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Allocating Awards Across Noncomparable Categories

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Abstract

Suppose an agency awards a …xed number of prizes to applicants in di¤erent categories such that the applicant-to-winner ratio is constant by category. It is demonstrated in a simple theoretical model that the number of awards in a category will typically be positively related to the degree of applicant uncertainty. The theoretical …ndings are related to awards data from the Social Sciences and Humanities Research Council of Canada doctoral fellowship competition.

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1. Introduction

Consider an agency charged with making scholarship awards. It may be difficult to make comparisons between applicants in different disciplines. With a multidisciplinary selection committee, the number of winning applicants in a discipline might depend less on applicant merit than upon the efforts of that discipline's committee representatives. One approach might be to allocate awards to different committees organized by discipline and then let each discipline committee choose the winners from that discipline's applicants. This paper studies the implications of such a solution if a committee's share of awards is set equal to its share of applicants. As potential applicants will not apply without sufficient probability of winning, in plausible circumstances the number of applicants and hence the share of awards will be a positive function of the uncertainty of assessment. Section 2 considers two special cases: the perfect certainty case where every applicant knows her own ability and the perfect uncertainty case, where the award is essentially by lottery. Section 3 describes the more formal model under the assumption that committees award entirely by perceived merit, states the main result as proved in the Appendix, and speculates on its implication for an alternative mode of committee behaviour. Section 4 discusses the Social Sciences and Humanities Research Council of Canada doctoral fellowship competition in light of the model. Section 5 concludes.

2. Two Special Cases

Begin by considering a discipline in which there is perfect certainty. Every individual knows her ranking in the applicant pool of her discipline and the number of awards in that discipline is a known fraction $\pi < .5$ of the number of applicants, with an integer value produced by conventional rounding. Given even a small cost of application, no one will apply and no awards will be made. To see this, suppose there are n students in a discipline. It can never be rational for the nth ranked individual to apply. If she is the only one to apply, she cannot win because under the rounding rule there will be no awards with only one application. If any combination of others apply, she is always beaten. Knowing that the nth ranked individual will not apply, the $(n - 1)$st ranked individual will not apply as well, realizing that she will either be the only applicant or lose as the last-ranked applicant. Following this logic to its conclusion, no one applies[1] and no one wins.[2]

Now consider the case where no individual has information about her ranking. Assuming the cost of application is sufficiently low, all individuals will apply. This result, in combination with the previous one, suggests that an increase in the uncertainty of assessment will tend to increase the application rate, a conjecture that is shown in the next section to hold under certain conditions.

3. The Model

3.1. Basic structure

We now turn to a model in which any randomly-selected individual in a particular discipline has some imperfect information about how her application will compare to others. We assume that, if she applies, the judging committee will grant her an award if $X + \sigma Y \geq S$, where X (which we call "known ability") is the publicly-known component of her ability, σY (which we call "measured ability") is the remaining component of the committee assessment that cannot be predicted by the applicant and S is the known committee-set standard required to win. X and Y are independent random variables with publicly-known density functions, the latter with mean zero and variance one so that σ (the "measurement variance")

[1] We can take this opportunity to relate our application to a more general literature. This competition for an award is a tournament as in Lazear and Rosen (1981) with effort a (0,1) variable (not to apply or to apply). As in their model and in Nalebuff and Stiglitz (1983), effort can be withdrawn as the uncertainty becomes small, because those with low ability are sure losers and decide not to play (just as here). Unlike these papers, however, we will largely take the design of the tournament as given and will not consider how applications could be stimulated by providing "handicaps" to the less able.

The example also makes it clear how the participation of others can make it more likely an individual should choose to participate, as in many games considered at a very general level by Milgrom and Roberts (1990).

[2] Assuming that every potential applicant assumes that if it is in her interest to apply, it will also be in the interest of all higher-ranked individuals to apply, it is possible to develop other solutions for different values of π and different rounding rules. The common element of all solutions is that each applicant wins an award, for under perfect certainty it would be irrational to apply and not win. If fractions are always rounded down, there will be no applicants for $\pi < 1$. For conventional rounding (rounding to the closest integer with a number ending with .5 rounded to the higher integer), the number of applicants will be $int[.5/(1-\pi)]$ where $int[x]$ is the largest integer in the closed interval $[0; x]$. For rounding up, the number of applicants will be $int(1/(1-\pi))$ where $int(x)$ is the largest integer in the open interval $(0; x)$. Under the last solution, there will always be at least one applicant for $\pi > 0$.

is a publicly-known indicator of applicant uncertainty.[3] We further assume that any individual will apply if

$$Prob(X + \sigma Y \geq S \mid X) \geq \alpha; \tag{3.1}$$

where probability α is the same across individuals and reflects the cost of applying. That is, the individual will apply if, given her known ability, the probability that her total ability will attain the winning standard is sufficient to offset the cost of applying.

If we assume that individuals are identical (and are treated identically by the committee) except for X and Y, then there will be some level of known ability E such that all individuals with $X \geq E$ will apply. Hence

$$Prob(E + \sigma Y \geq S) = \alpha \tag{3.2}$$

or

$$F_Y((S - E)/\sigma) = 1 - \alpha \tag{3.3}$$

where F_Y is the (monotonic) cumulative probability distribution of Y. Hence

$$E = S - \sigma a; \tag{3.4}$$

where $a = F_Y^{-1}(1 - \alpha)$.

Turning to the behavior of the judging committee, we assume the committee sets the standard S such that a fraction π of a large pool of applicants would win, that is, that a random draw from the applicant pool will have merit in excess of S with probability π:

$$\pi = Prob(X + \sigma Y \geq S \mid X \geq E) \tag{3.5}$$

We assume further that if it intends to reduce the proportion of successful applicants, the committee must increase the standard S, that is

$$\partial\pi/\partial S < 0. \tag{3.6}$$

It does seem natural that if the standard is tightened, a smaller fraction of applicants will win. However, because the standard is publicly-known before application, it is possible that increasing the standard will reduce the number of

[3] As a simple example, X could be based on transcript grades, where everyone agrees how a transcript can be distilled to a single number. σY could be the component the committee assigns to the new information in confidential reference letters, conditional upon transcript grades.

applicants so sharply that even though the number of winners is also reduced, the result will be a higher proportion of winners. We discuss this assumption further in the second theorem of Appendix A and find it follows from a necessary and sufficient condition on the probability distribution of known ability X. We also find a simpler sufficient condition, namely that (3.6) follows if the probability distribution of known ability X has everywhere increasing hazard in the relevant range. Many probability distribution functions, including the normal and the uniform, meet this condition.

3.2. Main Result

Under the above conditions we prove in Appendix A our conjecture that

$$dE/d\sigma < 0. \tag{3.7}$$

That is, an increase in the measurement variance σ (uncertainty) will lower the applicant threshold E and hence increase the number of applicants.

3.3. Computed Numerical Example

Figure 1 illustrates a computed numerical example where both X and Y are standard normal, which implies that (3.6) is satisfied. As the measurement error variance σ increases, the proportion of the population that applies increases monotonically. This proportion also increases when the win proportion π increases from .2 to .4 and decreases when the cost of applying increases as represented by an increase in α, the required win probability for the marginal applicant, from .05 to .1.

3.4. Alternative Committee Behavior

In our model there is an overall authority which gives each committee the right to award a fraction π awards to its (numerous) potential applicants: this could occur for example if there were a large number of committees and the authority had a fixed number of awards that it allocated by number of applications, as described in the Introduction. Up to now we have assumed that the committee chose the best applicants; we now point out that it might choose not to do so if it were interested in increasing the number of awards in its discipline. As an example, suppose the committee decides to award a certain publicly-known fraction ω of the awards by lottery, with the goal of attracting more applicants and hence increasing the discipline's share of total awards.

The win probability of the marginal applicant α now equals the sum of the probability of the marginal applicant winning "on ability" and the probability of being a "lucky loser" and winning by lottery:

$$\alpha = (1 - F_Y(a)) + \omega\pi(F_Y(a)) \tag{3.8}$$

so that solving for a

$$a = F_Y^{-1}\left(\frac{1-\alpha}{1-\omega\pi}\right) \tag{3.9}$$

if $\omega\pi$ is less than α. If ω increases, a increases since α and π are constant. It can be shown in the increasing hazard case that this will in turn imply a decrease in E and an increase in the number of applicants. Figure 2 continues our normal distribution example with $\sigma = .5$ and illustrates clearly how increases in ω increase the proportion of potential applicants who enter.

It is perhaps especially interesting to consider the case where $\omega\pi \geq \alpha$, as all potential applicants then apply. Consider the $\alpha = .01$, $\pi = .2$ case in Figure 2 where it can be seen that when ω is 0, about 25 per cent of potential applicants enter. That means a fraction equal to about $.25\pi = .05$ or about 5 per cent of potential applicants win. If ω is increased slightly to .05 ($\omega\pi = .01 = \alpha$), all potential applicants will enter and hence 20 per cent of potential applicants win (even though only one per cent of applicants will win via the lottery).[4]

4. SSHRC Doctoral Fellowship Competition

SSHRC doctoral fellowships are significant stipends (currently $17,700 per year) awarded to Ph.D. students in the social sciences and humanities who are citizens or Permanent Residents of Canada.[5] An award is normally renewed until the fourth

[4] An extension of this idea implies that if admission or editorial behavior has a capricious element, universities or academic journals cannot necessarily be ranked in perceived quality by using (the negative of) the admission or acceptance ratio.

[5] The Ontario Graduate Scholarship competition explicitly uses the method assumed in our model to allocate awards by discipline but does not publish suitable data. The National Science Foundation in the United States does not use this method but makes an institutional judgment as to where to allocate both awards and research funds. Until recently, the Australian Research Council awarded research funds by a method very similar to the one modelled here (see Fretz and Veall, 2001).

year of Ph.D. study. Applications consist of a form supplying academic history and other basic information, grade information, a very brief proposal for doctoral research and three academic reference letters. Before the 1995-96 competition, applications were made by students directly to SSHRC and were judged by about …fteen committees, each consisting of three to seven Canadian academics and organized by discipline. Awards were divided among the committees essentially as assumed in our model: each year the number of awards a committee could make was equal to about 20 per cent of the number of applications directed to that same committee.

A new system was introduced beginning with the 1995-96 competion. Applications are …rst judged at the university level (there is special provision for applicants not currrently enrolled at a Canadian university) and each university can only forward a given number to SSHRC for the second stage of judging. (The quota for each university is largely a function of the previous success of that university's applicants.) At the second stage there are now only …ve committees: Arts and Letters, Humanities, Civilization and the Environment, Cognitive Studies and Private and Public Policy Studies. The number of awards granted within each of these …ve areas is close to proportional to the number of applicants although there is some variation because each committee forwards some of its nonwinning applications for further consideration by a single supercomittee that makes further awards.

SSHRC publishes the number of applications and awards by discipline, although the discipline categories do change somewhat over time. We also need an estimate of the size of the potential applicant pool. We have obtained Statistics Canada data for the number of nonvisa full-time Ph.D. students by discipline.[6] However we need the numbers of Ph.D. students in the …rst four years of study (as more senior students are not eligible) and such data are not available.[7] Given that time to completion/withdrawal varies sharply across disciplines we decided to estimate (admittedly crudely) the number in the …rst four years using Ontario

[6] This excludes those Permanent Residents of Canada who can apply (provided they have a degree from a Canadian university) even though they are at a foreign university or are not currently at any university (respectively 9% and 8% of applications in 2001). There is no good way to estimate discipline categories for this part of the applicant pool. By leaving it out, our comparisons of relative application rates across disciplines will be misleading to the extent students in di¤erent disciplines apply di¤erentially from these other streams.

[7] Strictly speaking, the applicant pool consists of those who would be in their …rst four years of Ph.D. study in the following year when any award would apply. However taking the number of those currently in their …rst four years of Ph.D. studies seems a reasonable approximation.

Council of Graduate Studies (OCGS) data on entry/enrolment ratios by discipline. Appendix B describes this “four-year” adjustment as well as other aspects of the data, including the matches between SSHRC, Statistics Canada and OCGS categories that we used. Appendix B also includes results without the four-year adjustment.

Table 1 describes the data using 1990-1994 averages. The estimated number of eligible students in the first column is clearly imperfect: note for example that in the small categories Law and Classical Studies, the number of students who applied for SSHRC awards exceed our estimate of the number of eligible students.[8] We also note that the estimated number of eligible students in Geography and Psychology is misleadingly high because many students in these disciplines are ruled to be in natural rather than social sciences and are eligible for another fellowship program and not the SSHRC program. Finally, it might be argued that Education is a special case because most doctoral students in Education in Canada are at a few programs (the largest number at the Ontario Institute for Studies in Education) that attract a significant portion of post-career or late-career students, some of whom are educational practitioners as opposed to researchers. While we include Classical Studies, Law, Geography, Psychology and Education in our tables for completeness, we classify them as outliers and omit them from further discussion.

Turning more closely to Table 1, note that the application success rate (measured as the average number of winners divided by the average number of applications) does not vary much by discipline, ranging between 0.18 and 0.22 except for Anthropology/Archaelogy. There is however substantial variation in the application rate, even in disciplines with relatively clear boundaries. Note for example that while an estimated 34 per cent of eligible students in Management /Business Administration studies applied and an estimated 41 per cent of eligible students in Economics applied, the comparable figures in History, Anthropology/Archaelogy and Fine Arts are 75, 75 and 82 per cent respectively. The difference in application rates is reflected in the win rate (the average number of winners divided by the average estimated number of eligible students). Categories are ordered by win rates and it can be seen that this ordering reflects the differences in application

[8] In the case of Law, until 2001 SSHRC permitted students pursuing a Master’s degree to apply, provided they could demonstrate an intention to pursue an academic career. In the case of Classical Studies, the discrepancy is probably due to a mismatch between the Statistics Canada and SSHRC categorizations, as some students registered in other disciplines (e.g. History or Archaelogy) apply in Classical Studies.

rates: for example Fine Arts has a win rate more than double that of Economics. It has an estimated 27 per cent fewer eligible students but averaged 32 awards compared to just 20 in Economics. It might be argued that Economics and Management/ Business Administration are the most quantitative of the disciplines in the social sciences and humanities and hence their applicants have the best notion of how their abilities are perceived (i.e. ¾ is small). The relatively low application rates and win rates in these disciplines are therefore consistent with our model.[9]

There is a similar pattern in Table 2 which uses 1995-2001 data. While application success rates vary more than in the previous table, still it is obvious that in most cases a discipline's share of the awards depends largely on its application rate. Again Economics and Management/Business Administration have the lowest application and win rates, not counting disciplines we have classed as outliers.[10] We also note from Table 2 how much lower the application rates are in this second period as compared to the …rst period. We argue that this is consistent with our model: as described above from 1995 on, some applications (in practice 40% to 50%) were culled at the university level. Applicants who were removed from consideration at this stage were informed and hence became more aware of their perceived ability. In terms of our model, ¾ was reduced and hence the number of applications fell.[11]

[9] Anglin and Meng (2000) …nd that for …rst-year undergraduate classes in Ontario, grade variance in Economics is much higher than that in English, French, Philosophy, Political Science and Sociology, and moderately higher than in Psychology, although …rst-year undergraduate courses may not re‡ect marking practices at higher levels. Higher variation in grades could be associated with less uncertainty in one's perceived ability.

[10] A rival explanation is that Economics and Management/Business Administration students have a higher opportunity cost of time, perhaps because of better employment prospects. While we are not sure that the time required to write an application is so large as to make this explanation plausible, we cannot rule this possibility out. Neither can we rule out the "explanation" of discipline "culture". Regardless, our tables do show a strong association between applicant behaviour and the eventual allocation of awards. This result is of particular interest to those of us in Economics departments as not only does the result of current practice seem to be a low share for Economics but also the share has fallen recently, with only 7 doctoral awards in Economics in 2000-2001. Dixon and Rosson (undated) argue based on related data that students and faculty in business schools are receiving an inappropriately small portion of SSHRC student and research support.

[11] Unadjusted and adjusted enrolment increased in virtually all disciplines over this period, sometimes substantially. The number of applications fell in 12 of our 17 categories. This fall in application rates (which has led to a small increase in application success rates) is not easily explained by the opportunity cost explanation mentioned in the previous footnote.

5. Conclusions

We have presented a model of scholarship application and allocation in which an increase in an applicant's uncertainty about how her abilities will be judged may, under certain conditions, induce more individuals to apply. One reason this may be important is if the number of scholarships in a program or discipline are directly linked to the number of applications. While our examination of data for awards by the Social Sciences and Humanities Research Council of Canada is far from conclusive, we do present some findings which are consistent with our model, such as an apparent tendency for students in Fine Arts (whose performance may be difficult to measure) to apply more for awards than other students. The model suggests that committees interested in generating more applications, perhaps in the interest of gaining applicant share relative to other committees in the same program, may have an incentive to make some awards capriciously, such as by lottery.

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Table 1

SSHRC Doctoral Fellowships by Discipline: Estimated Number of Eligible Students, Actual Number of Applicants and Winners, Annual Averages, 1990-1994

Discipline	Est. # Eligible (A)	# Apply (B)	# Win (C)	Success Rate (C)¥(B)	Applic. Rate (B)¥(A)	#Win/ #Enroll (C)¥(A)
Law	43	72	15	0.20	1.67	0.35
Classical Studies	33	39	9	0.22	1.18	0.27
Fine Arts	180	148	32	0.22	0.82	0.18
Anthro./Archaelogy	176	132	30	0.23	0.75	0.17
History	408	301	63	0.21	0.75	0.15
Linguistics	109	69	15	0.22	0.63	0.14
Commun./Media Studies	68	48	9	0.20	0.71	0.13
Political Science	377	237	49	0.21	0.63	0.13
Religious Studies	164	116	21	0.18	0.71	0.13
Philosophy	300	165	34	0.21	0.55	0.11
Sociology	323	169	35	0.21	0.52	0.11
Modern Lang./Lit.	980	476	96	0.20	0.49	0.10
Economics	248	102	20	0.20	0.41	0.08
Mgmt./ Bus. Admin.	361	127	28	0.22	0.34	0.08
Psychology	991	333	66	0.20	0.34	0.07
Geography	249	83	15	0.18	0.33	0.06
Education	1382	258	50	0.19	0.19	0.04
Total (ex. Education)	5008	2616	538	0.21	0.52	0.11
Total (inc. Education)	6390	2873	588	0.20	0.45	0.09

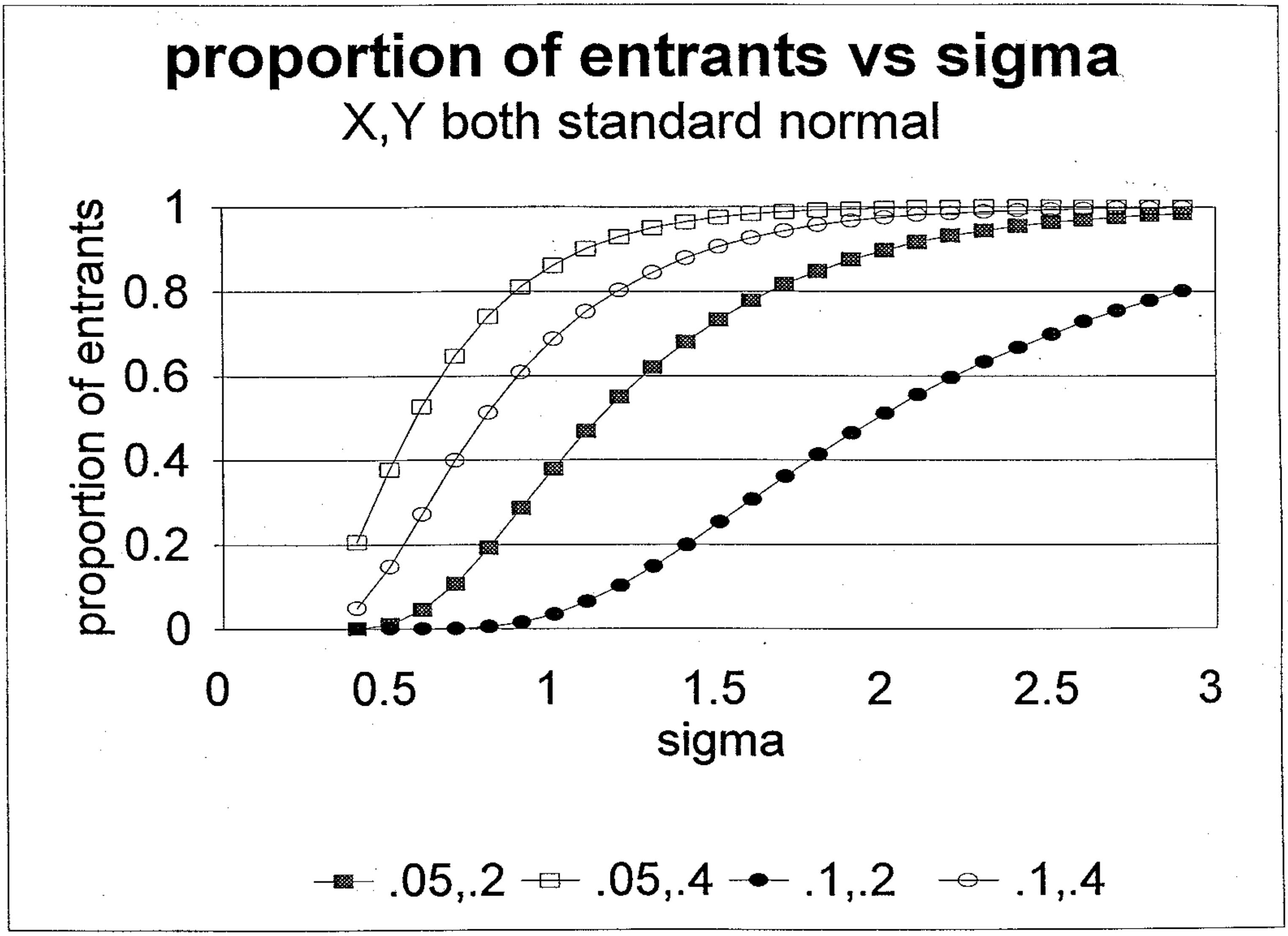
Categories are ranked by descending #Win/#Enroll values (last column). Values in the …rst three columns have been rounded to the nearest integer. Categories with an annual average of fewer than 25 applications have been omitted and totals re‡ect those omissions. See Appendix B for notes regarding underlying data.

Table 2
SSHRC Doctoral Fellowships by Discipline: Estimated Number of Eligible Students, Actual Number of Applicants and Winners, Annual Averages, 1995-2001

Discipline	Est. # Eligible (A)	# Apply (B)	# Win (C)	Success Rate (C)¥(B)	Application Rate (B)¥(A)	#Win/ #Enroll (C)¥(A)
Classical Studies	39	34	7	0.21	0.85	0.18
Anthro./Archaelogy	281	162	39	0.24	0.57	0.14
Law	126	64	16	0.25	0.51	0.13
Fine Arts	243	140	30	0.21	0.57	0.12
History	466	268	55	0.20	0.57	0.12
Philosophy	317	186	36	0.19	0.59	0.11
Commun./Media Studies	98	45	9	0.19	0.46	0.09
Political Science	424	214	39	0.18	0.51	0.09
Modern Lang./Lit.	1023	452	86	0.19	0.44	0.08
Linguistics	179	59	14	0.24	0.33	0.08
Sociology	426	165	31	0.19	0.39	0.07
Psychology	1242	365	77	0.21	0.29	0.06
Religious Studies	238	89	13	0.15	0.38	0.05
Geography	440	102	23	0.22	0.23	0.05
Economics	295	71	14	0.20	0.24	0.05
Mgmt./Business Admin.	462	96	16	0.16	0.21	0.03
Education	1679	261	40	0.15	0.16	0.02
Total (ex. Education)	6300	2512	505	0.20	0.40	0.08
Total (inc. Education)	7979	2773	544	0.20	0.35	0.07

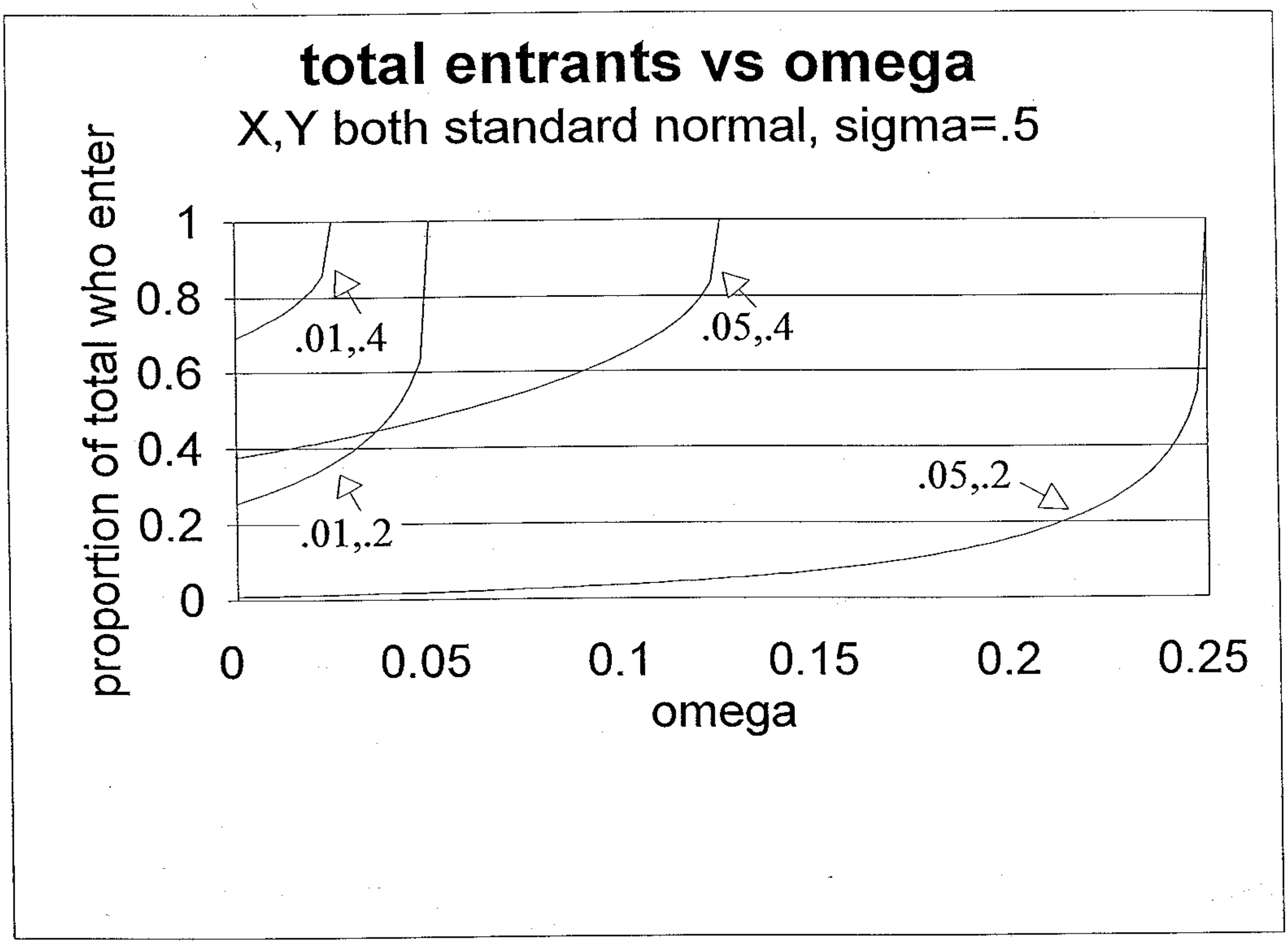
Notes to Table 1 apply. In addition note that average enrolment data are based on the 1995 to 1998 period, as more recent data adjusted for residence status in Canada are not available.

Figure 1



Notes to Figure 1: The horizontal axis "sigma" is the measurement variance σ. ".05, .2" in the legend indicates $\alpha = 0.05$, $\pi = 0.2$, with the other entries defined analogously.

Figure 2



Notes to Figure 2: The horizontal axis "omega" is the proportion of awards made by lottery. The label ".01, .4" in the legend indicates $\alpha = 0.01$, $\pi = 0.4$, with the other labels defined analogously.

Appendix A

Theorem 0.1. Using the notation and conditions given in the text, $dE/d\sigma < 0$ (that is an increase in applicant uncertainty will reduce the application threshold and hence increase the number of applications).

Proof. Begin by substituting (3.4) into (3.5) to obtain

$$
\begin{aligned}
\pi &= \mathrm{Prob}(X + \sigma(Y - a) \geq E \mid X \geq E) \\
&= \frac{\mathrm{Prob}(X + \sigma(Y - a) \geq E \text{ and } X \geq E)}{\mathrm{Prob}(X \geq E)} \\
&= \frac{\mathrm{Prob}(X + \sigma(Y - a) \geq E \text{ and } X \geq E \text{ and } Y \geq a)}{\mathrm{Prob}(X \geq E)} \\
&\quad + \frac{\mathrm{Prob}(X + \sigma(Y - a) \geq E \text{ and } Y < a)}{\mathrm{Prob}(X \geq E)} \\
&= \frac{\mathrm{Prob}(X \geq E \text{ and } Y \geq a) + \mathrm{Prob}(X + \sigma(Y - a) \geq E \text{ and } Y < a)}{\mathrm{Prob}(X \geq E)}
\end{aligned}
$$

and since independence implies that

$$\mathrm{Prob}(X \geq E \text{ and } Y \geq a) = \mathrm{Prob}(X \geq E) \cdot \mathrm{Prob}(Y \geq \alpha) = \alpha \cdot \mathrm{Prob}(X \geq E),$$

therefore

$$\pi = \alpha + \frac{\mathrm{Prob}(X + \sigma(Y - a) \geq E \text{ and } Y < a)}{\mathrm{Prob}(X \geq E)} \tag{0.1}$$

Note in passing that the implication that α must be less than or equal to π makes intuitive sense. Because π is the probability of winning for a randomly chosen applicant, it cannot be less than the probability of winning of the marginal applicant.

Now rewrite (0.1) as $\pi = \pi(E; \sigma)$ and differentiate totally (with a and α fixed):

$$d\pi = (\partial\pi/\partial E)dE + (\partial\pi/\partial\sigma)d\sigma$$

Setting $d\pi = 0$ and rearranging we obtain:

$$dE/d\sigma = -(\partial\pi/\partial\sigma) = (\partial\pi/\partial E) \tag{0.2}$$

By (3.4), $\partial\pi/\partial E = \partial\pi/\partial S$ which by assumption (3.6) is less than zero. (We will examine this assumption in the second theorem below.) Hence the sign of (0.2) will be the sign of $\partial\pi/\partial\sigma$.

To sign $\partial\pi/\partial\sigma$, begin by noting that (0.1) suggests a partition of (X, Y) space using the three lines:

$$X + \sigma(Y - a) = E \iff Y = a + (E - X)/\sigma \iff Y = (S - X)/\sigma \tag{0.3}$$

$$X = E \tag{0.4}$$

$$Y = a \tag{0.5}$$

(0.3) is the line corresponding to the committee's awarding rule: they award a scholarship if the sum of the known ability and the measured ability meets their standard S, that is if $X + \sigma Y \geq S$. (0.4) is the line corresponding to the individual's decision rule: all individuals with $X \geq E$ will apply. (0.5) is the level of normalized measured ability Y that the marginal applicant (with $X = E$) must have in order to win (and hence all applicants with that level of normalized measured ability will win).

Figure A1 illustrates these lines in (X, Y) space. A, B and C are the probabilities that the realizations (X, Y) will lie in the areas delineated by these lines. A is the probability that an applicant would have measured ability greater than a (and hence win) while B is the probability that an applicant will have measured ability below a but win anyway because of high known ability X. C is the probability that an applicant will not win. Hence the probability of an applicant winning is $\pi = (A + B)/(A + B + C)$ and the minimum probability for an applicant to apply is $\alpha = 1 - F_Y(a) = A/(A + B + C)$ where F_Y is the cumulative distribution function of Y (and f_Y is the density function and F_X and f_X are the corresponding functions for the independent random variable X). The last equality follows from the independence of X and Y.

Hence we can rewrite (0.1) as:

$$\pi = \alpha + \frac{B}{A + B + C} \tag{0.6}$$

so that the sign of (0.2) (which equals the sign of $\partial\pi/\partial\sigma$), is the sign of $\partial B/\partial\sigma$ because:

$$\partial\pi/\partial\sigma = \frac{\partial B/\partial\sigma}{A + B + C}$$

as a and $A + B + C = Prob(X \geq E)$ are not functions of σ. But from the diagram it can be seen that $\partial B/\partial\sigma$ must be negative as an increase in σ induces a counterclockwise rotation of line (0.3) around the point (E, a) which must necessarily reduce probability B. Signing $\partial B/\partial\sigma$ formally,

$$
\begin{aligned}
B &= Prob(X + \sigma(Y - a) \geq E \text{ and } Y < a) \qquad (0.7)\\
&= Prob(Y < a) \cdot Prob(X \geq E + \sigma(a - Y) \mid Y < a)\\
&= (1 - \alpha) \cdot Prob(X \geq E + \sigma(a - Y) \mid Y < a)\\
&= (1 - \alpha) \cdot E_Y[\int_{E+\sigma(a-Y)}^{\infty} f_X(x)dx \mid Y < a]
\end{aligned}
$$

where we have used the independence of X and Y and E_Y denotes the mathematical expectation over the density of Y. Therefore

$$\partial B/\partial\sigma = (1 - \alpha) \cdot E_Y[-(a - Y)f_X(E + \sigma(a - Y)) \mid Y < a] < 0$$

using Leibniz's rule and the fact that the expectation of a random variable that is always negative must also be negative. As discussed above this implies $\partial\pi/\partial\sigma$ is negative and hence that $dE/d\sigma$ is also negative.

Theorem 0.2. Remove assumption (3.6) but otherwise maintain the same notations and conditions. Then if the hazard function of X is monotonic, $dE/d\sigma$ has the opposite sign as the first derivative of the hazard function of X.

Proof. For convenience repeat

$$dE/d\sigma = -(\partial\pi/\partial\sigma) = (\partial\pi/\partial E)$$

and note that the previous proof established that $(\partial\pi/\partial\sigma) < 0$. Hence $dE/d\sigma$ will have the same sign as $\partial\pi/\partial E$ which is no longer simply assumed to be negative. Using (0.6)

$$
\begin{aligned}
\partial\pi/\partial E &= \frac{(A + B + C)\partial B/\partial E - B \cdot \partial(A + B + C)/\partial E}{(A + B + C)^2} \qquad (0.8)\\
&= \frac{(A + B + C)\partial B/\partial E + B \cdot f_X(E)}{(A + B + C)^2}
\end{aligned}
$$

where we have used the fact that $\partial(A + B + C)/\partial E = -f_X(E)$. To analyze $\partial B/\partial E$, it will help to define $Z = \sigma(a - Y)$ where $Y < a$ and 0 otherwise, so the density of Z is the density of $\sigma(a - Y)$ given $Y < a$. With this notation, we can write the fourth line of (0.7) as $B = (1 - \alpha) \cdot E_Z[1 - F_X(E + Z)]$ and hence $\partial B/\partial E = (\alpha - 1)\cdot E_Z[f_X(E+Z)]$. Therefore continuing from (0.8), using $A + B + C = 1 - F_X(E)$:

$$
\begin{aligned}
\partial\pi/\partial E &= \frac{(1-F_X(E))\cdot(\alpha-1)E_Z[f_X(E+Z)] + (1-\alpha)E_Z[1-F_X(E+Z)]f_X(E)}{(1-F_X(E))^2} \\
&= \frac{(1-\alpha)}{(1-F_X(E))^2}(E_Z[1-F_X(E+Z)]f_X(E) - (1-F_X(E))\cdot E_Z[f_X(E+Z)]
\end{aligned}
$$

or

$$
\begin{aligned}
\partial\pi/\partial E &= \frac{(1-\alpha)E_Z[(1-F_X(E+Z))h_X(E) - f_X(E+Z)]}{1-F_X(E)} \\
&= \frac{(1-\alpha)E_Z[(h_X(E) - h_X(E+Z))(1-F_X(E+Z))]}{1-F_X(E)}
\end{aligned}
$$

where $h_X(E) = f_X(E)/(1-F_X(E))$ is the hazard function of X at $X = E$. The sign of $\partial\pi/\partial E$ is the sign of this expectation which is the sign of $h_X(E) - h_X(E+Z)$ if this term has the same sign for all $Z > 0$: Thus if $h_X(E) = h_X(E+Z)$ for all $Z > 0$, (A8) is zero. This is the case of constant hazard. If $h_X(E) < h_X(E+Z)$ for all $Z > 0$ (everywhere increasing hazard such as if f_X is the normal density), $\partial\pi/\partial E$ is negative and hence from above $\partial E/\partial\sigma$ is negative and an increase in variance reduces the application threshold and hence increases the number of applicants and the number of scholarships awarded. In the decreasing hazard case where $h_X(E) > h_X(E+Z)$ for all $Z > 0$, an increase in variance increases the application threshold and reduces the number of applicants and the number of scholarships awarded.

Appendix B

The categories (whose titles we have sometimes abbreviated) are those used by SSHRC in its current annual statistics (Table 7.6). We have omitted categories with an annual average of fewer than 25 applications and combined Anthropology and Archaelogy. We use the current SSHRC category titles for comparability between Tables 1 and 2: there has been some renaming over time. Before 1997, there were separate SSHRC categories for different kinds of literature (e.g. American Literature, English-Canadian Literature). These have been combined in the Modern Languages and Literature category.

Most SSHRC categories had obvious counterparts in the Statistics Canada enrolment data. Less obvious matches include Education (which we matched to the total nonvisa Ph.D. enrolment in Statistics Canada categories Elementary-Secondary Teacher Training, Higher Education, Human Kinetics, Education–Non-Teaching Fields, Nursery and Kindergarten Education, Physical Education and Recreology/Recreation Administration), Fine Arts (matched to Statistics Canada categories Fine Arts, Music and Other Performing Arts), Linguistics (Linguistics and Translation and Interpretation), Management, Business and Administrative Studies (matched to Statisitics Canada categories Commerce, Management and Business Administration and Specialized Administration Studies) and Modern Languages and Literature (Statistics Canada categories English, French and Other Languages And/Or Other Literatures). SSHRC categories Geography and Urban, Regional and Environmental Studies are combined as one category ("Geography") and matched with enrolment data for Geography and Man/Environment Studies. As mentioned in the main text, the fields of Geography and Psychology may have significant numbers of students applying for Natural Science and Engineering Research Council of Canada postgraduate awards instead of SSHRCC doctoral awards.

As noted in the text, the Statistics Canada enrolment data were obtained specially to exclude visa students (who are ineligible for SSHRC awards) but they do not exclude students not eligible for SSHRC awards because they have been in doctoral studies for more than four years. Suitable data are not available to correct this shortcoming: we attempt a crude correction using Ontario Council of Graduate Studies (OCGS) data on the ratio of incoming full-time students to enrolled full-time students by discipline. (Call this the "OCGS ratio".) After constructing a Modern Languages and Literature category by adding together Comparative Literature, English, German, Romance Languages and French (the last sometimes but not always separate from Romance Languages) and using Anthropology as Anthropology/Archaelogy, there was a suitable OCGS ratio for all categories except Communications and Media Studies: in that last case the average OCGS ratio was used. The estimated number of eligible doctoral students is then taken as 4£OCGS ratio£the number of students reported by SSHRC.[12] This presumes a steady state in which full-time entrants stay 4 years after which they begin to graduate/withdraw/switch to part-time status. Note that the number "4" could be replaced by some other value and not a¤ect cross-discipline comparisons, except as noted in the footnote. Hence our approach could be recon…gured as comparing number of applicants and winners per discipline annual entrant.

While we feel that the di¤erences in completion and withdrawal behaviour across disciplines (that include students not eligible for SSHRC awards since they have been in doctoral studies more than 4 years) justify these admittedly crude adjustments, we provide the unadjusted data below in Tables A1 and A2. The rankings are not that di¤erent: in particular Economics and Management/Business Administration are still disciplines with low application and win rates and Fine Arts and History are disciplines with high application and win rates.

[12] For both periods for Law, as noted in the text an outlier in any case and for 1990-94 for Management/Business Administration, it was necessary to cap the estimate at total enrolment.

Table A1
Enrolment of Fulltime Doctoral Students (Net of International Students) in Canadian Universities and Number of SSHRCC Applicants and Winners, by Discipline, Annual Averages, 1990-1994

Discipline	# Enroll (A)	# Apply (B)	# Win (C)	Success Rate (C)¥(B)	Application Rate (B)¥(A)	#Win/ #Enroll (C)¥(A)
Law	43	72	15	0.20	1.68	0.34
Classical Studies	53	39	9	0.22	0.73	0.16
Fine Arts	213	148	32	0.22	0.70	0.15
Anthro./Archaelogy	201	132	30	0.23	0.66	0.15
Linguistics	114	69	15	0.22	0.60	0.13
Commun./Media Stud.	78	48	9	0.20	0.61	0.12
History	551	301	63	0.21	0.55	0.11
Political Science	446	237	49	0.21	0.53	0.11
Philosophy	348	165	34	0.21	0.47	0.10
Religious Studies	239	116	21	0.18	0.49	0.09
Sociology	416	169	35	0.21	0.41	0.08
Economics	252	102	20	0.20	0.41	0.08
Modern Lang./Lit.	1219	476	96	0.20	0.39	0.08
Mgmt., Bus.Admin.	361	127	28	0.22	0.35	0.08
Psychology	1292	333	66	0.20	0.26	0.05
Geography	316	83	15	0.18	0.26	0.05
Education	1462	258	50	0.19	0.18	0.03
Total (ex. Educ.)	6143	2616	538	0.21	0.43	0.09
Total (inc. Educ.)	7605	2873	588	0.20	0.38	0.08

Notes to Table 1 apply. As discussed in text, Table 1 uses an authors' estimate of enrollment by nonvisa Ph.D. students in their …rst four years of study. This table uses actual enrollment data, all years of study included.

Table A2
Enrolment of Fulltime Doctoral Students (Net of International Students) in Canadian Universities and Number of SSHRCC Applicants and Winners, by Discipline, Annual Averages, 1995-2001

Discipline	# Enroll (A)	# Apply (B)	# Win (C)	Success Rate (C)¥(B)	Application Rate (B)¥(A)	#Win/ #Enroll (C)¥(A)
Classical Studies	51	34	7	0.21	0.66	0.14
Law	126	64	16	0.25	0.51	0.13
Anthro./Archaelogy	347	162	39	0.24	0.47	0.11
Fine Arts	319	140	30	0.21	0.44	0.09
History	726	268	55	0.20	0.37	0.08
Philosophy	481	186	36	0.19	0.39	0.07
Political Science	572	214	39	0.18	0.37	0.07
Communication/Media Stud.	134	45	9	0.19	0.33	0.07
Linguistics	218	59	14	0.24	0.27	0.06
Modern Lang./Lit.	1548	452	86	0.19	0.29	0.06
Sociology	641	165	31	0.19	0.26	0.05
Economics	314	71	14	0.20	0.22	0.04
Geography	550	102	23	0.22	0.19	0.04
Psychology	1881	365	77	0.21	0.19	0.04
Religious Studies	330	89	13	0.15	0.27	0.04
Mgmt., Bus. Admin.	530	96	16	0.16	0.18	0.03
Education	2503	261	40	0.15	0.10	0.02
Total (ex. Educ.)	8768	2512	505	0.20	0.29	0.06
Total (inc. Educ.)	11271	2773	544	0.20	0.25	0.05

Notes to Table A1 and Table 2 apply.

Figure A1