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# Pro-cyclical Solow Residuals without Technology Shocks

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**Running Title**

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## **Abstract**

Most Real Business Cycle models have a hard time jointly explaining the twin facts of strongly pro-cyclical Solow residuals and extremely low correlations between wages and hours. We present a model that delivers both these results without using exogenous variation in total factor productivity (technology shocks). The key innovation of the paper is to add learning-by-doing to firms technology. As a result firms optimally vary their prices to control the amount of learning which in turn influences future productivity. We show that exogenous variation in labour wedges (preference shocks) measured from aggregate data deliver around fifty percent of the standard deviation in the efficiency wedge (Solow residual) as well as realistic second moments for key aggregate variables which is in sharp contrast to the model without learning-by-doing.

## **Keywords**

Business cycles; Learning-by-Doing; Productivity

# 1 Introduction

While the neoclassical growth model has proved to be an extremely effective tool for capturing the basic features of business cycles, two major areas of weakness have been identified in the literature, in terms of the ability of the model to capture the dynamics of the aggregate data. These problems have been elegantly re-interpreted in terms of “wedges” in the equations of the model in a recent paper by Chari et al. (2007). In an accounting exercise, the authors show that the wedges embedded in the hours first order condition and in the production technology, are essential for “explaining” the bulk of the movement seen in aggregate US data. In the business cycle literature, these wedges have been interpreted in the past as exogenous “shocks” which are the source of fluctuations. These interpretations of the two wedges as preference shocks and technology shocks respectively have been controversial, not least because they assume rather than explain a large fraction of the variation in the aggregate data. In this context, a number of authors have attempted to develop models that produce fluctuations in the labour wedge and account for a part of the movement in the efficiency wedge<sup>1</sup>. However much more work remains to be done in this area. As pointed out by Chari et al. (2007), “... our results suggest that future theoretical work should focus on developing models which lead to fluctuations in efficiency and labor wedges. Many existing models produce fluctuations in labor wedges. The challenging task is to develop detailed models in which primitive shocks lead to fluctuations in efficiency wedges as well.”<sup>2</sup>. The goal of this paper is to take a step in this direction.

Unlike most business cycle models, there are no exogenous movements in technology in our model. Instead, we rely on movements in the labour wedge to induce fluctuations in aggregate

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<sup>1</sup> See the discussion and references in King & Rebelo (1999).

<sup>2</sup> See Chari et al. (2007), p 828.

variables, and the general equilibrium structure of the model to generate dynamic correlations between these variables. We show that when fluctuations in the labour wedge are fed into a prototypical real business cycle model, it is unable to account for many of the key features of the data. However, when they are fed into our model it does quite well. In particular, the model does a good job of explaining fluctuations in the efficiency wedge (essentially the Solow residual) whereas the prototypical model would generate no endogenous fluctuations. These results are achieved with a simple modification to a standard real business cycle model: the presence of learning-by-doing effects in firm's technology which act as an endogenous propagation mechanism, converting shocks to the labour wedge into persistent movements in total factor productivity and output.

There is a large empirical literature documenting the pervasive influence of learning-by-doing in productive activities. Recent studies include Bahk & Gort (1993), Irwin & Klenow (1994), Jarmin (1994), Benkard (2000), Thompson (2001), Thornton & Thompson (2001), Cooper & Johri (2002), and Clarke (2007). These studies find that agents and organizations appear to become more productive as they gain experience at producing a particular product or service. A number of these studies also report spillover effects in learning across similar products, both within and across firms. Since there is evidence in favour of both internal and external learning-by-doing, and there is some controversy around which one is empirically more important, we look at both types of learning models and find that the results are not only qualitatively similar, the quantitative differences are quite small at the aggregate level.

There exist a small number of papers that introduce learning-by-doing into dynamic general equilibrium business cycle models. Both Cooper & Johri (2002) and Chang et al. (2002) emphasize the ability of learning-by-doing to propagate shocks. In the former study, a representative consumer has access to a production technology with learning effects. The latter offers a

specific decentralization in which workers learn and accumulate human capital. Our model offers a different decentralization in which firms accumulate production related knowledge but workers do not<sup>3</sup>. Firms in our model operate in a market with monopolistic competition and therefore have the power to set prices. None the less, our model is quite close to Cooper & Johri (2002) in that it directly borrows the way in which learning effects are introduced into the production technology<sup>4</sup>. As shall be clear later, these changes have a number of novel implications. The pricing decision facing these firms becomes dynamic because raising current prices lowers current production and learning which lead to future productivity decreases. Moreover, when firms are aware that they can learn and build up knowledge, price-cost markups can vary endogenously in an environment where they would be constant in the absence of learning-by-doing. In this sense, the model may be viewed as a new theory of time varying markups. These features are absent in the earlier papers. In contrast to our model, those models imply that labour supply is dynamic. The implications of dynamic labour supply are discussed more fully in Johri & Letendre (2007). While we believe that learning by workers is an important phenomenon, we choose to ignore it in this study in order to focus on the new implications of incorporating firm level learning in business cycle models.

Simulations from a linearized version of the model calibrated to US data show that substantial improvements to the existing literature can be achieved in terms of matching the usual set of “key” second moments. For example, the model can deliver a small positive correlation between average labour productivity and hours which has eluded most current business cycle models, either

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<sup>3</sup> There is considerable evidence that organizations accumulate intangible capital which is specific to the organization. Some (though not all) aspects of this are related to learning-by-doing. Recent attempts to quantify different aspects of organizational capital under varying names can be found in Brynjolfsson & Hitt (2000), Hall (2000), Lev & Radharkrishnan (2003), and Atkeson & Kehoe (2005) and the references therein.

<sup>4</sup> This paper also differs in that business cycles are not generated by technology shocks.

real or monetary. Moreover, the model generates a highly volatile and persistent efficiency wedge (Solow residual). Depending on the specification, we can account for around forty to fifty percent of the actual volatility in the efficiency wedge and over eighty percent of the movement in aggregate output, hours, consumption and investment. These results are obtained with fairly modest learning effects. In sharp contrast the results from the model without learning-by-doing are quite unrealistic: average labour productivity is negatively correlated with both output and hours<sup>5</sup>. In addition, investment and hours are too volatile relative to output. Moreover the model explains none of the volatility in the efficiency wedge and only sixty seven percent of the volatility in output. Unfortunately, while the simulated markups are time varying, the volatility is quantitatively too small to account for actual markup variations over the business cycle.

The next two sections of the paper presents the two models of learning-by-doing in order of decreasing complexity starting with the internal learning model. Section four presents results from simulating linearized versions of the model calibrated to the U.S. economy. The final section concludes.

## 2 A Decentralization of the Internal Learning Model

The model presented below offers a decentralization of the Cooper & Johri (2002) model. A common feature of both models is that learning occurs as a by-product of production. The key difference occurs in who gets to learn. In Cooper & Johri (2002), learning occurs at the household level while in the current model it is intermediate goods firms which learn by accumulating pro-

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<sup>5</sup> This counter-cyclical (and counter-factual) movement in average labour productivity has long troubled models of the business cycle driven by aggregate demand disturbances. See Rotemberg & Woodford (1991, 1999) for a discussion of the various attempts to overcome these problems.



duction related knowledge which we will refer to as organizational capital. These firms operate in a market with monopolistic competition which gives them the power to choose prices<sup>6</sup>. The firms control how much they wish to learn by varying the markup of price over marginal cost to maximize the present discounted value of profits. Since the behaviour of consumers is standard, we discuss firms first.

## Final and Intermediate Goods Producers

There are a large number of producers operating in a competitive industry that produce a final good  $Y_t$  using the following technology that converts intermediate goods  $Q_t(i)$  indexed by  $i \in [0, 1]$  into final goods:

$$Y_t = \left\{ \int_0^1 [Q_t(i)]^{1/\mu} di \right\}^\mu \quad (1)$$

Each period these firms choose inputs  $Q_t(i)$  for all  $i \in [0, 1]$ , and output  $Y_t$  to maximize profits.

The conditional demand for each intermediate input  $Q_t(i)$  that emerge from this exercise are:

$$f(v_t(i), Y_t) = v_t(i)^{\frac{\mu}{1-\mu}} Y_t \quad (2)$$

where  $v_t(i)$  is the relative price charged by the  $i$ th intermediate goods producer. The price elasticity of demand, faced by intermediate goods producer  $i$ , will be given by  $\mu/(1 - \mu)$ . A zero profit condition ties down the price charged by the final goods producer  $P_t$  as a function of intermediate good prices.

There is a continuum of intermediate goods producers, indexed by the letter  $i$  that operate in a monopolistically competitive economy. Each such producer produces a differentiated intermediate

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<sup>6</sup> The model outlined below takes the standard Blanchard & Kiyotaki (1987) model of monopolistic competition, commonly used in aggregate general equilibrium models, modified to allow for the accumulation of organizational capital. Neither Cooper & Johri (2002) nor Chang et al. (2002) include this feature.

good  $Q(i)$  according to the following production technology:

$$Q_t(i) = F [K_t(i), Z_t(i), H_t(i)] = K_t(i)^\theta H_t(i)^\alpha Z_t(i)^\varepsilon \quad 0 < \alpha, \theta, \varepsilon < 1 \quad (3)$$

where organizational capital  $Z(i)$  is combined with physical capital  $K(i)$  and labour  $H(i)$  to produce output  $Q(i)$ . The technology differs from a standard neo-classical production function because the firm carries a stock of organizational capital which is an input in the production technology. Organizational capital refers to the information accumulated by the firm, through the process of past production, regarding how best to organize its production activities and deploy its inputs<sup>7</sup>. As a result, the higher the level of organizational capital, the more productive the firm<sup>8</sup>. Note that there are diminishing returns to accumulating organizational capital, a feature often found in empirical studies of learning-by-doing.

While learning-by-doing is often associated with workers and modeled as the accumulation of human capital, a number of economists have argued that firms are also store-houses of knowledge. Atkeson & Kehoe (2005) note “At least as far back as Marshall (1930, bk.iv, chap. 13.I), economists have argued that organizations store and accumulate knowledge that affects their technology of production. This accumulated knowledge is a type of unmeasured capital distinct from the concepts of physical or human capital in the standard growth model.” Similarly Lev & Radharkrishnan (2003) write, “Organization capital is thus an agglomeration of technologies—business practices, processes and designs, including incentive and compensation systems—that enable some firms to consistently extract out of a given level of resources a higher level of product and at lower cost than other firms.” There are at least two ways to think about what constitutes organizational capital. Some,

<sup>7</sup> Atkeson & Kehoe (2005) model and estimate the size of organizational capital for the US manufacturing sector and find that it has a value of roughly 66 percent of physical capital. Although their broad interpretation of organizational capital is similar to ours, they do not allow for depreciation of past learning.

<sup>8</sup> Since we are focusing on movements at the business cycle frequency we ignore technical progress of any form.

like Rosen (1972), think of it as a firm specific capital good while others focus on specific knowledge embodied in the matches between workers and tasks within the firm. While these differences are important, especially when trying to measure the payments associated with various inputs, we believe they are not crucial to the issues at hand. As a result we do not distinguish between the two.

Learning at the firm level is modeled through an accumulation equation for organizational capital which is closely related to the empirical learning-by-doing literature in which each cumulative unit of past production contributes equally to the creation of knowledge. Our specification differs in that the contribution of production in any period to the current level of organizational capital is decreasing over time. Following Cooper & Johri (2002), who provide evidence on this specification, we write the accumulation technology as

$$Z_{t+1}(i) = Z_t(i)^\eta Q_t(i)^\gamma \tag{4}$$

or

$$\ln Z_t(i) = \eta^t \ln Z_0(i) + \gamma \sum_{k=0}^{t-2} \eta^k \ln Q_{t-1-k}(i)$$

where  $Z_t(i)$  denotes the stock of organizational capital available to the production unit at time  $t$ ,  $Z_0(i)$  denotes the initial endowment of organizational capital and  $Q_t(i)$  denotes the current level of output. This initial stock of organizational capital constitutes the inherited knowledge of the organization which might include a common component reflecting the prevailing best practice systems, structures, and procedures, and an idiosyncratic component reflecting more specific knowledge imparted by the organization's founders that becomes a durable feature of the organization.

This modification of the traditional specification of learning has a number of advantages. First, it allows for the sensible idea that production knowledge may become less and less relevant

over time as new techniques of production, new product lines and new markets emerge. Second, it allows in a general way for the idea that some match specific knowledge may be lost to the firm as workers leave or get reassigned to new tasks or teams within the firm. In addition, the knowledge accumulated through production experience will be a function of the current vintage of physical capital. The decision to replace physical capital will imply that the existing stock of organizational capital will be less relevant. Third, it allows for the existence of a steady state in which the stock of organizational capital is constant. In contrast, the traditional specification in the empirical learning-by-doing literature allows the stock of organizational capital to grow unboundedly. An alternative way to bound learning is to assume that productivity increases due to learning occur for a fixed number of periods. While this may be appropriate for any one task or worker within the firm, we think of the internal context of firms as an environment with an ever changing set of tasks, workers, teams, machines and information. In this context it may be better to model organizational capital as continually accumulating and depreciating.

The restriction  $\eta < 1$  is consistent with the empirical evidence supporting the hypothesis of depreciation of organizational capital often referred to as organizational forgetting. Argote et al. (1990) provide empirical evidence for this hypothesis of organizational forgetting associated with the construction of Liberty Ships during World War II. Similarly, Darr et al. (1995) provide evidence for this hypothesis for pizza franchises and Benkard (2000) provides evidence for organizational forgetting associated with the production of commercial aircraft. One difference between these studies and this paper is that the accumulation technology is log-linear rather than linear. Clarke (2006) shows that the additional curvature in this log-linear technology is unlikely to produce predictions for aggregate variables, in response to a technology shock, considerably different to those associated with a linear technology. It is the implied dynamic structure associated with the

accumulation of organizational capital, rather than any functional form assumptions that drives the results in Cooper & Johri (2002). We expect similar results to follow in the current context<sup>9</sup>.

It is useful to dissect the problem faced by an intermediate goods producer into two stages. In the first stage, the producer chooses the cost minimizing quantities of labour and physical capital, for a given stock of organizational capital, to solve the following static cost minimization problem:

$$C[w_t, r_t, Q_t(i), Z_t(i)] = \min_{H_t(i), K_t(i)} \left\{ w_t H_t(i) + r_t K_t(i) \mid K_t(i)^\theta H_t(i)^\alpha Z_t(i)^\varepsilon \geq Q_t(i) \right\} \quad (5)$$

Each intermediate goods producer is assumed to operate within perfectly competitive input markets such that the real rental price of physical capital  $r_t$  and the real wage  $w_t$  are taken as given. This cost minimization problem produces conditional factor demands that are a function of factor prices, the required level of output ( $Q_t$ ) and the stock of organizational capital ( $Z_t$ ). The cost function (5) will be a non-increasing function of the (given) stock of organizational capital.

In the second stage, the intermediate goods producer will solve a dynamic problem that selects the time path of output supply  $Q_t(i)$ , or the relative price  $v_t(i)$ , and the time path of organizational capital  $Z_t(i)$  that maximizes the real expected discounted present value of profits, subject to the demand function (2), the accumulation technology for organizational capital (4), and the initial stock of organizational capital  $Z_0(i)$ .

Each intermediate goods' producer solves the following maximization problem:

$$\max_{v_t(i), Z_{t+1}(i)} E_0 \sum_{t=0}^{\infty} D_t [v_t(i) \cdot f(v_t(i), Y_t) - C\{w_t, r_t, f(v_t(i), Y_t), Z_t(i)\}]$$

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<sup>9</sup> Another seemingly crucial feature of our model is that organizational capital is accumulated as a by-product of production. This ignores the considerable intentional investments made by firms in raising productivity. Hou & Johri (2007) show that allowing for intentional investments in organizational capital result in only small differences from the Cooper & Johri (2002) model.

subject to

$$Z_t(i)^\gamma [f(v_t(i), Y_t)]^\gamma = Z_{t+1}(i)$$

given some initial stock of organizational capital  $Z_0(i)$  and subject to an appropriate transversality condition on the stock of organizational capital. Since firms are owned by households,  $D_t$  is the appropriate endogenous discount rate for the firms<sup>10</sup>.

The solution to the (profit) maximization problem will satisfy the following first order conditions:

$$v_t(i)f'(v_t(i)) + f(v_t(i)) - mc_t(i)f'(v_t(i)) = -\lambda_t^f(i) \frac{\partial Z_{t+1}(i)}{\partial Q_t(i)} f'(v_t(i)) \quad (6)$$

and

$$\lambda_t^f(i) = E_t \left[ \frac{D_{t+1}}{D_t} \left\{ \lambda_{t+1}^f(i) \frac{\partial Z_{t+2}(i)}{\partial Z_{t+1}(i)} - \frac{\partial C_{t+1}(i)}{\partial Z_{t+1}(i)} \right\} \right] \quad (7)$$

where  $mc_t(i)$  denotes the marginal cost of producing output  $Q(i)$ . The term  $\lambda_t^f(i)$  denotes the Lagrange multiplier associated with the maximization problem defined above and represents the discounted value of an additional unit of organizational capital, in terms of marginal real profits.

The first order condition (6) captures the nature of the dynamic tradeoff that arises when intermediate goods producers face a downward sloping demand curve. Fundamentally, these producers face a tradeoff between maximizing current period profits and losing future productivity increases. The first order condition (6) determines the optimal relative price  $v_t$  to be set by the producer of the intermediate input  $i$ . Since the intermediate goods producer faces a downward sloping demand curve for their product, raising the output price by one unit causes demand for

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<sup>10</sup>Given our specification for preferences outlined in the next section:

$$D_{t+1} = \beta E \left[ \frac{C_t}{C_{t+1}} \right]$$

their product to fall. The first two terms in (6) captures the impact on current revenue of raising the relative price of output  $v_t$ . The third term represents the reduction in current period costs resulting from this corresponding lower level of output. The accumulation technology for organizational capital implies that a reduction in current output will lead to a reduction in the stock of organizational capital available in the next period. The term on the right hand side of (6) represents the value of this reduced (future) stock of organizational capital. The term  $f'(v_t(i))$  measures the reduction in current output due to the higher relative price and the term  $\partial Z_{t+1}/\partial Q_t$  represents the reduction in the stock of organizational capital resulting from the reduction in current period output, evaluated at the marginal value of organizational capital to the intermediate goods producer. The first order condition implies that the firm will set its optimal price by equating the current period benefit of increasing its relative price by one unit to the discounted future costs. This tradeoff, captured by the (dynamic) term on the right hand side of (6), will not appear in the standard model of monopolistic competition, without the accumulation of organizational capital

The first order condition (7) determines the value of an additional unit of organizational capital for use by the producer in period  $t + 1$ . This additional unit of organizational capital has a (marginal) value, in terms of profits, of  $\lambda_t^f$  to the producer. Since the cost function is decreasing in the stock of organizational capital, an additional unit of organizational capital reduces the cost of producing output level  $Q_{t+1}$ . The accumulation equation for organizational capital implies that an additional unit of organizational capital will increase the stock of organizational capital available in period  $t + 2$ . This higher stock of organizational capital has a value of  $\lambda_{t+1}^f$  to the producer. The condition (7) implies that organizational capital will be accumulated up to the point where the value of an additional unit of organizational capital today is equal to the discounted value of this organizational capital next period.

## Households

The economy is populated by a continuum of identical, infinitely lived households. At time  $t$ , the representative household has preferences over consumption of final goods  $C_t$  and leisure  $(T - H_t)$  where  $T$  represents the total time endowment. The representative household maximizes the expected discounted present value of utility by choosing consumption  $C_t$ , labour supply  $H_t$ , and investment in physical capital  $I_t$ , taking as given the real wage  $w_t$  and the real interest rate  $r_t$ . Consumers sell labour services and rent physical capital to the intermediate goods producers. As owners of the intermediate goods producers, they also receive the current profits of these producers. Physical capital is stored and accumulated by consumers according to a standard accumulation technology. The flow utility of the representative household is given by:

$$U(C_t, T - H_t) = \ln C_t + B_t \mathbf{V}(T - H_t) \quad (8)$$

where:

$$\mathbf{V}(T - H_t) = \begin{cases} \frac{1}{1 - \nu} (T - H_t)^{1-\nu} & \text{for } \nu \geq 0 \text{ and } \nu \neq 1 \\ \ln(T - H_t) & \text{for } \nu = 1 \end{cases}$$

and the term  $B_t$  evolves according to:

$$\ln B_t = (1 - \rho_b) \ln \bar{B} + \rho_b \ln B_{t-1} + v_{bt}$$

where  $v_{bt}$  is an *iid* random variable with mean zero and standard deviation  $\sigma_b$ .  $B_t$  is the only exogenous source of fluctuations in our model. It represents the idea of the “labour wedge” explored in Chari et al. (2007). We chose this formulation of the labour wedge over representing it as a labour income tax because it is also consistent with the idea of preference shocks which are increasingly



being used in business cycle models. Early references include Parkin (1988) and Baxter & King (1991). More recently they can be found in Chang et al. (2002) which is a business cycle model with only real shocks and Ireland (2004a, b) which have both preference and money shocks<sup>11</sup>.

An obvious question that arises in this context is what does the labour wedge represent? Our interpretation is that this is anything that exogenously shifts the aggregate labour supply at business cycle frequencies. Some possible sources of such shifts have been explored in the literature. For example, if the marginal utility of leisure is a function of omitted variables such as money balances or government spending, then variations in these might directly account for these shifts. See Parkin (1988) for an early discussion of this point. At a deeper level one might imagine these may be caused by changes in the amount of distortions in the economy caused by taxation and transfer payments, unionization and other shifts in market power. See, for example, Mulligan (2002) and Chari et al. (2002) or Cole & Ohanian (2002, 2004) for a model in which shifts in government policy towards monopolies in a model with unions can account for these preference shifts in the context of the Great Depression. Rotemberg & Woodford (1991, 1999) consider models of imperfect competition in which distortions vary over the business cycle. Alternatively a model with monetary shocks and sticky wages might account for them. Finally Hall (1997) has suggested that these shifts might be accounted for in a labour search model where the amount of aggregate hours spent searching for jobs varies with the cycle<sup>12</sup>.

Households own the imperfectly competitive firms and receive all (real) profits earned by

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<sup>11</sup> See Uhlig (2004) for the use of preference shocks in the context of an interesting model of labour hoarding involving workplace leisure.

<sup>12</sup> There is also a literature that suggests these shifts arise as a result of 'aggregation error' in a model with heterogenous agents. See Maliar & Maliar (2004), Chang & Kim (2007)

firms. The household budget constraint then becomes:

$$C_t + K_{t+1} - (1 - \delta) K_t = w_t H_t + r_t K_t + \int_0^1 \pi_t(i) di \quad (9)$$

The households problem may be represented by:

$$\max_{C_t, H_t, K_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t [\ln C_t + B_t \mathbf{V}(T - H_t)]$$

subject to the budget constraint and the usual transversality condition given an initial stock of physical capital  $K_0$ . The first order conditions associated with this problem are:

$$B_t \mathbf{V}'(T - H_t) = \frac{w_t}{C_t} \quad (10)$$

and

$$\frac{1}{C_t} = \beta E_t \left\{ \frac{1}{C_{t+1}} [r_{t+1} + (1 - \delta)] \right\} \quad (11)$$

where  $\delta$  represents the depreciation rate for physical capital. The interpretation of these first order conditions is quite standard. Condition (10) requires per-capita consumption and hours be chosen so that the marginal rate of substitution between consumption and leisure, for all  $t$ , is equal to the real wage rate. Condition (11) is the standard Euler equation for the accumulation of physical capital which requires that, at the optimal solution, the utility cost of giving up one unit of consumption must be equal to the present value (in terms of utility) of this unit of consumption tomorrow.

## Equilibrium Prices and Quantities

A competitive equilibrium consists of:

1. allocations  $C_t, H_t, K_{t+1}$  that solve the consumer's problem, taking prices as given

2. allocations  $K_{it}$ ,  $H_{it}$ ,  $Z_{i,t+1}$  for  $i \in [0, 1]$  that solve the firm's problem, taking all prices but their own output price as given
3. allocations  $Y_t$ ,  $Q_{it}$  that solve the final goods producer's problem, taking prices as given
4. prices  $w_t$ ,  $r_t$ ,  $v_{it}$  for  $i \in [0, 1]$

In addition, these allocations must satisfy the factor market clearing conditions and the aggregate resource constraint. Since the technology of the economy is assumed symmetric with respect to all intermediate goods producers, attention may be restricted to the symmetric equilibrium where all producers in the intermediate goods sector charge the same price and produce the same output. In this case:

$$Y_t = \left\{ \int_0^1 [Q_t(i)]^{1/\mu} di \right\}^\mu = Q_t$$

so that symmetry requires the relative price  $v_t = 1$  for all firms. Since all intermediate goods producers have the same level of technology and have the same initial endowment of organizational capital, it will be true that  $H_t(i) = H_t$ ;  $K_t(i) = K_t$ ; and  $Z_t(i) = Z_t$ . In this case, the total demand for hours and the total demand for physical capital will be given by:

$$\mathbf{H}_t^D = \int_0^1 H_t(i) di = H_t \quad \text{and} \quad \mathbf{K}_t^D = \int_0^1 K_t(i) di = K_t$$

with the aggregate stock of organizational capital given by:

$$\mathbf{Z}_t = \int_0^1 Z_t(i) di = Z_t$$

Using the factor market clearing conditions, the household's budget constraint in the symmetric equilibrium will be given by:

$$C_t + K_{t+1} - (1 - \delta)K_t = Y_t$$

### 3 An Aggregate Model with Learning Externalities

The most direct way to discuss the impact of learning externalities is to retain all features of the previous model except that firms no longer realize that production leads to the accumulation of organizational capital. Instead they take as given the economy-wide stock of organizational capital as an input in their production technologies. The obvious implication of this change in specification is that (7) is no longer part of the system of equations associated with equilibrium in the external model. Secondly (6), the pricing equation, simplifies to the standard condition found in models of monopolistic competition—price is set as a fixed markup over marginal cost. In other words the term on the right hand side of equation (6) equals zero.

An alternative way to proceed is to abandon the decentralization involving monopolistic competition and solve a representative agent model economy with external learning effects built directly into the technology. An advantage of this approach is that the model can be directly compared to some earlier work especially the closely related model presented in Cooper & Johri (2002). The two models then will differ in only two ways. First while the agent in Cooper & Johri (2002) internalizes the learning effects built into the technology, here they will not. Second, the source of fluctuations in Cooper & Johri (2002) were technology shocks whereas here they will be preference shocks.

The agent has preferences over consumption and leisure and has access to a Cobb-Douglas technology that produces a consumption\investment good ( $Y$ ) using three inputs: hours ( $H$ ), physical capital ( $K$ ) and organizational capital ( $Z$ )

$$Y_t = K_t^\theta H_t^\alpha Z_t^\varepsilon \tag{12}$$

The agent can use this good for consumption or to invest in physical capital. The specification for

the accumulation of physical capital is

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (13)$$

where  $\delta$  is the depreciation rate. The stock of organizational capital depends on past production as well as past organizational capital as follows:

$$Z_{t+1} = Z_t^\gamma Y_t^\eta \quad (14)$$

The maximization problem of the representative agent is given by:

$$\max_{C_t, H_t, K_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t [\ln C_t + B_t \mathbf{V}(T - H_t)]$$

subject to the aggregate resource constraint, the accumulation technology (14), the transversality condition  $\lim_{t \rightarrow \infty} \Lambda_t^h K_t = 0$  and an initial stock of physical capital  $K_0$

## 4 Solution Method and Calibration

An approximate linear solution, to the models outlined above, is obtained using the method outlined in King & Watson (2002). This solution is derived by linearizing the equations characterizing the competitive equilibrium in the neighbourhood of the steady state. It is necessary to specify values for the discount rate  $\beta$ , the depreciation rate of physical capital  $\delta$ , the production technology parameters  $\alpha$ ,  $\theta$ , and  $\varepsilon$ , the accumulation technology parameters  $\gamma$  and  $\eta$ , the preference parameter  $\nu$ , the persistence  $\rho_b$  and the volatility  $\sigma_b$  of the preference shock. In addition, for the internal learning model, the demand parameter  $\mu$  needs to be specified.

The following values for key parameters were used :  $\delta = 0.02$ ,  $\beta = 0.9926$  and  $\theta = 0.34$ . These values imply a steady state share of investment in output of 21.57 percent and a steady state (physical) capital to output ratio of 10.78. Previous attempts to measure the degree of returns to

scale in the aggregate production function using instrumental variable techniques have yielded quite a range of estimates often with large standard errors. Despite this, the consensus view (see the work of Basu (1996) and Basu & Fernald (1997)) is that the scale elasticity measured by  $(\alpha + \theta)$  is close to unity. Consequently, given  $\theta = 0.34$ , the model is calibrated with  $\alpha = (1 - \theta) = 0.66$ <sup>13</sup>. Turning to the parameters associated with learning-by-doing, we draw on the results of Cooper & Johri (2002). To illustrate the impact of varying the parameters associated with learning we report results for two different specifications of the learning technology that Cooper & Johri (2002) focussed on. For our baseline case, we set the elasticity of output with respect to organizational capital,  $\varepsilon = 0.264$ . This value is close to manufacturing industry estimates reported in Cooper & Johri (2002) and corresponds to a learning rate of twenty percent, commonly estimated in microeconomic studies of learning-by-doing that do not allow past experience to become less valuable to the firm<sup>14</sup>.

Two alternative values for the parameters of the accumulation technology are considered. The first set ( $\gamma = 0.5$  and  $\eta = 0.5$ ) correspond to manufacturing level estimates reported in Cooper & Johri (2002). Sensitivity analysis is conducted with a second set ( $\gamma = 0.2$  and  $\eta = 0.8$ ) of values which are based on economy-wide estimates reported in Cooper & Johri (2002). Our value for  $\mu$

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<sup>13</sup>Papers in the organizational capital literature often work with decreasing returns in labour and capital. Typically the implied capital output ratios are much lower than found in aggregate US data. Our results are not very sensitive to this assumption.

<sup>14</sup>This value is also (broadly) consistent with industry-level and plant-level estimates in Clarke (2007), based upon structural estimation of the firm side of the internal learning model outlined above. Note that Benkard (2000) has argued that allowing for “organizational forgetting” leads to higher estimates of the learning rate. For example in his work on aircraft production, the estimated learning rate rises to 39 percent once forgetting is allowed. Keeping this in mind, and the even higher estimate of  $\varepsilon = .49$  for the aggregate economy in Cooper & Johri (2002), it would not be unreasonable to allow for an  $\varepsilon = 0.4$  which corresponds to a learning rate of roughly 32 percent. Results for this case were reported in an earlier version of the paper and are available from the authors.

varies across specifications to deliver a constant capital output ratio and a steady state markup of fifteen per cent. We also report results for a much lower learning rate of ten percent. This specification sets ( $\gamma = 1$  and  $\eta = 0.5$ ) as in Cooper & Johri (2002).

While the calibration of the learning by doing parameters is based on estimates obtained in earlier studies, it may be useful to shed further light on the importance of organizational capital in the economy implied by these parameters. The level of organizational capital relative to output in the economy is governed by the accumulation equation (4). Since we have assumed (for the baseline case) constant returns to scale in this function, we end up with a steady state level of  $Z/Q = 1$ . By contrast, the ratio of physical capital to output is around 10 and Atkeson & Kehoe (2005) report that organizational capital is roughly 2/3 the size of physical capital, implying a  $Z/Q$  of roughly six. Our justification for a lower  $Z/Q$  is that we choose to focus on only one type of organizational capital, that which is accumulated as a by-product of production through learning-by-doing. The literature on organizational capital also emphasizes that firms intentionally invest conventional inputs in the creation of knowledge which may be captured in estimates of organizational capital. Finally organizational capital may also include intangibles such as “work-culture” which may have an effect on productivity. To the extent that these features do not vary over the business cycle, their exclusion from the model should not strongly influence the results. In this context note that if other aspects of organizational capital are not varying over the business cycle then the accumulation equation can be modified to include a constant term:

$$Z_{t+1}(i) = Z_0 Z_t(i)^\eta Q_t(i)^\gamma. \tag{15}$$

In steady state this implies  $Z(i) = Z_0(i)^{1/(1-\gamma)} Q(i)$ . In the current model  $Z_0 = 1$ . Given an estimate of the steady state ratio of organizational capital to output,  $Z_0$  could be calibrated to deliver that number. However, the interesting thing to note about the level of steady state

organizational capital is that it has little, if any, influence on the short run dynamics of the model. A simple (albeit incomplete) way to see this is to examine the linearized version of the accumulation equation where all lowercase variables are in deviations from steady state.

$$z_{t+1} = \gamma z_t + (1 - \gamma)y_t \tag{16}$$

Note that the level of organizational capital plays no role in it's short run dynamics and that  $Z_0$  drops out of the equation.

An alternative way to evaluate the impact of organizational capital on the economy is to compare the implied value for Tobin's  $q$  in the model with the data. Empirical estimates of average  $q$  vary quite widely. For example Barnett & Sakellaris (1999) provide an estimate of  $q=1.77$  on average for the manufacturing sector. This accords well with the average estimate of  $q=1.7$  reported in Summers et al. (1981). A much lower estimate of  $q=1.11$  can be found in Lang & Schultz (1994). In our model for the baseline calibration Tobin's  $q$  is equal to 1.643 which is in between the above estimates<sup>15</sup>.

One can also ask to what extent do firms invest in organizational capital in steady state? The model provides an indirect answer to this question since there are no direct investments in organizational capital by firms. However, since firms realize that producing more output yields additional future organizational capital, they choose to produce more than they would in the absence of learning-by-doing and more also than if they did not internalize the learning technology. How should we evaluate this additional "investment" in organizational capital? One way would be to calculate the costs associated with the extra output produced by firms that internalize learning effects. Recall, however, that the additional output also yields revenue, albeit at a lower price.

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<sup>15</sup> Business cycle models are typically calibrated to a markup of 11.11 %. The implied  $q$  is greater than 2 for a typical calibration.



Therefore it would appear that the best way to calculate the value of the investments in organizational capital is to calculate the difference in the steady state flow of profits between the internal and external learning models. Steady-state profits are 68% larger in the external learning model as compared to the internal learning model. Attributing this difference in steady-state profits to investments in learning, the steady-investment to output ratio for organizational capital is approximately 0.0359. This number may be compared to the income share of investments in intangible capital that are broadly similar to organizational capital. Corrado et al. (2006) refer to these as investments in firm-specific resources. Their estimates suggest that such investments account for approximately one third of all investments in intangible capital providing an income share of investments in intangible firm-specific resources of 5%.

For the data under consideration, the average per-capita total hours worked per quarter is given by approximately 300 hours per quarter<sup>16</sup>. With a calibrated total time endowment of 1369 hours per quarter, this provides average per-capita total hours, as a proportion of the total time endowment  $H/T$ , of 21%<sup>17</sup>.

The preferences of the representative household, given by (8), imply the following inter-temporal elasticity of labour supply (iels):

$$\text{iels} = \frac{1}{\nu} \frac{T - H_t}{H_t}$$

The value of  $\nu = 2.7226$  was picked to deliver an inter-temporal labour supply elasticity of 1.3088

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<sup>16</sup>We use aggregate US data at the quarterly frequency covering the period 1955:Q1–1992:Q4. We thank Michelle Alexopoulos for providing this data to us.

<sup>17</sup>See Christiano & Eichenbaum (1992), Burnside et al. (1993), or Burnside & Eichenbaum (1996)

as estimated in Chang et al. (2002)<sup>18</sup>. which is close to the upper end of estimates of the inter-temporal labour supply elasticity using microeconomic data<sup>19</sup>.

The first order conditions from the household's maximization problem imply the following relationship for the labour wedge:

$$\ln B_t = -\ln C_t + \nu(T - H_t) + \ln(W_t)$$

Consequently, an (implied) series for the exogenous process may be determined using data on consumption  $C_t$ , hours  $H_t$ , and wages  $W_t$ . This series  $B_t$  can then be used to estimate the second moments of the process according to:

$$\ln B_t = (1 - \rho_b) \ln \bar{B} + \rho_b \ln B_{t-1} + v_{bt}$$

Obviously, these moments will depend upon the value of the preference parameter  $\nu$ . Based upon the data, the model with  $\nu = 2.7226$  is calibrated with  $\rho_b = 0.9536$  and  $\sigma_b = 0.0107$ .

## 5 Simulation Results

### Response to movements in labour wedge

This section presents quantitative results from simulating the calibrated linearized versions of the learning models along with a baseline real business cycle model without learning effects. We begin with a discussion of the implications for productivity and then briefly discuss the implied second moments usually presented in the business cycle literature.

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<sup>18</sup>We also explored the impact of setting  $\nu = 1$  which, together with the steady state  $h/T$  implies an inter-temporal labour supply elasticity of 3.563. This value is commonly used in the business cycle literature. The results were similar and are available from the authors upon request.

<sup>19</sup>See Browning et al. (1999) for a review of this literature

	$\sigma_{sr}$	$\rho_{sr}$	$\rho_{h,alp}$	$\rho_{h,w}$	$\rho_{w,y}$
US data	0.0208	0.9457	0.0828	0.1900	0.7034
No Learning	0	n/a	-0.4316	-0.4316	-0.1217
20 % Learning 1 ( $\eta = 0.5$ )	0.0094	0.9951	0.2453	0.2487	0.6730
20 % Learning 2 ( $\eta = 0.8$ )	0.0084	0.9981	0.1416	0.1273	0.6039
10 % Learning ( $\eta = 0.5$ )	0.0103	0.9954	0.2695	0.2752	0.7042

Table 1: Productivity and Wages

Table 1 presents some key results for productivity and wages. The first two columns contain the standard deviation and first order autocorrelation coefficient of the Solow residual while the next two columns present the contemporaneous correlations of hours with average labour productivity and wages. The final column presents the contemporaneous correlation between output and wages. Comparing row two of Table 1 with row one, it is clear that the baseline business cycle model, without learning, generates results that are incompatible with US data on wages and productivity. Not only is there no mechanism for endogenously generating movements in the Solow residual (efficiency wedge), average labour productivity and wages are both highly negatively correlated with hours while the correlations in US data are weakly positive. Similarly wages are negatively correlated with output while they are highly positively correlated in the data.

The next three rows present results for three variants of the model with internal learning-by-doing. Rows 3 and 4 correspond to a 20 percent learning rate ( $\varepsilon = 0.26$ ) while the learning rate in row 5 is ten percent with  $\gamma = 1$  as is typical in the learning literature. Examination of these rows reveals that all three specifications generate considerable movement in the Solow residual. Depending on the specification, the model can account for 40 to 50 per cent of the volatility of the

Solow residual seen in the US data<sup>20</sup>. The model with learning also does quite well in replicating the low positive correlation of hours with wages and average labour productivity as well as the high positive correlation between output and wages. The results from the external learning model are quite similar except that in general the model generates a little more endogenous movement in the Solow residual and slightly higher correlations of productivity and wages with hours and output. These results are presented in an appendix with no discussion.

To understand why the learning models generate a small correlation between wages (average labour productivity) and hours it is useful to think in terms of a labour demand-labour supply diagram. The shock causes agents to shift their labour supply curves outwards leading to an increase in hours worked. Absent the learning channel, this would cause a movement down the labour demand curve leading to a sharp fall in wages and labour productivity, as in the model without learning. Once firms are allowed to learn from production, we get an offsetting second shift of the labour demand curve to the right induced by the increase in future productivity due to the accumulation of organizational capital. The net result of the two shifts is a large movement in hours, even if the supply curve is not very responsive to movements in the real wage, and a small movement in labour productivity. This analysis is reminiscent of Christiano & Eichenbaum (1992) but recall that in that model the two shifts occurred in response to two separate shocks: a government spending shock and a technology shock whereas here the only exogenous shift is in labour supply.

The analysis is also reminiscent of the discussion in Cooper & Johri (2002) of how the

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<sup>20</sup> Obviously the amount of variation in the Solow residual will depend on how productive organizational capital is in output production. In an earlier version of the paper we showed that even with a moderate learning rate of 32 % the model could account for 80% of the variation in the Solow residual.

representative agent responds to a technology shock. There, as is usual, the technology shock causes a rightward shift in labour demand while the wealth effect of the accumulation of organizational capital is to shift the labour supply curve to the left. Once again the model delivers a lower correlation of hours and average labour productivity than the baseline model but it also delivers a lower movement in hours which is counter-factual. This occurs because the labour supply curve shifts left, countering some of the rightward movement of labour demand.

Given this analysis, it is useful to compare the moments reported in Table 1 with similar moments generated by total factor productivity (tfp) shocks instead of preference shocks<sup>21</sup>. These are reported in Table 5<sup>22</sup>. As expected, the no learning model generates too high correlations of wages (and productivity) with hours and output. The actual values are .54 and .97 respectively. As discussed above, the internal learning by doing models are able to lower the correlation of wages with hours but the quantitative impact of this is small. As a result we conclude that the performance of the learning-by-doing model with preference shocks is superior to its performance with technology shocks.

## FIGURES 1 and 2 TO BE PLACED HERE

Figures 1 and 2 present impulse response plots for the high learning model (stars) as well as the baseline model without learning effects (solid lines). The impact of learning-by-doing on

<sup>21</sup>This requires some modification of the model outlined above. In the symmetric equilibrium, let the aggregate production technology be given by  $Y_t = A_t K_t^\theta H_t^\alpha Z_t^\varepsilon$ . The tfp process  $A_t$  evolves according to:

$$\ln A_t = (1 - \rho_a) \ln \bar{A} + \rho_a \ln A_{t-1} + v_{at}$$

The process is calibrated to deliver the observed autocorrelation and variance of the Solow residual (see Table 5) with  $\rho_a = 0.9457$  and  $\sigma_a = 0.0068$ .

<sup>22</sup>The value for  $\mu$  varies across specifications to deliver a constant capital output ratio and a steady state markup of fifteen per cent.

endogenous productivity movements is clearly visible in the strong hump-shaped response of the Solow residual. As expected, absent any learning mechanism, average labour productivity falls sharply when hours rise in the impact period and remain below steady state levels for four years. With learning, organizational capital is built up due to above steady state output. On its own this effect increases productivity thus counteracting the negative influence of the increase in hours worked. As a result labour productivity is below steady state for only six quarters as opposed to fourteen quarters.

	$\sigma_{C/Y}$	$\sigma_{I/Y}$	$\sigma_{H/Y}$	$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{H,Y}$	$\rho_Y$	$\sigma_Y$
US Data	0.79	1.76	0.72	0.92	0.83	0.78	0.97	0.0399
No Learning	0.78	2.79	1.10	0.94	0.86	0.95	0.97	0.0267
20 % Learning 1 ( $\eta = 0.5$ )	0.79	2.15	0.77	0.94	0.89	0.88	0.99	0.0358
20 % Low Learning 2 ( $\eta = 0.8$ )	0.81	2.05	0.80	0.95	0.89	0.87	0.99	0.0330
10 % Learning ( $\eta = 0.5$ )	0.80	2.12	0.74	0.94	0.89	0.88	0.99	0.0371

Table 2: Selected Second Moments

Table 2 presents selected second moments for the four models discussed above as well as for linearly de-trended aggregate U.S. data. Each row of the table is set up as before with the first three columns reporting standard deviations relative to output. The next three columns report contemporaneous correlations with output while the last two report the first order autocorrelation and standard deviation of output respectively. A glance at Table 2 suggests that all the models do a reasonable job of capturing the basic features of consumption and investment over the business cycle. As in the data, consumption, investment and hours are positively correlated with output and consumption is less volatile than output while investment is more volatile. Looking down the

second column one notices that learning tends to temper the relative volatility of investment which is too high compared to the data in the baseline model without learning. This moderating influence is clearly evident in Figure 1 where we see that investment rises by just over 2.5 percent in the impact period without learning while the rise is just under 1.5 percent in the model with learning. A small hump-shaped pattern is also evident in this case.

The impact of learning-by-doing shows up more starkly when we study the behaviour of aggregate hours. The baseline model generates a very high correlation between hours and output (0.95) which is reduced in all three specifications of the internal learning model. Similarly hours are too volatile in the baseline model. Without learning effects, the relative volatility of hours with respect to output is 1.1 whereas this falls to around .7 with learning. This is much closer to the US data which exhibits a relative volatility of 0.72. The last two columns of Table 2 display the effectiveness of learning-by-doing as a propagation mechanism. Not only does learning-by-doing increase the persistence of output, it also magnifies the effect of shocks as is evident in the increase in the volatility of output from 0.0267 in the baseline model to roughly 0.0358 in the 20 percent learning specification which is about 90% of the volatility of output seen in aggregate US data. The extra propagation effect of learning-by-doing can also be seen in the strong hump-shaped response of output in Figure 1.

The external learning model produces results that are very similar to those reported above both qualitatively and quantitatively. Tables 3 and 4 in the appendix provides the details. Here we quickly summarize the key differences. In general, given our specification for the learning parameters, the external model tends to magnify and propagate shocks more strongly. As a result, the volatility of productivity measures, output and investment tends to be higher. For the same reason the correlation of labour productivity and hours is somewhat higher as well.

Turning to the behaviour of the internal learning by doing model in the presence of technology shocks. We see from Table 6 in the Appendix that the moments largely mirror the results reported above. As widely documented in the literature, the key area of under-performance for both the models with and without learning effects is the volatility of hours. With persistent tfp shocks, hours do not move enough.

## 6 Conclusions

This paper presents two business cycle models with learning-by-doing that are quite successful in explaining the broad features of the US business cycle. The only shock to the economy comes from exogenous movement in the labour wedge which we interpret as an unmodeled fluctuation in labour supply. Our models endogenously generate pro-cyclical movements in labour productivity, wages and the Solow residual which are highly persistent and volatile. The learning-by-doing models also do well in generating a realistically small correlation between average labour productivity (wages) and hours, an elusive goal for most business cycle models. We show that the learning effects built into these models are key aspects of this success. Absent learning, the baseline model performs quite poorly: as expected, labour productivity is strongly counter-cyclical and there is no movement in the Solow residual. Moreover hours and investment are too volatile relative to output. Learning-by-doing is able to temper this excessive volatility. While the benchmark no-learning model can account for around 67 percent of the volatility of aggregate output seen in US data, the learning models can account for virtually all of it.

Chari et al. (2007) argue that wedges in the hours first order condition (labour wedge) and in the production function (efficiency wedge) of a prototypical real business cycle model are key ingredients in accounting for modern business cycle fluctuations. In this paper we show that



learning-by-doing can be an effective ingredient in developing an explanation of the efficiency wedge. Specifically we show that exogenous movements in the labour wedge can lead to endogenous movements in the efficiency wedge. The model can account for about fifty percent of the observed movement in the wedge.

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## Simulation Results—External Learning-by-Doing

	$\sigma_{sr}$	$\rho_{sr}$	$\rho_{h,alp}$	$\rho_{h,w}$	$\rho_{w,y}$
US data	0.0208	0.9457	0.0828	0.1900	0.7034
No Learning	0	n/a	-0.4316	-0.5782	-0.1217
20 % Learning 1 ( $\eta = 0.5$ )	0.0114	0.9960	0.2886	0.2886	0.7258
20 % Learning 2 ( $\eta = 0.8$ )	0.0103	0.9984	0.1967	0.1967	0.6814
10 % Learning ( $\eta = 0.5$ )	0.0126	0.9963	0.3104	0.3104	0.7530

Table 3: External Model: Productivity and Wages

	$\sigma_{C/Y}$	$\sigma_{I/Y}$	$\sigma_{H/Y}$	$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{H,Y}$	$\rho_Y$	$\sigma_Y$
US Data	0.79	1.76	0.72	0.92	0.83	0.78	0.97	0.0399
No Learning	0.78	2.79	1.10	0.94	0.86	0.95	0.97	0.0267
20 % Learning 1 ( $\eta = 0.5$ )	0.84	2.31	0.72	0.97	0.87	0.87	0.99	0.0434
20 % Learning 2 ( $\eta = 0.8$ )	0.85	2.18	0.75	0.97	0.88	0.85	0.99	0.0343
10 % Learning ( $\eta = 0.5$ )	0.84	2.28	0.69	0.97	0.87	0.86	1.00	0.0384

Table 4: External Model: Selected Second Moments

## Simulation Results—Technology Shocks

	$\sigma_{sr}$	$\rho_{sr}$	$\rho_{h,alp}$	$\rho_{h,w}$	$\rho_{w,y}$
US data	0.0208	0.9457	0.0828	0.1900	0.7034
No Learning	0.209	0.9457	0.5448	0.5448	0.9734
20 % Learning 1 ( $\eta = 0.5$ )	0.0339	0.9789	0.5312	0.5253	0.9822
20 % Learning 2 ( $\eta = 0.8$ )	0.0315	0.9763	0.4934	0.4872	0.9847
10 % Learning ( $\eta = 0.5$ )	0.0352	0.9804	0.5270	0.5203	0.9828

Table 5: Productivity and Wages

	$\sigma_{C/Y}$	$\sigma_{I/Y}$	$\sigma_{H/Y}$	$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{H,Y}$	$\rho_Y$	$\sigma_Y$
US Data	0.79	1.76	0.72	0.92	0.83	0.78	0.97	0.0399
No Learning	0.76	2.97	0.27	0.93	0.86	0.72	0.96	0.0361
20 % Learning 1 ( $\eta = 0.5$ )	0.77	2.26	0.23	0.93	0.89	0.68	0.99	0.0549
20 % Learning 2 ( $\eta = 0.8$ )	0.79	2.15	0.21	0.94	0.89	0.64	0.98	0.0502
10 % Learning ( $\eta = 0.5$ )	0.78	2.23	0.22	0.93	0.89	0.67	0.99	0.0570

Table 6: Selected Second Moments

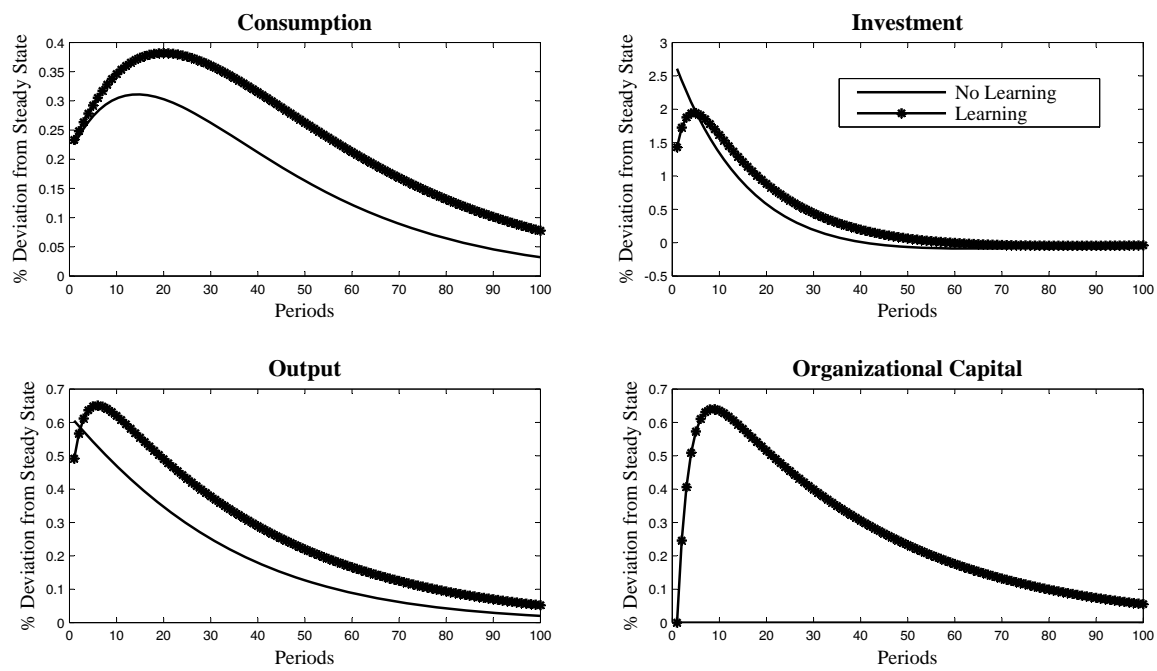


Figure 1: Internal Learning-by-Doing: Selected Impulse Responses to a 1% Shock to the Labour Wedge



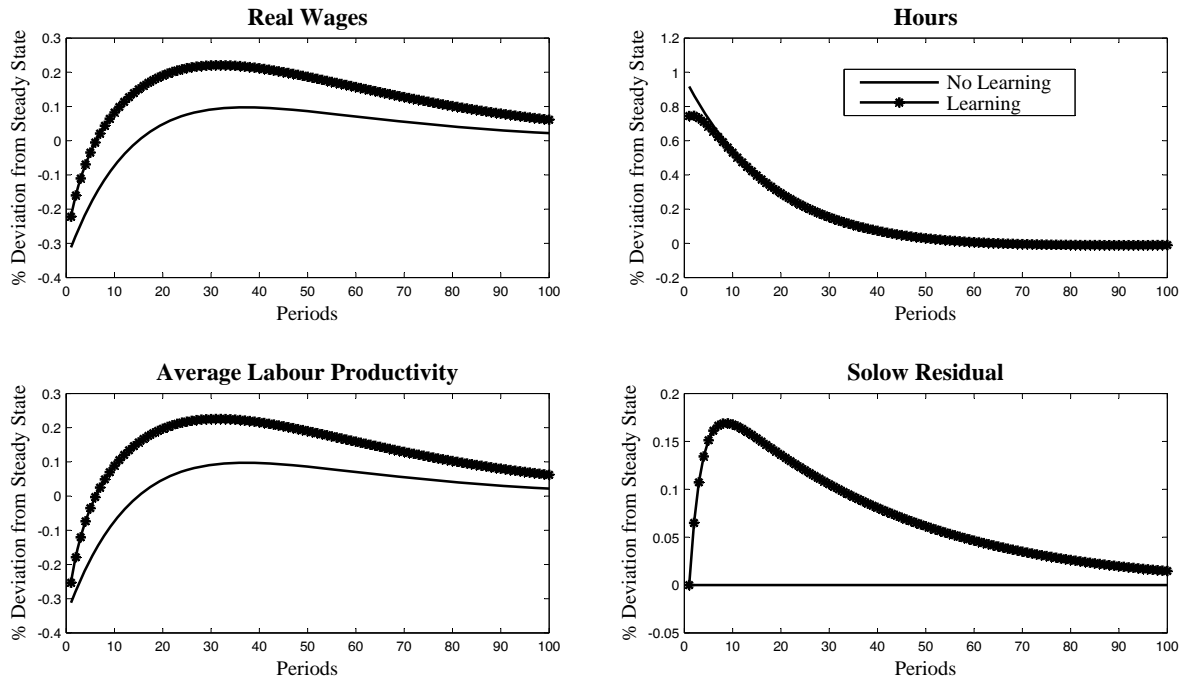


Figure 2: Internal Learning-by-Doing: Selected Impulse Responses to a 1% Shock to the Labour Wedge