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THE EFFECT OF RESTRICTING ASSET TRADE IN DYNAMIC EQUILIBRIUM MODELS

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Abstract

This paper shows that differences between the predictions of an international real business cycle model with complete markets and the predictions of a model where agents can only trade risk-free bonds depend heavily on the calibrated values for the degree of persistence in productivity shocks, the discount factor and the degree of international spillovers in productivity shocks. Since empirical work yields point estimates of the degrees of persistence and spillovers in productivity shocks that bear large standard errors, the outcomes of quantitative studies using only the point estimates of these parameters inherit the substantial uncertainty associated with the empirical estimates.

1. Introduction

The first attempt to model international real business cycles *(IRBC)* by Backus, Kehoe and Kydland (1992) pointed out a number of discrepancies between the data and the predictions of their model (see Backus *et al.*, 1995 and Ravn, 1997). One of these discrepancies comes from the fact that the cross-country correlation of output is higher than the cross-country correlation of consumption in the data, while the opposite is observed in *IRBC* models (the cross-country consumption correlation puzzle). In response to this shortcoming of the model some researchers have turned their attention toward *IRBC* models with incomplete asset markets to try solving the cross-country consumption correlation puzzle. Baxter and Crucini (1995) and Kollmann (1996) were among the first to consider incomplete asset markets in an *IRBC* framework.[1] Research on the difference between *IRBC* models with complete markets and incomplete markets is still ongoing. For instance, Kim, Kim and Levin (2000) compare complete and incomplete markets models using endowment economies, which enables them to derive analytical results. More generally, this paper is part of the growing literature that uses stochastic dynamic general equilibrium models to understand international business cycles and other issues in international finance .

This paper analyses two *IRBC* models similar to those of Kollmann (1996), Baxter and Crucini (1995) and Backus *et al.* (1995). In one model agents can trade a complete set of state-contingent assets (complete markets economy) and in the other agents can only trade one-period risk-free bonds (bond economy). The analysis focus on the differences of the two models for a large array of parameter values. The sensitivity analysis shows that the differences between the predictions of the complete markets economy and the bond economy depend primarily on the calibrated value for the degree of persistence in productivity shocks. The differences are also found to depend importantly on the calibrated values for the discount factor and the degree of international spillovers in productivity shocks. Previous empirical work found that estimates of the degrees of persistence and spillovers bear

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[1]See also Arvanitis and Mikkola (1996) and van Wincoop (1996).

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large standard errors. This uncertainty around the point estimates of the degrees of persistence and spillovers in productivity shocks implies that the outcomes of quantitative studies comparing complete market economies and bond economies using only the point estimates of these parameters inherit the substantial uncertainty associated with the point estimates. Since the models presented in this paper are often used as the backbones of more elaborate models with various types of shocks (*e.g.* Chari, Kehoe and McGrattan (2001) and Kollmann (2001)), the great importance of the calibration of the shocks process should not be forgotten. The sensitivity analysis performed in the paper also helps in understanding the different conclusions reached by Baxter and Crucini (1995) and Kollmann (1996) regarding the effect of restricting asset trade on the predictions of *IRBC* models with stationary productivity shocks.

2. Model and Calibration

The structure of the basic *IRBC* model is well known. The world economy is composed of two *ex ante* identical countries, denoted by $i = 1, 2$. There is a single homogeneous good. Each country is represented by a consumer who seeks to maximize

$$Eu_i = E_0 \sum_{t=0}^{\infty} \beta^t \frac{[c_{it}^{\mu}(1 - n_{it})^{1-\mu}]^{1-\sigma}}{1 - \sigma} \qquad 0 < \sigma, \quad 0 < \beta, \mu < 1 \qquad (1)$$

where c_{it} and n_{it} denote consumption and hours worked in country i and β is the agent's discount factor. Output in country i is given by the constant returns to scale production function

$$y_{it} = z_{it} k_{it}^{\theta} n_{it}^{1-\theta} \qquad 0 < \theta < 1 \qquad (2)$$

where z_{it} represents the stochastic level of productivity in country i and k_{it} the capital stock installed in country i. The law of motion for capital is,

$$k_{it+1} = x_{it} + (1 - \delta)k_{it} - \frac{\tau}{2}\left(\frac{x_{it}}{k_{it}} - \delta\right)^2 k_{it}, \qquad 0 < \delta < 1, \quad 0 \leq \tau \qquad (3)$$

where x_{it} denotes investment made by country i. This law of motion includes capital adjustment costs governed by τ and is such that there are no adjustment costs in steady state.

Productivity evolves according to the bivariate autoregressive process

$$\begin{bmatrix} \log z_{1t+1} \\ \log z_{2t+1} \end{bmatrix} = \begin{bmatrix} \rho_p & \rho_s \\ \rho_s & \rho_p \end{bmatrix} \begin{bmatrix} \log z_{1t} \\ \log z_{2t} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t+1} \\ \epsilon_{2t+1} \end{bmatrix} \tag{4}$$

where ρ_p measures the persistence in productivity shocks and ρ_s measures the degree of international spillovers. The variance in the innovations is denoted by σ_ϵ^2 and the covariance between ϵ_1 and ϵ_2 by $\sigma_{1,2}$.

The difference between the complete markets economy and the bond economy resides in the number of assets available to the representative agents. When markets are complete, then representative agents in both countries can trade a full set of contingent claims. Accordingly, agent i's budget constraint is

$$\sum_{\omega_t} P(\omega_{t+1}, \omega_t) b_i(\omega_{t+1}) + c_{it} + x_{it} = y_{it} + b_i(\omega_t) \tag{5}$$

where ω_t indicates the state in period t, $b_i(\omega_{t+1})$ denotes the quantity of contingent claims purchased in period t and paying off one unit of consumption the following period, conditional of the state of the world being ω_{t+1} next period. $P(\omega_{t+1}, \omega_t)$ denotes the price of these contingent assets.

In the bond economy, the representative agents in both countries can only trade one period risk-free bonds. Agent i's budget constraint is then

$$P_t^B b_{it+1} + c_{it} + x_{it} = y_{it} + b_{it} \tag{6}$$

where b_{it+1} denotes the quantity of discount bonds purchased in period t (each paying one unit of consumption in period $t+1$), and P_t^B denotes the price of these bonds.

Market clearing on the goods market requires that

$$c_{1t} + x_{1t} + c_{2t} + x_{2t} = y_{1t} + y_{2t}, \tag{7}$$

while the market clearing condition on the asset markets require

$$b_1(\omega_{t+1}) + b_2(\omega_{t+1}) = 0 \quad \forall \; \omega_{t+1} \qquad \text{or} \qquad b_{1t} + b_{2t} = 0 \tag{8}$$

depending on the market structure.

The benchmark calibration is mostly borrowed from Kollmann (1996): $\beta = 0.9828$, $\sigma = 2.0$, $\theta = 0.36$, $\delta = 0.021$, $\rho_p = 0.95$, $\rho_s = 0$, $\sigma_\epsilon = 0.007$ and $\sigma_{1,2} = 0.2$. The remaining two parameters are set endogenously. μ is set to that steady-state hours worked equal 0.25 and τ is set so that the standard deviation of investment is 3.3 times larger than the standard deviation of output (as in Kollmann). As is common in the business cycles literature, the model is solved using the technique explained in King, Plosser and Rebelo (1988). More details are provided in appendices 1 and 2.[2]

3. Quantitative Analysis

To identify the impact of each parameter on the size of the effect of restricting the number of assets traded, the moments predicted by the complete markets economy are compared to those predicted by the bond economy. To prevent a proliferation of tables the results are presented graphically. Each figure plots the difference between four moments from the complete markets economy and the same four moments from the bond economy, as a function of a given parameter. The moments considered are the cross-country correlations of output, consumption, hours and investment. Looking at more moments (for example, all moments presented in Table 1) would simply yield to a proliferation of graphs, without changing the inference. In each graph, the value of one parameter is changed and all other parameters are set to the values presented above.[3] The ranges for the parameters in the figures are chosen to cover empirically relevant values.

For instance, Figure 1 shows how the differences between the cross-country correlations

[2]Note that the linearized dynamic system derived from the bond economy has a unit root. Using an endogenous discount factor rather than a constant discount factor eliminates this unit root. However, Kim and Kose (2001) find that the dynamic properties of a small-open economy model with a constant discount factor are quite similar to those of a model with an endogenous discount factor.

[3]Note that for both models and all calibrations, τ and μ are set so that hours worked equal 0.25 in steady state and investment is 3.3 times more volatile than output.

in the complete markets economy and the cross-country correlations in the bond economy vary with the discount factor (β). If the difference between the moments from the complete markets economy and the bond economy did not depend on β, then the lines in Figure 1 would be horizontal. Figure 1 shows that the effects of imposing restrictions on the number of assets traded clearly depend on the discount factor. When β is close to one, both economies behave almost the same way. However, when $\beta = 0.96$ the difference in the cross-country correlations are sizeable (-0.42 for hours, 0.49 for consumption, -0.24 for output and -0.35 for investment).

A similar picture arises in Figure 2 where the difference in moments from both economies are plotted against the level of spillovers in productivity shocks (ρ_s). When spillovers are large the differences in moments are small whereas they are larger when there are no spillovers.[4] The largest impacts are on hours and consumption. The difference in the cross-country correlations of hours varies from -0.30 to -0.07. For consumption, the difference in the cross-country correlations ranges from 0.14 to 0.35.

Figure 3 clearly shows that the effects of imposing restrictions on the number of assets traded depend very importantly on the degree of persistence in productivity shocks (ρ_p). Whereas the difference in moments are relatively small for $\rho_p = 0.90$, they are very large as ρ_p approaches one. For instance, for $\rho_p = 0.995$, the difference in the cross-country correlations is -1.60 for hours, 0.93 for consumption, -0.55 for output and -0.31 for investment).

The sensitivity exhibited in Figures 2 and 3 is a cause for concern given that empirical estimates of ρ_p and ρ_s bear large standard errors. For instance, using the point estimates and associated standard errors reported in Backus *et al.*, the ninety-five percent confidence interval for the degree of persistence in productivity shocks in the US is $\rho_p \in [0.76, 1.05]$ and is $\rho_p \in [0.84, 0.98]$ for Europe. Confidence intervals for the spillovers are $\rho_s \in [-0.03, 0.13]$

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[4]Note that one of the conditions for the stochastic process in equation (4) to be stable is $\rho_p + \rho_s < 1$. Therefore, we must use values of ρ_s strictly less than 0.05.

(spillover from Europe to US) and $\rho_s \in [0.02, 0.27]$ (spillovers from US to Europe). Large standard errors around point estimates also appear in the work of Glick and Rogoff (1995) who report estimates of the degree of persistence in country specific productivity shocks for (Canada, France, Germany, Italy, Japan, the UK and the US). All the estimated autoregressive coefficients are above 0.89 and bear standard errors ranging from 0.02 to 0.082.

As explained by Baxter and Crucini (1995), the main factor driving a wedge between the predictions of a complete markets economy and a bond economy is the difference in wealth effects. In a bond economy, the extend of risk sharing is obviously smaller than in an economy with complete markets. This may imply different wealth effects in the two economies. The differences in wealth effects depend importantly on the discount factor and the degrees of persistence and international spillovers in productivity shocks. When agents are patient ($\beta = 0.984$), shocks are somewhat persistent ($\rho_p = 0.906$) and spillovers are relatively large ($\rho_s = 0.088$), Baxter and Crucini showed that wealth effects are almost identical in both economies. The intuition is that a one-period risk-free bond is a very good instrument to share risk when shocks are not close to permanent and rapidly spillover to the other country. A highly patient country experiencing a positive productivity shock is happy to lend to the other country for one period, the time needed for the shock to spillover to the other country. In such a case, the wealth effects in the bond economy are similar to those in the complete markets economy. When shocks are highly persistent and do not spillover, then the country experiencing a positive productivity shock in the bond economy enjoys a very large wealth effect while the other country experiences no wealth effect at all. The difference in wealth effects across countries is much smaller when markets are complete which explains the large differences in the predictions of the two economies when productivity shocks are highly persistent and do not spillover.

Figures 5 and 6 show that differences in cross-country correlations from both economies are not really sensitive to changes in the coefficient of risk aversion σ and the correlation in productivity innovations $\sigma_{1,2}$. Figure 4 shows that differences in cross-country correlations

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are modestly sensitive to the capital share θ. The largest impacts of changes in θ are on cross-country correlations of consumption and hours. The difference in the cross-country correlations of hours varies from -0.34 to -0.21. For consumption, the difference in the cross-country correlations ranges from 0.27 to 0.40.

In light of the results presented in Figures 1, 2 and 3, the different conclusions reached by Kollmann (1996) and Baxter and Crucini (1995) regarding the effects of restricting the number of assets traded (when technology shocks are stationary) is not unexpected. Kollmann, using larger β and ρ_p and a smaller ρ_s than Baxter and Crucini, finds significant effects while Baxter and Crucini do not. Table 1 shows some predicted moments from both economies under two different calibration. The *Benchmark* calibration where $(\beta, \rho_p, \rho_s) = (0.9828, 0.95, 0)$ as in Kollmann and the *Alternative* calibration where $(\beta, \rho_p, \rho_s) = (0.984, 0.906, 0.088)$ as in Baxter and Crucini. All other parameters are set to the benchmark values given in section 2. The moments in Table 1 reinforce the results of Figures 1, 2 and 3 regarding the importance of the calibrated values for β, ρ_p and ρ_s. For the Alternative calibration the differences in the moments from the two economies are trivial. The largest effect is on the cross-country correlation of consumption. This correlation is 0.90 in the complete markets economy and 0.86 in the bond economy. The effects of imposing restrictions on the number of assets available are much larger for the Benchmark calibration. Differences in cross-country correlations are especially large. For example, the cross-country correlation of hours worked (consumption) is -0.60 (0.88) in the complete markets economy and -0.30 (0.52) in the bond economy. The results presented in Table 1 further demonstrate how the difference between a complete markets economy and a bond economy depends on the degrees of persistence and spillovers in productivity shocks as well as on the discount factor. The size of the effects of imposing restrictions on the number of assets available documented in Table 1 are fully consistent with the significant effects found by Kollmann and the trivial effects found by Baxter and Crucini (when working with stationary productivity shocks). This strongly suggests that the different conclusions reached in these two papers depend importantly on the calibration employed in the quantitative analyses. The moments calculated with the

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Benchmark calibration suggest that incomplete markets take the *IRBC* closer to the data. However, when drawing such a conclusion, one must keep in mind that the difference between the predictions of a complete markets economy and a bond economy is very highly sensitve to small changes in the degree of persistence in the productivity shocks (Figure 3). The difference is also sensitive to small changes in the degree of international spillovers in productivity shocks (Figure 2) and in the discount factor (Figure 1).

4. Concluding Remarks

When comparing the predictions of an artificial economy where a complete set of financial assets are traded to an artificial economy where only risk-free bonds are traded, one must keep in mind that the outcome of the comparison is heavily influenced by the values selected for the discount factor as well as the degrees of persistence and international spillovers in productivity shocks. Since empirical estimates of the latter two parameters are far from precise, it should be recognized that the outcome of the comparison bears a very great deal of uncertainty.

The finding that the effect of restricting asset trade depends on the persistence in the shocks to income was also demonstrated in the asset-pricing literature. For instance, Telmer (1993) who specifies a labour income process with little persistence does not find much effect from asset markets restrictions whereas Constantinides and Duffie (1996) show that in a model where the shocks to income are random walks, an economy with incomplete markets is different from one with complete markets.

Appendix 1: Solving the Incomplete Markets Model

Agent in country i chooses sequences $\{c_{it}, n_{it}, k_{it+1}, b_{it}, x_{it}\}$ to solve the problem

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{[c_{it}^{\mu}(1 - n_{it})^{1-\mu}]^{1-\sigma}}{1 - \sigma}$$

subject to

$$c_{it} + x_{it} + P_t^B b_{it+1} = z_{it} k_{it}^{\theta} n_{it}^{1-\theta} + b_{it}$$

$$k_{it+1} = (1 - \delta)k_{it} + x_{it} - \frac{\tau}{2}\left(\frac{x_{it}}{k_{it}} - \delta\right)^2 k_{it}$$

Let λ_{it} and ν_{it} be the Lagrange multipliers attached to the budget constraint and transition equation for capital, respectively. The maximization problem yields the first-order conditions

$$(c_{it}): \qquad \mu\frac{[c_{it}^{\mu}(1 - n_{it})^{1-\mu}]^{1-\sigma}}{c_{it}} = \lambda_{it} \qquad (A1)$$

$$(n_{it}): \qquad (1 - \mu)\frac{[c_{it}^{\mu}(1 - n_{it})^{1-\mu}]^{1-\sigma}}{1 - n_{it}} = \lambda_{it}(1 - \theta)\frac{y_{it}}{n_{it}} \qquad (A2)$$

$$(x_{it}): \qquad \lambda_{it} = \nu_{it}\left[\tau\left(\frac{x_{it}}{k_{it}} - \delta\right) - 1\right] \qquad (A3)$$

$$(b_{it+1}): \qquad \lambda_{it}P_t^B = \beta E_t \lambda_{it+1} \qquad (A4)$$

$$(k_{it+1}): \nu_{it} + \beta E_t\left\{\lambda_{it+1}\theta\frac{y_{it+1}}{k_{it+1}} + \nu_{it+1}\left[\frac{\tau}{2}\left(\frac{x_{it+1}}{k_{it+1}} - \delta\right)^2 - (1 - \delta) - \tau\left(\frac{x_{it+1}}{k_{it+1}} - \delta\right)\frac{x_{it+1}}{k_{it+1}}\right]\right\} = 0 \qquad (A5)$$

$$(\lambda_{it}): \qquad c_{it} + x_{it} + P_t^B b_{it+1} = y_{it} + b_{it} \qquad (A6)$$

$$(\nu_{it}): \qquad k_{it+1} = (1 - \delta)k_{it} + x_{it} - \frac{\tau}{2}\left(\frac{x_{it}}{k_{it}} - \delta\right)^2 k_{it} \qquad (A7)$$

Therefore, the equilibrium system is composed of 18 equations: (A1) to (A7) for $i = 1, 2$,

both countries production functions and the market clearing conditions

$$b_{1t} + b_{2t} = 0. \qquad (A8)$$

$$c_{1t} + c_{2t} + x_{1t} + x_{2t} = y_{1t} + y_{2t}. \qquad (A9)$$

Note however that there are only 17 independent equations. Summing both budget constraints in equation $(A6)$ and imposing the market clearing condition $(A8)$ yields equation $(A9)$. I follow Baxter and Crucini (1995) and remove one of these four equations (country 1's budget constraint) from the system. Then, the system is simplified by using $(A8)$ to substitute out b_{1t} and $(A4)$ to substitute out P_t^B. Therefore, we are left with an equilibrium system in the endogenous variables $(c_1, c_2, n_1, n_2, x_1, x_2, k_1, k_2, b_2, \lambda_1, \lambda_2, \nu_1, \nu_2)$ composed of equations $(A1)$, $(A2)$, $(A3)$, $(A5)$ and $(A7)$ for both countries, equation $(A9)$, equation $(A6)$ for country 2 and

$$\frac{E_t\lambda_{1t+1}}{\lambda_{1t}} = \frac{E_t\lambda_{2t+1}}{\lambda_{2t}}. \qquad (A10)$$

The system can now be linearized by taking a first-order Taylor series approximation around its no-trade steady state (zeor asset holdings). After substituting out $(c_1, c_2, n_1, n_2, x_1, x_2, \lambda_1)$ using the linearized version of $(A1)$, $(A2)$, $(A3)$ and $(A9)$ we obtain the fundamental dynamic system

$$E_t \begin{bmatrix} \hat{k}_{1t+1} \\ \hat{k}_{2t+1} \\ \hat{b}_{2t+1} \\ \hat{\lambda}_{2t+1} \\ \hat{\nu}_{1t+1} \\ \hat{\nu}_{2t+1} \end{bmatrix} = W \begin{bmatrix} \hat{k}_{1t} \\ \hat{k}_{2t} \\ \hat{b}_{2t} \\ \hat{\lambda}_{2t} \\ \hat{\nu}_{1t} \\ \hat{\nu}_{2t} \end{bmatrix} + Q \begin{bmatrix} \hat{z}_{1t} \\ \hat{z}_{2t} \end{bmatrix} + R\, E_t \begin{bmatrix} \hat{z}_{1t+1} \\ \hat{z}_{2t+1} \end{bmatrix}$$

where hatted variables denote percent deviations from steady state. That is, if we let $\bar{z}_i$ be the steady-state value for z_i, then $\hat{z}_{it} = (z_{it} - \bar{z}_i)/\bar{z}_i$. Since asset holdings are assumed to be zero in steady state we define $\hat{b}_{2t} = b_{2t}/\bar{y}_2$. Matrix W is 6×6 and matrices Q and R are 6×2. $\hat{k}_{1t}$, $\hat{k}_{2t}$ and $\hat{b}_{2t}$ are predetermined at time t (state variables) while $\hat{\lambda}_{2t}$, $\hat{\nu}_{1t}$ and $\hat{\nu}_{2t}$ are not (co-state variables). Matrix W governs the system dynamics. For the system

to have a unique solution, W must have as many roots outside the unit circle as there are co-state variables. For the system to be stable, W must have as many roots on or outside the unit circle as there are co-state variables. Therefore we need W to have 3 eigenvalues greater than one (in absolute value) for uniqueness and 3 eigenvalues greater or equal to one (in absolute value) for stability. The roots are 0.9444, 0.9666, 1, 1.0175, 1.0526 and 1.0774. Therefore the system is unstable and has a unique solution given by Blanchard and Kahn (1980).

Appendix 2: Solving the Complete Markets Model

When financial markets are complete, the competitive equilibrium is Pareto optimal. Therefore, we can conveniently derive the equilibrium system using an equal weight planner problem. The planner maximizes the sum of expected lifetime utilities subject to the constraints

$$c_{1t} + x_{1t} + c_{2t} + x_{2t} = z_{1t} k_{1t}^\theta n_{1t}^{1-\theta} + z_{2t} k_{2t}^\theta n_{2t}^{1-\theta}$$

$$k_{it+1} = (1 - \delta)k_{it} + x_{it} - \frac{\tau}{2}\left(\frac{x_{it}}{k_{it}} - \delta\right)^2 k_{it} \qquad i = 1, 2.$$

Let λ_t, ν_{1t} and ν_{2t} be the Lagrange multipliers attached to the resource constraint and the transition equations for countries 1 and 2 respectively. The planner's maximization problem yields the first-order conditions

$$(c_{it}): \qquad \mu \frac{[c_{it}^\mu (1 - n_{it})^{1-\mu}]^{1-\sigma}}{c_{it}} = \lambda_t \qquad\qquad (A11)$$

$$(n_{it}): \qquad (1 - \mu) \frac{[c_{it}^\mu (1 - n_{it})^{1-\mu}]^{1-\sigma}}{1 - n_{it}} = \lambda_t (1 - \theta) \frac{y_{it}}{n_{it}} \qquad\qquad (A12)$$

$$(x_{it}): \qquad \lambda_t = \nu_{it} \left[\tau \left(\frac{x_{it}}{k_{it}} - \delta \right) - 1 \right] \qquad\qquad (A13)$$

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$$(k_{it+1}): \nu_{it}+\beta E_t\left\{\lambda_{t+1}\theta\frac{y_{it+1}}{k_{it+1}}+\nu_{it+1}\left[\frac{\tau}{2}\left(\frac{x_{it+1}}{k_{it+1}}-\delta\right)^2-(1-\delta)-\tau\left(\frac{x_{it+1}}{k_{it+1}}-\delta\right)\frac{x_{it+1}}{k_{it+1}}\right]\right\}=0 \tag{A14}$$

$$(\lambda_t): \qquad c_{1t}+x_{1t}+c_{2t}+x_{2t}=y_{1t}+y_{2t} \tag{A15}$$

$$(\nu_{it}): \qquad k_{it+1}=(1-\delta)k_{it}+x_{it}-\frac{\tau}{2}\left(\frac{x_{it}}{k_{it}}-\delta\right)^2 k_{it} \tag{A16}$$

Substituting out y_1 and y_2 from the above equations using the production functions, we get an equilibrium system composed of equations $(A11)$ to $(A16)$ in the endogenous variables $(c_1, c_2, n_1, n_2, x_1, x_2, k_1, k_2, \lambda, \nu_1, \nu_2)$. The system can now be linearized by taking a first-order Taylor series approximation around its steady state. After substituting out $(c_1, c_2, n_1, n_2, x_1, x_2, \lambda)$ using the linearized version of $(A11)$, $(A12)$, $(A13)$ and $(A15)$ we obtain the fundamental dynamic system

$$E_t\begin{bmatrix}\hat{k}_{1t+1}\\ \hat{k}_{2t+1}\\ \hat{\nu}_{1t+1}\\ \hat{\nu}_{2t+1}\end{bmatrix}=W\begin{bmatrix}\hat{k}_{1t}\\ \hat{k}_{2t}\\ \hat{\nu}_{1t}\\ \hat{\nu}_{2t}\end{bmatrix}+Q\begin{bmatrix}\hat{z}_{1t}\\ \hat{z}_{2t}\end{bmatrix}+R\,E_t\begin{bmatrix}\hat{z}_{1t+1}\\ \hat{z}_{2t+1}\end{bmatrix}$$

where hatted variables denote percent deviations from steady state. Matrix W is 4×4 and matrices Q and R are 4×2. $\hat{k}_{1t}$, $\hat{k}_{2t}$ are predetermined at time t (state variables) while $\hat{\nu}_{1t}$ and $\hat{\nu}_{2t}$ are not (co-state variables). Matrix W governs the system dynamics. For the system to have a unique solution, W must have as many roots outside the unit circle as there are co-state variables. For the system to be stable, W must have as many roots on or outside the unit circle as there are co-state variables. Therefore we need W to have 2 eigenvalues greater than one (in absolute value) for uniqueness and 2 eigenvalues greater or equal to one (in absolute value) for stability. The roots are 0.9492, 0.9675, 1.0516, and 1.0720. Therefore the system is stable and has a unique solution given by Blanchard and Kahn (1980).

References

Arvanitis, A.V. and A. Mikkola (1996) "Asset-market structure and international trade dynamics," *American Economic Review Papers and Proceedings* 86, 67–70.

Backus, D., Kehoe, P. and F. Kydland (1992) "International real business cycles," *Journal of Political Economy* 100, 745–775.

Backus, D., Kehoe, P. F. Kydland (1995) "International business cycles: theory and evidence," in: Cooley, T., (Ed.), Frontiers of Business Cycle Research, Princeton: Princeton University Press.

Baxter, M. and M. Crucini (1995) "Business cycles and the asset structure of foreign trade," *International Economic Review* 36, 821–854.

Chari, V.V., Kehoe, P. and H. McGrattan (2001) "Can sticky price models generate volatile and persistent real exchange rates?," Staff Report Number 277, Federal Reserve Bank of Minneapolis.

Constantinides, G.M. and D. Duffie (1996) "Asset pricing with heterogeneous consumers," *Journal of Political Economy* 104, 219–240.

Glick, R. and K. Rogoff (1995) "Global versus country-specific productivity shocks and the current account," *Journal of Monetary Economics* 35, 159–192.

Kim, J., Kim, S. and A. Levin (2000) "Patience, persistence and properties of two-country incomplete market models," Manuscript, University of Virginia, Brandeis University, Federal Reserve Board.

Kim, S. and A. Kose (2001) "Dynamics of open economy business cycle models: Understanding the role of the discount factor," Forthcoming in *Macroeconomic Dynamics*.

King, R., Plosser, C., and S. Rebelo (1988) "Production, growth and business cycles, I. The basic neoclassical model," *Journal of Monetary Economics* 21, 196–232.

Kollmann, R (1996) "Incomplete asset markets and the cross-country consumption correlation puzzle," *Journal of Economic Dynamics and Control* 20, 945–961.

Kollmann, R (2001) "Explaining international comovements of output and asset returns: The role of money and nominal rigidities," *Journal of Economic Dynamics and Control* 25, 1547–1583.

Ravn, M.O. (1997) "International business cycles in theory and in practice," *Journal of International Money and Finance* 16, 255–283.

Telmer, C.I. (1993) "Asset-pricing puzzles and incomplete markets," *The Journal of Finance* 48, 1803–1832.

van Wincoop, E. (1996) "A multi-country real business cycles model with heterogenous agents" *Scandinavian Journal of Economics* 98, 233–251.

Table 1
Moments of variables in complete and incomplete asset markets

	Alternative		Benchmark		Data
	CME	BE	CME	BE	
Standard deviations relative to output					
Hours	0.53	0.51	0.55	0.46	0.74
Consumption	0.41	0.42	0.36	0.42	0.50
Net exports over output	0.25	0.26	0.22	0.15	0.10
Cross-correlation with output					
Hours	0.94	0.94	0.96	0.98	0.59
Consumption	0.81	0.84	0.82	0.96	0.73
Investment	0.92	0.92	0.97	0.97	0.80
Net exports over output	0.00	-0.03	0.58	0.30	-0.23
Cross-country correlation					
Hours	-0.70	-0.68	-0.60	-0.30	0.40
Consumption	0.90	0.86	0.88	0.52	0.34
Investment	-0.11	-0.09	-0.27	-0.12	0.45
Output	-0.58	-0.57	-0.10	0.03	0.46

The moments are averages over 1,000 simulations. Each simulation is 150 periods long and the first 50 periods are dropped to remove the effect of initial conditions. Moments are computed using HP filtered percent deviations from steady state. CME refers to the moments from the complete markets economy while BE refers to the moments from the bond economy. The "Benchmark" and "Alternative" calibrations are explained on page 7. The "Data" column reports empirical statistics from the G-7 countries. The statistics reported are borrowed from Table 1 in Kollmann (1996).

Figure 1: Differences between Cross−Country Correlations in
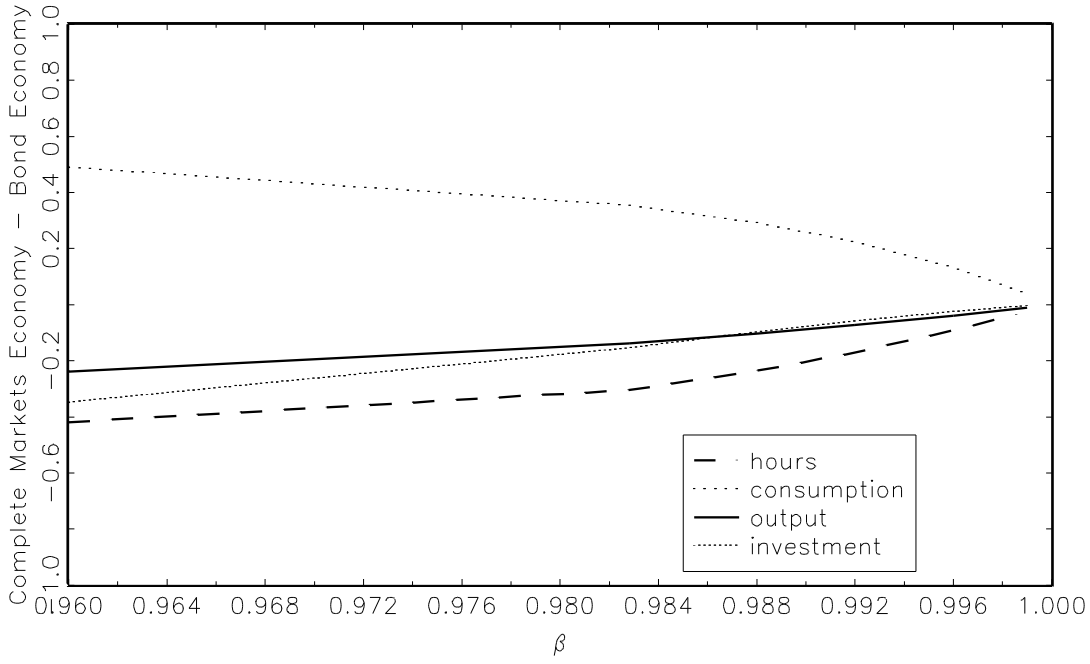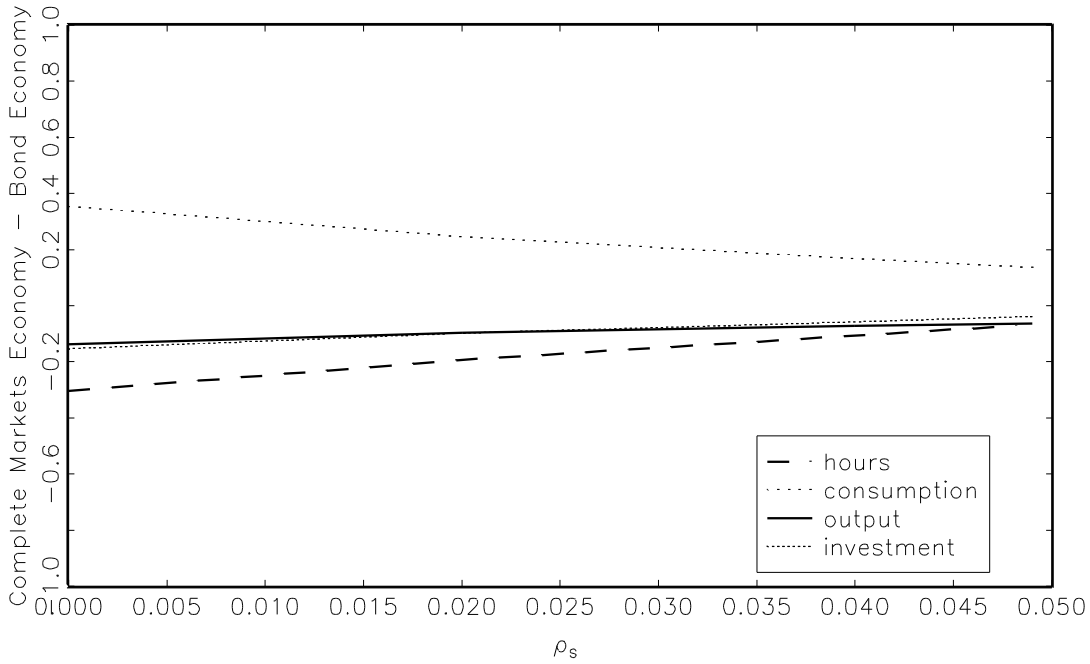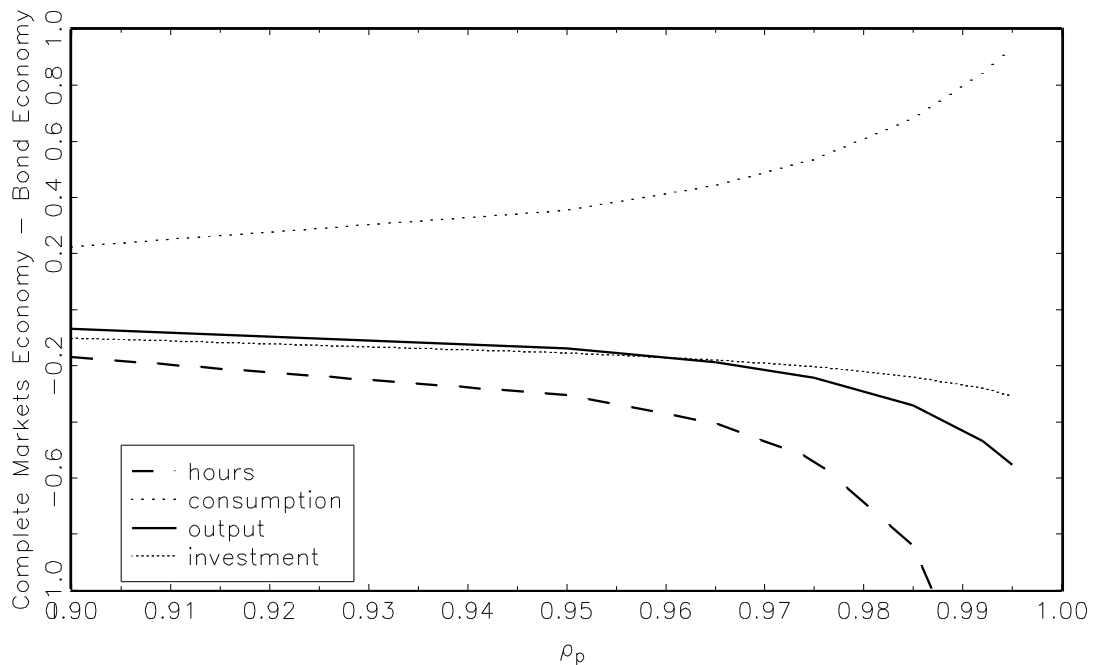Complete Markets Economy and Bond Economy

Figure 2: Differences between Cross−Country Correlations in
Complete Markets Economy and Bond Economy

The moments are averages over 1,000 simulations. Each simulation is 150 periods long and the first 50 periods are dropped to remove the effect of initial conditions. Moments are computed using HP filtered percent deviations from steady state.

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Figure 3: Differences between Cross–Country Correlations in
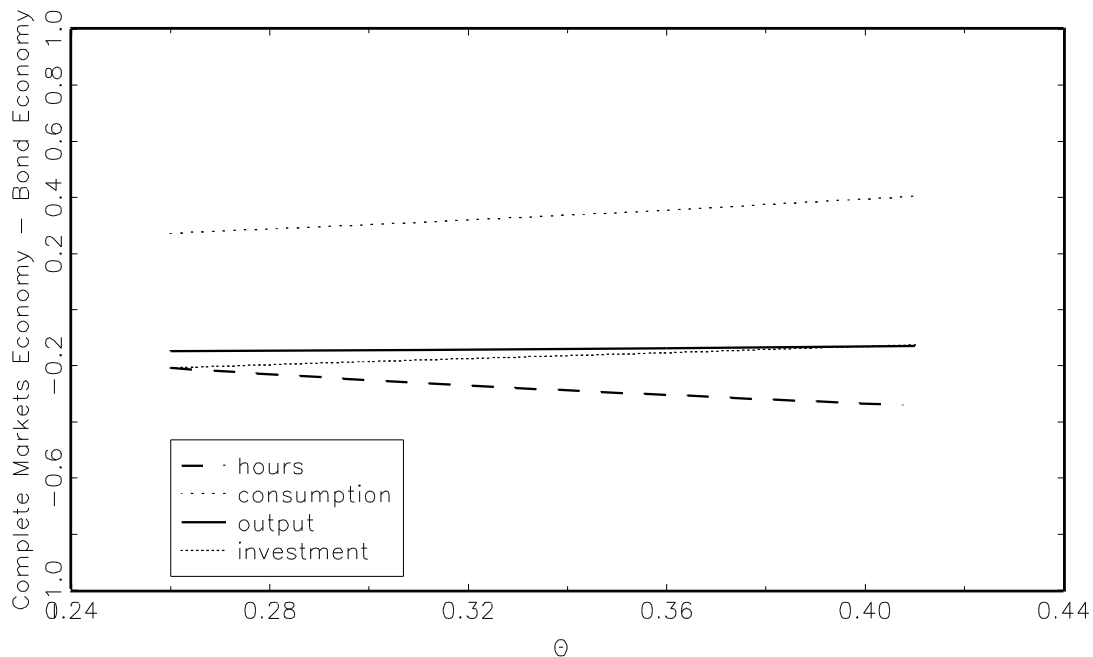Complete Markets Economy and Bond Economy

Figure 4: Differences between Cross–Country Correlations in
Complete Markets Economy and Bond Economy

The moments are averages over 1,000 simulations. Each simulation is 150 periods long and the first 50 periods are dropped to remove the effect of initial conditions. Moments are computed using HP filtered percent deviations from steady state.

Figure 5: Differences between Cross−Country Correlations in
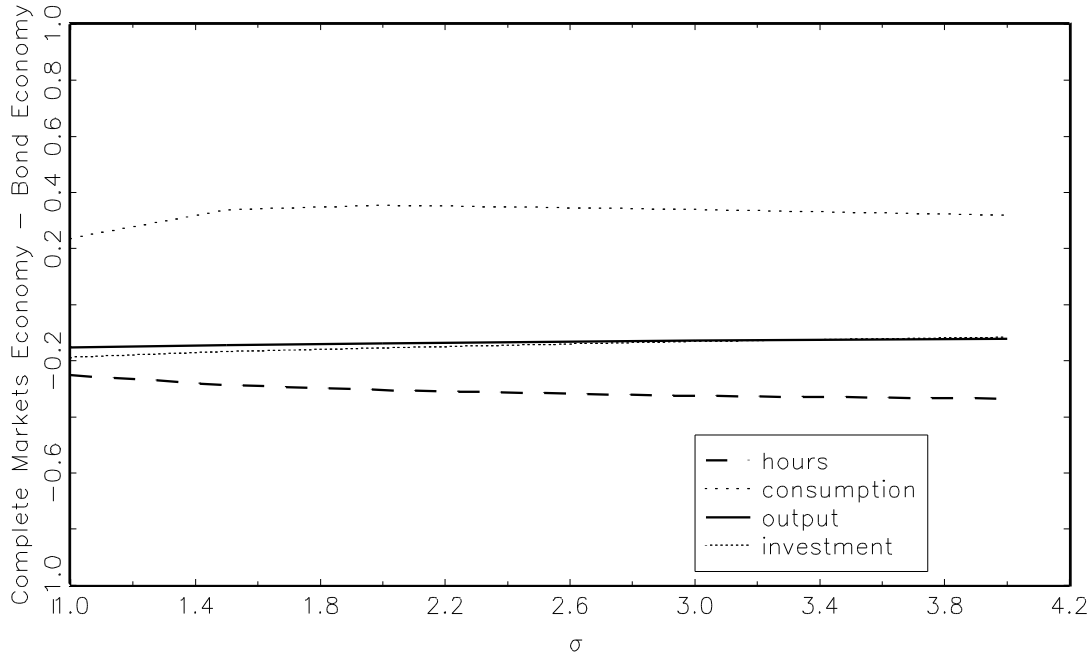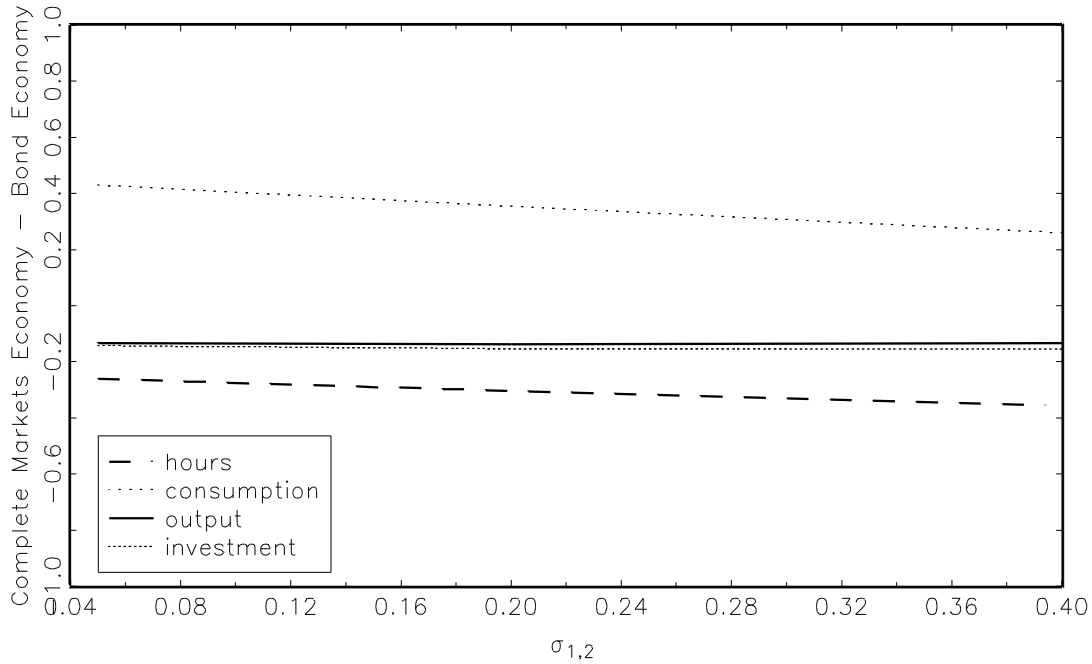Complete Markets Economy and Bond Economy

Figure 6: Differences between Cross−Country Correlations in
Complete Markets Economy and Bond Economy

The moments are averages over 1,000 simulations. Each simulation is 150 periods long and the first 50 periods are dropped to remove the effect of initial conditions. Moments are computed using HP filtered percent deviations from steady state.

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