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MIXED ESTIMATION WHEN THE MODEL AND/OR STOCHASTIC RESTRICTIONS ARE NONLINEAR

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QSEP Research Report No. 345

January 2000

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ABSTRACT

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The standard mixed estimation method allows the incorporation of linear stochastic constraints into the estimation of a linear regression model. The present paper shows how the method can be adapted and extended to accommodate nonlinearities in the model, in the constraints, or both. As an illustration, it shows how nonlinear constraints can be defined so as to impose strict bounds on parameters of the model, or functions of parameters.

MIXED ESTIMATION WHEN THE MODEL AND/OR STOCHASTIC RESTRICTIONS ARE NONLINEAR*

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1. INTRODUCTION

The mixed estimation method for incorporating prior stochastic information into the estimation of a linear regression model was developed by Theil and Goldberger (1961), following earlier work by Durbin (1953). It was extended by Theil (1963) and others over the years that followed. (Srivastava, 1980, provides an annotated bibliography covering the period to the late 1970s; for a subsequent interpretation and evaluation of the estimator, in a quadratic risk framework, see Mittelhammer and Conway, 1988.) Whether the method should be regarded as essentially Bayesian is a matter of debate. Goldberger (1989) observed that he was unaware of Bayesian ideas when the 1961 paper that he coauthored was written; Theil (1963, p. 401) refers to his paper as "more classical than Bayesian, although its subject is essentially the same: the combination of prior and sample information, both being stochastic but independent of each other." The method can in fact be developed within a Bayesian framework (Theil, 1971; Amemiya, 1985). But it can also be developed as a straightforward extension of classical regression procedures, one in which prior information is converted (implicitly or explicitly) into additional "observations" with a prespecified error covariance matrix, so that generalized least

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*My thanks to Lonnie Magee for helpful discussions, and for comments on an earlier version of this paper.

squares can be applied. Whatever its theoretical pedigree, the mixed estimation method is a useful, flexible, and easy-to-apply practical tool in regression analysis. It can be especially helpful in situations in which the sample size is small or there is strong multicollinearity.

The standard method proposed by Theil and Goldberger, and carried forward in the econometrics literature, assumes a linear model subject to one or more linear stochastic restrictions on the model's parameters. Kennedy (1991) has shown how the method can be extended to allow the coefficients of the stochastic restrictions to be themselves nonlinear functions of other unknown parameters for which there are unbiased estimates. The purpose of the present paper is to extend the method in a different direction, to encompass situations in which the basic model is nonlinear, the restrictions take the form of stochastic nonlinear constraints on the parameters of the basic model, or both.

The development begins with a review and then a reformulation of the standard stochastically constrained linear model. The model in its new form is then adapted so as to admit nonlinear functions. Of particular interest are nonlinearities in the parameter restrictions that place absolute bounds on individual coefficients or functions of coefficients, while remaining stochastic over some region of the parameter space. Examples are provided to illustrate the applicability of the general nonlinear mixed estimation method. They include estimation of AR(1) and AR(2) models subject to stationarity restrictions and estimation of a nonlinear production function subject to restrictions on returns to scale and the capital input parameter.

It is perhaps worth emphasizing (to avoid misunderstanding in what follows) that issues relating to the underpinnings of the mixed estimation method in subjective probability theory are not of concern in this paper. Issues of that kind that may arise in the context of the nonlinear version of the method are the same as those that may arise in the context of the linear version, and have therefore been discussed in the literature. (For a helpful review, see Mittelhammer and Conway, 1988.) No position is taken here on such issues. The aim is simply to extend the method's range of practical applicability.

2. THE STANDARD STOCHASTICALLY CONSTRAINED LINEAR MODEL

Using obvious notation, the basic (unrestricted) linear model is $y = X\beta + \varepsilon$, where y and ε are $n \mathrm{x} 1$, β is $m \mathrm{x} 1$, X is a nonstochastic $n \mathrm{x} m$ matrix of full rank $k < n$, $E(X'\varepsilon) = 0$, and $\varepsilon \sim N(0, \Sigma)$, with Σ positive definite. To this model are added r stochastic restrictions of the form $c = A\beta + v$, where c and v are $r \mathrm{x} 1$, A is $r \mathrm{x} m$, $v \sim N(0, \Omega)$, and the matrix of covariances between v and ε is null. A, c, and Ω are assumed to be known, and constitute the prior information brought to bear on the estimation of β. The complete model can then be written as

$$(1) \quad \begin{bmatrix} y \\ c \end{bmatrix} = \begin{bmatrix} X \\ A \end{bmatrix} \beta + \begin{bmatrix} \varepsilon \\ v \end{bmatrix}$$

$$\text{where} \quad \mathrm{Var} \begin{bmatrix} \varepsilon \\ v \end{bmatrix} = \begin{bmatrix} \Sigma & 0 \\ 0 & \Omega \end{bmatrix}$$

In effect, additional "observations" are appended to the sample data: c to the y vector, A to the X matrix. The model can be estimated by a straightforward application of generalized least squares. (Ω is assumed known; Σ must be estimated.) As with all procedures that incorporate prior information, the biasedness or unbiasedness of the estimator depends on the accuracy of that information. (Theil, 1963, suggests what he calls a compatibility statistic for checking for agreement or conflict between sample data and prior information; the statistic has a chi-square distribution under the null hypothesis of full agreement.) It can be shown that (as intuition would suggest) as the variances of the elements of v tend to zero, the mixed (stochastically constrained) estimator approaches the restricted (exactly constrained) GLS estimator (Amemiya, 1985). Conversely, it is easy to show that as the variances tend to infinity, the mixed estimator approaches the unrestricted GLS estimator. The suggested procedure for obtaining an initial estimate of Σ is to base it on the residuals from unrestricted GLS estimation of the model; a revised estimate can be based subsequently on the residuals corresponding to the elements of ε after the mixed estimator has been applied. If desired, the mixed estimator can be iterated, using successively revised estimates of Σ.

3. AN ALTERNATIVE FORM OF THE LINEAR MODEL

To pave the way for the subsequent introduction of nonlinearities, the mixed linear model can be rewritten as follows. Define $y^* = [y' \ c']'$, $X^* = [X' \ K']'$, $u = [\varepsilon' \ v']'$, with typical elements y_i^*, x_{ij}^*, u_i. K is an $r \times m$ matrix, each column of which consists of identical constant elements. Equation (1) can now be written as

$$(2) \quad y_i^* = \sum_{j=1}^{m} \beta_j x_{ij}^* \bar{d}_i + \sum_{k=1}^{r} \sum_{j=1}^{m} \beta_j a_{kj} d_{ik} + u_i \qquad (i=1,\ldots,n+r)$$

β_j and a_{kj} are typical elements of β and A; $\bar{d}_i$ and d_{ik} are dummy variables, defined as follows: $\bar{d}_i = 1$ if $i \le n$, zero otherwise; $d_{ik} = 1$ if $i = n+k$, zero otherwise. (Note that the elements of K could all be zero for the linear model, in which case $\bar{d}_i$ would be redundant. However, for models with nonlinear functions that may not be the case, and so the more general form is retained here.) Equation (2) can be estimated by GLS by combining terms involving the β_j and applying OLS, after transformations that convert $\mathrm{Var}(u)$ to a scalar matrix. As before, the unrestricted OLS residuals can be used in estimating the necessary transformations for observations $1,\ldots,n$.

4. INTRODUCING NONLINEARITIES

Suppose now that the basic model and the constraints are (possibly) nonlinear. Let the basic model be $y_i = g(\beta, x_i) + \varepsilon_i$ $(i=1,\ldots,n)$ and the constraints $c_k = h_k(\beta) + v_k$ $(k=1,\ldots,r)$, where g and the h_k are arbitrarily chosen functions. Adapting equation (2), the mixed model can be written as

$$(3) \quad y_i^* = g(\beta, x_i^*)\bar{d}_i + \sum_{k=1}^{r} h_k(\beta)\, d_{ik} + u_i \qquad (i=1,\ldots,n+r)$$

where y_i^*, u_i, $\bar{d}_i$, and d_{ik} are as defined previously, and x_i^* denotes the i^{th} row of X^*. Special cases of interest include those in which g is linear (in β), all of the h_k are linear, some of the h_k are linear and some nonlinear, and both g and all of the h_k are linear (in which case we are back at equation (2)). Note that for estimation to succeed the constants in the columns of the K matrix must be chosen so that $g(\beta, x_i^*)$ is defined for $i>n$, even though the values delivered by the function will be negated by the dummy variable $\bar{d}_i$. (One wants to avoid values that would cause the computer program used for estimation to attempt to take the logarithm of a nonpositive number, divide by zero, etc.)

As with all nonlinear equations, how GLS is applied to equation (3) depends on the nature of the nonlinearities and the pattern of the error variances and covariances. A general treatment is probably therefore not helpful. However, the problem is simplified greatly if the Σ and Ω matrices are diagonal.

Let Σ and Ω be diagonal, then, with typical diagonal elements σ_i^2 and ω_k^2. Define y_i^{**}, u_i^*, $\bar{d}_i^*$, and d_{ik}^* as y_i^*, u_i, $\bar{d}_i$, and d_{ik}, each divided by σ_i if $i \le n$, or by ω_k if $i = n+k$. (Where $\bar{d}_i$ and the d_{ik} are zero the transformations of them have no effect; including them in this way merely simplifies the description, and perhaps the programming of the calculations.) The transformed version of equation (3) is thus

$$(4) \quad y_i^{**} = g(\beta, x_i^{*})\bar{d}_i^{*} + \sum_{k=1}^{r} h_k(\beta) d_{ik}^{*} + u_i^{*} \qquad (i = 1, \ldots, n+r)$$

The error u_i^* is distributed N(0,1) for all i and (4) can be estimated by nonlinear least squares (NLS), using standard software, with σ_i replaced by an estimate based on the unrestricted NLS residuals. The method can be iterated in the same way as before.

Requiring Ω to be diagonal may seem highly restrictive, but it is not: nonzero covariances can be allowed for by incorporating additional stochastic constraints (and thus increasing the number of diagonal elements). For example, suppose that two parameters, β_1 and β_2, are constrained by $c_1 = h_1(\beta_1,\beta_2) + v_1$ and $c_2 = h_2(\beta_1,\beta_2) + v_2$, with $var(v_1)$ and $var(v_2)$ given. Now add a third constraint, $c_3 = h_3(\beta_1,\beta_2) + v_3$, where $c_3 = c_1 + c_2$, $h_3 = h_1 + h_2$, and $v_3 = v_1 + v_2$. The covariance of v_1 and v_2 is equal to $(var(v_3) - var(v_1) - var(v_2))/2$. A desired covariance can thus be introduced by an appropriate specification of var (v_3). As a practical matter, experience suggests that this way of accommodating covariances for the mixed estimator (whether linear or nonlinear) is in fact more natural than specifying them directly.

5. STRICT BOUNDARY CONSTRAINTS WITH NONLINEAR STOCHASTIC RESTRICTIONS

There are many types of nonlinear stochastic restrictions on the parameters of a model that might be appropriate in particular situations. Of special interest are restrictions that place strict bounds on a parameter or function of parameters. Suppose, for example, that β_1 must be strictly positive, with prior mean 0.1. A linear stochastic restriction would be $0.1 = \beta_1 + v_1$. In order that a nonpositive estimate of β_1 could occur only with very low probability, the distribution about 0.1 would have to be very tight: $var(v_1)$ would have to be very small. But that might not be realistic. Although the prior mean is 0.1, estimated values of β_1 substantially higher than that might be acceptable even though ones substantially lower were not. That is to say, a symmetric distribution about 0.1 might be too constraining. If nonlinear restrictions are admitted a log transformation can be used, suggesting a stochastic restriction of the form $\ln(0.1) = \ln(\beta_1) + v_1$, with $v_1 \sim N(0,\omega_1^2)$. The estimate of β_1 would then necessarily lie in $(0,\infty)$, thus imposing both the required lower bound and the desired right-side skewness. (In the transformed restriction, $\ln(0.1)$ is the prior mean of $\ln(\beta_1)$. Its choice is suggested by the prior mean in the linear restriction used in the example, but the mean log is of course not equal to the log of the mean. It will be convenient in what follows to refer to a value such as 0.1 as the prior "best guess" of β_1; $\ln(0.1)$ is then to be interpreted as the log transform of the "best guess.")

A general form of one-sided log-based restriction is $c_k = h_k(\beta) + v_k$, for some k, with $v_k \sim N(0,\omega_k^2)$ and h_k an appropriately chosen logarithmic function. To constrain a particular parameter, say β_1, to lie in the interval (θ,∞), set $h_k(\beta) = \ln(\beta_1 - \theta)$; to constrain it to lie in $(-\infty,\theta)$, set $h_k(\beta) = \ln(\theta - \beta_1)$; to constrain β_1 to be less than β_2, set $h_k(\beta) = \ln(\beta_2 - \beta_1)$; and so on.

A two-sided restriction can be imposed by employing a logit-type transformation. Suppose now that β_1 must lie in the interval (θ_1,θ_2). Set $h_k(\beta) = \ln\big((\beta_1-\theta_1)/(\theta_2-\beta_1)\big)$. This imposes a symmetric prior distribution on β_1 with absolute bounds at θ_1 and θ_2.

It is obvious, but perhaps worth emphasizing, that log and logit-type transformations are not the only transformations that can be used to impose boundary restrictions. They are simply convenient, and are used below to illustrate the more general method.

6. ILLUSTRATIVE EXAMPLES

Three examples are provided below for the mixed estimator with nonlinearities. The first two involve linear equations subject to nonlinear constraints; the third involves a nonlinear equation subject to a linear and a nonlinear constraint. The estimation results are shown in Table 1. For comparison, both constrained and unconstrained estimates are shown in each case.

The covariance matrix over the range $1, \ldots, n$ of actual observations is assumed to be scalar in all three examples; thus there is only a single value σ^2 to be estimated from the unconstrained residuals in order to implement the mixed estimation procedure.

SHAZAM, Version 8, was used for estimation (SHAZAM, 1997). As with all nonlinear estimation problems, some care is required in choice of convergence criterion, maximum number of iterations, choice of analytic vs. numerically calculated derivatives, and starting values for the parameters. (Problems that can be encountered in practice in nonlinear estimation are underscored by McCullough's 1999 review of some widely used econometric software.) Given the possibility of more than one (local) minimum point, the specification of starting values can be of considerable importance. However, with mixed estimation the constraints provide guidance: if a prior best guess is specified for some parameter in the form of a stochastic restriction, that provides an obvious starting value; if two parameters are jointly constrained by some prior (stochastic) function, that suggests a restriction to be imposed in the choice of starting values; and so on.

7. EXAMPLE 1: REQUIRING AN AR(1) MODEL TO BE STATIONARY

The first example involves the AR(1) model $x_t = \alpha_1 x_{t-1} + \varepsilon_t$, where ε_t is $N(0,1)$ white noise and the true value of α_1 is set at 0.98, close to the stationary/nonstationary boundary. There are 40 observations, generated randomly (plus one for the initial lag). It is assumed that for theoretical reasons the true model is known to be stationary. Unconstrained OLS estimation (1(a) in Table 1) produces an estimate of 1.0124 for α_1, an inadmissible value. (For purposes of the example, a few sets of random ε values were generated and one that resulted in an unconstrained estimate greater than one was selected.)

Now suppose that α_1 is believed to be high and positive. A best guess of 0.9 is chosen (in 1(b)), and a stochastic restriction of the form $\ln(0.1) = \ln(1-\alpha_1) + v_1$ is introduced, thus producing a left-skewed prior distribution, with α_1 strictly less than one. The standard deviation ω_1 of v_1 is set at 1.15, implying (under the assumption of normality) that about 95 percent of the prior probability distribution of $\ln(1-\alpha_1)$ lies in the range $\ln(0.01)$ to $\ln(0.9974)$, with the corresponding value of α_1 in the range 0.0026 to 0.99. Fitting the equation under this restriction produces an estimate of 0.9840 for α_1.

It is a good idea (as recommended in the literature on linear mixed estimation) to experiment with alternative best guesses and alternative values of the prior variance. Some experimentation with alternatives indicates that the results in the table are not very sensitive to moderate changes in the choices. However, since the present aim is simply to show how non-linearities can be introduced into the mixed estimation procedure that point is not emphasized here.

8. EXAMPLE 2: REQUIRING AN AR(2) MODEL TO BE STATIONARY

The second example follows similar lines to the first, except that now the model is an AR(2): $x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \varepsilon_t$. Again there are 40 observations, generated artificially from an N(0,1) distribution for ε. The true values of α_1 and α_2 are 0.70 and 0.28, respectively. The requirements for an AR(2) model to be stationary are $\alpha_2 > -1$, $\alpha_1 + \alpha_2 < 1$, and $\alpha_1 - \alpha_2 > -1$ (Hamilton, 1994, for example). The true model thus lies just within the stationary region.

As shown in 2(a) of Table 1, unconstrained OLS estimates α_1 at 0.6391 and α_2 at 0.3829, thus violating the stationarity requirement. The mixed estimation results, shown in 2(b), assume best guesses of zero for both α_1 and α_2, and log-based restrictions adapted from the three stationarity requirements stated above, with prior standard deviations $\omega_1 = \omega_2 = \omega_3 = 1.0$. The mixed estimator produces an estimate of 0.6618 for α_1 and 0.3278 for α_2, and hence a stationary model.

9. EXAMPLE 3: A CONSTRAINED NONLINEAR PRODUCTION FUNCTION

The third example uses actual U.S. data to estimate a nonlinear production function. The data consist of 33 annual observations on output, employment, and capital, covering the period 1921-1953, and taken from Table 3.1 of Klein (1962). The production function is Cobb-Douglas, and could be estimated by OLS in log-linear form. However, for purposes of the example it is assumed that the error term is additive, so that the function cannot be linearized by converting to logarithms. The function is thus of the form $Q_t = \alpha_0 K_t^{\alpha_1} L_t^{\alpha_2} e^{\alpha_3 t} + \varepsilon_t$. Estimation by unrestricted nonlinear least squares (3(a) in Table 1) produces values of α_1 and α_2, and of the associated returns-to-scale parameter $(\alpha_1 + \alpha_2)$, that are deemed to be too high (0.4148, 0.9160, and 1.3308, respectively). One problem is the collinearity among K, L, and the technical progress (time) variable t ; K and L, in particular, have a correlation coefficient of 0.97. (Although not shown in the table, estimation results for the log-linearized form of the function would also be deemed unacceptable by the same criteria.)

The first alternative, shown as 3(b), is re-estimation subject only to a constraint on returns to scale, of the form $1.0 = \alpha_1 + \alpha_2 + v_1$, with best guesses $\alpha_1 = 0.3$, $\alpha_2 = 0.7$, and a value of 0.05 for the standard deviation ω_1. That produces a more acceptable result for the returns to scale parameter, as one would expect, but the estimate of α_1 is (let us suppose, for the example) still deemed to be too high, and the estimate of α_2 too low, based on other studies and known national accounts factor shares. A second alternative, 3(c), is therefore tried.

The second alternative involves bounding α_1 by adding a logit-type restriction of the form $\ln(1.0) = \ln\left((\alpha_1 - 0.2)/(0.4 - \alpha_1)\right) + v_2$, thus requiring that α_1 lie in the interval (0.2, 0.4). That implies a nonlinear function constrained by both a linear and a nonlinear restriction. With a large value of 5.0 for ω_2 (reflecting a virtual lack of prior opinion about location within the interval), it produces estimates of 0.3547 for α_1 and 0.6485 for α_2, a rate of technical progress of 1.43 percent per annum, and a returns-to-scale parameter of 1.0032, all of which (let us assume) are more in conformity with prior information or beliefs.

10. CONCLUSION

The mixed estimation method can be adapted to allow nonlinear equations and nonlinear stochastic constraints, thus expanding greatly its range of applicability. In practice, an application requires no more than standard nonlinear econometric software. Of particular interest is the ability to specify nonlinear constraints that are stochastic but place strict bounds on parameters or functions of parameters. Log and logit-type transformations are convenient for imposing such constraints, and have been used in this paper for illustrative purposes.

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TABLE 1: ILLUSTRATIVE EXAMPLES

True Model	Estimation method (sample size)	Stochastic restrictions imposed in estimation	Estimated coefficients (standard errors)			
			α_0	α_1	α_2	α_3
1(a) $x_t = \alpha_1 x_{t-1} + \varepsilon_t$ ($\alpha_1 = 0.98,\ \sigma = 1.0$)	OLS (40)	No restrictions	-	1.0124 (0.0181)	-	-
(b) Same	MNGLS (40)	$\ln(0.1) = \ln(1-\alpha_1) + v_1$ ($\omega_1 = 1.15$)	-	0.9840 (0.0097)	-	-
2(a) $x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \varepsilon_t$ ($\alpha_1 = 0.70,\ \alpha_2 = 0.28,\ \sigma = 1.0$)	OLS (40)	No restrictions	-	0.6391 (0.1505)	0.3829 (0.1542)	-
(b) Same	MNGLS (40)	$\ln(1.0) = \ln(1+\alpha_2) + v_1$ ($\omega_1 = 1.0$) $\ln(1.0) = \ln(1-\alpha_1-\alpha_2) + v_2$ ($\omega_2 = 1.0$) $\ln(1.0) = \ln(1+\alpha_1-\alpha_2) + v_3$ ($\omega_3 = 1.0$)	-	0.6618 (0.1405)	0.3278 (0.1408)	-
3(a) $Q_t = \alpha_0 K_t^{\alpha_1} L_t^{\alpha_2} e^{\alpha_3 t} + \varepsilon_t$	NLS (33)	No restrictions	0.0821 (0.2381)	0.4148 (0.2464)	0.9160 (0.5846)	0.0093 (0.0077)
(b) Same	MNGLS (33)	$1.0 = \alpha_1 + \alpha_2 + v_1$ ($\omega_1 = 0.05$)	0.5095 (0.2570)	0.4281 (0.2270)	0.5749 (0.2323)	0.0141 (0.0021)
(c) Same	MNGLS (33)	$1.0 = \alpha_1 + \alpha_2 + v_1$ ($\omega_1 = 0.05$) $\ln(1.0) = \ln((\alpha_1 - 0.2)/(0.4-\alpha_1)) + v_2$ ($\omega_2 = 5.0$)	0.4441 (0.1530)	0.3547 (0.1136)	0.6485 (0.1240)	0.0143 (0.0018)

Note: OLS -- ordinary least squares; NLS --nonlinear least squares; MNGLS -- mixed nonlinear generalized least squares.

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