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**QSEP Research Report No. 425** 



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CONSUMER DEMAND ELASTICITIES

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**ABSTRACT** 

Errors introduced by using aggregate data in estimating a consumer demand model have long

been a concern. We study the effects of such errors on elasticity estimates derived from AIDS

and QUAIDS models. Based on a survey of published articles, a generic parameterization of the

income distribution, and the range of Gini coefficients reported for 28 OECD countries, we

generate and analyse a large number of "observations" on the differences between elasticities

calculated at the aggregate level and those calculated at the micro level. We suggest a procedure

for evaluating the likely range of aggregation error when a model is estimated with aggregate

data.

KEY WORDS: Aggregation error; Consumer demand elasticities; AIDS/QUAIDS models;

Income distribution.

JEL CLASSIFICATION: D12, C43

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#### 1. INTRODUCTION

In earlier days there was frequently no alternative to the use of aggregate time series data in estimating consumer demand models (Stone, 1954, to cite an early pioneering paper). Underlying the models was the notion of a representative consuming unit that maximized utility but aggregation blurred the relationship between micro theory and econometric practice. The likelihood of "aggregation bias" was well known but there was not much one could do about it. Later, as survey data for individual households became increasingly available (and increasingly rich in content), opportunities opened up for estimating micro-theoretic models using actual micro data. Nevertheless, it remains true today that micro data are not always available in particular contexts, or appropriate for particular research objectives. Survey data may be available in one country but not another, or available for broad categories of goods but not at a detailed level that may be required (food in total but not types of food, for example); a survey may fail entirely to provide certain variables of importance for a particular purpose; trends and dynamics may be of interest, thus necessitating the use of time series available only at the aggregate level. Whatever the reasons it is still the case that aggregate data are often used in estimating consumer demand models, and hence that aggregation bias remains on the list of concerns. (We report on a survey of 21 articles containing estimated models; 15 of the articles used aggregate data.) Other things equal (and sampling variability aside), elasticities calculated at the aggregate level will generally differ from those calculated at the micro level, even if the same model is used in both cases. The differences, how to calculate them, and what to do about them, are the subjects addressed in this paper.

We restrict our attention to two widely used models, Deaton and Muellbauer's (1980) "almost ideal demand system" (AIDS) and the quadratic extension of it (QUAIDS) proposed by Banks, Blundell, and Lewbel (1997). Aggregation of an AIDS micro model over households requires the introduction of an "aggregation parameter" that depends on the distribution of household total expenditure – on the "income distribution," as we shall call it for convenience, with slight inaccuracy; aggregation of a QUAIDS model requires two such parameters. We consider expenditure elasticities and own-price elasticities in the paper and there is, for each, a micro form and a corresponding macro form. This allows us to do a search for articles with AIDS/QUAIDS models that provide either micro or macro elasticities, calculate the corresponding macro or micro elasticities (under alternative assumptions about income distribution), and thus create a data set reflective of the types and magnitudes of aggregation

effects actually found in the empirical literature. Along the way we introduce some procedures for characterizing the income distribution in a generic form and (using data for OECD countries) establish a range of distributions according to degree of inequality. On that basis we are then able to arrive at what we think is a reasonable range for the aggregation parameters and study the effects on elasticities over that range.

#### 2. THE MODELS AT THE MICRO LEVEL

Assume K households, indexed by k, and I commodities, indexed by i (or by j if a supplementary index is required). Households face prices  $p_1, ..., p_I$ . Household k spends  $x_{ik}$  on commodity i and  $x_k$  on all commodities combined. Its expenditure share for commodity i is therefore  $w_{ik} = x_{ik} / x_k$ . AIDS and QUAIDS models are both of interest but AIDS is nested within QUAIDS, and so we focus on the latter. The QUAIDS model at the micro level is defined as follows:

$$w_{ik} = \alpha_i + \sum_{j=1}^{l} \gamma_{ij} \ln p_j + \beta_i \ln(x_k / q) + \lambda_i (\ln(x_k / q))^2 / Q \qquad (\forall i, k)$$
 (1)

$$q = \exp\{\sum_{i=1}^{I} \alpha_i \ln p_i + 1/2 \sum_{i=1}^{I} \sum_{j=1}^{I} \gamma_{ij} (\ln p_i) (\ln p_j)\}$$
 (2)

$$Q = \exp\{\sum_{i=1}^{I} \beta_i \ln p_i\}$$
(3)

The corresponding AIDS model is obtained by setting  $\lambda_i = 0$  ( $\forall_i$ ), and consequently dropping equation (3). Equations (1) might (and typically would) have additional linear terms representing household demographic characteristics, region of residence, etc., in which case the intercepts would be household-specific ( $\alpha_{ik}$  rather than  $\alpha_i$ ). However, that would have no fundamental bearing on the theoretical analysis, and we ignore it, for the moment.

There are different approaches to estimation. One is to simplify things by using

approximations to q and Q, rather than the strict specifications of equations (2) and (3): Deaton and Muellbauer (1980) used Stone's index to approximate q in estimating their AIDS model; Matsuda (2006) conducted experiments using the Stone index for q and Tornqvist, Laspeyres, and Paasche indexes as alternatives for Q, in estimating a QUAIDS model. A second approach is to retain the original specifications and use an iterative method: initial parameter values are chosen so as to obtain initial values of q and Q; equations (1) are then estimated, conditional on the initial values of those variables, thus obtaining new parameter values, and hence new calculated values of q and Q; and so the process goes until some convergence criterion is satisfied. This method was employed by Banks, Blundell, and Lewbel (1997), Blundell and Robin (1999), and Denton, Mountain, and Spencer (1999). (An additional level of iteration was included in the latter paper to allow for serial correlation in the error term, following Beach and MacKinnon, 1979.) A third approach is to substitute equations (2) and (3) into equations (1) and estimate the combined system of equations by some appropriate constrained nonlinear method. The resulting system can be quite large and complex and we are not aware of any published study in which this approach was actually used. We consider it further below, from the point of view of identification in the context of estimation with aggregate data. Whatever the approach taken the model would likely be estimated under theoretical restrictions on its parameters (homogeneity, symmetry), using a Zellner-type estimator.

#### 3. THE MODELS AT THE AGGREGATE LEVEL

Consumer demand models can also be estimated with macro data. That this may introduce aggregation error is a longstanding worry, assuming one wishes to interpret the estimates as applying to the underlying micro model (Gorman, 1953, Stoker, 1984, 1986, Blundell and Stoker, 2005).

The macro version of QUAIDS consistent with equations (1) is obtained as follows. Let  $X_i$  be aggregate expenditure on commodity i, X overall aggregate expenditure,

 $W_i = X_i / X$  the aggregate expenditure share, and  $\overline{x} = X / K$  mean expenditure per household. Equations (1) can then be rewritten as

$$w_{ik} = \alpha_i + \sum_{j=1}^{I} \gamma_{ij} \ln p_j + \beta_i (\ln(x_k / \overline{x}) + \ln(\overline{x} / q))$$

$$+ \lambda_i (\ln(x_k / \overline{x}) + \ln(\overline{x} / q))^2 / Q \tag{4}$$

and for a household for which  $x = \overline{x}$ , as

$$w_i = \alpha_i + \sum_{i=1}^{I} \gamma_{ij} \ln p_j + \beta_i \ln(\overline{x}/q) + \lambda_i (\ln(\overline{x}/q))^2 / Q$$
 (5)

The aggregate share equation corresponding to the micro share equation (5) is obtained by multiplying both sides of (4) by  $x_k / X$  and summing over k:

$$W_i = \alpha_i^* + \sum_{j=1}^I \gamma_{ij} \ln p_j + \beta_i^* \ln(\overline{x}/q) + \lambda_i (\ln(\overline{x}/q))^2 / Q$$
 (6)

where: 
$$\alpha_i^* = \alpha_i + \beta_i g + \lambda_i h / Q$$
;  $\beta_i^* = \beta_i + 2\lambda_i g / Q$ ;  
 $g = \sum_i (x_k / X) \ln(x_k / \overline{x})$ ;  $h = \sum_i (x_k / X) (\ln(x_k / \overline{x}))^2$ 

Equations (2) and (3) still hold at the macro level, with  $\alpha_i$  and  $\beta_i$  replaced by  $\alpha_i^*$  and  $\beta_i^*$ .

Two new parameters, g and h, now appear in the macro equation. In theory, both are identifiable. To see this, consider the QUAIDS model with two commodities (i = 1,2). Dropping the equation for the second commodity to avoid singularity of the system in estimation, substituting equations (2) and (3) into (6), and imposing homogeneity and symmetry restrictions, we obtain

$$W_1 = (\alpha_1 + \beta_1 g) + \beta_1 \ln \overline{x}^* + (\gamma_{11} - \beta_1 \alpha_1) \ln p_1^* - (\beta_1 \gamma_{11} / 2) (\ln p_1^*)^2$$

+ 
$$\lambda_1 h \exp\{-\beta_1 \ln p_1^*\} + 2\lambda_1 g \ln(\bar{x}/q) \exp\{-\beta_1 \ln p_1^*\}$$

$$+\lambda_1 (\ln(\overline{x}/q))^2 \exp\{-\beta_1 \ln p_1^*\} \tag{7}$$

where  $p_1^* = p_1 / p_2$ ,  $\overline{x}^* = \overline{x} / p_2$ , and q involves only the parameters  $\alpha_1$  and  $\gamma_{11}$ . A path to the determination of g and h is the following.  $\beta_1$  is estimated directly when equation (7) is fitted to the data and  $\gamma_{11}$  can then be calculated immediately. Given  $\beta_1$  and  $\gamma_{11}$ ,  $\alpha_1$  can be calculated, and then g. Given g,  $\lambda_1$  can be calculated, and given  $\lambda_1$ , h can be calculated. (In a larger system g and h would be restricted to being the same in all equations and there would be other paths to their determination.) One would like to exploit this identification property but unfortunately it is almost certain to be too weak to be useful, a fact confirmed by some experimentation with actual and artificial data. In the absence of other information the parameter estimates are too sensitive to small sampling errors to make them acceptable. This remains true even if the quadratic term is dropped, thus eliminating h and converting the model to the AIDS form. As a practical matter it is just not possible to extract reliably the  $\beta_i g$  component from the combined intercept term  $\alpha_i + \beta_i g$  and subsequent calculations of elasticities are almost certain to be unreliable. We therefore take a different approach, one that is likely to produce results at least within a reliable range.

#### 4. MICRO AND MACRO ELASTICITIES

We are interested in the effects of aggregation on calculated elasticities. To simplify what follows (without loss of generality) we normalize prices and incomes (as in Denton and Mountain, 2001, 2004) so that  $\ln p_i = 0$  for all i (hence q = Q = 1) and  $\ln \overline{x} = 0$ . The elasticities are invariant to the normalization, which amounts simply to a particular choice of measurement units. Note too that it has no effect on g and h; they are invariant to the scaling of income – to what Lewbel (1990, 1992) terms "mean scaling." (As an aside, the mean scaling property also contributes to the justification for assuming g and h to be constant when the income distribution changes and a model is estimated with aggregate time series data, just as the other parameters are typically assumed constant over time.) The expenditure elasticities are then given by

$$\varepsilon_i = 1 + \beta_i / \alpha_i \tag{8}$$

$$\overline{\varepsilon}_{i} = 1 + \beta_{i}^{*} / \alpha_{i}^{*} = 1 + (\beta_{i} + 2\lambda_{i}g) / (\alpha_{i} + \theta_{i})$$

$$(9)$$

where  $\varepsilon$  and  $\overline{\varepsilon}$  are the elasticities calculated from micro and macro data, respectively, and  $\theta_i = \beta_i g + \lambda_i h$ . The differences are thus

$$\overline{\varepsilon}_{i} - \varepsilon_{i} = (\beta_{i} + 2\lambda_{i}g)/(\alpha_{i} + \theta_{i}) - \beta_{i}/\alpha_{i} \tag{10}$$

The corresponding compensated price elasticities are

$$\eta_{ii} = -\delta_{ii} + \gamma_{ii} / \alpha_i + \alpha_i \tag{11}$$

$$\overline{\eta}_{ij} = -\delta_{ij} + \gamma_{ij} / \alpha_i^* + \alpha_j^* = -\delta_{ij} + \gamma_{ij} / (\alpha_i + \theta_i) + \alpha_j + \theta_j$$
(12)

and their differences are

$$\overline{\eta}_{ij} - \eta_{ij} = \gamma_{ij} (1/(\alpha_i + \theta_i) - 1/\alpha_i) + \theta_J \tag{13}$$

where  $\delta_{ij} = 1$  for i = j, zero otherwise.

Some points to note: (a) If the income distribution is uniform, g = h = 0, and the micro and macro elasticities are identical – both the expenditure and price elasticities. (b) The elasticities are also identical for the AIDS model if  $\beta_i = 0$ , and for the QUAIDS model if  $\beta_i = \lambda_i = 0$ , regardless of how income is distributed. (c) The effects of aggregation are particularly sensitive to variations of  $\alpha_i$ , if  $\alpha_i$  is small, and in the case of expenditure elasticities, to the  $\beta_i / \alpha_i$  ratio. (d) The elasticity differences induced by aggregation can be positive or negative. (e) A positive (negative) elasticity at the micro level could become negative (positive) at the macro level. (f) Similarly, a commodity that is in the elastic range at the micro level (elasticity greater than 1, in absolute value) could move into the inelastic range at the macro level, and vice versa.

#### 5. INCOME DISTRIBUTION AND INEQUALITY

The extent of aggregation error depends then on both the configuration of parameters in the underlying micro model and the distribution of income (strictly speaking, the distribution of total expenditure, but we are ignoring the difference). To move forward with our exploration we assume a particular type of distribution, the lognormal. In so doing we follow Bénabou (2000) in his study of income distribution and the social contract. To quote him, "The lognormal is a good

approximation of empirical income distribution, leads to tractable results, and allows for an unambiguous definition of inequality ..." (p. 98). Our purpose in the present paper is quite different from his but his three reasons for the choice of distribution function apply equally well.

Think for the moment of x as being a continuous variable. The lognormal distribution of income is defined implicitly by  $\ln x \sim N(\mu, \phi)$ , or explicitly by the density function

$$f(x) = \left(x\sqrt{2\pi\phi}\right)^{-1} \exp\left\{-\left(\ln x - \mu\right)^{2} / 2\phi\right\}$$
 (x > 0)

The function has two parameters,  $\mu = E(\ln x)$  and  $\phi = \text{var}(\ln x)$ . However, our sole use of the function is in the calculation of a range of values for g and h, both of which are invariant to the choice of measurement units. We are therefore at liberty in any calculation to choose units so that  $\mu = 0$ , leaving g and h to depend only on  $\phi$ . As  $\phi$  increases, so does the degree of income inequality; as  $\phi$  tends to zero, the distribution approaches a uniform distribution. From the definitions of g and h it is obvious that they too approach zero as  $\phi$  does, and hence that a model at the macro level tends to the corresponding model at the micro level. (We are of course ignoring differences in the estimates of the model's parameters because of the differences in the data being used; the point is simply that aggregation effects are no longer present when  $\phi = 0$ .)

We use the lognormal distribution in our subsequent calculations in the following way. We set  $\mu$  to zero (or to any other value, in light of the invariance property of g and h). We then choose a value for  $\phi$ , generate 50,000 random draws of  $\ln x$  from the  $N(0,\phi)$  distribution, exponentiate to get the corresponding 50,000 values of x, and then calculate the associated g and h values. (We experimented with 100,000 draws but found virtually no difference in accuracy.) The choice of  $\phi$  thus determines the values assigned to g and h. The process can be repeated for different degrees of income inequality. The question then is how to make a relevant and realistic choice of  $\phi$ . We base our choice on the Gini coefficient.

#### 6. GINI COEFFICIENTS IN 28 COUNTRIES

The Gini coefficient is a simple and time-honoured measure of income inequality. It is also a statistic that is available for a large number of countries, and a useful summary measure for

our purposes. We make use of recently published Gini coefficients available for 28 of the 30 member countries of the OECD, as provided by United Nations (2006, Table 15). The countries and coefficients are shown in Table 1. Collectively they represent a wide range of household income distributions, and a corresponding wide range of distributional inequality.

For households k=1,2,...,K, with K large and households arranged in nondecreasing order of income  $(x_k \geq x_{k-1})$ , the Gini coefficient (d, as we shall label it) can be expressed, to a close approximation, as

$$d = 1 - 2X^{-1} \sum_{k=1}^{K} (1 - k / K) x_k$$
 (15)

where  $X = \sum x_k$ . As K increases, the bounds of d approach 0 at one extreme (all incomes equal) and 1 at the other (all income held by one household). The countries in Table 1 are ordered from lowest to highest coefficient (from least to most unequal income distribution). At the low end is Denmark (d = .247); at the high end is Mexico (d = .495). The median (d = .3285) lies half way between France and Belgium.

#### 7. FROM GINI TO g AND h

The Gini coefficient serves for us as a bridge from the income distribution to the aggregation parameters  $\, g \,$  and  $\, h \,$ . What we have done is to take each of the 28 country-specific Gini coefficients, determine numerically the lognormal distribution that would generate the same coefficient (determine the  $\, \phi \,$  value, that is), and then calculate the  $\, g \,$  and  $\, h \,$  values that go with it . We should make it clear that the income distributions that we derive in this way are not intended as faithful representations of the actual distributions for the 28 countries. Rather they are the distributions that would generate the same Gini coefficients as the actual distributions if the actual ones were lognormal. We are not attempting to model the actual distributions of the 28 countries, merely to use the countries' Gini coefficients as a range of possible values from which to calculate realistic income distribution functions, and hence a realistic range of  $\, g \,$  and  $\, h \,$  values.

The g and h values thus calculated are shown in Table 1, spanning the Gini range from .247 to .495. A simple way of generating g and h values corresponding to any Gini value in that

range is afforded by the following recursive model (found by experimentation):

$$\hat{g} = -.038038 + .40125d + .075599d^2 + 2.2079d^3$$
 (16)

$$\hat{h} = .0012879 + 1.9893\hat{g} + 1.0718\hat{g}^2 - .072298\hat{g}^3 \tag{17}$$

The model was fitted to the d, g and h values in the table and used to calculate the predicted values  $\hat{g}$  and  $\hat{h}$ , also shown in the table. The model predictions are virtually perfect over the range of the table. Given an arbitrary Gini value in that range, g and h can thus be predicted with confidence, and used in subsequent calculations. With g and h determined, a model estimated with aggregate data can be converted to the corresponding micro form, and the micro and macro elasticities compared. The conversion can also go the other way: a model estimated with micro data can be converted to the corresponding macro form.

#### 8. A SELECTION OF MODELS FROM THE LITERATURE

We have done a rather extensive search of the literature for articles containing estimated AIDS and QUAIDS models and selected a total of 21 (fifteen for AIDS, six for QUAIDS). A criterion for selection was that an article must provide the estimated parameters of the model or sufficient other information (usually elasticities) to allow the parameters to be inferred. While the search was extensive we do not claim that it was exhaustive. If an article did not make it into our selection it may be that it did not provide sufficient numerical information to satisfy our criterion, or it may be that we simply missed it. Do not feel offended if your excellent article is not included.

The articles we have chosen include models estimated with either micro or aggregate data. For a model based on micro data we have derived both the micro expenditure and micro (compensated) own-price elasticities. (To keep the calculated results manageable for presentation purposes we do not concern ourselves with cross-price elasticities.) We have then assumed alternative values for the Gini coefficient, and hence for g and h, and calculated the corresponding macro elasticities based on equations (9) and (12). For a model estimated with aggregate data we have done the same thing in reverse, going from macro elasticities to corresponding micro elasticities, based on equations (8) and (11).

A typical example may be helpful in understanding how we have used information extracted from a published article. Consider an article in which an AIDS model was estimated using aggregate data. Suppose that the article provided, for some commodity i, the value  $\beta_i$  and the corresponding macro expenditure elasticity  $\overline{\mathcal{E}}_i$ , calculated (let us suppose) at the point of means. Suppose further that the model included linear terms representing household characteristics. The expenditure shares implicit in the calculation of the value of  $\overline{\mathcal{E}}_i$  reported in the article would then be the model's intercepts plus the means of the demographic terms. Since  $\overline{\mathcal{E}}_i$  must be equal to  $1+\beta_i/(\alpha_i+\beta_i g)$  (equation (9), with  $\lambda_i$  set to zero), and given an assumed value for  $\,g\,$  , a value for  $\,\alpha_i\,$  can be derived (under our normalization, at the point of means), thus completing the set of parameter values required for subsequent calculations. The idea is to take a macro elasticity reported in an article (and whatever other information is provided), extract or figure out the parameter values that give rise to that elasticity, and then calculate the corresponding implied micro elasticity for the assumed g value. The resulting pair of macro and micro elasticities gives us one "observation" on the difference between the two. The example assumes a model estimated with aggregate data. For a model estimated with micro data, and hence a reported micro elasticity, the calculations would go in the opposite direction, providing an implied macro expenditure elasticity, and again an "observation" on the difference. A similar approach is used to derive macro/micro differences for price elasticities.

The commodities for which models were estimated vary among the articles, both in number (from 4 to 11) and type (food, clothing, etc.). The associated elasticities are reported by authors at various reference points (at means, for particular years, etc.) and we have retained those reference points. (Intercepts are adjusted so that the elasticities remain the same as reported by the authors, under normalization.) Where elasticities were reported at two or more reference points in an article we have chosen only one for our calculations. (Example: Elasticities are reported for reference years 1962, 1977, and 1992 in Denton, Mountain, and Spencer, 1999; we chose the 1977 reference year.) The commodities differ, and so the commodity-specific elasticities may not be comparable from one article to another.

The elasticities that we extract from published articles are of course estimates subject to sampling error. However, that is not a concern here. We take the numbers at face value. The fact that they are not the "true" values (probability limits, if you prefer) still allows them to be

interpreted as representative of the distribution of estimated elasticities in the literature, and beyond that as an approximation to the distribution of the underlying true elasticities. Our study requires simply that we associate with each estimated micro (macro) value a corresponding implied macro (micro) value, conditional on the income distribution, and hence derive a realistic distribution of the differences induced by aggregation.

#### 9. COMPARISONS OF MICRO AND MACRO ELASTICITIES

We show, in Tables 2-4, the results, in summary form, of comparisons of the two types of elasticities. (The calculated elasticities are reported in full detail in the appendix tables.) Table 2 shows results of calculations for AIDS models, Table 3 for QUAIDS models, and Table 4 for both types of models combined. Of the 15 AIDS models, 12 were estimated using aggregate data with varying numbers of commodity categories, providing a total of 71 micro/macro pairs of elasticities. Three of the AIDS models were estimated using micro data, providing 20 pairs. Of the 6 QUAIDS models, 3 were estimated using aggregate data, providing 15 elasticity pairs, and 3 were estimated using micro data, providing 17 pairs. The articles themselves are coded A1, A2, etc. for AIDS models, Q1, Q2, etc. for QUAIDS models, and are identified by those codes in the list of references.

Table 4, which presents combined summary measures for all AIDS and QUAIDS models in our survey, is based on the largest number of observations, and we focus mainly on it. Considering first the expenditure elasticities, we note that the macro elasticities are lower than the micro elasticities in the great majority of cases, and concomitantly that the mean differences (macro minus micro) are generally negative, regardless of the degree of income inequality (the value of d). For all models combined, the mean absolute error ranges from about .05 to .31, depending on d. There are very few cases in which the macro and micro elasticities differ in sign and very few also in which one of them is less than 1 while the other is greater. Moreover, those few cases in which there are differences of either kind occur only when d is at its maximum.

In terms of averages, then, the effects of aggregation are relatively small when d is at its lower bound or median level. The effects are somewhat greater when d is at its upper bound, and enough so to suggest some concern. However, an examination of the individual observations indicates that much of the mean and mean absolute differences are attributable to a small number of outlier cases. To note some extremes, a macro elasticity of 4.4 based on a published model estimated with aggregate data converts to a micro elasticity of -5.5; a 2.7 macro elasticity

converts to a micro elasticity of 8.7; and a macro elasticity of 2.2 converts to a micro elasticity of 7.7. For the most part, extremes of this kind are associated with estimated elasticity values which are themselves large enough to warrant skepticism in their own right. Extreme differences result from particular parameter configurations interacting with a relatively high degree of income inequality. One would certainly like to flag those cases in which aggregation effects may cause quite misleading inferences. Fortunately there is a straightforward procedure for doing that, as discussed in the next section.

Focusing again on Table 4, the macro-micro differences for price elasticities are seen to be much smaller than those for expenditure elasticities. The overall mean difference is only about .01, even with maximum d, and the overall mean absolute difference is about .12, also for maximum d. There is some (rather weak) evidence that micro price elasticities tend to be smaller than the corresponding macro elasticities: for all models combined, the macro-minus-micro differences are negative in about 40 percent of all cases. (The percentages are closer to 50 percent for models estimated with micro data.) Difference in sign occur only seldom and greater-than-1/less-than-1 differences even less frequently. As with the expenditure elasticities there is the occasional horror story: a macro elasticity of -.47 that converts to a micro elasticity of 3.106, for example, and one would certainly want to be on guard for cases like that. For the most part, though, it appears that aggregation error is a relatively minor consideration in the interpretation of price elasticities. Given that prices are assumed to be independent of incomes in the AIDS/QUAIDS framework (and in consumer demand models generally) it is perhaps not surprising that their elasticities are affected rather little by income inequality.

#### 10. CHECKING AND ADJUSTING FOR AGGREGATION ERROR

The approach taken in this paper can be adapted to provide a straightforward way of checking and adjusting for aggregation error. For a model estimated with aggregate data one can calculate macro elasticities in the usual way, and then calculate the corresponding micro elasticities, based on an assumed Gini coefficient and the procedures underlying Table 1 for generating corresponding values of g and h. Of course, if the true income distribution is known the values of g and h can be calculated directly. However, information about the actual income distribution relevant for a particular model may be hard to come by (Stoker, 1993). If one can make a reasonable guess at the Gini coefficient, though, based on the 28-country information in Table 1, that can be used, and the associated values of g and h derived accordingly, using

equations (16) and (17). (The g and h values would still be estimates since there is no guarantee that the actual income distribution would be lognormal, as required by the Table 1 calculations.) Alternatively, one can do what we have done – do the calculations for the extreme values in the table to see how sensitive the results are to the choice of Gini coefficient. The analysis of our survey results suggests that in many cases the choice may make little difference. In any event, how much difference it does make is important information for judging and interpreting estimated elasticities. The person estimating a model can do the calculations but a person examining someone else's results – the reader of an article, say – can do them also, to assess their reliability.

Looking at it from the point of view of someone reading a published paper, the calculations might go as follows. Suppose that a QUAIDS model has been estimated with aggregate data and that the resulting values are reported for the slope coefficients  $\beta_i^*$  (as defined for equation (6)),  $\gamma_{ii}$ , and  $\lambda_i$ , as would often be the case. Choose a particular Gini coefficient (or alternative coefficients) in the range of Table 1, and hence particular values for g and h. Now suppose that elasticities are provided in the paper for some reference point in the variables space (averages over the data period, values in a particular year, etc.). The values of  $oldsymbol{eta}_i^*$  ,  $oldsymbol{\gamma}_{ij}$  , and  $oldsymbol{\lambda}_i$  are independent of the choice of reference point but if the estimated share intercepts incorporate linear additive terms for demographic, trend or other variables (as often they would), they will depend on the reference point values assigned to those variables in the elasticity calculations. The share intercepts will be affected also by any linear transformation of the variables (scaling, choice of measurement units). If their values are not provided (as we have found to be typical) they can be calculated by working backward from the expenditure elasticities or the price elasticities. Assuming the normalization that we have adopted ( $\ln \overline{x} = 0, \ln p_i = 0, \forall i$ ), the normalized intercepts,  $\alpha_i^*$ , can be found by inverting equation (9):  $\alpha_i^* = \beta_i^* / (\overline{\varepsilon}_i - 1)$ .(Alternatively, they can be calculated using equation (12).) With g and h given, the underlying parameters  $\, lpha_i \,$  and  $\, eta_i \,$  can then be obtained by inverting the equations that define  $\alpha_i^*$  and  $\beta_i^*$  for equation (6), with Q set to 1 because of the normalization of prices:  $\alpha_i = \alpha_i^* - \beta_i g - \lambda_i h$ ,  $\beta_i = \beta_i^* - 2\lambda_i g$ . All of the values for calculating elasticities

at the given reference point are now in place. The aggregate elasticities can be calculated (if one wants to redo the original calculations) using  $\alpha_i^*$ ,  $\beta_i^*$ ,  $\gamma_{ij}$ , and  $\lambda_i$ , along with the selected g and h values; the corresponding micro elasticities can be calculated using  $\alpha_i$ ,  $\beta_i$ ,  $\gamma_{ij}$ , and  $\lambda_i$ .

#### 11. CONCLUSION

Errors resulting from the use of aggregate data in model estimation have long been a concern in econometric consumer demand analysis. Such errors arise from the interaction of a model's parameters with the underlying distribution of incomes. In the QUAIDS framework, two parameters of the distribution determine the links between elasticities calculated with micro data and corresponding elasticities calculated with aggregate data; in the AIDS framework there is one such parameter. Assuming a lognormal function as a generic representation of the income distribution, and using the Gini coefficient as a summary measure of inequality, we have derived the distribution of the two aggregation parameters over a range of Gini values generated with data for 28 OECD countries. Based on a survey of the empirical AIDS and QUAIDS model literature we have extracted a large number of estimated expenditure and price elasticities and calculated the implied aggregation effects in those estimates for alternative Gini values. We conclude that on average the effects are relatively small, even for large Gini values. (To view them in a broader context, they may well be no greater than the effects of misspecifying the underlying theoretical model; see Denton, Mountain, and Spencer, 2006.) However, there are situations (model parameter configurations) in which the effects can in fact be large, and one would want to be on guard for such situations. We have proposed a simple procedure for evaluating the likely sensitivity of elasticity estimates to aggregation effects after estimating an AIDS or QUAIDS model with aggregate data. The procedure can be applied by someone who has estimated the model, or by the reader of a study in which the model is reported.

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Table 1: Gini Indexes and Associated Aggregation Parameters Based on Lognormal Income Distribution: 28 OECD Member Countries

		Aggregation parame		parameters	eters	
Country	Gini coefficient (d)	g	ĝ	h	ĥ	
1. Denmark	.247	.099	.099	.209	.209	
2. Japan	.249	.101	.101	.212	.212	
3. Sweden	.250	.102	.101	.215	.214	
4. Czech Republic	.254	.105	.105	.222	.222	
5. Norway	.258	.108	.108	.229	.230	
6. Slovokia	.258	.108	.108	.229	.230	
7. Hungary	.269	.118	.118	.252	.252	
8. Finland	.269	.118	.118	.252	.252	
9. Germany	.283	.132	.132	.281	.282	
10. Austria	.291	.140	.140	.299	.300	
11. Netherlands	.309	.158	.158	.342	.343	
12. South Korea	.316	.166	.166	.360	.361	
13. Canada	.326	.177	.177	.387	.387	
14. France	.327	.178	.178	.390	.390	
15. Belgium	.330	.182	.182	.399	.398	
16. Switzerland	.337	.190	.190	.417	.418	
17. Ireland	.343	.198	.198	.436	.436	
18. Greece	.343	.198	.198	.436	.436	
19. Poland	.345	.200	.200	.442	.442	
20. Spain	.347	.203	.203	.448	.448	
21. Australia	.352	.209	.209	.464	.463	
22. Italy	.360	.219	.219	.488	.488	
23. United Kingdom	.360	.219	.219	.488	.488	
24. New Zealand	.362	.222	.222	.494	.495	
25. Portugal	.385	.254	.254	.575	.574	
26. United States	.408	.288	.288	.661	.662	
27. Turkey	.436	.334	.334	.783	.783	
28. Mexico	.495	.447	.447	1.098	1.098	

Note: Gini coefficients are from United Nations (2006). Other figures are calculated by the authors.

Table 2: Differences Between Macro and Micro Elasticities: Summary Measures for AIDS Models

9	Expenditure elasticities			Own-price elasticities		
Summary measure	d=min	d=med	d=max	d=min	d=med	d=max
		mo	dels estimated	with macro da	ata	
Mean difference (macro-micro)	056	136	084	.002	006	.051
Mean absolute difference	.056	.136	.363	.020	.044	.114
% negative differences	100.0	100.0	98.6	40.8	42.3	40.8
% sign differences	0.0	0.0	0.0	4.2	5.6	7.0
% elast/inelast differences	0.0	0.0	1.4	1.4	2.8	2.8
Number of observations	71	71	71	71	71	71
		mo	dels estimated	with micro d	ata	
Mean difference (macro-micro)	006	011	030	.008	.015	.041
Mean absolute difference	.006	.011	.030	.009	.017	.046
% negative differences	100.0	100.0	100.0	30.0	30.0	30.0
% sign differences	0.0	0.0	0.0	0.0	10.0	10.0
% elast/inelast differences	0.0	0.0	0.0	0.0	0.0	0.0
Number of observations	20	20	20	20	20	20
		models	estimated with	h macro or mi	cro data	
Mean difference (macro-micro)	045	109	072	.003	001	.048
Mean absolute difference	.045	.109	.289	.018	.038	.099
% negative differences	100.0	100.0	98.9	38.5	39.6	38.5
% sign differences	0.0	0.0	0.0	3.3	6.6	7.7
% elast/inelast differences	0.0	0.0	1.1	1.1	2.2	2.2
Number of observations	91	91	91	91	91	91

Note: d is the Gini coefficient; min, med, and max are the minimum, median, and maximum values derived from Table 1. "% elast/inelast differences" is the percentage of cases in which one of the elasticities is greater than 1 in absolute value, while the other is not.

Table 3: Differences Between Macro and Micro Elasticities: Summary Measures for QUAIDS Models

G.	Exp	Expenditure elasticities		Own-price elasticities		
Summary measure	d=min	d=med	d=max	d=min	d=med	d=max
		mo	dels estimated	with macro d	ata	
Mean difference (macro-micro)	049	090	492	.010	.011	194
Mean absolute difference	.054	.101	.527	.030	.056	.324
% negative differences	86.7	86.7	86.7	26.7	26.7	26.7
% sign differences	0.0	0.0	0.0	6.7	6.7	13.3
% elast/inelast differences	0.0	0.0	6.7	0.0	0.0	6.7
Number of observations	15	15	15	15	15	15
		mo	dels estimated	l with micro d	ata	
Mean difference (macro-micro)	024	045	161	004	009	036
Mean absolute difference	.049	.089	.256	.017	.031	.092
% negative differences	64.7	64.7	64.7	70.6	70.6	58.8
% sign differences	0.0	0.0	5.9	0.0	0.0	0.0
% elast/inelast differences	0.0	0.0	23.5	0.0	0.0	0.0
Number of observations	17	17	17	17	17	17
		models	estimated wit	h macro or mi	cro data	
Mean difference (macro-micro)	035	066	316	.002	.000	110
Mean absolute difference	.051	.094	.383	.023	.043	.201
% negative differences	75.0	75.0	75.0	50.0	50.0	43.8
% sign differences	0.0	0.0	3.1	3.1	3.1	6.2
% elast/inelast differences	0.0	0.0	15.6	0.0	0.0	3.1
Number of observations	32	32	32	32	32	32
rumber of observations	32	32	32	32	32	32

Note: See note to Table 2.

Table 4: Differences Between Macro and Micro Elasticities: Summary Measures for AIDS and QUAIDS Models Combined

a.	Expenditure elasticities			Own-price elasticities		
Summary measure	d=min	d=med	d=max	d=min	d=med	d=max
		moe	dels estimated	with macro d	ata	
Mean difference (macro-micro)	055	128	155	.003	003	.008
Mean absolute difference	.055	.130	.391	.022	.046	.150
% negative differences	97.7	97.7	96.5	38.4	39.5	38.4
% sign differences	0.0	0.0	0.0	4.7	5.8	8.1
% elast/inelast differences	0.0	0.0	2.3	1.2	2.3	3.5
Number of observations	86	86	86	86	86	86
		mo	dels estimated	l with micro d	ata	
Mean difference (macro-micro)	014	027	090	.002	.004	.006
Mean absolute difference	.026	.047	.134	.013	.024	.067
% negative differences	83.8	83.8	83.8	48.6	48.6	43.2
% sign differences	0.0	0.0	2.7	0.0	5.4	5.4
% elast/inelast differences	0.0	0.0	10.8	0.0	0.0	0.0
Number of observations	37	37	37	37	37	37
		models	estimated wit	h macro or mi	cro data	
Mean difference (macro-micro)	042	098	136	.003	001	.007
Mean absolute difference	.046	.105	.314	.019	.040	.125
% negative differences	93.5	93.5	92.7	41.5	42.3	39.8
% sign differences	0.0	0.0	0.8	3.3	5.7	7.3
% elast/inelast differences	0.0	0.0	4.9	0.8	1.6	2.4
Number of observations	123	123	123	123	123	123

Note: See note to Table 2.

#### APPENDIX TABLES

Appendix Table 1: Macro Expenditure Elasticities Based on Published Articles, and Calculated Micro Elasticities

Article, commodity	Macro elasticity	Calculated micro elasticities			
	based on article	d=min	d=med	d=max	
AIDS models					
A1 - 1	.210	.267	.308	.416	
	2.000	2.110	2.220	2.808	
2 3	.300	.345	.378	.467	
4	1.670	1.718	1.762	1.956	
5	1.220	1.225	1.702	1.244	
6	1.230	1.235	1.240	1.256	
7					
8	1.210	1.214	1.218	1.232	
8	1.400	1.416	1.431	1.487	
A2 - 1	.342	.382	.412	.492	
2	2.141	2.286	2.436	3.329	
3	.574	.591	.604	.642	
4	2.269	2.451	2.645	3.932	
A3 - 1	.266	.316	.352	.447	
2	.041	.124	.182	.329	
3	1.636	1.679	1.718	1.889	
4	1.567	1.601	1.631	1.759	
5	1.577	1.612	1.644	1.778	
	270	407	42.4	500	
A4 - 1	.370	.407	.434	.508	
2 3	.240	.293	.331	.433	
	.750	.756	.761	.775	
4	.010	.098	.160	.314	
5	.620	.634	.644	.675	
6	.440	.469	.491	.552	
7	.310	.354	.386	.473	
8	4.420	6.171	9.897	-5.468	
9	1.130	1.132	1.133	1.138	
10	.710	.718	.724	.743	
11	.390	.425	.450	.521	
A5 - 1	1.394	1.410	1.424	1.478	
2	.853	.855	.857	.862	
3	.211	.268	.309	.417	
4	.314	.358	.389	.475	
A6 - 1	1.524	1.553	1.579	1.684	
2	.462	.489	.510	.566	
3	.860	.862	.863	.868	
4	.893	.894	.895	.898	

Article, commodity	Macro elasticity	Calculated micro elasticities			
	based on article	d=min	d=med	d=max	
A7 - 1	1.094	1.095	1.096	1.098	
2	1.006	1.006	1.006	1.006	
3	.891	.892	.893	.896	
4	.982	.982	.982	.982	
5	1.136	1.138	1.139	1.145	
<b>A</b> 8 - 1	.965	.965	.965	.966	
2	.884	.885	.886	.890	
3	.965	.965	.965	.966	
4	1.128	1.130	1.131	1.136	
5	1.193	1.197	1.200	1.211	
6	1.065	1.065	1.066	1.067	
7	1.202	1.206	1.210	1.222	
<b>A</b> 9 - 1	.702	.711	.717	.737	
	1.092	1.093	1.094	1.096	
2 3	.371	.408	.435	.509	
4	1.271	1.278	1.285	1.308	
5	.946	.946	.947	.947	
6	1.644	1.688	1.728	1.904	
7	1.364	1.378	1.390	1.435	
8	1.742	1.801	1.856	2.110	
A10 - 1	1.292	1.301	1.308	1.336	
2	1.149	1.151	1.153	1.160	
3	.767	.772	.776	.789	
4	.657	.668	.677	.703	
A11 - 1	.853	.855	.857	.862	
2	.631	.644	.654	.683	
3	.970	.970	.970	.970	
4	2.732	3.090	3.516	8.661	
A12 - 1	.978	.978	.978	.978	
2	1.602	1.640	1.675	1.824	
3	.712	.720	.726	.745	
4	.403	.436	.461	.529	
5	.272	.321	.356	.451	
/	1.821	1.094	1.903	2.291	
6 7	.876 1.821	.878 1.894	.879 1.963	.883 2.297	

Article,	Macro elasticity	Calculated micro elasticities				
commodity	based on article	d=min	d=med	d=max		
QUAIDS models						
Q1 - 1	.119	.180	.225	.340		
2	.460	.493	.518	.586		
3	.250	.300	.336	.433		
4	1.298	1.307	1.315	1.345		
Q2 - 1	1.040	1.129	1.192	1.369		
2	.604	.695	.757	.914		
3	1.456	1.464	1.471	1.492		
4	1.133	1.168	1.195	1.286		
5	2.169	2.358	2.593	7.681		
6	.362	.510	.605	.822		
Q3 - 1	.767	.776	.783	.804		
2	1.222	1.183	1.146	.984		
3	1.188	1.220	1.247	1.335		
4	.851	.872	.888	.935		
5	1.328	1.325	1.321	1.304		

Article, commodity	Micro elasticity	Calculated macro elasticities				
	based on article	d=min	d=med	d=max		
AIDS models						
A13 - 1	1.053	1.053	1.053	1.052		
2	.862	.860	.859	.853		
3	.960	.959	.959	.959		
4	1.031	1.031	1.031	1.030		
A14 - 1	.770	.765	.760	.744		
2	.890	.889	.888	.884		
3	.700	.691	.683	.654		
4	.760	.754	.749	.731		
5	.760	.754	.749	.731		
6	.610	.594	.581	.528		
7	.730	.723	.716	.693		
8	.790	.786	.782	.768		
9	.620	.605	.592	.542		
10	1.060	1.060	1.059	1.058		
11	1.100	1.099	1.098	1.096		
11	1.100	1.099	1.096	1.090		
A15 - 1	1.305	1.296	1.289	1.268		
2	1.149	1.147	1.145	1.140		
3	.745	.738	.733	.712		
4	.542	.520	.501	.424		
5	1.247	1.241	1.236	1.222		
QUAIDS models						
Q4 - 1	.608	.585	.565	.484		
2	2.290	2.145	2.048	1.818		
3	.838	.929	.995	1.162		
4	.917	.858	.802	.545		
3 4 5	1.201	1.210	1.216	1.233		
6	1.448	1.404	1.369	1.255		
7	.845	.867	.885	.942		
Q5 - 1	.568	.549	.532	.465		
2	.475	.447	.421	.315		
3	1.139	1.097	1.062	.923		
3 4	1.279	1.226	1.180	1.002		
5	1.260	1.263	1.265	1.270		
Q6 - 1	.788	.806	.821	.872		
2	1.445	1.436	1.428	1.405		
3	.839	.907	.958	1.098		
4	1.334	1.268	1.210	.963		
5	.825	.701	.569	398		

Article, commodity	Macro elasticity	Calculated micro elasticities				
	based on article	d=min	d=med	d=max		
AIDS models						
A1 - 1	.100	.052	.019	058		
	730	719	707	625		
2 3 4	150	189	217	286		
4	210	161	115	.091		
5	440	433	427	406		
6	-1.020	-1.027	-1.033	-1.053		
7	780	780	779	777		
8	720	721	722	722		
A2 - 1	310	278	252	161		
2	-1.016	-1.033	-1.049	-1.130		
3	773	777	780	788		
4	-1.264	-1.381	-1.495	-2.158		
A3 - 1	458	450	442	408		
2	811	808	805	790		
3	454	430	407	303		
4	438	425	412	352		
5	421	421	420	403		
A4 - 1	250	270	283	315		
2	210	258	292	381		
2 3	500	507	513	529		
4	150	202	236	313		
5	650	660	667	688		
6	300	328	349	404		
7	300	336	361	427		
8	100	.250	1.053	-2.525		
9	860	859	859	857		
10	130	148	162	203		
11	.080	.024	016	127		
. ~ 1	245	250	250	402		
A5 - 1	345	359	370	402		
2	598	596	594	587		
3 4	082	133	168	257		
4	163	202	230	302		
A6 - 1	420	425	429	431		
2	607	601	596	576		
3	720	718	717	712		
4	368	370	372	377		
•	.500	.570	.5 / 2	.5,,		

Article,	Macro elasticity	Calculated micro elasticities			
commodity	based on article	d=min	d=med	d=max	
A7 - 1	418	417	416	412	
2	058	058	057	056	
3	215	218	221	229	
4	133	133	134	135	
5	203	194	186	160	
A8 - 1	636	635	633	630	
2	912	912	911	910	
3	673	672	671	668	
4	891	892	893	895	
5	985	986	987	990	
6	954	954	955	956	
7	999	-1.001	-1.003	-1.008	
A9 - 1	709	703	699	684	
2	641	641	642	643	
3	220	250	272	327	
4	384	374	365	331	
5	355	357	359	364	
6	340	301	264	102	
7	005	.029	.059	.173	
8	927	934	940	961	
A10 - 1	-1.070	-1.093	-1.112	-1.179	
2	-1.393	-1.404	-1.413	-1.444	
3	730	720	712	686	
4	644	647	649	654	
A11 - 1	436	430	424	406	
2	484	483	483	478	
3	424	425	425	428	
4	768	758	739	394	
A12 - 1	405	406	407	409	
2	-1.363	-1.390	-1.415	-1.518	
3	793	797	801	811	
4	407	431	449	496	
5	121	136	145	157	
6	386	390	393	403	
7	014	.015	.047	.235	

Article, commodity	Macro elasticity	(	Calculated micro elasticities		
	based on article	d=min	d=med	d=max	
QUAIDS models					
Q1 - 1	341	388	421	508	
2	139	183	215	302	
3	298	312	320	333	
4	155	172	185	229	
Q2 - 1	235	265	282	312	
2	.003	013	020	016	
3	381	355	330	224	
4	458	466	471	482	
4 5	473	378	246	3.106	
6	172	254	303	402	
Q3 - 1	177	179	179	180	
2	546	521	498	412	
3	-1.320	-1.312	-1.307	-1.299	
4	208	219	226	246	
5	-1.088	-1.119	-1.145	-1.235	

Article, commodity	Micro elasticity	Calculated macro elasticities			
	based on article	d=min	d=med	d=max	
AIDS models					
A13 - 1	220	219	217	212	
2	497	495	493	486	
3	456	455	455	453	
4	849	849	849	849	
A14 - 1	710	707	704	693	
2	350	343	338	319	
3	030	001	.025	.118	
4	.000	.024	.045	.119	
5	070	048	289	.039	
6	040	002	.306	.158	
7	980	982	983	989	
8	-1.070	-1.073	-1.075	-1.084	
9	850	846	843	829	
10	050	052	053	057	
11	440	437	435	428	
11	+ + 0	437	+33	420	
A15 - 1	596	585	577	548	
2	418	423	426	437	
3	402	403	404	404	
4	706	703	700	684	
5	-2.088	-2.060	-2.037	-1.969	
QUAIDS models					
Q4 - 1	354	356	356	349	
2	-1.582	-1.504	-1.450	-1.317	
3	448	474	494	548	
4	526	508	489	386	
3 4 5	483	488	492	502	
6	554	553	552	542	
7	705	706	706	709	
Q5 - 1	782	798	811	858	
2	767	761	756	732	
3	961	965	968	985	
3 4	-1.649	-1.678	-1.708	-1.854	
5	028	026	023	008	
Q6 - 1	654	654	654	651	
2	382	398	408	434	
3	441	448	453	461	
4	-1.509	-1.553	-1.598	-1.831	
5	-1.029	-1.060	-1.091	-1.291	

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