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Equalization and the Decentralization of Revenue-Raising in a Federation[*]

by

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Abstract

We study federal economies in which regional governments have responsibility for delivering public services and redistributive objectives apply. The implications of these for the assignment of revenue-raising instruments and fiscal transfers, both vertical and horizontal, are considered. Models of heterogenous regions of varying degrees of complexity and generality are constructed. For each case, we determine what fiscal instruments must be given to the regions and what inter-governmental transfers must be made in order that the social optimum is achieved. With heterogenous households and regions, the social optimum can be decentralized by making regions responsible for redistribution and implementing equalization transfers that depend on the number of households of each type.

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1 Introduction

Decentralization is a fact of life in most economies. Several important public services, such as education, health care and social services, are provided by sub-national or 'regional' governments with more or less fiscal autonomy. These decentralized services are financed by varying combinations of own tax revenues and transfers from the 'central' government, implying a vertical fiscal imbalance. The taxes range from surtaxes on central taxes to separate regional taxes on payrolls, consumption or income. Transfer systems typically include some equalizing component intended to compensate for the differences in fiscal capacities that inevitably arise from decentralization. This paper investigates the appropriate assignment of taxes to regional governments and the accompanying set of transfers in standard fiscal federalism models.

The existing literature identifies inefficiencies and inequities caused by decentralization that can be countered by appropriate fiscal arrangements. On the one hand, various forms of fiscal externalities arise from the fact that one jurisdiction's policies will affect governments or households in other jurisdictions. These interactions can occur horizontally, as when regional governments compete for mobile tax bases, or vertically, as when a region's tax changes affect the central government's budget (Dahlby, 1996). When households differ in incomes, these externalities can result in non-optimal redistribution policies. On the other hand, fiscal decentralization, by creating differences in the capacity of the regions to provide public services, results in so-called fiscal inefficiency and fiscal inequity. The former occurs when the differences in fiscal capacity give rise to purely fiscal incentives to migrate from one region to the next, quite apart from productivity differences. The latter occurs between non-migrants of different regions: otherwise identical persons are treated differently by the public sector, so horizontal equity is violated.[1] These fiscal inefficiencies and inequities provide the rationale for equalization. While there is a relatively large literature on fiscal externalities, the literature on the theory of equalization is relatively limited, and tends to be confined mainly to models of homogeneous households with symmetric outcomes, with some exceptions (e.g. Burbidge and Myers 1994, Boadway, Marchand and Vigneault 1998). Two aspects of decentralization that are relevant in practice are thereby

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[1] The notions of fiscal inefficiency and fiscal inequity are due to Buchanan (1950, 1952). A general treatment of the circumstances in which they arise, and the forms of equalization that can deal with them is found in Boadway and Flatters (1982b).

neglected — redistribution at sub-national levels of government and equalization.

In our analysis, we assume there is some public good that by its nature is provided by regional governments. We focus on the implications of this assumed decentralization of expenditure responsibilities for the assignment of revenue-raising instruments and fiscal transfers, both vertical and horizontal. Our methodology involves constructing models of heterogenous regions of varying degrees of complication and generality. For each case, we characterize the planning optimum using the fictitious notion of a unitary state in which the central government takes all fiscal decisions on behalf of itself and the regions. Then, we consider how the unitary state optimum can be decentralized by a judicious choice of fiscal policy instruments for the regions, taking as given the expenditure responsibilities of the regional governments.

The appropriate method of decentralization will depend on the structure of the economy as well as on the assumed strategic interaction among the various actors in the model: the central government, the regional governments and the private agents. Following much of the literature on equalization, we focus on so-called Ricardian models in which labour mobility is the only variable factor of production and the main source of interdependency in the federation. Likewise, the timing of decision-making or strategic interaction is such that the central government moves first, followed by regional governments, and then by private agents. Regional governments act as Nash competitors with respect to each other's policies, and private agents are price-takers. Indeed, the location decision is the only real decision households take since we assume that their labour supplies are fixed. Moreover, we assume that regional public services have no spillover benefits to residents of other regions. These assumptions allow us to focus on the sorts of fiscal externalities that can be addressed by equalization transfers, those due to labour mobility.[2]

Heterogeneity can come from three sources in our models. Regions can be heterogeneous because of differences in production functions, say, because of differences in the

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[2] Some sources of fiscal externality that have been emphasized in the literature are effectively assumed away, including capital and commodity tax competition, inter-regional spillovers and vertical fiscal externalities. See Wilson (1999) for a general discussion of tax competition, especially capital tax competition. Lockwood (2001) synthesizes the various forms of commodity tax competition. Boadway and Keen (1996) consider the effects of vertical fiscal externalities.

endowment of fixed factors. Households can be heterogeneous for two reasons: first, because they have different costs of migration from one region to the other; and second, because they have different productivities. Throughout, we assume that governments are benevolent and that central and regional governments agree on the form of the social welfare function that satisfies three basic value judgments: i) the Pareto principle, ii) the symmetric and anonymous treatment of all citizens within the relevant jurisdiction, and iii) a non-negative aversion to inequality.[3] For simplicity, the aversion to inequality is taken to be zero (i.e. the least redistributive), so that the social welfare function is utilitarian.

We proceed as follows. We begin with the case where all households supply homogeneous labour. Given the heterogeneity of regions, decentralization will need to be accompanied by inter-regional redistribution, or equalization. We then briefly consider some extensions to this analysis in Section 3. First, we examine the consequences of full decentralization whereby all fiscal responsibilities, including inter-regional transfers, are devolved to the regional level. Second, we consider reversals in the order of decision-making, perhaps because of an inability of governments to commit to policies. Thus, we determine the implications of regional governments moving before the central government, and also of labour migration occuring before at least some policy decision-making. The homogenous labour case and its extensions generalize and synthesize existing results in the literature as well as serves as a useful basis for — and contrast to — the case we consider in Section 4 in which household productivities can differ. In this latter case, there will be both inter-regional and intra-regional redistribution. As we shall see, the latter possibility changes the nature of decentralization considerably.

2 Homogeneous Labour

The case of homogeneous labour is the one most commonly treated in the literature on equalization. Here, we use it as a useful benchmark to analyze the consequences of making the regions responsible for providing regional public goods and services to their residents. We indicate the fiscal arrangements necessary in a federation to ensure that the optimal

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[3] There is ample literature on federations in which governments adopt some non-benevolent decision rule. These range from models based on political decision-making, such as majority rule (see Persson and Tabellini 2000), to those that reflect the power of the bureaucracy (e.g. Brennan and Buchanan 1990).

resource allocation is achieved.

There are two regions $i = 1, 2$, each endowed with an initial population O_i.[4] House-holds may migrate across regions. The number of residents in region i after migration takes place is N_i, such that $O_1 + O_2 = N_1 + N_2 = N$, where N is total population. In what follows, we assume that the population is a continuum to simplify the analysis. Each household supplies one unit of labour to the region of its residence, so N_i is the labour supply of region i. Aggregate production functions for the two regions differ from one another and are given by $F_i(N_i)$, where $F_i' > 0 > F_i''$. For concreteness, we assume that $F_2(N) > F_1(N)$, $F_2'(N) > F_1'(N)$, so region 2 is the more productive one (perhaps because it has more of an underlying fixed factor).[5]

Migration may be costly. Costless migration, commonly assumed in the literature, is but a special case and yields qualitatively similar results when households are homogeneous. Migration costs take a non-pecuniary form which ensures that they have no resource implications. All households are equally happy in their original region. If they move, they incur varying degrees of non-pecuniary dissatisfaction: those with the least dissatisfaction from moving will move first.[6] Therefore, we can depict the migration cost of the marginal household by the migration cost function $k(O_i - N_i)$, where $O_i - N_i$ is the total number of migrants from region i to the other region and $k' \geq 0$. Migration will occur in one direction only, and that direction will depend on the initial allocation of population. For illustrative purposes, we assume that migration goes from region 1 to region 2, but that has no bearing on the qualitative results in this section.

Households have identical utility functions of the form $u(c_i) + b(g_i)$, where c_i is consumption of a composite private good in region i that serves as numeraire, and g_i rep-

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[4] Our notation convention is that aggregate variables and aggregate functions will be in upper case, while individual variables and functions are in lower case. Central government policies will be denoted using Roman letters, while regional ones will use Greek letters.

[5] To be more specific, we assume that the two regions have the same production function given by $G(T_i, N_i)$ where T_i is the amount of fixed factor in region i, $G_T, G_N > 0$, $G_{TT}, G_{NN} < 0$, and $G_{NT} > 0$. Therefore, $F_i(N) = G(T_i, N)$ and assuming $T_2 > T_1$ implies the above.

[6] This differs from the attachment-to-home concept used in Mansoorian and Myers (1993), where persons obtain differing levels of satisfaction in their region of residence. That is, even non-migrants obtain different levels of non-pecuniary satisfaction from being in their original location.

resents a public service whose benefits accrue equally to all residents of region i. The latter is related to public expenditures G_i in region i by $g_i = G_i/N_i^\alpha$, where α is a congestion parameter which can take values $0 \leq \alpha \leq 1$. For $\alpha = 0$, g is a pure public good, while for $\alpha = 1$, it is a publicly provided private good. Thus, α is also an index of privateness of the public good. We assume that the marginal rate of transformation between private and public goods is unity, so production for the nation as a whole is $F_1(N_1) + F_2(N_2) = N_1 c_1 + N_2 c_2 + G_1 + G_2$. Note that this assumes that all households within region i receive the same consumption — migrants and non-migrants alike. If migration goes from region 1 to 2, migration equilibrium (assuming an interior solution) satisfies: $u(c_1) + b(g_1) = u(c_2) + b(g_2) - k(O_1 - N_1)$. The marginal migrant will bear a non-pecuniary migration cost of $k(O_1 - N_1)$, while all $O_1 - N_1$ infra-marginal migrants bear a lower cost.

Unitary State Optimum

As a benchmark, we characterize the planning optimum. To do this, we characterize the optimum of a fictitious unitary state in which all fiscal instruments, including those that are region specific, are in the hands of the central government, although for heuristic purposes we differentiate between central and regional budgets. This case is fictitious in the sense that we assume that the central government can provide the regional public services g_i as efficiently as could the regions themselves. If this were literally the case, there would be no need to decentralize the provision of g_i to the regions. In fact, in the real world, there are numerous reasons why the regions may be more efficient at providing the public good that the central government.

Labour markets are perfectly competitive, so labour income of a resident in region i is given by the marginal product $F_i'(N_i)$. We assume that all rents go to the government, either because it owns the fixed factor or because it has access to a rent tax, which it uses to the fullest.[7] Rents in region i are given by: $R_i(N_i) = F_i(N_i) - N_i F_i'(N_i)$ where

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[7] The issue of who gets the rents in a federation is an important one. On the one hand, if rents accrue to regional governments, this constitutes a source of difference in tax capacity that can cause inefficiency, as discussed in Boadway and Flatters (1982a). We assume both in the budget assignment of the unitary state and in the decentralized outcome below that the regional governments collect the rents. An alternative is to assume that rents accrue to households nationwide. See Flatters, Henderson and Mieszkowski (1974). This introduces

$R_i'(N_i) = -N_i F_i''(N_i) > 0.$

The unitary state policy instruments include those that might be used in a decentralized setting. Those attributed to the central budget include a per person tax of t_i on each resident in region i and a transfer T_i to region i. Notional regional policies include the public good and three types of taxes — a surtax of σ_i imposed on t_i, a payroll tax at the rate π_i, and a consumption tax θ_i. Rents R_i are assumed to accrue to the region i budget. With migration going from region 1 to 2, the unitary state government problem (denoted by $(\mathcal{P})$) is:

$$\max_{\{t_i,\sigma_i,\pi_i,\theta_i,T_i,g_i,N_1\}} \quad N_1\left[u(c_1) + b(g_1)\right] + (N - N_1)\left[u(c_2) + b(g_2)\right] - \int_0^{O_1 - N_1} k(x)dx$$

subject to

$$N_1 t_1 + (N - N_1)t_2 - T_1 - T_2 \geq 0 \tag{λ}$$

$$T_1 + N_1(\sigma_1 t_1 + \pi_1 F_1'(N_1) + \theta_1 c_1) + R_1(N_1) - (N_1)^\alpha g_1 \geq 0 \tag{λ_1}$$

$$T_2 + (N - N_1)(\sigma_2 t_2 + \pi_2 F_2'(N - N_1) + \theta_2 c_2) + R_2(N - N_1) - (N - N_1)^\alpha g_2 \geq 0 \tag{λ_2}$$

$$u(c_2) + b(g_2) - k(O_1 - N_1) - u(c_1) - b(g_1) = 0 \tag{γ}$$

where $c_i = (F_i'(N_i)(1 - \pi_i) - (1 + \sigma_i)t_i)/(1 + \theta_i)$, and we have used the relationship $G_i = N_i^\alpha g_i$. The first three constraints are the notional budget constraints of the central government and the two regions, while the last constraint is the migration equilibrium condition. Notice that the central government is assumed to use N_1 as a control variable. This is purely artificial, and is the counterpart to adding the migration equilibrium condition as a constraint in the problem.[8] The equation labels (λ) and (γ) in problem $(\mathcal{P})$ refer to the Lagrange multipliers associated with the relevant constraints.

From the first-order conditions on T_1 and T_2, we obtain $\lambda_1 = \lambda_2 = \lambda$. This in turn implies that the first-order conditions on σ_i, π_i and θ_i are the same as those on t_i, which means that, in this homogeneous labour setting, the tax instruments assigned to the regions are redundant. Thus, starting from a set of policies $\{t_i, \sigma_i, \pi_i, \theta_i, T_i\}$ that satisfies

the possibility of source-based rent taxes, which can be a source of inefficiency in regional decision-making because of tax exporting (Boadway 1982).

[8] An alternative procedure would be to use the migration constraint to determine N_1 as a function of policies, and then take the latter as endogenous.

the first-order conditions, any other set that yields the same overall personal tax payment $(1 + \sigma_i)t_i + \pi_i F_i' + \theta_i c_i$ in the two regions, accompanied by the appropriate change in transfers T_i, will also satisfy the first-order conditions. Accordingly, the optimal *vertical fiscal imbalance* (VFI) — the total level of transfers to the regions — is indeterminate.

Using the above result and the fact that $R_i'(N_i) = -N_i F_i''(N_i)$, we can combine the conditions on t_i and g_i in each region to yield the standard public goods efficiency conditions:

$$\frac{N_1}{N_1^\alpha} \frac{b'(g_1)}{u'(c_1)} = 1, \qquad \frac{N_2}{N_2^\alpha} \frac{b'(g_2)}{u'(c_2)} = 1 \tag{1}$$

The conditions on t_1 and t_2 also give an interpretation of the nationwide marginal cost of public funds:

$$\frac{1}{\lambda} = \frac{N_1}{N} \frac{1}{u'(c_1)} + \frac{N_2}{N} \frac{1}{u'(c_2)} \tag{2}$$

The first-order condition on N_1 may be written as:

$$\left[(1 + \sigma_2)t_2 - \frac{\alpha G_2}{N_2} \right] - \left[(1 + \sigma_1)t_1 - \frac{\alpha G_1}{N_1} \right] = \frac{\gamma}{\lambda} k'(M) \tag{3}$$

where, using the conditions on t_1 and t_2,

$$\frac{\gamma}{\lambda} = \frac{N_1 N_2}{N} \left(\frac{1}{u'(c_2)} - \frac{1}{u'(c_1)} \right) \gtrless 0$$

It is important to note that the migration equilibrium constraint is generally binding here, so $\gamma \neq 0$, although it is not clear whether γ is positive or negative. This is true even if migration is costless. Therefore, according to the last relationship, consumption will not be equalized across the two regions nor will the level of public services. This is in contrast to the case with heterogeneous households and costless migration discussed below.

Note that in expression (3), $\alpha G_i / N_i$ is the increment in cost required to keep the level of public services constant when an additional migrant enters and imposes congestion on the others.[9] The term $(1 + \sigma_i)t_i - \alpha G_i / N_i$ therefore represents the net benefit to existing residents from a marginal migrant — the additional revenue raised less the congestion costs imposed.[10] Thus, the left-hand side of (3) is the *net fiscal externality* (NFE) resulting when a marginal migrant moves from region 1 to region 2. The right-hand side is the additional

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[9] Using the definition of $G_i = N_i^\alpha g_i$, we have $\partial G_i / \partial N_i |_{g_i = \text{constant}} = \alpha N^{\alpha - 1} g_i = \alpha G_i / N_i$.

[10] This term is standard. See Buchanan and Goetz (1972), Boadway and Flatters (1982a).

resource costs arising from the effect of the increased migration costs of the marginal migrant on the equal utility constraint. An additional migrant increases the marginal migration costs and increases the differential between the utility that must be provided in the two regions. If the NFE is positive then we have $c_2 > c_1$, and the opposite holds when the NFE is negative.

The optimal grant scheme must simultaneously satisfy the three government budget constraints (λ_1), (λ_2) and (λ), as well as the optimal allocation of labour equation (3). Combining the budget constraints for the two regions, we obtain:

$$\frac{T_1}{N_1} - \frac{T_2}{N_2} = \frac{G_1}{N_1} - \frac{G_2}{N_2} + \sigma_2 t_2 - \sigma_1 t_1 + \frac{R_2(N_2)}{N_2} - \frac{R_1(N_1)}{N_1}$$

This says that the difference in per capita transfers to the two regions should compensate for three sources of difference in fiscal capacities — differences in per capita expenditure needs, differences in per capita tax collections, and differences in per capita rents. However, that is a pure accounting interpretation. If we use the optimality condition on N_1 given by (3) and the central government budget constraint, we obtain:

$$T_1 = \frac{N_1 N_2}{N_1 + N_2} \left[\left(\frac{(1-\alpha)G_1}{N_1} - \frac{(1-\alpha)G_2}{N_2} \right) - \left(\frac{R_1(N_1)}{N_1} - \frac{R_2(N_2)}{N_2} \right) + \frac{\gamma}{\lambda} k'(M_1) \right] + N_1 t_1$$

$$= E_1 + N_1 t_1 \tag{4}$$

where E_1 is region 1's equalization entitlement. Similarly, for region 2,

$$T_2 = -E_1 + N_2 t_2 = E_2 + N_2 t_2$$

Note that by the central government's budget constraint, the VFI can be defined as VFI $\equiv T_1 + T_2 = N_1 t_1 + N_2 t_2$. The VFI is divided between the two regions according to the so-called principle of derivation,[11] and each region obtains an equalization entitlement E_i, where $E_1 + E_2 = 0$.

The equalization entitlement for, say, region 1 contains three terms reflecting the effect of an additional migrant moving from region 1 to 2. The first term is the difference in

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[11] This principle says that central tax revenues collected on behalf of regional expenditures are returned to the regions according to where they are collected. In other words, we could attribute the tax revenues raised by t_i to the regional budgets in the first place. In a more general setting, some central tax revenues would be used for central government expenditures, which we have assumed away here.

per capita resource costs of providing public goods less the congestion effect. Thus, an additional migrant contributes the per capita share of the cost of providing public goods, but reduces the benefits available for existing resident. The second term is the difference in per capita rents. Since rents are attributed to regional budgets, persons migrating to a jurisdiction effectively claims a share of the rents at the expense of existing residents. The final term is the shadow cost of an additional migrant on the migration equilibrium constraint.

Decentralizing the Unitary State Optimum

In our decentralized setting, regional governments have responsibility for the provision of public services g_i in their own region. The issue is: what financing arrangements will serve to ensure that the unitary state optimum is achieved? It turns out that in this simple economy with homogeneous labour, there is considerable freedom in the choice of tax instruments that can be given to the regions and the proportion of own revenues they can be expected to raise (which determines the VFI).

To be concrete, assume that the regions are able to levy their own per unit tax rates τ_i on households in their own region. We could equally have allowed the regions to levy any of the other taxes introduced above — a surtax σ_i, a payroll tax π_i, or a consumption tax θ_i — since in this setting, all are effectively equivalent.[12] As well, we assume that the rents accrue fully to the regional government, although our analysis carries through to the case where the central government has a share of the rents. It is necessary to be explicit about the sequence of events in the economy since we have two levels of government as well as private agents. We take as our standard case the following one:

Stage 1: *Central government policies:* The central government chooses $\{t_i, T_i\}$, anticipating regional government policies and private sector outcomes.

Stage 2: *Regional government policies:* Each regional government chooses $\{\tau_i, g_i\}$ taking as given central government policies and those of the other regional government,

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[12] For example, total per person tax liabilities are $t_i + \tau_i$ when regions levy their own tax and $(1+\sigma_i)t_i$ when regions levy a surtax. There will always be a level of surtax σ_i that replicates the overall tax rate achieved under the regional tax. The same will apply for the other taxes. Note that this will also be the case when the central government levies either a common per unit tax in both regions or a common proportional tax.

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and anticipating the private sector outcomes.

Stage 3: *Migration:* Given the policies announced by the two levels of government, house-holds choose their place of residence.

As mentioned, we assume that both levels of government use as their objective functions the sum of utilities of final residents in their own jurisdictions. The full solution to the decentralized problem is a sub-game perfect Nash equilibrium (SPNE), which is obtained by backward induction. It turns out that we need not fully solve the problem. To verify the conditions under which the SPNE will replicate the unitary state optimum, it will suffice to consider the Stage 2 problem of the regions.

Consider, for example, the problem of region 1 given that migration goes from region 1 to region 2. Assuming it maximizes the sum of utilities of final residents, its problem is:

$$\max_{\{\tau_1, g_1, N_1\}} N_1 \left[u \left(F_1'(N_1) - t_1 - \tau_1 \right) + b(g_1) \right]$$

subject to

$$T_1 + \tau_1 N_1 + R_1(N_1) - (N_1)^\alpha g_1 \geq 0 \qquad (\lambda_1)$$

$$u \left(F_2'(N - N_1) - t_2 - \tau_2 \right) + b(g_2) - k(O_1 - N_1) - u \left(F_1'(N_1) - t_1 - \tau_1 \right) - b(g_1) = 0 \quad (\gamma_1)$$

The region's policy instruments are its own tax rate τ_1 and its level of public services g_1. In addition, it uses as an artificial variable its population N_1, taking account of the migration equilibrium constraint (γ_1). As earlier, this is simply a way of accounting for the fact that it may recognize that its policies will affect the migration of labour. Precisely how it expects its policies will affect N_1 will depend upon how it assumes the other region's policies to be related to its own. It may adopt a Nash conjecture with respect to all the other region's policies, or it may be more sophisticated and treat either of τ_2 and g_2 to be given, with the other determined by budget balance. It turns out to be the case that whatever Nash conjecture the government of region 1 adopts, the same qualitative result will apply as long as the central government anticipates its behaviour. In fact, the decentralization results apply even if the region is myopic and takes N_1 as given. To see this, combine the first-order conditions on τ_1 and g_1 to obtain,

$$\frac{N_1}{N_1^\alpha} \frac{b'(g_1)}{u'(c_1)} = 1$$

which is the standard public good efficiency condition as given by (1). It holds regardless of how region 1 assumes the other region's policies are related to its own, and whether it is myopic. The first-order condition on N_1 essentially determines the value of γ_1 for region 1. In the myopic case, the region disregards the migration equilibrium constraint, so $\gamma_1 = 0$.

For region 2, a similar result applies. Assuming its objective is the aggregate utility of final residents, it maximizes:

$$N_2 \left[u\left(F_2'(N_2) - t_2 - \tau_2\right) + b(g_2)\right] - \int_0^{O_1 - N_1} k(x)dx$$

subject to its regional budget constraint and the migration equilibrium constraint, which are analogous to (λ_1) and (γ_1). Again, regardless of the conjecture adopted by the region with respect to the Nash behaviour of region 1, the efficiency conditions for the regional public good applies. Furthermore, it is straightforward to show that the same result holds if regional objective functions are the utilities of original residents, or per capita utility in the region.[13] Indeed, the two regions can even adopt different objective functions.

Given that the regions abide by the optimal decision rule for the provision of their public services, it is apparent that the central government by its choice of taxation and transfers can ensure that the unitary state outcome is replicated. It simply needs to get into the hands of the regional governments enough funds to ensure that when the efficiency rule for public goods is satisfied, the disposable income of households is sufficient to yield the unitary state optimal value of c_i. It is clear that there are alternative ways in which this may occur. As in the unitary state analysis, the VFI is indeterminate. Any system of central taxes and transfers to the regions that ensures the optimal allocation of population (3) and satisfies condition (4) and the central budget constraint will suffice. Each region obtains as a transfer the central taxes levied in its jurisdiction as well as an equalization transfer, which may be positive or negative. There is apparently no restriction on the taxes levied by the central government. For example, they could be uniform across regions. Moreover, whether or not migration is costly makes no difference to these conclusions: only the size of the equalization transfers are affected.

To summarize, with homogeneous labour, per capita levels of consumption and public

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[13] For region 1, the objective function when it cares about original residents is $N_1[u(c_1) + b(g_1)] + (O_1 - N_1)[u(c_2 + b(g_2)] - \int_0^{O_1 - N_1} k(x)dx$. The same constraints apply.

services and population will differ across the two regions in the unitary state (or, planning) optimum, and the migration equilibrium constraint will be binding. In the decentralized federation when the regions are responsible for public good provision, the unitary state optimum can be decentralized in a variety of ways. As long as the optimal equalization scheme is in place which takes account of the net welfare effects of migration, any of the various taxes can be assigned to the regions. Regional governments will abide by the efficiency conditions for public goods provision given they maximize the sum of utilities regardless of their conjectures about migration responses. The VFI will be indeterminate. That is, the central government can occupy whatever share of the tax room it chooses, provided it transfers to each region an amount reflecting the sum of the tax revenues raised in the region by the federal government and the region's equalization entitlement.

We now turn to various extensions of the homogeneous labour case.

3   Extensions to the Homogeneous Labour Case

In this section, we retain the assumption of homogeneous labour and consider various extensions to the above analysis. We begin by allowing the regions full fiscal responsibility and show that although voluntary equalization transfers are feasible, they are generally not sufficient to take the decentralized economy to the unitary state optimum. Next, we examine the consequences of changing the order of decision-making among the various players in the economy — the central government, the regional governments, and the households. Unless the central government is the first-mover, the unitary state optimum cannot be decentralized.

Voluntary Inter-Regional Transfers

In the costless migration case, Myers (1990) has shown that there is no need for the central government to impose equalization transfers as they will be made voluntarily from one region to another. In the absence of migration costs, the utility possibility frontier (UPF) consists of a single point since households will always achieve the same utility level. Therefore, efficiency and equity coincide and voluntary transfers, which are efficient in this setting, will also be fully optimal. However, with migration costs, there is an equity-efficiency trade-off. Along the UPF, migration from region 1 to region 2 makes persons in region 1 worse off and those infra-marginal in region 2 better off. Voluntary transfers

may still reach a point on the national UPF in this case, as Mansoorian and Myers (1997) show. More important for our purposes, they will not reach the socially optimal point. We examine the role of equalization transfers here and illustrate that centrally imposed transfers will be necessary to take the economy to the unitary state optimum, and will fully crowd out voluntary transfers rendering the latter irrelevant.

To simplify our analysis, we assume, following Mansoorian and Myers (1993), that there are no public goods. All rents accrue to the regions, so the source of potential inefficiency is that in-migrants obtain a share of the rents in addition to their marginal product, implying that they obtain their average product. Let T_2 be a transfer from region 1 to region 2. It may be made by the central government or voluntarily by one of the regions. Feasibility requires that $c_1 = (F_1(N_1) - T_2)/N_1$ and $c_2 = (F_2(N - N_1) + T_2)/(N - N_1)$, and, assuming migration from region 1 to 2, migration equilibrium requires $u(c_1) = u(c_2) - k(O_1 - N_1)$. Together with the feasibility conditions, this condition yields $N_1(T_2)$ where $dN_1/dT_2 < 0$.[14] We can then define the aggregate utility of final residents in regions 1 and 2 as $\mathcal{V}_1(T_2)$ and $\mathcal{V}_2(T_2)$, respectively.[15] Consider then the central government's objective, which is $\mathcal{W}(T_2) \equiv \mathcal{V}_1(T_2) + \mathcal{V}_2(T_2)$. The optimal transfer, say T_2^*, satisfies $\mathcal{W}'(T_2^*) = \mathcal{V}_1'(T_2^*) + \mathcal{V}_2'(T_2^*) = 0$.[16]

Suppose both regions agree that a transfer should be made by region 1, i.e., at $T_2 = 0$ $\mathcal{V}_1'(0) > 0$ and $\mathcal{V}_2'(0) > 0$. Region 1 would optimally choose T_2 such that $\mathcal{V}_1'(T_2) = 0$. Now, at $\mathcal{V}_1'(T_2) = 0$, $\mathcal{W}'(T_2) = \mathcal{V}_2'(T_2)$, which may be positive or negative. If it is positive, then the central government would like to increase transfers. Doing this crowds out regional transfers on a one-for-one basis. The central government increases the transfers until $\mathcal{W}'(T_2^*) = 0$ and therefore, $\mathcal{V}_1'(T_2^*) < 0$ and $\mathcal{V}_2'(T_2^*) > 0$, implying that voluntary transfers would not suffice to achieve the unitary state optimum. If instead it is negative, then the level of voluntary transfers that region 1 wants to make will not be accepted by region 2. Region 2 will only accept transfers until $\mathcal{V}_2'(T_2) = 0$, while at this point, $\mathcal{W}'(T_2) =$

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[14] Differentiating, $dN_1/dT_2 = -[u'(c_1)(R_1 - T_2)/N_1 + u'(c_2)(R_2 + T_2)/N_2 + k']^{-1} [u'(c_1)/N_1 + u'(c_2)/N_2] < 0$.

[15] To be precise, $\mathcal{V}_1(T_2) \equiv N_1(T_2)u([F_1(N_1(T_2)) - T_2]/N_1(T_2))$ and $\mathcal{V}_2(T_2) \equiv (N - N_1(T_2))u([F_2(N - N_1(T_2)) + T_2]/(N - N_1(T_2))) - \int_0^{O_1 - N_1(T_2)} k(x)dx$.

[16] It is assumed that the second-order condition of the central government's problem is satisfied.

$\mathcal{V}'_1(T_2) > 0$ and a conundrum arises. Both the central and region 1 governments would like to increase transfers, but region 2 will not accept them! Unless region 2 can be forced to accept the central government's transfers, the only way for the unitary state outcome to be achieved is for the central government to rely solely on the interpersonal tax-transfer system and set $t_1 > 0$ and $t_2 < 0$ such that $t_1 N_1 = -t_2 N_2 = T_2^*$.

Suppose instead that the two regions are in disagreement about the direction of the transfer. There are two possible cases. First, at $T_2 = 0$, it may be that $\mathcal{V}'_1(0) > 0$ and $\mathcal{V}'_2(0) < 0$, so region 1 would like to make transfers, but region 2 will not accept any. No transfers are possible, and presumably this is can also be a problem for the central government. Again, the central government would have to implement the transfer via the interpersonal redistribution system. Second, it could be that at $T_2 = 0$, $\mathcal{V}'_1(0) < 0$ and $\mathcal{V}'_2(0) > 0$. In this case, region 2 would like to receive transfers, but region 1 is not willing to make them. While the central government may not be able to force region 1 to make a transfer, it could achieve the equivalent outcome by using its power to tax, and make a transfer to region 2, which would be willingly accepted.

We can conclude that if the central government adopts a policy that implements the unitary state optimum, then any voluntary transfers will be completely crowded out. Of course, the above discussion is only suggestive since we have not analyzed the circumstances in which the various options would occur. Further work is needed to characterize fully the relationship between the allocations under voluntary transfers with those of the unitary state optimum. Nonetheless, it is clear that the scope for voluntary transfers is drastically limited when the UPF does not consist of a single point.

Different Timing I: Migration First

It might be argued that since migration is a relatively long-term decision, it may take place before policies are enacted. Equivalently, governments may not be able to commit to future fiscal policies. Mitsui and Sato (2001) have analyzed this problem for the case of costless mobility. Following their analysis, we reverse the order of decision-making so that migrants move in Stage 1 anticipating government policy, and in the second stage, the unitary state government chooses its policies, taking labour allocation across regions as given. We again use backward induction.

The problem of the unitary state government is identical to problem $(\mathcal{P})$ in Section 2 without the migration equilibrium condition (γ). Taking labour allocations as predetermined the unitary state government no longer selects N_1 as an artifical variable and therefore, is not constrained by the migration equilibrium. The first-order conditions are identical to those in problem $(\mathcal{P})$ except with $\gamma = 0$. Therefore, the conditions on t_i and g_i imply that the standard public good efficiency conditions (1) are satisfied, $c_1 = c_2$ and, for $\alpha < 1$, $g_1 \gtreqqless g_2$ as $N_1 \gtreqqless N_2$.

Households anticipate that consumption will be equalized across regions, $c_1 = c_2$. Therefore, they have an incentive to migrate to the region with the highest public good provision, which will be the most populous one. In the special case where there are no migration costs, all households will move to a single region (the Mitsui-Sato result). If migration is costly, not all households will necessarily locate in one region. Given that $c_1 = c_2$, the migration equilibrium condition for an interior solution will be $b(g_1) = b(g_2) + k(O_1 - N_1)$. Obviously, the unitary state optimum as described in Section 2 cannot be achieved when migration occurs before government policy, and likewise in the decentralized case. However, if the interregional equalization transfers T_i takes place first, followed by migration and the choice of regional tax and expenditure policies, then the problem is overcome. Hence, what is critical for decentralizing the unitary state optimum from Section 2 is that the central government move first.

Different Timing II: Regions Move before Central Government

Suppose that regions choose g_i before central policies are chosen. This might be reasonable if one supposes that actual expenditure decisions are of a longer-term nature than tax and transfer decisions. We revert to the assumption that migration occurs after policies are implemented. It is straightforward to see that the unitary state optimum cannot be decentralized regardless of how public expenditures are financed. To see this, consider the simplest case in which all financing is done by central taxes, and funds are simply transferred to the regions to cover the costs of the public goods. This is the extreme case of a soft budget constraint.

Since regions move first anticipating the induced central government behaviour, it is necessary to first consider the central government's problem. It can be characterized using problem $(\mathcal{P})$ from Section 2 and treating g_1 and g_2, which are chosen by the regions, as

given. As well, we ignore all the possible regional taxes considered in the previous section and assume only central taxes are imposed. From this problem we can obtain the first order conditions which together with the constraints determine the optimal values for the choice variables $\{t_1, t_2, T_1, T_2, N_1\}$ in terms of the exogenous variables $\{g_1, g_2\}$. As before, the first-order conditions on central taxes and transfers ensure the marginal cost of public funds are equalized across regions. Likewise, we obtain an interpretation of the nationwide marginal cost of public funds as represented by (2). From the condition on N_1, we also obtain

$$\left[t_2 - \frac{\alpha G_2}{N_2}\right] - \left[t_1 - \frac{\alpha G_1}{N_1}\right] = \frac{N_1 N_2}{N}\left(\frac{1}{u'(c_2)} - \frac{1}{u'(c_1)}\right)k'(O_1 - N_1) \tag{5}$$

which is analogous to (3) and determines the optimal allocation of population. Thus, federal behaviour is characterized by the choice of $\{t_1, t_2, T_1, T_2, N_1\}$ that satisfies conditions (2) and (5) and its constraints, for given values of $\{g_1, g_2\}$. It is this characterization that the regional governments anticipate.

Consider now the optimizing behaviour of region 1. Assuming it behaves as a Nash competitor with respect to the other region, it chooses g_1, given g_2 and anticipating central government behaviour. The regions can choose the central government policy variables artificially by incorporating the equations governing central behaviour into their problem. Therefore, region 1's problem is as follows, where we have eliminated the central-regional transfers T_i by consolidating the three budget constraints:

$$\max_{\{g_1, t_1, t_2, N_1\}} \quad N_1\left[u(c_1) + b(g_1)\right]$$

subject to

$$N_1 t_1 + (N - N_1)t_2 + R_1(N_1) + R_2(N - N_1) - N_1^\alpha g_1 - (N - N_1)^\alpha g_2 \geq 0 \tag{λ_1}$$

$$u(c_2) + b(g_2) - k(O_1 - N_1) - u(c_1) - b(g_1) = 0 \tag{γ_1}$$

$$t_1 - \alpha N_1^{\alpha-1}g_1 - t_2 + \alpha N_2^{\alpha-1}g_2 + \frac{(N - N_1)N_1}{N}\left(\frac{1}{u'(c_2)} - \frac{1}{u'(c_1)}\right)k'(O_1 - N_1) = 0 \tag{δ}$$

where $c_i = F_i'(N_i) - t_i$. Combining the first-order conditions with respect to g_1 and t_1 yields:

$$\frac{N_1 b'(g_1)}{N_1^\alpha u'(c_1)} = \frac{\lambda_1 + \delta\alpha/N_1}{\lambda_1 + \delta(N - N_1 N_2 u''(c_1)k'(O_1 - N_1)/u'(c_1)^2)} \tag{6}$$

16

In general, the migration equilibrium constraint will be binding ($\delta \neq 0$) and the right-hand side of (6) will not be unity. Therefore, the efficiency conditions for public goods (1) will not be satisfied, and since the sign of δ is ambiguous, there can be over- or under-supply. In any case, the unitary state optimum cannot be decentralized when regions move before the central government.

4 Heterogeneous Labour

In this section, households are assumed to differ in their labour productivity. This introduces the possibility of intra-regional as well as inter-regional redistribution. Unlike with homogenous labour, the case of costless migration has qualitatively different results from the costly case and therefore, we treat each of the two cases in turn.

Costless Migration

Households are assumed to be of two types — high-ability types denoted by h and low-ability types denoted by ℓ. They supply respectively a^h and a^ℓ efficiency units of labour each, where $a^h > a^\ell$ and are assumed to be perfect substitutes in production. The production function in region i is $F_i(A_i)$, where $A_i = a^h N_i^h + a^\ell N_i^\ell$ is the total effective labour supply in region i. As before, the wage rate per efficiency unit of labour is equal to the marginal product $F_i'(A_i)$ and regional rents are given by $R_i(A_i) = F_i(A_i) - A_i F_i'(A_i)$, where $R_i'(A_i) = -A_i F_i''(A_i)$. With competitive labour markets, type-h workers receive a labour income of $a^h F_i'(A_i)$, and type-ℓ workers receive $a^\ell F_i'(A_i)$. We again assume that region 2 is the more productive one: $F_2(A) > F_1(A)$, $F_2'(A) > F_1'(A)$.

Household utility is now $u(c_i^j) + b(g_i)$ for $j = h, \ell$ and $i = 1, 2$. Migration equilibrium with costless migration requires (assuming an interior equilibrium):

$$u(c_1^j) + b(g_1) = u(c_2^j) + b(g_2) \qquad\qquad j = h, \ell$$

With costless migration, the initial allocation of labour is irrelevant. Only the total populations matter.

Unitary State Optimum

Following the method of the previous section, we characterize the optimum of a fictitious unitary state government. We maintain separate budgets for the central government

17

and the regions purely for expositional purposes. Assume for simplicity that taxes are notionally levied at the central level, with the VFI being made up by transfers to the regions T_i. Taxes are lump-sum and imposed on each type of households in each region, t_i^j ($j = h, \ell$; $i = 1, 2$). The problem of the unitary state government is:

$$\max_{\{T_i, t_i^j, g_i, N_1^j\}} N_1^h \left[u \left(a^h F_1'(A_1) - t_1^h \right) + b(g_1) \right] + N_1^\ell \left[u \left(a^\ell F_1'(A_1) - t_1^\ell \right) + b(g_1) \right]$$

$$+(N^h - N_1^h) \left[u \left(a^h F_2'(A_2) - t_2^h \right) + b(g_2) \right] + (N^\ell - N_1^\ell) \left[u \left(a^\ell F_2'(A_2) - t_2^\ell \right) + b(g_2) \right]$$

subject to:

$$t_1^h N_1^h + t_1^\ell N_1^\ell + t_2^h N_2^h + t_2^\ell N_2^\ell - T_1 - T_2 = 0 \qquad (\lambda)$$

$$T_i + R_i(A_i) - (N_i^h + N_i^\ell)^\alpha g_i = 0 \qquad\qquad i = 1, 2 \qquad (\lambda_i)$$

$$u \left(a^j F_2'(A_2) - t_2^j \right) + b(g_2) - u \left(a^j F_1'(A_1) - t_1^j \right) - b(g_1) = 0 \qquad\qquad j = h, \ell \qquad (\gamma^j)$$

The first-order conditions on g_i and t_i^j yield the analogue of the efficiency conditions (1) for this setting given by:

$$\frac{N_1^h}{N_1^\alpha} \frac{b'(g_1)}{u'(c_1^h)} + \frac{N_1^\ell}{N_1^\alpha} \frac{b'(g_1)}{u'(c_1^\ell)} = 1 \qquad \frac{N_2^h}{N_2^\alpha} \frac{b'(g_2)}{u'(c_2^h)} + \frac{N_2^\ell}{N_2^\alpha} \frac{b'(g_2)}{u'(c_2^\ell)} = 1 \qquad (7)$$

It is straightforward to show that the solution to this problem, assuming it is interior, is one in which the migration equilibrium conditions are not binding. To see this, suppose for a while that $\gamma^h = \gamma^\ell = 0$. Then, the conditions on t_1^j and t_2^j yield full consumption equality: $c_1^h = c_1^l = c_2^h = c_2^\ell = c$. Subtracting the condition on N_1^h from N_1^ℓ, we obtain that $F_1'(A_1) = F_2'(A_2)$, which then implies from these conditions that $N_1^{\alpha-1} g_1 = N_2^{\alpha-1} g_2$, or, using the condition on g_i, $g_1 b'(g_1) = g_2 b'(g_2)$.[17] This will be solved by $g_1 = g_2$, which in turn implies that $N_1 = N_2$.[18] Note also that since we have assumed region 2 to be the more productive, $A_2 > A_1$. Thus, although the total populations are identical in the two regions, region 2 must have a higher proportion of type-h residents. Finally, note that

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[17] The first order conditions on g_i ($i = 1, 2$) and N_1^j ($j = h, \ell$) are given respectively by $(N_i^h + N_i^\ell) b'(g_i) - \lambda N_i^\alpha = 0$ and $a^j \left[F_1'(A_1) - F_2'(A_2) \right] - (c_1^j - c_2^j) - \alpha \left[N_1^{\alpha-1} g_1 - N_2^{\alpha-1} g_2 \right] = 0$.

[18] This may not be a unique solution, depending on the form of the function $b(g)$. In what follows, we assume it to be unique.

this symmetric solution satisfies the migration equilibrium conditions (γ^h), (γ^ℓ), so full consumption equality holds in the unitary state optimum.

This solution is very different from the homogeneous labour case, where an asymmetric equilibrium is the norm ($N_1 \neq N_2$, $c_1 \neq c_2$, $g_1 \neq g_2$). It is apparent that this result can be generalized to many regions and many types. The migration equilibrium constraint will not be binding whenever the number of ability-types is at least as great as the number of regions. In our setting of full information and lump-sum taxes, this implies that all households will enjoy the same levels of consumption and public services. Of course, as with the two-region case of this section, this requires that the assumptions of our model apply. In particular, different ability-types of labour must be perfect substitutes in production.

Although the unitary state optimum has full consumption equality and common levels of public services in the two regions, equalization payments between regions will generally be required. Since $G_1 = G_2$ in the unitary state optimum, combining the two regional budget constraint yields:

$$T_1 - T_2 = R_2(A_2) - R_1(A_1) > 0$$

which follows from $A_2 > A_1$ and implies that an equalization transfer is made from the more to the less productive region. This result that transfers should equalize for the difference in rents in the high- and low-productivity regions follows from the fact that we have allocated rents to the regional budgets. If all rents went to the central government, the transfer to the two regions would be the same since it is the sole source of finance of public goods expenditure.

The results of this section depend on the solution being an interior one for both types of labour. If the populations of the two types of labour do not allow for an interior solution, then the outcome will be quite different. One of the regions will generally have only one type of labour. In the region with a heterogeneous population, there will be full equality of consumption between types. However, between regions, populations will generally differ and so too will the level of public services and consumption. The nature of this solution will be as in the homogeneous case.

Decentralizing the Unitary State Optimum

The regional governments have control over the spending G_i in their regions. The issue is

what financing instruments can be given to the regions so that the unitary state optimum can be replicated. The unitary state optimum requires that a) consumption be equalized for all households, b) the efficiency conditions for public goods be satisfied in each region, and c) populations be equalized across the two regions so that the actual level of public services is the same. It is straightforward to show that if the regions have access to either a surtax on central government taxes or a payroll tax, decentralization cannot achieve the unitary state optimum. However, the other two regional tax instruments can be used to decentralize the unitary state optimum — the consumption tax or a differentiated lump-sum tax on households.[19]

To see this, consider the problem of, say, regional government 1 when it has access to all four of these taxes:

$$\max_{\{\sigma_1, \pi_1, \theta_1, \tau_1^j, g_1, N_1^j\}} \quad N_1^\ell \left[u(c_1^\ell) + b(g_1) \right] + N_1^h \left[u(c_1^h) + b(g_1) \right]$$

subject to:

$$T_1 + \sigma_1 \left(t_1^h N_1^h + t_1^\ell N_1^\ell \right) + \pi_1 \left(a^j F_1'(A_1) N_1^h + a^j F_1'(A_1) N_1^\ell \right)$$

$$+\theta_1 \left(c_1^h N_1^h + c_1^\ell N_1^\ell \right) + \tau_1^h N_1^h + \tau_1^\ell N_1^\ell + R_1(A_1) - (N_1^h + N_1^\ell)^\alpha g_1 = 0 \qquad (\lambda_1)$$

$$u(c_2^j) + b(g_2) - u(c_1^j) - b(g_1) = 0 \qquad\qquad j = h, \ell \qquad (\gamma_1^j)$$

where $c_i^j = [(1 - \pi_1)a^j F_1'(A_1) - (1 + \sigma_1)t_1^j - \tau_1^j]/(1 + \theta_i)$. Rents are assumed to accrue to the regions, but this is inessential since the central government can undo the effect of differential rents in the two regions using its transfers T_i.

The solution to this problem depends on what region 1 assumes about the Nash behaviour of region 2, that is, what is assumed to determine the other region's policies. Since we cannot say that in advance, our results throughout this section must apply whatever the conjecture region 1 makes (and, of course, vice versa for region 2). It suffices to consider the first-order conditions on the relevant tax instruments and the amount of public

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[19] A uniform poll tax would also be sufficient, but that is a special case of household-specific lump-sum taxes. The central government can always choose its policies such that the regions choose to impose uniform lump-sum taxes on their residents.

good g_1. The first-order condition on g_1 is the same regardless of the tax instrument(s) available to the regional government and is given by:

$$(N_1^h - \gamma_1^h + N_1^\ell - \gamma_1^\ell)b'(g_1) - \lambda_1 N_1^\alpha = 0 \tag{g_1}$$

First, consider the case when the regional government has access to either a surtax tax σ_1 or a payroll tax π_1. The relevant first-order conditions are:

$$-(N_1^h - \gamma_1^h)t_1^h u'(c_1^h) - (N_1^\ell - \gamma_1^\ell)t_1^\ell u'(c_1^\ell) + \lambda_1(t_1^h N_1^h + t_1^\ell N_1^\ell) = 0 \tag{σ_1}$$

$$-(N_1^h - \gamma_1^h)a^h F_1'(A_1)u'(c_1^h) - (N_1^\ell - \gamma_1^\ell)a^\ell F_1'(A_1)u'(c_1^\ell) + \lambda_1(a^h F_1'(A_1)N_1^h + a^\ell F_1'(A_1)N_1^\ell) = 0 \tag{π_1}$$

where the values of the Lagrange multipliers depend not only on central government policies, but also on what the regions perceive to be the migration responses as a result of their policies. It is clear that, even if consumption levels within a region are equalized as in the unitary state, the efficiency conditions for public good provision (7) cannot be obtained from condition (g_1) under either tax. This follows from the facts that $a^h > a^\ell$ and to achieve equal consumption the central government must set $t_1^h \neq t_1^\ell$. Since the unitary state optimum requires that the efficiency conditions for public goods be satisfied, regional financing by a surtax or a payroll tax will not suffice, unlike in the homogeneous labour case.

Next, consider the case where the regional government has access only to a consumption tax θ_1. The first-order condition on θ_1 is:

$$-(N_1^h - \gamma_1^h)c_1^h u'(c_1^h) - (N_1^\ell - \gamma_1^\ell)c_1^\ell u'(c_1^\ell) + \lambda_1(c_1^h N_1^h + c_1^\ell N_1^\ell) = 0 \tag{θ_1}$$

The central government can select personal tax rates within each region such that consumption is equalized between high-and low-productivity types. In that case, the above first-order condition together with (g_1) will reduce to the efficiency conditions for public goods (7). Then, the central government can equalize consumption levels between regions and ensure that the regions provide equal levels of public services by an appropriate choice of equalization transfers. Thus, the unitary state optimum will be achieved. Moreover, the VFI will be indeterminate since the central government can perfectly substitute per capita

tax rates and transfers to the regions.[20] Moreover, this will be true whatever conjecture regions make about migration responses.

A similar result applies if the regions are allowed to use redistributive taxes on individuals, τ_i^j. In this case, the regions decide not only how much it should spend on public services, but also how much tax to raise from each type of person. The central government must induce the regions to behave optimally in each of these dimensions. The first-order conditions on τ_1^j is:

$$-(N_1^j - \gamma_1^j)u'(c_1^j) + \lambda_1 N_1^j = 0 \qquad\qquad j = h, \ell \qquad (\tau_1^j)$$

In this case, the efficiency condition for the public good (7) is obtained directly by combining the three first-order conditions (τ_1^ℓ), (τ_1^h) and (g_1). Central government policy must then induce each region to set their tax rates $\{\tau_i^h, \tau_i^\ell\}$ optimally, that is, so that consumption is equalized across types within the region. This will only apply if the migration equilibrium constraints are not binding $(\gamma_i^j = 0)$. Obviously, that will be the case if the regions are myopic with respect to migration. In the case of non-myopic behaviour on the part of regional governments, the central government must ensure that each region has the resources such that when it optimizes, the migration equilibrium constraints are not binding. The optimal equalization scheme of the unitary state optimum will suffice to ensure that.

When the optimum is decentralized by allowing the regions to use redistributive taxes τ_i^j, central government policies are indeterminate in two dimensions. First, the size of the VFI is indeterminate: what is important for ensuring that the migration equilibrium constraints will not be binding is that the relative amounts of resources available in the two regions be optimal. Second, central government intra-regional redistribution policies will be irrelevant. Regional redistribution policies are perfect substitutes for central policies, and regions move after the central government. Thus, the regions can be made solely responsible for redistributive policies.

To summarize, the unitary state optimum with costless migration will equalize consumption and public services for households across types and regions. Population will be

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[20] We are assuming that migration responses are stable and converge to the unitary state optimum.

equalized between regions, and the migration equilibrium constraints for the two types will not be binding. There will be an equalization transfer from region 2 to region 1 to the extent that regions have access to rents. The unitary state optimum can be decentralized by allowing the regions to finance their public goods using either a general consumption tax or redistributive taxes on the two types of households, and instituting the optimal equalization transfer. When regional consumption taxes are used, the central government retains control of redistribution, while if the regions can use redistributive taxes, central government redistribution policy becomes irrelevant. In either case, the VFI is indeterminate.

Costly Migration

Now suppose that there are different non-decreasing migration costs. Since there are two types of migrants, we can let them have the following migration cost functions: $k^h(O_1^h - N_1^h)$ and $k^\ell(O_1^\ell - N_1^\ell)$, where $O_1^j - N_1^j$ for $j = h, \ell$ are the numbers of migrants of each type. We continue to assume that migration of both types goes from region 1 to region 2. The issue now is whether full intra-regional equality of consumption will still apply. We begin by showing that in general the migration equilibrium constraints will be binding in the unitary state optimum. This implies that $\gamma^h, \gamma^\ell \neq 0$, although in general they can take either sign. Then, we investigate the consequences of this for the pattern of consumption and resource allocation in the unitary state. Finally, we consider the possibility of decentralizing the unitary state allocation when the regions have responsibility for providing the public good.

To see that the migration equilibrium constraints will be binding when migration is costly, we investigate the problem of the unitary state government when these constraints are not imposed. Following the same procedure as in the costless migration case, assume again that the personal taxes are attributed to the national budget. The problem may be written as:

$$\max_{\{t_i^j, g_i, N_i^j\}} \quad N_1^h \left[u(a^h F_1'(A_1) - t_1^h) + b(g_1) \right] + N_1^\ell \left[u(a^\ell F_1'(A_1) - t_1^\ell) + b(g_1) \right]$$

$$+ (N^h - N_1^h) \left[u(a^h F_2'(A_2) - t_2^h) + b(g_2) \right] + (N^\ell - N_1^\ell) \left[u(a^\ell F_2'(A_2) - t_2^\ell) + b(g_2) \right]$$

$$- \int_0^{O_1^h - N_1^h} k^h(x) dx - \int_0^{O_1^\ell - N_1^\ell} k^\ell(z) dz$$

subject to:

$$t_1^h N_1^h + t_1^\ell N_1^\ell + t_2^h(N^h - N_1^h) + t_2^\ell(N^\ell - N_1^\ell) - T_1 - T_2 = 0 \qquad (\lambda)$$

$$T_i + R_i(A_i) - (N_i^h + N_i^\ell)^\alpha g_i = 0 \qquad\qquad i = 1, 2 \qquad (\lambda_i)$$

The first-order conditions on (T_i) yield $\lambda_1 = \lambda_2 = \lambda$ and the first-order conditions on (t_i^j) yield $c_1^h = c_1^\ell = c_2^h = c_2^\ell$. Finally, the first-order conditions on N_1^j for $j = h, \ell$ reduce to:

$$b(g_1) - b(g_2) + k^j(M_1^j) + \lambda a^j(F_1'(A_1) - F_2'(A_2)) - \lambda\alpha(g_1 N_1^{\alpha-1} - g_2 N_2^{\alpha-1}) = 0 \quad (N_1^j)$$

The first three terms represent the utility differential between region 1 and region 2 for the last migrant (since $c_1^j = c_2^j$), which will be zero if the two migration equilibrium conditions are satisfied. Thus, the migration equilibrium conditions will be satisfied — and therefore not binding — if and only if $a^j(F_1'(A_1) - F_2'(A_2)) - \alpha(g_1 N_1^{\alpha-1} - g_2 N_2^{\alpha-1}) = 0$ for $j = h, \ell$. These will generally not be satisfied in the presence of migration costs.[21] The exception is the special case where migration costs are constant $(k^{j'} = 0)$ and the public good is pure $(\alpha = 0)$.[22]

The next step will be to characterize the allocation of consumption and public services in the unitary state optimum given that the migration equilibrium constraints are binding. We show that in an interior solution (when both types of persons are present in both regions), consumption will generally not be equalized either within or between regions. This is in contrast to the case of costless migration.

Unitary State Optimum

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[21] Subtracting (N_1^ℓ) from (N_1^h), we obtain: $k^h(O_1^h - N_1^h) - k^\ell(O_1^\ell - N_1^\ell) + \lambda(a^h - a^\ell)(F_1'(A_1) - F_2'(A_2)) = 0$. Therefore, if $k^h(O_1^h - N_1^h) > (<)k^\ell(O_1^\ell - N_1^\ell)$ then $F_1'(A_1) < (>)F_2'(A_2)$. (By chance, $k^h(O_1^h - O_1^\ell)$ could equal $k^\ell(O_1^\ell - N_1^\ell)$, but in general we can neglect that possibility.) From (g_1) and (g_2), there are only two possible outcomes, $g_1 > g_2$ and $N_1 > N_2$ or $g_1 < g_2$ and $N_1 < N_2$. In either case, the migration equilibrium constraints will not be satisfied.

[22] Subtracting (N_1^ℓ) from (N_1^h), we obtain: $(a^h - a^\ell)(F_1'(A_1) - F_2'(A_2)) = 0$ Therefore, since $a^h > a^\ell$, $F_1'(A_1) = F_2'(A_2)$, which implies from (N_1^h) and (N_1^ℓ) that $b(g_1) - b(g_2) + k = 0$ and the migration equilibrium conditions will be satisfied for both types of households. Note that this result depends on the fact that migration is going in the same direction for both types. If they move in opposite directions, migration equilibrium cannot be satisfied for both at the same time. Therefore, we must have either a corner solution, or one in which both types move in the same direction.

As shown above, the migration equilibrium conditions will generally be binding with costly migration. Therefore, the problem of our fictitious unitary state government is the same as the one given above except with the two migration equilibrium conditions as additional constraints. Assuming that migration goes from region 1 to region 2 for both types of worker, these conditions are:

$$u\left(a^j F_2'(A_2) - t_2^j\right) + b(g_2) - u\left(a^j F_1'(A_1) - t_1^j\right) - b(g_1) - k^j(O_1^j - N_1^j) = 0 \quad j = h, \ell \quad (\gamma^j)$$

Solving this problem, it is straightforward to show that, in general, the solution will be asymmetric — c_i^j differs between both household types and regions, and g_i, $F_i'(A_i)$ and N_i differ between regions. This is demonstrated in the Appendix.

Combining the first-order conditions on t_i^j, we obtain conditions governing inter-region and intra-region redistribution, the analogue to (2) for the homogeneous case (where the only redistribution was between persons in different regions):

$$\frac{1}{\lambda} = \frac{N_1^h}{N^h} \frac{1}{u'(c_1^h)} + \frac{N_2^h}{N^h} \frac{1}{u'(c_2^h)} = \frac{N_1^\ell}{N^\ell} \frac{1}{u'(c_1^\ell)} + \frac{N_2^\ell}{N^\ell} \frac{1}{u'(c_2^\ell)} \tag{8}$$

Then, substituting the conditions on t_i^j into the conditions for the g_i's, we obtain the standard public goods efficiency conditions given by (7). The optimal distribution of population is governed by two conditions, one each for the high- and low-ability persons:

$$\left(t_2^j - \frac{\alpha G_2}{N_2}\right) - \left(t_1^j - \frac{\alpha G_1}{N_1}\right) = \frac{\gamma^j}{\lambda} k^{j\prime}(M_1^j) \qquad j = h, \ell \tag{9}$$

where, as mentioned, $\gamma^j \gtrless 0$. These conditions have similar interpretations as before. Migration will occur until the difference in NFE is just equal to the social value of the increment in resources required to keep the migration equilibrium constraint in balance.

The unitary state government can implement this optimum by its choice of policy instruments $\{t_i^j, g_i, T_i\}$. From the regional budget constraints, the difference in per capita transfers at the optimum can be written as:

$$\frac{T_1}{N_1} - \frac{T_2}{N_2} = \frac{R_2(A_2)}{N_2} - \frac{R_1(A_1)}{N_1} + \frac{G_1}{N_1} - \frac{G_2}{N_2}$$

This accounting identity states that the equalization transfer compensates for differences in tax capacity, per capita rents, and expenditure needs. The equalization scheme combined

with the other fiscal policies must ensure that the optimal population allocation conditions (N_1^j) are satisfied. These conditions for the two types of persons imply that

$$t_1^\ell - t_2^\ell + \frac{\gamma^\ell}{\lambda} k^{\ell\prime}(M_1^\ell) = t_1^h - t_2^h + \frac{\gamma^h}{\lambda} k^{h\prime}(M_1^h)$$

Moreover, using the migration optimality conditions, an explicit expression for the equalization transfer to region 1 can be obtained as:

$$T_1 = \frac{N_1 N_2}{N_1 + N_2} \left[\left(\bar{t}_2 - t_2^j \right) - \left(\bar{t}_1 - t_1^j \right) + \left(\frac{(1-\alpha)G_1}{N_1} - \frac{(1-\alpha)G_2}{N_2} \right) \right.$$
$$\left. - \left(\frac{R_1(N_1)}{N_1} - \frac{R_2(N_2)}{N_2} \right) + \frac{\gamma_j}{\lambda} k^{j\prime}(M_1^j) \right] \qquad j = h, \ell \qquad (10)$$

where $\bar{t}_i$ is average per capita tax revenues raised in region i.

Decentralizing the Unitary State Optimum

The unitary state optimum requires that within each region both the optimal amount of redistribution between high- and low-ability persons and the optimal output of the regional public good be achieved. These requirements are challenging for designing the financial arrangements that must accompany decentralization. It is straightforward to show that, whatever conjecture the region adopts about the effect of its policies on migration, the regional governments must have the ability to redistribute between high and low-ability types within their jurisdictions. This is necessary in order that the efficiency conditions for public goods (7) be satisfied. We begin by establishing that. Then, we turn to the issue of how to ensure that the regions will use their redistributive instruments to achieve the optimal amount of redistribution within their respective jurisdictions. It turns out that a simple equalization transfer will not suffice. An incentive must be introduced to influence the way in which the regions use their redistributive responsibilities.

To see that the regional governments must be given redistributive responsibilities, we follow the same procedure as in the costless migration case and set up the regional government's problem with all four possible tax instruments – a surcharge on central taxes σ_i, a consumption tax θ_i, a payroll tax π_i, and differentiated lump-sum taxes on households τ_i^j. Allowing for all of these, the problem for the government of region 1 is:

$$\max_{\{\sigma_1, \pi_1, \theta_1, g_1, N_1\}} \quad N_1^h \left[u(c_1^h) + b(g_1) \right] + N_1^\ell \left[u(c_1^\ell) + b(g_1) \right]$$

26

subject to

$$T_1 + N_1^h(\sigma_1 t_1^h + \pi_1 a^h F_1'(A_1) + \theta_1 c_1^h + \tau_1^h)$$

$$+N_1^\ell(\sigma_1 t_1^\ell + \pi_1 a^\ell F_1'(A_1) + \theta_1 c_1^\ell + \tau_1^\ell) + R_1(A_1) - (N_1)^\alpha g_1 \geq 0 \qquad (\lambda_1)$$

$$u(c_2^j) + b(g_2) - k^j(O_1^j - N_1^j) - u(c_1^j) - b(g_1) = 0 \qquad\qquad j = h, \ell \qquad (\gamma_1^j)$$

where $c_1^j = (a^j F_1'(A_1)(1 - \pi_1) - (1 + \sigma_1)t_1^j - \tau_1^j)/(1 + \theta_1)$ for $j = h, \ell$.

First, suppose the regional government has access to all the taxes except the lump-sum tax on households. The first-order conditions with respect to fiscal variables are:

$$(N_1^h - \gamma_1^h + N_1^\ell - \gamma_1^\ell)b'(g_1) - \lambda N_1^\alpha = 0 \qquad\qquad (g_1)$$

$$-(N_1^h - \gamma_1^h)u'(c_1^h)t_1^h - (N_1^\ell - \gamma_1^\ell)u'(c_1^\ell)t_1^\ell + \lambda_1(t_1^h N_1^h + t_1^\ell N_1^\ell) = 0 \qquad (\sigma_1)$$

$$-(N_1^h - \gamma_1^h)u'(c_1^h)c_1^h - (N_1^\ell - \gamma_1^\ell)u'(c_1^\ell)c_1^\ell + \lambda_1(c_1^h N_1^h + c_1^\ell N_1^\ell) = 0 \qquad (\theta_1)$$

$$-(N_1^h - \gamma_1^h)u'(c_1^h)a^h F_1'(A_1) - (N_1^\ell - \gamma_1^\ell)u'(c_1^\ell)a^\ell F_1'(A_1) + \lambda_1(a^h F_1'(A_1)N_1^h + a^\ell F_1'(A_1)N_1^\ell) = 0$$

$$(\pi_1)$$

None of the above first-order conditions on the three tax instruments (σ_1), (θ_1), and (π_1) will ensure that condition (g_1) reduces to the efficiency condition for public goods provision (7) whether or not the regional government is myopic with respect to migration $(\gamma_1^j = 0$ or $\gamma_1^j \neq 0)$.

Suppose then that the regional governments are able to levy redistributive taxes $\{\tau_i^h, \tau_i^\ell\}$ on high- and low-skilled households. The first-order conditions are:

$$-(N_1^j - \gamma_1^j)u'(c_1^j) + \lambda_1 N_1^j = 0 \qquad\qquad j = h, \ell \qquad (\tau_1^j)$$

where the levels of consumption c_1^j reflect the fact that the central government may also levy redistributive taxes t_i^j. Similar conditions apply for region 2 although the problem of the government is slightly different since it must account for the migration costs of the persons who migrate into the region. Combining the conditions (τ_1^ℓ), (τ_1^h), and (g_1) we obtain the public goods efficiency conditions (7) confirming that in the heterogeneous case with costly migration, the regions *must* be given the authority to redistribute income.

There are a number of features of regional behaviour that are worth noting. First, the above results will apply regardless of the conjecture that regions make about migration

responses. Second, the VFI is indeterminate for the same reason as in earlier problems. A decrease in t_1^j accompanied by a compensating decrease in central transfers will not affect the solution to the regional problem. The regional government will fully offset the decline in central taxes by increasing their own rates. Third, as the above problem is stated, redistribution will be decided entirely by the regions. Any revenue-neutral attempt by the central government to change inter-personal redistribution by adjusting the relative tax rates t_1^h and t_1^ℓ will be completely offset by the regions. This is a consequence of the regions being second movers. In fact, this inability of the central government to influence the distribution of consumption within a region turns out to pose a difficulty in decentralizing the unitary state optimum, as we shall now see.

Consider the problems of the two regions. To be concrete, assume that each region behaves as a Nash competitor with respect to the policies of the other region as well as the central government.[23] As just discussed, central government taxes are redundant in this decentralized setting in the sense that their use does not add anything to the ability of the central government to control regional government behavior. Therefore, we can suppress them from the following analysis and assume that the only central government policy instruments are transfers to the regions T_1 and T_2. The problem of region 1 is as above and the first-order conditions determine the values of the choice variables $\{\tau_1^h, \tau_1^\ell, g_1, N_1^h, N_1^\ell\}$ as well as the Lagrangian multipliers $\{\lambda_1, \gamma_1^h, \gamma_1^\ell\}$ as functions of the central government policies and those of region 2 $\{T_1, \tau_2^h, \tau_2^\ell, g_2\}$. Likewise, we can solve for a similar problem for region 2 taking into account the migration costs borne by some of its final residents. The solution to this problem will be values for $\{\tau_2^h, \tau_2^\ell, g_2, N_2^h, N_2^\ell, \lambda_2, \gamma_2^h, \gamma_2^\ell\}$ as functions of $\{T_2, \tau_1^h, \tau_1^\ell, g_1\}$.

In a Nash equilibrium, the solutions to these two sets of equations must be simultaneously satisfied, and the values of N_1^j desired by both regions must be compatible. A Nash equilibrium will therefore be the set of regional policies $\{\hat{\tau}_i^h, \hat{\tau}_i^\ell, \hat{g}_i, \hat{N}_2^h, \hat{N}_2^\ell\}$ and associated Lagrangian multipliers $\{\hat{\lambda}_i, \hat{\gamma}_i^h, \hat{\gamma}_i^\ell\}$ for the two regions that solve the two sets of first-order

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[23] This implies that the regions do not take into account that budget constraints of other governments. An alternative approach would be to assume that one policy variable of the other government is determined by budget balance. The choice of endogenous variable affects the Nash equilibrium outcome (Wildasin 1988). Adopting this alternative modeling strategy will not affect our results.

conditions, given central policies $\{T_1, T_2\}$. Given these policies, the allocation of resources will be determined for the two regions: $\{\widehat{c}_i^h, \widehat{c}_i^\ell, \widehat{g}_i, \widehat{N}_2^h, \widehat{N}_2^\ell\}$.

It can be seen that the central government does not have enough instruments to ensure that the regions choose both the optimal allocation of resources between public goods and private goods and the distribution of consumption goods between the two ability-types. Both these need to be satisfied if labour is to be allocated optimally between the two regions. In the homogeneous labour case, the region's only discretion was with respect to the allocation of regional income between public and private sectors. This was assured by the fact that the regions abided by the efficiency rule for public goods provision. Then, inter-regional transfers were sufficient to ensure that regional incomes were such that the levels of consumption and public goods were optimal in the two regions. Given that, population allocation would be optimal. In this case, the regions will optimally choose the division of output between private goods and public goods, provided they are given the power to redistribute. However, it is not sufficient to use equalization transfers to achieve the correct distribution of incomes between the two regions. The regions will generally not choose the optimal distribution of private goods between the two type of households.

More formally, for consumption allocations to be optimal, the values $\widehat{c}_i^j$ that solve (τ_i^j) in the Nash equilibrium must correspond with those that solve the corresponding first-order conditions in the central government's problem, (t_i^j). That, in turn, requires that the relative values of the Lagrange multipliers in the central and regional problems be aligned in a particular way, and that cannot be achieved with the available central government instruments. Intuitively, the use of $\{T_1, T_2\}$ can change the relative values of the shadow value of public funds in the two regions, $\{\lambda_1, \lambda_2\}$, but it cannot be used to manipulate the relative values of $\{\gamma_i^h, \gamma_i^\ell\}$ within region i, which is what determines the relative values of $\{c_i^h, c_i^\ell\}$, that is, intra-regional redistribution. As can be seen, the two regional problems are substantially different from one another and from the problem of the unitary state, and the relative values of $\{\gamma_i^j\}$ can take arbitrarily different values depending on the structure of preferences and production functions in the economy.

In order to decentralize the unitary state optimum, the central government must have policy instruments that enable it to influence the incentives of the regions to redistribute, that is to affect the relative values of $\{\gamma_i^h, \gamma_i^\ell\}$ within each region. As we have noted,

redistributive personal taxes will not work because they will be undone by regional re-distribution policies. Instead, policies must be able to work independently on the two migration equilibrium constraints facing regional governments, since it is those that deter-mine the multipliers $\{\gamma_i^j\}$. Moreover, the policies must be differentiated by region.[24] Such a policy would be to refine the system of equalization transfers so that they are contingent on the numbers of each type of person in the region.

Let T_i^j be the transfer to region i per type-j person in the region. Then, region i's total transfer is $T_i^h N_i^h + T_i^\ell N_i^\ell$. The problem of, say, region 1 can then be written as:

$$\max_{\{\tau_1^j, g_1, N_1^j\}} N_1^h \left[u \left(a^h F_1'(A_1) - \tau_1^h \right) + b(g_1) \right] + N_1^\ell \left[u \left(a^\ell F_1'(A_1) - \tau_1^\ell \right) + b(g_1) \right]$$

subject to:

$$T_1^h N_1^h + T_1^\ell N_1^\ell + \tau_1^h N_1^h + \tau_1^\ell N_1^\ell + R_1(A_1) - (N_1^h + N_1^\ell)^\alpha g_1 = 0 \qquad (\lambda_1)$$

$$u \left(a^j F_2'(A_2) - \tau_2^j \right) + b(g_2) - u \left(a^j F_1'(A_1) - \tau_1^j \right) - b(g_1) - k^j (O_1^j - N_1^j) = 0 \quad j = h, \ell \qquad (\gamma_1^j)$$

The first-order conditions on (g_1) and (τ_1^j) are as before and the first-order conditions on N_1^j are given by:

$$u(c_1^j) + b(g_1) + \lambda_1 \left(T_1^j + \tau_1^j - \alpha g_1 N_1^{\alpha-1} \right) + \gamma_1^j \left(k^{j\prime} - u'(c_2^j) a^j F_2''(A_2) a^j \right)$$

$$-\gamma_1^j u'(c_1^j) a^j F_2''(A_2) a^j = 0 \qquad j = h, \ell \qquad (N_1^j)$$

Since T_1^h and T_1^ℓ enter separately in the above two first-order conditions, it is clear that they can be used to affect the tightness of the migration constraints and therefore the relative sizes of their multipliers, $\{\gamma_1^h, \gamma_1^\ell\}$. This provides the central government the needed degrees of freedom in each region to induce the regions to divide their aggregate consumption between c_i^h and c_i^ℓ optimally.

We can now summarize the results for the heterogeneous labour case. The unitary state optimum differs in this case depending on whether or not migration is costly. When there is costless migration and an interior solution for both types of migrants, then the

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[24] Thus, differential migration subsidies on the two types of persons would not suffice because they do not affect the migration constraints differentially in the two regions.

migration equilibrium constraints are not binding. As a result, per capita levels of consumption and the level of public services will be equalized across and within regions. As well, population will be the same in both regions, but the more productive one will have a higher proportion of high-ability persons. It turns out that the set of instruments that can be used to decentralize the unitary state optimum is strictly smaller than in the homogeneous labour case. Allowing regions to use either a surtax on central government taxes or a payroll tax will not result in the unitary state optimum whereas allowing them to use either a consumption tax or differential lump-sum taxes on households will. In the latter case, the central government through its choice of personal tax rates and equalization transfers or simply equalization transfers can ensure that the unitary state optimum is achieved.

These results are in sharp contrast to those obtained when migration is costly. In this case, the migration constraints will necessarily be binding in the unitary state optimum. As a result, per capita levels of consumptions will generally differ both across and within regions when there are both types of workers in each region. As well, the level of public services will generally be different in the two regions. To decentralize the unitary state optimum, regional governments must have the ability to redistribute between the different types of workers in their respective region. This will ensure that the standard public good efficiency conditions are satisfied. The central government must then intervene and introduce some incentive to ensure that the regional governments undertake the optimal amount of redistribution within their jurisdictions. In other words, a standard equalization system is not sufficient to decentralize the unitary state optimum. The equalization transfers must be made contingent on the numbers of each type of persons in the region in order to achieve the unitary state optimum.

5 Conclusion

The possible extensions to the general framework developed in this paper have by no means been exhausted. We conclude by discussing some of them. In the homogeneous labour case, several extensions could be considered some of which have already appeared in the literature. For example, we could assume that migration costs are prohibitive. This is the extreme case of no mobility. We could also have allowed for central-regional bargaining over inter-regional transfers as in Sato (1998). Allowing for tax exporting by having a source-based tax on rent owned both by residents and non-residents would be another interesting

extension. A national public good with voluntary contributions by the region as in Cornes and Silva (2000) or Ihori (2001) could also be introduced. A more novel extension would be to explore the implications of having idiosyncratic regional shocks. There are also several extensions which have received less attention in the literature. For example, it was assumed in the heterogeneous labour case that labour is perfectly substitutable in the production function. Perhaps a more reasonable assumption would be to treat high and low ability workers as two different inputs in the production process. It seems likely that the symmetric outcome in the case of costless migration would no longer be obtained. We could also have allowed for some aversion to inequality. Finally, involuntary unemployment resulting from either frictions in the labour market, efficiency wages, or union-bargaining could be incorporated into the model.

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Appendix

We show that, when migration costs are increasing for each ability-type and migration goes from region 1 to region 2, consumption will differ across households, and populations, marginal products and public services will differ across regions.

Suppose we eliminate the Lagrange multipliers from the first-order conditions for the unitary state optimum problem. The four equations (t_1^j) can be reduced to one by eliminating three variables $\gamma^h, \gamma^\ell, \lambda$ to give:

$$\frac{N_1^h}{N^h}\frac{1}{u'(c_1^h)} + \frac{N^h - N_1^h}{N^h}\frac{1}{u'(c_2^h)} = \frac{N_1^\ell}{N^\ell}\frac{1}{u'(c_1^\ell)} + \frac{N^\ell - N_1^\ell}{N^\ell}\frac{1}{u'(c_2^\ell)} \qquad (c)$$

where

$$\gamma^h = \frac{u'(c_1^h) - u'(c_2^h)}{u'(c_1^h)/N_1^h + u'(c_2^h)/N_2^h} \qquad \gamma^\ell = \frac{u'(c_1^\ell) - u'(c_2^\ell)}{u'(c_1^\ell)/N_1^\ell + u'(c_2^\ell)/N_2^\ell}$$

$$\frac{1}{\lambda} = \frac{N_1^h}{N^h}\frac{1}{u'(c_1^h)} + \frac{N^h - N_1^h}{N^h}\frac{1}{u'(c_2^h)} = \frac{N_1^\ell}{N^\ell}\frac{1}{u'(c_1^\ell)} + \frac{N^\ell - N_1^\ell}{N^\ell}\frac{1}{u'(c_2^\ell)}$$

Note that:

$$\frac{\gamma^h}{\lambda} = \frac{N_1^h N_2^h}{N^h}\frac{u'(c_1^h) - u'(c_2^h)}{u'(c_1^h)u'(c_2^h)} \qquad \frac{\gamma^\ell}{\lambda} = \frac{N_1^\ell N_2^\ell}{N^\ell}\frac{u'(c_1^\ell) - u'(c_2^\ell)}{u'(c_1^\ell)u'(c_2^\ell)}$$

We also have the public goods efficiency conditions which are given by (7). Finally, using the expressions for γ^h/λ and γ^ℓ/λ, (N_1^h) and (N_1^ℓ) become:

$$a^h(F_1'(A_1) - F_2'(A_2)) - (c_1^h - c_2^h) - \alpha\left(g_1 N_1^{\alpha-1} - g_2 N_2^{\alpha-1}\right)$$

$$+\frac{N_1^h N_2^h}{N^h}\frac{u'(c_1^h) - u'(c_2^h)}{u'(c_1^h)u'(c_2^h)}k^{h'}(O_1^h - N_1^h) = 0 \qquad (N_1^h)$$

$$a^\ell(F_1'(A_1) - F_2'(A_2)) - (c_1^\ell - c_2^\ell) - \alpha\left(g_1 N_1^{\alpha-1} - g_2 N_2^{\alpha-1}\right)$$

$$+\frac{N_1^\ell N_2^\ell}{N^\ell}\frac{u'(c_1^\ell) - u'(c_2^\ell)}{u'(c_1^\ell)u'(c_2^\ell)}k^{\ell'}(O_1^\ell - N_1^\ell) = 0 \qquad (N_1^\ell)$$

This leaves us with 8 equations — $(c), (7), (N_1^h), (N_1^\ell), (\lambda), (\gamma^h), (\gamma^\ell)$ — in 8 variables — $c_1^h, c_1^\ell, c_2^h, c_2^\ell, g_1, g_2, N_1^h, N_1^\ell$.

Now suppose $c_1^h = c_1^\ell = c_1$ and $c_2^h = c_2^\ell = c_2$. We show that this incompatible with the above conditions and constraints? From (c), we obtain

$$\left(\frac{N_1^h}{N^h} - \frac{N_1^\ell}{N^\ell}\right)\frac{1}{u'(c_1)} = \left(\frac{N_1^h}{N^h} - \frac{N_1^\ell}{N^\ell}\right)\frac{1}{u'(c_2)}$$

There are two options. Either $c_1 = c_2 = C$, or $N_1^h/N_1^\ell = N^h/N^\ell$. Consider these in turn.

Case 1: $c_1 = c_2 = c$

Then, (γ^h), (γ^ℓ), (N_1^h) and (N_1^ℓ) reduce to:

$$k^h(O_1^h - N_1^h) = k^\ell(O_1^\ell - N_1^\ell), \qquad F_1'(A_1) = F_2'(A_2)$$

These two equations determine N_1^h and N_1^ℓ, and therefore, N_1, N_2, A_1 and A_2. Given the allocation of labour thus determined, the following must be satisfied by choice of c, g_1 and g_2:

$$b(g_1) - b(g_2) = k^H(\cdot) = k^L(\cdot) \tag{γ}$$

$$N_1^{\alpha-1}g_1 = N_2^{\alpha-1}g_2 \tag{N}$$

$$N_1^{1-\alpha}b'(g_1) = N_2^{1-\alpha}b'(g_2) \tag{g}$$

$$F_1(A_1) + F_2(A_2) - NC - N_1^\alpha g_1 - N_2^\alpha g_2 = 0 \tag{λ}$$

Therefore, the system is underdetermined: we do not have enough variables left to solve these equations. This implies that consumption cannot generally be equalized within and across regions when migration is costly.

Case 2: $N_1^h/N_1^\ell = N^h/N^\ell$

Now, (γ^h) and (γ^ℓ) reduce to:

$$k^H(O_1^h - N_1^h) = k^L(O_1^\ell - N_1^\ell)$$

This combined with $N_1^h/N_1^\ell = N^h/N^\ell$ determines N_1^h and N_1^ℓ, and therefore A_1 and A_2. We are left with (g_1), (g_2), (N_1^h), (N_1^ℓ), (λ) and:

$$u(c_2) + b(g_2) - u(c_1) - b(g_1) = k^H(\cdot) = k^L(\cdot) \tag{γ}$$

We only have four variables left to determine: c_1, c_2, g_1, g_2. The system is again underdetermined.

The upshot is that consumption cannot generally be equalized within regions when migration costs are increasing.

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