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EFFICIENT INTRA-HOUSEHOLD ALLOCATIONS: A GENERAL CHARACTERISATION AND EMPIRICAL TESTS

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ABSTRACT

The neo-classical theory of demand applies to individuals yet in empirical work it is usually taken as valid for households with many members (or even for whole economies!). This paper explores what the theory of individuals implies for households which have more than one member. We make minimal assumptions about how the individual members of the household resolve conflicts. All we assume is that however decisions are made, outcomes are efficient. We refer to this as the collective setting.

The main result we derive shows that in the collective setting household demands must satisfy a symmetry and rank condition on the Slutsky matrix. This condition includes the usual symmetry condition as a special case. We also present some further results on the effects on demands of variables that do not modify preferences but that do affect how decisions are made. The prime candidates for such variables are the incomes of the individual in the household.

We apply our theory to a series of surveys of household expenditures from Canada. We use a flexible quadratic log demand system as our maintained model. The tests of the usual symmetry conditions are rejected for two person households but not for one person households. Moreover we show that the estimates for the two person households, assuming a single utility function for the household, display problems that are absent for single person households.

We then test for our collective setting conditions on the couples data. None of the collective setting restrictions are rejected. We conclude that the collective setting is a plausible and tractable next step to take in the analysis of household demand and labour supply behaviour.

I. INTRODUCTION

When considering household behaviour and welfare it almost universally assumed that the many person household can be treated as though it has a single set of goals. The adoption of this 'unitary' model is very convenient. Methodologically, however, it stands on weak grounds. Neo-classical utility theory applies to individuals and not to households. There is also mounting empirical evidence that the unitary model does not hold. In particular the fundamental observable implication of utility theory - symmetry of the Slutsky matrix - is regularly rejected on household data (see, for example, Blundell *et al* (1992) and Browning and Meghir (1991))[1]. Usually this rejection has either been seen as a rejection of utility theory or it has been attributed to technical problems (inadequate functional forms, inappropriate separability assumptions, misspecification of the stochastic structure and so on). Thus it has been concluded either that utility theory is false or that it is untestable.

Our answer to these 'problems' with neo-classical utility theory is completely different. We claim that the theory has not been taken seriously enough. We start from the premise that utility theory does apply, but only to individuals and not to households. In this paper we present a general characterisation of an alternative model of household behaviour to the unitary model, namely the 'collective' model suggested in Chiappori (1988 and 1992). The two assumptions of the collective model are that each person in the household has his or her own preferences and that collective decisions are Pareto efficient.

We do not present a formal justification for the assumption of efficiency but it has a good deal of intuitive appeal. First, axiomatic models of bargaining with symmetric information generally assume efficiency. Second, the household is one of the preeminent examples of a repeated 'game' so that we feel justified in assuming that each person knows the preferences of the other

[1] Another important implication of the unitary model - income pooling- is also routinely rejected; see, for example, Bourguignon *et al* (1993), Thomas (1990) and Phipps and Burton (1992).

people in the household. Given this symmetry of information and the fact that the game is repeated it is plausible that agents find mechanisms to support efficient outcomes even if behaviour is non-cooperative. Finally, efficiency includes as a special case the unitary model. This is not to say, however, that we cannot envision circumstances that would lead to non-efficient outcomes. Clearly, if there is asymmetric information (for example, one partner can consume some goods without the other partner knowing) then the case for efficiency is weakened. In the end this is an empirical matter: what does the collective setting imply for household behaviour and are these predictions rejected by the data? This paper is directed to these issues.

In previous papers with other co-authors (Bourguignon *et al* (1993), Browning *et al* (1994) and Bourguignon *et al* (1994)) we investigated what could be learned from conventional family expenditure data about what goes on inside the household. In this analysis we used only cross-section variation in the data; that is, we did not exploit any price variation. This paper investigates what can be learned when we also have price variation. This has implications for two areas: demand analysis on time series of family expenditure surveys (for example, the U.S. CES) and the analysis of labour supply on cross-sections (or panel data) where the prices that vary across individuals are wages. Although the latter is the more important application, we have chosen initially to concentrate on the former since the analysis of labour supply for individuals raises many problems that are less pressing in the demand case (for example, wages may be non-linear, endogenous and unobserved for some individuals).

We make minimal assumptions in the analysis presented here. For example, we do not assume that the econometrician can determine which goods are private and which public within the household. Similarly, we do not assume anything about individual preferences except that they can be represented by conventional utility functions. Despite this we find that we can make very specific predictions about household behaviour. The principal result in section II is that the collective setting implies that the Slutsky matrix need not satisfy symmetry; rather it must be equal to a symmetric matrix plus a rank one matrix. In section III we extend the analysis in three different directions. The most important of these extensions is to allow for variables that do not affect individual preferences directly but that do affect distribution within the household. We term such variables distribution factors. It turns out that the collective model implies that there is a close relationship between the influence of such variables on demand and price responses.

In section IV we present a flexible parametric demand system and derive the implications of the predictions of the previous sections for the parameters of this system. This includes a novel analysis of testing for the rank of a matrix in our context. In section V we use Canadian Family Expenditure Survey (FAMEX) data on single person households and households containing just a married couple. We first show that Slutsky symmetry is not rejected for singles but it is for couples. To the best of our knowledge this is the first time that anyone has shown that symmetry is not rejected for singles. We then go on to test the predictions of the collective setting derived in sections II and III on the couples data. We do not reject any of these restrictions. This provides strong, though preliminary, support for our view that the collective model is a viable alternative to the unitary model. In the concluding section we discuss some possible areas of future research.

II. THEORY - THE GENERAL CASE

II.1 The collective setting.

We consider a two person (A and B) household. Household purchases are denoted by the n-vector $\mathbf{q}$ with associated market price vector $\mathbf{p}$[2]. Household demands are divided between three uses: private consumptions by each person, $\mathbf{q}^A$ and $\mathbf{q}^B$ and public consumption $\mathbf{Q}$. The constraint for this intra-household allocation is

[2] Formally $\mathbf{q}$ could include leisure (so that $\mathbf{p}$ includes the wages (or virtual wages for non-participants) of A and B). As already indicated we shall not be emphasising the implications of our results for labour supply. Also, in the following we assume that all goods are non-durable; the extension to durable goods along the usual lines is straight-forward.

$$\mathbf{q}^A+\mathbf{q}^B+\mathbf{Q} = \mathbf{q} \tag{2.1}$$

Thus any good can be consumed privately or as a public good. For example, expenditures on 'telephone services' includes a public element (the rental) and a private element (the actual use of the telephone). The household budget constraint is given by:

$$\mathbf{p}'\mathbf{q} = x \tag{2.2}$$

where x is total household expenditure.

We adopt a Beckerian framework in which each person has his or her own preferences over the goods consumed in the household. Whether consumption of a particular good by a particular person is, by nature, private, public or both is irrelevant for our results. Also, each members' preferences can depend on both members' private and public consumption; this allows for altruism, externalities or any other preference interactions. Our results are consistent with all possible situations. Thus the two members of the household have preferences over $(\mathbf{q}^A,\mathbf{q}^B,\mathbf{Q})$. We make conventional assumptions about preferences:

> *Assumption 1*. Preferences can be represented by utility functions $u^A = \upsilon^A(\mathbf{q}^A,\mathbf{q}^B,\mathbf{Q})$ and $u^B = \upsilon^B(\mathbf{q}^A,\mathbf{q}^B,\mathbf{Q})$. These utility functions are quasi-concave, non-decreasing and strictly increasing in at least one argument.

Given preferences and the budget constraint (2.2) we need to specify the mechanism that the household uses to decide on what to buy. Of course, if $\upsilon^A(.)$ and $\upsilon^B(.)$ represent the same ordinal preferences then we have the conventional 'unitary' model. Alternatively we could assume that one or other of the partners can impose his or her preferences and use the corresponding utility function in the usual way; this also returns us to the unitary model. More ambitiously, we could model the interaction as some (cooperative or non-cooperative) game.

As discussed in the introduction, we adopt a minimalist description of the household decision process. Our approach is axiomatic; specifically we only postulate that the outcome is always efficient. Following Chiappori (1992) we refer to models that allow for different preferences with efficiency as the 'collective' setting. The necessary conditions we derive must hence be satisfied whenever the decision process satisfies the efficiency property. This characterises a large class of models including the unitary model and bargaining models (at least with symmetric information).

Definition. Let $(\mathbf{q}^A, \mathbf{q}^B, Q)$ be given (demand) functions of $(\mathbf{p}, x)$. These functions are compatible with **collective rationality** if and only if there exists two utility functions $\upsilon^A(\mathbf{q}^A, \mathbf{q}^B, Q)$ and $\upsilon^B(\mathbf{q}^A, \mathbf{q}^B, Q)$ such that for every $(\mathbf{p}, x)$, $(\mathbf{q}^A, \mathbf{q}^B, Q)$ is Pareto-efficient for the household, subject to the constraints (2.1) and (2.2).

Our first result captures the familiar result that efficient outcomes can be characterised as the outcome of a weighted utilitarian maximisation problem.

Proposition 1. Let $(\mathbf{q}^A, \mathbf{q}^B, Q)$ satisfy the collective setting conditions. Then there exists a function $\mu(\mathbf{p}, x)$ such that $(\mathbf{q}^A, \mathbf{q}^B, Q)$ is the solution to:

$$\max_{(\mathbf{q}^A, \mathbf{q}^B, Q)} \mu\upsilon^A(\mathbf{q}^A, \mathbf{q}^B, Q) + (1-\mu)\upsilon^B(\mathbf{q}^A, \mathbf{q}^B, Q)$$

$$\text{subject to } \mathbf{p}'(\mathbf{q}^A + \mathbf{q}^B + Q) = x.$$

The $\mu(.)$ function is of fundamental importance for the collective setting. It has an obvious interpretation as a 'distribution of power' function. If $\mu = 1$ then the household behaves as though A always gets their way and if $\mu = 0$ then it is as though B is the effective dictator. For intermediate values the household behaves as though each person has some decision making power. Note that $\mu(.)$ must be allowed to depend on prices and total expenditure since these influence the distribution of 'power' within the household. We make the following assumptions about the distribution function[3]:

> *Assumption 2.* The distribution function $\mu(.)$ is continuously differentiable and zero homogeneous in $(\mathbf{p},x)$.

The zero homogeneity assumption is uncontroversial. The smoothness assumption is made for convenience; it is unclear to us how restrictive it is. In the next section we shall allow the distribution function $\mu(.)$ to depend on other factors as well (for example, the individual incomes of the two partners) but for now we explore further the basic model.

In the unitary model there is a well defined household utility function which takes as its arguments the household demands $\mathbf{q}$. It turns out that the same is true of the collective setting except that the household utility function now also includes the $\mu(.)$ function.

> *Definition.* In the collective setting the **household utility function** is defined by $u = \upsilon(\mathbf{q},\mu) =$
>
> $$\max_{(\mathbf{q}^A,\mathbf{q}^B,Q)} \mu\upsilon^A(\mathbf{q}^A,\mathbf{q}^B,Q) + (1-\mu)\upsilon^B(\mathbf{q}^A,\mathbf{q}^B,Q)$$
>
> $$\text{subject to} \quad \mathbf{q}^A+\mathbf{q}^B+Q = \mathbf{q}.$$

Thus the direct household utility function now includes prices and total expenditure[4]. The critical point to note, however, is that the latter enter only through the single index $\mu(.)$. The same will also be true of any other variables that affect distribution within the household but not preferences.

[3] It would be more satisfactory to make more primitive assumptions on preferences and derive these conditions as conclusions but we are content here to simply assume them.

[4] Our approach can readily be extended to allow for household production. To do this we simply include an extra constraint for the production in this definition. The household utility function thus obtained will still be a function of $\mathbf{q}$ and μ, which is the main requirement for the results below to hold true.

From this restriction flows all of the results below. The maximisation of the household utility function $\upsilon(\mathbf{q},\mu)$ subject to the budget constraint (2.2) gives the same household demands as the optimisation problem in proposition 1.

Given utility functions for the two people we can define some dual representations for 'household' preferences. For any μ, define the household indirect utility function $V(\mathbf{p},x,\mu)$ as the maximand of the optimisation problem in Proposition 1. For fixed μ this function has all of the conventional properties of an indirect utility function. Since V(.) is strictly increasing in x, we can define the cost function $E(\mathbf{p},u,\mu)$ implicitly by $V(\mathbf{p},E(\mathbf{p},u,\mu),\mu) = u$ or by:

$$E(\mathbf{p},u,\mu) =$$

$$\min_{(\mathbf{q}^A,\mathbf{q}^B,Q)} \{\mathbf{p}'(\mathbf{q}^A+\mathbf{q}^B+Q) \mid \mu\upsilon^A(\mathbf{q}^A,\mathbf{q}^B,Q) + (1-\mu)\upsilon^B(\mathbf{q}^A,\mathbf{q}^B,Q) \geq u\} \qquad (2.3)$$

Equivalently, we could define dual representations $\tilde{V}(\mathbf{p},x,u^B)$ and $\tilde{E}(\mathbf{p},u^A,u^B)$ in the obvious way. There is, of course, a one-one relationship between any two of the representations $\upsilon(.)$, $V(.)$, $E(.)$, $\tilde{V}(.)$ and $\tilde{E}(.)$.

II.2 Restrictions on demands.

Given preferences and the budget constraint we can derive compensated and uncompensated demands. Applying the Envelope Theorem to (2.3) we have compensated household demands $\mathbf{q} = \mathbf{h}(\mathbf{p},u,\mu)$. Substituting $u = V(\mathbf{p},x,\mu)$ we have uncompensated household demands $\mathbf{q} = \mathbf{f}(\mathbf{p},x,\mu)$ for private and public goods. If μ is constant then these demands satisfy all the usual properties of demand in $(\mathbf{p},x)$ (that is: adding up; zero homogeneity; symmetry and negativity). We make the following assumptions on demands:

> *Assumption 3.* The demand functions $\mathbf{f}(\mathbf{p},x,\mu)$ are twice continuously differentiable.

In general the weight μ is not given but depends on $(\mathbf{p},x)$ as given in

Proposition 1. Of course, we cannot observe the demands f(.) directly since μ is not observed. What we do observe is the values taken by f(.) for a particular μ. Taking this into account the uncompensated demands are of the form $\mathbf{q} = f(\mathbf{p},x,\mu(\mathbf{p},x)) = \xi(\mathbf{p},x)$. Given assumption 2 we have one immediate result:

Proposition 2. In the collective setting demands $\xi(.)$ are zero homogeneous in $(\mathbf{p},x)$.

Thus demands satisfy the familiar homogeneity property even in the collective model.

We can also derive other properties of demands in the collective setting. These generalise Slutsky symmetry and negativity. Given the potentially observable demand functions $\xi(.)$ we define the Pseudo-Slutsky matrix:

$$S = \xi_{\mathbf{p}} + \xi_x \xi' \tag{2.4}$$

where $\xi_{\mathbf{p}}$ is the (n x n) Jacobian matrix of partials of $\mathbf{q}$ with respect to $\mathbf{p}$ and ξ_x is the gradient of $\mathbf{q}$ with respect to x. In the unitary model S is symmetric and negative semi-definite. The next Proposition gives the properties of S in the collective setting.

Proposition 3. In the collective setting S is the sum of a symmetric, negative semi-definite matrix Γ and an outer product:

$$S = \Gamma + \mathbf{uv}'$$

where $\mathbf{u}$ and $\mathbf{v}$ are n-vectors of functions.

Although $\mathbf{u}$ and $\mathbf{v}$ in proposition 3 are not observable they do have interpretations: $u_i = \dfrac{\partial q_i}{\partial \mu}$ and $v_i = \left\{ \dfrac{\partial \mu}{\partial p_i} + \dfrac{\partial \mu}{\partial x} q_i \right\}$ (see the proof for demonstrations).

In our empirical work below we shall not test the negativity condition. For testing of symmetry it turns out to be easier to work with the following condition:

Corollary 1 In the collective setting, S is the sum of a symmetric matrix and a matrix that has at most rank one (denoted SR1).

This SR1 ('symmetric plus rank one') condition includes the special case in which S is symmetric; that is, the unitary case. This condition is reminiscent of the Diewert-Mantel aggregation restriction for economies with more goods than agents (see Shafer and Sonnenschein (1982) for a discussion and references).

We can immediately note one feature of SR1 here: any square matrix S of dimension three or less is SR1 (the proof is left to the reader). Thus we only have restrictions if we have more than three goods in the collective setting with two people and only prices and total expenditure in the distribution function $\mu(.)$. In the next section we discuss the implications of relaxing these assumptions.

How can we test for SR1? A first remark is that if $S = \Gamma + \mathbf{uv}'$ then $M = (S-S') = \mathbf{uv}' - \mathbf{vu}'$ has at most rank 2. Moreover, M has a specific structure as the difference between a matrix and its transpose. We now present some results for such matrices.

II.3 Properties of symmetric difference matrices.

In the empirical work below it turns out to be much easier to work with the symmetric difference matrix (S-S') rather than S itself. To facilitate later derivations we gather together in this section some relevant results on such matrices[5]. Formally:

[5] We are not claiming that the linear algebra results are novel; we simply list the results we need and provide proofs.

Definition. A real matrix M is a *symmetric difference* matrix if and only if $m_{ij} = -m_{ji}$ for all i and j.

This of course implies that $m_{ii} = 0$ for all i.

The following lemma is analogous to the result that all of the roots of a real symmetric matrix are real:

Lemma 1. Any real symmetric difference matrix M has only imaginary roots. That is, if a+b*i* is a root of M then a = 0.

This has an immediate (and rather odd) implication:

Lemma 2. Any real symmetric difference matrix M has even rank.

Since we shall be testing whether a particular symmetric difference matrix M has rank two or less, this Lemma implies that we need only test for M being rank two or zero. Since it is trivial to test for rank zero we need only develop tests for rank 2.

Although there are general tests of rank in the literature we prefer here to follow a simpler route that exploits the fact that the matrix M is a symmetric difference and that we are only concerned with testing for rank 2. The basic idea behind our test can be seen by considering the 4x4 symmetric difference matrix:

$$M = \begin{bmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{bmatrix}.$$

By Lemma 2 this matrix has rank 0, 2 or 4. Thus it has at most rank two if and only if it has one non-zero element and it is singular. If M is not rank zero then we can, without loss of generality, take the parameter a to be non-zero.

By computing the determinant we see that M is singular if and only if:

$$f = \frac{be - cd}{a}.$$

This condition can be generalised to any size matrix; the implied conditions are given in the next Lemma.

Lemma 3. Let m_{ik} be the (i,k)th element of a symmetric difference matrix M. Let m_{12} be non-zero. M has rank 2 if and only if for all (i,k) such that k > i > 2 we have:

$$m_{ik} = \frac{m_{1i}m_{2k} - m_{1k}m_{2i}}{m_{12}} \qquad (2.5)$$

Thus the elements in rows 3 to n of the upper triangle of M are functions of the elements in the first two rows (and similarly for columns in the lower triangle). Since this set of restrictions involves parametric restrictions of the familiar sort, it is easy to test. Note that for an n×n matrix M we have (n-2)(n-3)/2 restrictions; fairly obviously the condition given in (2.5) only imposes restrictions if M has dimension greater than 3 (see the discussion following corollary 1 above).

The next result is fundamental to our alternative characterisation of the collective setting:

Lemma 4. S is SR1 if and only if (S-S′) has rank zero or two.

Thus to check for SR1 we need only test whether the rank of S is zero or two. If S has rank 2 then a natural question to ask is whether the decomposition S = Σ + **uv′** is unique. The answer is no; what is 'structural' is the column space of M = (S-S′). We denote the sub-space spanned by the columns of M Im(M). Precisely:

Lemma 5. (i) Let S be SR1 and denote M = (S-S′). If S =

$\Sigma + \mathbf{uv}'$ where Σ is symmetric and $\mathbf{u}$ and $\mathbf{v}$ are not co-linear then M has rank 2 and $\mathbf{u}$ and $\mathbf{v}$ are in Im(M).

(ii) Conversely, suppose that S is SR1 and let M = (S-S') with rank(M) = 2. If we take any two arbitrary non co-linear vectors $\mathbf{u}$ and $\mathbf{v}$ in Im(M) then we can write:

$$M = a(\mathbf{uv}' - \mathbf{vu}')$$

for some scalar a.

Thus in the collective/non-unitary case where rank(S-S') is two, we can choose an infinite number of non co-linear vectors $\mathbf{u}$ and $\mathbf{v}$ that satisfy the Sr1 condition. This implies that we shall need extra restrictions if we are to recover the dependence of the distribution factor $\mu(.)$ on its arguments.

Using the above results we have the following generalisation of proposition 3:

Proposition 3'. Denote the pseudo-Slutsky matrix by S and let M = (S-S'). In the collective setting:

(i) M has rank zero or two.

(ii) If M has rank 2 then for any vector $\mathbf{x}$ such that $M\mathbf{x} = 0$, we have $\mathbf{x}'S\mathbf{x} \leq 0$.

Fairly obviously the case where M has rank zero is the unitary case so that if M has rank two then the unitary case does not hold but we are still in the collective setting.

III. THEORY - EXTENSIONS

In this section we present three extensions to the basic theory of the last section. The first of these extends the theory to many person households. The second extension allows for *distribution factors*, i.e. variables that affect the distribution function $\mu(.)$ but not preferences directly. The final extension puts some restrictions on the way prices enter $\mu(.)$.

III.1 Many person households.

If there are more then two people in the household then the class of demands admitted in the collective setting will generally be wider. The exact conditions are given in the next Proposition:

Proposition 4. Assume that the household has k+1 members where $k < (n-1)$. In the collective setting the Pseudo-Slutsky matrix S is the sum of a symmetric matrix Γ and a matrix of rank no greater than k (SRk).

Fairly obviously all of the previous analysis goes through with $(\mu_1 \ldots \mu_k)$ replacing μ everywhere. This rank condition includes the unitary case and also the two person collective setting.

One possible field of application is to households with children present. To illustrate, suppose the child is named C and let $\upsilon^C(.)$ be her utility function. Formally we can test whether the household behaves as a one, two or three person decision unit by testing for symmetry, SR1 and SR2 respectively. If we reject symmetry but not SR1 (so that SR2 is also not rejected) then it is *as though* the household is composed of two decision makers. One obvious choice would be mother and father; this is not to say, of course, that neither parent cares about the child but simply that the child does not have a direct influence on the decision making process. They may, however, have an indirect effect since each parents' preferences over (q^A, q^B, q^C, Q) may take into account the child's preferences. Other interpretations are also possible: for example, mother and daughter have the same preferences and father differs.

Identifying intra-household interactions requires more structure than we have so far imposed (see Bourguignon, Browning and Chiappori (1993) for a discussion in the cross-section case) but even the possibility of determining the effective number of decision makers in a household leads to interesting issues. For example, in the adult equivalence scale literature statements are often made about the amount of income needed to make one household as well off as another. Since it is people and not households that have welfares this equating of household welfares is sometimes somewhat murky (but not in all formulations; see, as an exemplary counter-example, Blackorby and Donaldson (1988)). Within the collective framework we can, of course, define household welfare as being the weighted sums of particular utilities. Whether or not we actually want to make this identification between weights that rationalise demands and weights in a social (family) welfare index is another matter. Knowing that father acts as a dictator and discounts the welfare of mother and daughter may not lead us to do the same.

In the multi-person household above we restricted the number of members to be at least two less than the number of goods. Indeed the necessary condition in proposition 4 is no longer restrictive for $k \geq n-1$, since any nxn matrix can be written as the sum of a symmetric matrix and a matrix of rank (n-1). Though the condition in proposition 4 is only necessary, it is indeed the case that if we have as many people as goods minus one then the collective setting does not impose any restrictions on demand, as stated in the following result:

Proposition 5. (Chiappori (1990)). Assume that the household has at least (n-1) members. For any finite set of prices and demands, one can find preferences for which observed behaviour is compatible with the collective setting.

The proof relies on known results on aggregate demands for private goods.

III.2 Including distribution factors.

The next extension to the basic model that we discuss in this section is the inclusion of variables that affect the distribution function $\mu(\mathbf{p},x)$. The obvious examples here are the incomes of the two partners but these variables could also include a host of Extra-Environmental Parameters (EEP's) to use the terminology of McElroy (1992). For example, it might be that changes in divorce law or discrimination against women in the work place have an impact on intra-household decision making. In defining such variables it is most important to identify variables that may affect the $\mu(.)$ function but that do not affect preferences directly (that is, that do not enter each person's utility function). We term such variables *distribution factors.* We distinguish such variables from *preference factors* which are variables that affect preferences directly[6].

To take an example, suppose that it is the case that there are fixed costs of going to work that are independent of the wage. Then participation in the labour force could be considered a preference factor and earnings would be a candidate for a distribution factor since demand should not depend on earnings, once we condition on total expenditure. Of course, if the costs of going to work do depend on the wage (for example, high wage jobs require more expensive clothing or higher travel costs) then we cannot take earnings as a distribution factor.

We begin with the case of a single distribution factor y, so that $\mu = \mu(\mathbf{p},x,y)$. As already discussed this means that y only enters the household utility function through the same index as prices and total expenditure: $u = \upsilon(\mathbf{q},\mu(\mathbf{p},x,y))$. Household demands take the form $\xi(\mathbf{p},x,y)$. Denoting the response of demands to changes in y by ξ_y, we have the following conditions on the way this factor can affect demands:

Proposition 6. **Distribution factor linearity.** In the collective setting, we have the following equivalent conditions:

(i) the Pseudo-Slutsky matrix takes the form:

$$S = \Gamma + \xi_y \mathbf{v}'$$

[6] For convenience we assume that there is no overlap between preference and distribution factors. Thus all variables that affect demands (other than prices and total expenditure) are classified as one or other of these sorts of factors.

where Γ is symmetric.

(ii) ξ_y can be written as a linear combination of the columns of (S-S').

Since S and the vector ξ_y are observable we can use condition (ii) to test for this restriction. Of course, we can only test for condition (ii) conditional on imposing SR1 on S; without this (S-S') can have full rank and condition (ii) would be satisfied trivially.

Proposition 6 is an unusual result since it relates the response to a change in the distribution factor to price effects 'purged' of the usual Slutsky symmetry. Outside the collective setting there is no particular reason why responses to, say, changes in the relative earnings of the two partners should be related to price responses. Thus this proposition offers a potentially powerful test of the collective setting.

Proposition 6 also has an interesting converse. Suppose that we have some variable y that we are sure would affect demands if the collective model holds but the unitary model does not hold. If we find that this variable does not affect demands (that is, $\xi_y = \mathbf{0}$, the zero n-vector) then we cannot reject the unitary model. To illustrate, if there is no effect of relative incomes on demand then it must be that households behave as though they are maximising a single utility function (since Γ is symmetric). Of course, this test relies on our maintaining that if anything is going to affect intra-household allocation but not preferences then it is relative incomes; if we do not maintain this then this is not a test of the unitary model (that is, ξ_y zero is only necessary for the unitary model, it is not sufficient). This parallels the tests of the unitary model which test for 'income pooling' (that is, the absence of any effect of incomes on allocation) that have now been performed by many people (see, for example, Thomas (1990), Bourguignon *et al* (1993) and Phipps and Burton (1992)).

If we do not observe price variation then the presence of a single distribution factor does not impose any restrictions on demands (strictly, Engel curves). Intuitively, this can be seen by noting that the condition in Proposition 6(ii) requires an estimate of S which is only identified if we have price variation. Thus proposition 6 adds to the conditions that are present if we observe price variation. If we add more distribution factors so that **y** is now an m-vector then the collective setting imposes further restrictions. In Bourguignon *et al* (1994) the following is proved:

Proposition 7. **Distribution factor proportionality.** In the collective setting we have:

$$\xi_{y_i} = k_i \xi_{y_1} \quad \text{for } i = 2,3\ldots m.$$

Thus the responses to different distribution factors are co-linear; this is very simple to test (see Bourguignon *et al* (1993)). The extra distribution factors do not, however, impose any more restrictions on the Pseudo-Slutsky matrix S. Thus the testing of restrictions in proposition 7 constitute an independent series of tests of the collective model (which can be applied in the non-price context) to those developed above. Thus we can test for distribution factor proportionality (proposition 7) and for SR1 (proposition 3) independently. If neither is rejected then we can test for distribution factor linearity (proposition 6) with both SR1 and distribution factor proportionality imposed.

III.3 Restricting the dependence of distribution on prices.

We can also impose alternative structure on the distribution function $\mu(.)$. For example, suppose that we restrict prices to enter only through a *known* linear homogeneous price index $\pi(\mathbf{p})$. This assumption smacks of *ad hocery* but it does cut down on the way price variation can affect demands a great deal. This case is particularly interesting if some of the distribution factors are money variables since in this case we can normalise $\pi = 1$ and make all monetary values real. We then have $\mu = \mu(x,y)$. This gives:

Proposition 8. If there is only a single distribution factor and $\mu = \mu(x/\pi, y/\pi)$ then the Pseudo-Slutsky matrix takes the form:

$$S = \Gamma + k\xi_y\xi'$$

where k is a constant.

Since the two components of the outer product on the right hand side are observable this gives an immediate test of the collective model with a known linear homogeneous price index and a single distribution factor. Note that we need to know the price index *a priori* to deflate x and y. The condition given in proposition 8 is a special case of the condition given in proposition 6 above (the vector **v** is replaced by $k\xi$) which is, in turn, a restriction on the conditions given in proposition 3.

IV. A PARAMETRIC DEMAND SYSTEM

IV.1 A quadratic log demand system.

In this section we take a parameterisation for the demand system and derive the implications of the restrictions implied by the collective setting. Our attention will focus on tests of symmetry and 'symmetry plus rank one' (SR1) (proposition 3) and the restrictions imposed for distribution factors (propositions 6 and 7). We do not test for the conditions for many person households in propositions 4 and 5 since we consider here only one or two person households. When choosing a demand system it is important to allow for as much flexibility as possible since tests of symmetry may be biased if the parameterisation is too restrictive *a priori*. Thus we start with the Quadratic Almost Ideal Demand System (QUAIDS) of Banks, Blundell and Lewbel (1993) (BBL). This system takes the AI demand system, which includes a term in log deflated total expenditure, and adds a quadratic term in log deflated total expenditure to it. The parameterisation chosen admits of different shaped Engel curves even when the integrability conditions are imposed (formally, it is rank three in the sense of Lewbel (1991)). The QUAIDS of BBL is not the only generalisation of the AI model that has this property (see, for example, the quadratic AI model of Fry and Pashardes (1992) or the fixed cost AI model of Browning and Xie (1994)) but in the absence of any evidence that any one of these is better than any other we choose to work with it[7].

For given budget share n-vector ω, an n-vector of log prices $\mathbf{p}$ and total expenditure x, the QUAIDS demand system takes the vector form:

$$\omega = \alpha + \Gamma\mathbf{p} + \beta(\ln(x)-a(\mathbf{p})) + \lambda\frac{(\ln(x)-a(\mathbf{p}))^2}{b(\mathbf{p})} \tag{5.1}$$

where α, β and λ are n-vectors of parameters and Γ is an n×n matrix of parameters[8]. The price indices $a(\mathbf{p})$ and $b(\mathbf{p})$ are defined as:

$$a(\mathbf{p}) = \alpha_0 + \alpha'\mathbf{p} + \frac{1}{2}\mathbf{p}'\Gamma\mathbf{p} \tag{5.2}$$

and

$$b(\mathbf{p}) = \exp(\beta'\mathbf{p}) \tag{5.3}$$

Note that (5.1) reduces to the AI model if the λ vector is zero. In all that follows we shall always impose adding up and homogeneity[9]. To do this we drop the last equation to accommodate adding up and work with homogeneous prices (that is, prices divided by the price of the n th. good). Then we estimate the parameters of the (n-1)-vectors (α,β,λ) without their last elements and the parameters of the (n-1)x(n-1) matrix Γ without its last row and column. To cut down on notation, we now take n to be the number of goods minus one and $(\alpha,\beta,\lambda,\Gamma)$ to be these reduced vectors and matrices.

We derive the the Pseudo-Slutsky matrix for the parameterisation in (5.1) using the budget share form:

$$S = \omega_{\mathbf{p}} + \omega_x\omega' \tag{5.4}$$

where $\omega_{\mathbf{p}}$ is the n×n Jacobian matrix of partial derivatives of the budget shares with respect to log prices and ω_x is the gradient of ω with respect to

[7] In fact, the empirical results with the different parameterisations are very similar.

[8] In our empirical implementation below we shall let these parameters be functions of demographics but that changes nothing in this section.

[9] One of the encouraging results of moving from testing on aggregate data to testing on micro data is that homogeneity is not usually rejected; see Blundell *et al* (1993) or Browning and Meghir (1991), for example. Tests for homogeneity on the data used below (not reported) also fail to reject.

x. Applying this to (5.1)-(5.3) we have (the derivation is given in the Appendix):

$$S = \Gamma - \frac{1}{2}(\beta + 2\frac{y}{b(\mathbf{p})}\lambda)\mathbf{p}'(\Gamma-\Gamma') + y(\beta\beta' + \frac{y}{b(\mathbf{p})}(\lambda\beta'+\beta\lambda') + \left\{\frac{y}{b(\mathbf{p})}\right\}^2\lambda\lambda') \quad (5.5)$$

where $y = (\ln(x)-a(\mathbf{p}))$. Since all of the parameters in (5.5) are identified from the system (5.1) we can use this for testing.

IV.2 Testing for symmetry and SR1.

We are now in a position to give the necessary and sufficient conditions for symmetry and 'symmetry plus rank one' (SR1) for our parameterisation.

Proposition 9.

(i) S is symmetric for all $(\mathbf{p},x)$ if and only if Γ is symmetric.

(ii) S is SR1 for all $(\mathbf{p},x)$ if and only if Γ is SR1.

Thus the Γ matrix of parameters inherits the symmetry and SR1 properties of S. This makes testing relatively easy; all we need to do is test for parametric restrictions on the estimated Γ.

IV.3 Testing for other implications of the collective model.

In the demand system given in (5.1) we conditioned only on prices and total expenditure but other observable factors also have an important influence on demand patterns. Following the distinction made in section 3 we designate these other variables as either 'preference factors', **z**, or 'distribution factors' **y**. We include the preference factors in the conventional way by allowing them to modify the parameters of the indices $a(\mathbf{p})$ and $b(\mathbf{p})$:

$$a(\mathbf{p},\mathbf{z}) = \alpha_0 + \alpha(\mathbf{z})'\mathbf{p} + \mathbf{p}'\Gamma\mathbf{p} \quad (5.2')$$

and

$$b(\mathbf{p},\mathbf{z}) = \exp(\beta(\mathbf{z})'\mathbf{p}) \quad (5.3')$$

Note that we follow most other investigators and assume that the price response terms Γ are the same for all households within any given sample; this is largely imposed on us by the limited price variation in the data. It is important to state, however, that in our empirical work below we stratify fairly finely and estimate separate demand systems for different strata. Thus we only impose that price responses are the same within strata and not across the whole population[10]. In particular, we shall allow the Γ matrix to vary across households of different sizes.

To incorporate the distribution factors we note that propositions 6 and 7 refer to the derivatives of demand with respect to such factors. Thus it is convenient to include these in the constant term in (5.1):

$$\omega = \alpha(\mathbf{z}) + \Theta\mathbf{y} + \Gamma\mathbf{p} + \beta(\mathbf{z})(\ln(x)-a(\mathbf{p})) + \lambda\frac{(\ln(x)-a(\mathbf{p},\mathbf{z}))^2}{b(\mathbf{p},\mathbf{z})} \quad (5.6)$$

where $\mathbf{y}$ is an m-vector of distribution factors and Θ is an nxm matrix of parameters. We denote the k th. column of Θ by θ^k.

The next condition we are interested in testing is the distribution factor proportionality condition given in proposition 7. For our parameterisation this is equivalent to Θ having rank 1. This is most easily tested by testing for the following condition on the columns of Θ:

DISTRIBUTION FACTOR PROPORTIONALITY: $\theta^k = \mu_k\theta^1$ for $k = 2,..n$ (5.7)

If this condition is not rejected then we can replace the m distribution factors $\mathbf{y}$ by the index of factors $y = (y_1 + \mu_2 y_2 + \ldots + \mu_n y_n)$. We can then apply the restriction from proposition 6 to this index.

10 In the present context imposing that Γ is the same for single person and multiple person households would be very odd since the former should have symmetric Γ's whereas the latter may not have.

We now derive the testable conditions on our parameters for the single distribution factor restriction given in proposition 6. This states that the (observable) vector of the derivatives of demand with respect to the a single factor y, ξ_y, be a linear combination of the columns of the matrix M. If the SR1 condition holds then ξ_y must be a linear combination of the first two columns of M (with SR1 imposed). Denoting the ith. column of M as m^i we have:

$$\text{DISTRIBUTION FACTOR LINEARITY:} \quad \xi_y = \lambda_1 m^1 + \lambda_2 m^2 \tag{5.8}$$

This restriction has (n-2) degrees of freedom. Note that this condition imposes restrictions even if we have only three goods; it will be remembered that SR1 only imposes restrictions if we have more than three goods.

In this section we have derived a flexible demand system ((5.7)) and a series of tests of conditions implied by the unitary and collective model. These are tests for 'symmetry' and 'symmetry plus rank 1' (see proposition 9); 'distribution factor proportionality' (condition (5.7)) and 'distribution factor linearity' (condition (5.8)). We turn now to testing these conditions on individual household data.

V. EVIDENCE FROM THE CANADIAN FAMEX

V.1 A description of the data.

To test and estimate the collective model we need several features in the data. First, we of course need information on (household) demands; thus we have to use micro/household data. We also need enough price variation to allow us to estimate the price responses reliably. This already rules out many data sets since this requires either a long time series of cross sections or a shorter time series with some observable cross-section price dispersion within the period. Effectively then, we were restricted to using data from only three countries: Canada, the UK and the US[11]. All three of these have family

[11] We did not consider using data from developing countries in this study.

expenditure surveys that provide very reliable and detailed information on the purchases of commodities by households and sufficient price variation to allow us to estimate price responses. Unfortunately, the US Consumer Expenditure Survey (CES) does not provide reliable income information for the individual members of the household. Since we regard the individual incomes of the members as the prime candidates for distribution factors this rules out testing the 'proportionality' and 'single distribution factor' conditions on this survey. Thus the effective choice is between the UK Family Expenditure Survey (FES) and the Canadian Family Expenditure Survey (FAMEX) for our full range of tests. We chose to use the latter since we can associate different prices with different regions which increases the price dispersion.

The Canadian FAMEX is a survey of annual purchases by households. The FAMEX is not run every year so that we only have surveys for the years 1978, 1982, 1984, 1986 and 1990[12]. Fortunately, there is also significant price variation within Canada (due to different provincial tax rates and transport costs) so that we can estimate reliable price responses even when we allow for cross-country taste differences. We model the demand for seven non-durables: food; household operations (sometimes referred to as services); men's clothing; women's clothing; transport (excluding the purchase of vehicles); recreation and vices (tobacco and alcohol). We assume the preferences for these goods are separable form all other goods except the ownership of a car. We allow for non-separabilities between goods and leisure by conditioning on labour force status (see Browning and Meghir (1991)); that is, we do not assume that preferences for our seven goods are separable from labour supply

For our core analysis we select households comprising only a married couple, both of whom are in full time employment (defined as at least fifty weeks of full time work in the survey year). We also present a complementary analysis of the demands of households comprising a single male or a single female. These are of interest since the unitary theory should apply to such households. Here we also condition on full-time employment. For couples, the

[12] FAMEX were conducted in 1969, 1972, 1974 and 1976. Only the former of these has been released as Public Use tape. We consider the price data associated with the 1969 FAMEX to be too unreliable to allow us to use this survey.

FAMEX records the net income of the household and the gross earnings of husband and wife. Although it would be preferable to use the net (after tax) earnings of each person in our analysis we feel that the gross earnings are sufficiently correlated with these to pose no problem. We also select on age (all members aged less than 65) and drop a few households with implausible values for some variables[13]. The sample sizes for couples, single females and single males are 1758, 1562 and 1505 respectively. A detailed description of the data is given in the Data Appendix.

V.2 Testing the unitary model.

We first present a conventional demand analysis for the three strata. That is, an analysis assuming that the unitary model holds for all households. The purpose of this is to illustrate some of the problems that motivated the analysis presented in this paper. To do this we estimate the parameters of the system given in (5.6) without the Θ matrix. We have to address some econometric issues. First, we must allow for unobservable heterogeneity. Although it would be desirable to derive the stochastic formulation by allowing for heterogeneity in preferences and distribution functions we follow usual practice and simply add a (heteroscadestic) error term to each equation in (5.6). As already discussed, we drop one equation (that for vices) to allow for adding-up. We also allow for observable heterogeneity by letting the α_k's vary with demographics (see (5.2')). The household variables included are dummies for region; home ownership; living in a city and car ownership. The variables for individuals are dummies for francophone; non-francophone or anglophone; education and occupation and the age and age squared of the husband. The precise list of preference factors used can be found in the Data Appendix.

We also allow for the possible endogeneity of total expenditure in (5.6) by instrumenting. Since the tests of the validity of these instruments plays an important role in what follows we discuss them explicitly. The usual reason for assuming that total expenditure might be endogenous in a demand system is

[13] For example, three men recorded zero gross earnings even though they reported being in full time employment.

that unusually high (or low) expenditure on one good by a particular household will affect both the error for that household and total expenditure; thus infrequency (or lumpiness of purchases) will induce a correlation between total expenditure and the errors in the system. Measurement error for individual expenditures also induces endogeneity. The usual instrument suggested to correct for this is net income. This is correlated with total expenditure but is usually assumed to be uncorrelated with any infrequency of purchase or measurement error. The critical point here is that within the unitary model, income should not affect demand once we condition on total expenditure. Thus it should be excluded from the right hand side of the system and is available as an instrument. The same applies to the individual incomes of the two members in the couples households. The specific instruments we use are log net real income; the square of this variable and the logs of the individual gross earnings for the couples data. We also have one other instrument since we impose homogeneity everywhere by deflating all prices by the price of vices. Thus we can also use the log of this price as an instrument.

The final difficulty in estimating (5.6) is that it is non-linear. Note, however, that if we have estimates of the indices $a(\mathbf{p},z)$ and $b(\mathbf{p},z)$ in (5.2′) and (5.3′) then we can estimate (5.6) as a system of linear equations. The obvious estimates of a(.) and b(.) to use are the values constructed using estimates of the α, Γ and and β in the definitions of these indices. These in turn can be derived from estimates of the system. Thus we only need starting estimates of the a(.) and b(.) indices; we use a Stone price index for the linear homogeneous a(.) and unity for the zero homogeneous b(.). This 'iterated moment' estimator is discussed more fully in Browning and Meghir (1991) and Blundell and Robin (1992). In practice, it works well and usually converges after three or four iterations. The only parameter that is not identified in this way is α_0 in the a(.) index. To find this we use a grid search with the iterated moment estimator. In this grid search we hold constant the weighting matrix for the GMM estimation (that is, the usual '$\hat{V}$' from the first stage). At the minimum, the criteria that is minimised is a χ^2 statistic for the validity of the over-identifying restrictions.

The tests of the conditions given in the last section are all performed using minimum chi-squared methods (see Browning and Meghir (1991) for a detailed account of min-χ^2 tests in this context). Thus we first estimate the parameters and covariance matrix of the parameters of the system (5.6) with no restrictions; denote these by τ and C respectively. Then we impose the restrictions by solving:

$$\underset{\eta}{\text{minimise}}\ (\tau-f(\eta))'C^{-1}(\tau-f(\eta))$$

where $f(\eta)$ is the mapping from the restricted parameters η to the unrestricted parameters τ. The value of this minimand gives the χ^2 statistic for the restriction.

In Table 1 we present the tests for symmetry and for the validity of the over-identifying restrictions for our three strata. Note that according to the FAMEX single males never buy women's clothing (and *vice versa*) so that for singles we have only six goods as against the seven for couples. Thus symmetry on the 6x6 Γ matrix for couples involves 15 restrictions but there are only 10 restrictions on the 5x5 Γ matrices for singles.

TABLE 1: TESTS OF THE UNITARY MODEL

Test for:	COUPLES # = 1758	SINGLE MALES # = 1505	SINGLE FEMALES # = 1562
over-identification	76.40 (42) [0.09]	5.65 (5) [34.18]	4.78 (5) [44.31]
symmetry	34.78 (15) [0.26]	18.25 (10) [5.08]	11.17 (10) [34.45]

Notes. The values given are:

χ^2 statistic
(degrees of freedom)
[% probability under the null]

The results for couples given in Table 1 are representative of the results usually presented in the literature on demand analysis on micro data: symmetry and the over-identifying restrictions are both rejected. On the other hand the results for the two single strata do not display any any signs of misspecification[14]. This is obviously consistent with the view developed in this paper that we cannot necessarily apply the unitary model to two person households. We now turn to testing the implications of our proposed alternative, the collective model.

V.3 Testing the collective model.

The results presented in Table 1 suggest that there are some problems with imposing the unitary model on the couples data that do not appear for singles. Thus we now estimate the collective model. To do this we include the log of the husband's and wife's gross income on the right hand side of the demand equations, see (5.6). We present all of the parameter estimates for the

14 As far as we are aware no one else has ever estimated a demand system for singles on their own. Usually they are pooled with many person households wit some dummies to allow for their differences. Significantly this pooling involves imposing that singles and couples have the same price responses. Thu the restriction that singles satisfy Slutsky symmetry implies that couples have to as well.

unrestricted demand system in the Appendix. The tests of particular interest are presented in Table 2.

TABLE 2: TEST OF THE COLLECTIVE MODEL

Test for:	COUPLES # = 1758
overidentification	47.80 (30) [2.07%]
exclusion of individual income variables	30.75 (12) [0.22%]
symmetry	40.57 (15) [0.04%]
SR1	6.86 (6) [33.38%]
distribution factor proportionality	7.99 (5) [15.67%]
SR1 and distribution factor linearity and proportionality	23.27 (15) [7.85]

As can be seen, the test of the over-identification restrictions still indicates that there may be some scope for including yet more of the instruments on the right hand side of the system. Despite this, the value of the probability under the null that the instruments are orthogonal to the error terms is a good deal higher than in Table 2. Put another way, the fall in the test statistic for the over-identifying restrictions implies that the two individual income variables should be included in the system. The next row of Table 2 presents direct evidence on this: this is a test for excluding the two income measures from the system. We conclude that individual incomes are important in the demands by couples. Referring back to Table 1 we see that this is not the case for singles since income is one of the excluded variables used to identify the model and the over-identification restrictions are not rejected for singles.

The next two rows in Table 2 test for symmetry and 'symmetry plus rank one'. Comparing the test statistics for symmetry in Tables 1 and 2 we see that adding the individual income variables actually increases the test statistic. The SR1 condition, however, is not rejected. Thus the price responses are consistent with the collective model. One objection to this non-rejection might be that there is not sufficient price variation to estimate the Γ parameters precisely enough to reject SR1. This is not, however, consistent with the fairly decisive rejection of symmetry.

The next row presents the test for distribution factor proportionality. As already discussed this restriction is independent of the test for SR1. The proportionality test does not reject. Finally, then, we can go on to testing for SR1, distribution factor proportionality and distribution factor linearity together. The test statistic for this is given in the final row of table 2. As can be seen these restrictions are not rejected. We conclude that the collective setting is consistent with these data.

VI. CONCLUSIONS

In the above we presented a general characterisation of the collective model. We showed that the collective model can be be completely captured by using a household utility function $v(.)$ that depends on household purchases $\mathbf{q}$ and a distribution index $\mu(.)$. If the latter is a constant then we have the usual unitary model. Generally, however, the function $\mu(.)$ depends on prices $\mathbf{p}$, total expenditure x and distribution factors $\mathbf{y}$: $\mu(\mathbf{p},x,\mathbf{y})$. The fact that all non-preference influences have to act through this index puts strong restrictions on household behaviour. In sections II and III we presented some of these restrictions.

In the empirical section we estimated the parameters of a demand system and then tested for some of the predictions of the unitary and collective models. Although we made minimal assumptions in the theory section we necessarily had to make stronger assumptions in this empirical work. For example, we have assumed that preferences over the non-durables modeled are separable from other goods (except leisure and the ownership of a car). We have also assumed that the labour supply decision is exogenous for the demand system. More fundamentally, we have assumed that the marriage decision is given; that is we do not control for selection in to couples or singles. Conditional on these reservations the results are unambiguous: the predictions of the unitary model are not rejected for single people but they are rejected for couples. The predictions of the collective model are not rejected by the data for couples. This encourages us that the collective setting is worth further investigation.

As mentioned in the introduction, one of the other important areas where the results presented here can be applied directly is to the joint labour supply decision of husband and wife. The theoretical results presented in section II and III have implications for such work on cross-sectional data. Since there is no cross-section variation in prices for goods we can only define a single composite commodity, consumption, and then analyse the three 'good' system for male and female labour supply and consumption. The cross-section variation in wages gives the (relative) 'price' variation that we have exploited in this paper. Referring back to the discussion following corollary 1, however, we see that without further restrictions, the collective setting does not have any implications for price responses in a three good model. Essentially any Slutsky responses are consistent with the collective setting. Thus the restrictions from proposition 7 (the factor proportionality restrictions) are the only restrictions that the collective model imposes in this context (see also Chiappori (1990)). Additional restrictions (see, for example, Chiappori (1988, 1992)) may be derived, but only under additional assumptions (typically, privateness of leisure and consumption and restrictions on preferences).

The power of thinking about the collective model in terms of a distribution function is shown by the ease with which we derived the results in sections II and III. Just as importantly, this way of looking at things is likely to facilitate future work that undertakes more structural analyses of household behaviour. In particular, there are important decisions that individuals make that pre-date the allocation decisions within marriage. This obviously includes the marriage decision itself but also education and human capital decisions. If the collective setting is indeed appropriate for decision making once a union is formed then the distribution function is a useful 'sufficient statistic' for the importance of these earlier decisions in the division of the gains to marriage.

It may also be the case that assuming the collective setting allows a more precise determination of empirical effects. To give an example, suppose that it is posited that changes in law governing the division of assets on divorce leads to shifts in 'power' within the household. If we have households that are observed in different policy regimes then it may be possible to incorporate a variable capturing these differences in environment in the $\mu(.)$ function. The fact that reactions to this variable are closely related to reactions to other distribution factors and to price effects means that we may be able to determine the effects of such changes more precisely. Of course, this gain in precision comes at the expense of maintaining the collective model but we regard this as being acceptable given the foregoing.

Another area that deserves systematic exploration is the use of the distribution function in the analysis of intra-household welfare. Once we accept that households do not have a single welfare index we need to allow for differences in distribution within the household. It is likely that any such extensions that maintain the collective setting will use the distribution function even though at present it is unclear how this will be achieved since the distribution function depends on the normalisation of the utility functions used.

As emphasised in the introduction we regard the collective setting as a tractable and plausible next step in the analysis of the behaviour and welfare of many person households. The implications of the collective model are significantly weaker than those of the unitary model but not so weak as to impose no restrictions on observables. In this paper we have restricted attention to demand behaviour but it is clear that the collective framework can be extended to the analysis of labour supply; fertility; savings; portfolio choice and other areas of household behaviour.

APPENDIX

Proof of Proposition 1.

If $(\hat{\mathbf{q}}^A, \hat{\mathbf{q}}^B, \hat{\mathbf{Q}})$ satisfies the collective setting conditions then it a solution to:

$$\max_{(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q})} \ \upsilon^A(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q}) + \lambda\upsilon^B(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q})$$
$$\text{subject to } \mathbf{p}'(\mathbf{q}^A + \mathbf{q}^B + \mathbf{Q}) = x.$$

for some $\lambda(\mathbf{p},x) \geq 0$. If we multiply the maximand by $\mu = (1+\lambda)^{-1}$ the program is unchanged and we have the form given in the proposition.

Proof of Proposition 2.

For any positive scalar σ we have:

$$\xi(\sigma\mathbf{p},\sigma x) = \mathbf{f}(\sigma\mathbf{p},\sigma x,\mu(\sigma\mathbf{p},\sigma x)) = \mathbf{f}(\mathbf{p},x,\mu(\mathbf{p},x))$$

since $\mathbf{f}(\mathbf{p},x,.)$ is zero homogeneous in $(\mathbf{p},x)$ for fixed μ and $\mu(..)$ is zero homogeneous by assumption.

Proof of Proposition 3.

Since $\xi(\mathbf{p},x) = \mathbf{f}(\mathbf{p},x,\mu(\mathbf{p},x))$ we have:

$$S = \xi_{\mathbf{p}} + \xi_x\xi' = \mathbf{f}_{\mathbf{p}} + \mathbf{f}_\mu\mu_{\mathbf{p}}' + (\mathbf{f}_x + \mathbf{f}_\mu\mu_x)\mathbf{f}'$$
$$= (\mathbf{f}_{\mathbf{p}} + \mathbf{f}_x\mathbf{f}') + \mathbf{f}_\mu(\mu_{\mathbf{p}} + \mu_x\mathbf{f})'$$

Since $\mathbf{f}(\mathbf{p},x,\mu)$ is a conventional uncompensated demand function for fixed μ we have that $\Gamma = (\mathbf{f}_{\mathbf{p}} + \mathbf{f}_x\mathbf{f}')$ is symmetric and negative semi-definite. Denoting $\mathbf{u} = \mathbf{f}_\mu$ and $\mathbf{v} = (\mu_{\mathbf{p}} + \mu_x\mathbf{f})$ we have the result given in the proposition.

Proof of Lemma 1.

Let S be any n×n real matrix. Let $(a+ib)$ be an eigen-root of $M = (S-S')$ (where $i = \sqrt{-1}$). Let $\mathbf{x}+i\mathbf{y}$ be a corresponding eigen-vector; note that at least one element of $\mathbf{x}$ or $\mathbf{y}$ is non-zero. We have:

$$M(\mathbf{x}+i\mathbf{y}) = (a+ib)(\mathbf{x}+i\mathbf{y})$$

Pre-multiply both sides by $(\mathbf{x}-i\mathbf{y})$. Some calculation gives that the left hand side equals:

$$2i(m_{12}(y_2x_1-y_1x_2) + \dots + m_{1n}(y_nx_1-y_1x_n) + \dots + m_{(n-1)n}(y_nx_{n-1}-y_{n-1}x_n))$$

where m_{ij} is the (i,j)th element of M (= $-m_{ji}$). Thus the left hand side has no real part. The right hand side equals:

$$(a+ib)(\mathbf{x}'\mathbf{x} + \mathbf{y}'\mathbf{y}).$$

Since $(\mathbf{x}'\mathbf{x} + \mathbf{y}'\mathbf{y}) > 0$ equating the real part of the two sides implies that a = 0.

Proof of Lemma 2.

Let S be any real n×n matrix. The eigen-values of any matrix come in conjugate pairs $a\mp ib$. Since a = 0 for (S-S') (by lemma 1) this implies that we have an even number of non-zero eigen-values. Hence (S-S') has even rank.

Proof of Lemma 3.

Given any symmetric difference matrix M, if $m_{12} \neq 0$ and

$$m_{ik} = \frac{m_{1i}m_{2k} - m_{1k}m_{2i}}{m_{12}}$$

for all (i,k) such that k > i > 2 then row i for i > 2 can be written:

$$(m_{13}m^2 - m_{23}m^1)/m_{12}$$

where m^i is the i th. row of M. Hence M has rank 2.

Conversely, if M has rank 2 then (taking $m_{12} \neq 0$) the i th. row of m can be written:

$$m^i = \lambda m^1 + \mu m^2.$$

Since M is a symmetric difference matrix we have $m_{13} = -m_{31}$ and $m_{23} = -m_{32}$ so that:

$$\lambda = -\frac{m_{2i}}{m_{12}} \quad \text{and} \quad \mu = \frac{m_{1i}}{m_{12}}.$$

This yields

$$m_{ik} = \lambda m_{1k} + \mu m_{2k} = \frac{m_{1i}m_{2k} - m_{1k}m_{2i}}{m_{12}}$$

for all (i,k) such that k > i > 2.

Proof of Lemma 4.

Let S be SR1, so that $S = \Sigma + \mathbf{uv}'$ for some vectors $\mathbf{u}$ and $\mathbf{v}$. The symmetric difference $(S-S') = \mathbf{uv}' - \mathbf{vu}'$ has even rank by lemma 2. Since it is the sum of two outer products it has at most rank 2. Thus it has rank zero or two.

Conversely, suppose that for a given real square matrix S, $(S-S')$ has rank zero or two. If this has rank zero then S is symmetric and hence is SR1. If $(S-S')$ has rank 2, take $m_{12} \neq 0$ and define $\Sigma = S - \mathbf{uv}'$ where:

$$\mathbf{u} = \begin{bmatrix} 0 \\ m_{12} \\ \cdot \\ \cdot \\ \cdot \\ m_{1n} \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} -1 \\ 0 \\ m_{23}/m_{12} \\ \cdot \\ \cdot \\ m_{2n}/m_{12} \end{bmatrix}.$$

where $m_{ij} = (s_{ij}-s_{ji})$. We now prove that Σ is symmetric so that S is SR1.

Let σ_{ij} be the (i,j)th. element of Σ. Clearly the diagonal of Σ is irrelevant for the symmetry of Σ so that we need only consider the off-diagonal elements of Σ. For the first column we have:

For $i > 1$: $\sigma_{i1} = s_{i1}+m_{1i} = s_{i1}+(s_{1i}-s_{i1}) = s_{1i} = \sigma_{1i}$.

For the second row we have:

For $i > 2$: $\sigma_{2i} = s_{2i}-m_{2i} = s_{2i}-(s_{2i}-s_{i2}) = s_{i2} = \sigma_{i2}$.

For all other rows and columns we have:

For $i > k > 2$: $\sigma_{ik}-\sigma_{ki} = (s_{ik} - \dfrac{m_{1i}m_{2k}}{m_{12}}) - (s_{ki} - \dfrac{m_{1k}m_{2i}}{m_{12}}) = 0$

since for $i > k > 2$ we have:

$$s_{ki} - s_{ik} = \frac{m_{1k}m_{2i} - m_{1i}m_{2k}}{m_{12}}$$

by lemma 3. Hence Σ is symmetric and S is SR1.

Proof of Lemma 5.

(i) Let $S = \Gamma + \mathbf{uv}'$ where Γ is symmetric. If $\mathbf{u}$ and $\mathbf{v}$ are not co-linear then choose some vector λ that is orthogonal to $\mathbf{u}$ but not to $\mathbf{v}$. We have:

$$(S-S')\lambda = M\lambda = \mathbf{u}\mathbf{v}'\lambda - \mathbf{v}\mathbf{u}'\lambda = (\mathbf{v}'\lambda)\mathbf{u}.$$

Thus **u** is in the column space of M. Similar reasoning establishes that **v** is also in Im(M).

(ii) M is symmetric difference and rank 2, with $m_{12} \neq 0$. Thus the first two columns of M are linearly independent. We first show that $M = (1/m_{12})D$ where $D = (\mathbf{m}^2\mathbf{m}^{1\prime} - \mathbf{m}^1\mathbf{m}^{2\prime})$. We have $d_{ij} = (m_{i2}m_{ji} - m_{i1}m_{j2})$.

For $j = 1$: $d_{i1} = m_{i2}m_{11} - m_{i1}m_{12} = -m_{i1}m_{12} = m_{i1}m_{21}$.

For $j = 2$: $d_{i2} = m_{i2}m_{21} - m_{i1}m_{22} = m_{i2}m_{21}$.

For $j > 2$: $m_{ij} = (-m_{j2}m_{i1} + m_{j1}m_{i2})/m_{21}$

from lemma 4. Thus $m_{ij} = d_{ij}/m_{21}$ for all i, j.

Now take arbitrary linearly independent vectors **u** and **v** in the column space of M. Since **u** and **v** are linearly independent and are in Im(M) we can write $\mathbf{m}^1 = \mu_1\mathbf{u}+\theta_1\mathbf{v}$ and $\mathbf{m}^2 = \mu_2\mathbf{u}+\theta_2\mathbf{v}$. Using $M = (\mathbf{m}^2\mathbf{m}^{1\prime} - \mathbf{m}^1\mathbf{m}^{2\prime})/m_{21}$ and substituting for $\mathbf{m}^1$ and $\mathbf{m}^2$ we have (after some manipulation):

$$M = (\mu_1\theta_2 - \theta_1\mu_2)(\mathbf{u}\mathbf{v}' - \mathbf{v}\mathbf{u}')/m_{12} = a(\mathbf{u}\mathbf{v}' - \mathbf{v}\mathbf{u}').$$

Proof of Proposition 3′.

(i) Follows immediately from corollary 1 and lemma 4.

(ii) If M has rank 2 then **u** and **v** are not co-linear. For any **x** such that $M\mathbf{x} = 0$ we have $(S-S')\mathbf{x} = 0$ which in turn implies $\mathbf{u}\mathbf{v}'\mathbf{x} - \mathbf{v}\mathbf{u}'\mathbf{x} = 0$. Now $\mathbf{v}'\mathbf{x} = 0$ since otherwise we can write $\mathbf{v} = \mathbf{u}\left(\frac{\mathbf{u}'\mathbf{x}}{\mathbf{v}'\mathbf{x}}\right)$ which contradicts **u** and **v** not co-linear. Thus:

$$\mathbf{x}'S\mathbf{x} = \mathbf{x}'\Gamma\mathbf{x} + \mathbf{x}'\mathbf{u}\mathbf{v}'\mathbf{x} = \mathbf{x}'\Gamma\mathbf{x} \le 0$$

since Γ is negative semi-definite.

Proof of Proposition 4.

If the household has (k+1) members then $\xi(\mathbf{p},x) = \mathbf{f}(\mathbf{p},x,\mu^1(\mathbf{p},x),..\mu^k(\mathbf{p},x))$ where the μ^j's are the weights on the utility functions. We have

$$S = \xi_{\mathbf{p}} + \xi_x \xi'$$
$$= \mathbf{f}_{\mathbf{p}} + \mathbf{f}_{\mu 1}\mu_{\mathbf{p}}^{1\,\prime} + \ldots + \mathbf{f}_{\mu k}\mu_{\mathbf{p}}^{k\,\prime} + \mathbf{f}_x\mathbf{f}' + \mathbf{f}_{\mu 1}\mu_x^1\mathbf{f}' + \ldots \mathbf{f}_{\mu k}\mu_x^k\mathbf{f}'$$
$$= (\mathbf{f}_{\mathbf{p}} + \mathbf{f}_x\mathbf{f}') + \mathbf{f}_{\mu 1}(\mu_{\mathbf{p}}^1 + \mu_x^1\mathbf{f})' + \ldots \; \mathbf{f}_{\mu k}(\mu_{\mathbf{p}}^k + \mu_x^k\mathbf{f})'$$

$\Gamma = (\mathbf{f}_{\mathbf{p}} + \mathbf{f}_x\mathbf{f}')$ is symmetric and negative semi-definite. Thus S is the sum of Γ and k outer products and hence SRk.

Proof of Proposition 6

(i) From the proof of proposition 3 we have:

$$S = \Gamma + f_\mu(\mu_p + \mu_x\mathbf{q})'$$

From $\xi(\mathbf{p},x,y) = f(\mathbf{p},x,\mu(\mathbf{p},x,y))$ we have:

$$\xi_y = f_\mu \mu_y.$$

Thus:

$$S = \Gamma + \xi_y(\mu_p + \mu_x\mathbf{q})'\mu_y^{-1} = \Gamma + \xi_y\mathbf{v}'$$

(ii) If (S−S′) has rank 2 then ξ_y and $\mathbf{v}$ in part (i) are not co-linear. Thus we can find a vector λ that is orthogonal to ξ_y but not to $\mathbf{v}$. We have:

$$(S-S')\lambda = \xi_y\mathbf{v}'\lambda - \mathbf{v}\xi_y'\lambda = \xi_y\mathbf{v}'\lambda$$

since $\xi_y'\lambda = 0$. Thus ξ_y is in the column space of (S−S′).

Proof of Proposition 7.

From $\xi(\mathbf{p},x,y_1,\ldots y_m) = f(\mathbf{p},x,\mu(\mathbf{p},x,y_1,\ldots y_m))$ we have:

$$\xi_{y_i} = f_\mu \mu_{y_i}.$$

Hence:

$$\xi_{y_i} = \frac{\mu_{y_i}}{\mu_{y_1}}\,\xi_{y_1}.$$

Proof of Proposition 8.

Simply note that if $\mu = \mu(x,y)$ then $\mathbf{v} = (\mu_p + \mu_x\mathbf{q}) = \mu_x\mathbf{q}$ so that $\mathbf{v}$ is co-linear to $\mathbf{q}$.

Derivation of (5.5)

Starting with the terms on the right hand side of (5.4) and (5.1)-(5.3) we have (setting $y = \ln(x)-a(\mathbf{p})$):

$$\omega_{\mathbf{p}} = \Gamma - \beta a_{\mathbf{p}}' - 2\frac{y}{b(\mathbf{p})}\lambda a_{\mathbf{p}}' - \left\{\frac{y}{b(\mathbf{p})}\right\}^2 \lambda b_{\mathbf{p}}'$$

$$= \Gamma - \frac{1}{2}\left\{\beta + 2\frac{y}{b(\mathbf{p})}\lambda\right\}(\alpha' + \mathbf{p}'(\Gamma+\Gamma')) - \frac{y^2}{b(\mathbf{p})}\lambda\beta' \qquad \text{(A1)}$$

and

$$\omega_x\omega' = \left\{\beta + 2\frac{y}{b(\mathbf{p})}\lambda\right\}(\alpha' + \mathbf{p}'\Gamma' + y\beta' + \frac{y^2}{b(\mathbf{p})}\lambda') \qquad \text{(A2)}$$

Combining (A1), (A2) and (5.4) and re-arranging, we have (5.5).

Proof of Proposition 9.

From (5.5) we have that S takes the form:

$$S = \Gamma - \mathbf{R}(\Gamma-\Gamma') + \Sigma$$

where $\mathbf{R} = \frac{1}{2}(\beta + 2\frac{y}{b(\mathbf{p})}\lambda)\mathbf{p}'$ and Σ is symmetric. Note that $\mathbf{R}$ is an outer product and hence has at most rank 1.

(i) If S is symmetric for all $(\mathbf{p},x)$, set prices equal to unity so that $\mathbf{R}$ is the zero matrix (remember that $\mathbf{p}$ are log prices). This gives $S = (\Gamma + \Sigma)$ and hence Γ is also symmetric.

Conversely, S is obviously symmetric if Γ is symmetric.

(ii) Suppose now that S is SR1 for all $(\mathbf{p},x)$. Set $\mathbf{R}$ equal to the zero matrix by taking unity prices. Thus $S = \Gamma + \Sigma$ and hence Γ is also SR1.

Conversely, suppose that Γ is SR1 so that $\Gamma = \Sigma^* + \mathbf{uv}'$ where Σ^* is symmetric. Then:

$$M = (S-S') = \mathbf{uv}' - \mathbf{vu}' - \mathbf{Ruv}' + \mathbf{Rvu}' + \mathbf{vu}'\mathbf{R}' - \mathbf{uvR}'$$
$$= (\mathbf{I}-\mathbf{R})(\mathbf{uv}'-\mathbf{vu}') - (\mathbf{uv}'-\mathbf{vu}')\mathbf{R}'$$

Since $\mathbf{R}$ has at most rank 1, $(\mathbf{uv}'-\mathbf{vu}')\mathbf{R}'$ has at most rank 1 (since $\text{rank}(AB) \le \min(\text{rank}(A),\text{rank}(B))$). Thus M is the sum of matrices with at most rank 2 and 1 respectively (since $(\mathbf{uv}'-\mathbf{vu}')$ has at most rank 2) and hence M has at most rank three. But M is a symmetric difference matrix so that by lemma 2 it has even rank. Hence the rank of M is at most 2. Consequently, by lemma 4, S is an SR1 matrix.

DATA APPENDIX

TABLE D1: DEMOGRAPHICS FOR COUPLES

Variable	Mean	Minimum	Maximum
Living in Atlantic provinces	0.14	0	1
Living in Quebec	0.19	0	1
Living in Ontario	0.265	0	1
Living in Prairie provinces	0.30	0	1
Living in British Columbia	0.105	0	1
Living in city (urban area with population > 30,000)	0.80	0	1
Owning a house	0.66	0	1
Owning a car	0.94	0	1
Age of husband	37.0	20	64
Age of wife	34.8	18	64
Husband has more then high school education	0.18	0	1
Wife has more then high school education	0.16	0	1
Husband has white collar job	0.36	0	1
Wife has white collar job	0.32	0	1
Husband's mother tongue is French	0.20	0	1
Wife's mother tongue is French	0.20	0	1
Husband's mother tongue is neither English nor French	0.11	0	1
Wife's mother tongue is neither English nor French	0.10	0	1

TABLE D2: INCOMES AND BUDGET SHARES FOR COUPLES

Variable	Mean	Minimum	Maximum	Number of zeros
Net real income	51,965	9,787	176,183	0
Real total expenditure	22,514	5,579	67,720	0
Husband's gross income	38,520	774	221,887	0
Wife's gross income	27,404	864	95,135	0
Food budget share	0.307	0.07	0.78	0
Men's clothing budget share	0.055	0	0.28	7
Women's clothing budget share	0.086	0	0.37	2
Services budget share	0.125	0	0.34	1
Recreation budget share	0.107	0	0.58	7
Transport budget share	0.244	0	0.72	7
Vices budget share	0.077	0	0.44	57

Note: all expenditures and incomes are given in 1990 dollars.

TABLE D3: DEMOGRAPHICS FOR SINGLE FEMALES

Variable	Mean	Minimum	Maximum
Living in Atlantic provinces	0.15	0	1
Living in Quebec	0.18	0	1
Living in Ontario	0.24	0	1
Living in Prairie provinces	0.33	0	1
Living in British Columbia	0.11	0	1
Living in city (urban area with population > 30,000)	0.84	0	1
Owning a house	0.22	0	1.
Owning a car	0.64	0	1
Age	37.9	17	64
Has more then high school education	0.18	0	1
Has white collar job	0.41	0	1
Mother tongue is French	0.18	0	1
Mother tongue is neither English nor French	0.09	0	1

TABLE D4: INCOMES AND BUDGET SHARES FOR SINGLE FEMALES

Variable	Mean	Minimum	Maximum	Number of zeros
Net real income	24,528	4,037	92,491	0
Real total expenditure	11,457	2,152	38,918	0
Food budget share	0.306	0.01	0.82	0
Women's clothing budget share	0.151	0	0.62	2
Services budget share	0.170	0.03	0.64	0
Recreation budget share	0.099	0	0.61	19
Transport budget share	0.209	0	0.61	3
Vices budget share	0.065	0	0.49	175

Note: all expenditures and incomes are given in 1990 dollars.

TABLE D5: DEMOGRAPHICS FOR SINGLE MALES

Variable	Mean	Minimum	Maximum
Living in Atlantic provinces	0.14	0	1
Living in Quebec	0.15	0	1
Living in Ontario	0.25	0	1
Living in Prairie provinces	0.32	0	1
Living in British Columbia	0.14	0	1
Living in city (urban area with population > 30,000)	0.81	0	1
Owning a house	0.28	0	1
Owning a car	0.79	0	1
Age	35.8	19	64
Has more then high school education	0.23	0	1
Has white collar job	0.38	0	1
Mother tongue is French	0.16	0	1
Mother tongue is neither English nor French	0.10	0	1

TABLE D6: INCOMES AND BUDGET SHARES FOR SINGLE MALES

Variable	Mean	Minimum	Maximum	Number of zeros
Net real income	29,495	3,036	175,572	0
Real total expenditure	14,181	2,247	48,962	0
Food budget share	0.323	0.02	0.89	0
Men's clothing budget share	0.084	0	0.49	20
Services budget share	0.102	0.005	0.45	0
Recreation budget share	0.123	0	0.64	21
Transport budget share	0.248	0	0.75	16
Vices budget share	0.120	0	0.63	81

Note: all expenditures and incomes are given in 1990 dollars.

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