g}_{R,L}) + s(\vartheta' + \hat{g}_{S,L})$, and the rhs will be

$$(1-s)(\vartheta' + \hat{g}_{R,H}) + s(\vartheta' + \hat{g}_{S,H}).$$

Since $\hat{g}_{R,H} > \hat{g}_{R,L}$ and $\hat{g}_{S,H} > \hat{g}_{S,L}$, average

consumption across the two wage groups will not be equal.

If $\vartheta' \leq \vartheta'_{S,L}$ then the lhs of (6a) will be $(1-s)\vartheta' + s(\vartheta' + \hat{g}_{S,L})$, and the rhs of (6a) will be

$$(1-s)(\vartheta' + \hat{g}_{R,H}) + s(\vartheta' + \hat{g}_{S,H}).$$

Since $\hat{g}_{R,H} > 0$ and $\hat{g}_{S,H} > \hat{g}_{S,L}$, average

consumption across the two wage groups will not be equal.

If $\hat{g}_{S,L} \leq \hat{g} < \hat{g}_{R,H}$, then the lhs of (6a) reduces to \hat{g} , and the rhs of (6a) will be

$(1-s)(\hat{g} + \hat{g}_{R,H}) + s(\hat{g} + \hat{g}_{S,H})$. Since $\hat{g}_{R,H} > 0$ and $\hat{g}_{S,H} > 0$, average consumption across the two wage groups will not be equal.

If $\hat{g}_{R,H} \leq \hat{g} < \hat{g}_{S,H}$, then using (6a), $\hat{g} < (1-s)\hat{g} + s(\hat{g} + \hat{g}_{S,H})$, or consumption will not be equalized across wage groups.

If $\hat{g}_{S,H} \leq \hat{g}$. Only here is consumption equalized across wage groups and (6b) is (weakly) satisfied.

Appendix 4

Derivation of the condition for a welfare comparison between differential pricing and uniform provision when preferences are homogeneous.

Assuming that $w_H = aw_L$, where $a \geq 1$, then under either policy,

$$\sum_{j=L,H} N_j V_j = N_L V_L + N_H V_H = N_L V_L + N_H a^{\frac{1}{2+\theta}} V_L = [N_L + N_H a^{\frac{1}{2+\theta}}] V_L. \quad (\text{A.6})$$

$$\text{Thus, } \sum_{j=L,H} N_j V_j(P_{g,j}^*) \geq \sum_{j=L,H} N_j V_j(\bar{g}^*, t^*) \text{ if and only if } V_L(P_{g,L}^*) \geq V_L(\bar{g}^*, t^*), \quad (\text{A.7})$$

or substituting in the functional forms, if and only if

$$\left(\frac{T}{2+\theta} \right)^{\frac{1}{2+\theta}} \left(\frac{w_L T}{2+\theta} \right)^{\frac{1}{2+\theta}} \left(\frac{\theta}{P_{g,L}^*} \frac{w_L T}{2+\theta} \right)^{\frac{\theta}{2+\theta}} \geq \left(\frac{T}{2} \right)^{\frac{1}{2+\theta}} \left(\frac{1}{1+\theta} \frac{w_L T}{2} \right)^{\frac{1}{2+\theta}} \left(\frac{\theta}{1+\theta} \frac{\bar{w} T}{2} \right)^{\frac{\theta}{2+\theta}} \quad (\text{A.8})$$

where $\bar{w} = [(N_L/N)w_L + (N_H/N)aw_L]$. Simplifying, this is equivalent to

$$\left(\frac{4(1+\theta)}{(2+\theta)^2} \right)^{\frac{1}{2+\theta}} \left(\frac{a \frac{N_H}{N}}{\left(a \frac{N_H}{N} + \frac{N_L}{N} \right)} \frac{2(1+\theta)}{2+\theta} \right)^{\frac{\theta}{2+\theta}} \geq 1, \text{ or getting } a \text{ by itself,} \quad (\text{A.9})$$

$$a = \frac{w_H}{w_L} \geq \frac{N_L / N_H}{\left(\frac{2}{2+\theta}\right)^{\frac{2+\theta}{\theta}} (1+\theta)^{\frac{1+\theta}{\theta}} - 1}. \quad (\text{A.10})$$

To show that the right hand side of (A.10) is falling in θ , define

$$z = \left(\frac{2}{2+\theta}\right)^{\frac{2+\theta}{\theta}} (1+\theta)^{\frac{1+\theta}{\theta}}. \quad \text{We show that a monotonic transformation, } \ln z, \text{ is rising in } \theta.$$

$$\frac{d \ln z}{d\theta} = \frac{1}{\theta^2} [2 \ln(2+\theta) - \ln(1+\theta) - \ln 4]. \quad (\text{A.11})$$

We consider the sign of (A.11) when $\theta \rightarrow 0$, $0 < \theta < 2$, and $2 \leq \theta$.

$$\text{As } \theta \rightarrow 0, \text{ by L'hospital's rule, } \frac{d \ln z}{d\theta} = \frac{f'(\theta)}{g'(\theta)} \Big|_{\theta=0} = \frac{2}{2+\theta} - \frac{1}{1+\theta} = \frac{0}{0} \text{ at } \theta=0 \quad (\text{A.12})$$

$$\text{Reapplying L'hospital's rule, } \frac{f''(\theta)}{g''(\theta)} \Big|_{\theta=0} = \frac{-2}{(2+\theta)^2} + \frac{1}{(1+\theta)^2} > 0 \text{ at } \theta=0. \quad (\text{A.13})$$

For $0 < \theta < 2$, from (A.11) $\ln(2+\theta) - \ln(1+\theta) > \ln(2+\theta) - \ln(4)$, so $\frac{d \ln z}{d\theta} > 0$.

For $2 \leq \theta$, from (A.11) $\ln(2+\theta) - \ln(1+\theta) > 0$ and $\ln(2+\theta) - \ln(4) \geq 0$, so $\frac{d \ln z}{d\theta} > 0$.

Appendix 5

Derivation of the welfare comparison between differential pricing and uniform provision when preferences are heterogeneous

In the special case where $\rho = 0$, optimal prices adjust under differential pricing such

that $g_{R,L}^* = g_{R,H}^*$ and $g_{S,L}^* = g_{S,H}^*$. Then, just as under uniform provision,

$$V_{R,H} = a^{\frac{1}{2+\theta_r}} V_{R,L} \quad \text{and} \quad V_{S,H} = a^{\frac{1}{2+\theta_s}} V_{S,L}$$

Social welfare under either policy can then be expressed as

$$\sum_i \sum_j V_{i,j} = (1-s)(N_L + a^{\frac{1}{2+\theta_R}} N_H) V_{R,L} + s(N_L + a^{\frac{1}{2+\theta_S}} N_H) V_{S,L} \quad (\text{A.14})$$

Social welfare will be higher under differential pricing than uniform provision if and only if

$$(1-s)(N_L + a^{\frac{1}{2+\theta_R}} N_H)(V_{R,L}(P_{g,L}^*) - V_{R,L}(\bar{g}^*, t^*)) + s(N_L + a^{\frac{1}{2+\theta_S}} N_H)(V_{S,L}(P_{g,L}^*) - V_{S,L}(\bar{g}^*, t^*)) \geq 0 \quad (\text{A.15})$$

The two expressions $(V_{i,L}(P_{g,L}^*) - V_{i,L}(\bar{g}^*, t^*))$ in (A.15) will be non-negative if and only if

$$\frac{V_{i,L}(P_{g,L}^*)}{V_{i,L}(\bar{g}^*, t^*)} \geq 1 \Leftrightarrow \frac{a(A_i - 1)N_H / N}{N_L / N} - 1 \geq 0 \quad (\text{A.16})$$

where $A_i = \left(\frac{2}{2+\theta_i}\right)^{\frac{2+\theta_i}{\theta_i}} (1+\theta^*)^{\frac{1+\theta_i}{\theta_i}} \frac{\theta_i}{\theta^*}$, $i = R$ or S , and θ^* is that weight on the target

good that for a given wage disparity made the two policies welfare equivalent under homogeneity (A.10). Substituting these two expressions into (A.15) and dividing

through by $(N_L + a^{\frac{1}{2+\theta_R}} N_H)$, differential pricing achieves higher social welfare than uniform provision if and only if

$$(1-s) \left(\frac{a(A_R - 1)N_H / N}{N_L / N} - 1 \right) + s\delta \left(\frac{a(A_S - 1)N_H / N}{N_L / N} - 1 \right) \geq 0 \quad (\text{A.17})$$

where $\delta = [N_L + a^{\frac{1}{2+\theta_S}} N_H] / [N_L + a^{\frac{1}{2+\theta_R}} N_H] \leq 1$. We proceed by showing that the left hand side of (A.17) is increasing in a mean-preserving spread in preferences around θ^* , starting from efficiency equivalence under homogeneity.

With the distribution of preferences identical across income groups, a mean preserving spread from θ^* can be defined as $(1-s)\theta_R + s\theta_S = \theta^*$, where $\theta_R \equiv \theta^* - \varepsilon$, and

$\theta_S \equiv \frac{\theta^* - (1-s)\theta_R}{s} \equiv \theta^* + \frac{(1-s)}{s}\varepsilon$. Thus a decrease in heterogeneity can be represented

by $\varepsilon \rightarrow 0$, and an increase by $\varepsilon \rightarrow \theta^*$. Focusing on A_R and A_S in A.17,

$$\begin{aligned} \frac{\partial A_R}{\partial \varepsilon} &= A_R \left[\frac{1}{(\theta_R)^2} \ln \frac{4(1+\theta^*)}{(2+\theta_R)^2} \right] && \text{and} \\ \frac{\partial A_S}{\partial \varepsilon} &= -A_S \left[\left(\frac{1-s}{s} \right) \frac{1}{(\theta_S)^2} \ln \frac{4(1+\theta^*)}{(2+\theta_S)^2} \right] \end{aligned} \quad (\text{A.18})$$

While $\frac{\partial A_S}{\partial \varepsilon} > 0$, $\frac{\partial A_R}{\partial \varepsilon} < 0$ for $\varepsilon \rightarrow 0$, but $\frac{\partial A_R}{\partial \varepsilon} > 0$ for $\varepsilon \rightarrow \theta^*$. It can be shown that

$\frac{\partial A_R}{\partial \varepsilon}$ is monotonically increasing in ε ,¹⁵ so there exists a unique degree of

heterogeneity where $\frac{\partial A_R}{\partial \varepsilon} = 0$, denoted by ε^* . Since both $\partial A_i / \partial \varepsilon$ are monotonically

increasing, it follows that the partial derivative of the left hand side of (A.17) with

respect to ε will be minimized at $\varepsilon = 0$. It can be shown that the evaluated value of the

derivative at $\varepsilon = 0$ is zero. Initial increases in $\varepsilon < \varepsilon^*$ lower the relative utility that

differential pricing delivers to regular strength individuals, but this is outweighed by

higher relative utility gained by strong preference individuals. Differential pricing thus

raises overall social welfare over uniform pricing. For $\varepsilon \geq \varepsilon^*$, individuals with both

preference strengths gain welfare from differential pricing relative to uniform

provision.

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Notes

¹ This section draws heavily on our earlier paper (Clark and Kim (2004)), that focuses exclusively on the re-distributional properties of differential money/time pricing.

² Our preferences over 3 goods are more general than in Alexeev and Leitzel (2001), who assume equal weight Cobb Douglas preferences between a single good and leisure. We are less general, however, than O'Shaughnessy (2000), who assumes general concave utility, though also between one good and leisure.

³ We adopt the convention of lower case letters for demand, and upper case letters for supply.

⁴ We assume that individuals must queue once per unit purchased, and that everyone in a given outlet will wait an identical period of time per unit purchased. This is a common way of modeling the time cost of queuing (Barzel (1974), Sah (1987), Suen (1989), Polterovich (1993), O'Shaughnessy (2000) and Alexeev and Leitzel (2001)). Alternatives have been proposed, such as queuing time depending on show-up time (Holt and Sherman (1982)), or fixed time costs for any quantity of purchase (Weitzman (1991)).

⁵ We assume the policy maker can successfully prevent a black market in G from forming directly between producers and individual buyers. Alternatively, the government could allow private firms to sell G directly to individuals, offering a subsidy to those who sell at a prescribed below-market price, and taxing those who sell it at market price.

⁶ A single resource constraint (for g) is sufficient because two of the three prices in the model have already been determined. Alternatively, the resource constraint for y will be identical at market clearing prices.

⁷ However allocated between G and Y production, labour hours supplied can be represented as a function of full price $P_{g,j}$ via each individual's time constraint, $T - l - hg$. By (6a), $P_{g,L}$ can further be written as an increasing function of $P_{g,H}$, such that the entire resource constraint can be written as a function of $P_{g,H}$. Uniqueness of full price is then satisfied in that the entire expression is monotonically decreasing in $P_{g,H}$, with infinite excess demand at zero price, and finite excess supply at infinite price.

⁸ As drawn, the "isoprice" lines for high and low ability individuals cross in the positive quadrant, so that the time and money prices for low ability individuals are both positive. In cases where $\rho < 0$ the intersection may occur at a positive time price, but *negative* money price (or subsidy). See Clark and Kim (2004) for further details.

⁹ One plausible feasible allocation of labour across sectors would be for everyone to work in the production of Y in proportion to his or her expenditures on y out of total expenditures.

¹⁰ The constraint (21) is closed, and the objective function (20) is bounded for all values contained in the constraint.

¹¹ Less willingness to substitute would require greater queuing time and a negative money price at the low wage outlet. Greater willingness would require less queuing time and a positive money price.

¹² While we do not illustrate it, we get analogous results if the wage disparity remains at 5.4, and we reduce N_L/N_H from 1 to 1/3, or raise it to 3. A decrease in the proportion of the poor increases the range of ρ over which differential pricing achieves specific egalitarianism more efficiently than uniform provision.

¹³ Our alternative policies relying on income tax also require knowledge of each person's total income.

¹⁴ For example, when $\rho = 0$, $w_L = 1$ and $w_H = 5.4$, all possible combinations of s_L and s_H are feasible when $N_L/N = N_H/N = .5$ and $\theta_S = 1.5$ and $\theta_R = .5$. If, however, $N_L/N = .9$ and $N_H/N = .1$, then if $s_H = 1$, s_L must exceed .083 for h_L to be non-negative. Or returning to the baseline, if $\theta_R = .1$ and $\theta_S = 1.9$, then if $s_H = 0$, s_L cannot exceed .476 for h_L to be non-negative.

¹⁵ A detailed proof is available from the authors upon request.