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# TIME SERIES PROPERTIES AND STOCHASTIC FORECASTS: SOME ECONOMETRICS OF MORTALITY FROM THE CANADIAN LABORATORY

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# TIME SERIES PROPERTIES AND STOCHASTIC FORECASTS: SOME ECONOMETRICS OF MORTALITY FROM THE CANADIAN LABORATORY*

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## ABSTRACT

Methods for time series modeling of mortality and stochastic forecasting of life expectancies are explored, using Canadian data. Consideration is given first to alternative indexes of aggregate mortality. Age-sex group system models are then estimated. Issues in the forecasting of life expectancies are discussed and their quantitative implications investigated. Experimental stochastic forecasts are presented and discussed, based on nonparametric, partially parametric, and fully parametric methods, representing alternatives to the well known Lee-Carter method. Some thoughts are offered on the interpretation of historical data in generating future probability distributions, and on the treatment of demographic uncertainty in long-run policy planning.

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## 1. INTRODUCTION

This paper is an exploratory study in the time series modeling of mortality rates and the stochastic forecasting of life expectancies. Stochastic demographic forecasting was pioneered and developed by Ronald Lee and his associates (Lee, 1992, Lee and Carter, 1992, Lee and Tuljapurkar, 1994, to note three of the earliest papers; Lee, 2000, Lee and Miller, 2000, to note two recent ones). Our forecasting explorations fall into that general area, and we (like anyone else working in the area) owe a large debt to its originators. The Lee-Carter approach, as it has been called, provides a particular, interesting way of using historical mortality data to estimate probability distributions of future life expectancies. It has attracted considerable attention and has had various applications in stochastic forecasts for the U.S. and other countries. (See Lee and Miller, 2000, for a review of comments on the approach and refinements of it, and an assessment of its accuracy by means of ex post and simulated ex ante forecasting.) However, in the interest of exploring alternatives we take a different approach in this paper – indeed, a number of different approaches. Working with Canadian time series we investigate several other methods of introducing probabilistic elements into the modeling of mortality and the quantification of uncertainty about future life expectancies. Our aim is not primarily to produce predictions for Canada, although that is the form that the numerical product of our work takes. Rather it is to investigate possible ways of making predictions, whether for Canada or other countries or regions. It is essentially an investigation of competing techniques and assumptions. Working with Canadian data is convenient for us but we hope that our results and discussion will be of interest to persons concerned more generally with mortality modeling and stochastic forecasting.

Our principal data base is a set of annual age-sex group mortality rates for the 71-year period 1926 to 1996, based on numbers of deaths and population figures from various Statistics

Canada sources. (The population figures were adjusted to place them at July 1 in each year and to make them as consistent as possible with current Statistics Canada definitions.) The set includes rates for males and females in 19 age categories: under 1, 1-4, and then by 5-year groups up to 80-84, plus an open ended group, 85 and over. There are thus 38 age-sex groups, and 2698 observations in total. We analyse the time series behaviour of the rates and use the data and analysis as a basis for the subsequent forecasts of life expectancies. The forecasts are made within an overall period framework, as opposed to a cohort framework. A cohort framework might well be preferable but would introduce complexities that we have thought it best to avoid, at least for the present. In that regard our study is similar to others.

A sketch of the paper is as follows. We begin at the aggregate level by constructing overall summary measures of mortality. We have experimented with a range of alternative index formulas familiar in economics in the context of price/quantity index theory, and in some cases also in demography. The calculated indexes are all generally similar in their time series patterns and we have selected four of them for presentation, discussion, and modeling. (In logarithmic form the indexes provide information about long-run trends similar to that provided by the first singular value series extracted by the matrix decomposition procedure used in the Lee-Carter approach to stochastic forecasting.)

We turn next to an examination of the time series patterns of the age group rates. We present two alternative system models of those rates, and then consider some issues in forecasting – in particular the future lengthening of the life span, lower bounds on mortality rates and the possible long-run slowing of the rates of decline, and the related issue of the "rectangularization" of the survival function. We explore the implications of how those issues are dealt with by presenting a range of nonstochastic forecasts of life expectancies based on different

assumptions about oldest surviving age and changes in long-run trajectories. We then move on to stochastic forecasting. The stochastic procedures include nonparametric bootstrap methods, partially parametric methods in which a time series system model is coupled with bootstrap procedures for generating random variation, and a strictly parametric method in which the random variation is generated by drawing disturbances from a multivariate normal distribution. The forecasts are interpreted, and in a final section the problems and issues in stochastic forecasting are reviewed and some additional thoughts offered.

## 2. INDEX FORMULAS AND HISTORICAL REGULARITY AT THE AGGREGATE LEVEL

Index-type calculations have a long history in demography, often under the heading of "standardization" (Kitagawa, 1964, 1966, Shryock and Siegel, 1980, Keyfitz, 1985, Chapter 3, to take some examples from the literature). But the history in economic statistics is longer, dating from known applications in the construction of price indexes as far back as the 18th century (Diewert, 1987). Numerous formulas have been suggested for calculating indexes of the "general price level" and its quantity level counterpart, and the transference to mortality indexes is straightforward: age-sex group ( $x$ ) replaces commodity, number of deaths ( $D$ ) replaces expenditure, population ( $N$ ) replaces quantity, and mortality rate ( $m$ ) replaces price. The simple identity $D_x = m_x N_x$ then holds, and the stage is set for application of any of the available index formulas.

We have experimented with a range of mortality indexes, including arithmetic indexes with fixed population weights (based on various actual or stationary life table populations), "current weighted" or Paasche indexes, indexes with geometric population weights, Fisher's

"ideal" index and the Törnqvist index (various forms in both cases), and a Divisia index. (General treatments of index formulas can be found in Banerjee, 1975, and Diewert, 1987.) All of the indexes display roughly the same time series patterns and from the larger set we have chosen four for presentation here. The four include three of the arithmetic indexes with fixed population weights – weights based on actual populations in 1931, 1966, and 1991, which cover the pre-baby-boom, immediate post-baby-boom, and advanced baby-bust periods, with their markedly different age distributions. The fourth is the Divisia index, representing a discrete approximation to a theoretically continuous chain index. The four indexes are plotted as logarithmic time series in Figure 1 for the period 1926 to 1996, along with the first differences of the logarithms. The fixed weight arithmetic indexes are labeled fwaP31, fwaP66, and fwaP91 in the figure; the Divisia index is labeled div. In explicit form, the formula for the fixed weight indexes can be written as

$$(1)\quad M_t = \sum_x n_{x0} m_{xt} \Big/ \sum_x n_{x0} m_{x0}$$

where $n_{xt}$ is the proportion of the population in age-sex group x in year t and year 0 is a chosen base year; the formula for the Divisia index can be written as

$$(2)\quad M_t = \exp\left\{ \sum_{s=1}^{t} \sum_x w_{x,s-1} \Delta \ln m_{xs} \right\}$$

where $w_{xs} = D_{xs} / D_s$, $D_s$ being the total number of deaths in year s, all age-sex groups combined. Note that the formula for the fixed-weight index in equation (1) can be converted to

an equivalent form that also uses $w$ weights:

(3) $$M_t = \sum_x w_{x0}\left(m_{xt} / m_{x0}\right)$$

All four of the log indexes show long-run downward paths that are roughly linear over the seven decades and constitute the dominant time series pattern; in each case a simple linear trend function fitted to the log series accounts for over 99 percent of the total variation. The consistency of the trends might be thought remarkable, given all of the developments that have occurred over the seven decades in the science and practice of medicine, in health care and illness prevention more broadly defined, and in the socioeconomic circumstances of the population. (Why should the history of such developments have produced such a regular pattern of decline over such a long period?) However, the trends are consistent with those that have been found in studies employing the Lee-Carter singular value decomposition method; the time series corresponding to the first singular values identified in those studies show similar such trends in the United States and a number of other countries. Tuljapurkar, Li, and Boe (2000) analysed mortality data for the G7 countries using the Lee-Carter method and reported that the first singular value accounted for about 96 percent of the total variation of the matrix of age-sex log mortality rates in Canada, and from 94 to 97 percent in the other six countries. (For another application of the Lee-Carter method to Canadian data, with similar results, see Lee and Nault, 1993.) The first singular value series in the Tuljapurkar et al. study correspond essentially to long-run trends, and plots of those series show the same predominantly linear time paths for all countries as our indexes do for Canada. Against that background, the long-run time series pattern of our indexes is not so surprising.

The short-run fluctuations of the mortality indexes are represented by the annual first differences plotted in Figure 1. A first glance suggests negative autocorrelation, and that is confirmed by the subsequent fitting of autoregressive models. It is apparent too that there was a shift in the degree of volatility around 1946: the earlier fluctuations are more pronounced than the later ones. The fluctuations of the four series of log differences are quite similar, as one might expect.

## 3. TIME SERIES MODELS OF THE AGGREGATE INDEXES

Estimates of three alternative models of the indexes are provided in Table 1. (These models and others in the paper were estimated using SHAZAM, Version 9; see SHAZAM, 2001.) The models are autoregressive in the log first differences – AR(0), AR(1), and AR(2) models. Equivalently, the AR(1) and AR(2) models can be thought of as ARIMA(1,1,0) and ARIMA(2,1,0) models of the undifferenced log series, and the AR(0) one as a random walk model. The models were estimated by a generalized least squares procedure to allow for the apparent downward shift in error variance starting about 1946. (Variances for the period prior to 1946 and the period 1946 and after were estimated from ordinary least squares residuals, and then used in a second stage to construct a modified covariance matrix for implementation of the GLS procedure.) There is little to choose among the models. The autocorrelation evident in the plots of the log difference series manifests itself now in negative first-lag coefficients in all four. The differences in the standard errors of estimate (S) are small. The Akaike Information and Schwartz Criterion statistics (not shown in the table) give an edge to the AR(1) model, but again the differences are slight. Our choice for any further work with an aggregate model is AR(1), and our index of choice is the 1991 fixed weight index (fwaP91), but both choices are by narrow

margins.

## 4. TRENDS AND FLUCTUATIONS AT A DISAGGREGATED LEVEL

We leave the aggregate indexes behind now and go down to the level of the 38 individual age-sex groups. The first differences of the logarithms of the group mortality rates are plotted in Figure 2. The plots suggest negative autocorrelation at the group level, as they did at the aggregate level, and again that is confirmed by subsequent formal modeling. Also they indicate an age pattern in the degree of short-term volatility: the extent of annual fluctuations is most pronounced among children and teenage groups, falls off during the younger adult and middle age years, and is least pronounced in the older age groups. That pattern is confirmed by the model-based residual variances. Finally, the average proportionate decline (mean log first difference) in mortality rates over the period 1926 to 1996 also falls off with age; that can be seen in the plots, but more clearly in calculations provided below.

## 5. DISAGGREGATED TIME SERIES MODELS

We have experimented with different time series models of the first differences of the log mortality rates at the age-sex group level, including an AR(1), an AR(2), and what we refer to as a "quasi-vector autoregressive" (QVAR) model. Standard criteria give an edge to the AR(2) over AR(1) model, and we therefore choose it for presentation, along with the QVAR model. The two selected models are displayed in Tables 2 and 3.

The 38 equations of each model were estimated as a system, using an iterated form of the seemingly unrelated regression estimator (SURE) to allow for cross-equation error correlations. (Convergence required 69 iterations with the AR(2) model, 34 with the QVAR model.) We refer

to the two models as system models because of the simultaneous estimation procedure employed to estimate their equations in each case, and because we had in mind treating them as systems in using them to generate stochastic forecasts. As it turned out, we chose the AR(2) system and used it exclusively for the model-based forecasts presented below. However, we present both systems as viable modeling alternatives.

The AR(2) system equations in the log first differences (or equivalently, ARIMA(2,1,0) in the undifferenced logs) are of the form

$$(4)\quad \Delta \ln m_{xt} = \beta_{0x} + \beta_{1x}\Delta \ln m_{x,t-1} + \beta_{2x}\Delta \ln m_{x,t-2} + \varepsilon_{xt} \qquad (x \in X)$$

where $X$ denotes the set of 38 age-sex groups and $\varepsilon$ is a white noise error. In theory an alternative model that would capture both the short-run and intergroup dynamics of mortality rates is a vector autoregressive model (VAR) in which the lagged rates for all groups appear in all of the equations. But of course without severe restriction the estimation of such a model is infeasible with anything like the length of time series that would be available. Our practical approximation is a "quasi" version of a VAR model – a QVAR model, as we label it – in which the equations include an own-lagged variable plus a variable that is a function of all the other variables, in lagged form. Our choice for that function is an aggregate index, $M$, represented by the fwaP91 index of section 2. ( $M$ in fact includes the mortality rate for the individual age-sex group whose log differences are being modeled in any particular equation, as well as the other 37 rates. But that is inconsequential, and allows the same index variable to be used in all equations.) The resulting system that we considered as an alternative to the AR(2) system is a QVAR(1), of the form

$$(5)\quad \Delta \ln m_{xt} = \gamma_{0x} + \gamma_{1x}\Delta \ln m_{x,t-1} + \gamma_{2x}\Delta \ln M_{t-1} + \eta_{xt} \qquad (x \in X)$$

where the white noise errors are now represented by $\eta$ .

Both models have system $R^2$ values in excess of 0.99 (see Berndt, 1991, for definition) but that should be discounted rather heavily; it is known that the system $R^2$ formula can produce very high values, which ought to be interpreted with caution (SHAZAM, 2001). Nevertheless both models seem to perform reasonably well. The t statistics (interpreted as asymptotic normal statistics because of the method of estimation) are generally high in the AR(2) model – greater than 2.0 in all but one case out of 38 for the one-period lag coefficients, and in all but eight for the two-period lag coefficients. The AR(2) model performs somewhat better than the QVAR(1) model by that criterion, but an equation-by-equation comparison of the standard errors of estimate yields 28 cases out of 38 in which the QVAR(1) model performs better, though usually by only a small margin. The equations of both models are stochastically stable (stationary). On balance there seems to be little to choose between the two models, and the overall system $R^2$ measures provide no basis for discrimination. The AR(2) model is a little more convenient to work with for stochastic forecasting purposes, and that is therefore our choice for the experiments presented below. Whichever model one looks at, though, the negative autocorrelation of the log differences stands out clearly: in the AR(2) model the coefficient of $\Delta \ln m_{x,t-1}$ is negative in 37 of the 38 equations; in the QVAR(1) model it is negative in 36 of the 38.

The steady state rate of change of an age-sex group mortality rate can be calculated easily for the AR(2) model by setting $\Delta \ln m_{xt} = \Delta \ln m_{x,t-1} = \Delta \ln m_{x,t-2} = \Delta \ln m_x$ and $\varepsilon_{xt} = 0$ .

Letting $\alpha$ be the steady state rate, $\alpha_x = \beta_{0x} / \left(1 - \beta_{1x} - \beta_{2x}\right)$. The $\alpha$ values for the model are shown in Table 4, together with the means of the $\Delta \ln m$ series over the period 1926 to 1996. The two sets of values are close, as one would expect. For age groups 10-14 and older the rates of decline, by both measures, are greater for females than for males. The long-run proportionate rate of decline is seen generally to fall off with age, by both measures, and for adult ages that means that it falls off as the mortality rate itself increases. (Mean $\Delta \ln m$ and $\ln m$ are plotted together in Figure 3 for males and females, by age group.) We make use of the $\alpha$ values in one of the stochastic forecasting experiments by randomizing their assignment to age-sex groups to allow for possible distributional shifts in the effects of future medical advances or other mortality-reducing developments.

## 6. ISSUES IN THE FORECASTING OF MORTALITY RATES AND LIFE EXPECTANCIES

A general (and totally obvious) principle in forecasting is that one looks for past regularities as a guide to the prediction of future events. The regularities we have focused on are the fact of continuous decline of mortality in every age-sex group, and beyond that the actual patterns of decline over seven decades, at both the aggregate and group levels. Continuous decline of mortality is by no means a law of nature; mortality rates could in fact go up, and that of course has happened in some countries in some periods. A widespread major epidemic could cause at least a temporary reversal and continuous degradation of the environment could have longer-run effects, to say nothing of wars and sudden natural disasters. But a common though implicit assumption in demographic forecasting is that such reversals will not occur over the period of the forecast, so let us make that assumption, and consider how to make use of the

observed past regularities. For experimental purposes we are interested here in forecasts that extend a long way into the future – as far as the year 2100, in fact, with some stops in between – so the question is how much reliance can we put on past behaviour in looking that far ahead. We do not want to rely on the simple maintenance of past average rates of change; that would ignore the obvious and important role of uncertainty, negate any interest in stochastic forecasting, and rob this paper of its reason for existence. However, even in stochastic forecasting the question arises as to whether the distribution of past rates of change can be viewed as a stable probability distribution for the purpose of making probabilistic statements about the future of mortality and life expectancies. To put it more sharply, can the mean of the distribution (or some other location parameter) be assumed fixed, even though we cannot know just where in the distribution the future will actually lie?

Suppose that we do assume the mean to be fixed, if we are doing stochastic forecasting, or more simply that observed average rates of decline over some period will continue, if we are doing nonstochastic forecasting. (Not that nonstochastic forecasters need to make such an assumption; they can make judgemental choices, of course.) The problem then is that mortality rates at all ages will approach zero asymptotically, either as probability limits in the one case, or as strict limits in the other – and if we hold fixed the oldest life table age (more on that shortly) the survival function will approach a rectangular shape. The prospect of an ultimate situation in which everyone survives to, say, 106 (the oldest age in the 1991 Canadian life table), and then everyone dies in one year, is not appealing on grounds of realism, so something has to give. The oldest age can be increased, but even then it would seem reasonable to stipulate lower bounds on mortality at every age. Even at younger ages, where mortality rates are very low, one would suppose that there would always be deaths by misadventure, if not disease, and that there should

be some bounds. But placing lower bounds on mortality rates means that the regularities observed in the historical patterns of decline of those rates cannot continue forever. Sooner or later the pace of decline must be reduced (unless one believes in an ultimate perfectly rectangular survival function). For the mortality forecaster the question then is "how soon is sooner?", or more specifically, "does it fall within my forecast period?".

Let us turn to the issue of the lengthening of the life span, or in life table terms the increase in the oldest age at which there are any survivors. The issue has become a hot one, a fact publicized by the recent announcement of a bet between the biodemographer Jay Olshansky and the zoologist Steven Austad. Austad bet that in the year 2150 someone now living will still be alive, at the age of 150; Olshansky bet that that would not happen, arguing that 130 is the longest attainable life span. The bet was $150, which with compound interest was predicted to be worth half a million by 2150 (in the dollars of 2150, one presumes, not constant dollars), payable to the winner's heirs (Globe and Mail, 2001, University of Idaho, 2001). On a more academic level, a recent special issue of the journal Population contains twelve articles on "biodemographic perspectives on human longevity" which collectively demonstrate the vitality of the research and debate that is going on. (Vaupel, 2001, provides a summary of the current state of knowledge and opinion on the subject, and an overview of the articles in the special issue. Also, see Olshansky, Carnes, and Butler, 2001, for an interesting article on the aging of the human body and the problems that have to be overcome if the life span is to be lengthened.) We have nothing to contribute to the debate about longevity but we cannot avoid considering its implications for the forecasting of mortality and life expectancies, stochastic or otherwise.

There is no point in pretending that for our purposes we can reach a convincing decision on either the question of when mortality declines will taper off, or the questions of how much the

life span will lengthen, and how fast. The jury is going to be out on those questions for a very long time, one would think. What we can do – the best we can do – is to make different assumptions and explore their numerical implications for forecasts. We do that in a nonstochastic setting before turning our attention to alternative methods of stochastic forecasting.

## 7. SOME EXPLORATORY NONSTOCHASTIC FORECASTS

We present in Table 5 alternative forecasts of mean and median life expectancies to 2050 and 2100. The forecasts are obtained by applying rates of change in group mortality rates ($\Delta \ln m$) to the single-age rates in the 1991 male and female life tables. The group rates are treated as step functions for this purpose: the $\Delta \ln m$ value for the 50-54 age group is assumed to apply to all life table ages between exact age 50 and exact age 55, for example; the $\Delta \ln m$ value for 85+ is assumed to apply to exact age 85 and all older ages. Thus the age structures of the 1991 life table mortality rates and survival functions are preserved as starting points for the forecasts. Separate forecasts of male and female survival functions are calculated, but to reduce the detail for presentation the two are combined to form a "unisex" function and the life expectancies in Table 5 are derived from that. (The combination of the male and female functions is based on the 1986-1995 average Canadian male/female birth ratio of 1.05377.)

Nonstochastic forecasts are presented for combinations of three scenarios for the tapering off of the rates of decline of mortality rates and five for the lengthening of the life span. There are thus fifteen combinations in total. The tapering off scenarios are as follows: (a) none, meaning that $\Delta \ln m$ remains equal to its average1926-1996 value in every age-sex group throughout the whole of the forecast period; (b) rates of decline are reduced to half the 1926-1996 mean values

by 2100, with $\Delta \ln m$ values for other years calculated by linear interpolation; and (c) rates of decline are reduced to zero by 2100 (again with linear interpolation). The maximum life span scenarios assume an oldest age of 106 throughout the forecast period (as noted, 106 is the oldest age in the 1991 life tables), and alternatively oldest ages of 110, 120, 130, and 140, to be attained by the year 2100. The oldest ages for years between 1991 and 2100 are calculated as the integer values of linearly interpolated ages (an interpolated value of 115.7 would yield a maximum age of 115, for example).

It is of some importance to understand how mortality rates at the oldest ages are treated. Initially the oldest age is 106 for all scenario combinations, the life table rate of mortality at that age is obviously 1.0, and the mortality rates for ages beyond that are undefined. Age 106 retains a mortality rate of 1.0 as long as it continues to be the oldest age. However, if the oldest age rises at some point in the forecast period to 107, the mortality rate for 107 becomes 1.0 and the mortality rate at age 106 is treated thereafter like the rates for all other ages 85 and over; that is to say it is reduced in accordance with the forecast 85+ values of $\Delta \ln m$. This treatment preserves the necessary inflexibility of the assumed mortality rate at the oldest age but respects the requirement that mortality rates at all ages short of the oldest be allowed to decline. Any awkwardness in the treatment is an unavoidable consequence of a truncated rather than continuous survival function in the conventional life table framework.

Let us look then at the forecasts in Table 5, keeping in mind that they are for the sole purpose of exploring the effects on life expectancies of altering the assumptions about long-term changes in rates of mortality decline and length of life span. Mean and median expectancies are shown for ages 0, 65, and 80 in the years 2000, 2050, and 2100, for each of the fifteen

combinations of assumptions. The first point to note is that changes in life span, considered in isolation, have negligible effects, even at age 80 (where the effects would be expected to be greatest), and even as far out as 2100. The largest effect on the age 80 mean life expectancy in the table is a difference of 0.14 years (13.81 compared with 13.95) in moving from a forecast that assumes a maximum age in 2100 of 106 to one that assumes a maximum of 140, under the assumption in both cases that the 1926-1996 rates of change in mortality rates will continue. The largest effect on the age 0 expectancy in 2100 is 0.11 years. For 2050 the differences are much smaller: the largest difference anywhere in the table at any of the three ages is 2/100 of a year, again holding the rate of decline assumption constant. Even though the age span is allowed to lengthen considerably over the century or half-century there are just not enough survivors in the extended age interval to have a significant effect on the means. As for the medians, they are not affected at all: the median age is the age at which the survivor count is equal to half of the initial population, and that depends only on the mortality rates at younger ages. The sizes of the effects on the means are of course a product of the way in which we have treated mortality rates at the oldest ages, as described above. We think that what we have done is as reasonable a choice as one could make but other choices may be possible. Nevertheless the insensitivity of mean expectancies to even large increases in life span suggests to us that assumptions about the maximum age are well down on the list of things to worry about in making life expectancy forecasts.

The effects of assuming a long-term slowing of the rates of mortality decline are more important. Moving from assuming a continuation of the average 1926-1996 $\Delta \ln m$ values to assuming that $\Delta \ln m$ will fall to zero in all age groups by 2100 – from (a) to (c) in the table,

with any of the life span assumptions – reduces the mean life expectancy at age 0 by somewhat less than five years, the median life expectancy by somewhat more than four. At age 65 the reductions are less than four years by both measures, and at age 80 they are less than three. The differences in 2050 are of course smaller – about 1.6 years at the most at age 0, a little over one year at age 65, and less than a year at age 80. A century is a very long time in any demographic forecast, even for social security planning where looking far into the future can be necessary. Half a century is very long too, but a somewhat more realistic horizon, and the fact that the difference in life expectancy in 2050 between the two most extreme assumptions is never greater than 1.6 years at age 0, and only a little more than one year at age 65 (an age neighbourhood important for social security planning), may give some cause for comfort. In any event, in the stochastic forecasts to which we turn next we shall hold the oldest age of survival constant at 106, assume that the possibility of a systematic long-term drift toward zero means in the distributions of rates of mortality decline can be ignored for the 21st century, and proceed to treat the historical distributions as probability distributions for randomized forecasting experiments and the drawing of statistical inferences. In doing so we are following the same path as earlier investigators, who used the Lee-Carter method, although the methods that we experiment with are different.

## 8. STOCHASTIC FORECASTS

We present below the results of seven stochastic forecasts of life expectancies, based on seven different methods. As we have emphasized, the results should be regarded as experimental. We are interested in the theoretical structures of the alternative methods and in the similarities and differences among the forecasts that they produce. Forecasts of mean and median life

expectancies are provided, represented by the $5^{th}$ , $50^{th}$ and $95^{th}$ percentiles. The forecast distributions were generated in each case by 10,000 replications. (Experiments with random number sequences generated from different seeds indicated that the distributions were quite stable at 10,000 replications.)

The methods fall into three categories: nonparametric, partially parametric, and fully parametric. There are three nonparametric forecasts, based entirely on bootstrap procedures. Their advantage is that they do not require any formal model or assumptions about the exact form of underlying probability distributions. The partially parametric procedures, of which there are also three, do require a formal model, and for that purpose we use the AR(2) system presented previously. They use bootstrap methods to deal with the random selection of disturbances, drawing for that purpose from the residuals generated in fitting the model, and thus they too require no exact specification of probability distributions. There is a single fully parametric procedure, of a more traditional Monte Carlo form. It again uses the AR(2) system, but assumes that the disturbances come from a multivariate normal distribution, and samples them accordingly. This method is the most demanding in the prior assumptions that it brings to bear on the generation of forecasts. The seven alternative methods are identified as (a), (b), (c), (d), (e), (f), and (g). Their characteristics are summarized in Table 6 and described in detail in what now follows. (For stochastic forecasting in a different context, based on Monte Carlo procedures with a specified probability distribution, see Denton and Spencer, 1988; for applications of bootstrap forecasting methods, see Bernard and Veall, 1987, Denton and Spencer, 1991. A general review of time series bootstrap methods is provided in Horowitz, forthcoming.)

Nonparametric Methods

Method (a) employs a vector variant of the overlapping block bootstrap procedure, with

fixed block length 25. (As described in the statistics literature, blocks may be overlapping – see Hall, 1985, Künsch, 1988 – or nonoverlapping – see Carlstein, 1986. Also they may be of fixed length, or of random length as in the stationary bootstrap of Politis and Romano, 1994.) This first method of ours is an entirely model-free one that preserves the age-sex correlation structure of changes in mortality rates, allows (within blocks) for serial correlation of the rates of change (both autocorrelation and cross-group serial correlation), and avoids having to stipulate a particular type of probability distribution for sampling purposes. (A characteristic of block bootstrap procedures is that they preserve serial correlation within blocks but the blocks drawn randomly in each sample are placed in sequence, and there are discontinuities of serial correlation where one block meets another.) Let $v_t$ denote the 38-element vector of $\ln m_x$ values in year t and let $\Delta v_t = v_t - v_{t-1}$ denote the corresponding vector of first differences. Now let $\Delta V_{t+1}$ denote the sequence $\Delta v_{t+1}, \Delta v_{t+2}, \ldots, \Delta v_{t+25}$ ; $\Delta V_{t+1}$ is referred to as a vector block – or simply block, if the meaning is clear – with starting point t+1. There are 46 such starting points, and hence 46 vector blocks spanning the period 1926 to 1996. The year 2000 single-age life table mortality rates are given for purposes of the experiments; they are the nonstochastic ones calculated in generating Table 5. The stochastic forecasts for years subsequent to 2000 are then calculated as follows: (1) Choose a $\Delta V$ block at random from the 46 historical blocks. (2) Employing the step function procedure used for the nonstochastic projections, apply the sequence of 25 $\Delta v$ vectors in the sample block to project the single-age rates to 2025. (3) Now choose another $\Delta V$ block at random from the 46 historical blocks (the sampling is with replacement, so the same block as before could be chosen) and use the vectors of the newly selected block to

move the single-age rates to 2050, and so on for 2075 and 2100. (4) Use the predicted single-age mortality rates to derive life expectancies in the chosen years. (5) Repeat steps (1) to (4) 10,000 times and use the distributions of life expectancies thus generated to calculate percentiles (or any other summary measures that might be of interest).

The procedure just described assumes that output is required only at 25-year intervals, as in our tables. If output is required for shorter intervals the $v$ vectors can be moved ahead one year at a time, or 5 or 10 years at a time, say, even though a new block is chosen only every 25 years. We have set the block length at 25 on the assumption that the implementation of health care innovations, or other relevant developments, may have mortality-reducing effects that last for a quarter of a century, but of course that is an arbitrary assumption. One could experiment with other block lengths.

Block bootstrap methods in the statistics literature are used as a device for drawing inferences from time series that are subject to serial correlation, the optimum length of block depending on the sample size and the particular application. Our application can be viewed in that way, although we do not know what the optimum length is, and simply assume one. An important consideration in using method (a) is that by sampling the historical $\Delta \ln m_x$ values as vectors (instead of treating each age-sex group independently) the structure of correlations among age-sex group rates of change in mortality rates is preserved, without having to stipulate a particular type of probability distribution.

Method (b), the second of the two model-free methods, takes a different approach. It too can be thought of as a vector block bootstrap method, with block length 25 years, and the elements of the blocks representing $\Delta \ln m_x$ values. However the assignment of the rows of the

blocks among age-sex groups is randomized. Each sample vector block, in its original form (the form defined for method (a)), can be thought of as a 38 x 25 matrix, a row of which corresponds to one of the 38 age-sex groups. The block is then transformed randomly, as follows. The first row of the transformed matrix is assigned a number from 1 to 38, and the row of the original matrix corresponding to that number becomes the first row of the transformed matrix; the second row of the transformed matrix is then assigned a number from 1 to 38, and the row of the original matrix corresponding to that number becomes the second row of the transformed matrix; and so on. (The sampling of rows is with replacement, and hence a given row of the original matrix may be assigned to more than one row of the transformed matrix.) The transformed matrix having been created in this way, it then plays the same role in the subsequent calculations as $\Delta V$ does in method (a).

The rationale for method (b) is the idea that reductions of mortality rates may have causes which in any period are specific to a particular sex or a particular age range, or at least have larger impacts on some age-sex groups than on others. Developments in medical knowledge or practice that reduce infant mortality rates may have little effect on the rates of the elderly; developments that affect the mortality rates of the elderly may have little effect on the mortality rates of women at childbirth; developments in the treatment of prostate cancer affect mainly older men and have no direct effect on the mortality rates of women, and similarly developments that affect female mortality from breast cancer have no direct effects on men; legislation and enforcement designed to reduce motor vehicle fatalities may have proportionately different effects on different age groups, depending on their frequencies of motor vehicle use and their relative accident risks. Extrapolating from that general notion, life saving or life prolonging developments in future decades (whatever the developments turn out to be) may alter the patterns

of mortality reduction among age-sex groups in ways different from the past, and unknowable today. Groups that had rapid mortality declines in the past may have slow declines in the future, and conversely. Method (b) thus takes a position that is different from that of method (a): in method (a), care is taken to preserve past relations among age-sex groups, assuming that they will hold in the future, while randomizing the matching of historical periods (blocks) with future periods; in method (b), the intergroup relations are deliberately randomized, as well as the period matching, on the basis of the argument just given. Both positions may have justification, in greater or lesser degree. We do not advocate one over the other; we simply explore their implications for forecasting.

Method (c) is the same as method (b) except that the first four age groups for each sex, representing ages 0 to 14, are removed from the set of possible choices in making the random group assignments. Each of the 38 age-sex groups is now matched, for the future, with one of the 30 15-and-over groups. One of the great achievements of the 20th century was the lowering of child mortality rates: the four youngest male and female age groups were the ones with the highest rates of mortality reduction in our data set, which covers seven decades of that century. The assumption made in method (c) is that those rates of reduction were so large as to be unique – that such rates cannot again be achieved for any group. We do not assert that to be the case; we merely make the assumption in order to see how it alters the forecasts obtained by method (b).

Partially Parametric Methods

Method (d), the first of the three under this heading, is a forecasting application of the sieve or autoregressive bootstrap (Bühlmann, 1997; see Choi and Hall, 2000, for a comparative study of sieve and block bootstrap estimators). It uses the AR(2) system of equations to project, year by year, from the 2000 mortality rates, with the rates for 2000 and 1999 serving as initial lag

values in the projection. In each year the vector of AR(2) predicted changes in mortality rates is disturbed by adding to it a vector chosen randomly, with equal probabilities, from the set of 68 residual vectors arising out of the estimation of the AR(2) equations. As before, the sampling is with replacement, so that a given residual vector may be chosen more than once. The method thus combines model-based predictions with a vector bootstrap procedure, and so both the historical parametric structure and the intergroup error correlation structure are preserved. (Autocorrelation in the errors is presumed to have been removed by the AR(2) filtering of the mortality rates.) Employing the same step function procedure as before, the predicted changes in group rates are used to move the single-age mortality rates forward, year by year.

Method (e) is the same as method (d), except that the steady state values of $\Delta \ln m_x$ are randomized across age-sex groups every 25 years, thus shifting the distributions of the rates of mortality decline up or down to allow for possible differential effects of medical innovations or other mortality-reducing developments. The steady state values are the $\alpha$ values in Table 4. Randomizing their assignment induces randomization in the constant terms ( $\beta_{0x}$ ) of the AR(2) equations through the transformation $\beta^*_{0x} = \alpha^*_x(1 - \beta_{1x} - \beta_{2x})$ , where the asterisk indicates a randomized parameter. The long-run rate of mortality decline is thus shifted randomly within each group while the group's autocorrelation structure is preserved by retaining the same values for the $\beta_1$ and $\beta_2$ parameters. The contemporaneous intergroup error correlation structure is also preserved (as in method (d)) by employing the vector bootstrap procedure for selecting residuals.

Method (f) stands in relation to method (e) as (c) stands to (b), in that the under-15 groups are removed from the set of possible choices for random reassignment of $\alpha$ values. The purpose

is the same as for method (c): to see the effects of assuming that the high rates of mortality reduction achieved for children in the last century cannot again be achieved for any age-sex group, whether child or adult.

Fully Parametric Method

Method (g) is the only fully parametric method of the seven we have chosen to experiment with. It is a traditional type of Monte Carlo method familiar in econometrics. It assumes again the AR(2) equation system but now stipulates a multivariate normal distribution for the disturbances, with parameters based on the error covariance matrix calculated in estimating the system. Letting $\Sigma$ denote the 38 x 38 covariance matrix, a vector of disturbances is drawn from $N(0,\Sigma)$ in each forecast year. The historical intergroup error correlation structure is thus once again maintained, but now in the form of a specific joint probability distribution. In one way method (g) is similar to methods (a) and (d). It preserves the historical correlation structure among age-sex groups, as do they; there are no random transfers among groups.

## 9. INTERPRETING THE FORECASTS

The stochastic forecasts of mean life expectancies are provided in Table 7 and two selected ones are plotted in Figure 5; the forecasts are represented in both the table and the figure by the 5$^{th}$, 50$^{th}$ and 95$^{th}$ percentiles. The forecasts of median life expectancies (similarly represented) are provided in Table 8. The 90 percent probability intervals for the means and medians (differences between the 5$^{th}$ and 95$^{th}$ percentiles) are shown in Table 9. As noted previously, the percentiles were calculated from the distributions generated by 10,000 random

replications.

Our seven methods fall into two broad categories, determined by two competing views of how the historical series of declines in mortality rates should be interpreted when they are used as a basis for constructing stochastic forecasts. The competing views are as follows:

(1) The observed age-sex distribution of mortality declines represents an inherent (stochastic) pattern that can be expected to hold among age-sex groups in the future, and should be respected in making forecasts.

(2) The observed age-sex distribution of mortality declines was a consequence of the history of mortality-reducing developments during the data period, and while the overall numerical distribution of rates of decline is indicative of the future distribution its historical identification with particular groups need not be maintained.

Methods (a), (d) and (g) fall into category (1), methods (b), (c), (e) and (f) into category (2). (We would put the Lee-Carter method in category (1); it models the relationships between age-sex rates and an overall stochastic mortality index, and assumes that the relationships will continue to hold, stochastically, throughout the forecast period.) At this point we take no position on which of the categories is "better" but note that forecasting results can be quite different, depending on which of the two views one adopts.

Our category (2) forecasts of life expectancies (both mean and median) are appreciably higher than our category (1) forecasts, based on the $50^{th}$ percentiles. Randomly reassigning actual rates of decline among age-sex groups (methods (b) and (c)) or steady state rates (methods (e) and (f)) means that in some proportion of the 10,000 forecast runs the historically higher rates of decline among younger age groups will replace the historically lower ones among older groups, where the mortality rates themselves are much higher. Of course, in some proportion too the rates

of decline will be low for all age groups, but on balance the random effects of faster declines replacing slower ones dominate the random effects of slower declines replacing faster ones in the calculation of life expectancies.

The year 2000 mean life expectancy at age 0 (as calculated by us for the experiments) is 78.96. The highest 50$^{th}$ percentile forecast for the year 2100 is produced by method (e), at 98.54, the lowest by method (a), at 88.68 – a difference of 9.86 years. The range is even greater for the 50$^{th}$ percentile forecasts of median life expectancy: 101.40 by method (e), 90.47 by method (a) (compared with 81.75 in 2000); thus a forecast difference of 10.93 years. The spreads are substantial also within a given forecast distribution, as measured by the 90 percent intervals in Table 9. In 2100 the interval for mean life expectancy at age 0 ranges from 3.46 years (method (a)) to 6.18 years (method (b)); for median life expectancy it ranges from 3.14 (method (a)) to 10.30 (method (f)). The 50$^{th}$ percentiles and 90 percent intervals at ages 65 and 80 differ among the forecasts in a similar fashion. In the absence of conviction as to which of the broad categories defined above is the "correct" one, our results suggest a very high degree of uncertainty about future life expectancies a hundred years from now – even leaving aside the questions of lower bounds on mortality rates and possible shifts of the probability distributions, as considered in sections 5 and 6.

The situation is altered greatly if one accepts category (1) as the "correct" one. The choice then is restricted to methods (a), (d) and (g), and the differences among the 50$^{th}$ percentile forecasts are quite small, even as far into the future as 2100: the largest difference is 0.38 years at age 0 for mean life expectancy in that year, and only 0.05 for median life expectancy. Such small differences among 100-year forecasts generated by such different methods seem quite remarkable. Much is therefore riding on whether one can accept the category (1) view of how

historical distributions should be interpreted for forecasting purposes.

Hundred-year forecasts (like the Austad-Olshansky debate about a 150 year life span) are of academic interest, perhaps fun to think about, but hardly of much practical value. We have chosen a maximum forecast horizon of 100 years in order to draw out clearly the implications of different assumptions and the use of different methods, and for that it serves well. Reducing the horizon to 50 years is a move in the right direction from a policy forecasting point of view if one is concerned with the future of a social security system; people entering the labour force today may well be paying into such a system, and expecting to be drawing benefits, a half century and more from now. But even that is pretty far into the future for most practical purposes. A 25-year horizon is likely to be of more general interest: for long-run planning of capital investment in the energy sector; for the construction of schools and hospitals; for the training of physicians, teachers, and other skilled professionals to meet predicted future demands; and so on. With that in mind we now focus on the shortest of our forecast periods, 25 years, and see what our results tell us about life expectancies in the year 2025.

The 50$^{th}$ percentile forecast of mean life expectancy at age 0 in 2025 ranges from 81.85 to 84.55, a difference of 2.70 years; the corresponding forecast of median life expectancy ranges from 84.31 to 86.95, a difference of 2.64. Taken at face value, those differences suggest considerable uncertainty about 2025 expectancies, quite aside from the additional uncertainty introduced by the stochastic nature of the forecasts – by not knowing where in the probability distribution for each forecast an expectancy will actually lie. On the other hand, if one can convince oneself that category (1) is the correct choice – that historical age-sex distribution patterns are somehow inherent characteristics that should be maintained – then the actual choice of method is much less important. The choice then is among (a), (d) and (g), and for those the

largest difference in 50th percentile forecasts is only 0.23 years for mean expectancy, 0.13 for median expectancy. So again it is important how one views the historical distributions of mortality declines.

The 90 percent intervals for the method (a) mean and median forecasts are smaller than the intervals for the corresponding (d) and (g) forecasts, which are very close to each other. That is true of all three ages shown in the tables, and it is true also for the later forecast years, as well as 2025. All in all, the (d) and (g) forecasts are remarkably similar, both in central location (50th percentile) and in range (90 percent interval). (If a smaller block length were used in method (a) one would expect its 50th percentiles to be little changed but its 90 percent intervals to be closer to those of the other two; we have confirmed by numerical experimentation that that is in fact the case. Choi and Hall, 2000, demonstrate, in a different setting, that block bootstrap confidence intervals can be sensitive to block length – more so than sieve bootstrap intervals are to model lag length.) One result that comes through clearly, then, is that it makes almost no difference whether one assumes a normal distribution for the disturbances (as in (g)) or uses a bootstrap method, which avoids any specification of the type of distribution (as in (d)). That is similar to a result found by Denton and Spencer (1991) in a quite different stochastic forecasting context. One can of course imagine cases in which a similar result would not hold, because of a pronounced departure of the error distribution from normality. In practice, though, it is our impression that the result is not an uncommon one.

As final observations to conclude this section we note the following. The 50th percentile forecast of mean life expectancy at age 0 reported by Tuljapurkar, Li and Boe (2000) for Canada in 2050, based on the Lee-Carter method, is 85.26. That compares with our category (1) forecasts of 84.26 (for method (a)) and 84.67 (for both (d) and (g)) – differences of one year or less. (The

category (1) forecasts are the ones most comparable with Lee-Carter forecasts.) The 90 percent interval reported by Tuljapurkar et al. is 2.78, which compares with our intervals of 2.50, 3.44 and 3.32 for (a), (d), and (g), in that order. Tuljapurkar et al. compare their forecasts with ones based on the assumptions underlying the "official" forecasts made in 1998 for the Canada Pension Plan (see Office of the Chief Actuary, 1998): their forecast 50$^{th}$ percentile is higher than the official medium forecast for 2050 and their 90 percent interval is smaller than the official (nonstochastic, judgemental) high-low range. Our category (1) forecasts lie between the two, in both respects, but our 50$^{th}$ percentile forecasts are closer to the Tuljapurkar et al. one.

## 10. FINAL THOUGHTS

There are various aggregate indexes that can be calculated to represent the long-run historical time path of mortality. One is the Lee-Carter singular value index; others can be taken from the economic statistics price/quantity indexing tool kit, as we have done. Whatever the formula, though, the indexes, in log form, show similar near-linear trajectories. That is true for Canada over the seven decades of our data period, and it is true also for a number of other countries over comparably long periods. There are various ways too of modeling the decreases in mortality rates, both at the aggregate level and at the level of individual age-sex groups. Again the Lee-Carter method represents one way, and we have experimented with others. Of the two system models that we estimated at a disaggregated level, we chose one for subsequent forecasting experiments but the other would be a candidate for further forecasting experimentation. It brings into play an aggregate index in the forecasting of age-sex mortality rates – as does the Lee-Carter method, though in a different way. There is little to choose between our two models on statistical grounds.

The stochastic forecasting methods we have explored fall into two broad categories: (1) those that preserve age-sex distribution patterns of mortality decline, and (2) those that randomize the association of rates of decline across age-sex groups. Category (1) forecasts of life expectancies are lower, on average, than category (2) forecasts, have shorter forecasting ranges (90 percent intervals) and are in close agreement, based on 50$^{th}$ percentiles. Within category (1) it makes little difference whether one uses a nonparametric (bootstrap) method, a partially parametric method or a fully parametric method: the 50$^{th}$ percentiles are very close in all three cases and the 90 percent intervals are generally close too, and for the partially and fully parametric forecasts they are almost the same. (Were we to use a length shorter than 25 years with the block bootstrap method the interval for that method would be brought closer to the ones for the other two.)

There are other issues to consider in making forecasts of life expectancies besides those associated with stochastic methods. One is the issue of the future lengthening of the life span, or increasing of the oldest age of survival. Our experimental calculations suggest that while that is an interesting matter to consider it is not quantitatively important for forecasting life expectancies, at least for ages short of the very oldest. We considered expectancies up to age 80 and found hardly any effect when the life span to be achieved by 2100 was allowed to increase to as much as 140 years. The other issue is that of lower bounds on mortality rates and the eventual slowing of the rates of decline, and that proved to be more important, and difficult to deal with since the history of mortality declines offers little help. In making our stochastic forecasts we followed the example of others and assumed that the issue could be ignored, at least for forecasting within the 21$^{st}$ century. But of course that is just an assumption.

Our aim in this paper has been simply to experiment with different forecasting methods

and assumptions to see how much difference they would make. But given the range of possibilities, what should one do in practice when the forecasts are intended for application in real policy planning situations – in long-run pension planning based on population projections that require mortality projections, say, or the planning of physical investment, or of human resource investment where education/training programs may put people into the labour force who will not retire for three to five decades? In essence such a situation is no different from any number of other policy forecasting situations: it is a classical planning-under-uncertainty situation. It may be difficult at present to visualize the application of formal optimization theory. However, by introducing the notion of probability distributions into mortality forecasting, and into demographic forecasting more generally, the work of Ronald Lee and others represents an important step in that direction. In the mortality and life expectancy context, observed historical distributions provide an objective criterion for stochastic forecasting as long as one assumes that the distributions will hold in the future, and can be treated as probability distributions. To take a simple alternative though, in order to make a point, suppose that one thinks it possible that the rate of decline in mortality rates will fall to zero by the end of the 21$^{st}$ century. Entering the realm of subjective probabilities – and there is no avoiding that – one might attach probabilities to the two possibilities: 0.6 and 0.4, perhaps, or some other combination. As a matter of arithmetic it would then be a simple matter to combine the subjective and objective probabilities so as to generate stochastic forecasts that reflected both. The forecast range – represented by 90 percent intervals, say – would then be wider, reflecting the additional uncertainty introduced into the forecasting framework. Taking a more formal Bayesian-type approach, one could easily allow instead for a (subjective) probability distribution over the full space of possible shifts in distributions, rather than allowing for only the two possibilities. At the other extreme, one could

attach probability 1 to the continuation of the historical distributions, probability 0 to all other possibilities. In effect that is what we did in our experimental stochastic forecasts, and what others have done. Another approach inspired by the same concern – that mortality rates cannot decline forever – would be to set a lower bound on the mortality rate in every age-sex group and use a model in which the rates approach the lower bounds (rather than zero), asymptotically. But the choice of ultimate lower bounds would have to have a subjective element, and one can conceive of treating them stochastically too, by imposing subjective probability distributions.

Whether or not subjective probabilities are brought into play explicitly, stochastic demographic forecasting allows the possibility (at least at a theoretical level) of defining a loss function in a particular demographically based planning situation, a corresponding risk function, and then minimizing the latter as a guide to policy choice – and if the choice framework is dynamic, reoptimizing from period to period as new information becomes available, and new stochastic forecasts are produced. A feedback effect on demographic variables would make the problem even more interesting. All of that takes us rather far from our much more modest goal in this paper of exploring alternative approaches to the stochastic forecasting of mortality rates and life expectancies, and so we leave the matter there. The introduction of random behaviour into demographic forecasting does, though, open the door to possible future applications of optimization theory and methods, an area in which economists should feel comfortable working with their demographer cousins.

## REFERENCES

Banerjee, Kali S. (1975), Cost of Living Index Numbers: Practice, Precision, and Theory, New York: Marcel Decker, Inc.

Bernard, J.-T. And M. R. Veall (1987), "The Probability Distribution of Future Demand: The Case of Hydro Quebec," Journal of Business and Economic Statistics, 5, 417-424.

Bühlmann, P. (1997), "Sieve Bootstrap for Time Series," Bernoulli, 3, 123-148.

Carlstein, E. (1986), "The Use of Subseries Methods for Estimating the Variance of a General Statistic from a Stationary Time Series," Annals of Statistics, 14, 1171-1179.

Choi, Edwin and Peter Hall (2000), "Bootstrap Confidence Regions Computed from Autoregressions of Arbitrary Order," Journal of the Royal Statistical Society, Series B, 62, 461-477.

Denton, Frank T. and Byron G. Spencer (1988), "Inflation Protection and Economic Security in Retirement," Task Force on Inflation Protection for Employment Pension Plans: Research Studies, Volume 1, Toronto: Queens Printer for Ontario.

Denton, Frank T. and Byron G. Spencer (1991), "Resampling a Time-Series Process: A Method of Estimating the Probabilities Associated with Alternative Plans for Protecting Pensions Against

Inflation," Journal of Applied Econometrics, 6, 303-314. See also "Erratum" (1992), 7, 211-213, for correction of printing errors in tables.

Diewert, W. E. (1987), "Index Numbers," in J. Eatwell, M. Milgate and P. Newman (eds.), The New Palgrave: A Dictionary of Economics, Volume 2, 767-780.

Globe and Mail (2001), "Let's Talk Longevity," The Globe and Mail, April 4 issue.

Hall, Peter (1985), "Resampling a Coverage Pattern," Stochastic Process Applications, 20, 231-246.

Horowitz, Joel L. (forthcoming), "The Bootstrap," in J. Heckman and E. Leamer (eds.), Handbook of Econometrics, Volume 5, Elsevier (available in electronic form at http://www.biz.uiowa.edu/faculty/horowitz/index.html).

Keyfitz, Nathan (1985), Applied Mathematical Demography, Second Edition, New York: Springer-Verlag.

Künsch, H. R. (1989), "The Jackknife and the Bootstrap for General Stationary Observations," Annals of Statistics, 17, 1217-1261.

Kitagawa, Evelyn M. (1964), "Standardized Comparisons in Population Research," Demography, 1, 296-315.

Kitagawa, Evelyn M. (1966), "Theoretical Considerations in the Selection of a Mortality Index, and Some Empirical Comparisons," Human Biology, 38, 293-308.

Lee, Ronald D. (1992), "Stochastic Demographic Forecasting," International Journal of Forecasting, 8, 315-327.

Lee, Ronald D. (2000), "The Lee-Carter Method for Forecasting Mortality, with Various Extensions and Applications," North American Actuarial Journal, 4, 80-93.

Lee, Ronald D. and Lawrence R. Carter (1992), "Modeling and Forecasting U. S. Mortality," Journal of the American Statistical Association, 87, 659-671.

Lee, Ronald D. and Timothy Miller (2000), "Evaluating the Performance of Lee-Carter Mortality Forecasts," unpublished paper, University of California at Berkeley (available in electronic form at www.demog.berkeley.edu).

Lee, Ronald D. and François Nault (1993), "Modeling and Forecasting Canadian Mortality: An Application of the Lee-Carter Approach, with Extensions," paper presented at the International Conference of the IUSSP, Montreal.

Lee, Ronald D. and Shripad Tuljapurkar (1994), "Stochastic Population Forecasts for the United States: Beyond High, Medium and Low," Journal of the American Statistical Association, 89, 1175-1189.

Office of the Chief Actuary (1998), Canada Pension Plan, Seventeenth Actuarial Report, Office of the Superintendent of Financial Institutions , Canada, 1998.

Olshansky, S. Jay, Bruce A. Carnes and Robert N. Butler (2001), "If Humans Were Built to Last," Scientific American, 284, 50-55 (March issue).

SHAZAM (2001), SHAZAM User's Reference Manual Version 9, Northwest Econometrics.

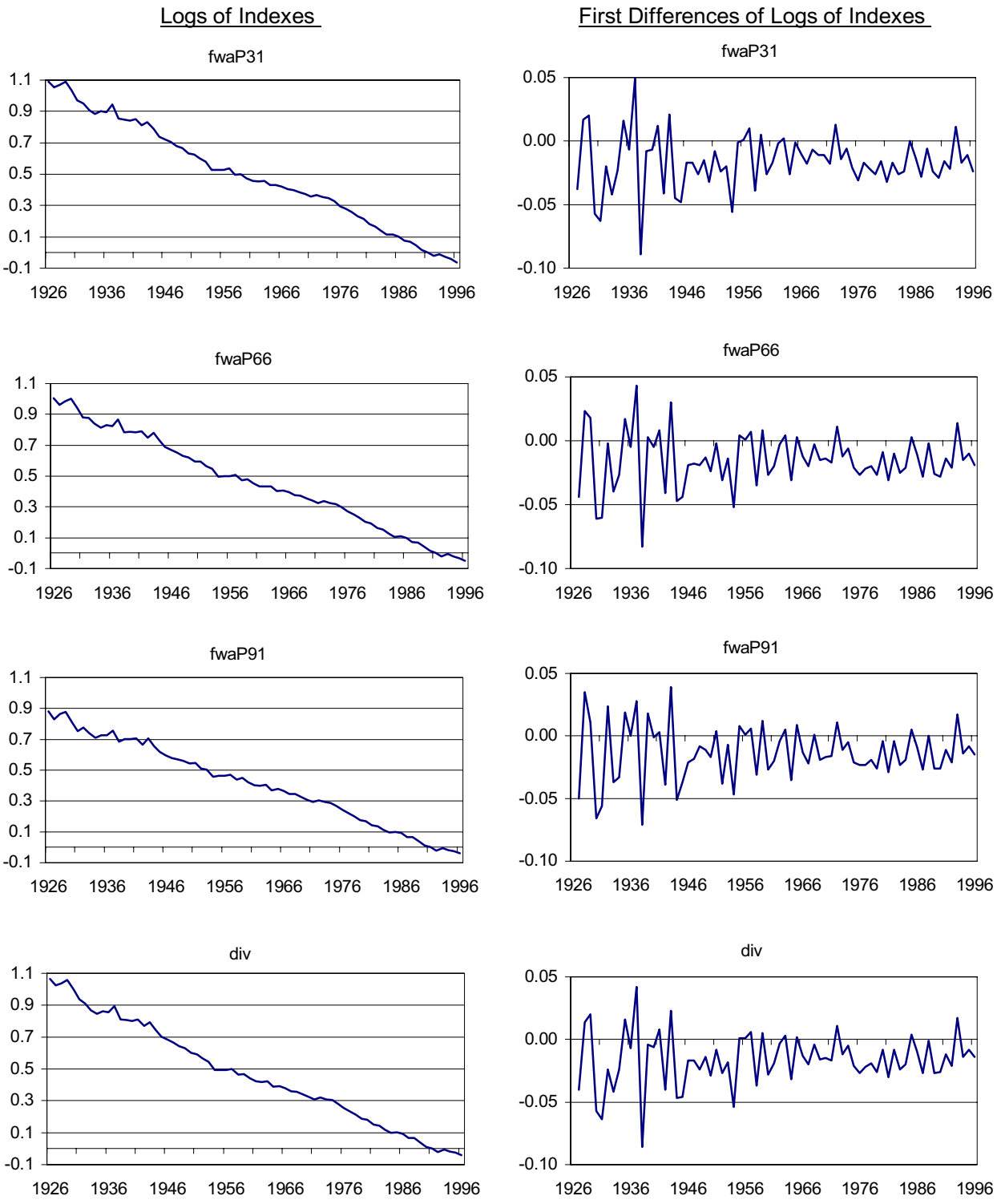
Shryock, Henry S. and Jacob S. Siegel (1980), The Methods and Materials of Demography, U.S. Department of Commerce, Bureau of the Census.

Tuljapurkar, Shripad, Nan Li and Carl Boe (2000), "A Universal Pattern of Mortality Decline in the G7 Countries," Nature, 405, 789-792.

University of Idaho (2001), "Scientists Bet $500 Million on Whose Aging Theory Will Prove Best in 2150," electronic news release, University of Idaho College of Letters and Science (www.its.uidaho.edu).

Vaupel, James (2001), "Demographic Insights into Longevity," Population, An English Selection, 13, 245-259. (The special issue of the journal is entitled "Biodemographic Perspectives on Human Longevity".)

FIGURE 1: LOGS AND FIRST DIFFERENCES OF LOGS OF SELECTED AGGREGATE MORTALITY INDEXES, 1926-1996




Note: See text for index formulas.

FIGURE 2: FIRST DIFFERENCES OF LOGS OF AGE GROUP MORTALITY RATES, 1926-1996

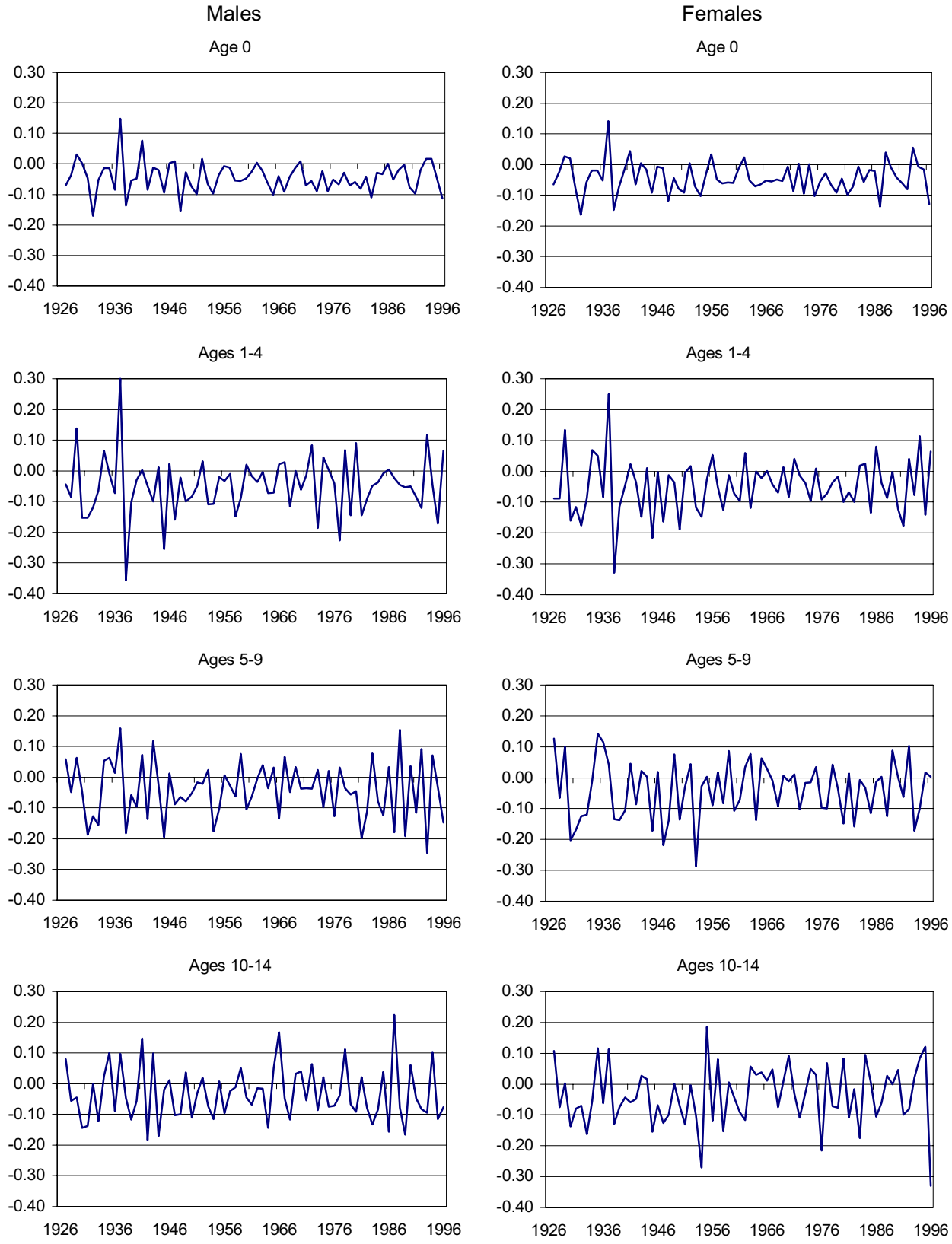


FIGURE 2: CONTINUED

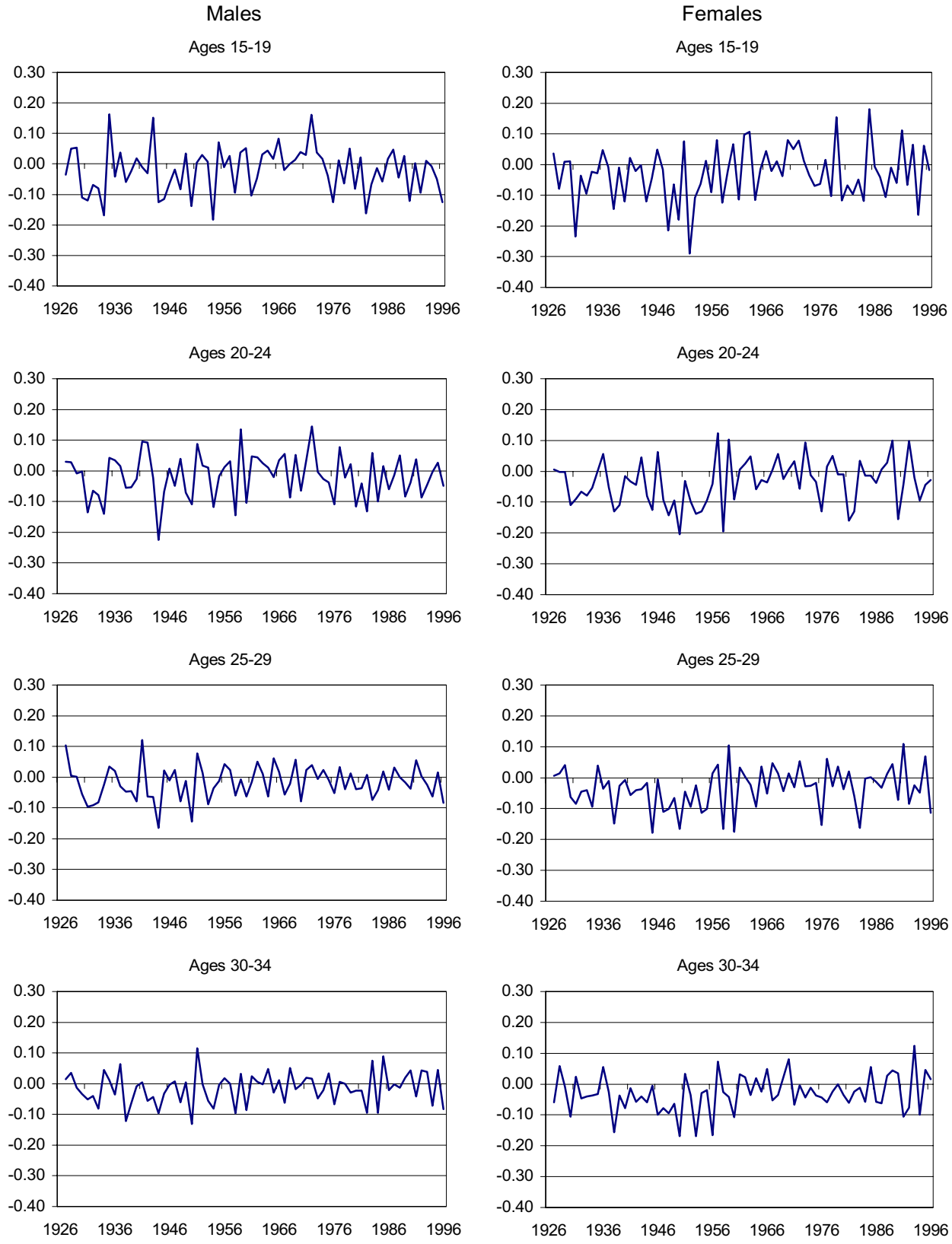


FIGURE 2: CONTINUED

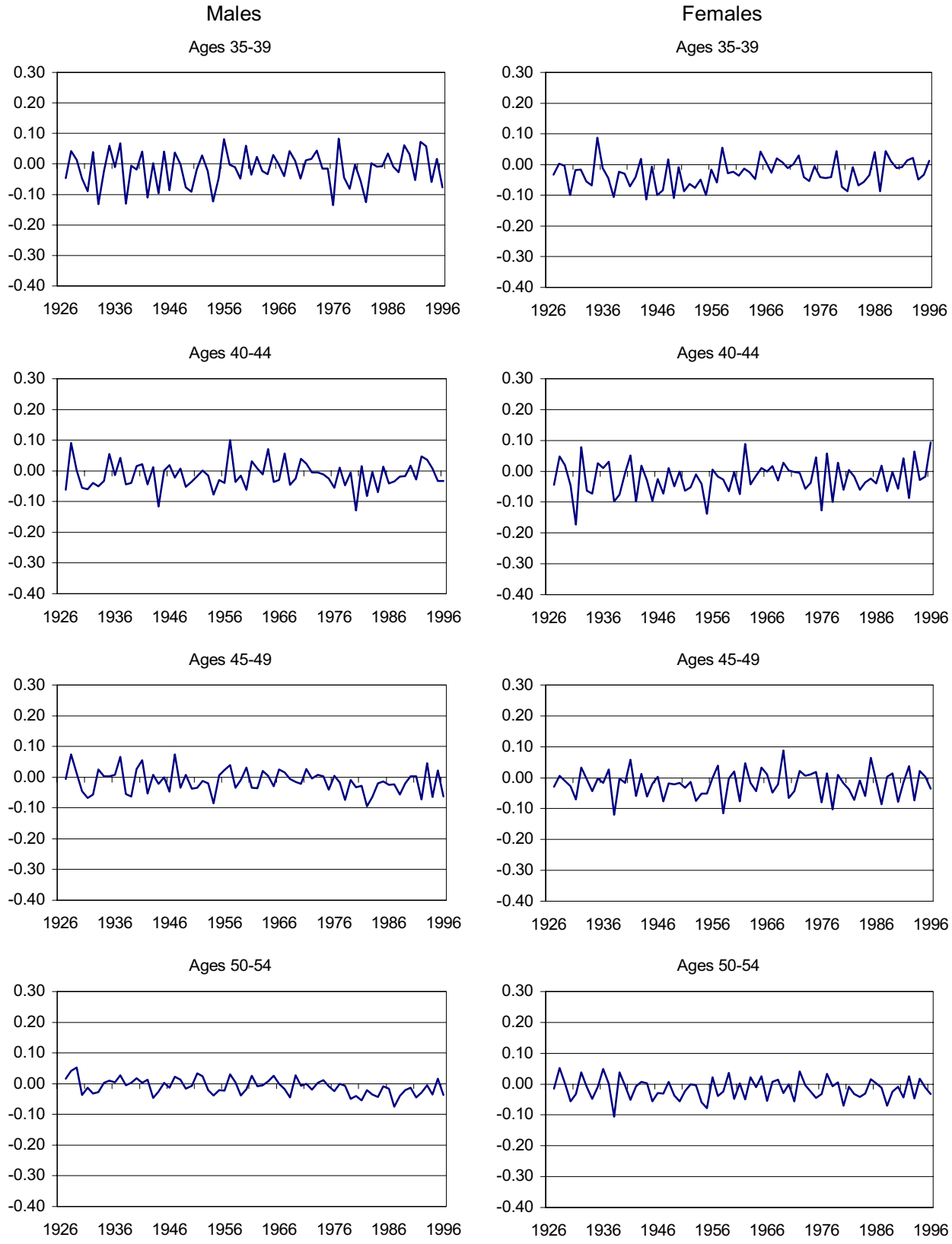


FIGURE 2: CONTINUED

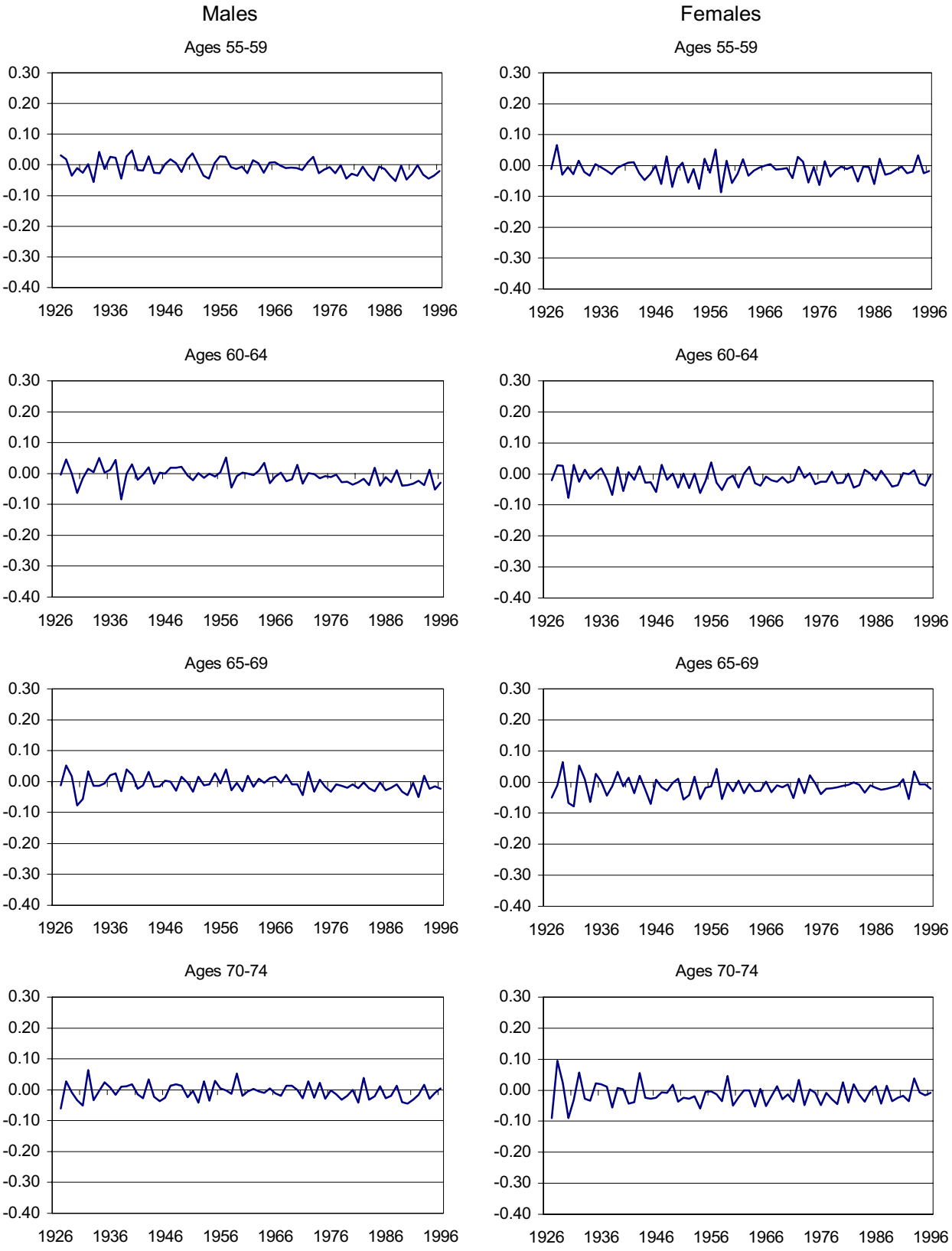


FIGURE 2: CONCLUDED

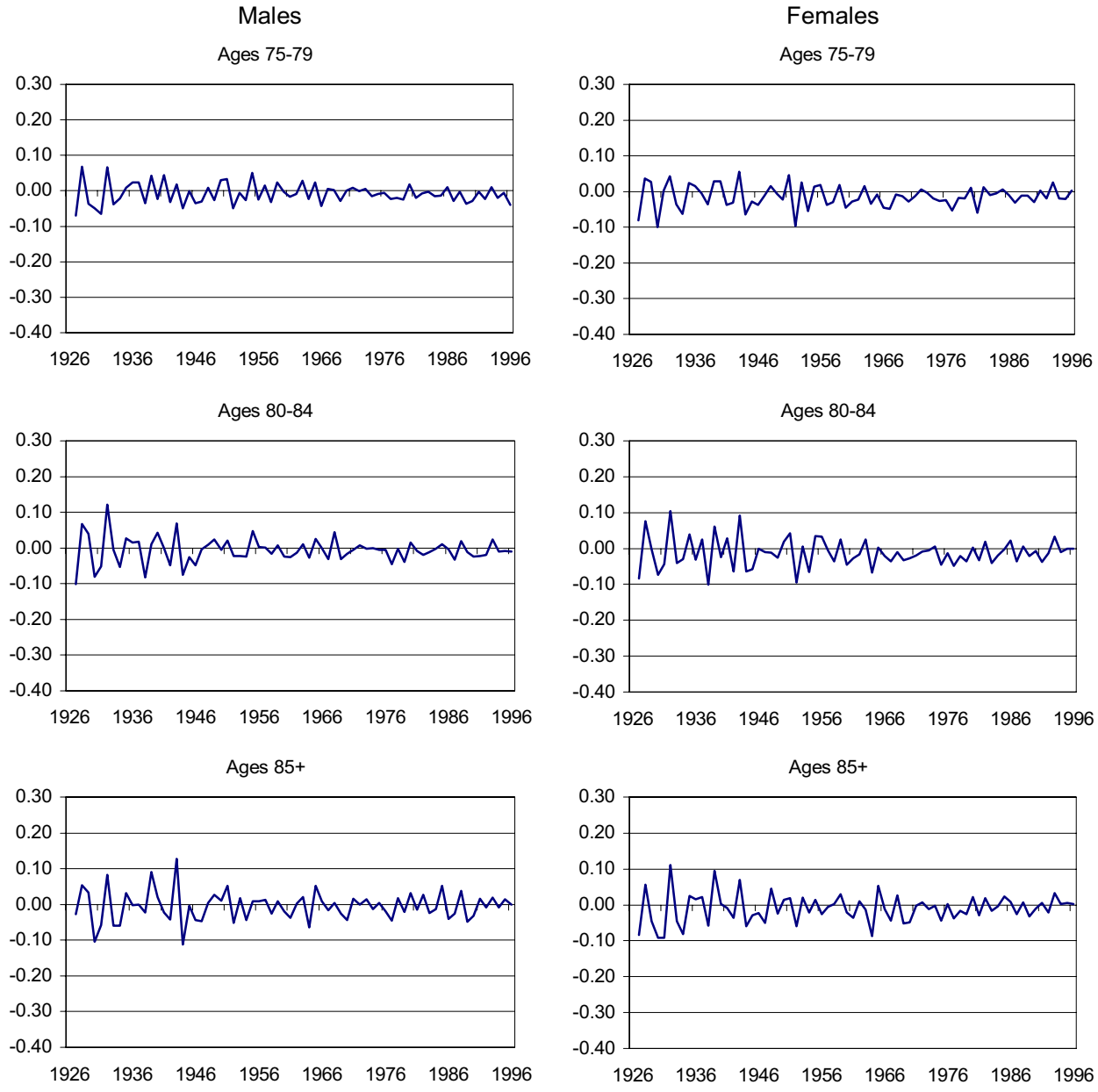
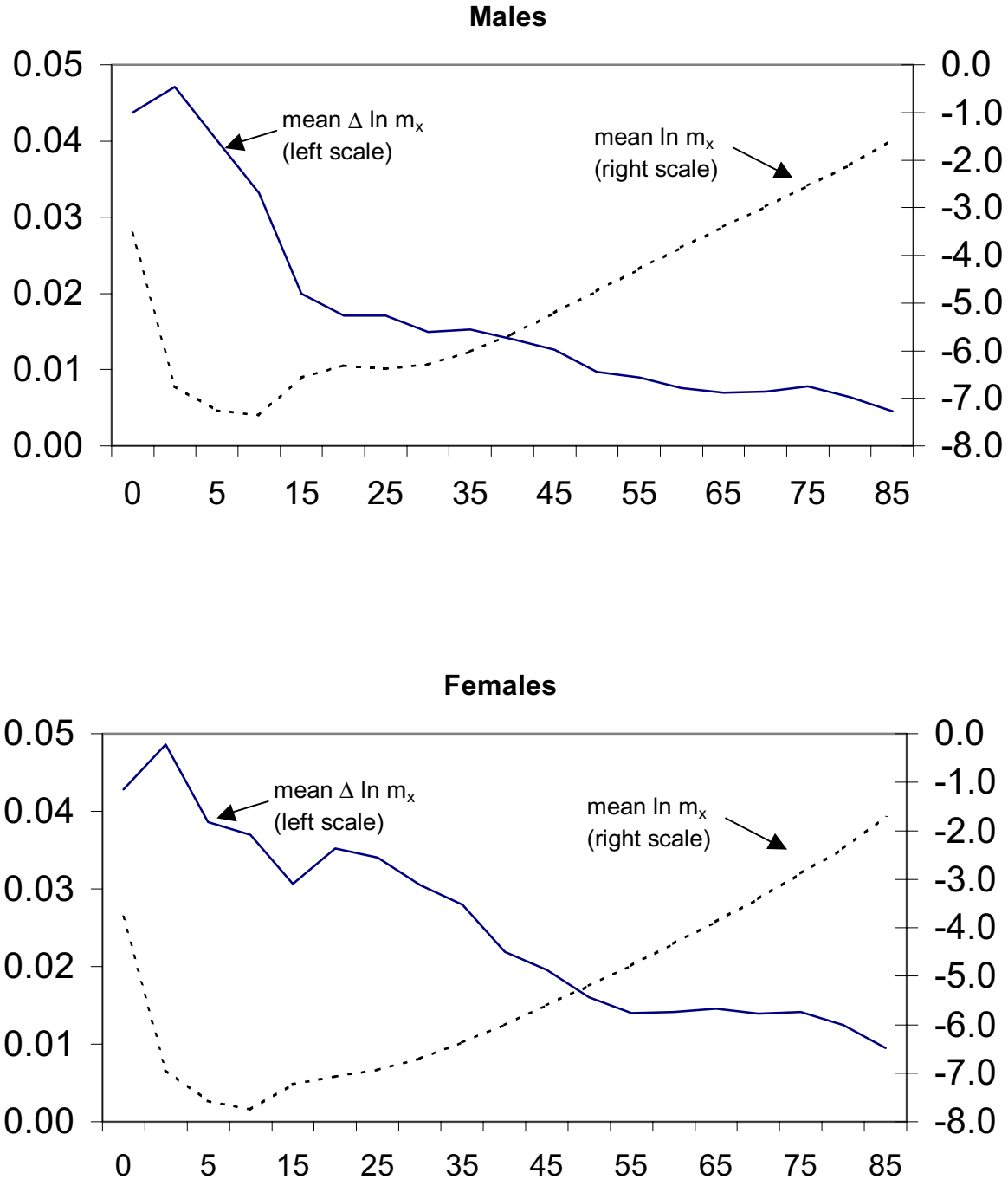
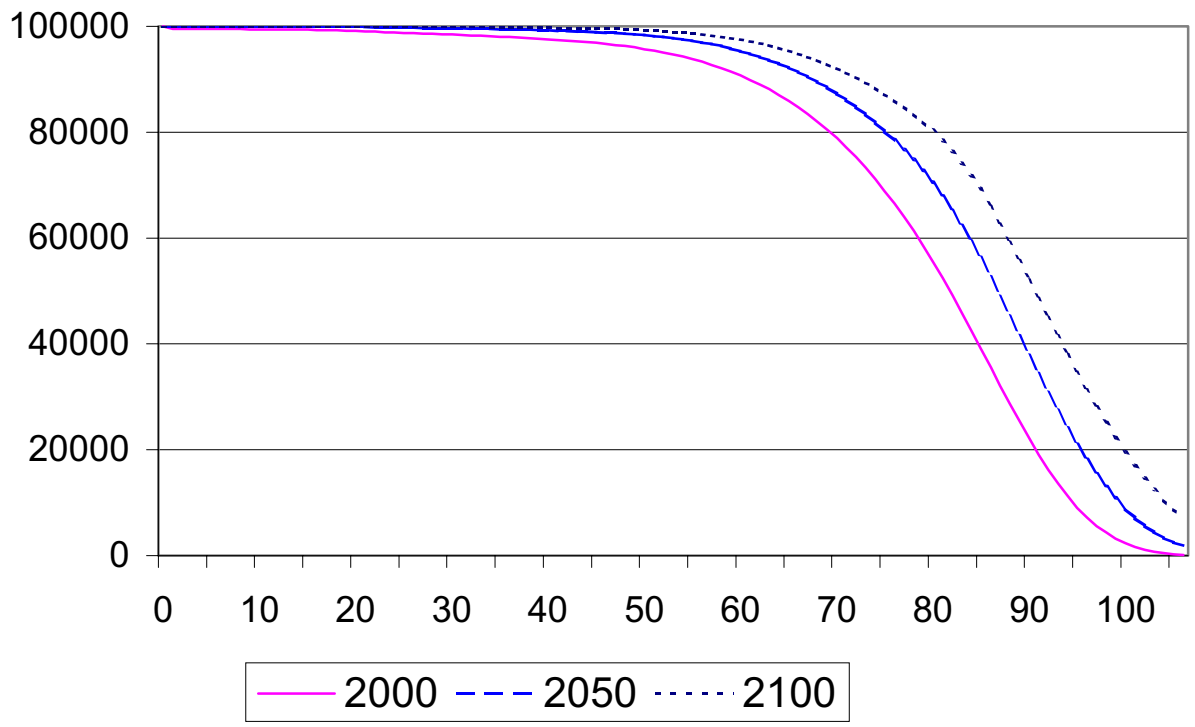


FIGURE 3: MEAN LOGS OF MORTALITY RATES AND MEAN FIRST DIFFERENCES, BY AGE GROUP



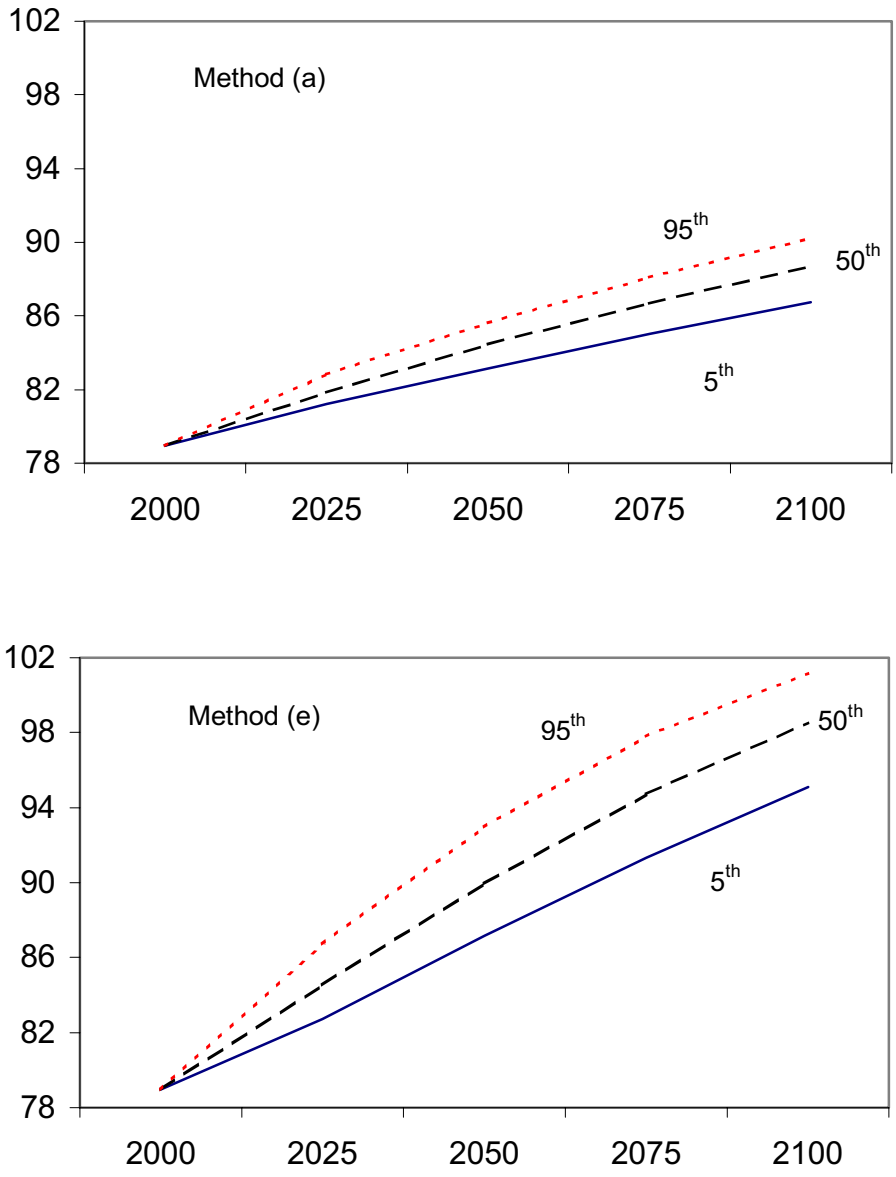

Note: Mean Δ ln $m_x$ series are plotted with sign changed.

FIGURE 4: NONSTOCHASTIC FORECASTS OF SURVIVAL FUNCTIONS, BOTH SEXES COMBINED, 2000, 2050, AND 2100




Note: The forecasts assume continuation of 1926-1996 average rates of decline in ln $m_x$ series (assumption (a) in Table 5) with maximum age 106 in all years.

FIGURE 5: SELECTED STOCHASTIC FORECASTS OF MEAN LIFE EXPECTANCY AT AGE 0, BOTH SEXES COMBINED: 5th, 50th, AND 95th PERCENTILES




Note: Methods (a) and (e), respectively, result in the forecast distributions with the lowest and highest 50th percentiles.

TABLE 1: ESTIMATED AR MODELS OF FIRST DIFFERENCES OF LOGS OF MORTALITY INDEXES ($\Delta \ln M_t$)

| Index (M) and model | Coefficients, t statistics | | | | | | S | $R^2$ |
|---|---|---|---|---|---|---|---|---|
| | Constant | t | $\Delta \ln M_{t-1}$ | t | $\Delta \ln M_{t-2}$ | t | | |
| fwaP31 | | | | | | | | |
| AR(0) | -0.0159 | 8.7 | -- | -- | -- | -- | 0.0133 | -- |
| AR(1) | -0.0185 | 7.0 | -0.1701 | 1.4 | -- | -- | 0.0132 | 0.0290 |
| AR(2) | -0.0175 | 5.0 | -0.1538 | 1.2 | 0.0489 | 0.4 | 0.0134 | 0.0290 |
| fwaP66 | | | | | | | | |
| AR(0) | 0.0145 | 7.9 | -- | -- | -- | -- | 0.0134 | -- |
| AR(1) | -0.0185 | 7.5 | -0.2750 | 2.4 | -- | -- | 0.0129 | 0.0764 |
| AR(2) | -0.0182 | 5.4 | -0.2611 | 2.1 | 0.0091 | 0.1 | 0.0132 | 0.0708 |
| fwaP91 | | | | | | | | |
| AR(0) | 0.0128 | 6.6 | -- | -- | -- | -- | 0.0141 | -- |
| AR(1) | -0.0173 | 7.5 | -0.3591 | 3.2 | -- | -- | 0.0132 | 0.1315 |
| AR(2) | -0.0184 | 5.8 | -0.3617 | 3.0 | -0.0652 | 0.5 | 0.0136 | 0.1230 |
| div | | | | | | | | |
| AR(0) | -0.0148 | 8.0 | -- | -- | -- | -- | 0.0135 | -- |
| AR(1) | -0.0180 | 7.1 | -0.2262 | 1.9 | -- | -- | 0.0132 | 0.0516 |
| AR(2) | -0.0174 | 5.1 | -0.2124 | 1.7 | 0.0293 | 0.2 | 0.0133 | 0.0493 |

Note: The equations were estimated by GLS to allow for apparent shift in error variances around 1946. S is standard error of estimate; $R^2$ is coefficient of determination calculated as in Buse (1973); t statistics are reported with signs omitted. The data are annual for the period 1926-1996 (before loss of observations because of differencing and lags). See text for index formulas.

TABLE 2: ESTIMATED AR(2) SYSTEM MODEL OF FIRST DIFFERENCES OF LOGS OF AGE GROUP MORTALITY RATES ($\Delta \ln m_{xt}$)

| Age group | Coefficients, t statistics | | | | | | |
|---|---|---|---|---|---|---|---|
| (x) | Constant | t | $\Delta \ln m_{x,t-1}$ | t | $\Delta \ln m_{x,t-2}$ | t | S |
| Males | | | | | | | |
| Under 1 | -0.0609 | 8.6 | -0.3397 | 6.5 | -0.0736 | 1.4 | 0.0503 |
| 1 - 4 | -0.0953 | 8.1 | -0.6934 | 13.1 | -0.3158 | 6.1 | 0.0904 |
| 5 - 9 | -0.0612 | 5.6 | -0.5100 | 10.0 | 0.0139 | 0.3 | 0.0863 |
| 10 - 14 | -0.0660 | 6.4 | -0.5300 | 9.7 | -0.4267 | 7.9 | 0.0815 |
| 15 - 19 | -0.0246 | 2.6 | -0.2383 | 4.1 | 0.0310 | 0.5 | 0.0763 |
| 20 - 24 | -0.0211 | 2.4 | -0.2027 | 4.3 | 0.0499 | 1.1 | 0.0724 |
| 25 - 29 | -0.0229 | 3.4 | -0.2883 | 5.2 | 0.0904 | 1.7 | 0.0537 |
| 30 - 34 | -0.0307 | 4.8 | -0.7632 | 17.4 | -0.2448 | 5.6 | 0.0519 |
| 35 - 39 | -0.0238 | 3.4 | -0.4048 | 6.6 | -0.1668 | 2.8 | 0.0559 |
| 40 - 44 | -0.0221 | 4.1 | -0.3508 | 6.1 | -0.1976 | 3.5 | 0.0430 |
| 45 - 49 | -0.0240 | 4.7 | -0.4403 | 7.3 | -0.3789 | 6.2 | 0.0411 |
| 50 - 54 | -0.0118 | 3.8 | 0.1403 | 2.5 | -0.2420 | 4.3 | 0.0254 |
| 55 - 59 | -0.0126 | 3.9 | -0.1376 | 2.6 | -0.1539 | 2.9 | 0.0259 |
| 60 - 64 | -0.0115 | 3.5 | -0.2504 | 4.4 | -0.1949 | 3.5 | 0.0269 |
| 65 - 69 | -0.0092 | 3.1 | -0.0499 | 1.0 | -0.1500 | 3.0 | 0.0242 |
| 70 - 74 | -0.0090 | 3.0 | -0.2852 | 4.3 | -0.0395 | 0.6 | 0.0236 |
| 75 - 79 | -0.0101 | 3.2 | -0.2501 | 5.6 | -0.0680 | 1.6 | 0.0253 |
| 80 - 84 | -0.0098 | 2.8 | -0.2967 | 7.8 | -0.3529 | 9.9 | 0.0289 |
| 85+ | -0.0069 | 1.5 | -0.3198 | 7.3 | -0.0940 | 2.2 | 0.0385 |
| Females | | | | | | | |
| Under 1 | -0.0707 | 10.2 | -0.3751 | 8.4 | -0.2966 | 6.6 | 0.0517 |
| 1 - 4 | -0.0905 | 7.7 | -0.6497 | 12.1 | -0.2185 | 4.1 | 0.0895 |
| 5 - 9 | -0.0739 | 5.9 | -0.4446 | 7.7 | -0.3689 | 6.6 | 0.0983 |
| 10 - 14 | -0.0687 | 5.5 | -0.3547 | 6.3 | -0.5117 | 9.3 | 0.1002 |
| 15 - 19 | -0.0636 | 4.8 | -0.4925 | 9.3 | -0.5290 | 9.9 | 0.1066 |
| 20 - 24 | -0.0524 | 5.2 | -0.2666 | 4.6 | -0.1826 | 3.1 | 0.0793 |
| 25 - 29 | -0.0640 | 7.0 | -0.6342 | 11.5 | -0.2154 | 3.8 | 0.0711 |
| 30 - 34 | -0.0554 | 7.0 | -0.4710 | 8.5 | -0.2957 | 5.4 | 0.0618 |
| 35 - 39 | -0.0454 | 6.6 | -0.6183 | 8.3 | 0.0184 | 0.2 | 0.0502 |
| 40 - 44 | -0.0490 | 7.0 | -0.7898 | 11.1 | -0.3392 | 4.8 | 0.0526 |
| 45 - 49 | -0.0441 | 8.0 | -0.7838 | 11.5 | -0.4764 | 7.0 | 0.0414 |
| 50 - 54 | -0.0297 | 7.4 | -0.4685 | 7.9 | -0.3319 | 5.6 | 0.0309 |
| 55 - 59 | -0.0293 | 8.5 | -0.5928 | 11.6 | -0.4147 | 8.1 | 0.0265 |
| 60 - 64 | -0.0281 | 8.1 | -0.7261 | 12.0 | -0.2285 | 3.7 | 0.0260 |
| 65 - 69 | -0.0205 | 6.0 | -0.1229 | 2.2 | -0.3221 | 5.7 | 0.0262 |
| 70 - 74 | -0.0221 | 6.7 | -0.3217 | 6.2 | -0.2558 | 5.2 | 0.0255 |
| 75 - 79 | -0.0205 | 5.6 | -0.3710 | 7.4 | -0.1126 | 2.3 | 0.0285 |
| 80 - 84 | -0.0216 | 5.0 | -0.4061 | 7.9 | -0.3238 | 6.5 | 0.0348 |
| 85+ | -0.0137 | 3.0 | -0.2245 | 4.8 | -0.2434 | 5.4 | 0.0368 |

Note: The equations for the 38 age-sex groups were estimated as a system using iterated SURE to allow for cross-equation error correlations. The system $R^2$ is 0.9988 (see Berndt, 1991, p. 468). See note to Table 1 for other information.

TABLE 3: ESTIMATED QVAR(1) SYSTEM MODEL OF FIRST DIFFERENCES OF LOGS OF AGE GROUP MORTALITY RATES ($\Delta \ln m_{xt}$)

| Age group | Coefficients, t statistics | | | | | | |
|---|---|---|---|---|---|---|---|
| (x) | Constant | t | $\Delta \ln m_{x,t-1}$ | t | $\Delta \ln M_{t-1}$ | t | S |
| Males | | | | | | | |
| Under 1 | -0.0481 | 6.8 | -0.2679 | 4.9 | 0.5091 | 1.9 | 0.0486 |
| 1 - 4 | -0.0560 | 4.7 | -0.6060 | 12.1 | 1.5821 | 3.4 | 0.0840 |
| 5 - 9 | -0.0505 | 4.3 | -0.5591 | 10.3 | 0.9523 | 2.1 | 0.0838 |
| 10 - 14 | -0.0467 | 4.0 | -0.3522 | 5.7 | -0.0337 | 0.1 | 0.0825 |
| 15 - 19 | -0.0163 | 1.5 | -0.3078 | 4.6 | 0.6979 | 1.6 | 0.0756 |
| 20 - 24 | -0.0191 | 1.9 | -0.2373 | 4.1 | 0.2012 | 0.5 | 0.0721 |
| 25 - 29 | -0.0236 | 3.2 | -0.2106 | 3.9 | -0.0987 | 0.3 | 0.0525 |
| 30 - 34 | -0.0221 | 3.2 | -0.5412 | 10.7 | 0.0619 | 0.2 | 0.0489 |
| 35 - 39 | -0.0177 | 2.3 | -0.3732 | 6.0 | 0.1886 | 0.6 | 0.0557 |
| 40 - 44 | -0.0164 | 2.8 | -0.2694 | 4.5 | 0.0516 | 0.2 | 0.0423 |
| 45 - 49 | -0.0113 | 2.1 | -0.0457 | 6.1 | 0.5224 | 2.4 | 0.0390 |
| 50 - 54 | -0.0077 | 2.2 | 0.1510 | 2.4 | 0.0776 | 0.6 | 0.0249 |
| 55 - 59 | -0.0115 | 3.2 | -0.1245 | 1.9 | -0.0596 | 0.4 | 0.0257 |
| 60 - 64 | -0.0102 | 2.6 | -0.3280 | 5.5 | -0.0121 | 0.1 | 0.0277 |
| 65 - 69 | -0.0100 | 2.9 | 0.0482 | 0.8 | -0.2555 | 1.8 | 0.0247 |
| 70 - 74 | -0.0101 | 3.1 | -0.3085 | 4.0 | -0.1082 | 0.8 | 0.0235 |
| 75 - 79 | -0.0119 | 3.4 | -0.3418 | 4.9 | -0.1918 | 1.3 | 0.0252 |
| 80 - 84 | -0.0093 | 2.0 | -0.1111 | 1.7 | -0.2709 | 1.4 | 0.0328 |
| 85+ | -0.0095 | 1.7 | -0.3234 | 5.3 | -0.2884 | 1.2 | 0.0391 |
| Females | | | | | | | |
| Under 1 | -0.0523 | 7.1 | -0.4331 | 8.8 | 0.6297 | 2.2 | 0.0512 |
| 1 - 4 | -0.0554 | 4.8 | -0.5609 | 11.5 | 1.5858 | 3.6 | 0.0810 |
| 5 - 9 | -0.0358 | 2.9 | -0.1978 | 3.5 | 0.9860 | 2.1 | 0.0880 |
| 10 - 14 | -0.0476 | 3.7 | -0.3620 | 5.0 | 0.2496 | 0.5 | 0.0922 |
| 15 - 19 | -0.0376 | 3.0 | -0.2592 | 3.7 | 0.1530 | 0.3 | 0.0881 |
| 20 - 24 | -0.0384 | 3.6 | -0.1432 | 2.1 | 0.1900 | 0.5 | 0.0753 |
| 25 - 29 | -0.0470 | 5.1 | -0.5262 | 9.6 | 0.3799 | 1.0 | 0.0659 |
| 30 - 34 | -0.4250 | 5.0 | -0.3338 | 5.2 | -0.1506 | 0.5 | 0.0602 |
| 35 - 39 | -0.0423 | 6.1 | -0.5179 | 6.7 | 0.0292 | 0.1 | 0.0481 |
| 40 - 44 | -0.0251 | 3.8 | -0.7111 | 10.7 | 1.0083 | 3.9 | 0.0467 |
| 45 - 49 | -0.0235 | 4.2 | -0.5955 | 7.6 | 0.5641 | 2.5 | 0.0401 |
| 50 - 54 | -0.0211 | 4.5 | -0.4508 | 7.5 | 0.1529 | 0.8 | 0.0336 |
| 55 - 59 | -0.0212 | 5.7 | -0.4901 | 8.8 | -0.0233 | 0.2 | 0.0262 |
| 60 - 64 | -0.0210 | 5.7 | -0.5813 | 8.3 | 0.0981 | 0.7 | 0.0259 |
| 65 - 69 | -0.0142 | 3.5 | -0.0153 | 0.2 | 0.0036 | 0.0 | 0.0287 |
| 70 - 74 | -0.0190 | 4.6 | -0.2007 | 2.8 | -0.2541 | 1.4 | 0.0298 |
| 75 - 79 | -0.0198 | 4.8 | -0.2898 | 4.2 | -0.1897 | 1.1 | 0.0293 |
| 80 - 84 | -0.0220 | 4.5 | -0.0547 | 0.7 | -0.7490 | 3.3 | 0.0348 |
| 85+ | -0.0166 | 3.2 | -0.2165 | 3.3 | -0.4636 | 2.1 | 0.0366 |

Note: See note to Table 2. The system $R^2$ is 0.9965.

TABLE 4: STEADY STATE VALUES OF $\Delta \ln m_x$ BASED ON AR(2) SYSTEM MODEL ($\alpha$ VALUES) AND MEAN VALUES CALCULATED FROM 1926-1996 DATA

| Age group (x) | $\alpha$ value | | Mean $\Delta \ln m_x$, 1926-1996 | |
|---|---|---|---|---|
| | Males | Females | Males | Females |
| Under 1 | -0.0431 | -0.0423 | -0.0437 | -0.0428 |
| 1 - 4 | -0.0474 | -0.0484 | -0.0471 | -0.0486 |
| 5 - 9 | -0.0409 | -0.0408 | -0.0400 | -0.0386 |
| 10 - 14 | -0.0337 | -0.0368 | -0.0332 | -0.0369 |
| 15 - 19 | -0.0204 | -0.0315 | -0.0200 | -0.0306 |
| 20 - 24 | -0.0183 | -0.0361 | -0.0171 | -0.0353 |
| 25 - 29 | -0.0191 | -0.0346 | -0.0171 | -0.0341 |
| 30 - 34 | -0.0153 | -0.0314 | -0.0150 | -0.0305 |
| 35 - 39 | -0.0151 | -0.0284 | -0.0153 | -0.0280 |
| 40 - 44 | -0.0143 | -0.0230 | -0.0140 | -0.0219 |
| 45 - 49 | -0.0132 | -0.0195 | -0.0126 | -0.0195 |
| 50 - 54 | -0.0107 | -0.0165 | -0.0097 | -0.0160 |
| 55 - 59 | -0.0097 | -0.0146 | -0.0090 | -0.0140 |
| 60 - 64 | -0.0080 | -0.0144 | -0.0076 | -0.0141 |
| 65 - 69 | -0.0077 | -0.0142 | -0.0070 | -0.0146 |
| 70 - 74 | -0.0068 | -0.0140 | -0.0072 | -0.0139 |
| 75 - 79 | -0.0077 | -0.0138 | -0.0078 | -0.0141 |
| 80 - 84 | -0.0059 | -0.0125 | -0.0064 | -0.0125 |
| 85+ | -0.0049 | -0.0094 | -0.0046 | -0.0095 |

Note: See text for calculation of $\alpha$ values.

TABLE 5: NONSTOCHASTIC FORECASTS OF LIFE EXPECTANCIES, BOTH SEXES COMBINED

| Assumed rates of decline of ln $m_x$ | Assumed maximum age attained by 2100 | Life expectancies | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | At age 0 | | | At age 65 | | | At age 80 | | |
| | | 2000 | 2050 | 2100 | 2000 | 2050 | 2100 | 2000 | 2050 | 2100 |
| | | | | | -- means -- | | | | | |
| (a) Continuation of 1926-96 average rates of decline | 106 | 78.96 | 84.68 | 89.09 | 18.53 | 22.23 | 25.66 | 8.94 | 11.38 | 13.81 |
| | 110 | 78.96 | 84.69 | 89.16 | 18.53 | 22.24 | 25.74 | 8.94 | 11.39 | 13.90 |
| | 120 | 78.96 | 84.70 | 89.20 | 18.53 | 22.24 | 25.78 | 8.94 | 11.40 | 13.94 |
| | 130 | 78.96 | 84.70 | 89.20 | 18.53 | 22.24 | 25.78 | 8.94 | 11.40 | 13.95 |
| | 140 | 78.96 | 84.70 | 89.20 | 18.53 | 22.25 | 25.79 | 8.94 | 11.40 | 13.95 |
| (b) Rates of decline fall to ½ of those in (a) by 2100 | 106 | 78.93 | 83.89 | 86.80 | 18.52 | 21.66 | 23.83 | 8.93 | 10.99 | 12.50 |
| | 110 | 78.93 | 83.90 | 86.83 | 18.52 | 21.67 | 23.86 | 8.93 | 11.00 | 12.53 |
| | 120 | 78.93 | 83.90 | 86.84 | 18.52 | 21.67 | 23.87 | 8.93 | 11.00 | 12.55 |
| | 130 | 78.93 | 83.90 | 86.84 | 18.52 | 21.67 | 23.88 | 8.93 | 11.00 | 12.55 |
| | 140 | 78.93 | 83.90 | 86.84 | 18.52 | 21.67 | 23.88 | 8.93 | 11.00 | 12.55 |
| (c) Rates of decline fall to zero by 2100 | 106 | 78.91 | 83.07 | 84.24 | 18.51 | 21.08 | 21.91 | 8.92 | 10.60 | 11.16 |
| | 110 | 78.91 | 83.07 | 84.25 | 18.51 | 21.09 | 21.91 | 8.92 | 10.60 | 11.17 |
| | 120 | 78.91 | 83.07 | 84.25 | 18.51 | 21.09 | 21.92 | 8.92 | 10.61 | 11.17 |
| | 130 | 78.91 | 83.07 | 84.25 | 18.51 | 21.09 | 21.92 | 8.92 | 10.61 | 11.17 |
| | 140 | 78.91 | 83.07 | 84.25 | 18.51 | 21.09 | 21.92 | 8.92 | 10.61 | 11.17 |
| | | | | | -- medians -- | | | | | |
| (a) As above | 106 | 81.75 | 86.74 | 90.59 | 18.86 | 22.80 | 26.23 | 8.26 | 10.75 | 13.36 |
| (b) As above | 106 | 81.73 | 86.06 | 88.56 | 18.85 | 22.22 | 24.39 | 8.25 | 10.35 | 11.92 |
| (c) As above | 106 | 81.71 | 85.36 | 86.36 | 18.83 | 21.63 | 22.47 | 8.24 | 9.95 | 10.52 |

Note: Age 106 is the oldest age at which there are any survivors in the 1991 Canadian life table. The present table is based on separate projections for males and females, combined assuming a male/female ratio at birth of 1.05377. Median life expectancies are unaffected by altering the maximum age.

TABLE 6: SUMMARY OF FORECAST METHODS

| Method | Parametric, nonparametric classification | Model used | Distribution of disturbances | Time period randomization | Age-sex group randomization |
|---|---|---|---|---|---|
| (a) | Nonparametric | None | Not specified | 25-year vector block bootstrap applied to $\Delta \ln m_x$ series | None |
| (b) | Nonparametric | None | Not specified | 25-year vector block bootstrap applied to $\Delta \ln m_x$ series | $\Delta \ln m_x$ blocks reassigned randomly |
| (c) | Nonparametric | None | Not specified | 25-year vector block bootstrap applied to $\Delta \ln m_x$ series | $\Delta \ln m_x$ blocks reassigned randomly,but under-15 groups excluded |
| (d) | Partially parametric | AR(2) | Not specified | Vector bootstrap applied each year to model residuals | None |
| (e) | Partially parametric | AR(2) | Not specified | Vector bootstrap applied each year to model residuals | $\alpha$ values reassigned randomly |
| (f) | Partially parametric | AR(2) | Not specified | Vector bootstrap applied each year to model residuals | $\alpha$ values reassigned randomly, but under-15 groups excluded |
| (g) | Fully parametric | AR(2) | Multivariate normal | Disturbance vector drawn each year from $N(0,\Sigma)$ | None |

TABLE 7: STOCHASTIC FORECASTS OF MEAN LIFE EXPECTANCIES, BOTH SEXES COMBINED

| Forecasting procedure | Year | Selected percentiles of mean life expectancy distributions | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | At age 0 | | | At age 65 | | | At age 80 | | |
| | | 5th | 50th | 95th | 5th | 50th | 95th | 5th | 50th | 95th |
| Nonparametric | | | | | | | | | | |
| (a) Vector block | 2000 | -- | 78.96 | -- | -- | 18.53 | -- | -- | 8.94 | -- |
| bootstrap applied | 2025 | 81.20 | 81.85 | 82.82 | 19.64 | 20.49 | 20.96 | 9.58 | 10.30 | 10.67 |
| to historical | 2050 | 83.14 | 84.46 | 85.64 | 21.06 | 22.25 | 23.06 | 10.60 | 11.58 | 12.30 |
| $\Delta$ ln $m_x$ series | 2075 | 85.02 | 86.68 | 88.08 | 22.56 | 23.94 | 24.98 | 11.67 | 12.86 | 13.77 |
| | 2100 | 86.76 | 88.68 | 90.22 | 23.97 | 25.54 | 26.75 | 12.75 | 14.10 | 15.13 |
| (b) like (a) but random | 2000 | -- | 78.96 | -- | -- | 18.53 | -- | -- | 8.94 | -- |
| reassignment of | 2025 | 82.63 | 84.16 | 86.29 | 20.88 | 22.49 | 24.78 | 10.09 | 11.78 | 14.61 |
| blocks among | 2050 | 86.82 | 89.30 | 92.21 | 24.03 | 26.68 | 29.75 | 12.24 | 15.09 | 18.53 |
| age-sex groups | 2075 | 90.76 | 93.93 | 97.07 | 27.35 | 30.58 | 33.79 | 14.68 | 18.16 | 21.50 |
| | 2100 | 94.37 | 97.70 | 100.55 | 30.43 | 33.83 | 36.71 | 17.13 | 20.66 | 23.51 |
| (c) Like (b) but under-15 | 2000 | -- | 78.96 | -- | -- | 18.53 | -- | -- | 8.94 | -- |
| groups excluded in | 2025 | 81.83 | 82.88 | 84.57 | 20.35 | 21.46 | 23.29 | 9.83 | 10.98 | 13.34 |
| reassignment of | 2050 | 85.11 | 86.86 | 89.24 | 22.80 | 24.63 | 27.15 | 11.42 | 13.41 | 16.42 |
| blocks | 2075 | 88.32 | 90.64 | 93.45 | 25.37 | 27.77 | 30.70 | 13.29 | 15.88 | 19.04 |
| | 2100 | 91.36 | 94.03 | 96.92 | 27.93 | 30.66 | 33.63 | 15.25 | 18.13 | 21.31 |
| Partially parametric | | | | | | | | | | |
| (d) AR(2) model; vector | 2000 | -- | 78.96 | -- | -- | 18.53 | -- | -- | 8.94 | -- |
| bootstrap applied to | 2025 | 80.92 | 82.08 | 83.29 | 19.28 | 20.39 | 21.57 | 9.16 | 10.14 | 11.19 |
| model residuals | 2050 | 82.99 | 84.67 | 86.43 | 20.55 | 22.19 | 23.90 | 9.91 | 11.34 | 12.90 |
| | 2075 | 84.96 | 86.97 | 89.12 | 21.92 | 23.90 | 26.06 | 10.80 | 12.54 | 14.51 |
| | 2100 | 86.70 | 89.05 | 91.52 | 23.22 | 25.56 | 28.02 | 11.64 | 13.74 | 15.98 |
| (e) like (d) but random | 2000 | -- | 78.96 | -- | -- | 18.53 | -- | -- | 8.94 | -- |
| reassignment of $\alpha$ | 2025 | 82.70 | 84.55 | 86.73 | 20.97 | 22.82 | 25.07 | 10.16 | 12.10 | 14.58 |
| values among | 2050 | 87.15 | 89.97 | 92.97 | 24.40 | 27.26 | 30.34 | 12.53 | 15.52 | 18.67 |
| age-sex groups | 2075 | 91.32 | 94.73 | 97.81 | 27.83 | 31.31 | 34.37 | 15.15 | 18.67 | 21.68 |
| | 2100 | 95.10 | 98.54 | 101.16 | 31.13 | 34.60 | 37.18 | 17.76 | 21.20 | 23.67 |
| (f) Like (e) but under-15 | 2000 | -- | 78.96 | -- | -- | 18.53 | -- | -- | 8.94 | -- |
| groups excluded in | 2025 | 81.78 | 83.31 | 84.98 | 20.33 | 21.81 | 23.51 | 9.86 | 11.30 | 13.12 |
| reassignment of | 2050 | 83.30 | 87.60 | 89.99 | 22.96 | 25.28 | 27.69 | 11.61 | 13.93 | 16.45 |
| $\alpha$ values | 2075 | 88.71 | 91.57 | 94.40 | 25.73 | 28.60 | 31.45 | 13.67 | 16.52 | 19.29 |
| | 2100 | 91.99 | 95.16 | 97.96 | 28.49 | 31.67 | 34.42 | 15.76 | 18.89 | 21.52 |
| Fully parametric | | | | | | | | | | |
| (g) AR(2) model; error | 2000 | -- | 78.96 | -- | -- | 18.53 | -- | -- | 8.94 | -- |
| vectors drawn from | 2025 | 80.91 | 82.07 | 83.25 | 19.29 | 20.39 | 21.53 | 9.17 | 10.14 | 11.16 |
| multivariate normal | 2050 | 83.08 | 84.67 | 86.40 | 20.64 | 22.17 | 23.88 | 9.96 | 11.35 | 12.87 |
| distribution | 2075 | 85.01 | 86.97 | 89.08 | 21.98 | 23.92 | 26.02 | 10.82 | 12.56 | 14.48 |
| | 2100 | 86.78 | 89.06 | 91.48 | 23.32 | 25.58 | 28.02 | 11.70 | 13.78 | 15.94 |

TABLE 8: STOCHASTIC FORECASTS OF MEDIAN LIFE EXPECTANCIES, BOTH SEXES COMBINED

| Forecasting procedure | Year | Selected percentiles of median life expectancy distributions | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | At age 0 | | | At age 65 | | | At age 80 | | |
| | | 5th | 50th | 95th | 5th | 50th | 95th | 5th | 50th | 95th |
| Nonparametric | | | | | | | | | | |
| (a) Vector block | 2000 | -- | 81.75 | -- | -- | 18.86 | -- | -- | 8.26 | -- |
| bootstrap applied | 2025 | 83.43 | 84.31 | 85.27 | 20.15 | 21.01 | 21.52 | 8.94 | 9.66 | 10.02 |
| to historical | 2050 | 85.40 | 86.66 | 87.67 | 21.75 | 22.86 | 23.64 | 9.98 | 10.96 | 11.75 |
| $\Delta$ ln $m_x$ series | 2075 | 87.14 | 88.63 | 89.84 | 23.27 | 24.59 | 25.64 | 11.07 | 12.33 | 13.37 |
| | 2100 | 88.75 | 90.47 | 91.89 | 24.69 | 26.27 | 27.58 | 12.19 | 13.71 | 15.02 |
| (b) like (a) but random | 2000 | -- | 81.75 | -- | -- | 18.86 | -- | -- | 8.26 | -- |
| reassignment of | 2025 | 84.75 | 86.51 | 88.67 | 21.23 | 22.88 | 25.34 | 9.43 | 11.15 | 14.51 |
| blocks among | 2050 | 88.78 | 91.54 | 96.58 | 24.59 | 27.52 | 32.88 | 11.67 | 15.02 | 21.06 |
| age-sex groups | 2075 | 92.36 | 96.93 | 103.67 | 27.87 | 32.56 | 39.06 | 14.30 | 19.19 | 24.82 |
| | 2100 | 95.30 | 100.82 | 105.31 | 30.64 | 36.09 | 40.36 | 16.58 | 21.81 | 25.54 |
| (c) Like (b) but under-15 | 2000 | -- | 81.75 | -- | -- | 18.86 | -- | -- | 8.26 | -- |
| groups excluded in | 2025 | 84.09 | 85.40 | 86.96 | 20.63 | 21.90 | 23.53 | 9.16 | 10.36 | 12.71 |
| reassignment of | 2050 | 87.21 | 89.12 | 92.02 | 23.28 | 25.26 | 28.49 | 10.82 | 13.01 | 17.23 |
| blocks | 2075 | 90.25 | 93.07 | 98.23 | 25.95 | 28.93 | 34.40 | 12.83 | 16.13 | 21.90 |
| | 2100 | 93.15 | 97.21 | 103.32 | 28.67 | 32.83 | 38.76 | 15.10 | 19.47 | 24.73 |
| Partially parametric | | | | | | | | | | |
| (d) AR(2) model; vector | 2000 | -- | 81.75 | -- | -- | 18.86 | -- | -- | 8.26 | -- |
| bootstrap applied to | 2025 | 83.30 | 84.44 | 85.59 | 19.86 | 20.90 | 22.04 | 8.49 | 9.49 | 10.60 |
| model residuals | 2050 | 85.17 | 86.69 | 88.43 | 21.18 | 22.73 | 24.54 | 9.22 | 10.72 | 12.47 |
| | 2075 | 86.71 | 88.65 | 91.04 | 22.47 | 24.45 | 26.91 | 10.04 | 11.98 | 14.39 |
| | 2100 | 88.05 | 90.51 | 93.58 | 23.62 | 26.13 | 29.25 | 10.86 | 13.32 | 16.34 |
| (e) like (d) but random | 2000 | -- | 81.75 | -- | -- | 18.86 | -- | -- | 8.26 | -- |
| reassignment of $\alpha$ | 2025 | 84.96 | 86.95 | 89.28 | 21.35 | 23.30 | 25.86 | 9.52 | 11.51 | 14.60 |
| values among | 2050 | 89.12 | 92.40 | 97.32 | 24.89 | 28.31 | 33.51 | 11.97 | 15.66 | 21.17 |
| age-sex groups | 2075 | 93.02 | 98.08 | 104.11 | 28.51 | 33.62 | 39.42 | 14.87 | 19.98 | 24.99 |
| | 2100 | 96.00 | 101.40 | 105.43 | 31.37 | 36.63 | 40.46 | 17.21 | 22.13 | 25.59 |
| (f) Like (e) but under-15 | 2000 | -- | 81.75 | -- | -- | 18.86 | -- | -- | 8.26 | -- |
| groups excluded in | 2025 | 84.26 | 85.83 | 87.52 | 20.74 | 22.29 | 24.08 | 9.21 | 10.69 | 12.75 |
| reassignment of | 2050 | 87.53 | 89.97 | 93.06 | 23.49 | 26.04 | 29.28 | 11.01 | 13.68 | 17.22 |
| $\alpha$ values | 2075 | 90.67 | 94.26 | 99.20 | 26.35 | 30.06 | 35.11 | 13.26 | 17.08 | 22.19 |
| | 2100 | 93.95 | 98.87 | 104.25 | 29.44 | 34.39 | 39.57 | 15.84 | 20.69 | 25.06 |
| Fully parametric | | | | | | | | | | |
| (g) AR(2) model; error | 2000 | -- | 81.75 | -- | -- | 18.86 | -- | -- | 8.26 | -- |
| vectors drawn from | 2025 | 83.30 | 84.43 | 85.55 | 19.86 | 20.90 | 22.02 | 8.50 | 9.48 | 10.56 |
| multivariate normal | 2050 | 85.24 | 86.66 | 88.41 | 21.25 | 22.71 | 24.51 | 9.26 | 10.72 | 12.45 |
| distribution | 2075 | 86.78 | 88.64 | 91.01 | 22.51 | 24.45 | 26.87 | 10.08 | 12.00 | 14.34 |
| | 2100 | 88.15 | 90.52 | 93.53 | 23.73 | 26.16 | 29.21 | 10.95 | 13.35 | 16.29 |

TABLE 9: 90 PERCENT PROBABILITY INTERVALS DERIVED FROM STOCHASTIC FORECASTS OF MEAN AND MEDIAN LIFE EXPECTANCIES, BOTH SEXES COMBINED

| Forecasting procedure | Year | Mean life expectancies | | | Median life expectancies | | |
|---|---|---|---|---|---|---|---|
| | | At age 0 | At age 65 | At age 80 | At age 0 | At age 65 | At age 80 |
| Nonparametric | | | | | | | |
| (a) Vector block | 2025 | 1.62 | 1.32 | 1.09 | 1.84 | 1.37 | 1.08 |
| bootstrap applied | 2050 | 2.50 | 2.00 | 1.70 | 2.27 | 1.89 | 1.77 |
| to historical | 2075 | 3.06 | 2.42 | 2.10 | 2.70 | 2.37 | 2.30 |
| $\Delta$ ln $m_x$ series | 2100 | 3.46 | 2.78 | 2.38 | 3.14 | 2.89 | 2.83 |
| (b) like (a) but random | 2025 | 3.66 | 3.90 | 4.52 | 3.92 | 4.11 | 5.08 |
| reassignment of | 2050 | 5.39 | 5.72 | 6.29 | 7.80 | 8.29 | 9.39 |
| blocks among | 2075 | 6.31 | 6.44 | 6.82 | 11.31 | 11.19 | 10.52 |
| age-sex groups | 2100 | 6.18 | 6.28 | 6.38 | 10.01 | 9.72 | 8.96 |
| (c) Like (b) but under-15 | 2025 | 2.74 | 2.94 | 3.51 | 2.87 | 2.90 | 3.55 |
| groups excluded in | 2050 | 4.13 | 4.35 | 5.00 | 4.81 | 5.21 | 6.41 |
| reassignment of | 2075 | 5.13 | 5.33 | 5.75 | 7.98 | 8.45 | 9.07 |
| blocks | 2100 | 5.56 | 5.70 | 6.06 | 10.17 | 10.09 | 9.63 |
| Partially parametric | | | | | | | |
| (d) AR(2) model; vector | 2025 | 2.37 | 2.29 | 2.03 | 2.29 | 2.18 | 2.11 |
| bootstrap applied to | 2050 | 3.44 | 3.35 | 2.99 | 3.26 | 3.36 | 3.25 |
| model residuals | 2075 | 4.16 | 4.14 | 3.71 | 4.33 | 4.44 | 4.35 |
| | 2100 | 4.82 | 4.80 | 4.34 | 5.53 | 5.63 | 5.48 |
| (e) like (d) but random | 2025 | 4.03 | 4.10 | 4.42 | 4.32 | 4.51 | 5.08 |
| reassignment of $\alpha$ | 2050 | 5.82 | 5.94 | 6.14 | 8.20 | 8.62 | 9.20 |
| values among | 2075 | 6.49 | 6.54 | 6.53 | 11.09 | 10.91 | 10.12 |
| age-sex groups | 2100 | 6.06 | 6.05 | 5.91 | 9.43 | 9.09 | 8.38 |
| (f) Like (e) but under-15 | 2025 | 3.20 | 3.18 | 3.26 | 3.26 | 3.34 | 3.54 |
| groups excluded in | 2050 | 6.69 | 4.73 | 4.84 | 5.53 | 5.79 | 6.21 |
| reassignment of | 2075 | 5.69 | 5.72 | 5.62 | 8.53 | 8.76 | 8.93 |
| $\alpha$ values | 2100 | 5.97 | 5.93 | 5.76 | 10.30 | 10.13 | 9.22 |
| Fully parametric | | | | | | | |
| (g) AR(2) model; error | 2025 | 2.34 | 2.24 | 1.99 | 2.25 | 2.16 | 2.06 |
| vectors drawn from | 2050 | 3.32 | 3.24 | 2.91 | 3.17 | 3.26 | 3.19 |
| multivariate normal | 2075 | 4.07 | 4.04 | 3.66 | 4.23 | 4.36 | 4.26 |
| distribution | 2100 | 4.70 | 4.70 | 4.24 | 5.38 | 5.48 | 5.34 |

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