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CAPITAL TAX COMPETITION AND RETURNS TO SCALE

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Abstract. There is a gap between the predictions of capital tax competition models and the reality they purport to describe. In a standard capital-tax model, with head taxes, capital-importing regions tax capital and capital-exporting regions subsidize capital. In the real-world, competing regions appear to subsidize capital whether or not they are capital importers. We show that by relaxing the standard assumption of constant returns to scale symmetric regions in a Nash equilibrium may all subsidize capital.We also prove that any inefficiencies in a non-symmetric Nash equilibria arise entirely from regions' incentives to manipulate the terms of trade, and not from increasing returns.We also compare our results to those in captial tax competition models without head taxes.

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1. Introduction

Today's newspapers are filled with articles relating the subsidization of capital investment by state, provincial, and municipal governments. Consider the following quotation from a recent issue of the *Wall Street Journal*:

> In 2000, Mississippi went hog wild in outbidding neighboring states for the Nissan factory by offering a fat package of close to $300 million in subsidies and tax breaks. The deal included $80 million from the state to train Nissan's new workers. Also included was a pledge to 'quick-take' the property of three families and give it to Nissan so it could build a parking lot and access road for the factory. (extracted from "Mississippi Churning", Friday 4th January 2002, p. A12)

What does the capital tax competition literature have to say about such regional government behaviour? This literature, based on the work of Zodrow and Mieszkowski (1986) and Wilson (1986), demonstrates that price-taking regional governments restricted to the use of source-based capital taxes will set inefficiently low positive tax rates when they act as Nash competitors for mobile capital. Since the tax revenue raised on capital is used to publicly provide a private good, the provision of this good will also be inefficient. This literature was extended to allow for non-price taking behaviour on the part of regional governments (Wildasin, 1988) and to asymmetric regions (Bucovetsky (1991) and Wilson (1991)). In both extensions, the Nash equilibria are still inefficient and capital tax rates are positive in equilibrium.[1] Another model of capital tax competition (Hamada (1966) or Burbidge et al. (1997)), which is a close relative of the theoretical model of non–cooperative tariff determination (Johnson, 1953), allows regions to use a wage tax which is equivalent to a lump-sum tax since labour is assumed to be supplied inelastically. The inefficiency of the provision of the public good disappears in these models, but each region still uses its capital tax to turn the terms-of-trade in its favour. Capital-importing regions tax capital to reduce the price they have to pay for the capital they import and capital-exporting regions subsidize capital to drive up the price they receive for their exports. Although, such models can explain the subsidization of firms by capital exporters its results contrast starkly with the observations of governments of capital-importing jurisdictions subsidizing firms.

All of these capital-tax competition models assume each jurisdiction's production function exhibits constant returns to scale. In this short paper, we show that with increasing returns to scale each region may have an incentive to subsidize capital. Moreover, if one sets up the initial conditions so that no jurisdiction has an incentive to manipulate the terms-of-trade then the Nash equilibrium is efficient. In this sense,

[1] See Wilson (1999) for a comprehensive survey of the standard capital tax competition model and its various extensions.

each region uses its capital tax (subsidy) properly to correct the production externality induced by increasing returns to scale. We describe the model and characterize the efficient allocation in the next section. Then, in Section 3, we derive our results with competing regions and relate them to the existing literature. Finally, in Section 4 we make some concluding remarks.

2. The Model

Consider an economy with two regions, $i = 1, 2$. Denote the fixed population in each region by N_i, the number of units of capital operating in each region by K_i, and the capital owned by the residents of each region by $\overline{K_i}$. All individuals supply one unit of labour inelastically, so N_i is also the units of labour in region i. We define the economy-wide endowment of capital and labour as $\overline{K} \equiv \overline{K_1} + \overline{K_2}$ and $\overline{N} \equiv N_1 + N_2$, respectively. There is full employment of capital, so $\overline{K} = K_1 + K_2$. Both regions have the same production technology and total output of region i is given by $F(N_i, K_i)$. We denote first and second derivatives by F^i_N, F^i_K, F^i_{KN} and so on. Output in each region can either be consumed or used to produce a publicly provided private good. One unit of output is required to produced a unit of the public good. Households derive utility from private consumption c and the public good g. Preferences are the same in both regions and can be represented by the well-behaved utility function $U(c_i, g_i)$.

2.1. Increasing Returns to Scale. We want to think about capital tax competition models without constant returns to scale. Consider a regional production function $F(N, K)$ that is homogeneous of degree $\gamma \geq 1$. When $\gamma = 1$ the production function exhibits constant returns to scale (CRTS); when $\gamma > 1$ it has increasing returns to scale (IRTS).We assume that the production technology is such that $F_N, F_K, F_{KN} > 0$ and $F_{KK}, F_{NN} < 0$. Although, there are increasing returns we are focus on the case when these returns are not that large, so marginal products are still downward-sloping. We know that

$$F(\lambda N, \lambda K) = \lambda^\gamma F(N, K), \ \forall \lambda > 0. \tag{1}$$

Differentiating with respect to λ and then setting λ equal to unity yields (by Euler's Theorem)

$$F_N N + F_K K = \gamma F(N, K). \tag{2}$$

Since F is homogeneous of degree γ, F_K is homogeneous of degree $\gamma - 1$,

$$F_{KN} N + F_{KK} K = (\gamma - 1) F_K \tag{3}$$

or

$$F_{KN} = (\gamma - 1) F_K / N - F_{KK} K / N. \tag{4}$$

One of the attractions of CRTS is that with competitive markets each factor is paid the value of its marginal product, the value of output equals the sum of the payments to each factor and it does not matter whether capital hires labour or labour hires capital. If we drop the CRTS assumption at least one factor will not be paid the value of its marginal product. It may then matter which factor (if any) receives its marginal product. For example, in a model in which immobile workers hire perfectly mobile capital, absent taxes and under competitive conditions, capital will be paid the value of its marginal product. On the other hand, when perfectly mobile firms/capital hire labour and the labour market is competitive, then labour receives the value of its marginal product, and it is the mobile factor — firms — who may not earn their marginal product (Boadway et al. (2002)). Another reason capital may not receive the value of its marginal product is the existence of production or Marshallian externalities.[2] In particular, one could imagine a situation in which an individual firm's production depends positively both on the capital it employs and on the total capital stock utilized in the region.[3] Competitive firms would take both factor prices and the total capital stock as given when they make their production decisions. As a result, the return to capital would differ from the value of its regional marginal product.[4] It turns out that whether the mobile or immobile factor receives the value of its marginal product affects the equilibrium capital tax rates, but not the efficiency of the resulting allocation when regions are symmetric. In particular, regions have an incentive to subsidize capital only when labour receives the value of its marginal product. We focus on this case in our paper. Before turning to the outcome with competing governments, we characterize the efficient allocation.

2.2. Efficient Allocation. To characterize the efficient allocation, assume that there is some social planner who maximizes the utility of a representative individual in region 1, $U(c_1, g_1)$, subject to some minimum utility level for a representative individual in region 2, $U(c_2, g_2) \geq \overline{U}$, the economy-wide resource constraint, $N_1c_1 + N_2c_2 + N_1g_1 + N_2g_2 = F(N_1, K_1) + F(N_2, K_2)$, and the capital supply capital constraint $\overline{K} = K_1 + K_2$.[5] The first-order conditions can be written as:

$$\text{MRS}_i \equiv \frac{\partial U(c_i, g_i)/\partial g_i}{\partial U(c_i, g_i)/\partial c_i} = 1 \text{ for } i = 1, 2 \tag{5}$$

[2] See Fujita and Thisse (2002) for a extensive discussion of Marshallian externalities.

[3] See Garcia-Mila and McGuire (2001) for a model in which firm production depends positively on the regional capital-labour ratio. In their paper, regional governments are price-takers and supply both a public good to residents and a public input to firms.

[4] If the externality instead depended on total regional labour supply, then capital would be paid the value of its marginal product.

[5] The planner is also constrained by the immobility of labour. With increasing returns to scale and mobile labour and capital, the planner would agglomerate all factors in one region to maximize total output. However, given the dispersion of labour which is immobile across the two regions the planner would agglomerate capital in one region only if increasing returns are sufficiently strong. We assume that this is not the case. In addition, the interior solution is a global maximum since $F_{KK} < 0$.

$$\frac{\partial F(N_1, K_1)}{\partial K_1} = \frac{\partial F(\overline{N} - N_1, \overline{K} - K_1)}{\partial(\overline{K} - K_1)} \tag{6}$$

Expression (5) is the Samuelson condition for the efficient provision of the public good. The marginal rate of substitution between the private and the public good should be equal to the marginal rate of transformation which is unity. Condition (6) implicitly defines the efficient allocation of capital denoted by K_1^{eff}. The first thing to note from (6) is that $K_1^{eff}$is independent of capital endowments and depends only on regional populations. Second, if regions have the same population (are symmetric) then they will also utilize the same amount of capital. In the efficient allocation with symmetric regions $K_1/N_1 = K_2/N_2 = \overline{K}/\overline{N}$.

What happens when the regions are not symmetric? Consider an increase in N_1 starting from the symmetric outcome. Differentiating (6), we obtain the following expression for the change in the *efficient* level of K_1 with respect to N_1.

$$\frac{dK_1^{eff}}{dN_1} = -\frac{F_{KN}^1 + F_{KN}^2}{F_{KK}^1 + F_{KK}^2} \tag{7}$$

Substituting (4) for each region into (7) and evaluating the resulting expression at the symmetric outcome, we obtain

$$\frac{dK_1^{eff}}{dN_1} \geq \frac{\overline{K}}{\overline{N}} \text{ as } \gamma \geq 1. \tag{8}$$

Result 1: In a neighbourhood of the symmetric efficient outcome, the proportional increase in the efficient number of units of capital exceeds (equals) the proportional increase in the size of the population with increasing (constant) returns to scale.

With CRTS, the marginal product of capital depends only on the capital-labour ratio in each region. Starting from a symmetric outcome increasing the population in region 1 (and simultaneously reducing the population in region 2) results in a proportional increase in the efficient level of capital in region 1. The capital-labour ratio is constant in any efficient allocation. This is no longer true when there is IRTS. In this case, starting from a symmetric outcome, an increase in the population of region 1 results in a greater proportional increase in capital utilized in that region. The capital-labour ratio in region 1 increases to capture the scale economies. We now turn to the market allocation with competing regional governments.

3. Competing Regions

Two cases are considered. First, we allow the regional governments to have access to a head tax. This eliminates the inefficiency of the public good provision that arises in the standard capital tax competition models. Second, we adopt the standard model and assume that regional governments can use only source-based capital taxes to finance the publicly provided private good. In both cases, we illustrate how increasing returns affect the market outcome.

3.1. Increasing Returns with Both Capital and Head Taxes. The government's budget constraint in region i is given by:

$$N_i g_i = t_i N_i + \tau_i K_i, \tag{9}$$

where τ_i is the specific capital tax or subsidy rate in region i and t_i is a head tax. We suppose that labour receives the value of its marginal product. Therefore, the return to a unit of capital is given by:

$$r = \frac{F(N,K) - NF_N}{K} \leq F_K = \frac{\gamma F(N,K) - NF_N}{K} \tag{10}$$

where

$$\frac{dr}{dK} = -F_{NK}\frac{N}{K} + (\gamma - 1)\frac{F(N,K)}{K^2} \tag{11}$$

The inequality in (10) follows from (2). With CRTS, the return to a unit of capital is equal to its marginal product which is decreasing in the amount of capital in that region. With IRTS, capital earns less than its marginal product and it is possible that (11) is non-negative. In what follows, we assume that the increasing returns are not so large that $dr/dK < 0$.[6]

Free mobility of capital implies that it earns the same after-tax return wherever it locates, that is,

$$r_1 - \tau_1 = r_2 - \tau_2. \tag{12}$$

The free mobility condition (12) implicitly defines the equilibrium allocation of capital as a function of regional capital tax rates, $K_i(\tau_i, \tau_j)$, keeping in mind that $K_i + K_j = \overline{K}$. Totally differentiating it, we obtain

$$\frac{dK_i}{d\tau_i} = \frac{1}{dr_i/dK_i + dr_j/dK_j} < 0, \frac{dK_i}{d\tau_j} = -\frac{1}{dr_i/dK_i + dr_j/dK_j} > 0, \text{ for } j \neq i \tag{13}$$

Increases in the regional capital tax rate drive capital out of that region and into the other region and the location equilibrium will be stable.

In each region, workers own an equal per capita share of the region's capital endowment. They receive their marginal product or wage and the after-tax return on their share of the capital endowment. Consumption for a representative worker in region i is given by,

[6]This assumption puts an implicit upper bound on γ of $1 + NKF_{NK}/F$. This ensures stability of the location equilbrium.

$$c_i = F_N^i + (r_i - \tau_i)\frac{\overline{K}_i}{N_i} - t_i. \tag{14}$$

Since all individuals are identical in each region, we assume that the regional government maximizes the utility of a representative worker subject to its budget constraint (9) and the free mobility condition (12). There are three stages to the decision-making. First, regional governments play a Nash game and select their tax policies.[7] Second, given the capital tax rates capital owners decide where to employ their capital and thirdly, production decisions are made. Substituting out t_i using (9) and substituting out F_N^i using (10), the government's problem can be written as:

$$\underset{g_i, \tau_i}{\text{Max}} \; U\left(\frac{F(N_i, K_i(\tau_i, \tau_j))}{N_i} - (r_i(K_i(\tau_i, \tau_j)) - \tau_i)\frac{K_i(\tau_i, \tau_j) - \overline{K}_i}{N_i} - g_i, g_i\right)$$

The above says that the residents of each region consume what is produced in the region less the after-tax return paid to non-resident capital owners. The first order conditions of this problem can be written as:

$$\text{MRS}_i \equiv \frac{\partial U(c_i, g_i)/\partial g_i}{\partial U(c_i, g_i)/\partial c_i} = 1 \tag{15}$$

and

$$\tau_i = r_i - F_K^i - (K_i - \overline{K_i})\frac{dr_j}{dK_j}, \text{ for } i \neq j. \tag{16}$$

The first condition is simply the Samuelson condition (5). Allowing regional governments access to a head tax ensures that they provide the public good efficiently. The second condition (16) tells us region i's optimal capital tax rate given the other region's tax policy. What is the intuition for this tax formula? Consider region 1. For region 1 (16) says

$$\tau_1 = r_1 - F_K^1 - (K_1 - \overline{K_1})\frac{dr_2}{dK_2}. \tag{17}$$

Ignore region 2 for the moment. What is region 1 trying to do? F_K^1 is the marginal benefit of having another unit of capital in region 1 and r_1 is the marginal cost of having another unit of capital in region 1. Therefore, region 1 is picking τ_1 to get the "correct" (from its point of view) number of units of capital. With increasing returns this effect (see (10)) causes region 1 to subsidize capital. The next term appears to fit the "standard" CRTS model. Given the allocation of capital across regions resulting from a stable equilibrium $dr_2/dK_2 < 0$, a capital-importing region $(K_1 - \overline{K_1} > 0)$ will want to tax capital and a capital-exporting region $(K_1 - \overline{K_1} < 0)$ will want to subsidize capital.

[7] Each regional government takes the capital tax rate in the other region as given when it selects its own tax rate. In a Nash equilibrium, the regional capital tax rates are mutual best responses.

Is the Nash equilibrium efficient, that is, do the tax rates chosen by the each region maximize total output in the two regions? In this case we know that in the efficient allocation $F_K^1 = F_K^2$ (from (6)). Consider symmetric regions, that is, regions that have identical endowments of both capital and labour. In this case, we have the following result.

Result 2: The Nash equilibrium of symmetric regions will be efficient. With homogeneous production functions that exhibit increasing returns to scale each region will be subsidizing capital at the same rate.

This result follows from (12) and (16). Neither region imports or exports capital in the Nash equilibrium with symmetric regions. Therefore, with CRTS, the capital tax rates will be zero. With IRTS, the tax rates will be negative and equal to the difference between the return to capital and its marginal product.

What happens when regions are asymmetric? There are at least two ways to think about asymmetric regions. First, regions could have different populations, but the same total capital endowments. Second, all individuals could own an equal per capita share of the total capital stock but one region could have a larger population. Only in the latter case does the more populated region have a larger capital endowment. What then is the relationship between the Nash equilibrium level of capital, K_1^{ne}, and the efficient level of capital, K_1^{eff}, as we move N_1 away from the symmetric equilibrium? Combining the free mobility condition (12) and the optimal tax rules (16), we obtain

$$F_K^1 + (K_1 - \overline{K_1})\frac{dr_2}{dK_2} = F_K^2 + (K_2 - \overline{K_2})\frac{dr_1}{dK_1}. \tag{18}$$

Since $K_2 = \overline{K} - K_1$, we can think of (18) as an equation in which K_1 is the only endogenous variable. First, consider the case where the capital endowments are kept constant. Then, implicitly differentiating this equation with respect to N_1 and evaluating the derivative in a neighbourhood of the symmetric equilibrium yields

$$\frac{dK_1^{ne}}{dN_1} = \frac{F_{KN}^1 + F_{KN}^2}{-(F_{KK}^1 + F_{KK}^2 + dr_1/dK_1 + dr_2/dK_2)}. \tag{19}$$

The only difference in the right-hand sides of (7) and (19) are the dr_i/dK_i terms in the denominator of (19) Since $dr_i/dK_i < 0$, we have

$$0 < \frac{dK_1^{ne}}{dN_1} < \frac{dK_1^{eff}}{dN_1} \tag{20}$$

in a neighbourhood of the symmetric equilibrium. Starting in a symmetric setting, where we know that the Nash equilibrium is efficient, a reallocation of population towards region 1 will cause the Nash equilibrium number of units of capital to increase in region 1 but by less than efficient number. The reason is that region 1 becomes a capital-importer and through the usual manipulating the-terms-of-trade effect it will tax capital, or subsidize it at an inefficiently low rate. Region 2 reinforces this effect by subsidizing capital too much. The actions of both regions cause the Nash equilibrium number of units of capital in region 1 to be inefficiently low. This proves Result 3.

Result 3: In a neighbourhood of the symmetric Nash equilibrium, a reallocation of population towards region 1 will increase the Nash equilibrium number of units of capital in region 1 by less than the efficient number when regional capital endowments are kept constant.

Recall that the efficient level of K_1 is independent of the endowment of capital across the two regions — $dK_1^{eff}/d\overline{K_1} = 0$. From (18) we can deduce that

$$\frac{dK_1^{ne}}{d\overline{K_1}} = \frac{dr_1/dK_1 + dr_2/dK_2}{F_{KK}^1 + F_{KK}^2 + dr_1/dK_1 + dr_2/dK_2} > 0 \tag{21}$$

So if we increase N_1 and simultaneously raise $\overline{K_1}$ to keep it equal to the new efficient level of K_1 then the associated Nash equilibrium will coincide with the efficient allocation. Thus it is easy to construct non-symmetric Nash equilibria that are efficient. More importantly, the reason most Nash equilibria in this case are not efficient is that the capital-importing region wants to tax capital (or subsidize it at an inefficiently low rate) and the capital-exporting region oversubsidizes capital.

Suppose now individuals own an equal share of the total capital stock. Again, we can think of (18) as an equation in which K_1 is the only endogenous variable, but now treat the capital endowment in region 1 as a function of regional population, that is, $\overline{K_1} = N_1(\overline{K}/\overline{N})$. In this case, the capital endowment to labour ratio doesn't change with population.[8] Implicitly differentiating (18) with respect to N_1 and evaluating the derivative in a neighbourhood of the symmetric equilibrium yields

$$\frac{dK_1^{ne}}{dN_1} = \frac{F_{KN}^1 + F_{KN}^2 - \left(\overline{K}/\overline{N}\right)(dr_1/dK_1 + dr_2/dK_2)}{-\left(F_{KK}^1 + F_{KK}^2 + dr_1/dK_1 + dr_2/dK_2\right)}. \tag{22}$$

With CRTS, $dr/dK = -F_{KN}(N/K) = F_{KK}$ and (22) becomes

$$\frac{dK_1^{ne}}{dN_1} = \frac{dK_1^{eff}}{dN_1} = \frac{\overline{K}}{\overline{N}} \tag{23}$$

Reallocating workers to region 1 causes the Nash equilibrium level of K_1 to increase by the efficient amount. The reason is the change in the capital endowment in region 1 coincides with the efficient allocation when there is constant returns to scale and symmetric regions. With increasing returns to scale, the increase in the regional

[8] Given $\overline{K}_i = N_i(\overline{K}/\overline{N})$, we have $\overline{K_i}/N_i = \overline{K}/\overline{N}$, which is independent of N_i.

capital endowment is less than the increase in the efficient level of capital in region 1 and again we have the terms-of-trade effect.

$$\frac{\overline{K}}{\overline{N}} < \frac{dK_1^{ne}}{dN_1} < \frac{dK_1^{eff}}{dN_1} \tag{24}$$

Result 4: In a neighbourhood of the symmetric Nash equilibrium with equal per capita ownership of the total capital stock, a reallocation of population towards region 1 will increase the Nash equilibrium number of units of capital in region 1 by (less than) the efficient number when there is constant returns to scale (increasing returns to scale).

All of these results have been derived in a tax competition model where each region can use two different tax instruments - a capital tax (or subsidy) and a head tax. Most of the capital tax literature reviewed by Wilson (1999) assumes that each region can only use a capital tax and that this tax finances the publicly-provided private good. We now relate our results to those obtained from these models.

3.2. Increasing Returns with Capital Taxes Only. Restricting head taxes to be zero, the government's budget constraint can be written as

$$g_i = \tau_i K_i / N_i. \tag{25}$$

Assuming labour receives the value of its marginal product, consumption for a representative worker in region i is

$$c_i = F_N^i + (r_i - \tau_i)\frac{\overline{K}_i}{N_i}. \tag{26}$$

The government in region i selects its tax rate to maximize the utility of a representative individual living in region i where F_N^i has been substituted out using the expression for r_i (see 10).

$$\underset{\tau_i}{\text{Max}}\; U\left(\frac{F(N_i, K_i(\tau_i, \tau_j))}{N_i} - r_i(K_i(\tau_i, \tau_j))\frac{K_i(\tau_i, \tau_j)}{N_i} + (r_i(K_i(\tau_i, \tau_j)) - \tau_i)\frac{\overline{K}_i}{N_i}, \tau_i \frac{K_i(\tau_i, \tau_j)}{N_i}\right).$$

The first-order condition for τ_i can be written as

$$\text{MRS}_i \equiv \frac{\partial U_i/\partial g_i}{\partial U_i/\partial c_i} = \frac{1}{1 + \frac{\tau_i}{K_i}\frac{dK_i}{d\tau_i}}\left(1 + \left(\frac{K_i - \overline{K}_i}{K_i}\frac{d(r_i - \tau_i)}{d\tau_i}\right) + \frac{r_i - F_K^i}{K_i}\frac{dK_i}{d\tau_i}\right). \tag{27}$$

The first factor on the right-hand side is equal to one over one plus the own tax price elasticity of capital in region i. We make the standard assumption that the capital tax revenue is increasing in τ. This implies that the absolute value of the tax price elasticity of capital lies between zero and one and that the denominator of the first factor is positive and less than one. Therefore, the first factor in the above expression is greater than one. If there were no terms-of-trade effect and capital earned the value of its marginal product, then the marginal rate of substitution between consumption and the publicly provided private good would be greater than one, that is, the public good would be under-provided. An important question then is how does the terms-of-trade effect and increasing returns affect this under-provision result?

The terms-of-trade effect works as before. First note that the after-tax rate of return on capital is decreasing in the tax rate.[9] Therefore, the under-provision of the public good ($\mathrm{MRS}_i > 1$) is worsened (reduced) if the region exports (imports) capital. In other words, a capital-exporting region has an incentive to subsidize capital and a capital importing region has an incentive to tax capital. Since the region has only one instrument to attract capital and finance its provision of the public good a lower tax rate results in lower provision of the public good.

Perhaps the clearest way to see the effect of increasing returns is to focus on symmetric equilibria. Here (27) can be written as

$$\frac{\partial U_i/\partial g_i}{\partial U_i/\partial c_i} = \frac{1}{1+\frac{\tau_i}{K_i}\frac{dK_i}{d\tau_i}}\left(1+\frac{r_i - F_K^i}{K_i}\frac{dK_i}{d\tau_i}\right). \tag{28}$$

In the standard capital tax competition model which assumes CRTS, $r_i = F_K^i$, and $\mathrm{MRS}_i > 1$. The capital tax rate by construction must be positive. However, regional governments compete the tax rate down in the Nash equilibrium and there is an under-provision of the public good relative to the efficient outcome. In this model, the introduction of increasing returns to scale, which makes $r_i - F_K^i < 0$, would appear to push τ_i lower and widen the gap between the Nash equilibrium level of τ_i and its efficient level. However, such a conclusion would be misplaced. We show in the appendix that the tax rate in the symmetric Nash equilibrium can go up or down. The reason is that increasing returns also affects the marginal rate of substitution between the public and the private good as well as the elasticity of the capital stock with respect to the tax rate. We illustrate in Figure 1 a case with Cobb-Douglas preferences and technology where the tax rate (or equivalently, the amount of public good provided) in the symmetric Nash equilibrium increases with the introduction of increasing returns. With CRTS, the symmetric Nash equilibrium is at point NE_1. Slight increasing returns pushes out the region's production possibility frontier and in the symmetric Nash outcome the region ends up at point NE_2 with higher private consumption and public good provision.

Does the increase in the tax rate mean that the Nash equilibrium is 'more efficient' with increasing returns? Not necessarily. Introducing increasing returns also changes the efficient outcome. In Figure 1, we move from point E_1 to E_2. What really matters is whether the Nash outcome is any 'closer' to the efficient outcome when we introduce

[9] Differentiating, $d(r_i - \tau_i)/d\tau_i = (dr_i/dK_i)(dK_i/d\tau_i) - 1 = (dr_i/dK_i)/(dr_i/dK_i + dr_j/dK_j) - 1 < 0$.

increasing returns. To answer this question, we need to look at what happens to the utility level in the Nash outcome relative to the efficient outcome. One measure of this is the proportional change in the ratio of utility in the Nash outcome to the utility level in the efficient outcome. If the proportional change is positive when we introduce increasing returns then we can say that increasing returns improves welfare in the Nash relative to the efficient level, that is, the Nash equilibrium becomes 'more efficient'. If, on the other hand, the proportional change is negative then increasing returns makes the Nash outcome 'less efficient'. From Figure 1, it is clear that the proportional change in the utility ratio is negative. We can show more generally that with homothetic preferences and any CES production technology, increasing returns reduces the ratio of utility in the Nash to the efficient outcome. Result 5 is proven in the appendix.

Result 5: Starting from a symmetric Nash equilibrium with CRTS and without head taxes, introducing increasing returns to scale with homothetic preferences and a CES production technology will result in a proportional reduction in the ratio of utility in the Nash equilibrium to utility in the efficient allocation.

It is worth pointing out that this result holds only starting from the Nash equilibrium with CRTS. It's possible to imagine a situation in which there is already increasing returns to scale and an increase in these returns results in a proportional increase in the utility ratio.

Relating the results of this subsection with the previous one we can see that the reason regions do not deal with increasing returns to scale efficiently in the usual tax competition model of Wilson's (1999) survey is that regions start with the handicap of too few instruments — they lack a head tax to provide the efficient level of the public good. If we remove this handicap by adding a head tax to the model then we see that regions *do* handle increasing returns efficiently. All Nash equilibria of symmetric regions are efficient and some Nash equilibria of asymmetric regions are efficient. The only reason some Nash equilibria are inefficient is not that increasing returns to scale exist but rather usual trade-literature phenomenon that each region wants to turn the terms of trade in its favour — capital-importing regions want to tax capital and capital-exporting regions want to subsidize capital.

4. Concluding Remarks

We have shown that within the standard capital tax competition model *all* regional governments may have an incentive to subsidize firms. With increasing returns to scale in production, the marginal return to capital in a given region may be greater than what firms pay to employ the capital. Regional governments, recognizing this discrepancy, will fight harder to attract capital to their regions. We have identified one avenue in which capital may have additional benefits.

This stark capital tax competition model, however, is still far removed from what we read about in newspapers. In particular, capital investment is not continuous, but rather it's lumpy nor are firms perfectly competitive. Extensions in these directions have recently been made using Krugman's (1991) economic geography model (see Ludema and Wooton (2000) and Kind, Knarvik and Schjelderup (2001)). In addition, there may be other external benefits of capital investment such as reduced unemployment We leave these avenues for future research.

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Appendix

What happens to the utility ratio, U^{ne}/U^{eff}, in a symmetric outcome, as we move from a situation of CRTS to one with IRTS? Figure 1 illustrates that if preferences are homothetic then the sign of the proportional change in this ratio, $(\partial(U^{ne}/U^{eff})/\partial\gamma)/(U^{ne}/U^{eff})$, matches the sign of the proportional change in the public good to consumption ratio, $(\partial(g/c)/\partial\gamma)/(g/c)$, where we omit the regional designation to reduce notational clutter. Using the definitions of $g = \tau K/N$ and $c = (F - \tau K)/N$ in the Nash outcome, it is straightforward to see that the sign of $(\partial(g/c)/\partial\gamma)/(g/c)$ is the same as the sign of $\partial\tau/\partial\gamma - (\tau/F)\partial F/\partial\gamma$. Below, we derive this expression and show that its sign is negative when production has a constant elasticity of substitution.

Write MRS (the absolute value of the marginal rate of substitution of g for c) as $f(c/g)$. Then

$$\frac{\partial \mathrm{MRS}}{\partial\gamma} = \frac{f'F}{\tau K}\left(\frac{\partial F}{\partial\gamma}\frac{1}{F} - \frac{\partial\tau}{\partial\gamma}\frac{1}{\tau}\right)$$

where $f(c/g) > 1$ and $f' > 0$. From (10), we have

$$(r - F_K)|_{\gamma=1} = 0 \text{ and } \left.\frac{\partial(r - F_K)}{\partial\gamma}\right|_{\gamma=1} = -\frac{F}{K}.$$

From (11), we know that

$$\left.\frac{dr}{dK}\right|_{\gamma=1} = -\frac{N}{K}F_{NK} \text{ and } \left.\frac{\partial(dr/dK)}{\partial\gamma}\right|_{\gamma=1} = -\frac{N}{K}\frac{\partial F_{NK}}{\partial\gamma} + \frac{F}{K^2}.$$

In a symmetric Nash equilibrium without head taxes

$$\tau = \frac{1}{\mathrm{MRS}}\left(r - F_K + 2K\frac{dr}{dK}\right) - 2K\frac{dr}{dK},$$

so

$$\tau\,|_{\gamma=1} = -2NF_{NK}\frac{1 - \mathrm{MRS}}{\mathrm{MRS}}$$

and

$$\begin{aligned}\left.\frac{\partial\tau}{\partial\gamma}\right|_{\gamma=1} &= -2K\frac{dr}{dK}\frac{1}{\mathrm{MRS}^2}\left.\frac{\partial \mathrm{MRS}}{\partial\gamma}\right|_{\gamma=1} + \\ &\quad \frac{1}{\mathrm{MRS}}\left.\frac{\partial(r - F_K)}{\partial\gamma}\right|_{\gamma=1} + \frac{2K(1 - \mathrm{MRS})}{\mathrm{MRS}}\left.\frac{\partial(dr/dK)}{\partial\gamma}\right|_{\gamma=1}.\end{aligned}$$

Using the above expressions we can write

$$\left.\frac{\partial \tau}{\partial \gamma}\right|_{\gamma=1} = \frac{B}{A},$$

where

$$A \equiv 1 + 2\frac{N}{K}F_{NK}\frac{f'F}{\mathrm{MRS}^2\tau^2} > 0$$

and

$$B \equiv 2\frac{N}{K}F_{NK}\frac{f'}{\mathrm{MRS}^2\tau}\frac{\partial F}{\partial \gamma} + \frac{F(1-2\mathrm{MRS})}{K.\mathrm{MRS}} + \frac{\tau}{F_{NK}}\frac{\partial F_{NK}}{\partial \gamma} \lesseqgtr 0.$$

Starting from a symmetric Nash equilibrium with CRTS, introducing increasing returns can increase, decrease, or not affect the equilibrium tax rate. From here one can deduce that

$$\left.\left(\frac{\partial \tau}{\partial \gamma} - \frac{\tau}{F}\frac{\partial F}{\partial \gamma}\right)\right|_{\gamma=1} = \frac{1}{A}\left\{\frac{F(1-2\mathrm{MRS})}{K.\mathrm{MRS}} + \tau\left(\frac{\partial F_{NK}/\partial\gamma}{F_{NK}} - \frac{\partial F/\partial \gamma}{F}\right)\right\}.$$

Since $A > 0$ the sign of the left-hand side of the above expression is the same as the sign of the expression in brackets. To say more we need to assume a specific production function.

If the production technology exhibits a constant elasticity of substitution, then F can be written in a general CES form,

$$F = (\beta K^\rho + (1-\beta)N^\rho)^{\gamma/\rho}, \ \rho < 1.$$

We then have

$$F\,|_{\gamma=1} = (\beta K^\rho + (1-\beta)N^\rho)^{1/\rho},$$

$$\left.\frac{\partial F/\partial\gamma}{F}\right|_{\gamma=1} = \frac{1}{\rho}\ln(\beta K^\rho + (1-\beta)N^\rho),$$

$$F_{NK} = \beta(1-\beta)\gamma(\gamma-\rho)K^{\rho-1}N^{\rho-1}(\beta K^\rho + (1-\beta)N^\rho)^{\gamma/\rho-2},$$

$$F_{NK}\,|_{\gamma=1} = \beta(1-\beta)(1-\rho)K^{\rho-1}N^{\rho-1}F^{1-2\rho},$$

and

$$\left.\frac{\partial F_{NK}/\partial\gamma}{F_{NK}}\right|_{\gamma=1} = 1 + \frac{1}{1-\rho} + \frac{1}{\rho}\ln(\beta K^\rho + (1-\beta)N^\rho).$$

Therefore,

$$\left.\frac{\partial F_{NK}/\partial\gamma}{F_{NK}}\right|_{\gamma=1} - \left.\frac{\partial F/\partial\gamma}{F}\right|_{\gamma=1} = \frac{2-\rho}{1-\rho}.$$

Using the expression for τ and F_{NK} when $\gamma = 1$, we have

$$\begin{aligned}\left.\left(\frac{\partial\tau}{\partial\gamma} - \frac{\tau}{F}\frac{\partial F}{\partial\gamma}\right)\right|_{\gamma=1} &= \frac{1}{A}\left\{\frac{F(1-2\text{MRS})}{K\text{MRS}} - 2\beta(1-\beta)K^{\rho-1}N^{\rho}F^{1-2\rho}\frac{(1-\text{MRS})}{\text{MRS}}(2-\rho)\right\} \\ &= \frac{1}{A}\frac{F}{K\text{MRS}}\left\{\begin{array}{c}(1-\text{MRS})F^{-2\rho}[F^{2\rho} - 2\beta(1-\beta)K^{\rho}N^{\rho}] \\ -\text{MRS}(1-\frac{\tau K}{F})\end{array}\right\} < 0\end{aligned}$$

where the inequality comes from the following three observations:

1. The first factor is positive since F, K,MRS, $A > 0$.
2. The first term in the brackets is negative since $MRS > 1$ and

$$F^{2\rho} = \beta^2 K^{2\rho} + (1-\beta)^2 N^{2\rho} + 2\beta(1-\beta)K^{\rho}N^{\rho} > 2\beta(1-\beta)K^{\rho}N^{\rho}.$$

3. The second term in the brackets, counting the $-$MRS, is negative since $\tau K = Ng < F$.

Utility in the symmetric Nash equilibrium relative to utility in the symmetric efficient outcome goes down as the economy moves away from CRTS and towards IRTS given homothetic preferences and a CES production function (including Cobb-Douglas, $\rho = 0$).

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Figure 1: A Cobb-Douglas Example