DEPARTMENT OF ECONOMICS

WORKING PAPER SERIES

2002-01

McMASTER UNIVERSITY

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Semi-parametric predictions of the intertemporal approach to the current account

December 2001

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Abstract

This paper uses dynamic programming by GMM (DP by GMM) to solve a dynamic model of a small-open economy calibrated to Canada. The predicted paths for consumption, output, investment and trade balance (over output) are highly correlated with their historical counterparts. Moreover, the variance of the predicted trade balance-output ratio matches the variance observed in the data. The latter result contrasts with earlier research on the intertemporal approach to the current account using linear-quadratic models that found that the sample paths for the current account predicted by the theory are less volatile than the historical paths in Canadian data. It is shown that a special case of the model with a constant world interest rate does not match the historical path of the trade balance as well.

Keywords: intertemporal approach, current account, predicted paths

JEL classification: F32, E21, C63

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1. Introduction

This paper solves a small-open economy model calibrated to the Canadian economy using dynamic programming by GMM (DP by GMM). This semi-parametric solution method described in Letendre and Smith (2001) does not require the specification and calibration of the transition equations for the exogenous variables. This turns out to be a very interesting feature of DP by GMM when employed to solve a small-open economy model.

Glick and Rogoff (1995) study the dynamic properties of a small-open economy model including productivity shocks. They assume that the exogenous shocks follow a first-order autoregressive process with degree of persistence ρ. One striking finding of Glick and Rogoff is that for large values of ρ, small changes in the value of ρ greatly modify the response of the current account to productivity shocks. This result, obtained in a framework with quadratic utility and zero capital depreciation carry over to a model with constant relative risk aversion utility and non-zero capital depreciation.[1] Since empirical estimates of the level of persistence in productivity shocks often fall within the range of values for which current account dynamics is sensitive, the use of a semi-parametric solution method is very appealing. Moreover, contrary to numerical solution methods widely applied in macroeconomics (*e.g.* log-linear approximation and linear-quadratic approximation techniques), DP by GMM does not impose certainty equivalence. This is also an appealing feature of the method in the light of the work of Ghosh and Ostry (1997) who find that precautionary saving motives may be important to understand current account behavior.

This paper compares predicted and actual paths to evaluate the performance of the model. This pointwise comparison is a stronger test than moment matching for predicted and historical series can have identical moments and still be uncorrelated. As Smith and

[1] See Letendre (2002)

Zin (1997) put it: "Having a zero variance of the difference between two series is a much stricter criterion than having a zero difference of their variances." A number of researchers have compared predicted and historical paths to test their models. For example, Christiano (1988), Plosser (1989), Hansen and Prescott (1993), Smith and Zin (1997) and Chow and Kwan (1998) perform such tests for the US economy in the context of closed-economy general equilibrium models whereas Yi (1993) and Kollmann (1998) do this in the context of two-country general equilibrium models. Comparing historical and predicted path is also common in the small-open economy literature.

In fact, the intertemporal approach to the current account has been tested in (at least) two ways in the literature. On one hand, Sheffrin and Woo (1990), Otto (1992), Ghosh (1995), Obstfeld and Rogoff (1996, chapter 2) and Nason and Rogers (2001a) perform econometric tests, in a vector autoregression framework, of the intertemporal approach for Canada and other OECD countries using linear-quadratic small-open economy models. Their tests show that there is little support for the model in the Canadian data. However, graphical comparisons of predicted and actual paths for the Canadian current account show that the model can predict the dynamics of the current account to some extent even though the predicted current account series are not volatile enough. The work of Bergin and Sheffrin (2000) shows that adding world interest rate and exchange rate movements increases significantly the magnitude of the fluctuations in the predicted current account while Nason and Rogers (2001b) find that adding fiscal and world interest rate shocks take the model closer to the data. In this paper, the small-open economy experiences productivity, fiscal and world interest rate shocks.

On the other hand, Cardia (1991) and Mendoza (1991) use stochastic equilibrium small-open economy models with multiple exogenous shocks and more general utility functions to test the intertemporal approach.[2] They find that their models are able to match the volatility in the current account data. Intuitively we expect Mendoza's model to generate more

[2]Papers on two-country stochastic equilibrium models with incomplete markets which started with the work of Baxter and Crucini (1995) and Kollmann (1996) also use the intertemporal approach framework (but make the world interest rate endogenous).

realistic dynamics in the current account (the difference between savings and investment) because it models the savings decisions better than linear-quadratic models by using a variable world interest rate and a utility function that allow for precautionary savings. In theory, productivity and government shocks (among others) that are common to all countries, or that hit large open countries only, affect the world real interest rates. Therefore, a model with variable world interest rates accounts for the effects, on a small-open country, of the shocks occurring in the rest of the world. Even though the models of Cardia and Mendoza match the current account volatility, they do not produce predicted paths. It is then natural to ask whether this class of models can replicate the dynamics found in the data.

The small-open economy model considered in this paper includes productivity shocks, government spendings shocks as well as a variable world interest rate. The leading model investigated in the paper has constant labor supply and is solved by estimating approximate decision rules for consumption and investment that are linear in the elements of the state vector. While the correlation between the historical and predicted paths is high for investment and the trade balance, it is very high for consumption and output. Moreover, statistical tests also lend support to the model. Overidentifying restriction tests do not reject the model and the linear decision rules while the difference between the variance of the historical and predicted trade balance-output ratio is not statistically significant. This is an interesting finding since previous research trying to match the path of the Canadian current account, using linear-quadratic models, generated predicted paths that were significantly less volatile.

A number of variations of the leading model are considered. One notable variation is a special case of the leading model where the world interest rate is constant. Evidence are mixed on the effect of allowing variations in the world interest rate in small-open economy models. Mendoza (1991), Correia, Neves and Rebelo (1995) and Schmitt-Grohé (1998) find a very modest role for variations in the world interest rate while Blankenau, Kose and Yi (2001) find the opposite. The results in this paper are in line with the results of Blankenau *et al.*. The fit of the trade balance-output ratio is significantly reduced by the removal of the fluctuations in the world interest rate. Also, the overidentifying restriction test rejects the model with constant world interest rate. Other variations include models with endogenous labor supply and more flexible decision rules. When the model is solved using approximate decision rules that are second-order polynomials in the elements of the state vector, the predicted output, consumption and investment fit the data better. However, the decision rules estimated involves forty parameters that must be estimated using a sample of 172 observations.

This paper is organized as follows. Section 2 introduces the small-open economy model and explain how the structural parameters are estimated. Section 3 explains how the solution method DP by GMM is used to solved the model. Section 4 presents the empirical results. Section 5 concludes.

2. Model and estimation of structural parameters

2.1 Leading small-open economy model

The leading small-open economy model has a representative agent who seeks to maximize lifetime expected discounted utility

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\alpha}}{1-\alpha} \tag{1}$$

where $0 < \beta < 1$ and $0 < \alpha$. β is a discount factor, α is the coefficient of relative risk aversion and C_t is consumption. In contrast with quadratic utility functions, *CRRA* utility functions rule out negative consumption and are consistent with a balanced growth path.

In this economy, output is given by the aggregate production function

$$Y_t = e^{\xi} z_t K_t^{\theta} (n\gamma^t)^{1-\theta} \tag{2}$$

with $0 < \theta < 1$ and $\gamma \geq 1$. ξ is a scale parameter, z_t is a productivity shock and K_t is the stock of capital at the beginning of period t. θ is the capital share and γ is the gross rate of labor-augmenting technical progress. In the leading model, labor is supplied inelastically and is normalized to one (*i.e.* $n = 1$). The domestic capital stock evolves according to the law of motion

$$K_{t+1} = (1-\delta)K_t + X_t \tag{3}$$

where $0 \leq \delta \leq 1$, and where X_t is investment in period t and δ is the rate of depreciation of capital.

The representative agent can use international loans (A) and domestic capital to smooth consumption over time. In doing so, she faces the budget constraint

$$C_t + X_t + A_{t+1} = Y_t + (1+r_t)A_t - G_t, \tag{4}$$

where A_t is the holding of foreign assets at the end of period $t-1$ and r_t is the return on those assets. G_t is government consumption in period t. Also, in order to rule out Ponzi schemes, the optimal program must satisfy the transversality condition

$$\lim_{T\to\infty} \frac{A_{t+T+1}}{(1+r_{t+1})(1+r_{t+2})\dots(1+r_{t+T})} = 0. \tag{5}$$

2.2 Estimating the structural parameters

As an alternative to calibration, Christiano and Eichenbaum (1992) propose using the first-order conditions of a model and some steady-state conditions to derive moment conditions that can be used to estimate the structural parameters of stochastic dynamic general equilibrium models by GMM. I apply their techniques to estimate all the structural parameters of the model. The parameter vector includes δ, γ, ξ, θ and α. Note that the discount factor is not estimated because it is not a free parameter. As is well known, the small-open economy model has some undesirable asymptotic properties for arbitrary parameter values. Let $\bar{r}$ denote the mean of the world interest rate. Unless $\beta(1+\bar{r}) = \gamma^{\alpha}$, in the long run the country will always be a net borrower or a net lender. Therefore, we impose the condition $\beta(1+\bar{r}) = \gamma^{\alpha}$ when estimating the parameters.[3]

[3] This is a common assumption in the small-open economy literature. See Correia, Neves and Rebelo (1995) for example.

Since the Euler equation corresponding to the optimal intertemporal allocation of loans involves the world interest rate and that the sample equivalent of this Euler equation is used in the estimation exercises, we have to calculate the world interest rate. I measure it as the real return earned by a Canadian investor on a US three-month T-bill. Let i_{t+1}^h and π_{t+1}^h denote the nominal return on a US three-month T-bill and the Canadian inflation rate between periods t and $t+1$. Also, let e_t^h denote the spot nominal exchange rate (C$ per US$) in period t.[4]

The *ex post* real return earned by a Canadian investing in period t in a US three-month T-bill is

$$r_{t+1}^h = \frac{1+i_{t+1}^h}{1+\pi_{t+1}^h}\frac{e_{t+1}^h}{e_t^h} - 1. \tag{6}$$

In a model with rational agents, this *ex post* rate of return differs from the *ex ante* rate of return by a zero-mean forecast error that cannot be predicted using any variables in the agents information set. This measure of world interest rates is also used by Schmitt-Grohé (1998).

Following Burnside, Eichenbaum and Rebelo (1993) as well as Burnside and Eichenbaum (1996), I specify a just-identified GMM estimator to estimate the five parameters δ, γ, ξ, θ and α. The sample moment conditions used in the estimation are

$$\frac{1}{T}\sum_{t=1}^{T}\left(\frac{\gamma^\alpha}{1+\bar{r}}\right)\left(\frac{C_t}{C_{t+1}}\right)^\alpha (1+r_{t+1}) = 0 \tag{7}$$

$$\frac{1}{T}\sum_{t=1}^{T}\left(\frac{\gamma^\alpha}{1+\bar{r}}\right)\left(\frac{C_t}{C_{t+1}}\right)^\alpha \left(1+\theta\frac{Y_{t+1}}{K_{t+1}} - \delta\right) = 0 \tag{8}$$

$$\frac{1}{T}\sum_{t=1}^{T}\left(1-\delta+\frac{X_t}{K_t} - \frac{K_{t+1}}{K_t}\right) = 0 \tag{9}$$

$$\frac{1}{T}\sum_{t=1}^{T}\left[\ln\left(\frac{Y_t}{K_t^\theta n_t^{1-\theta}}\right) - \ln\xi - ((1-\theta)\ln\gamma)\times t\right]\times 1 = 0 \tag{10}$$

$$\frac{1}{T}\sum_{t=1}^{T}\left[\ln\left(\frac{Y_t}{K_t^\theta n_t^{1-\theta}}\right) - \ln\xi - ((1-\theta)\ln\gamma)\times t\right]\times t = 0 \tag{11}$$

[4] The data used in this paper are described in the data appendix. They cover the period 1955 to 1997 at a quarterly frequency. The total number of observations is 172. Note that throughout the paper we use the superscript h to denote historical data.

where T denotes the sample size.

The first two conditions are the sample equivalent of the Euler equations for capital and loans. The third condition says that the the law of motion for capital holds on average. The last two equations are the normal equations associated with the estimation of ξ and γ in the linear regression

$$\ln\left(\frac{Y_t}{K_t^{\theta}n_t^{1-\theta}}\right) = \ln\xi + ((1-\theta)\ln\gamma) \times t + \ln z_t \qquad (12)$$

The estimation of the structural parameters is done by iterated GMM.[5]

It may appear unusual that I allow for fluctuations in hours in computing the productivity shocks and in estimating ξ and γ while the theoretical model does not. Actually, the paper focus on the model with constant labor because the model with endogenous labor does not perform very well. However, as shown in Table 1, GMM estimation of the structural parameters of the model with endogenous labor yields estimates of γ, ξ, θ and δ almost identical to those estimated using the constant labor model. The point estimates of α differ but are not statistically different. The similarity of the two sets of estimates implies that the difference between the fit of the model with endogenous labor and the model with constant labor cannot be explained by the fact that the parameter values used in the empirical work below are more suited to the latter.

The estimation results presented in Table 1 show that in both models, we cannot reject the null hypothesis $\alpha = 1$. Therefore, I set $\alpha = 1$ (logarithmic utility) in the computations. Other parameter values are set at $\delta = 0.045$, $\gamma = 1.0058$, $\xi = 1.9797$ and $\theta = 0.18$. The point estimates of δ and $(1-\theta)$ are higher than the values usually encountered in the literature. Obviously, parameter estimates depend on the dataset used and the model estimated. Since previous estimation exercises were estimating parameters of closed-economy model using US data, we do not expect to obtain the same estimates.

[5] I use the iterated GMM algorithm written for GAUSS by Lars P. Hansen, John C. Heaton, and Masao Ogaki.

3. Solving the model – DP by GMM

3.1 The detrended model

The leading model presented in section 2.1 includes labor-augmenting technical progress which introduces a deterministic trend in the model. In order to work with a stationary economy, I divide all growing variables by γ^t and let lower case letters denote detrended variables. Therefore we have: $c_t = C_t/\gamma^t$, $x_t = X_t/\gamma^t$, $a_t = A_t/\gamma^t$, $k_t = K_t/\gamma^t$, $g_t = G_t/\gamma^t$. The adjusted discount factor is then $\tilde{\beta} = \beta\gamma^{1-\alpha}$.

In this transformed economy, the representative agent's stationary problem is then to maximize

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \frac{c_t^{1-\alpha}}{1-\alpha} \tag{1'}$$

subject to

$$\gamma k_{t+1} = (1-\delta)k_t + x_t \tag{3'}$$

$$c_t + x_t + \gamma a_{t+1} = e^{\xi} z_t k_t^{\theta}(1-\lambda_t) + (1+r_t)a_t \tag{4'}$$

where $\lambda_t \equiv G_t/Y_t$ and the initial conditions k_0 and a_0 are given.[6]

3.2 Exogenous state variables

The solution method, DP by GMM, allows the use of the realizations of the state variables when estimating approximate decision rules. This section explains how the realized shocks are calculated.

[6]Baxter (1995) lists the three most important elements for thinking about the determinants of the trade balance and current account in general equilibrium: intertemporal optimization by consumers, optimal investment decisions by producers, and endogenous determination of the interest rate. Despite its simplicity, the model utilized in this paper contains the first two elements. Even though the model is not general equilibrium in nature (the interest rate is not determined endogenously) the use of actual data for the interest rate seems more realistic than a constant interest rate.

The measurement of the world interest rate series (r_t^h) was explained in section 2.2 while λ_t^h is measured as the ratio of government consumption to GDP, $\lambda_t^h = G_t^h/Y_t^h$. Given the production function (2) and parameter values, productivity shocks are recovered as follows

$$z_t^h = \frac{Y_t^h}{e^{\xi} K_t^{h\,\theta} (n_t^h \gamma^t)^{1-\theta}}. \tag{13}$$

The three series are graphed in Figure 1 and various moments of the exogenous variables are presented in Table 2. Figure 1 suggests that the exogenous variables are stationary. The serial correlations in λ^h, r^h and z^h are 0.93, 0.39 and 0.98 respectively. This lends support to the assumption that the exogenous variables are stationary first-order Markov processes. However, the serial correlation in productivity shock is close to one and for this reason, a model where productivity shocks follow a second-order process is also investigated.

It is important to kept in mind that the moments presented in Table 2 are given only for completeness. No transition equations for the shocks have to be specified in order to solve the model using DP by GMM. The historical paths plotted in Figure 1 are used directly in the solution method. It is interesting to note that the 95% confidence interval for the serial correlation in productivity shocks is [0.954, 1.014], a region where the dynamic properties of the model are highly sensitive to the degree of persistence in the productivity shocks.[7] This strengthen the case for using DP by GMM to solve the model.

A last point regarding the exogenous state variables. The measure of productivity shocks coming out of the estimation of equation (13) incorporates both a country-specific component and a world component. Models of the intertemporal approach usually interpret z_t as a country-specific shock. Glick and Rogoff (1995) find that for six G7 countries, the hypothesis that world productivity shocks do not affect the current account cannot be rejected (the exception being the UK). They also find that for five G7 countries, the hypothesis that country-specific shocks do not affect the current account is rejected (exceptions being Germany and France). Glick and Rogoff use an average of productivity measures across countries to generate a series for world productivity shocks. As an alternative, Gregory and Head (1999) use dynamic factor analysis and Kalman filtering to identify a world component

[7] See Letendre (2002) and Table 9 in Glick and Rogoff (1995).

in productivity and investment for G7 countries. The world component they estimate tracks very closely the US investment series, which indicates that shocks to the US economy tend to be global shocks. This is consistent with findings by Elliott and Fatás (1996) and Canova and Marrinan (1998). Gregory and Head's estimates show that both world and country-specific components have small effects on current accounts of G7 countries and that most of the dynamics in current accounts come from country-specific changes in investment.

One effect of world productivity shocks, compared to country-specific shocks, is to affect the world interest rate. For example, consider a highly persistent positive productivity shock that is common to all (symmetric) countries. Following this shock, all countries want to borrow on the world market. But that cannot be an equilibrium since there are no lender countries. Therefore, the world interest rate increases so that the world borrowing market reaches an equilibrium.

As explained above, my measure of world interest rates uses US interest rates. Since common shocks to productivity come primarily from the US and since US shocks obviously affect US interest rates, my measure of the world interest rates can control (to a certain extent) for the world component included in the measured productivity shocks.

3.3 DP by GMM

The solution to the model is a pair of decision rules (for consumption and investment) such that the representative agent maximizes (1′) subject to constraints (3′) and (4′). However, because the model does not allow for closed-form solutions to those decision rules, a numerical solution method must be used.

From dynamic programming we know that agent's decision rules depend on state variables. As is routinely done in the literature, I assume that z, r and λ follow a stationary first-order vector Markov process. Therefore, the state vector for this economy includes k_t, a_t, z_t, r_t and λ_t. Since I use a GMM estimator to estimate the approximate decision-rule parameters any correlation among z_t, r_t and λ_t is allowed.

The state vector implies that the (unknown) decision rules giving optimal values for time t consumption and investment are $c(a_t, k_t, z_t, r_t, \lambda_t)$ and $x(a_t, k_t, z_t, r_t, \lambda_t)$ respectively. Substituting these rules in the budget constraint yields a rule for a_{t+1} and substituting the investment decision rule in the law of motion for capital yields a rule for k_{t+1}.

I use DP by GMM to identify approximate decision rules that satisfy the first-order conditions of the agent's problem. The latter conditions are the Euler equations

$$\mathrm{E}_t \left\{ \tilde{\beta}(1 + r_{t+1}) \frac{c(a_t, k_t, z_t, r_t, \lambda_t)^\alpha}{c(a_{t+1}, k_{t+1}, z_{t+1}, r_{t+1}, \lambda_{t+1})^\alpha} - \gamma \right\} = 0 \qquad (14)$$

$$\mathrm{E}_t \left\{ \tilde{\beta}(1 + \theta z_{t+1} k_{t+1}^{\theta-1} - \delta) \frac{c(a_t, k_t, z_t, r_t, \lambda_t)^\alpha}{c(a_{t+1}, k_{t+1}, z_{t+1}, r_{t+1}, \lambda_{t+1})^\alpha} - \gamma \right\} = 0 \qquad (15)$$

the law of motion for capital

$$\gamma k_{t+1} = (1 - \delta) k_t + x(a_t, k_t, z_t, r_t, \lambda_t) \qquad (16)$$

and the budget constraint

$$\gamma a_{t+1} = (1 - \lambda_t) e^\xi z_t k_t^\theta + (1 + r_t) a_t - c(a_t, k_t, z_t, r_t, \lambda_t) - x(a_t, k_t, z_t, r_t, \lambda_t). \qquad (17)$$

The first step in solving the model by DP by GMM is to guess a form for the approximate decision rules. In what is referred to as the "leading model," the approximate decision rules for consumption and investment are assumed to be linear functions of the state vector. Formally, the guess is

$$c_t = H_t b_c \equiv \check{c}(b_c, k_t, a_t, z_t, \lambda_t, r_t) \qquad x_t = H_t b_x \equiv \check{x}(b_x, a_t, k_t, z_t, \lambda_t, r_t) \qquad (18)$$

where $H_t = [\,1 \quad k_t \quad a_t \quad z_t \quad \lambda_t \quad r_t\,]$ and b_c and b_x are six-dimensional vectors of unknown coefficients to be estimated. In addition, all unknown parameters are collected in the vector $b = [\,b_c \quad b_x\,]$. Substituting the guess in the law of motion (16) and the budget constraint (17) yields

$$k_{t+1} = \frac{1}{\gamma}\left[(1-\delta)k_t + H_t b_x\right] \equiv \check{k}'(b_x, a_t, k_t, z_t, r_t, \lambda_t) \tag{19}$$

$$a_{t+1} = \frac{1}{\gamma}\left[e^{\xi} z_t k_t^{\theta}(1-\lambda_t) + (1+r_t)a_t - H_t b_c - H_t b_x\right] \equiv \check{a}'(b, a_t, k_t, z_t, r_t, \lambda_t) \tag{20}$$

The second step is to substitute approximate decision rules for unknown optimal rules in the Euler equations (14) and (15). Doing so introduces an approximation error which I assume has mean zero and is unforecastable. After substitutions, Euler equations (14) and (15) are

$$\mathrm{E}_t\left\{\frac{\tilde{\beta}(1+r_{t+1})\ \check{c}(b_c, a_t, k_t, z_t, r_t, \lambda_t)^{\alpha}}{\check{c}[b_c, \check{a}'(b, a_t, k_t, z_t, r_t, \lambda_t), \check{k}'(b_x, a_t, k_t, z_t, r_t, \lambda_t), z_{t+1}, r_{t+1}, \lambda_{t+1}]^{\alpha}} - \gamma\right\} = 0 \tag{14$'$}$$

$$\mathrm{E}_t\left\{\frac{\tilde{\beta}(1+\theta z_{t+1} k'(b_x, a_t, k_t, z_t, r_t, \lambda_t)^{\theta-1} - \delta)\ \check{c}(b_c, a_t, k_t, z_t, r_t, \lambda_t)^{\alpha}}{\check{c}[b_c, \check{a}'(b, a_t, k_t, z_t, r_t, \lambda_t), \check{k}'(b_x, a_t, k_t, z_t, r_t, \lambda_t), z_{t+1}, r_{t+1}, \lambda_{t+1}]^{\alpha}} - \gamma\right\} = 0 \tag{15$'$}$$

Equations (14′) and (15′) both depend only on current endogenous state variables as well as current values and forecasts of future exogenous variables.

Then, the expectations in the Euler equations (14′) and (15′) are replaced by their *ex post* counterparts. The difference between the *ex ante* values and the *ex post* values is a forecast error which the model predicts has a mean of zero and cannot be forecast using information available in period t. Euler equation errors are

$$\eta_{a,t+1} = \frac{\tilde{\beta}(1+r_{t+1}^h)\ \check{c}(b_c, k_t^h, a_t^h, z_t^h, \lambda_t^h, r_t^h)^{\alpha}}{\check{c}[b_c, \check{k}'(b, k_t^h, a_t^h, z_t^h, \lambda_t^h, r_t^h), \check{a}'(b_x, k_t^h, a_t^h, z_t^h, \lambda_t^h, r_t^h), z_{t+1}^h, \lambda_{t+1}^h, r_{t+1}^h]^{\alpha}} - \gamma \tag{21}$$

$$\eta_{k,t+1} = \frac{\tilde{\beta}(1+\theta z_{t+1}^h k'(b_x, k_t^h, a_t^h, z_t^h, \lambda_t^h, r_t^h)^{\theta-1} - \delta)\ \check{c}(b_c, k_t^h, a_t^h, z_t^h, \lambda_t^h, r_t^h)^{\alpha}}{\check{c}[b_c, \check{k}'(b, k_t^h, a_t^h, z_t^h, \lambda_t^h, r_t^h), \check{a}'(b_x, k_t^h, a_t^h, z_t^h, \lambda_t^h, r_t^h), z_{t+1}^h, \lambda_{t+1}^h, r_{t+1}^h]^{\alpha}} - \gamma \tag{22}$$

For arbitrary values of the parameter vector b, we do not expect the approximate decision rules to satisfy the agent's first-order conditions. Therefore, I use iterated GMM to identify b. The GMM procedure is searching for values of the vector b that: (1) satisfy the system of equations of the model[8] and; (2) minimize the distance between historical and predicted paths for consumption and investment. More precisely, I use data on consumption, investment and all state variables to jointly estimate the Euler equations (14') and (15') and the approximate decision rules (18).[9]

The moment conditions used in the estimation and the GMM estimates are discussed in the next section.

4. Empirical results

4.1 The leading model

Equations (14') and (15') imply that η_{at+1} and η_{kt+1} are orthogonal to any variable included in the period t information set. This provides instruments to be used in the GMM estimation. The estimation of the vector b is performed with a variety of moment restrictions. The first set of moment restrictions includes the sample equivalent of equations (14') and (15')

$$\frac{1}{T}\sum_{t=1}^{T}\eta_{a,t+1}\times 1=0 \qquad \frac{1}{T}\sum_{t=1}^{T}\eta_{k,t+1}\times 1=0 \tag{23}$$

[8] The system of equations includes the law of motion for capital (3'), the budget constraint (4'), and the Euler equations (14') and (15').

[9] This estimation exercise is related to the work of Smith and Zin (1997) and Blankenau, Koe and Yi (2001). Smith and Zin use DP by GMM to estimate approximate decision rules to solve a closed-economy real business cycle model. Blankenau *et al.* use GMM to estimate approximate decision rules for next period capital stock and assets as part of a procedure to back out the exogenous shocks of their small-open economy model.

as well as the normal equations corresponding to the regression of consumption and investment on the state vector H_t

$$\frac{1}{T}\sum_{t=1}^{T}[c_t^h - H_t b_c] \times H_t = \mathbf{0} \qquad \frac{1}{T}\sum_{t=1}^{T}[x_t^h - H_t b_x] \times H_t = \mathbf{0} \tag{24}$$

where **0** denotes a six-dimensional vector of zeros.

Estimates of b obtained with the above moment restrictions are reported in the second column of Table 3 (column heading (I)). Most estimates are of the expected sign and only one parameter estimate is not statistically significant (the coefficient on r_t in the consumption rule). Larger physical capital stocks and assets increase consumption. A positive productivity shock increases consumption whereas government consumption reduces it. In theory, the effect of a change in the interest rate on consumption depends on a substitution effect, an income effect, and a wealth effect. In general, the income effect works against substitution and wealth effects producing an ambiguous net effect. With log utility, income and substitution effects cancel out leaving only the wealth effect. Accordingly, in the estimation consumption is found to depend negatively on the interest rate. A smaller capital stock and a positive productivity shock increase investment. Theory predicts that country-specific government consumption shocks should have no effect on investment and that world transitory shocks should decrease investment since they increase the interest rate. We see in Table 3 that an increase in the interest rate tends to reduce investment, and so are government consumption shocks.[10]

Since fourteen moment restrictions are used to estimate twelve parameters, it is possible to test the overidentifying restrictions using Hansen's J-test.[11] The values of the test statistics are reported at the bottom of Table 3. The test for overidentifying restrictions has the χ^2 distribution asymptotically. This distribution is used to calculate the p-values associated with the test statistics (also reported in the table). The null hypothesis of the test is that

[10] Although the point estimates suggest that consumption reacts more strongly to productivity than investment, elasticities calculated using the no-trade steady state show otherwise. A one percent increase in z_t^h increases consumption by six percent and investment by 8.5 percent.

[11] Blankenau *et al.* (2001) also report overidentifying restrictions tests while Bergin (2001) test the intertemporal model using a likelihood ratio test.

the economic model is correct, the instruments are valid and the approximate decision rules are correct. The p-value of the J-test is just above five percent, so we cannot reject the null at the one and five percent levels.

Euler equations (14') and (15') imply that the residuals $\eta_{a,t+1}$ and $\eta_{k,t+1}$ must be orthogonal to all variables part of the time t information set. Therefore, the moment restrictions (23) and (24) may be augmented by more moment restrictions. To verify the robustness of the estimates obtained, I redo the estimation using more overidentifying restrictions. In the third column of Table 3 (column heading (II)), the following two additional moment restrictions are used

$$\frac{1}{T}\sum_{t=1}^{T}\eta_{a,t+1}\times\frac{c_t}{c_{t-1}}=0 \qquad \frac{1}{T}\sum_{t=1}^{T}\eta_{k,t+1}\times\frac{c_t}{c_{t-1}}=0 \tag{25}$$

while in the fourth column of Table 4 (column heading (III)), the following two additional moment restrictions are used

$$\frac{1}{T}\sum_{t=1}^{T}\eta_{a,t+1}\times\frac{y_t}{y_{t-1}}=0 \qquad \frac{1}{T}\sum_{t=1}^{T}\eta_{k,t+1}\times\frac{y_t}{y_{t-1}}=0. \tag{26}$$

We see that the parameter estimates are very similar across the three sets of results and that the model and decision rules are never rejected at the five percent level.

Lastly, the parameter vector is estimated using moment restrictions (23) and the orthogonality conditions

$$\frac{1}{T}\sum_{t=1}^{T}\eta_{a,t+1}\times H_t=\mathbf{0} \qquad \frac{1}{T}\sum_{t=1}^{T}\eta_{k,t+1}\times H_t=\mathbf{0} \tag{27}$$

for a total of 24 moment restrictions (column heading (IV) in Table 3). The estimates are similar to the estimates obtained in the first three experiments but the numerous overidentifying restrictions are soundly rejected.[12]

Figure 2 plots historical and predicted paths for consumption, output, investment and trade balance over output. Given the small differences in the points estimates in Table 3, the

[12] The time t information set includes a very large number of variables. So in principle we could use very many instruments in the GMM estimation. However, Monte Carlo work by Hansen, Heaton and Yaron (1996), Kocherlakota (1990) and Smith (1999) suggests using small instrument sets in small samples.

predicted paths are virtually identical for all four sets of estimates in Table 3. Accordingly, I only present paths calculated using the first set of estimates reported in Table 3.

As usual, the trade balance is defined as

$$tb_t = y_t(1 - \lambda_t) - c_t - x_t. \tag{28}$$

Historical data on output, consumption and investment (*per capita*) are linearly detrended. In the discussion below, all references to historical data refer to detrended *per capita* historical data, except for the trade balance, which is not detrended.

Most papers using stochastic general equilibrium models compare statistical moments computed in the data with their artificial economy counterparts. A few selected moments are reported in Table 4 together with the correlations between historical and predicted paths plotted in Figure 2.[13] The second column of Table 4 presents a set of moments computed in Canadian data using *GMM*. Standard errors are in parentheses. The remaining columns present moments computed using artificial data generated by the four sets of estimated decision rules in Table 3. These moments do not have standard errors. The reason is that we have only one realization of historical exogenous variables. Since I do not specify their distribution, I cannot resample from that distribution and therefore I have only one set of moments (and sample paths) conditional on the realization of the exogenous variables.

Figure 2 and Table 4 demonstrate that the simple small-open economy model with linear approximate decision rules generate predicted paths that track historical paths fairly closely. There are five points to take away. First, the model predicts consumption exceptionally well. The correlation between historical and predicted paths is 0.96 and the coefficient of variation of consumption is 0.045 which is slightly lower than the coefficient of variation of consumption in Canadian data (0.0466).[14]

[13]Note that in the case of tb/y, predicted and historical paths have slightly different means so Figure 1 presents the paths in deviation from their means. The difference in means is expected since the calculation of net exports in national accounts include more elements (*e.g.* changes in inventories) than equation (28).

[14]I use the coefficient of variation as a of measure volatility instead of the standard deviation because the various series have different magnitudes. For example, since output is much larger than investment the

Second, the correlation between historical and predicted paths for output is very high (0.91). This is not surprising for two reasons: (i) in a model with constant labor supply, all the short-run fluctuations in output come from fluctuations in productivity shocks; (ii) productivity shocks are measure using output data. However, the predicted path for output is not as volatile as the historical path.

Third, the model predicts investment reasonably well (correlation of 0.70). Just like predicted output, predicted investment is less volatile than in the Canadian data. As shown in the next section, adding endogenous labor supply increase the volatility of output and investment (as expected) but at a significant cost.

Fourth, the ratio trade balance over output is reasonably well match by the model.[15] The correlation between predicted and historical paths is 0.61. This is somewhat lower than the correlation obtained with linear-quadratic models. Otto finds a correlation of 0.735 over the sample 1950 to 1987 whereas Ghosh finds an astonishing 0.98 over the sample 1960 to 1988. This is not very surprising since we do not expect that a structural model like the one used here outperforms an unrestricted *VAR* model in terms of dynamics. One interesting finding is that historical and predicted trade balance over output have the same standard deviation. This contrasts with the results of Otto (1992), Ghosh (1995), and Obstfeld and Rogoff (1996, chapter 2) who use vector autoregressions to generate predicted paths for the current account of many countries. Their predicted paths for Canada are always less volatile than the historical paths. For instance, Otto finds that the ratio of the standard deviation of the predicted current account to the standard deviation of the historical current account is 0.15 over the sample 1950 to 1987 whereas Ghosh finds a ratio of 0.39 over the sample 1960 to 1988. Also, the autocorrelation in the predicted and historical trade balance over

standard deviation of output is larger than the standard deviation of investment. This is obviously going against known empirical evidence. The coefficient of variation is a better measure of volatility when series have different means.

[15]The measure of external balance used to evaluate the model is the ratio of the trade balance to output. The trade balance is used instead of the current account because the latter is a function of past trade balances. Therefore, discrepancies between historical and predicted trade balances in the past enhance discrepancies between historical and predicted current account.

output are not statistically different at the five percent level.

Finally, another prediction of interest concerns the relation between savings and investment.[16] Since the work of Feldstein and Horioka (1980) on saving-investment correlations, a lot of attention has been given to the issue of international capital mobility. For instance, Baxter and Crucini (1993) use a two-country stochastic general equilibrium model to demonstrate that a high saving-investment correlation is consistent with perfect international capital mobility. The prediction of the current model regarding this correlation is consistent with their result, and the results of Cardia (1991). Even if the small-open economy has access to a frictionless international capital market, the saving-investment correlation is high, 0.43. This is very close to the correlation observed in the Canadian data, 0.44.

For all practical purposes, the fit of the model is extremely similar when log-linear rules rather than linear rules are estimated. The moments are extremely close to those reported in Table 4. The p-values associated with the J-test are slightly smaller than those reported in Table 3 but are all close to five percent.[17]

One short coming of the model identified in this section is the lack of volatility in output and investment. We expect that adding endogenous labor would increase the volatility of investment and output. Section 4.2 looks at a model with endogenous labor.

4.2 Endogenous labor

In the model with endogenous labor, periodic utility is given by

$$u(c,n) = \frac{\left[c^{\mu}\ (1-n)^{1-\mu}\right]^{1-\alpha}}{1-\alpha} \tag{29}$$

while the production function is

$$Y_t = e^{\xi} z_t K_t^{\theta} (n_t \gamma^t)^{1-\theta} \tag{30}$$

[16] In the data, savings equal *GDP* minus private consumption and government consumption. In the model, $s_t = y_t(1-\lambda_t) + r_t a_t - c_t$.

[17] Results available upon request.

The estimates of the structural parameters of the model with endogenous labor are presented in Table 2. Based on these estimates, μ is set to 0.093.

The addition of endogenous labor does not modify the state vector so the approximate decision rules estimated are

$$c_t = H_t b_c \qquad x_t = H_t b_x \qquad n_t = H_t b_n \tag{31}$$

where $H_t = [\,1 \quad k_t \quad a_t \quad z_t \quad \lambda_t \quad r_t\,]$ and b_c, b_x and b_n are six-dimensional vectors of unknown coefficients to be estimated. The vector of parameters $b = [\,b_c \quad b_x \quad b_n\,]$ is estimated by GMM using the equivalent of moment restrictions (23)-(24) in addition to

$$\frac{1}{T}\sum_{t=1}^{T} \eta_{n,t} \times 1 = 0 \tag{32}$$

and

$$\frac{1}{T}\sum_{t=1}^{T} [n_t^h - H_t b_n] \times H_t = \mathbf{0} \tag{33}$$

where $\mathbf{0}$ denotes a six-dimensional vector of zeros and $\eta_{n,t}$ denotes the residual associated with the labor first-order condition

$$\eta_{n,t} = \frac{1-\mu}{\mu}\frac{n_t}{1-n_t} - (1-\theta)\frac{y_t}{c_t} \tag{34}$$

The estimated decision rules (moments) are reported in the second (third) column of Table 5 (Table 6). The correlations between historical and predicted paths are very low. Predicted hours and trade balance are actually negatively correlated with their historical counterparts. The poor fit of the model comes from the counterfactual form of the labor first-order condition. The very poor fit of this condition is demonstrated in Johri and Letendre (2001). It is so at odds with the data that it is not possible for the consumption, output and hours predicted paths to match the historical paths and at the same time satisfy the labor first-order condition in a satisfactory fashion. For this reason, the J-test soundly rejects the model and the predicted paths do not track the historical paths very well.[18] As

[18] However, the small sample properties of the J-test may not be very good because of the high persistence in the residuals associated with the labor first-order condition. Essentially, estimating the covariance matrix of the empirical moments (used in calculating the J-test statistic) is difficult with a short sample of persistent data [see Andrews (1991), Altonji and Segal (1996), Christiano and den Haan (1996)].

expected, output and investment are more volatile in the economy with endogenous labor. In this sense, the model with endogenous labor improves on the leading model. However, correlations between historical paths and predicted paths strongly suggest otherwise. The correlations are negative for hours and the trade balance. These results suggest that a better modeling of the labor market side of the model is necessary to improve its fit. Therefore, the small-open economy model is subject to the same difficulty as early closed-economy business cycles models and asset pricing models. Accordingly, the focus of the paper is on the model with constant labor.

The leading model presented in section 2.1 allows for variations in the world interest rate. Evidence on the effects of interest rate variations on the fit of the small open economy model are mixed. While Mendoza (1991), Correia, Neves and Rebelo (1995) as well as Schmitt-Grohé (1998) finds no major role for it, Blankenau, Kose and Yi (2001) do. In the next section, a special case of the model with a constant world interest rate is investigated.

4.3 Constant interest rate

The importance of allowing for fluctuations in the world interest rate can be verified by studying the predictions of a special case of the leading model where the interest rate is constant. Table 5 (third column) reports the estimated approximate decision rules, Table 6 (fourth column) gives the moments obtained in this model, and Figure 3 shows the trade balance for Canada, the leading model and the model with constant interest rate.

The model with variable world interest rate dominates the model with constant interest rate. First, comparing Tables 4 and 6, we see that the correlations between historical and predicted paths are never larger in the model with constant interest rate. The largest impact is on the fit of the trade balance. As can be seen in Figure 3, the fit of the trade balance is better in the model with a variable interest rate (correlation of 0.61 vs. 0.31). This result is explained by the poorer fit of investment and consumption in the model with constant interest rate.

Second, from a statistical point of view, the J-test provides much stronger support to the model with variable interest rate. The p-value (reported in Table 5) indicates that the constant interest rate model is rejected at the one percent level whereas the leading model is not rejected at the five percent level.[19] Therefore, the analysis strongly suggests that a model with variable world interest rate is more in line with Canadian data. This result contrasts with the findings of Mendoza (1991), Correia, Neves and Rebelo (1995) and Schmitt-Grohé (1998) who find a modest role for a variable world interest rate but it is in line with the findings of Nason and Rogers (2001b) and Blankenau, Kose and Yi (2001). Blankenau *et al.* find that variations in the world interest rate (depending on the ordering of the variables in their VAR exercises) explain between four and twenty-nine percent of the variation in consumption, between six and thirty-two percent of the variation in investment and between fifteen and sixty-two percent of the variation in net exports. Our results are in line with these findings. Comparing the results in Table 4 with the fourth column of Table 6 shows that turning off the variation in the world interest rate worsen the fit of the trade balance more than the fit of consumption and investment.

4.4 More flexible decision rules

The analysis performed in the previous section has focussed on relatively simple decision rules. Despite their simplicity, these decision rules are not rejected by the data and they generate predicted paths that are highly correlated with historical paths. This section investigates the fit of the small-open economy model of section 3.1 when it is solved by DP by GMM using more elaborate approximate decision rules.

In the first case, it is assumed that productivity shocks follow a second-order Markov process.[20] The state vector must now include z_t and z_{t-1} since the current realization as

[19]When we add the projections of the Euler equation residuals on lagged consumption or output growth (as is done for the leading model), the p-values associated with the J-test are of order 10^{-12} and the moments are very similar to those reported in Table 6.

[20]We could allow all exogenous variables to follow a multivariate second-order process. In such a case, the

well as the first lag are necessary to predict future productivity shocks. The approximate decision rules are

$$c_t = H_t' \, b_c \qquad x_t = H_t' \, b_x \tag{35}$$

where $H_t' = [1 \quad k_t \quad a_t \quad z_t \quad \lambda_t \quad r_t \quad z_{t-1}]$ and b_c and b_x are seven-dimensional vectors of unknown coefficients to be estimated. The moment restrictions used are given by equations (23) and (24) where H_t is replaced by H_t'. The fourth column of Table 5 and the fifth column of Table 6 report the estimated decision rules and the moments. Expanding the state vector to include z_{t-1} slightly increases the correlation between historical and predicted trade balance and reduces its persistence. Overall, the results are very similar to those from the leading model. For space considerations, the predicted paths are not presented. The test for overidentifying restrictions reported in the fourth column of Table 5 has an associated p-value of 0.0444, very close to the p-value associated with the estimation of the leading model (0.0544). Note that since H_t' differs from H_t, the estimation of the approximate decision rules with lag productivity shocks do not necessarily yields a p-value higher than in the leading model.

In the second case, a second-order decision rule is estimated. The approximate decision rules estimated are denoted

$$c_t = H_t'' \, b_c \qquad x_t = H_t'' \, b_x \tag{36}$$

where H_t'' includes all elements of H_t plus squares and cross-products of k_t, a_t, z_t λ_t and r_t. Note that it was not possible to estimate the approximate decision rules when a_t^2 was part of H_t''. For this reason, a_t^2 was left out and b_c and b_x each have 20 elements. The moment restrictions used are given by equations (23) and (24) where H_t is replaced by H_t''. The parameter estimates are reported in Table 5. Evidently, estimating 40 parameters by GMM using a sample of 172 data points is a difficult task. It is not surprising to see that many (twenty-five) of the parameter estimates are not statistically significant. The test of overidentifying restrictions do not reject the model and the decision rules at the five percent level (although it is not know how reliable inference based on asymptotic theory is when so many parameters are estimated in a small sample like the one used here). The last column

results are very similar to those where we only allow productivity shocks to be second-order.

of Table 6 reports the moments while Figure 4 presents the historical and predicted paths for consumption, output, investment and the trade balance from the leading model (first-order rules) and the model with second-order rules. The results suggest that the second-order decision rules dominate the first-order rules (but not overwhelmingly). The first and second-order rules fit consumption and output equally well. The second-order rules fit investment better (correlation of 0.89 vs. 0.70) while the first-order rules fit the trade balance better (correlation of 0.61 vs. 0.53). Since the second-order rules predict output, consumption and investment at least as well as the the first-order rules, it is puzzling that they do not predict the trade balance as well. This implies that a small part of the correlation observed between the historical and predicted trade balance for the leading model comes from the correlation between predicted investment and some factors entering the definition of the trade balance in Canadian national accounting but not in the model. Obviously, if the definition of the trade balance used in the paper [equation (28)] matched exactly the definition in national accounts, the model with second-order rules would fit the trade balance data better. In this sense, the second-order rules are better because first-order rules fit the trade balance better by "capturing" elements of the data not included in the theoretical model. However, for a number of reasons, I prefer to make the first-order rules part of the leading model: (i) the results based on first or second-order rules are similar; (ii) estimating a large number of parameters is difficult in a small sample; (iii) estimating the second-order rules yields many insignificant parameter estimates and; (iv) the reliability of the inference based on asymptotic theory is questionable.

5. Concluding comments

The intertemporal approach to the current account uses dynamic-optimizing models to explain movements in current accounts of open economies. The small-open economy model presented here is an example of such a model and its predictions are compared to the Canadian history.

The solution method adopted here, DP by GMM, is based on estimation. It makes weak distributional assumptions and uses historical paths for exogenous variables contrary to many other numerical solution methods that calibrate parametric stochastic processes for exogenous variables. This study does not rely on calibration since all structural parameters are estimated using the method suggested by Christiano and Eichenbaum (1992).

The empirical analysis shows that a simple small-open economy model with constant labor and solved by first-order linear approximation rules accords well with the data. First, comparisons of historical and predicted paths shows that the model replicates the dynamics of output, consumption, investment and trade balance fairly well. Second, a formal statistical test does not reject the model and the decision rules at the five percent level. However, predicted output and investment paths are less volatile than their empirical counterparts. A model with endogenous labor decisions is shown to increase the volatility in output and investment. However, due to the counterfactual form of the labor first-order condition, the predicted paths for output consumption and hours cannot at the same time track the historical paths closely and satisfy the labor first-order condition. As a result, the data reject the model and the decision rules and the predicted paths do not match the historical paths. This suggests that the small-open economy model is subject to the same difficulty as early closed-economy business cycles models and asset pricing models. A more empirically realistic modeling of the labor market side of the model is probably necessary to further improve the predictions of the model.

A special case of the model with a constant world interest rate is found to be dominated by the model with a variable world interest rate. Compared to the model with a constant interest rate, the predicted paths of the model with a variable interest rate are always at least as highly correlated with historical paths. Also, the model with constant interest and the decision rules are statistically rejected by the data.

Even though the small-open economy models investigated in this paper do a reasonable job at reproducing the fluctuations in the Canadian trade balance, the correlation between historical and predicted paths is never larger than 0.64. So there is plenty of room for improvements. A first research avenue is an improvement of the labor side of the model. Another avenue is the introduction of other types of shocks that could be important determinants of the trade balance. The work of Bergin and Sheffrin (2000) suggests that exchange rate shocks is a leading candidate. Finally, various trade frictions could be added to the model. Bergin (2001) finds stronger support for an intertemporal model including costs of borrowing internationally.

Acknowledgements

This paper is based on a chapter of my Ph.D. thesis at Queen's University. I thank the Social Sciences and Humanities Research Council of Canada for support of this research. I thank Gregor Smith for his advice. Martin Boileau, Michel Normandin, Paul Bergin, Robert Kollmann and seminar participants at University of Toronto, Queen's University, McMaster University, Boston College, Université de Montréal, Bank of Canada, University of Alberta, Concordia University, University of Guelph, York University, Carleton University, University of Ottawa, the Canadian Economics Association meetings, the Theories and Methods in Macroeconomics 1999 conference and the Midwest Macroeconomics Spring 1999 conference provided helpful comments. Comments from two anonymous referees greatly improved the paper.

DATA APPENDIX

Data for the United States

Interest rate: three-month treasury bill rate (secondary market) averages of daily closing bid prices. From the Federal Reserve Bank of St. Louis (FRED).

Data for Canada

Unless otherwise specified, Canadian series are from CANSIM. Labels are in parentheses.

output: real GDP (D20463).

capital: net stock of real fixed non-residential capital, total for all industries (D886842). This series is annual. It is converted to quarterly using `interpol` in *RATS*. Using linear interpolation yields similar results.

total hours: employment times average weekly hours in manufacturing times 13. The series on employment was constructed using three series. Two series for employment 15 years and older (D980595 and D767608) and one for employment 14 years and older (D762363). The series D762363 is from the *Canadian Statistical Review Annual Supplement to Section 1*, 1974. The series on average weekly hours in manufacturing was constructed using two data sources. The series D97800 is from CANSIM. The remaining data come from the *Canadian Statistical Review Annual Supplement to Section 1*, 1964 and 1982.

government consumption: government sector real current expenditure on goods and services (D20465).

consumption: real personal expenditure on nondurables (D20498) and services (D20504).

investment: sum of real business investment in residential construction (D20469), non-residential construction (D20470) and machinery and equipment (D20471).

net exports: real exports of goods and services (D20476) minus real imports of goods and services (D20480).

assets: Assets are calculated using the equation $A_{t+1} = (1 + r_t)A_t + TB_t$.

population: quarterly estimates of population for Canada (D1).

nominal exchange rate: Canada-US nominal noon spot exchange rate. In Canadian dollars (B3400).

price index: GDP implicit price index (D20556).

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Table 1
Estimation of Structural Parameters

Parameter	Constant Labor	Endogenous Labor
α	0.0333 (1.1127)	-13.9799 (30.1755)
μ		0.0931 (0.0006)
γ	1.0058 (0.0001)	1.0058 (0.0008)
ξ	1.9797 (0.0215)	1.9804 (0.0221)
θ	0.1765 (0.0053)	0.1767 (0.0054)
δ	0.0447 (0.0006)	0.0447 (0.0007)

Notes: Canadian data are *per capita* quarterly. The data appendix gives a complete description of the data. Standard errors are in parentheses. For the model with endogenous labor, the periodic utility function is $u(c,n) = [c^{\mu}(1-n)^{1-\mu}]^{1-\alpha}/(1-\alpha)$.

Table 2
Moments of Exogenous Variables

	z_t^h	λ_t^h	r_t^h	y_t^h
Mean	0.9866	0.2083	0.0056	2.2548
	(0.0071)	(0.0019)	(0.0020)	(0.0253)
Coefficient of variation	0.0429	0.0571	3.0711	0.0667
	(0.0026)	(0.0063)	(0.1979)	(0.0044)
Standard deviation	0.0423	0.0119	0.01689	0.1504
	(0.0036)	(0.0013)	(0.0010)	(0.0099)
Correlation matrix				
z_t^h	1	0.3867	-0.2292	0.9145
λ_t^h		1	-0.1147	0.1528
r_t^h			1	-0.1689
y_t^h				1

Notes: z_t denotes productivity shocks; λ_t denotes the ratio of government spendings to GDP; r_t denotes the world interest rates; y_t^h denotes detrended GDP. The calculation of the exogenous shocks is explained in section 3.2. Standard errors are in parentheses.

Table 3
Approximate Decision Rules: Leading Model

	(I)	(II)	(III)	(IV)
		Consumption		
1	0.8898 (0.0528)	0.8577 (0.0466)	0.8821 (0.0511)	0.8947 (0.0483)
k_t^h	0.0150 (0.0040)	0.0153 (0.0038)	0.0141 (0.0037)	0.0276 (0.0043)
a_t^h	0.0011 (0.0001)	0.0011 (0.0001)	0.0011 (0.0001)	0.0012 (0.0001)
z_t^h	0.5565 (0.0461)	0.5751 (0.0442)	0.5655 (0.0453)	0.5147 (0.0469)
λ_t^h	-1.3770 (0.1159)	-1.3568 (0.1142)	-1.3605 (0.1140)	-1.6148 (0.1154)
r_t^h	-0.0903 (0.0692)*	-0.0723 (0.0678)*	-0.0877 (0.0682)*	-0.1748 (0.0436)
		Investment		
1	0.8910 (0.0843)	0.8813 (0.0836)	0.8802 (0.0835)	0.8642 (0.0788)
k_t^h	-0.0537 (0.0100)	-0.0504 (0.0101)	-0.0544 (0.0098)	-0.0425 (0.0090)
a_t^h	0.0009 (0.0001)	0.0009 (0.0001)	0.0009 (0.0001)	0.0009 (0.0001)
z_t^h	0.2323 (0.0756)	0.2149 (0.0758)	0.2360 (0.0754)	0.1771 (0.0705)
λ_t^h	-0.5793 (0.1866)	-0.5801 (0.1855)	-0.5316 (0.1824)	-0.6132 (0.1737)
r_t^h	-0.5297 (0.1093)	-0.5397 (0.1102)	-0.5395 (0.1080)	-0.5861 (0.1084)
J [df]	5.8232 [2]	9.2288 [4]	9.2289 [4]	52.7464 [12]
(p-value)	(0.0544)	(0.0556)	(0.0556)	(4.6E-7)

Notes: The column headings (I)-(IV) represents various instrument sets described in section 4.1. J denotes the value of the test for overidentifying restrictions. This test statistic is asymptotically distributed χ^2 with df degrees of freedom. A * indicates that the estimate is not statistically significant at the five percent level. Standard errors are in parentheses.

Table 4
Statistical Moments

	Data	(I)	(II)	(III)	(IV)
Panel 1: Correlation between predicted and historical paths					
c		0.9580	0.9578	0.9580	0.9551
y		0.9139	0.9139	0.9139	0.9139
x		0.6972	0.6965	0.6975	0.6900
tb/y		0.6110	0.6068	0.6047	0.6103
Panel 2: Coefficients of variation					
c	0.0466 (0.0037)	0.0450†	0.0449†	0.0447†	0.0485†
y	0.0667 (0.0044)	0.0500	0.0500	0.0500	0.0500
x	0.0863 (0.0091)	0.0593	0.0584	0.0590	0.0579
Panel 3: Trade balance over output					
Std Dev	0.0181 (0.0015)	0.0181†	0.0176†	0.0180†	0.0173†
Autocor	0.9079 (0.0326)	0.9502†	0.9490†	0.9483†	0.9359†

Notes: The column headings (I)-(IV) correspond to the column headings in Table 3. Canadian data are *per capita* quarterly. The data are linearly detrended. The data appendix gives a complete description of the data. Numbers in parentheses are *GMM* standard errors computed using two lags of the autocovariances of the moment conditions. † indicates that the predicted moment is not statistically different from the historical moment at the five percent level.

Table 5
Alternative Models and Decision Rules

	Endogenous n	Constant r	z lagged
		Consumption	
1	-4.4073 (0.2102)	0.8662 (0.4060)	0.8201 (0.0527)
k_t^h	0.1997 (0.0331)	-0.0002 (0.0052)*	0.0146 (0.0045)
a_t^h	0.0129 (0.0004)	0.0037 (0.0018)	0.0010 (0.0001)
z_t^h	2.9370 (0.2074)	1.1180 (0.0639)	0.1014 (0.1333)*
λ_t^h	5.6497 (0.6484)	-0.9053 (0.1424)	-1.3704 (0.1213)
r_t^h	1.1647 (0.4062)		-0.0713 (0.0666)*
z_{t-1}^h			0.5128 (0.1382)
		Investment	
1	-4.7566 (0.2307)	4.5032 (0.4137)	0.8595 (0.0858)
k_t^h	0.1641 (0.0359)	-0.0985 (0.0097)	-0.0502 (0.0123)
a_t^h	0.0144 (0.0006)	0.0190 (0.0019)	0.0009 (0.0001)
z_t^h	2.6269 (0.1997)	0.3979 (0.0669)	-0.0078 (0.2301)*
λ_t^h	7.1745 (0.7441)	0.1303 (0.1797)*	-0.6562 (0.2174)
r_t^h	0.6969 (0.4235)*		-0.5419 (0.1102)
z_{t-1}^h			0.2531 (0.2394)*
		Hours	
1	-0.7468 (0.0380)		
k_t^h	0.0383 (0.0053)		
a_t^h	0.0025 (0.0001)		
z_t^h	0.4446 (0.0333)		
λ_t^h	0.9547 (0.1115)		
r_t^h	0.1832 (0.0705)		
J [df]	80.2958 [3]	55.8327 [2]	6.2286 [2]
(p-value)	(2.7e-11)	(7.5e-13)	(0.0444)

Table 5 (continued)
Alternative Models and Decision Rules

Second-order polynomials		
	Consumption	**Investissement**
1	-0.7883 (2.2754)*	5.6184 (2.6590)
k_t^h	0.5380 (0.2869)*	-3.6338 (0.5609)
a_t^h	0.0014 (0.0024)*	0.0128 (0.0028)
z_t^h	1.2716 (3.2206)*	19.5590 (4.3253)
λ_t^h	-7.9218 (7.6430)*	10.3151 (8.7905)*
r_t^h	1.5240 (4.2243)*	0.6348 (5.8225)*
${z_t^h}^2$	3.7570 (2.0907)*	-13.7739 (2.7091)
${k_t^h}^2$	0.0245 (0.0107)	0.0822 (0.0150)
${\lambda_t^h}^2$	55.2630 (9.4828)	-1.3673 (14.2776)*
$z_t^h \times k_t^h$	-0.8024 (0.3172)	1.5962 (0.4957)
$a_t^h \times k_t^h$	-0.0008 (0.0000)	-0.0030 (0.0005)
$\lambda_t^h \times k_t^h$	-1.2931 (0.6288)	0.6959 (0.9816)*
$z_t^h \times \lambda_t^h$	-4.7873 (7.5475)*	-14.2738 (8.9771)*
$z_t^h \times a_t^h$	0.0046 (0.0023)	0.0093 (0.0038)
$\lambda_t^h \times a_t^h$	0.0092 (0.0086)*	0.0143 (0.0110)*
${r_t^h}^2$	0.0820 (3.4284)*	-4.0723 (4.7428)*
$r_t^h \times k_t^h$	-0.0737 (2.2701)*	0.7514 (0.3845)*
$r_t^h \times a_t^h$	0.0024 (0.0051)*	0.0012 (0.0062)*
$r_t^h \times z_t^h$	-0.1164 (2.8866)*	-6.0533 (4.2909)*
$r_t^h \times \lambda_t^h$	-2.2192 (7.5581)*	-3.3609 (11.0502)*
J [df]	5.5415 [2]	
(p-value)	(0.0626)	

Note: Standard deviations in parentheses. See notes to Table 6.

Table 6
Statistical Moments

	Data	Endog. n	Constant r	z lagged	2^{nd} order
Panel 1: Correlation between predicted and historical paths					
c		0.1084	0.8854	0.9622	0.9655
y		0.3908	0.9139	0.9137	0.9139
x		0.0153	0.6451	0.6958	0.8893
tb/y		-0.1466	0.3088	0.6400	0.5266
n		-0.2244			
Panel 2: Coefficients of variation					
c	0.0466 (0.0037)	0.0808	0.0456†	0.0453†	0.0448†
y	0.0667 (0.0044)	0.0935	0.0500	0.0502	0.0500
x	0.0863 (0.0091)	0.2742	0.0661§	0.0594	0.0774†
n	0.0309 (0.0031)	0.0967			
Panel 3: Trade balance over output					
Std Dev	0.0181 (0.0015)	0.0418	0.0162†	0.0175†	0.0220§
Autocor	0.9079 (0.0326)	0.8746†	0.9474†	0.9057†	0.9143†

Notes: Column headings: *Endog. n* refers to the model with endogenous labor; *Constant r* refers to the model with a constant world interest rate; *z lagged* refers to the leading model solved assuming productivity shocks follow a second-order Markov process; *2nd order* refers to the leading model solved using approximate decision rules that are second-order polynomials in the elements of the state vector. Canadian data are *per capita* quarterly. The data are linearly detrended. The data appendix gives a complete description of the data. Numbers in parentheses are *GMM* standard errors computed using two lags of the autocovariances of the moment conditions. § indicates that the predicted moment is not statistically different from the historical moment at the one percent level and † at the five percent level.

Figure 1: Exogenous Variables

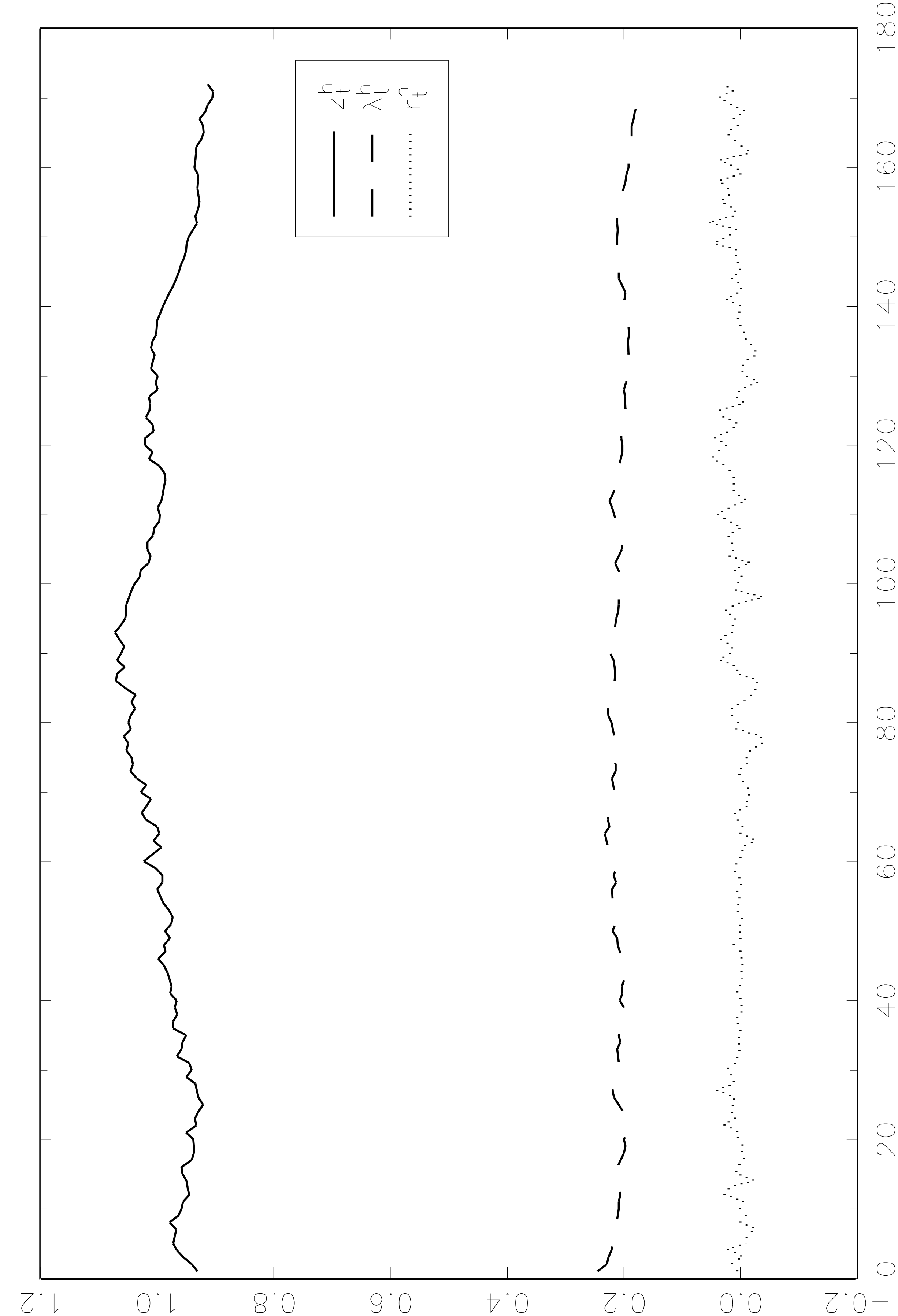


Figure 2a: Historical and Predicted Consumption

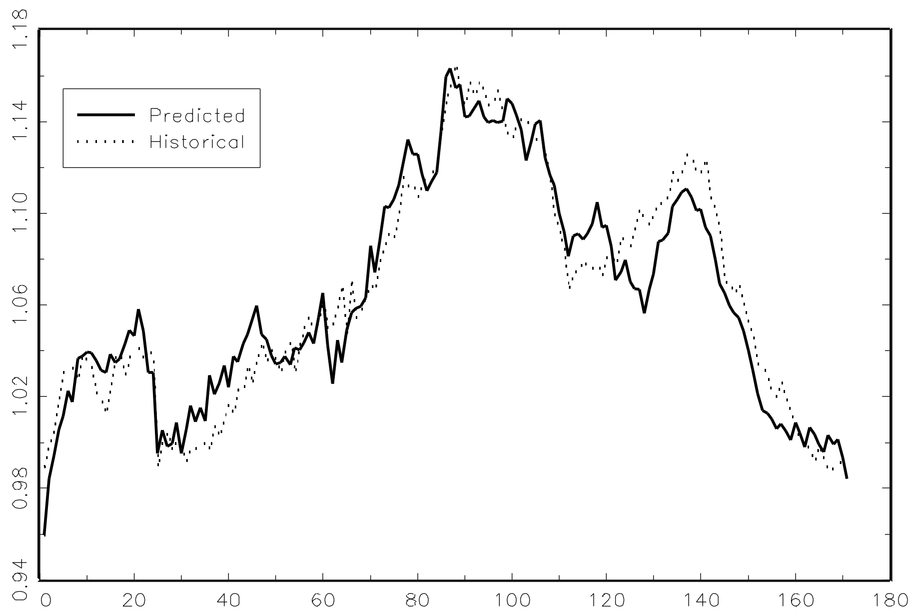


Figure 2b: Historical and Predicted Output

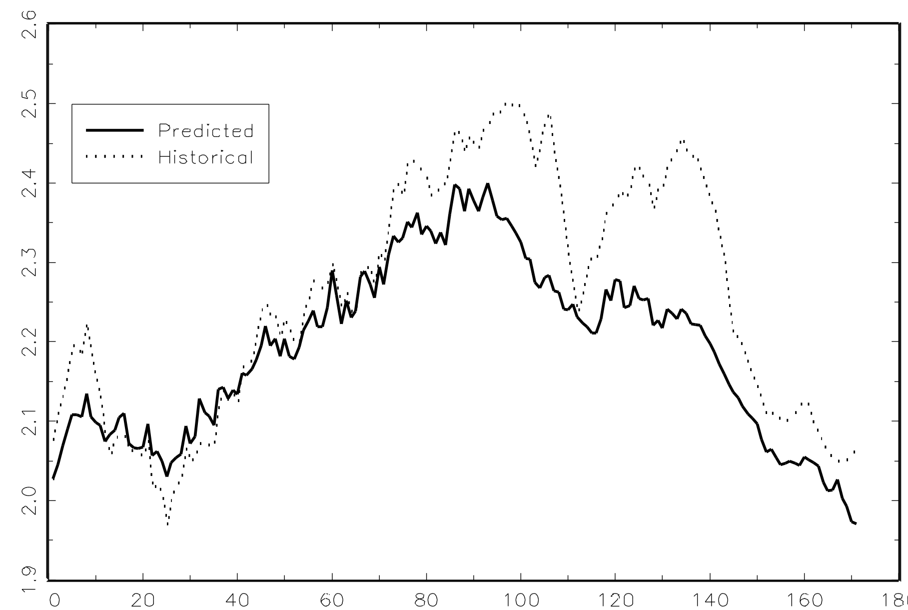


Figure 2c: Historical and Predicted Investment

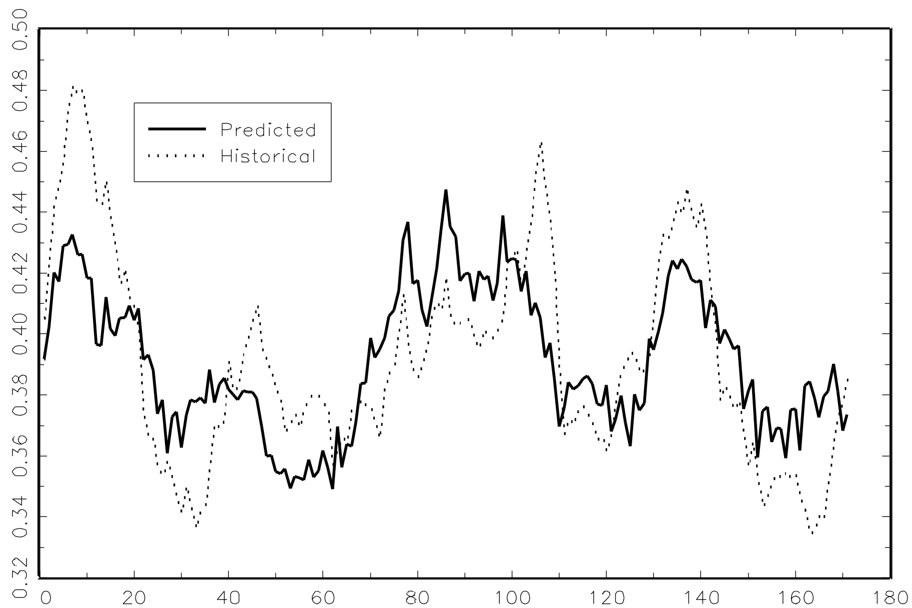
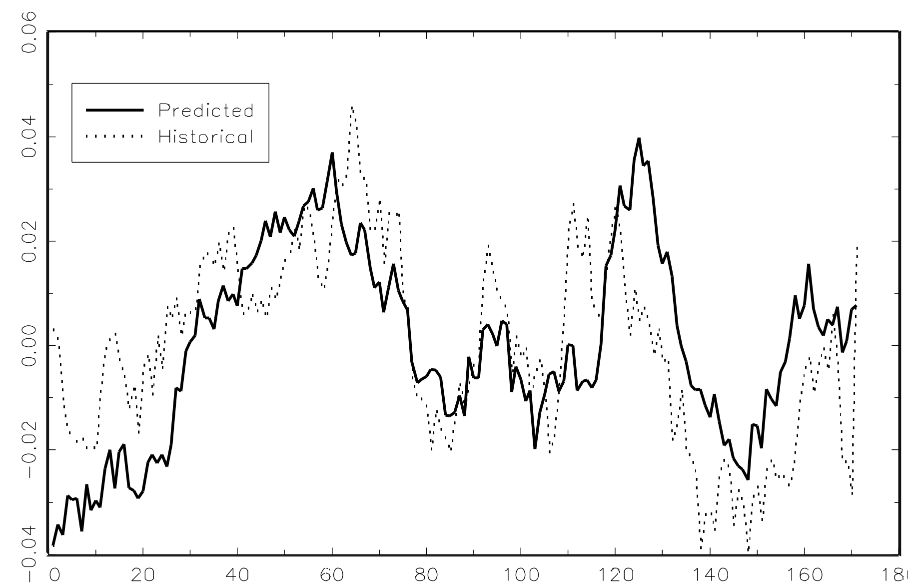


Figure 2d: Historical and Predicted Trade Balance over Output



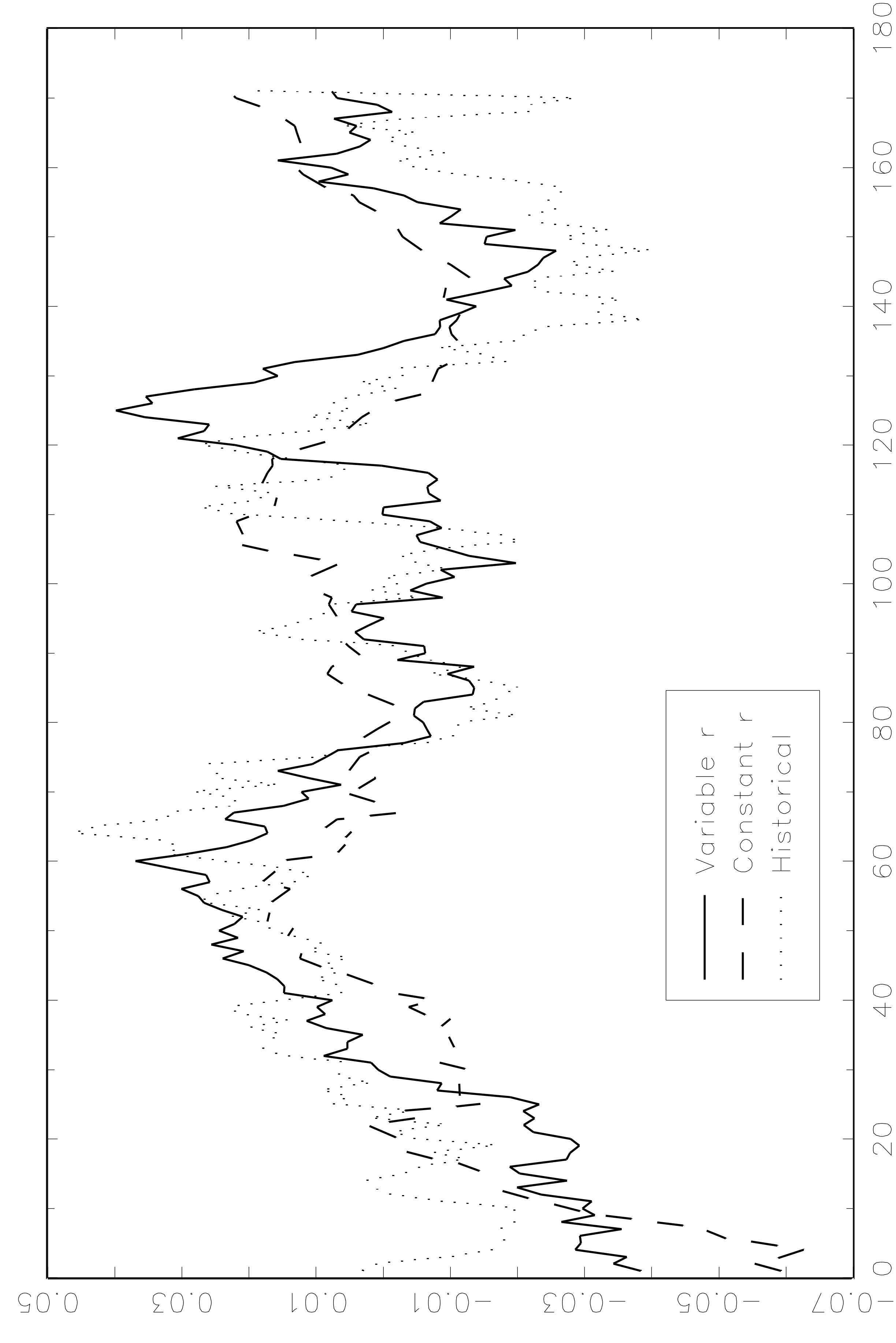


Figure 3: Historical and Predicted Trade Balance over Output

Figure 4a: Historical and Predicted Consumption

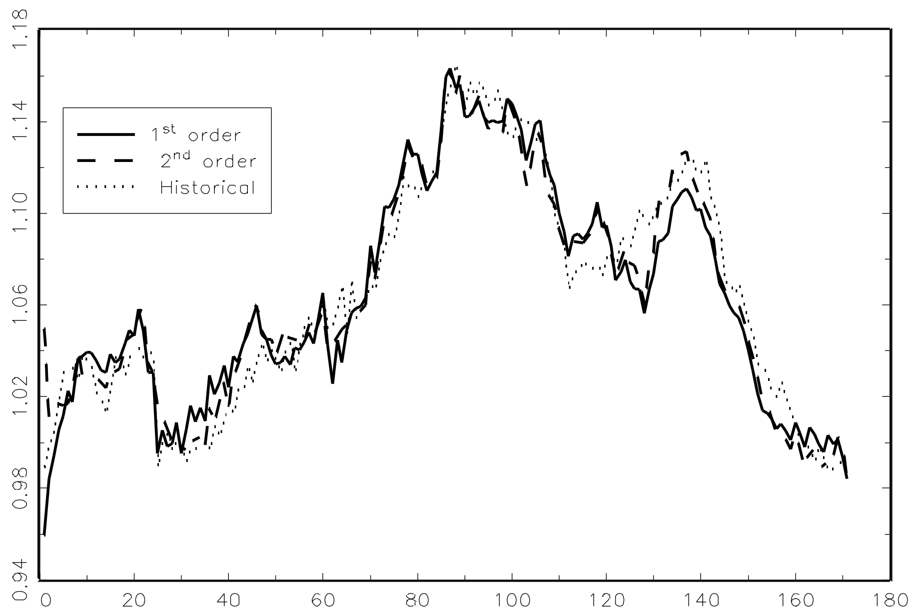


Figure 4b: Historical and Predicted Output

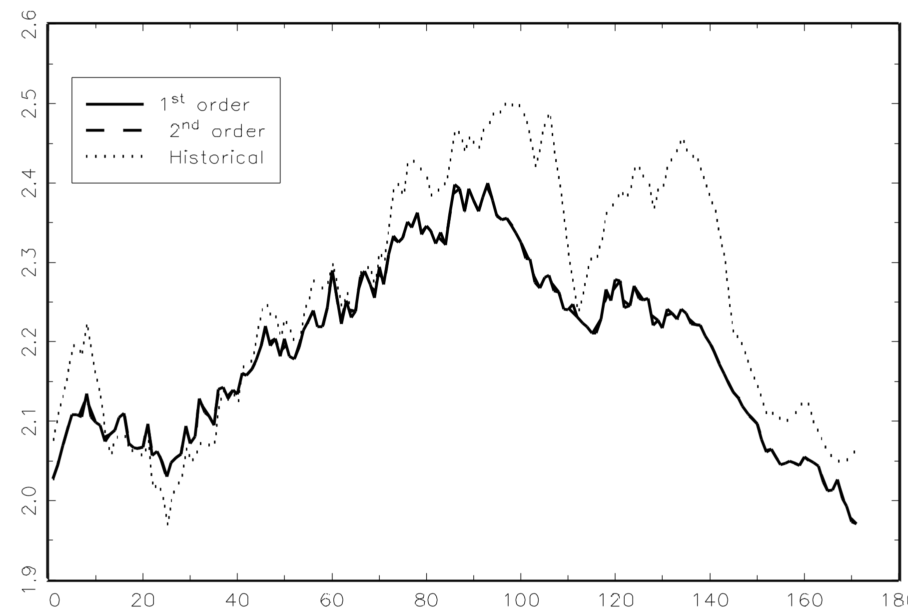


Figure 4c: Historical and Predicted Investment

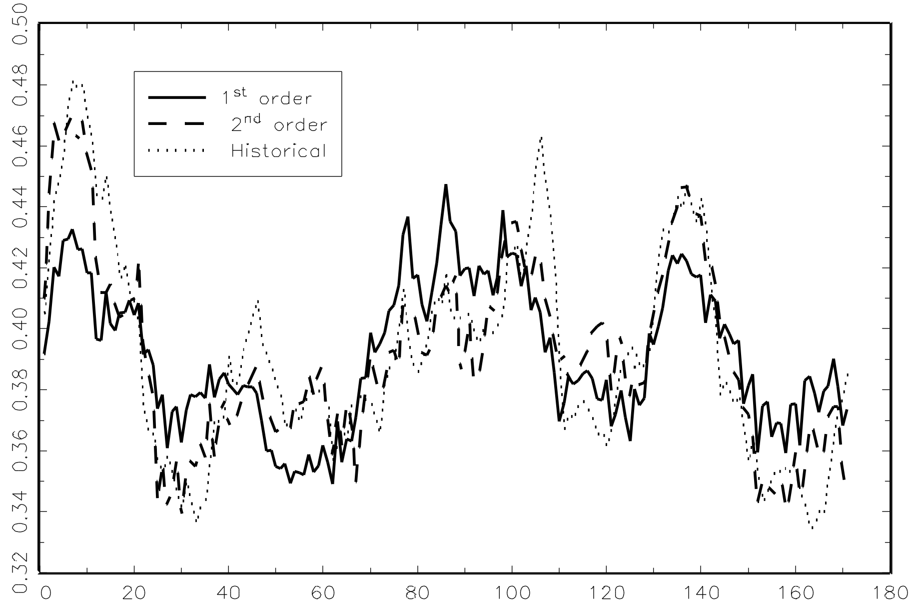
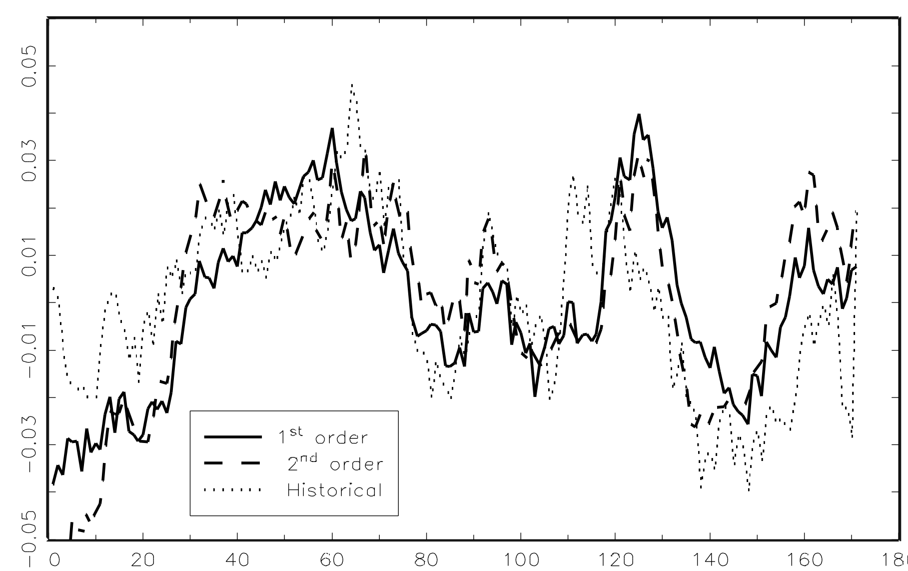


Figure 4d: Historical and Predicted Trade Balance over Output



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