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A Note on Income Distribution and Growth

William Scarth*

June 2007

Abstract:

Many analysts expect the aging population to lead to a reduction in the growth of living standards. Income inequality – a problem that has been accentuated by the payroll tax hikes that were necessary to fund the public pension as the population ages – is becoming an increasing challenge at the same time. As a result, policy-makers need to pursue initiatives that can simultaneously address both our efficiency and our equity objectives. With the challenge of the aging population, it is all the more important that we not rely on fiscal policies that involve a trade-off between growth and equality. This paper identifies a strategy for tax policy that meets these objectives.

Keywords: fiscal policy, endogenous growth, efficiency and equity

JEL Classifications: E10, E60, H30, O40

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1. Introduction

The aging population has raised at least two challenges for economic policy. First, as Denton and Spencer (1998) have shown, the ongoing rate of increase in material living standards can be expected to slow if there is not an increase in productivity growth. Second, as Souare (2003) has shown, the increase in payroll tax rates that has been the Canadian solution to the fact that the aging population would have otherwise left our public pension system under funded, can be expected to raise Canada's natural unemployment rate. This development accentuates our unequal income-distribution problem. The suggested policy response to the first challenge is usually to argue for tax cuts for those individuals who do most of the saving and investing – that is, for high-income Canadians. The standard suggestion for the income-distribution challenge is to argue for tax cuts targeted to low-income Canadians. Since all tax cuts need to be financed, these two responses appear to conflict with one another. We seem to face a trade-off between our efficiency and our equity objectives; either taxes on the rich need to be raised to finance our equity-oriented policies, or taxes on the poor need to be raised to finance our growth-oriented initiatives. The purpose of this note is to draw attention to a literature that suggests that we may not face such a trade-off after all.

The remainder of the paper is organized as follows. In section 2, a simple growth model containing both rich and poor households is explained. In section 3, this model is used as a vehicle for addressing this debate. A revenue-neutral tax substitution is considered, and the conditions that must be satisfied for a tax cut for the poor to be both

pro-growth and pro-equity are identified. It is argued that these conditions are likely to be met in the actual Canadian economy. Concluding remarks, and reference to related studies, are offered in section 4.

2. A Simple Growth Model with Both Rich and Poor Households

We follow a suggestion in Barro and Sala-i-Martin (1995, p. 144-146) and consider a model with both physical and human capital, and we assume that the same production process can be used to produce all items in the economy (both forms of capital, private consumption goods, and government services). That production function is Cobb-Douglas:

$$Y = \gamma K^\alpha H^\beta (qJ)^{1-\alpha-\beta}$$

where the variables are: Y – output, K – physical capital (owned entirely by the rich), H – human capital owned by the rich, J – human capital owned by the poor, and q – the quality index that defines the effectiveness of each unit of the poor’s human capital on the job. The details concerning this quality index are explained below when the behaviour of firms is discussed.

The economy’s resource constraint is

$$Y = C + E + G + \dot{K} + \dot{H} + \dot{J}$$

This equation states that output takes the form of: C – consumption by rich households, plus E – expenditures by poor households, plus G – government programs, plus capital accumulation (increases in K , H and J (the dots indicate time derivatives)).

The government budget constraint is

$$G = \tau(rK + vH) + \lambda wJ$$

This equation states that there is no government debt, and that the budget is balanced at each point in time. Program spending is financed by a proportional tax levied on the income of the rich (the first term on the right-hand side) and a proportional tax on the income of the poor (the second term on the right-hand side). The new notation is: r – the rental rate earned by physical capital, v – the rent earned by human capital owned by the rich, w – the rent earned on the human capital owned by the poor, τ – the tax rate levied on the rich, and λ – the tax rate levied on the poor.

The remaining equations of the model define optimal behaviour for households and firms. Rich households operate as ever-lasting dynasties. They enjoy their work, so there is no labour-leisure choice and no potential shirking on the job. Their utility (U) function is simple and standard:

$$U = \int \ln C_t e^{-\rho t} dt$$

where ρ is the rate of time preference. Utility maximization leads to two conditions. The first is the Ramsey (1928) condition, which is the solution to the consumption-savings choice. Households save if the after-tax return on capital exceeds their rate of impatience, and saving makes positive growth in consumption possible. Hence:

$$\dot{C} / C = r(1 - \tau) - \rho.$$

The second optimizing rule is that each household's portfolio of assets must be in equilibrium, and since the rich pay the same tax rate on income from both physical and human capital, this requires that

$$r = v.$$

There are two differences between rich and poor households. First, the poor do not like their jobs, so they are tempted to shirk while renting out their human capital in

the workplace. Shirking results in a lower value of the worker's quality index, q . Second, these individuals are impatient. The implication of the shirking is explained when the behaviour of profit-maximizing firms is explained in the next paragraph. The implication of the impatience is straightforward. For these individuals, since their time-preference rate exceeds the after-tax return on saving, it is never rational to save. Thus, these households do not acquire the ownership of any physical capital, and they accumulate human capital only because they have to. (It is assumed that there is compulsory attendance in school, so even poor households must invest in the human capital that is required to keep a job on an ongoing basis (in a balanced-growth equilibrium).) The consumption function for these households is simply their budget constraint; they consume all their current resources at each point in time, and so (in Mankiw's (2000) terminology) they live "hand-to-mouth". This expenditure function is:

$$E = w(1 - \lambda)J - \dot{J}.$$

We complete the specification of the model by describing firms' behaviour. Profit maximization leads to four conditions. The first three are standard optimal hiring rules – that each factor be hired up to the point that its marginal product just equal its rental price:

$$\alpha Y / K = r$$

$$\beta Y / H = v$$

$$(1 - \alpha - \beta)Y / J = w$$

The final rule firms must follow to maximize profits is that they must pick the wage offered to poor workers that delivers the profit-maximizing level of worker effort on the job. We assume that Solow's (1979) simple specification applies; that is, we assume that

worker quality is a function of the after-tax wage: $q = q(w(1 - \lambda))$. This specification leads to the proposition that firms keep the after-tax wage constant. Hence, the following relationship completes the model:

$$w(1 - \lambda) = \Omega$$

where Ω is constant, and this implies that q is unaffected by tax policy.

This model can be specified in a more compact form, and the remainder of this section is devoted to explaining how. First, the equal-yield condition for the rich and the two optimal hiring rules for their factors imply

$$H / K = \beta / \alpha.$$

Second, the constancy of the poor's after-tax wage and the optimal hiring rule for their factor imply

$$J / K = ((1 - \alpha - \beta)(1 - \lambda) / \Omega)(Y / K).$$

Third, the production function can be divided through by K , and then these last two relationships can be substituted in. The result is

$$Y = AK$$

where

$$A = \phi(1 - \lambda)^\theta$$

$$\theta = (1 - \alpha - \beta) / (\alpha + \beta)$$

$$\phi = \gamma^{1/(\alpha+\beta)} (\beta / \alpha)^{\beta/(\alpha+\beta)} (q(1 - \alpha - \beta) / \Omega)^\theta$$

We see that the model has the detailed structure that has been outlined above, while at the same time, it can be solved as simply as the traditional “AK” model. Scarth (2007, p. 238-239) explains a standard property of this class of models – that there is no transitional dynamics. The system is always in its balanced-growth equilibrium. Balanced growth

means that Y, C, E, K, H and J all grow at the same rate. Several of the model's equations can be re-written so that this balanced-growth condition,

$\dot{C}/C = \dot{K}/K = \dot{H}/H = \dot{J}/J = n$, can be substituted in. First, divide the poor household's expenditure function through by K , and substitute in the (J/K) expression, $(Y/K) = A$, and the balanced-growth assumption. The result is

$$e = (1 - \lambda)(1 - \alpha - \beta)A(1 - n/\Omega) \quad (1)$$

where $e = E/K$. The economy's resource constraint can be re-written in a similar manner. We divide this relationship through by K , then substitute in the (H/K) , (J/K) and e expressions (and the balanced-growth condition) to get

$$A(1 - g) = c + (1 - \lambda)(1 - \alpha - \beta)A + n(1 + (\beta/\alpha)) \quad (2)$$

where $g = G/Y$ is the ratio of government program spending to *GDP*.

The government budget constraint is simplified by dividing through by Y and substituting in the optimal hiring rules. The result is

$$g = (\alpha + \beta)\tau + (1 - \alpha - \beta)\lambda. \quad (3)$$

The consumption function of the rich can be re-written as

$$n = r(1 - \tau) - \rho. \quad (4)$$

The model we solve in the next section is a six-equation system: equations (1) through (4) along with

$$r = \alpha A \quad (5)$$

and

$$A = \phi(1 - \lambda)^\theta \quad (6)$$

3. Policy Analysis

The system that was summarized at the end of the last section determines how six endogenous variables (n, c, e, r, A and λ) respond when there is an assumed change in any of the exogenous variables or parameters ($g, \tau, \alpha, \beta, \rho, \phi, \theta$, and Ω). Since we are interested in a revenue-neutral switch in taxes between the rich and the poor, we consider a once-for-all increase in the tax rate applied to the rich, τ , that makes possible a cut in the tax rate applied to the poor, λ . We examine the effects of this tax substitution on the *growth rate* of living standards that is shared by all individuals in the economy, n . This is the *slope* of the (log of the) per-capita consumption time path. We also check for the existence of any one-time adjustment in the *level* of this per-capita consumption time path (its *intercept*). Since the physical capital stock cannot jump at a point in time, this intercept-shift effect can be determined by assessing whether either c or e respond to the tax substitution.

The reason that a formal model is needed to sort things out can be best appreciated by focusing on equation (4), which is repeated here for convenience:

$$n = r(1 - \tau) - \rho. \tag{4}$$

The direct effect of raising the tax rate on the rich households is that the economy's growth rate is reduced. Other things equal, the increase in τ lowers the net yield on saving, so rich households accumulate less capital. But other things are not equal. The government uses the revenue to cut the tax rate that poor workers must pay. Each firm responds to the reduction in shirking by cutting the pre-tax level of the wage, and this leads firms as a group to rent more human capital (that is owned by the poor) each period. Since physical capital then has more of this complementary factor to work with, its pre-

tax rate of return, r , is pushed up. If the “good news” development (the higher r) dominates the “bad news” development (the lower $(1 - \tau)$), this pro-equity tax substitution can be pro-growth (pro-efficiency) as well.

To check on this no-trade-off possibility, and on the one-time consumption level effects, we derive the appropriate policy multipliers:

$$dn / d\tau = r(\lambda - \tau) / (1 - \lambda) \quad (7)$$

$$dc / d\tau = 0 \quad (8)$$

$$(de / e) / d\tau = [(1 / (1 - \alpha - \beta)) - (r(\lambda - \tau) / (\Omega - n))] / (1 - \lambda) \quad (9)$$

The effects of the tax substitution depend on which tax rate is the bigger one before the initiative takes place. If the tax rate applied to the rich is initially greater than the tax rate applied to the poor, then raising that tax rate further involves a trade-off. In this case, a higher tax rate on the rich hurts the rich; it causes no one-time change in per-capita consumption, but it reduces the growth rate of consumption. The poor may be helped, since they enjoy a one-time increase in per-capita consumption. However, the poor lose in that their consumption grows more slowly. If the poor households’ rate of impatience is high enough, the short-term gain dominates the longer-term pain, and they are better off. But there is still a trade-off, since the other segment of the population is hurt.

The outcomes are quite different if the tax rate on the poor is the bigger levy in the first instance. This may seem like an unappealing case to consider. However, it is the case that, with the claw-backs involved in welfare programs, most low-income Canadians face an effective marginal tax rate of at least 80%. The only parameter (in this model) that can be set to reflect this “welfare wall” is λ . To do so, we consider $\lambda > \tau$. With this initial condition, an increase in the tax rate applied to the rich helps the rich – since the

ongoing growth rate of consumption rises. This result emerges since the higher tax on the rich – while a distortion in itself – allows the government to remove an even more damaging pre-existing distortion (the higher tax applied to the poor). This is an example of the general theorem of the second-best, and of the Bhagwati-Ramaswami (1963) proposition – that it is best to remove the biggest pre-existing distortion and to remove it as close as possible to its source. (For a fuller discussion, see Scarth (2005).)

The preceding discussion provided the intuition behind why it makes sense that a tax cut for the poor increases efficiency (the economy's ongoing growth rate). But what about equality? It may seem obvious that the poor would benefit from this development. But policy result (9) indicates that when the initial λ exceeds the initial τ , it is theoretically possible for the poor households to be worse off in a one-time-level sense. Why is this? The direct effect of the lower tax rate for these households is favourable (it raises disposable income). But there is an indirect effect. The higher growth rate means that a higher proportion of that disposable income must be set aside to finance the household's investment in human capital. Nevertheless, for plausible parameter values, this indirect effect is clearly dominated by the direct effect, so $(de/d\tau)$ is negative. To appreciate this, it is instructive to focus on the expression for the poor households' income – before taxes but after investment expenses – as a proportion of *GDP*:

$E/Y = (1 - \alpha - \beta)(\Omega - n)/\Omega$. Representative parameters values are: $\alpha = \beta = 1/3$, $n = 0.02$, and $\Omega = 0.10$. These parameter values imply that the poor households receive a third of *GDP* before they pay their investment expenses and their taxes, and that those investment expenses constitute 20 percent of that pre-tax income. These values make expression (9) negative by a wide margin.

4. Conclusions

Several recent studies have emphasized similar outcomes. Osberg (1995) surveyed a number of papers (and he paid particular attention to the endogenous growth literature) to stress that there are good theoretical reasons to expect that we may not need to trade-off our equity and our efficiency objectives. Scarth (2005) followed up by providing a specific straightforward analysis that illustrated the direct applicability of this view. Scarth showed that an employment subsidy (as championed by Phelps (1997)) can simultaneously lower unemployment and raise the productivity growth rate. He explained how it is the reduction in the magnitude of a pre-existing distortion – unemployment caused by unions – that makes a Pareto improvement occur (both rich and poor become better off). Kappius (2007) has extended Scarth’s analysis to make it more relevant for the Canadian economy. In particular, she considered an open (not a closed) economy, and she allowed both rich and poor households to be unemployed – via Summers’ (1988) efficiency-wage specification. She found that only the employment subsidy granted to firms – not a personal income tax cut for the poor (such as the federal government’s new Working Income Tax Benefit) – raised the productivity growth rate with this specification of efficiency wages. Since Kappius found little difference in results as she shifted from the closed, to the open, economy, we have limited our attention here to the closed-economy case. Our analysis – which is based on Solow’s specification of efficiency wages, not Summers’ – shows that the effects of the tax substitution on the growth rate are model-specific. So, comparing the results of these three closely related studies – Scarth, Kappius and the present paper – we conclude that more analytical and

empirical study is needed before this endogenous growth literature can be said to lead to clear policy advice.

Despite this acknowledgement of limited understanding at this stage, however, some optimism and sense of accomplishment is warranted. The paper has used an entirely mainstream and simple model to show that – in this setting at least – a conventional tax-substitution policy is sufficient to generate progress on both efficiency and equity fronts. The previous analyses indicated that an employment-subsidy program offered to firms must be set up to achieve this joint outcome. It is important that an administratively feasible initiative (such as a change within the existing income tax system – not the creation of a whole new administrative structure) be identified as sufficient to meet our objectives. After all, actual policy arrangements need to be in place before the pressures of the aging population make the need for progress on both fronts ever more compelling.

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