QSEP RESEARCH INSTITUTE FOR QUANTITATIVE STUDIES IN ECONOMICS AND POPULATION

ON THE SENSITIVITY OF AGGREGATE PRODUCTIVITY GROWTH RATES TO NOISY MEASUREMENT

FRANK T. DENTON

QSEP Research Report No. 415

McMaster University
SOCIAL SCIENCES

ON THE SENSITIVITY OF AGGREGATE PRODUCTIVITY GROWTH RATES TO NOISY MEASUREMENT

FRANK T. DENTON

QSEP Research Report No. 415

May 2007

Frank T. Denton is a QSEP Research Associate and a faculty member in the McMaster University Department of Economics.

This report is cross-listed as No. 192 in the McMaster University SEDAP Research Paper Series.

ON THE SENSITIVITY OF AGGREGATE PRODUCTIVITY GROWTH RATES TO NOISY MEASUREMENT

Frank T. Denton
McMaster University

Abstract: Aggregate rates of productivity growth are among the most closely watched indicators of economic performance. They are also among the most difficult to measure accurately. This paper explores the sensitivity of such rates to random measurement error using a simple generic model. The model allows for errors in the input and output components of the productivity ratio, with different variances, and for serial and cross correlation of the errors. The effects of the errors are considered from the point of view of growth rates themselves, changes in growth rates, and comparisons between rates in different countries.

Keywords: productivity; growth rates; measurement error

JEL Classification: O47

ON THE SENSITIVITY OF AGGREGATE PRODUCTIVITY GROWTH RATES TO NOISY MEASUREMENT[1]

Frank T. Denton
McMaster University

1. INTRODUCTION

"Productivity growth, a key driver of the economic expansion ... slowed in the second quarter, the government reported today ... . Business productivity, which is measured by dividing the economy's total output by the number of hours worked, rose 2.2 percent, down from 3.2 percent in the first quarter ... . The report came hours before the Federal Reserve, which is increasingly worried about the effect of rising wages and slowing productivity on inflation and the economy as a whole, is widely expected to raise its benchmark short-term interest rate a quarter point, to 3.5 percent." (Article on the New York Times web site by Vikas Bajaj, August 9, 2005.)

"Labour productivity in the Canadian business sector edged up 0.2% in the first three months of 2005 ... . In the United States, productivity increased 0.6%, three times the rate of growth in Canada." (Article in Statistics Canada's news release publication, The Daily, June 9, 2005.)

Did these changes actually occur? Did the productivity growth rate really decrease by one percentage point in the U.S., at least to a close approximation? Did productivity really rise three times as fast in the U.S. as in Canada? Maybe, maybe not. The purpose of this paper is to show how sensitive calculated productivity growth rates can be to statistical noise, and for that reason how great is the risk of overinterpreting them and giving them too much weight in policy decisions.

Aggregate rates of productivity growth (labour or multifactor) are among the most closely watched indicators of economic performance. They are also among the most difficult to measure accurately. Measurement issues arise on both the input and output sides. [2] On the input side are questions of how to allow for changes in the educational/occupational characteristics of the work force and its supply of hours, how to weight different inputs, how to measure the capital stock (more correctly, the services of the stock) – including especially how to choose a depreciation function – how to allow for quality change in the deflation of investment expenditure in perpetual inventory calculations, and so on. On the output side are all the national accounting issues that arise in the definition and calculation of GDP and its deflator indexes. These issues – conceptual, definitional, practical – are important, and dealing with them is in many cases far from straightforward. (Dealing with the ever-present problem of quality change in price deflators is most certainly not straightforward.) An issue that is easier to grasp is the role of noisy measurement.

Suppose there were unanimous agreement as to exactly what should be measured and the way in which productivity indexes should be calculated. There would still remain the actual observations on inputs and outputs and the associated errors. Errors of that kind are intellectually less interesting for an economist but can be quantitatively important. They can affect significantly how calculated productivity growth rates are judged and their implications for economic performance and policy decisions.

The cleanest way to begin to think about the errors and their implications is to imagine a labour productivity index in which both the input and output indexes come from well designed sample surveys. (On the input side the Canadian Labour Force Survey and the U.S. Current Population Survey are obvious examples.) Input and output are thus both subject to random sampling errors. The errors may be correlated with each other or serially correlated, and that has to be taken into account. Suppose initially though that there is no such correlation, that there is a single input and a single output, and that the productivity measure is the simple ratio of the two. The ratio is subject to two independently distributed random errors. Measurement of the *change* in the ratio from one period (a quarter, a year) to the next – the productivity growth rate – is then at the mercy of four random errors, two in each period. Going further, since one is likely to be interested in how much the growth rate has increased or decreased, the *change in the change* of the ratio from one period to the next is at the mercy of six random errors. The potential sensitivity of productivity growth rates is obvious in this simple case, and hence the need for care in interpreting fluctuations. Care is needed also (for similar reasons) in comparing growth rates in two countries, or in two regions or industries.

Calculated productivity growth rates are subject to pronounced fluctuations. To be sure, there are real shocks (strikes and weather variations are obvious examples). A large change in growth rate may therefore be real. But it may also be spurious. The nature of the productivity growth rate calculation implies the possibility of a high degree of sensitivity to error.

This paper then is about the effect of noisy measurement on the rate of growth of aggregate productivity. The point of view for which this is important is that of an analyst (or journalist, or the general public) concerned with short-run changes in the economy rather than longer-run trends. (The effects of pure noise can be smoothed out by longer-run averaging.) The paper focuses on pure observational error rather than definitional or conceptual errors (measuring an inappropriate variable or using an inappropriate theoretical framework for the construction of an index). The approach is generic and uncomplicated. We consider a hypothetical productivity ratio with assumed error properties in its input and output components, work out the resulting error properties of the growth rate of the ratio, and then calculate the probabilities of errors of different magnitudes in the growth rate.[3]

2. SETTING THE STAGE: A THEOREM ON ECONOMIC STATISTICS

As background for the subsequent argument I offer the following theorem on economic statistics, and four corollaries.

Theorem on Economic Statistics: With the exception of some financial and administrative statistics, all economic statistics are subject to measurement error. (If anyone can supply a verifiable counter example I will be happy to modify the theorem slightly.)

Proof: Go and work for a good central statistical agency and observe closely the problems and processes associated with the production of economic statistics over a wide range. (This was the method of proof employed originally by the author in an earlier professional incarnation.) Alternatively, simply be an informed, experienced and perceptive user of a wide variety of economic statistics.

Corollary 1: Input and output statistics are subject to measurement error.

Corollary 2: *A fortiori*, productivity statistics are subject to measurement error.

Corollary 3: *A fortiori squared*, productivity growth rates are subject to measurement error.

Corollary 4: *A fortiori to the fourth power*, changes in productivity growth rates are subject to measurement error.

The theorem could be expanded to encompass other types of statistics (for example, population censuses, which are certainly prone to measurement error). However it is sufficiently general in the above form for present purposes.

I hasten to add that the foregoing, and everything that follows in this paper, should not be taken as criticism of the producers of official productivity statistics. Much progress has been made over the years in the development and improvement of measures of productivity growth and continuing lively debate augurs well for further improvements in the future.[4] The target of this paper is not estimated productivity growth rates themselves but the tendency to overinterpret them – to read too much into changes in ratios that are inherently very sensitive to small errors in numerators and denominators.

3. FRAMEWORK

Consider a simple model. There is one observed output variable Q, one observed input variable X, and a calculated productivity ratio $P = Q / X$. (The situation could be complicated by considering multiple inputs and outputs but the importance of measurement error can be demonstrated with this simple case.) Now suppose that the input and output variables are subject to random measurement error.[5] Denoting logarithms (convenient for what follows) by lower case letters and attaching a time subscript, the observed log values of Q_t and X_t are $q_t = q_t^* + u_t$ and $x_t = x_t^* + v_t$, where an asterisk indicates a true value and u and v are the errors. Assume the time interval to be sufficiently short (a quarter, a year) that log differences can be interpreted as period-to-period rates of growth. Assume also (for simplicity) that true output is growing at a constant rate α , true input at a constant rate β, and true productivity therefore at a constant rate $\gamma = \alpha - \beta$. The observed productivity growth rate is then $\Delta p_t = \gamma + w_t$, where Δ denotes a backward first difference and $w_t = \Delta u_t - \Delta v_t$.

Serial and cross correlations among the errors are potentially important. To allow for their effects suppose u and v to be generated by the AR(1) processes $u_t = \rho u_{t-1} + \varepsilon_t$ and $v_t = \lambda v_{t-1} + \eta_t$, where ε and η are white noise variables with mean zero, variances σ_ε^2 and σ_η^2, and contemporaneous correlation $corr(\varepsilon_t, \eta_t) = \xi$.

4. ERROR VARIANCE: THE PERIOD-TO-PERIOD GROWTH RATE

The variance of the error in the period-to-period productivity growth rate is determined by the properties of the input and output error distributions. Denoting a variance by var and a covariance by cov,

(1) $\mathrm{var}(w_t) = \mathrm{var}(\Delta u_t) + \mathrm{var}(\Delta v_t) - 2\,\mathrm{cov}(\Delta u_t, \Delta v_t)$

Solving for the variances and covariances in terms of the model’s underlying parameters, equation (1) can be restated as

(2) $$\sigma_w^2 = 2\Big(\sigma_\varepsilon^2 / (1+\rho) + \sigma_\varepsilon^2 / (1+\lambda) - \xi\sigma_\varepsilon\sigma_\eta(2-\rho-\lambda) / (1-\rho\lambda)\Big)$$

or equivalently as

(3) $$\sigma_w^2 = 2\Big((1-\rho)\sigma_u^2 + (1-\lambda)\sigma_v^2 - \theta\sigma_u\sigma_v(2-\rho-\lambda\Big)$$

where σ_w^2, σ_u^2 and σ_v^2 are now symbols for the (constant) variances and $\theta = corr(u,v)$.

It is helpful in interpreting equation (3) to consider the special case in which u and v have a common variance $\bar{\sigma}^2$ and a common serial correlation coefficient $\bar{\rho}$. For this case the number of parameters is reduced from five to three and the equation becomes

(4) $$\sigma_w^2 = 4(1-\theta)(1-\bar{\rho})\bar{\sigma}^2$$

It is now easier to see what is going on. If the input and output errors are both serially uncorrelated and uncorrelated with each other the variance of the error in the productivity growth rate is 4 times the common input and output variance. Serial and cross correlation reduce σ_w^2 as long as the correlations are positive. If there is perfect positive correlation of either kind σ_w^2 becomes zero. In that case the input and output errors can have very large variances but produce zero variance in the productivity growth rate error. At the other extreme, if there is perfect *negative* correlation of both kinds the growth rate variance is 16 times the common input and output error variance. What the correlations are likely to be in practice depends on the measurement processes. If the input and output series are generated by independent surveys the cross correlation of errors may be zero and the question is then how much serial correlation there is. On the other hand, if both series come from the same survey (a survey of manufacturing establishments, say) one might expect the errors to be positively correlated with each other, and possibly positively serially correlated as well, thus reducing the productivity growth rate error variance substantially. As a general statement though, the possible range of error variance is very wide.

Could the errors in fact be negatively correlated? Possibly. That could happen if there were errors in timing because of faulty memory by respondents in a survey, or perhaps because of business accounting practices. Some payroll or hours worked might be attributed to the end of period t for accounting reasons when in fact the associated physical production occurred in period t+1. There would thus be a positive error in reported input in the first period and a corresponding negative error in the second. With quarterly productivity calculations the peculiarities of the seasonal adjustment process might produce a similar result. One of Lovell’s (1963) desirable properties for a method of seasonal adjustment is the preservation of products (and hence ratios) by the adjustment procedure but there is no guarantee that that would hold in practice if the numerator and denominator were adjusted separately.[6] Even if the productivity ratio or its growth rate were seasonally adjusted directly there could still be overadjustment in one period and underadjustment in the next. The productivity growth rate would already be a delicate statistic in unadjusted form. The uncertain reliability of seasonal adjustment with respect to small quarter-to-quarter changes is an additional concern.

5. ERROR VARIANCE: THE CHANGE IN THE GROWTH RATE

The error in the change in productivity growth rate from one period to the next is Δw_t and its variance is

$$(5)\ \operatorname{var}(\Delta w_t) = \operatorname{var}(w_t) + \operatorname{var}(w_{t-1}) - 2\operatorname{cov}(w_t, w_{t-1})$$

The variances are the same at t and $t-1$. Replacing them with the right side of equation (3) and working out the covariance), equation (5) can be rewritten as

$$(6)\ \sigma^2_{\Delta w} = 2\Big((1-\rho)(3-\rho)\sigma^2_u + (1-\lambda)(3-\lambda)\sigma^2_v$$
$$- ((1-\rho)(3-\rho) + (1-\lambda)(3-\lambda))\theta\sigma_u\sigma_v\Big)$$

Again the result can be simplified by considering the case in which the input and output errors have the same variance and the same serial correlation coefficient:

$$(7)\ \sigma^2_{\Delta w} = 4(1-\theta)(1-\bar{\rho})(3-\bar{\rho})\bar{\sigma}^2$$

If there is no serial or cross correlation in this case the variance of Δw is 12 times $\bar{\sigma}^2$. The variance is reduced by introducing positive serial and/or cross correlation and becomes zero if either type of correlation is perfect. The extreme in the other direction occurs if there is perfect negative correlation of both kinds, in which case the variance is 64 times $\bar{\sigma}^2$. The range of variance for the error in the first difference of the productivity growth rate is much wider than the range for the error in the growth rate itself.

6. ILLUSTRATIVE CALCULATIONS

Table 1 provides some calculations of error variances for the productivity growth rate under alternative parameter configurations. The calculations are based on equation (4) and assume u and v to be jointly normally distributed. The table shows for each configuration the value of σ_w and the probability that the absolute error in the percentage growth rate will exceed some specified level, ranging from 0.1 to 2.0 percentage points. To take an example, if $\overline{\sigma} = .01$ ($\overline{\sigma}^2 = .0001$) and both types of correlation are zero, the probability that w will exceed 1 percentage point is .617. Thus if the true growth rate is 2 percent the reported rate will be less than 1 percent or greater than 3 percent with probability .617. (To put it differently, the reported growth rate will overstate or understate the true rate by half, with that probability.) The probabilities are lower when $\overline{\sigma}$ is set at .005 ($\overline{\sigma}^2 = .000025$) and decline with the introduction of positive serial and/or cross correlation. High positive correlation is needed though to drive the probabilities of large errors close to zero.

Table 2 shows similar calculations for Δw, based on equation (7). For a given parameter configuration the variance and the corresponding error probabilities are of course greater for Δw than for w. The range of error in a reported change in growth rate is wider, in a probability sense.

The parameter configurations in Tables 1 and 2 allow for negative as well as positive correlation. The variances and error probabilities are much greater with negative correlation than with positive correlation, as the previous argument implies they must be.

7. LONG-TERM AVERAGES

The error variance is reduced markedly by averaging the growth rates over a long period. For annual growth rates the mean error over the n years ending with year t is

$$(8)\quad \overline{w}_t = (1/n)\sum_{k=0}^{n-1} w_{t-k} = (1/n)((u_t - u_{t-n}) - (v_t - v_{t-n}))$$

Assuming n is large enough that ρ^n and λ^n are effectively zero, $\text{var}(\overline{w}_t)$ is then given by

$$(9)\quad \sigma_{\overline{w}}^2 = (2/n^2)(\sigma_u^2 + \sigma_v^2 - 2\theta\sigma_u\sigma_v)$$

With common variance $\overline{\sigma}^2$ in the input and output error distributions this becomes

$$(10)\quad \sigma_{\overline{w}}^2 = (4/n^2)(1-\theta)\overline{\sigma}^2$$

If $n = 10$, $\theta = 0$, and $\overline{\sigma}^2 = .0001$, the variance of $\overline{w}$ is .000004 and the probability of an error of half a percentage point or more in the mean productivity growth rate is only .012 (assuming a normal distribution, as before). Even in the extreme case of perfect negative correlation ($\theta = -1$) the probability is still only .071. There may be other measurement difficulties associated with long-term averages but pure statistical noise is not likely to be a major problem.

8. INTERCOUNTRY COMPARISONS

Assume now two countries, A and B, with productivity growth rate errors w_A and w_B. The variance of w_A is given by equation (3) with the A subscript added, and similarly for w_B. It is reasonable to assume that the measurement errors in the two countries are uncorrelated, in which case $w_A - w_B$ (the error in the intercountry difference in growth rates) has variance $\sigma_A^2 + \sigma_B^2$. If also the parameters of the error distributions are the same in the two countries (and the variances and serial coefficients are the same for inputs and outputs) the variance reduces to

$$(11) \quad \sigma_{AB}^2 = 8(1-\bar{\rho})(1-\bar{\theta})\bar{\sigma}^2$$

where a bar again indicates a common parameter. For $\bar{\rho} = \bar{\theta} = 0$ and $\bar{\sigma}^2 = .0001$, σ_{AB}^2 is .0008. The probability of an error of half a percentage point or greater in the difference between the two productivity growth rates is then .86 and the probability of an error of one percentage point or greater is .73. In the absence of high values of $\bar{\rho}$ and/or $\bar{\theta}$ the errors in intercountry comparisons of short-run productivity growth rates can be very large.

Intercountry comparisons of productivity growth rates are common among OECD countries, for example. In Canada, comparisons with the United States are given special attention. (See the study by Baldwin et al., 2005.) Comparisons of that kind are interesting and reasonable if the scope for error is properly taken into account but the statement cited at the beginning of this paper – that U.S. business productivity grew three times as fast as Canadian business productivity in a particular quarter of the year – is surely a good example of overconfidence in the data.

9. SUMMING UP

The model used in this paper is simple and results based on it are intended merely for illustration. However, the results make it clear that small errors of measurement in inputs and outputs can translate into productivity growth rates with very large errors.[7] More caution is warranted than is generally exercised when interpreting such rates. Only if the input and output errors are known to be highly positively serially correlated and/or highly correlated with each other is it safe to attribute a high degree of accuracy to the rates, and more especially to changes in them. Negative serial correlation can have an opposite and large effect on accuracy. Long-term averaging can reduce the errors markedly but then problems other than simple statistical noise may come into play, such as those discussed in Diewert and Fox (1999). Productivity growth rates are viewed as among the most important indicators of economic performance. Unfortunately they may also be among the least reliable indicators in assessing short-run performance.

FOOTNOTES

1. The work underlying this paper was carried out as part of the SEDAP (Social and Economic Dimensions of an Aging Population) Research Program. On-going support for SEDAP is provided by the Social Sciences and Humanities Research Council of Canada. Bill Scarth provided helpful comments on an earlier version of the paper.

2. A good idea of measurement problems that have received attention is provided by the special issue of the Canadian Journal of Economics edited and summarized by Diewert, Nakamura and Sharpe (1999). The focus of the articles in that issue is the service sector of the economy and the "productivity paradox" – the apparent decline in productivity growth rates in spite of obvious rapid advances in technology. The issues discussed relate mainly to longer-run growth, as is appropriate to the focus. However, the high degree of year-to-year volatility exhibited by the rates for OECD countries in Tables 1 and 2 of the article by Diewert and Fox (1999) suggests that noisy measurement may be important in interpreting shorter-run variations, as argued in the present paper.

3. Van Biesebroeck (2002, 2004) explores alternative methods of estimating productivity and considers the role of measurement error in each method, theoretically and by simulation.

4. See OECD (2001) for a comprehensive treatment of concepts and procedures in the calculation of productivity indexes. See Diewert, Nakamura and Sharpe (1999) for a good summary of issues under debate.

5. To refer to the measurement error as random is simply to say that it is unknown or unpredictable. If it were known or predictable the obvious thing to do would be to remove it.

6. If the series being adjusted were in logarithmic form another of Lovell's desirable properties, the preservation of sums, would be the relevant one.

7. Only single-output/single-input productivity measures have been considered in the paper. The analysis becomes more complicated if multiple outputs and inputs are allowed for. The variances then depend on the aggregator functions – on whether they involve arithmetic or geometric weighting, and of course on the weights themselves. Under some weight configurations the variances are increased, under others they are decreased. The error correlations among individual outputs and inputs also play a role. But the general conclusions as to the sensitivity to measurement error still hold.

REFERENCES

Baldwin, John R., Jean-Pierre Maynard, Marc Tanguay, Fanny Wong and Beiling Yan (2005), A Comparison of Canadian and U.S Productivity Levels: An Exploration of Measurement Issues, Statistics Canada Research Paper, Catalogue 11F0027MIE, Ottawa: Industry Canada..

Diewert, W. Erwin and Kevin J. Fox (1999), "Can Measurement Error Explain the Productivity Paradox?," Canadian Journal of Economics: Special Issue on Service Sector Productivity and the Productivity Paradox, 32, 251-280.

Diewert, W. Erwin, Alice O. Nakamura and Andrew Sharpe (1999), "Introduction and Overview," Canadian Journal of Economics: Specialy Issue on Service Sector Productivity and the Productivity Paradox, 32, v-xv.

Lovell, Michael C. (1963), "Seasonal Adjustment of Economic Time Series and Multiple Regression Analysis," Journal of the American Statistical Association, 58, 993-1010.

OECD (2001), Measuring Productivity: Measurement of Aggregate and Industry-Level Productivity Growth, Paris: Organisation for Economic Co-operation and Development.

Van Biesebroeck (2002), "Impact of Measurement Error on Productivity Estimates," unpublished material for presentation at CIRANO workshop, University of Toronto, October 10, 2002.

Van Biesebroeck (2004), "Robustness of Productivity Estimates," National Bureau of Economic Research, Working Paper 10303.

Table 1: Distribution Characteristics of the Error in the Productivity Growth Rate

Parameter configuration	σ_w	Probability that absolute error exceeds r percentage points					
		r = 0.1	r = 0.2	r = 0.5	r = 1.0	r = 1.5	r = 2.0
No cross or serial correlation							
$\bar{\sigma} = .005, \theta = 0, \bar{\rho} = 0$	.0100	.920	.841	.617	.317	.134	.046
$\bar{\sigma} = .01, \theta = 0, \bar{\rho} = 0$	.0200	.960	.920	.803	.617	.453	.317
Cross but no serial correlation							
$\bar{\sigma} = .01, \theta = .5, \bar{\rho} = 0$	.0141	.944	.888	.724	.480	.289	.157
$\bar{\sigma} = .01, \theta = .9, \bar{\rho} = 0$	.0063	.874	.752	.429	.114	.018	.002
Serial but no cross correlation							
$\bar{\sigma} = .01, \theta = 0, \bar{\rho} = .5$	.0141	.944	.888	.724	.480	.289	.157
$\bar{\sigma} = .01, \theta = 0, \bar{\rho} = .9$	.0063	.874	.752	.429	.114	.018	.002
$\bar{\sigma} = .01, \theta = 0, \bar{\rho} = -.5$	.0245	.967	.935	.838	.683	.540	.414
$\bar{\sigma} = .01, \theta = 0, \bar{\rho} = -.9$	.0276	.971	.942	.856	.717	.586	.468
Serial and cross correlation							
$\bar{\sigma} = .01, \theta = .5, \bar{\rho} = .5$	.0100	.920	.841	.617	.317	.134	.046
$\bar{\sigma} = .01, \theta = .5, \bar{\rho} = .9$	.0045	.823	.655	.264	.253	.001	.000
$\bar{\sigma} = .01, \theta = .9, \bar{\rho} = .5$	.0045	.823	.655	.264	.253	.001	.000
$\bar{\sigma} = .01, \theta = .9, \bar{\rho} = .9$	.0020	.617	.317	.012	.000	.000	.000
$\bar{\sigma} = .01, \theta = .5, \bar{\rho} = -.5$	.0173	.954	.908	.773	.564	.386	.248
$\bar{\sigma} = .01, \theta = .5, \bar{\rho} = -.9$	.0195	.959	.918	.798	.608	.442	.305
$\bar{\sigma} = .01, \theta = .9, \bar{\rho} = -.5$	.0077	.897	.796	.519	.197	.053	.010
$\bar{\sigma} = .01, \theta = .9, \bar{\rho} = -.9$	.0087	.909	.819	.566	.251	.085	.022

Note: Calculations are based on equation (4).

Table 2: Distribution Characteristics of the Error in the First Difference of the Productivity Growth Rate

Parameter configuration	$\sigma_{\Delta w}$	Probability that absolute error exceeds r percentage points					
		r = 0.1	r = 0.2	r = 0.5	r = 1.0	r = 1.5	r = 2.0
No cross or serial correlation							
$\bar{\sigma} = .005, \theta = 0, \bar{\rho} = 0$	.0173	.954	.908	.773	.564	.386	.248
$\bar{\sigma} = .01, \theta = 0, \bar{\rho} = 0$	.0346	.977	.954	.885	.773	.665	.564
Cross but no serial correlation							
$\bar{\sigma} = .01, \theta = .5, \bar{\rho} = 0$	.0245	.967	.935	.838	.683	.540	.414
$\bar{\sigma} = .01, \theta = .9, \bar{\rho} = 0$	.0110	.927	.855	.648	.362	.171	.068
Serial but no cross correlation							
$\bar{\sigma} = .01, \theta = 0, \bar{\rho} = .5$	.0224	.964	.929	.823	.655	.502	.371
$\bar{\sigma} = .01, \theta = 0, \bar{\rho} = .9$	.0092	.913	.827	.585	.275	.102	.029
$\bar{\sigma} = .01, \theta = 0, \bar{\rho} = -.5$	.0458	.983	.965	.913	.827	.743	.663
$\bar{\sigma} = .01, \theta = 0, \bar{\rho} = -.9$	.0544	.985	.971	.927	.854	.783	.713
Serial and cross correlation							
$\bar{\sigma} = .01, \theta = .5, \bar{\rho} = .5$	.0158	.950	.899	.752	.527	.343	.206
$\bar{\sigma} = .01, \theta = .5, \bar{\rho} = .9$	.0065	.877	.758	.440	.123	.021	.002
$\bar{\sigma} = .01, \theta = .9, \bar{\rho} = .5$	.0071	.888	.777	.480	.157	.034	.005
$\bar{\sigma} = .01, \theta = .9, \bar{\rho} = .9$	.0029	.730	.490	.084	.001	.000	.000
$\bar{\sigma} = .01, \theta = .5, \bar{\rho} = -.5$	.0324	.975	.951	.877	.758	.643	.537
$\bar{\sigma} = .01, \theta = .5, \bar{\rho} = -.9$	.0385	.979	.959	.897	.795	.697	.603
$\bar{\sigma} = .01, \theta = .9, \bar{\rho} = -.5$	.0145	.945	.890	.730	.490	.301	.168
$\bar{\sigma} = .01, \theta = .9, \bar{\rho} = -.9$	.0172	.954	.908	.771	.561	.384	.245

Note: Calculations are based on equation (7).

Number	Title	Author(s)
(2003)		
No. 380:	Population Aging, Productivity, and Growth in Living Standards	W. Scarth
No. 381:	The Transition from Good to Poor Health: An Econometric Study of the Older Population	N.J. Buckley F.T. Denton A.L. Robb B.G. Spencer
No. 382:	The Evolution of High Incomes In Canada, 1920-2000	E. Saez M.R. Veall
No. 383:	Population Change and Economic Growth: The Long-Term Outlook	F.T. Denton B.G. Spencer
No. 384:	The Economic Legacy of Divorced and Separated Women in Old Age	L. McDonald A.L. Robb
No. 385:	National Catastrophic Drug Insurance Revisited: Who Would Benefit from Senator Kirby's Recommendations?	T.F. Crossley P.V. Grootendorst M.R. Veall
No. 386:	Wages in Canada: SCF, SLID, LFS and the Skill Premium	A.L. Robb L. Magee J.B. Burbidge
No. 387:	Socioeconomic Influence on the Health of Older People: Estimates Based on Two Longitudinal Surveys	N.J. Buckley F.T. Denton A.L. Robb B.G. Spencer
No. 388:	An Invitation to Multivariate Analysis: An Example About the Effect of Educational Attainment on Migration Propensities in Japan	A. Otomo K-L. Liaw
(2004)		
No. 389:	Financial Planning for Later Life: Subjective Understandings of Catalysts and Constraints	C.L. Kemp C.J. Rosenthal M. Denton
No. 390:	Exploring the Use of a Nonparametrically Generated Instrumental Variable in the Estimation of a Linear Parametric Equation	F.T. Denton
No. 391:	Borrowing Constraints, the Cost of Precautionary Saving, and Unemployment Insurance	T.F. Crossley H.W. Low

Number	Title	Author(s)
No. 392:	Healthy Aging at Older Ages: Are Income and Education Important?	N.J. Buckley F.T. Denton A.L. Robb B.G. Spencer
(2005)		
No. 393:	Where Have All The Home Care Workers Gone?	M. Denton I.S. Zeytinoglu S. Davies D. Hunter
No. 394:	Survey Results of the New Health Care Worker Study: Implications of Changing Employment Patterns	I.S. Zeytinoglu M. Denton S. Davies A. Baumann J. Blythe A. Higgins
No. 395:	Unexploited Connections Between Intra- and Inter-temporal Allocation	T.F. Crossley H.W. Low
No. 396:	Measurement Errors in Recall Food Expenditure Data	N. Ahmed M. Brzozowski T.F. Crossley
No. 397:	The Effect of Health Changes and Long-term Health on the Work Activity of Older Canadians	D.W.H. Au T.F. Crossley M. Schellhorn
No. 398:	Population Aging and the Macroeconomy: Explorations in the Use of Immigration as an Instrument of Control	F.T. Denton B.G. Spencer
No. 399:	Users and Suppliers of Physician Services: A Tale of Two Populations	F.T. Denton A. Gafni B.G. Spencer
No. 400:	MEDS-D Users' Manual	F.T. Denton C.H. Feaver B.G. Spencer
No. 401:	MEDS-E Users' Manual	F.T. Denton C.H. Feaver B.G. Spencer

Number	Title	Author(s)
No. 402:	Socioeconomic Influences on the Health of Older Canadians: Estimates Based on Two Longitudinal Surveys (Revised Version of No. 387)	N.J. Buckley F.T. Denton A.L. Robb B.G. Spencer
No. 403:	Population Aging in Canada: Software for Exploring the Implications for the Labour Force and the Productive Capacity of the Economy	F.T. Denton C.H. Feaver B.G. Spencer
(2006)		
No. 404:	Joint Taxation and the Labour Supply of Married Women: Evidence from the Canadian Tax Reform of 1988	T.F. Crossley S.H.Jeon
No. 405:	The Long-Run Cost of Job Loss as Measured by Consumption Changes	M. Browning T.F. Crossley
No. 406:	Do the Rich Save More in Canada?	S. Alan K. Atalay T.F. Crossley
No. 407:	The Social Cost-of-Living: Welfare Foundations and Estimation	T.F. Crossley K. Pendakur
No. 408:	The Top Shares of Older Earners in Canada	M.R. Veall
No. 409:	Estimating a Collective Household Model with Survey Data on Financial Satisfaction	R. Alessie T.F. Crossley V.A. Hildebrand
No. 410:	Physician Labour Supply in Canada: a Cohort Analysis	T.F. Crossley J. Hurley S.H. Jeon
No. 411:	Well-Being Throughout the Senior Years: An Issues Paper on Key Events and Transitions in Later Life	M. Denton K. Kusch
No. 412:	Satisfied Workers, Retained Workers: Effects of Work and Work Environment on Homecare Workers' Job Satisfaction, Stress, Physical Health, and Retention	I.U. Zeytinoglu M. Denton
(2007)		
No. 413:	Gender Inequality in the Wealth of Older Canadians	M. Denton L. Boos
No. 414:	Which Canadian Seniors are Below the Low-Income Measure?	M. Veall

Number	Title	Author(s)
No. 415:	On the Sensitivity of Aggregate Productivity Growth Rates to Noisy Measurement	F.T. Denton