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Learning-by-doing and Endogenous Price-level Inertia

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Abstract

This paper presents a DGE model in which aggregate price level inertia is generated endogenously by the optimizing behaviour of price setting firms. All the usual sources of inertia are absent here ie., all firms are simultaneously free to change their price once every period and face no adjustment costs in doing so. Despite this, the model generates persistent movements in aggregate output and inflation in response to a nominal shock. This occurs because firms temper their desire to raise prices after a positive money growth shock in order to learn and lower future costs.

Key words: Endogenous price stickiness, Business Cycles, Inflation, Nominal rigidities, Learning-by-doing, Propagation mechanisms.

1 Introduction

There has been a recent surge in interest in dynamic general equilibrium models in which firms adjust their prices infrequently. Many of these models employ one of two classes of time-dependent pricing rules associated with Taylor (1999) and Calvo (1983).[1] In the former, prices are set for a given number of periods and the opportunity to adjust prices is staggered so that not all firms can adjust prices in any given period. In the latter, firms face a fixed probability of being able to adjust prices in each period. While the duration for which prices are fixed is uncertain for an individual firm, the average duration is known and in the aggregate a constant fraction of all firms will adjust prices every period as in the Taylor model.

While these models have had some success in generating empirically plausible business cycles in response to monetary shocks the pricing arrangements embedded in the models are theoretically unappealing.[2] This theoretical weakness arises from the exogenous nature of the pricing arrangements imposed upon firms which determine both the length of time for which prices cannot be re-optimized as well as the degree of synchronization among firms. This can have important consequences for the ability of these models to predict the response of the economy to changes in the economic environment, especially to changes in monetary policy. While one might expect that the optimal pricing arrangements of firms may respond to policy, they cannot in the model. Since the duration of price stickiness and the degree of staggering of pricing decisions influence the response of aggregate variables in the model one may not end up with sensible predictions regarding how the economy will respond to these changes.

Staggered price setting models were popular despite this well understood weakness because staggering was viewed as a critical element, along with long periods of price stickiness, in generating an inertial response of the price level and aggregate output to monetary shocks. However recent work has questioned the centrality of these two phenomena in propagating nominal shocks. Chari, Kehoe and McGrattan (2000) forcefully argue that staggering of pricing decisions is ineffective in propagating nominal shocks beyond the assumed duration for which prices are fixed. In addition, Christiano,

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[1] There are too many sticky price dynamic general equilibrium models to list here. Some examples are King and Watson (1996), Yun (1996), Cho et al (1997), Chari et al. (2000), Bergin and Feenstra (2000), Erceg et al. (2000) Huang and Liu (2001).

[2] See Christiano, Eichenbaum and Evans (2005) for example.

Eichenbaum and Evans (2005) show that price staggering is not crucial to generating realistic impulse responses. Finally Bils and Klenow (2005) show that on average prices change much more quickly in US data than has been assumed in the sticky price literature.[3]

The goal of this paper is to show that it is not necessary to retain either unappealing element in order to generate inertia in the aggregate price level. To make this point forcefully, the paper restricts the amount of exogenous price rigidity to one period, i.e., all firms set prices simultaneously at the beginning of each period. None the less, the aggregate price level will adjust slowly to a money growth shock because all firms optimally choose to adjust their prices slowly[4]. In the standard one-period sticky price model, firms wish to adjust prices in proportion with the expected change in marginal costs next period. The novel feature of this paper is that firms may actually choose to adjust prices less than proportionally to expected marginal cost. As a result the model can generate prices that adjust very slowly. Since the firm could in principle adjust fully to expected future inflation after the first period, any subsequent sluggishness in the price level is endogenous. Indeed, in the standard one period sticky price model, almost all of the adjustment in a firm's price occurs in the first period after the shock.

The key mechanism that is responsible for this novel feature is learning-by-doing. Firms desire to accumulate production experience because it is an input in the production technology and reduces future costs.[5] Accumulation of additional experience requires additional production. In a monopolistically competitive environment, firms must lower prices in order to sell this extra output. As a result, the presence of learning-by-doing may lead firms to set lower prices compared to standard models in which they equate expected marginal revenue with expected marginal cost. Moreover, any event that leads to an increase in the demand for the firm's product (such as a money growth shock), creates a favorable environment for learning because the demand curve is more responsive to a change in price. Essentially, a unit reduction in price yields more bang for the buck by generating more production, more experience and greater future cost reductions. By itself, this

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[3]Bils and Klenow suggest that half of the prices studied lasted no more than 4.3 months which is much shorter than the assumed duration of price rigidity (typically about 12 months) in the literature.

[4]This is not induced by imposing menu costs on the firms.

[5]See Arrow (1962) and especially Rosen (1972) for early discussions of learning-by-doing as a by-product of production experience.

mechanism makes firms want to lower prices when the economy is hit with an unexpected increase in the growth rate of money. In addition, the usual mechanism is in place:higher expected inflation and hence higher marginal costs lead firms to want to raise prices. The net effect of these two forces yield prices that increase less than expected increases in marginal cost. A further reason that prices rise slowly is the effect of learning-by-doing on costs. The accumulated production experience lowers real costs and acts to dampen the increase in nominal marginal costs.

The modelling of firm-level learning closely follows the specification in Cooper and Johri (2002) which introduces an additional input in the production technology referred to as organizational capital[6]. The accumulation of organizational capital depends on past production with the property that recent production contributes more to the stock of organizational capital than does older production. There is a vast empirical literature that documents the pervasive presence of learning-by-doing effects in the economy. Some recent evidence and references to other studies can be found in Thornton and Thompson (2001). There is less work on the idea that older production experience is less valuable than recent experience in reducing costs. One recent study of this phenomenon is Benkard (2000). Cooper and Johri also present both aggregate and plant level estimates of the specific form of learning-by-doing assumed in this paper.

The effectiveness of learning-by-doing in generating inertia in the aggregate price level is demonstrated in the context of a dynamic general equilibrium model with real money balances in preferences. Simulations from a linearized version of the model, calibrated to the US economy show that the model is capable of generating considerable inertia in the aggregate price level. The model is also very successful in generating additional persistence in output dynamics which is reminiscent of the results in Cooper and Johri.

There are few previous studies that look at the impact of learning-by-doing on economic fluctuations.[7] The closest to this paper is work by Cooper and Johri (2002) who show that learning-by-doing is extremely effective at propagating technology shocks in a real business cycle framework. Cooper and Johri use a representative agent framework and are agnostic about the issue of who actually learns from past production: workers or firms, and

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[6] Prescott and Visscher (1980) offer some interpretations of organizational capital. See also Atkeson and Kehoe (2001).

[7] There is a larger literature on the impact of learning-by-doing on economic growth with an emphasis on learning externalities. See Stokey (1988) or Romer (1986) for example.

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thus offer no account of possible decentralizations. In complementary work, Chang, Gomes and Schorfheide (2002) focus solely on learning that is embodied in workers and is fully captured in wages. They estimate the aggregate learning rate for the US using data from the PSID and incorporate this into a dynamic general equilibrium model with real shocks. They too are able to generate a persistent response of output to real shocks. A related literature studies the impact of learning externalities in dynamic general equilibrium models. Cooper and Johri (1997) show that these are very effective at propagating technology and preference shocks while Cook (1999) investigates nominal shocks. While potentially very important, externalities are ignored in this paper to focus on the impact of internal learning-by-doing on pricing decisions.

The next section presents the model. Section 3 discusses the calibration of key parameters, section 4 presents the simulation results. The final section concludes.

2 The Model

This section describes a monetary economy populated by many identical, infinitely lived consumers. The economy closely resembles that in Chari, Kehoe and McGrattan (2000) with the major departure occurring in the technological environment of firms to incorporate learning-by-doing. Each period, the economy finds itself in one of finitely many states, s_t. Let $s^t = (s_{0,...,}s_t)$ be the history of these states of the world. Along with labour and a good that is used both for consumption and investment, the commodities in the economy are money, a continuum of intermediate goods, and organizational capital.

There are a large number of final good producers who behave competitively and use the following technology for converting intermediate goods indexed by $i \in [0, 1]$ into final goods.

$$Y(s^t) = [\int Y_i^\eta(s^t)di]^{\frac{1}{\eta}} \tag{1}$$

Each period they choose inputs $Y_i(s^t)$ for all $i \in [0, 1]$, and output $Y(s_t)$ to maximize profits given by

$$\max P(s^t)Y(s^t) - \int P_i(s^{t-1})Y_i(s^t)di \tag{2}$$

subject to (1) where $P(s^t)$ denotes the price of the final good at history s^t, while $P_i(s^{t-1})$ is the price paid for the ith intermediate good in period t. Note that these prices are set before the realization of the period t shock. The solution to this problem gives us the input demand functions:

$$Y_i^d(s^t) = (\frac{P(s^t)}{P_i(s^{t-1})})^{\frac{1}{1-\eta}} Y(s^t). \qquad (3)$$

The zero profit condition can be used to infer the level of final goods prices from the intermediate good prices:

$$P(s^t) = [\int P_i^{\frac{\eta}{\eta-1}}(s^t) di]^{\frac{\eta-1}{\eta}}. \qquad (4)$$

There are a large number of intermediate goods producers, indexed by the letter i who operate in a Dixit-Stiglitz style imperfectly competitive economy. Each such producer produces intermediate goods with a technology subject to learning-by-doing which is described by the following equations:

$$Y_i(s^t) = F(K_i(s^t), N_i(s^t), H_i(s^{t-1}), A(s^t)) \qquad (5)$$

where $N_i(s^t)$ is the amount of labor hired, $K_i(s^t)$, the amount of physical capital hired by the firm to produce output, $Y_i(s^t)$ and $A(s^t)$ is a common term governing the level of total factor productivity. In addition to these conventional inputs, the firm carries a stock of organizational capital, $H_i(s^{t-1})$ which is an input in the production technology. Organizational capital refers to the information accumulated by the firm in the process of past production regarding how best to organize its production activities and deploy its inputs.[8] As a result, the higher the level of organizational capital, the more productive the firm. The production function is increasing in all three inputs. Organizational capital is accumulated using output and the current stock of organizational capital:

$$H_i(s^t) = \Phi(H_i(s^{t-1}), Y_i(s^t)) \qquad (6)$$

with Φ increasing in both arguments. Here all producers begin life with a positive and identical endowment of organizational capital. This form of

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[8] Atkeson and Kehoe (2001) model and estimate the size of organizational capital for the US manufacturing sector and find that it has a value of roughly 66 percent of physical capital. While the broad interpretation of organizational capital and its accumulation is similar, the details are quite different, especially there is no depreciation of past learning.

learning-by-doing and its implications for aggregate output in response to technology shocks are discussed in detail in Cooper and Johri (2002). The paper offers a detailed justification for the modelling assumptions and a number of estimates of the learning technology at different levels of aggregation.

Each intermediate goods producer faces a downward sloping demand function for his product (3) which comes from the profit maximization problem of the final goods producers discussed above. Prices are set by all producers at the beginning of each period before the realization of the event s_t and cannot be changed during the period once set. Thus there are two differences between the intermediate goods firm's problem in Chari-Kehoe-McGrattan (2000) and this paper. First, the technology has been modified to incorporate learning-by-doing. Second, firms set prices for one period in a synchronized way which is a special case of their N period overlapping contracts structure with N=1.

In period t, each producer chooses a price $P_i(s^{t-1})$ and the level of organizational capital for period $t + 1$, $H_i(s^t)$, to maximize discounted profits:

$$\max \sum_{t=1}^{\infty} \sum_{s^\tau} Q(s^t \mid s^{t-1})[P_i(s^{t-1})Y_i(s^t) - P(s^t)V(s^t)] \qquad (7)$$

subject to (3) and (6) where $Q(s^t \mid s^{t-1})$ is the appropriate discount rate to use to price revenue and costs in adjoining periods which is determined in the household problem. $V(s^t)$ denotes the real cost function which has the real wage,$w(s^t)$,the real rental rate on capital, $r(s^t)$, $H_i(s^{t-1})$ and $Y_i(s^t)$ as arguments. The cost function is obtained from the cost minimization exercise:

$$V_i(s^t) \quad = \quad \min_{N_{it,}K_{it}} w(s^t)N_i(s^t) + r(s^t)K_i(s^t) \qquad (8)$$

subject to

$$F(K_i(s^t), N_i(s^t), H_i(s^{t-1}), A(s^t)) \geq \overline{Y}. \qquad (9)$$

The solution to this minimization problem implies

$$\frac{w(s^t)}{r(s^t)} = \frac{F_N(s^t)}{F_K(s^t)} \qquad (10)$$

and

$$F(K_i(s^t), N_i(s^t), H_i(s^{t-1}), A(s^t)) = \overline{Y} \qquad (11)$$

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from which the input demands can be obtained. Substituting these into (8) yields the cost function. Taking $V_i(s^t)$ as given, the solution to the maximization problem above implies two first order conditions:

$$\lambda^F(s^t) = \sum_{s^{t+1}} Q(s^{t+1} \mid s^t) \left\{ \lambda^F(s^{t+1})\Phi'_H(H_i(s^{t+1}), Y_i(s^{t+1})) - P(s^{t+1})V'_{Hi}(s^{t+1}) \right\}$$

(12)

and

$$\sum_{s^t} Q_t[Y_{it}(\frac{-\eta}{1-\eta}) + P_t V'_{Yit}(\frac{1}{1-\eta})\frac{Y_{it}}{P_{it}(s^{t-1})} - \lambda^F_t \Phi'_Y(H_{it}, Y_{it})(\frac{1}{1-\eta})\frac{Y_{it}}{P_{it}(s^{t-1})}] = 0$$

(13)

where $\lambda^F(s^t)$ is the Lagrange multiplier associated with the organizational capital accumulation equation once (3) has been used to substitute out for $Y_i(s^t)$. The latter first order condition determines the optimal level of prices to be set by the producer. Note that the state has been suppressed in this equation except where it is needed to avoid confusion. Raising prices by one unit causes output to fall since producers face downward sloping demand curves for their product. The first term captures the net impact on current revenue of the higher price but lower output, while the second term represents the current cost savings from producing less. The third term appears because the producer realizes that he faces a forward looking problem due to learning by doing. The accumulation equation for organizational capital implies that a reduction of current period output will lead to a reduction in organizational capital available tomorrow. The third term captures the value of this organizational capital lost to the firm and is made up of three parts. The term $(\frac{1}{1-\eta})\frac{Y_{it}}{P_{it}}$ represents the reduction in output due to the higher price, while $\Phi'_Y(H_{it}, Y_{it})$ represents the reduction in H_{it+1} due to the reduction in output which must be evaluated at λ^F_t, the marginal value of organizational capital to the firm.

Equation (12) determines the value of having available an additional unit of organizational capital for use by the firm in the following period. First, the additional organizational capital improves profits by reducing costs, as captured by the second term on the right hand side (recall V'_{Hit+1} is negative). Second, it adds to the ability of the organization to learn from production thus raising future organizational capital. This additional organizational capital has a value of λ^F_{t+1} for the firm. All this must be discounted by the price of one dollar in period t+1 in units of period t dollars. Alternatively

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one could say that the firm sets prices so that the value of accumulating an additional unit of organizational capital today is just equal to the discounted value of organizational capital tomorrow.

The intuition in (12) and (13) suggests that firms face a trade-off between current profits and future profits which is not present in the traditional price setting problem. Charging a higher price today lowers the amount of organizational capital available tomorrow which raises future costs and lowers future profits. As a result, firms will optimally select a lower price in the presence of learning by doing than they would otherwise set. This can be seen by re-writing (13) as

$$P_{it}(s^{t-1}) = \frac{\sum_{s^t} Q_t Y_{it}(P_t V'_{Yit} - \lambda_t^F \Phi'_{Yit})}{\eta \sum_{s^t} Q_t Y_{it}} \tag{14}$$

and noting that the second term appears only when the learning-by-doing mechanism is present.

Note that each intermediate good firm earns positive profits even in the presence of a constant returns to scale technology due to the accumulation of organizational capital. However there is no entry or exit in this industry by assumption.

2.1 Consumers

The economy is populated by a large number of identical consumers whose preferences are defined over consumption of final goods $(C(s^t))$, leisure $(L(s^t))$ and real money balances $(M(s^t)/P(s^t))$. Each consumer maximizes the sum of discounted expected utility subject to a sequence of budget constraints (given below) by choosing the optimal quantity of these goods to consume, the amount of hours to work and how much to invest in physical capital $(K(s^t))$ and one-period nominal bonds $B(s^{t+1})$ each period. They take as given prices $(P(s^t))$, wages $(w(s^t))$ and interest rates $(r(s^t))$. If $0 < \beta < 1$ is the discount factor, then the household's problem is

$$\max \sum_{t=0}^{\infty} \sum_{S_t} \beta^t \pi(s^t) U(C(s^t), L(s^t), M(s^t)/P(s^t)) \tag{15}$$

subject to the sequence of budget constraints:

$$P(s^t)(C(s^t) + K(s^t)) + M(s^t) + \sum_{s_{t+1}} Q(s^{t+1} \mid s^t) B(s^{t+1}) \leq M(s^{t-1}) \tag{16}$$

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$$+B(s^t) + \Pi(s^t) + P(s^t)(T(s^t) + w(s^t)N(s^t) + [r(s^t) + 1 - \delta]K(s^{t-1}))$$

and

$$N(s^t) + L(s^t) \leq 1. \tag{17}$$

In addition we need the sequence of borrowing constraints $B(s^{t+1}) \geq B^u$ for some large negative value of B^u. Consumers lend out their stock of physical capital and labor services to intermediate goods producers and receive wages and interest income. Each of the nominal bonds, $B(s^{t+1})$, provides one dollar in state s^{t+1} at the expense of $Q(s^{t+1} \mid s^t)$ dollars in state s^t. In addition they receive $\Pi(s^t)$, the current profits of intermediate goods producers as owners of all firms and $T(s^t)$ the current real net transfers from the monetary authority. The initial conditions $K(s^{-1})$, $M(s^{-1})$, $B(s^0)$ are also given.

The first order conditions associated with the household problem are:

$$-U_L(s^t) = U_C(s^t)w(s^t) \tag{18}$$

$$U_{C_t}(s^t) = \beta \sum_{s_{t+1}} \pi(s^{t+1} \mid s^t)\{U_C(s^{t+1})(1 - \delta + r(s^{t+1}))\} \tag{19}$$

$$U_{M/P}(s^t) = U_C(s^t) - \beta \sum_{s_{t+1}} \pi(s^{t+1} \mid s^t)\frac{P(s^t)}{P(s^{t+1})}U_C(s^{t+1}) \tag{20}$$

$$Q(s^{t+1} \mid s^t) = \beta\pi(s^{t+1} \mid s^t)\frac{U_C(s^{t+1})P(s^t)}{U_C(s^t)P(s^{t+1})} \tag{21}$$

The interpretation of these first order conditions is quite standard. Equation (18) implies that the marginal rate of substitution between consumption and leisure should equal the real wage rate while equation (19) is the usual euler equation. Equation (20) states that the consumer should choose to save nominal balances to the point that the current net benefit of saving an additional dollar (which is made up of the marginal utility lost due to lower current consumption minus the marginal utility gained due to higher money balances) is just equal to the discounted expected benefit next period(composed of the marginal utility of the extra consumption that can be bought next period which in turn depends on the expected value of inflation over this interval).

The nominal money supply process is

$$M(s^t) = \mu(s^t)M(s^{t-1}) \tag{22}$$

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where $\mu(s^t)$ is a stochastic process. Consumers receive lump sum transfers of new money balances which satisfy:

$$P(s^t)T(s^t) = M(s^t) - M(s^{t-1}). \tag{23}$$

In addition to these first order conditions from the consumer and firm problem we have market clearing conditions which require that the total stock of capital supplied by consumers is equal to the sum of capital rented by all intermediate goods firms. Similarly the total hours of labor supplied by consumers should equal the sum of labor hours demanded by all intermediate goods firms. Recall that while prices are chosen by firms before uncertainty about shocks is resolved, factor demands are chosen afterwards. Bond market clearing requires that $B(s^{t+1}) = 0$. The resource constraint for the economy is

$$C(s^t) + K(s^t) = Y(s^t) + (1 - \delta)K(s^{t-1}). \tag{24}$$

An equilibrium is a collection of allocations for consumers, $C(s^t), N(s^t)$, $K(s^{t-1})$, $B(s^{t+1})$ and $M(s^t)$; allocations for intermediate goods firms: $N_i(s^t)$, $K_i(s^t), H_i(s^{t-1})$ for all i $\in [0,1]$; allocations for final goods firms: $Y(s^t), Y_i^d(s^t)$ for all i $\in [0,1]$; together with prices $w(s^t), r(s^t), Q(s^{t+1} \mid s^t), P(s^t), P_i(s^{t-1})$, for all i $\in [0,1]$ that satisfy the following conditions: i) taking prices as given the consumer allocations solve the consumer's problem; ii)taking all prices but its own as given, each intermediate goods producer's price and stock of organizational capital satisfies (12) and (13); iii) taking prices as given the final goods producers allocations solve the final goods producer problem; iv) the factor market conditions and resource constraint hold and the bond market clears. Only symmetric equilibria in which all consumers and producers behave identically are studied.

3 Computation method and calibration

The model is solved using the method outlined in King and Watson (2002) using a linear approximation to the system of equations including the first order conditions of the intermediate goods producers problem, the first order conditions from the consumers problem, the production function, the resource constraint for the economy and the accumulation equation for physical and organizational capital. Some variables are growing in steady state - they are rendered stationary by dividing by the stock of money in the economy.

In order to simulate the economy, functional forms have to be specified. The specification for preferences is similar to Chari, Kehoe and McGrattan (2000):

$$U(C, L, M/P) = \frac{1}{1-\sigma}[(\omega C^{(\theta-1)/\theta} + (1-\omega)(\frac{M}{P})^{(\theta-1)/\theta})^{\frac{\theta}{\theta-1}} L^{\psi}]^{1-\sigma} \qquad (25)$$

which also provides values for the parameters related to preferences. Following Chari, Kehoe and McGrattan (2000) I set $\omega = .94$, and $\theta = .39$ and set ψ so that the fraction of the time endowment spent on working in steady state is .3. Following Cooper and Johri (2002) β, the discount factor is set to .984 while δ, the depreciation rate is set to .021, the value estimated in Johri and Letendre (2004).[9] Turning to the specification of technology, intermediate goods producers are assumed to use a Cobb Douglas production function to produce output, given by

$$Y_i(s^t) = N_i^{\alpha}(s^t)K_i^{1-\alpha}(s^t)H_i^{\varepsilon}(s^{t-1})A(s^t). \qquad (26)$$

The accumulation equation for organizational capital is

$$H_i(s^t) = H_i^{\gamma}(s^{t-1})Y_i^{\phi}(s^t). \qquad (27)$$

Since the learning-by-doing technology is at the heart of the mechanism generating the sluggish response of prices I explore a range of values for the parameters ε, γ and ϕ. These values are based on available estimates of these parameters. Cooper and Johri (2002) provide estimates of a number of specifications of the technology in (26) and (27) for the US manufacturing sector. The estimates of ε range from around .1 to .49 depending on whether constant returns to scale was imposed on (27). The estimates of γ were concentrated around .5 and went as high as .79 while ϕ was either set to 1-γ or to 1. In a related specification of learning in the aircraft industry, Benkard (2000) finds that a doubling of production related experience can lead to a 36 percent fall in unit labour requirements which corresponds to roughly $\varepsilon = .45$. By comparison the benchmark learning rate in the traditional learning literature (see Irwin and Klenow (1994)) is around 20 percent corresponding to $\varepsilon = .263$. However as discussed in Cooper and Johri and in Benkard, the traditional learning model imposes quite strong restrictions on the production

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[9]Simulations show that the results are not sensitive to small variations in the above parameters.

of experience, corresponding to a linear version of (27) with all parameters set to unity:

$$H(t+1) = H(t) + Y(t).$$

I set the labour share, $\alpha = .6$ in all specifications explored. The steady state capital output ratio is a function of the technological parameters as well as β, δ and η. As the learning-by-doing parameters are varied, η is set to maintain the steady state capital output ratio at 10.6. The associated steady state markup varies around 1% above marginal cost. The persistence in the growth rate of money was set to .68, taken from Chari, Kehoe and McGrattan (2002).

4   Results

4.1   The dynamics of the price level

The main question addressed in this section is how much additional inertia in the aggregate price level is generated by adding learning-by-doing to the benchmark one period sticky price model when the economy is hit by money growth shocks. I will focus on two measures that describe the additional inertia added by these models over and above the benchmark one-period sticky price model. The first measure is the autocorrelation between inflation and its first lag (ρ_π). While this measure hides a lot of information about differences in the dynamics of prices across models, it is a useful summary measure commonly reported in other studies. In addition, I will look at the response of the price level to a one percent increase in the growth rate of money. In the benchmark model, prices jump by 2.6 percent in the quarter after the shock. In order to highlight the inertia generated by learning-by-doing (lbd), I will report the number of quarters that it takes for the price level to rise 2.6 percent above steady state for various specifications of the lbd model.

Table 1 reports the first order autocorrelation coefficient of inflation and the quarter in which the price level rises 2.6 percent above steady state for various specifications of the model with learning-by-doing (lbd) as well as the benchmark model without lbd. Row 1 of the table corresponds to the benchmark model which generates a huge amount of inertia in inflation with $\rho_\pi = .85$. However, this result must be interpreted with caution because of

the unrealistic behavior of investment in physical capital in response to an increase in the growth rate of money. Impulse responses show that consumers respond to a one percent increase in the rate of growth of money with a 176 percent increase in investment. The consequent rise in the capital stock sharply curtails the rise in marginal costs for firms which therefore raise prices slowly. In order to curtail this unrealistic surge in investment, I add adjustment costs to the model of the following form used in Chari, Kehoe and McGrattan:

$$\frac{v}{2}(\frac{x(t)}{k(t)} - \delta)^2$$

where x(t) corresponds to investment in physical capital in period t. In all specifications of the model, the adjustment cost parameter ν was set to keep the ratio of the standard deviations of investment and output equal to their value in the US data. This modification will ensure that the results are not contaminated by an unrealistic increase in investment.

Row 2 of table 1 reports results for the benchmark model with adjustment costs. Now $\rho_\pi = .06$, which shows the well known inability of the one-period sticky price model to generate inflation inertia. As discussed above, the second column reports that prices jump by 2.6 percent in quarter 2. The next three rows look at various lbd specifications with increasing amounts of persistence in inflation. Row 3 sets $\varepsilon = .263$, which correspond to a learning rate of 20 percent and sets $\eta = \phi = .5$ while row 4 raises ε to .45. Finally row 5 lowers ε back to .263 but raises ϕ to 1. The addition of learning-by-doing to the sticky price model raises the inflation autocorrelation coefficient by 166 percent to .16. This rises further in row 4 to .24 and .26 in row 5. Learning-by-doing generates a slow moving price level for two reasons. First, the additional output produced by firms to meet the increase in demand for their product leads to the accumulation of organizational capital. This prevents future marginal costs from rising too quickly. Second, firms wish to take advantage of the high demand for their product to learn, and therefore raise prices by less than marginal cost. The cumulative impact of these effects can be seen clearly in the response of prices to a money shock. The third column of table 1 shows that it takes three additional quarters for the price level to rise by 2.6% in row three compared to the benchmark model. The specification in row 4 generates even more inertia in the price level: it takes until quarter 8 for prices to rise by that much. This rises to quarter 11 in row 5, two and a half years after the initial shock to the

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economy. These results are quite sensitive to the curvature of the utility function, controlled by σ. For example, raising σ from 2 to 5, raises the autocorrelation of inflation in the specification in rows 4 and 5 to .63 and .88 (numbers reported in column two of table 1) respectively. The dramatic rise in the persistence of inflation in response to σ can be understood as follows. An increase in σ, makes consumers more sensitive to the money shock leading to a larger increase in demand for intermediate goods. A by-product of the large increase in production of intermediate goods is the creation of a large amount of organizational capital which puts downward pressure on real costs causing nominal marginal costs to rise very slowly.

It is interesting to note the contrast in the behaviour of the firm as we raise σ : when it is low, firms choose to raise prices less than marginal cost increases whereas when it is high firms raise prices by more than increases in marginal cost. In order to understand this, recall that in the impact period prices are fixed so firms increase output to meet demand, creating organizational capital for the next period. Organizational capital is valued both because it reduces costs and because it improves the ability of the firm to learn. Depending on the size of the shock and the curvature in the learning technology, the value of organizational capital to the firm can rise or fall which will lead the firm to set the change in prices by more or less than marginal cost as suggested by equation (14). When the shock is small, firms desire to learn even more and further temper increases in prices to take advantage of the temporarily high demand. In contrast when the shock is large so much organizational capital has been accumulated in the impact period that it is more profitable to raise prices by more than marginal costs and slow down the amount of learning. This attempt by the firm to mitigate the effect of the demand shock by raising prices may be seen as a form of a firm level "wealth effect". Moreover we see that firms respond to exogenous shocks to demand by using prices to control the amount of learning that occurs.

4.2   Output and real variables

This section discusses the behaviour of real variables with an emphasis on the dynamics of output. I begin by asking how much persistence in output is generated by the lbd model when the only shock is a persistent increase in the growth rate of money. As highlighted by Chari, Kehoe and McGrattan (2000), models with staggered price setting have difficulty generating realistic output dynamics in response to money shocks. I then briefly ask if the

lbd model is capable of generating second moments for real aggregate variables comparable to those seen in the US data. This discussion occurs in an environment with both money and technology shocks.

Table 2 reports the autocorrelation of output with its first lag (ρ_y). It is evident from Row 1 that the benchmark model generates no persistence in aggregate output ($\rho_y = .006$). The lbd models all do a much better job of propagating output over time. Row 2, ($\varepsilon = .263, \gamma = \phi = .5$) generates an autocorrelation coefficient of .21 while raising ε to .45 in row three raises ρ_y to .44. In row 4 ε is reduced back to .263 while ϕ is raised to 1 which generates $\rho_y = .56$. The increase in persistence of output is reminiscent of the results in Cooper and Johri (2002) and are easy to understand. The additional organizational capital created by firms in responding to the increase in demand for their product raises future firm productivity thus propagating the effects of the money shock forward through time.

Table 3 reports unfiltered theoretical second moments for the benchmark model as well as the three specifications of the lbd model discussed above. The corresponding moments for log-linearly detrended US data (taken from Cooper and Johri (2002)) are reported in the last row. In all cases, the economy is hit with both money and total factor productivity shocks which are uncorrelated with each other. The tfp shock has a first order autocorrelation coefficient of .95 and a standard deviation of .007 while the money shock has a standard deviation of .00498 which is the value used in Nelson (1998). Looking across the rows of table 3, all the models do a good job of capturing the basic features of business cycles. Consumption, hours and investment are all procyclical and there is evidence of consumption smoothing. The various models also inherit some common problems: in all cases consumption is more volatile than in the US data and too highly correlated with output. In fact the behaviour of consumption is virtually identical in all rows of table 3. Similarly, investment is too highly correlated with output in all the models of table 3 relative to the data. The first clear differences across models appear when we study the behaviour of hours. The baseline model without learning-by-doing generates too much relative volatility compared to the data. The introduction of learning-by-doing in row 2 lowers the relative volatility of hours from .9 to .67 which is much closer to the value seen in US data (.52). Raising either ε as in row 3 or ϕ as in row 4, further lowers the value of this moment to about .5. This improvement comes at the expense of worsening the correlation between hours and output which was already too low in the benchmark model (.5) relative to the data (.71). The other stark difference

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between the models is apparent in their ability to magnify and propagate shocks over time. The autocorrelation of output, ρ_y is .75 in the benchmark, takes a value of .88 in row 2 and further rises to .94 in rows 3 and 4 which is very close to the value found in US data (.96). Similarly. the volatility of output increases from .0325 in the benchmark case to .117 in row 4.

5 Conclusions

Learning-by-doing is introduced into a monetary dynamic general equilibrium model. In order to highlight the ability of the model to generate inertia in the aggregate price level, all other sources of inertia commonly used in the literature such as menu costs, staggered price or wage contracts are ignored. The model therefore relies on the minimal amount of price stickiness needed: prices are chosen before the shocks occur. A calibrated version of the model generates considerable inertia in both inflation and output dynamics in response to money growth shocks. The model also does reasonably well in matching moments that capture key features of the US business cycle.

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	$\rho_\pi, \sigma = 2$	$\rho_\pi, \sigma = 5$	quarter
no lbd, no adjustment costs	.85		
benchmark: no lbd	.06		2
$\varepsilon = .263, \phi = \gamma = .5$	.16	.04	5
$\varepsilon = .45, \phi = \gamma = .5$	.24	.63	8
$\varepsilon = .263, \phi = 1, \gamma = .5$	.26	.88	11

Table 1: Price-level inertia

	ρ_y
benchmark: no lbd	.006
$\varepsilon = .263, \phi = \gamma = .5$	.21
$\varepsilon = .45, \phi = \gamma = .5$	.44
$\varepsilon = .263, \phi = 1, \gamma = .5$	.56

Table 2: Output Persistence

	$\sigma_{C/Y}$	$\sigma_{I/Y}$	$\sigma_{H/Y}$	$\rho_{C/Y}$	$\rho_{I/Y}$	$\rho_{H/Y}$	ρ_Y	σ_Y
benchmark: no lbd	0.92	1.31	0.90	0.99	0.98	0.50	0.75	0.0325
$\varepsilon = .263, \phi = \gamma = .5$	0.92	1.30	0.67	0.99	0.99	0.43	0.88	0.048
$\varepsilon = .45, \phi = \gamma = .5$	0.93	1.30	0.49	0.99	0.96	0.35	0.94	0.083
$\varepsilon = .263, \phi = 1, \gamma = .5$	0.92	1.30	0.48	0.99	0.98	0.37	0.94	0.117
US Data	0.69	1.30	0.52	0.89	0.60	0.71	0.96	0.04

Table 3: Second Moments