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Department of Economics 1280 Main Street West Hamilton, Ontario, Canada L8S 4M4

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### Asymmetric Labor Adjustment, Organizational Capital and Aggregate Job Flows \*

Francisco M. González $^{\dagger}$  and Alok Johri $^{\ddagger}$ 

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#### Abstract

This paper illustrates how the destruction of firm-specific organizational capital associated with changes in firm-level employment can influence the behavior of aggregate job flows, even in the presence of heterogeneity across firms and even in the absence of aggregate shocks. Our analysis highlights the potential importance of the distinction between adjustment costs that are associated with a loss of output (*output-costs of labor adjustment*) and those associated with a loss of organizational capital (*OC-costs of labor adjustment*). In particular, the analysis indicates how this link between organizational capital and labor demand can shape the behavior of net employment growth and gross job reallocation when conventional hiring and firing costs of adjustment may be unable to do so.

**Keywords:** job flows, organizational capital, adjustment costs, idiosyncratic shocks, heterogeneity, aggregation

JEL classification: E00, J00

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<sup>&</sup>lt;sup>†</sup>Department of Economics, University of British Columbia, #997–1873 East Mall, Vancouver, B.C. V6T 1Z1, Canada.

<sup>&</sup>lt;sup>‡</sup>Department of Economics, McMaster University, 1280 Main Street West, Hamilton, Ontario L8S 4M4, Canada.

#### **1** Introduction

The observations of infrequent employment adjustment and bursts of job creation and destruction reflect the complex dynamic nature of the labor demand decision. The standard explanation for this behavior is that employment changes entail adjustment costs. These costs are typically viewed as involving the loss of current output and the lapse of time, whether they are associated with the flow of workers or that of jobs. <sup>1</sup> However, changes in firm-level employment are often associated with some disruption and re-organization in the production process and therefore accompanied by productivity losses. These productivity losses may be associated with both the creation and the destruction of employment as they are accompanied by changes in teams of workers or in matches of workers to jobs. For example, "... expansion of the work force may result in difficulties scheduling the flow of work across sites within an establishment, problems that in turn reduce average efficiency. Adding a few employees to a work crew may require senior workers to spend time training their new co-workers; hiring replacement workers for a work crew whose size is unaltered may have the same effect, and cutting employment may reduce the morale of the remaining employees and lower their efficiency." (Hamermesh (1993, Chapter 6, page 207)). The phenomenon that productivity losses may occur as adjustments are made can be understood best in the context of the view of the firm as a storehouse of organizational capital. However, while considerable research has focused on the accumulation of organizational capital, little research effort has been devoted to understanding the potential destruction of organizational capital associated with employment changes.<sup>2</sup>

In this paper we model organizational capital as being influenced by the creation and the destruction of jobs at the firm level. Specifically, our analysis highlights the potential loss of organizational capital associated with job creation and destruction decisions. In this context, we

<sup>&</sup>lt;sup>1</sup>Hamermesh (1993) and Hamermesh and Pfann (1996) discuss much of the theoretical and empirical work in this area.

<sup>&</sup>lt;sup>2</sup>Several different aspects of the accumulation of this type of firm-specific knowledge have been discussed in the literature. For example, in Arrow (1962), Rosen (1972) and Ericson and Pakes (1995) organizational capital is accumulated by endogenous learning-by-doing. Jovanovic (1979) and Prescott and Visscher (1980) view organizational capital as embodied in workers, in teams of workers and in their matches to tasks within the firm. Recent attempts at evaluating the importance of organizational capital are Jovanovic and Rousseau (2001) and Atkeson and Kehoe (2001). Cooper and Johri (2001) and Benkard (2000) emphasize the potential importance of the depreciation of experience.

show that the firm's labor demand decision involves an inherent asymmetry between the creation and the destruction of jobs. Furthermore, we show how this microeconomic asymmetry can influence the behavior of aggregate job flows even in the presence of heterogeneity across firms and even in the absence of aggregate shocks. In this respect, our analysis follows Caballero's (1992) seminal work in that it addresses the possibility that the influence of microeconomic asymmetries may be undone in the process of aggregation. Our analysis highlights the potential importance of the distinction between adjustment costs that are associated with a loss of output, which we label *output-costs of adjustment* and those associated with a loss of organizational capital, which we label organizational-capital costs of adjustment or simply *OC-costs of adjustment*. <sup>3 4</sup> In particular, the analysis indicates how this link between organizational capital and labor demand can shape the behavior of aggregate employment when conventional hiring and firing costs of adjustment may be unable to do so.

The structure of labor adjustment costs in our model implies that the optimal labor demand decision by each firm is characterized by a (S,s) rule, which involves a state-dependent decision to adjust together with a choice of the magnitude of the adjustment. Intuitively, when labor productivity has fallen sufficiently the firm optimally chooses to destroy jobs so as to achieve a target increase in labor productivity. Conversely, when labor productivity has risen sufficiently the firm optimally chooses to create jobs until a target decline in labor productivity is reached. At intermediate productivity levels, employment at the firm remains optimally unchanged while organizational capital is subject to stochastic growth.

The distinguishing feature of the OC-costs, as opposed to the output-costs, of labor adjustment is that they link changes in employment and changes in organizational capital. In this sense optimal job creation and destruction by firms reflect their continuing effort to re-organize the production process and their scale of operation efficiently. During job destruction episodes,

<sup>&</sup>lt;sup>3</sup>Although there is substantial evidence of non-linear employment adjustment at the plant-level, it seems fair to state that the extent to which non-linear labor adjustment matters for the dynamics of aggregate employment is still an open question. See, for example, Caballero and Engel (1993), Hamermesh (1993), Caballero, Engel and Haltiwanger (1997) and Cooper and Willis (2001).

<sup>&</sup>lt;sup>4</sup>Our emphasis on the distinction between output-costs and OC-costs of adjustment should be understood as complementing Hamermesh's (1995) distinction between gross and net adjustment costs.

when labor productivity is low, the firm's productivity increases because employment falls, but the resulting loss of organizational capital by itself lowers labor productivity. Thus, in order to achieve any target increase in labor productivity, more jobs have to be destroyed when job destruction is accompanied by the loss of organizational capital. That is, the elasticity of labor productivity with respect to employment at destruction times is less than one. Our analysis will illustrate that the behavior of this elasticity greatly influences the behavior of job flows. It is interesting to compare this to the situation where the creation of jobs is also associated with re-organizations and therefore with some loss of organizational capital. In this case job creation occurs when labor productivity starts out high but falls both because employment increases and because organizational capital is destroyed. Thus, in order to achieve any target decline in labor productivity less jobs need to be created when job creation is accompanied by the loss of organizational capital than when it is not. In other words, the elasticity of labor productivity with respect to employment at creation times is larger than one in this case.

To better understand the significance of OC-costs of labor adjustment we compare the aggregate implications of our model to those of a benchmark model in which firms face output-costs instead of OC-costs of adjustment. The benchmark model is a natural extension of Bentolila and Bertola's (1990) model of hiring and firing costs to allow for lumpy, in addition to infrequent, job creation and destruction. In each case we study the average behavior of a large number of firms which face independent shocks but otherwise solve identical problems regarding the choice of labor demand. Our analysis focuses on the behavior of standard measures of aggregate job creation and job destruction, as defined by Davis and Haltiwanger (1992). In particular, we focus on the behavior of net job creation, that is, the difference between the job creation and destruction rates —a measure of aggregate employment growth— and the behavior of gross job reallocation, that is, the sum of the job creation and destruction rates —a measure of the intensity of job reallocation.

We find that the benchmark model with output-costs of adjustment has strong implications for the behavior of these standard measures of aggregate flows. Specifically, our analysis indicates that

- higher output-costs of adjustment cause gross job reallocation to fall, but do not significantly influence net job creation,
- higher trend growth in productivity causes net job creation to rise, but does not significantly influence gross job reallocation.

Thus, asymmetries in the output-costs of labor adjustment appear not to influence net job creation whereas asymmetries in the cross-sectional distribution of firms, which are induced by the trend rate of growth in productivity, appear not to influence gross job reallocation. These two observations further imply that neither asymmetries in the costs of labor adjustment nor asymmetries in the cross-sectional distribution of firms on their own can explain a relationship between net employment growth and the intensity of job reallocation in the benchmark model with output-costs of labor adjustment. Underlying these results is the intensity-frequency tradeoff emphasized by Caballero (1992), whereby the behavior of the cross-sectional distribution of firms may undo the effect of microeconomic asymmetries on the behavior of the individual firm.

In contrast, our analysis of the aggregate implications of OC-costs of labor adjustment indicates that

- higher OC-costs associated with job destruction cause net job creation and gross job reallocation to fall
- higher OC-costs associated with job creation cause gross job reallocation to fall, but do not have a significant impact on net job creation
- higher trend growth in the stock of organizational capital is associated with higher net job creation and lower gross job reallocation.

The influence of the OC-costs of adjustment on net job creation is subtle. To understand the underlying mechanism, consider an increase in the OC-costs associated with job destruction. Not only does this make job destruction more expensive, but any target increase in productivity requires more job destruction. That is, the elasticity of labor productivity with respect to employment at the job destruction margin tends to fall as job destruction becomes more costly. In contrast, an increase in the OC-costs associated with job creation, makes job creation more expensive but it requires less job creation in order to achieve any given target decline in productivity. That is, the elasticity of labor productivity with respect to employment at the job creation margin tends to rise as job creation becomes more costly. It is the distinct behavior of these elasticities that underlies the result that the behavior of net employment growth is sensitive to the OC-costs of job destruction but not to the OC-costs of job creation.

The effect of differences in the (exogenous) trend growth rate in organizational capital the analog of trend growth in productivity in the benchmark model— is also interesting. Such differences induce a negative relationship between net employment growth and the intensity of job reallocation in the model with OC-costs of labor adjustment. As will become clear below, trend growth in organizational capital influences the intensity of job reallocation because the elasticities of labor productivity with respect to employment at job creation and destruction times are different from unity.

It should be noted that the actual trend growth rate in the stock of organizational capital is jointly determined by the (exogenous) trend growth rate of firm-specific organizational capital and by the (endogenous) destruction of organizational capital associated with employment changes. We define a measure of this endogenous component, the *OC destruction rate*, as the sum of the rates of destruction of organizational capital induced by job creation and destruction. When we study the aggregate implications of the optimal labor demand decisions for the destruction of organizational capital, we find that

- the OC destruction rate tends to increase with the OC-costs of adjustment, which in turn induces a negative relationship between the job reallocation rate and the OC destruction rate,
- the OC destruction rate tends to fall with the (exogenous) trend growth in organizational capital, which in turn induces a positive relationship between the gross job reallocation rate and the OC destruction rate.

The remaining sections of the paper are structured as follows. Section 2 presents the model under study. Section 3 characterizes the optimal labor demand policies and highlights the difference between job creation and job destruction episodes at the firm level. Section 4 discusses our approach to aggregation and in turn examines the aggregate implications of the outputcosts of labor adjustment and the OC-costs of labor adjustment. Section 5 concludes. Technical derivations are relegated to an appendix.

#### 2 The Model

In this section we set up a model of the labor demand decision by price-taking firms. <sup>5</sup> We model their production technology as combining two complementary inputs: labor and organizational capital. In order to focus on the influence of the destruction of organizational capital on the firms' labor demand decisions, we model the accumulation of firm-specific organizational capital rather simply as a random process with drift. We assume that all employment adjustments are associated with some re-organization in the firm's production process and therefore with some destruction of organizational capital.

Firms use their stock of labor together with their stock of organizational capital to produce output. The operating profit of the firm, that is, revenue minus the cost of labor, is given by

$$\Pi(z_t, n_t) = z_t^{\alpha} n_t^{1-\alpha} - w n_t, \qquad 0 < \alpha < 1,$$
(2.1)

where  $z_t$  denotes the firm's stock of organizational capital at time t,  $n_t$  denotes the firm's size at time t as measured by the stock of jobs at that time, and w is the real cost of a job. In addition to the wage bill, the firm incurs a fixed cost whenever it creates or destroys jobs. We assume that the fixed cost of adjustment is proportional to the size of the firm,  $cn_t$ , which ensures that the firm cannot grow out of the fixed cost. This homogeneity assumption, together with the assumption that the firm's revenue function is homogeneous of degree one in  $z_t$  and  $n_t$ , ensures the form of the firm's problem does not change with the firm's size.

We assume that, as long as the stock of jobs at the firm remains unchanged, the stock of organizational capital is a geometric Brownian motion with constant mean growth rate  $\mu$  and standard deviation  $\sigma > 0$ 

<sup>&</sup>lt;sup>5</sup>There is no meaningful plant/firm distinction in this model, and we refer to firms for expositional convenience.

$$dz_t = \mu z_t dt + \sigma z_t dW_t, \quad \text{if} \quad dn_t = 0 \tag{2.2}$$

where  $W_t$  is a standard Wiener process with independent, normally distributed increments. Thus, exogenous changes in the stock of organizational capital are composed of the deterministic contribution  $\mu dt$  and the stochastic shock  $\sigma dW_t$ . The first component reflects the accumulation of firm-specific organizational capital over time. The second component introduces randomness and heterogeneity in the accumulation process across firms. There is a simple link between the firm's stock of organizational capital,  $z_t$ , and its labor productivity,  $(1 - \alpha) (z_t/n_t)^{\alpha}$ , so that the latter grows with  $z_t$  as long as the labor force is kept constant. Note that whereas productivity is expected to grow at the exponential rate  $\mu$ , the actual rate of growth is random.

The critical assumption we make is that some of the accumulated organizational capital is lost whenever the firm chooses to expand or to contract. We formalize this idea simply: the loss is proportional to the jobs being created or destroyed,

$$\Delta z_t = -\tau_c \,\Delta n_t, \qquad \text{if} \quad \Delta n_t > 0, \tag{2.3}$$

$$\Delta z_t = \tau_d \,\Delta n_t, \qquad \text{if} \quad \Delta n_t < 0, \tag{2.4}$$

where  $\{n_t\}$  is the cumulative job turnover process ( $\Delta n_t > 0$  when the firm expands and  $\Delta n_t < 0$ when the firm contracts), and  $\tau_c$  and  $\tau_d$  parameterize the influence of job creation and destruction, respectively, on organizational capital. <sup>6</sup> In order to focus on the potential loss of organizational capital associated with employment adjustments, our analysis focuses on the case where  $\tau_c > 0$  and  $\tau_d > 0$ .

The process we propose for the accumulation and destruction of organizational capital generates a learning curve in the sense of inducing its characteristic concavity. For instance, when

<sup>&</sup>lt;sup>6</sup>Unlike the conventional firing and hiring costs, both  $\tau_d$  and  $\tau_c$  are arguably closely related to the costs of internal reorganization at the firm and less to institutional features of the labor market. Thus, our emphasis is on technological differences across firms and industries.

a firm tries to take advantage of the increased productivity generated by the accumulation of organizational capital, its very actions (job creation) limit this accumulation, since the destruction of organizational capital is triggered. The firm can control the magnitude of the loss of organizational capital that occurs as well as how frequently it occurs by choosing when to change employment and by how much.

Finally, we assume that the firm maximizes the expected present value of cash flows discounted at a positive rate  $r > \mu$ .<sup>7</sup>

#### 3 Destruction of Organizational Capital and Labor Demand

The section is organized in two parts. In the first part, we characterize a firm's optimal labor demand strategy. In the second part, we further discuss the role of the endogenous destruction of organizational capital in determining the dynamics of job creation and job destruction.

#### 3.1 Optimal Labor Demand

In this section we characterize the firm's optimal behavior. Let V(z, n) denote the current value of the firm after it has already paid all the costs associated with job creation and destruction. Given that the firm makes its labor demand decisions in order to maximize the present value of profits, the Bellman equation associated with the problem of the firm is

$$rV(z,n) = \Pi(z,n) + \mu z V_z(z,n) + \frac{1}{2}\sigma^2 z^2 V_{zz}(z,n).$$
(3.1)

Since the firm faces increasing returns to labor adjustment (due to the fixed cost), the optimal labor demand strategy will involve infrequent and lumpy job creation and destruction. Because adjusting  $n_t$  whenever  $z_t$  changes will be too costly, the firm allows  $z_t$  to rise up to a maximum value or to fall to a minimum value before adjusting employment. In other words, firms create jobs when labor productivity,  $(1 - \alpha) (z_t/n_t)^{\alpha}$ , is high enough to justify incurring the costs of adjustment; and similarly, firms destroy jobs when labor productivity is low enough. For a firm

<sup>&</sup>lt;sup>7</sup>The restriction  $r > \mu$  ensures that the value of the firm is bounded when  $c = \tau_c = \tau_d = 0$ .

of size n, let  $z_c(n)$  denote the creation margin, that is, the maximum level of organizational capital which a firm of size n is willing to accumulate before creating jobs. Similarly, let  $z_d(n)$ denote the firm's destruction margin, that is, the minimum stock of organizational capital which triggers a bout of job destruction. The firm's optimal behavior when its stock of jobs is n will be such that the firm does nothing so long as its stock of organizational capital is in the interior of the interval  $[z_d(n), z_c(n)]$ . Whenever the creation margin is reached, the firm increases its employment from n to  $N_c(n)$ , and the associated destruction of organizational capital brings  $z_c(n)$  down to  $Z_c(n)$ . Similarly, when the job destruction margin is reached,  $z_d(n)$  falls to  $Z_d(n)$ and employment falls from n to  $N_d(n)$ .

In addition to satisfying the Bellman equation, V(z, n) must satisfy the following six boundary conditions. <sup>8</sup> First, at the creation margin, a firm of size n must be indifferent between creating jobs and doing nothing, that is, the value matching condition

$$V(Z_{c}(n), N_{c}(n)) = V(z_{c}(n), n) + c n$$
(3.2)

must hold. Similarly, at the destruction margin the benefits and the costs of job destruction must exactly balance each other,

$$V(Z_d(n), N_d(n)) = V(z_d(n), n) + c n.$$
(3.3)

Furthermore, for the creation and destruction margins to be optimal, it must be that the smooth pasting conditions

$$V_n(z_c(n), n) - \tau_c V_z(z_c(n), n) + c = 0$$
(3.4)

and

$$V_n(z_d(n), n) + \tau_d V_z(z_d(n), n) + c = 0$$
(3.5)

<sup>&</sup>lt;sup>8</sup>See Dumas (1991) and Dixit (1991) for a clear discussion of the value matching and the smooth pasting conditions used here.

are satisfied. Equation (3.4) requires that job creation be postponed until the net benefit from delaying job creation is exhausted, which happens when the marginal present value of job creation,  $(dV(z_c(n), n)/dn) + c$ , is zero. The marginal job created contributes  $V_n(z_c(n), n)$  to the value of the firm and it induces a shadow cost  $\tau_c V_z(z_c(n), n)$  associated with the loss of organizational capital. In addition, optimal job creation takes into account the fact that larger firms incur a proportionally larger fixed cost when they create jobs. Similarly, equation (3.5) says that job destruction will be postponed until the value of retaining the marginal job,  $(dV(z_d(n), n)/dn) + c$ , is exhausted. Delaying job destruction at the margin retains the contribution  $V_n(z_d(n), n)$  to the value of the firm while it avoids incurring the shadow cost  $\tau_d V_z(z_d(n), n)$ . As in the case of job creation, optimal job destruction takes into account that the marginal job at the firm raises the fixed cost of job destruction by an amount c.

Finally, two additional conditions are needed in order to ensure that the optimal amount of job creation and destruction occurs. First, upon reaching the creation margin, the firm will expand until the marginal value of further expansion,  $dV(Z_c(n), N_c(n))/dn$ , is exhausted,

$$V_n(Z_c(n), N_c(n)) - \tau_c V_z(Z_c(n), N_c(n)) = 0.$$
(3.6)

Similarly, upon reaching the destruction margin, the firm will contract until the value of the last job retained,  $dV(Z_d(n), N_d(n))/dn$ , is zero,

$$V_n(Z_d(n), N_d(n)) + \tau_d V_z(Z_d(n), N_d(n)) = 0.$$
(3.7)

Given our homogeneity assumptions it is easy to show (the details are relegated to an appendix) that the process of job turnover is in effect the same for all firms, in the sense that each firm views the ratio z/n as its state variable and the firm's problem does not change with its size. <sup>9</sup> Thus, it will be convenient to define  $x \equiv z/n$ . While x is the firm's stock of organizational capital per job, we will often refer to x as labor productivity since true labor productivity is a simple transformation of x, namely,  $(1-\alpha) x^{\alpha}$ . Accordingly, the creation and destruction trigger

<sup>&</sup>lt;sup>9</sup>The value of the firm, however, is proportional to its level of employment.

points  $x_c(n) \equiv z_c(n)/n$  and  $x_d(n) \equiv z_d(n)/n$ , and the return points  $X_c(n) \equiv Z_c(n)/N_c(n)$  and  $X_d(n) \equiv Z_d(n)/N_d(n)$  are the same for all firms. Let us denote them by  $x_c$  and  $x_d$  and  $X_c$  and  $X_d$  respectively. Consequently, optimal labor demand decisions are such that

$$\frac{\Delta n}{n} = \begin{cases} \frac{x_d - X_d}{X_d - \tau_d} < 0 & \text{if } x = x_d \\ 0 & \text{if } x_d < x < x_c \\ \frac{x_c - X_c}{X_c + \tau_c} > 0 & \text{if } x = x_c. \end{cases}$$
(3.8)

Figure 1 depicts the optimal labor demand policy as a function of the firm's current stock of labor,  $n_t$ , and its current stock of organizational capital,  $z_t$ . In addition, the figure shows the optimal job creation and destruction intensities by a firm of arbitrary size n, in the case where  $\tau_c > 0$  and  $\tau_d > 0$ . When the firm's stock of organizational capital  $z_t$  reaches  $z_d(n)$  the firm destroys jobs so that its labor productivity increases from  $x_d$  to  $X_d$ . The horizontal dotted line from  $x_d$  to  $X_d$  indicates the amount of job destruction that would be needed to achieve this target increase in productivity if the stock of organizational remained equal to  $z_d(n)$ . The stock of labor, however, falls by a larger amount, from n to  $N_d(n)$ . Intuitively, during job destruction episodes, when the firm seeks to raise labor productivity, there are two opposing forces. Job destruction directly increases labor productivity, but the resulting destruction of organizational capital by itself reduces labor productivity. Thus a higher job destruction intensity is needed to achieve any given increase in labor productivity when job destruction involves the loss of organizational capital than when it does not. Similarly, when the firm's stock of organizational capital  $z_t$  reaches  $z_c(n)$  the firm creates jobs until its labor productivity falls from  $x_c$  to  $X_c$ . The horizontal dotted line from  $x_c$  to  $X_c$  indicates the amount of job creation that would be needed to achieve this target decline in productivity if the stock of organizational remained equal to  $z_c(n)$ . The stock of labor, however, increases by a smaller amount, from n to  $N_c(n)$ . Intuitively, during job creation episodes, labor productivity falls both because employment increases and because organizational capital is lost. Thus, a lower job creation intensity is needed to achieve any given increase in productivity when job creation is accompanied by the loss of organizational capital than when it does not.

#### [FIGURE 1]

It is useful to observe how the loss of organizational capital associated with job creation and destruction influences the costs of creating and destroying jobs. Using the definition  $V(z,n) \equiv nv(x(n))$  and the fact that the firm's optimal behavior depends on z and n only through x, the optimality conditions (3.4) and (3.5) (compare with (A.6) and (A.8) in the Appendix) imply that

$$\tau_c V_z \left( z_c(n), n \right) = \left( 1 - \frac{x_c}{x_c + \tau_c} \right) \left( v \left( x_c \right) + c \right)$$
(3.9)

and

$$\tau_d V_z (z_d(n), n) = -\left(1 - \frac{x_d}{x_d - \tau_d}\right) \left(v (x_d) + c\right).$$
(3.10)

These are the shadow costs of job creation and destruction, evaluated at the creation and the destruction margins, respectively. They amount to a constant fraction of the average value of the firm (adjusted to account for the dependence of the fixed cost of adjustment on the size of the firm) at job creation and job destruction times, respectively. More importantly, these fractions are determined by the elasticities of labor productivity (as measured by x) with respect to employment at creation times,  $-(x_c + \tau_c)/x_c$ , and destruction times,  $-(x_d - \tau_d)/x_d$ , respectively. To see this, note that

$$\frac{dx}{x} = \frac{dz}{z} - \frac{dn}{n} = \begin{cases} \frac{-\tau_c dn}{z} - \frac{dn}{n} & \text{if } dn > 0\\ \mu dt + \sigma dW & \text{if } dn = 0\\ \frac{\tau_d dn}{z} - \frac{dn}{n} & \text{if } dn < 0 \end{cases}$$
(3.11)

and recall that  $x_c$  is the optimal creation margin and  $x_d$  is the optimal destruction margin to obtain

$$\frac{dx/x}{dn/n} = \begin{cases} -\left(\frac{x_c + \tau_c}{x_c}\right) & \text{if } dn > 0\\ -\left(\frac{x_d - \tau_d}{x_d}\right) & \text{if } dn < 0. \end{cases}$$
(3.12)

Furthermore, we note that

$$\frac{x_c + \tau_c}{x_c} > 1 > \frac{x_d - \tau_d}{x_d} \tag{3.13}$$

when  $\tau_c > 0$  and  $\tau_d > 0$  (as in Figure 1). In this case, the elasticity of labor productivity at job creation times is larger (in absolute value) than one, reflecting the fact that, when creating jobs, the firm lowers labor productivity both through an increase in employment and a loss of organizational capital. However, the elasticity of labor productivity with respect to employment at the destruction margin is less (in absolute value) than one. When destroying jobs there are two opposing forces. Job destruction directly increases labor productivity, but the resulting destruction of organizational capital reduces labor productivity, thus requiring larger job destruction to achieve any given increase in labor productivity.

As long as neither creation nor destruction take place at the firm, x is simply a linear function of z, in which case, by Ito's Lemma, the process  $\{x_t\}$  follows a geometric Brownian motion with drift  $\mu$  and standard deviation  $\sigma$ . This process, together with the optimal labor demand policy (3.8) completely characterizes the dynamics of job creation and job destruction at the firm level.

#### 3.2 The Frequency and the Intensity of Job Creation and Destruction

Next, we re-examine the smooth pasting conditions (3.4) and (3.5) in order to highlight the influence of the endogenous destruction of organizational capital on the timing as well as the intensity of job creation and job destruction. In addition, this exercise serves to motivate our subsequent analysis of the aggregate behavior that is implied by the model.

Letting  $t_c$  be a job creation time and  $T_d$  be the first job destruction time after  $t_c$ , the smooth pasting condition (3.4) can be written as

$$E_{t_c} \left\{ \int_{t_c}^{T_d} e^{-r(s-t_c)} x_s^{\alpha} ds \right\} + E_{t_c} \left\{ e^{-r(T_d-t_c)} \int_{T_d}^{\infty} e^{-r(s-T_d)} x_s^{\alpha} ds \right\} - \frac{w}{r}$$
  
=  $\tau_c V_z \left( z_c(n), n \right) - c.$  (3.14)

The left-hand side of equation (3.14) is the shadow price of labor, or the partial derivative of V with respect to  $n_{tc}$ . By the envelope theorem, the currently marginal job at the firm is viewed as the marginal job throughout the future. The firm knows that this equation, together with the other optimality conditions will continue to hold at all times, which in turn defines the probability distribution of future employment levels. It is with respect to this distribution that the expectation in equation (3.14) is taken.

By the strong Markov property of (controlled) Brownian motion processes, the random variable (as of time  $t_c$ )  $T_d$  and the stochastic process  $\{x_s; s > T_d\}$  are independent. Therefore, the second expectation in the left-hand side of equation (3.14) can be written as the product of two expectations. Taking iterated expectations and rearranging we can then write

$$E_{t_c} \left\{ \int_{t_c}^{T_d} e^{-r(s-t_c)} x_s^{\alpha} ds \right\} = \frac{w}{r} \left( 1 - E_{t_c} \left\{ e^{-r(T_d - t_c)} \right\} \right)$$
$$+ \tau_c V_z \left( z_c(n), n \right) - c - E_{t_c} \left\{ e^{-r(T_d - t_c)} \right\} E_{t_c} \left\{ E_{T_d} \left\{ \int_{T_d}^{\infty} e^{-r(s-T_d)} x_s^{\alpha} ds \right\} - \frac{w}{r} \right\}. \quad (3.15)$$

Since  $T_d$  is a destruction time, we have, from the smooth pasting condition (3.5)

$$E_{T_d} \left\{ \int_{T_d}^{\infty} e^{-r(s-T_d)} x_s^{\alpha} \, ds \right\} - \frac{w}{r} = -\tau_d \, V_z \left( z_d(n), n \right) - c. \tag{3.16}$$

Noting that the right-hand side of (3.16) is non-stochastic, we can rewrite (3.15) as

$$E_{t_c} \left\{ \int_{t_c}^{T_d} e^{-r(s-t_c)} x_s^{\alpha} \, ds \right\} = \frac{w}{r} \left( 1 - E_{t_c} \left\{ e^{-r(T_d - t_c)} \right\} \right) + \tau_c \, V_z \left( z_c(n), n \right) - c \, V_z \left$$

+ 
$$E_{t_c} \left\{ e^{-r(T_d - t_c)} \right\} \left( \tau_d V_z \left( z_d(n), n \right) - c \right).$$
 (3.17)

Thus, job creation will take place when the revenue that an additional job created (at time  $t_c$ ) is expected to contribute before the job is destroyed is equal to the total cost of creating the job. In turn, this cost has the following components. First, there is the present value of the wage bill associated with that job. Second, there is the shadow cost associated with the destruction of organizational capital induced by the creation of the job. Third, there is the shadow cost associated with the future destruction of organizational capital induced by the creation as of time  $t_c$ ) destruction time  $T_d$ . Finally, the terms -c and  $-E_{t_c} e^{-r(T_d-t_c)} c$  simply reflect the homogeneity assumption that larger firms incur a proportionally larger fixed cost whenever they adjust their employment.

Similarly, letting  $t_d$  be a job destruction time and  $T_c$  be the first job creation time after  $t_d$ , equation (3.5) can be written as

$$E_{t_d} \left\{ \int_{t_d}^{T_c} e^{-r(s-t_d)} x_s^{\alpha} \, ds \right\} = \frac{w}{r} \left( 1 - E_{t_d} \left\{ e^{-r(T_c - t_d)} \right\} \right) - \tau_d \, V_z \left( z_d(n), n \right) - c - E_{t_d} \left\{ e^{-r(T_c - t_d)} \right\} \left( \tau_c \, V_z \left( z_c(n), n \right) + c \right).$$
(3.18)

Thus, job destruction will take place when the revenue foregone by destroying the marginal job (at time  $t_d$ ) until the next creation time equals the opportunity cost of retaining the marginal job. In turn, by retaining the marginal job the firm pays the corresponding wage bill, but it avoids incurring the shadow costs associated with the loss of organizational capital which is induced by the current destruction of the job and by the future job creation. As before, this opportunity cost also accounts for the fact that the fixed cost of adjustment is proportional to the size of the firm.

#### 4 Implications for the Behavior of Aggregate Job Flows

This section is organized in three parts. We begin by explaining our approach to evaluating the model's implications for the behavior of aggregate job flows. In the second part, we use this approach to study the benchmark model which is identical to ours except that labor adjustments do not influence organizational capital, and instead firms face conventional hiring and firing costs, as in Bentolila and Bertola (1990). The analysis of this benchmark illustrates the process by which aggregation tends to undo the influence of microeconomic asymmetries. In the last part, we discuss the implications of the endogenous destruction of organizational capital for the behavior of aggregate job flows in our model.

#### 4.1 Aggregate Job Flows

In order to understand the aggregate behavior associated with the dynamics of firm-specific organizational capital, we study the average behavior of many heterogeneous firms which face idiosyncratic shocks but otherwise face identical problems regarding the choice of labor demand. Our analysis exploits the fact that the firm-specific labor productivity process  $\{x_t\}$  associated with the optimal (S,s) policy (3.8) never leaves the interval  $[x_d, x_c]$  and reaches every point in that interval with probability one, hence it possesses an invariant, ergodic distribution. In this context, it is well understood that the cross-sectional distribution of firm productivity will settle into a time-invariant distribution that mimics each firm's individual distribution. <sup>10</sup>

To better assess the properties of aggregate job flows, we simulate the aggregate dynamics that are generated by the long-run distribution of firms. To that end, first we calculate the optimal creation and destruction margins,  $x_c$  and  $x_d$  and the corresponding return points  $X_c$ and  $X_d$ . <sup>11</sup> Then, we approximate the exogenous Brownian motion given by (2.2) in discretetime space and study the dynamics of 30,000 firms for 200 periods after the cross-sectional

 $<sup>^{10}</sup>$ See Bertola and Caballero (1990) and Caballero (1992) for a clear discussion of this approach to aggregation. The procedure for calculating the ergodic distribution associated with a Brownian motion regulated by a (S,s) rule is well-known and it is explained in Bertola and Caballero (1990). The details of the calculation in our case are relegated to the Appendix.

<sup>&</sup>lt;sup>11</sup>In the Appendix, we show that the Bellman equation together with the optimal boundary conditions give rise to a system of non-linear equations that can be solved numerically for the four margins that compose the optimal (S,s) policy.

distribution has settled into the ergodic distribution of labor productivity  $\{x_t\}$  when each firm uses the common optimal labor demand policy.

Specifically, we simulate the behavior of standard measures of job creation and destruction, as defined by Davis and Haltiwanger (1990), namely, the job creation rate

$$POS_t = \frac{2}{L_t + L_{t-1}} \sum_{i \in I^+} \left( n_{i,t} - n_{i,t-1} \right), \tag{4.1}$$

and the job destruction rate

$$NEG_t = \frac{2}{L_t + L_{t-1}} \sum_{i \in I^-} |n_{i,t} - n_{i,t-1}|, \qquad (4.2)$$

where  $n_{i,t}$  is the employment level at firm *i* in period *t*,  $L_t$  is aggregate employment at time *t*, and  $I^+$  and  $I^-$  denote the set of firms that are expanding and contracting, respectively.

The job creation and destruction rates can also be expressed as a size-weighted average of the individual firms' growth rates at creation and destruction times, respectively. In our model, optimal behavior implies that those growth rates are independent of the size of the firm, as is clear from the optimal labor demand policy (3.8). The job creation and destruction rates are then given by

$$POS_t = \left(\frac{x_c - X_c}{X_c + \tau_c}\right) \left(\frac{2}{L_t + L_{t-1}} \sum_{i \in I^+} n_{i,t}\right),\tag{4.3}$$

and

$$NEG_{t} = \left| \frac{x_{d} - X_{d}}{X_{d} - \tau_{d}} \right| \left( \frac{2}{L_{t} + L_{t-1}} \sum_{i \in I^{-}} n_{i,t} \right).$$
(4.4)

Thus, these rates are functions of the absolute value of the employment growth rate of expanding firms or contracting firms, which we term the *job creation intensity* and the *job destruction intensity*, respectively, and the sum of the employment levels of all expanding or contracting firms during a given period, normalized by the average of aggregate employment in the current and previous period. It should be noted that this second element of creation and destruction rates depends both on the cross-sectional distribution of labor productivity, which determines the constant number of firms creating and destroying jobs at all times, and the size distribution of firms. Furthermore, while the distribution of labor productivity is stationary, the processes of organizational capital and employment at each firm are non-stationary. Consequently, the non-stationary dynamics of the size distribution may in principle have some influence on aggregate employment dynamics. This issue is addressed by the normalization used in calculating the creation and destruction rates which makes them insensitive to random changes in the size distribution of firms. Note, in particular, that the growth rate of employment has a stationary distribution.

When studying labor market behavior, two other related measures of job change are often used. The rate of net job creation

$$NET_t = POS_t - NEG_t \tag{4.5}$$

measures net employment growth from t-1 to t, and the gross job reallocation rate

$$SUM_t = POS_t + NEG_t \tag{4.6}$$

provides a measure of the gross change in jobs from t - 1 to t. Because the rates  $POS_t$  and  $NEG_t$  that are implied by the firms' average behavior are independent of time, then the average behavior is such that net employment growth, as measured by  $NET_t$ , is constant, although not necessarily equal to 0. Our numerical analysis focuses on the implied stationary values for the measures of employment changes already discussed. We denote them by POS, NEG, NET and SUM, respectively.

#### 4.2 A Benchmark: Aggregate Implications of Output-Costs of Adjustment

Before exploring the behavior of aggregate job flows in our model, it is useful to introduce a benchmark model of labor demand which is identical to our own except for two changes. First, there is no loss of organizational capital when jobs are created or destroyed; and second, each firm incurs adjustment costs of H units of current output for every job created when it expands employment, and F units of current output for every job destroyed when it contracts employment. We label these hiring and firing costs the *output-costs of labor adjustment* in order to distinguish them from those adjustment costs associated with the loss of organizational capital. This benchmark model is a standard model of dynamic labor demand in which a firm faces both fixed and proportional costs of hiring and firing. In particular, it is an extension of the model in Bentolila and Bertola (1990) to include fixed costs of adjustment. Besides its role as a benchmark, the model's implications for standard measures of aggregate job flows are of interest to the extent that microeconomic adjustment costs are frequently used to explain the behavior of employment dynamics. <sup>12</sup>

It is well understood that the firm's optimal labor demand policy in this context takes the form of a (S,s) rule. The corresponding Bellman equation is still given by (3.1). The difference between this problem and ours is reflected in the optimal boundary conditions, whereby (3.2)–(3.7) must be changed to account for the exogenous proportional hiring and firing costs instead of the endogenous productivity loss at creation and destruction times. <sup>13</sup> Accordingly, a version of equations (3.17) and (3.18) also holds, whereby the endogenous costs of adjustment are replaced by the exogenous costs of adjustment in the obvious way. It should be noted that the firm continues to target labor productivity, as given by the ratio of organizational capital to employment, although the dynamics of organizational capital,  $\{z\}$ , are exogenous in this case. Consequently, the employment dynamics associated with optimal labor demand decisions differ across the two models since the dynamics of organizational capital are different and, therefore, the optimal (S,s) policy in the benchmark model also differs from that in our model.

We begin by summarizing the main implications of the benchmark model, before we discuss them in turn. Table 1 illustrates the impact of the hiring costs H and the firing costs F on the behavior of job flows in the aggregate. Columns two and three report the net employment

 $<sup>^{12}</sup>$ Recent examples include Foote (1998) and Campbell and Fisher (2000).

 $<sup>^{13}</sup>$ The appropriate optimality conditions are discussed by Bertola and Caballero (1990), Dumas (1991) and Dixit (1991), among others.

growth rate NET and the rate of gross job reallocation SUM in the absence of trend growth in the stock of organizational capital ( $\mu = 0$ ). The last two columns of Table 1 repeat the exercise for a much higher trend rate of growth ( $\mu = 0.045$ ).<sup>14</sup>

#### [TABLE 1]

Several remarkable features of the benchmark model are immediately apparent in Table 1. First, as one might expect, the higher the cost of adjusting the size of the labor force, the less the total amount of adjustment that occurs in the aggregate. This is confirmed by looking down column three of Table 1. As H and F both increase by twelve times from 0.1 to 1.2, <sup>15</sup> SUM falls roughly 58 percent from 1.62% to 0.68%. Interestingly the total amount of adjustment as captured by SUM appears to depend only on the sum of H and F. The middle two rows of column three reveal that SUM is constant at about 0.8% (abstracting from approximation error) as we vary H and F, keeping their sum equal to 1.3. Second, looking down column two, one is struck by a stronger invariance result. NET, the difference between POS and NEG, is invariant (once again ignoring approximation error) to the value of H and F at a value of about -0.26%. Furthermore, Table 1 indicates that the previous results hold even when  $\mu \neq 0$ . Third, and somewhat surprisingly, SUM is independent of the size of  $\mu$ . This can be seen by looking along any row at the numbers for SUM in columns three and five. Finally, comparing the numbers for NET in columns two and four, we see that NET is increasing in  $\mu$ . Note, however, that NET is significantly different from zero along the second column in Table 1, even though  $\mu = 0$ . This bias in the aggregate flows towards NEG seems odd, given the result that neither H nor Finfluence NET. As will become clear, such a bias is driven directly by the assumed accumulation process for the stock of organizational capital. Next, we discuss these results in turn.

The reason why adjustment costs do not influence net employment growth can be understood

<sup>&</sup>lt;sup>14</sup>In our simulations we approximate the theoretical long-run distribution of labor productivities. As a result, there is a small amount of variation in the number of firms that are adjusting their employment in each period. The standard deviation associated with the approximation error in POS, NEG, NET and SUM is always less than 0.02 percent, which is an order of magnitude less than the corresponding means.

<sup>&</sup>lt;sup>15</sup>A value of the firing cost F equal to 0.1 implies that a firm incurs total firing costs which are 1.13 percent of the current wage bill in each firing episode, whereas this number rises to 12.19 percent when F is equal to 1.2. These numbers exclude the small fixed cost  $c \cdot n$  equal to 0.05 percent of the wage bill.

by considering an increase in F, from 0.1 to 1.2, holding H constant at 0.1. Figure 2 plots the target change in labor productivity at creation and destruction times for the case of F = 0.1 and F = 1.2. Observe that raising F causes destroying firms to adjust their productivity by a much smaller magnitude than before while creating firms hardly change the amount of their adjustment. Of course, this also applies to changes in employment, and not just productivity, at creation and destruction times, since x is simply the ratio z/n and z does not change with employment adjustments in this benchmark model. Next, consider the response of the cross-sectional distribution of firms to the increase in F. Figure 3 plots the theoretical long-run cumulative distribution. Note that the creation and destruction margins spread out, increasing the firms' region of inaction. This leads to a spreading out of the distribution which by itself tends to lower the number of firms hitting either margin. In addition, an increase in F causes the mass in the distribution of firms to shift towards the destruction margin. To isolate this latter change, we have calculated the theoretical distribution associated with the proportional distance from the destruction margin (see Appendix), which in turn is defined by

$$u = \frac{x - x_d^\star}{x_c^\star - x_d^\star},\tag{4.7}$$

where  $x_c^{\star}$  and  $x_d^{\star}$  are the optimal creation and destruction margins associated with the benchmark model. Figure 4 shows that this distribution shifts uniformly to the left as F is raised from 0.1 to 1.2. This, in turn, clearly shows that the increase in F induces relatively more bunching of firms towards the destruction margin.

Thus, the increase in F has made all firms more reluctant to adjust their labor force, which translates into less frequent job creation and destruction and also lower magnitudes of labor adjustment. Hence, the reduction in SUM. At the same time, the magnitude of job destruction relative to job creation has fallen. But NET has remained unchanged because the frequency of job destruction relative to job creation has increased. This mechanism reflects, in essence, Caballero's (1992) argument that the effect of asymmetries in the costs of adjustment on microeconomic behavior may be undone by aggregation.

#### [FIGURE 2, FIGURE 3, FIGURE 4]

It should be noted how the underlying intensity-frequency trade-off operates directly in the space of labor productivity. In particular, note that it is the cross-sectional distribution of firm productivity, not that of employment, that determines the bunching of firms towards the creation or destruction margins. This is because firms target an optimal level of labor productivity, not an optimal size. Anticipating our results in the following section regarding the influence of the endogenous destruction of organizational capital, we note that each firm's productivity and employment move together during creation and destruction episodes in this benchmark model, while this will not be the case if employment changes influence organizational capital. In turn, this allows the intensity-frequency trade-off to fully cancel the microeconomic influence of adjustment costs on NET. Furthermore, the trade-off operates in terms of absolutes changes in productivity and employment, not in terms of proportional changes as measured by (3.8). The precise manner in which the intensity-frequency trade-off operates is relevant because the aggregation process influences different macroeconomic aggregates differently. In turn, this indicates the potential difficulty in understanding aggregate behavior solely through individual incentives, as emphasized by Caballero (1992). Concerning this point, it is interesting to consider the simulation results in Table 2, regarding the increase of F from 0.1 to 1.2. Comparing columns two and four, we see that POS and NEG fall by the same amount, 0.39, lowering SUM = POS + NEG while leaving NET = POS - NEG essentially unchanged. Table 2 also displays detailed information about the intensities as well as the frequencies of job creation and destruction.<sup>16</sup>

#### [TABLE 2]

For example, comparing columns two and four in Table 2, we see that the creation intensity falls by more than the destruction intensity does and the creation frequency also falls by more than the destruction frequency does, even though the firing costs have risen substantially. At first pass, one may expect NET to fall, but NET remains unchanged. Moreover, the lack of

<sup>&</sup>lt;sup>16</sup>Note that job creation and destruction intensities are calculated from the optimal labor demand policy (3.8), whereas the corresponding frequencies are obtained from simulating the cross-sectional distribution of firms.

a significant response of NET to the substantial increase in the firing cost F is not explained by changes in the size-distribution of firms. This is confirmed in our simulations. As explained above, what actually happens is that the absolute amount of job creation by each expanding firm hardly changes, whereas the amount of job destruction per contracting firm falls substantially. The behavior of the job creation and destruction intensities in percentage terms hides this fact. The intensity-frequency trade-off is quite apparent when we consider the impact of switching Hand F between the values of 0.1 and 1.2 (see columns three and four of Table 2). Abstracting from the approximation error, we see that not only are NET and SUM invariant to which cost is bigger, but POS and NEG themselves remain unchanged. This result is obtained by frequencies moving in the opposite direction to intensities to just offset any change in the aggregate: while the number of firms creating and destroying both fall, the percentage change in number of jobs created or destroyed by individual firms both increase. The last column shows the impact of increasing both H and F simultaneously, in which case adjustment on all margins is reduced causing POS and NEG to fall by the same amount.

Table 3 repeats the simulations associated with changing H and F when  $\mu = 0.045$ .<sup>17</sup> The patterns discussed above are in evidence here as well. Rather than repeat the analysis, we turn to the influence of  $\mu$  on aggregate job flows.

#### [TABLE 3]

The higher value of  $\mu$  implies that the exogenous accumulation of organizational capital tends to push firms up against the job creation margin. In the absence of any reaction by firms, this would lead to a higher frequency of job creation and lower frequency of job destruction. On the other hand, firms will optimally respond to the higher productivity trend by altering their behavior. First, they can expand, contract or shift the region of inaction by moving the creation and destruction margins,  $x_c$  and  $x_d$ , and second they can control the frequency and intensity of

<sup>&</sup>lt;sup>17</sup>Note that the implied productivity growth is 0.45 times the rate at which the future is discounted, r. It is also equal to 0.5 times the variance of the firm's productivity process,  $\sigma^2$ , when there is neither creation nor destruction of jobs. Also see the discussion following equation (4.8) below concerning the relation between  $\mu$  and  $\sigma^2$ .

adjustment by moving the return points,  $X_c$  and  $X_d$ , closer to or further away from the creation and destruction margins. Figures 5 and 6 quite clearly show the response of firms.

#### [FIGURE 5, FIGURE 6]

The intensity of creation is increased while the intensity of destruction is reduced. At the same time the cumulative distribution of firms (in terms of the proportional distances to the destruction margin) shifts clearly towards the creation margin. This is confirmed by the simulations reported in Table 3. There is an increase in the number of firms creating jobs and a corresponding fall in the number of firms destroying jobs. This is accompanied by slightly more intense creation and slightly less intense destruction as measured by the percentage of jobs created or destroyed relative to the size of the firm. These results indicate that it is optimal for the firm to not resist large changes in the exogenous trend rate of growth  $\mu$ . Since the firm is largely indifferent between paying adjustment costs for creation or destruction, when  $\mu$  rises the firm just trades-off adjustment costs incurred at the creation and the destruction margins. This intuition is confirmed by comparing the aggregate magnitude of employment adjustments, as measured by SUM, in tables 2 and 3. We see that SUM remains essentially constant at any given firing and hiring costs, that is, the large increase in POS is exactly offset by a fall in NEG. To reiterate this point, SUM remains constant as trend employment growth in the aggregate goes from -26 percent to +24 percent. It is worth emphasizing that this result should not be understood as the mechanical effect of productivity trends on the distribution of firms. For firms are choosing labor demand optimally and they account for the influence of trend growth in productivity as well as asymmetries in the costs of labor adjustment. Given this, the lack of response of SUM to productivity trends in this benchmark model is quite remarkable.

Up to this point we have illustrated how firing and hiring costs influence SUM but not NET, and how trend growth in the stock of organizational capital influences NET but not SUM. Interestingly, these results imply that neither asymmetries in the costs of labor adjustment nor asymmetries in the cross-sectional distribution of firms on their own explain a relationship between net employment growth, NET = POS - NEG, and the intensity of job reallocation,

SUM = POS + NEG. This is a remarkable implication of the benchmark model, to the extent that microeconomic adjustment costs and shifts in the cross-sectional distribution of firms, as described by this model, are frequently used to explain the behavior of POS and NEG.<sup>18</sup>

Before we turn to the implications of the destruction of organizational capital associated with employment adjustments, a special feature of the exogenous process of accumulation of organizational capital deserves attention. As it was noted above, our simulations reveal that NET is significantly different from zero when  $\mu = 0$ . This odd feature was apparent in the values of NET in the second column of Table 1. This is because there is a built-in asymmetry in the model associated with the assumption that organizational capital is accumulated according to a geometric Brownian motion. In turn, this implies that  $\{\log z_t\}$  is a linear Brownian motion with drift  $\mu - \sigma^2/2$ , which implies the relationship

$$\frac{dz_t}{z_t} = d\log z_t + \frac{1}{2}\sigma^2 \, dt.$$
(4.8)

This feature explains why we have chosen to present our results for the particular cases in which  $\mu = 0$  and  $\mu = \sigma^2/2$ . The former is the case where the stock of organizational capital exhibits no trend growth. The latter is the case where the logarithm of the stock of organizational capital exhibits no trend growth. What is interesting to note here is the different implications of each case. When  $\mu = 0$  firms recognize that organizational capital,  $z_t$ , and therefore labor productivity, x, are expected to grow at the rate of 0. From their viewpoint, the process exhibits no asymmetry, and this is reflected in the optimal choice of job creation and destruction intensities. This can be seen in the second column in Table 2 , for the case where H = F = 0.1. In this case, there are no asymmetries and the hiring and firing costs are small, in which case the optimal job creation and destruction intensities are roughly equal, as one would expect. On the other hand, note that the frequencies of job creation and destruction are significantly different. This reflects the property of the geometric Brownian motion that  $z_t$  exhibits random fluctuations about a long-term exponential decay. <sup>19</sup> Note that log  $z_t$  is

 $<sup>^{18}{\</sup>rm See}$  Hamermesh and Pfann (1996) for a discussion of the literature, and Foote (1998) for an interesting example.

<sup>&</sup>lt;sup>19</sup>See Taylor and Karlin (1998, pages 514–516).

expected to grow at the rate  $-\sigma^2/2$  in this case. This introduces a bias in the long-run crosssectional distribution of firms. Instead, when  $\mu = \sigma^2/2$ ,  $\log z_t$  is expected to grow at a zero rate and there is no bias in the cross-sectional distribution. This is confirmed by looking at the second column in Table 3 and noting that the frequencies of creation and destruction are roughly equal in this case. However, individual firms now understand that productivity itself is trending upwards at the expected growth rate of  $\sigma^2/2$ . This explains the higher creation intensity relative to the destruction intensity.

Thus, the previous asymmetry biases NET downwards when  $\mu = 0$  and upwards when  $\mu = \sigma^2/2 = 0.045$ . It should also be noted that this is not entirely a technical issue. An implication of our analysis is that the properties of the underlying stochastic process are important in order to understand the induced aggregate behavior. In this context, it is interesting to observe that the properties of the driving process determine whether the corresponding asymmetry may show up in intensities or, instead, in frequencies.

#### 4.3 The OC-Costs of Labor Adjustment

The foregoing analysis illustrates the process by which asymmetries in microeconomic labor adjustment as well as asymmetries in the cross-sectional distribution of firms may be undone by aggregation in the absence of aggregate shocks. <sup>20</sup> It also illustrates how this process may affect specifically net job creation, as measured by NET = POS – NEG, and gross job reallocation, as measured by SUM = POS + NEG. Here, we turn to the implications of our model for the behavior of aggregate job flows. Recall that, in this context, there are no firing and hiring costs and, instead, employment changes influence the stock of organizational capital.

In light of the strong aggregate implications of the benchmark model with output-costs of labor adjustment, it is remarkable how the destruction of organizational capital associated with job flows influences aggregate behavior, as the following analysis illustrates. As emphasized in the Introduction of the paper, this indicates the potential importance of the distinction between adjustment costs that are associated with a loss of output, which we have labeled

 $<sup>^{20}</sup>$ Bertola and Caballero (1990) and Caballero (1992) discuss why this average effect continues to be relevant in the presence of aggregate shocks.

*output-costs of adjustment*, and those associated with a loss of organizational capital, which we label organizational-capital costs of adjustment or simply *OC-costs of adjustment*.

We begin by summarizing the main implications of the model for the behavior of job flows, which we then discuss in turn. Table 4 displays the impact of varying  $\tau_c$  and  $\tau_d$  on NET and SUM, for the same values of  $\mu$  used before, that is,  $\mu = 0$  and  $\mu = 0.045$ .<sup>21</sup>

#### [TABLE 4]

First, it is apparent that SUM falls with the OC-costs of adjustment  $\tau_c$  and  $\tau_d$ . Moreover, it appears that SUM is influenced by  $\tau_c$  and  $\tau_d$  through their sum, just as SUM was influenced by H + F in the benchmark model. Two other features of Table 4 stand out immediately. First, looking along columns two and four, we note that NET falls significantly with the OC-costs of adjustment, irrespective of the value of  $\mu$ . Furthermore, all the action comes from raising  $\tau_d$ , while  $\tau_c$  has no noticeable impact on NET. Second, SUM falls significantly when  $\mu$  increases from 0 to 0.045, for any given value of  $\tau_c$  and  $\tau_d$ . Furthermore, differences in  $\mu$  induce a negative relation between NET and SUM. For example, looking along the third row in Table 4, we see that SUM falls from 1.01% to 0.79% while NET rises from -0.28% to 0.21%, as  $\mu$  rises.

In order to understand the mechanism that links microeconomic adjustment costs and aggregate job flows, we begin our analysis by considering the impact of an increase in the OC-cost of job destruction. Raising  $\tau_d$  discourages both job creation and destruction, and as a result the optimal region of inaction widens. As before, the relative bunching towards creation or destruction is more clearly seen in the cross-sectional distribution of the firms' distance to the destruction margin, given by  $(x - x_d)/(x_c - x_d)$ . As illustrated in Figure 8, the increase in  $\tau_d$ 

<sup>&</sup>lt;sup>21</sup>One way to measure the size of the OC-costs of adjustment is in terms of the lost output resulting purely from the destruction of organizational capital. For our baseline parameterization in Table 5, when  $\tau_c = \tau_d = 0.1$ the stock of organizational capital falls by 0.1 percent during job creation and 0.3 percent during job destruction. This implies an output loss purely due to the destruction of organizational capital of 0.07 percent on the creation margin and 0.21 percent on the destruction margin, whereas creation and destruction episodes involve increasing or decreasing the number of employees by 9.38 percent and 10.51 percent, respectively. Increasing  $\tau_c$  and  $\tau_d$  to 0.6 raises these output losses to 0.20 percent on the creation margin and 2.93 percent on the destruction margin, while the job creation intensity falls to 7.21 percent and job destruction intensity increases to 13.85 percent.

causes relatively more bunching towards the destruction of jobs. The magnitude of this effect is illustrated in Table 5. For example, when  $\tau_d$  increases from 0.1 to 0.6 keeping  $\tau_c$  constant at 0.1, the number of expanding firms falls by 60 percent, from 906 to 360, whereas the number of contracting firms drops by 22 percent, from 2,226 to 1,731. Figure 7, then, illustrates how the optimal change in labor productivity that is targeted by expanding firms increases slightly whereas the optimal productivity change that is targeted by contracting firms falls substantially. Together, figures 7 and 8 illustrate the intensity-frequency trade-off underlying the aggregation of individual behavior, very much like in the benchmark model.

#### [FIGURE 7, FIGURE 8]

Nevertheless, increasing  $\tau_d$  does influence the behavior of aggregate job flows. As shown in Table 5, POS falls by more than 50 percent, from 0.36% to 0.17%, while NEG falls by 17 percent, from 0.65% to 0.54%, resulting in an overall decline in NET. Just as in the benchmark model, the behavior of aggregate job flows here cannot be understood without reference to the process of aggregation. Consequently, arguments based solely on individual incentives, while being intuitive, are insufficient to understand the behavior of the aggregates. Nonetheless, it is possible to gain additional insight into the basic mechanism at work. First, recall that the intensity-frequency trade-off operates in the space of labor productivity, since it is productivity that firms target. Yet, unlike the output-costs of adjustment, the OC-costs of adjustment decouple the changes in employment and productivity at creation and destruction times. More importantly, a higher  $\tau_d$  requires a larger amount of job destruction to achieve a given change in productivity. Comparing columns two and four in Table 5, one notices that while expanding firms reduce the job creation intensity, contracting firms actually increase the job destruction intensity. The changes in the magnitudes of job creation and destruction and the changes in the cross-sectional distribution of firms reinforce each other, rather than canceling each other out.

#### [TABLE 5]

Further insight is gained by noting why it is only  $\tau_d$ , and not  $\tau_c$ , that influences the behavior

of NET significantly. On the one hand, an increase in the OC-cost of job creation,  $\tau_c$ , makes firms reluctant to create jobs, as job creation is more expensive. On the other hand, a higher  $\tau_c$ makes job creation more effective in achieving a target change in productivity. This is because, when  $\tau_c$  is higher, every job created induces a larger fall in labor productivity through the larger loss in organizational capital. As a result, expanding firms do not need to create relatively as many jobs to achieve their target productivity levels when  $\tau_c$  is higher. In contrast, we saw that raising  $\tau_d$  made firms more reluctant to destroy jobs and it also required more destruction in order to achieve their target productivity. In other words, the elasticity of productivity with respect to employment at creation times increases with  $\tau_c$  whereas the corresponding elasticity at destruction times falls with  $\tau_d$ .

Although it is useful to think of the influence of  $\tau_c$  and  $\tau_d$  on job flows in terms of the asymmetry in the elasticities of productivity with respect to employment between creation and destruction times, these elasticities are determined by the firms' optimal labor demand policy, and are therefore endogenous to the model. Further intuition can be gained by examining the implied costs in terms of the destruction of organizational capital. Thus, we define the *OC* destruction rate as the sum of the rates of destruction of organizational capital induced by job creation and destruction, given respectively by

OC destruction from POS = 
$$\frac{2}{Z_t + Z_{t-1}} \sum_{i \in I^+} \left( z_{i,t} - z_{i,t-1} \right),$$
 (4.9)

and

OC destruction from NEG = 
$$\frac{2}{Z_t + Z_{t-1}} \sum_{i \in I^-} |z_{i,t} - z_{i,t-1}|,$$
 (4.10)

where (recalling that time is discrete and the number of firms finite in our simulations)  $z_{i,t}$  is the stock of organizational capital at firm *i* in period *t*,  $Z_t$  is aggregate stock of organizational capital at time *t*, and  $I^+$  and  $I^-$  denote the set of firms that are expanding and contracting, respectively.

The last three rows of Table 5 show how these measures of the loss of organizational capital

vary as a function of the OC-costs of labor adjustment  $\tau_c$  and  $\tau_d$ . Looking at the last row, we see that  $\tau_c$  has no noticeable effect on the OC destruction rate, just as it did not have a noticeable effect on NET. Instead, raising  $\tau_d$  induces a three-fold increase in the OC destruction rate, from 0.06% to 0.23%. Interestingly, this result explains a negative relation between the destruction of organizational capital and the intensity of job reallocation. Specifically, our analysis so far indicates that an increase in  $\tau_d$  causes the OC destruction rate to increase, but it causes SUM to fall.

To further illustrate the distinct influence of  $\tau_c$  and  $\tau_d$ , Table 6 provides information about the elasticities of labor productivity with respect to employment and the corresponding shadow costs of job creation and destruction at the firm level, for different values of  $\tau_c$  and  $\tau_d$ .

#### [TABLE 6]

Columns 2 and 4 reveal that as  $\tau_d$  rises from 0.1 to 0.6, keeping  $\tau_c$  constant at 0.1, the elasticity of labor productivity with respect to employment at job destruction times falls substantially, from 0.97 to 0.70, which implies that substantially more jobs must be destroyed to achieve a unit change in x. It should be noted that this elasticity is optimally controlled by the firms, and therefore they could, in principle, choose to adjust their behavior so as to lower this elasticity. Since this behavior would not be optimal, it indicates that raising  $\tau_d$  does effectively increase the costs of destroying jobs. Instead, the same increase in  $\tau_d$  does not have a noticeable impact on the corresponding elasticity at creation times, which suggests that the effective cost of creating jobs hardly changes. This is confirmed in the last two rows of Table 6, which show that the shadow cost of job destruction increases from 0.4 to 2.7, while the shadow cost of job creation is hardly influenced.

To sum up, the key to our argument lies in the inherent asymmetry between job creation and destruction associated with the accompanying loss of organizational capital. In particular, not only the elasticities of productivity with respect to employment at the creation and the destruction margins are different from unity, but the elasticity at creation times increases with  $\tau_c$  whereas the elasticity at destruction times falls with  $\tau_d$ . By contrast, recall that in the benchmark model with output-costs of adjustment, the corresponding elasticities at job creation and destruction times are always equal and equal to one so it is equally easy to create and destroy jobs.

We now turn to a discussion of our last result that the presence of OC-costs of labor adjustment facilitates a channel for  $\mu$  to influence SUM. Table 7 describes the simulation results for the case in which  $\mu = 0.045$  for different values of  $\tau_c$  and  $\tau_d$ . Our previous results regarding the influence of  $\tau_c$  and  $\tau_d$  remain valid and so it is unnecessary to go into the details here. The feature that we wish to highlight at this point is that an increase in  $\mu$  causes an increase in NET (POS – NEG) and a decline in SUM (POS + NEG), thereby inducing a negative relationship between NET and SUM.

#### [TABLE 7]

The influence of the exogenous trend rate of growth in the stock of organizational capital  $\mu$ on NET is easily understood. The higher value of  $\mu$  induces more frequent job creation and less frequent job destruction by firms, since firms now have an incentive to grow faster and it is optimal to take advantage of the effect of the higher trend on the frequency margins. Consequently, the higher value of  $\mu$  tends to shift the cross-sectional distribution of firms towards the creation margin, away from the job destruction margin. Just as in the benchmark model, NET increases with  $\mu$ , reflecting the powerful effect of asymmetries in the cross-sectional distribution. More subtle is the effect of  $\mu$  on SUM. A column by column comparison of tables 5 and 7 shows that, for each pair of  $\tau_c$  and  $\tau_d$ , NEG falls by a significantly larger magnitude than POS increases, as  $\mu$  rises, resulting in a reduction in SUM.

Even though  $\tau_c$  does not significantly influence NET, as was explained above, the channel which allows  $\mu$  to influence SUM is present whenever either  $\tau_c$  or  $\tau_d$  is positive. A comparison of the third column in tables 5 and 7 illustrates the effect of the higher trend for the case where  $\tau_c = 0.6$  and  $\tau_d = 0.1$ . Here we see that the increase in POS does not offset the decline in NEG associated with the increase in  $\mu$ . This can be understood in comparison with the benchmark model, where an increase in  $\mu$  results in an increase in POS which just offsets the decline in NEG, leaving SUM unchanged. In the model with OC-costs of adjustment, a positive value of  $\tau_c$  implies that reaching a given target reduction in labor productivity at job creation times is relatively easy, as compared with the case where  $\tau_c = 0$ . Thus, firms can save on the shadow costs of job creation by increasing job creation by less than they reduce job destruction. This logic is also supported by our simulation results when  $\tau_d = 0$  and  $\tau_c > 0$  (not shown in tables).

Now consider the impact of a positive value of  $\tau_d$  when  $\mu$  is increased. Since the destruction of jobs is accompanied by the loss of organizational capital, firms choose to save on the OC-costs of adjustment by reducing job destruction significantly more than increasing job creation. We can trace this result back to the fact that the elasticity of labor productivity with respect to employment at job destruction times is less than one. A comparison of column 4 in tables 5 and 7 illustrates the effect of the higher trend for the case where  $\tau_c = 0.1$  and  $\tau_d = 0.6$ . We note the sharp increase in job destruction intensity, from 13.94% to 40.08%, which in turn contributes to a decline in NEG from 0.54% to 0.17%, whereas POS increases from 0.17% to 0.32%. As a result, SUM falls from 0.71% to 0.49%. For SUM to have remained constant at 0.71%, the firm would have had to target a much higher labor productivity when destroying jobs, but that would have required a much higher intensity of job destruction than 40.08% and a correspondingly larger loss of organizational capital. In turn, we note that the OC destruction rate from job creation remains roughly unchanged whereas the OC destruction rate associated with job destruction falls from 0.22% to 0.07%, causing overall a decline in the OC destruction rate from 0.23% to 0.09% as  $\mu$  increases.

A further implication of the previous result is that differences in  $\mu$  induce a positive relationship between the destruction of organizational capital and the intensity of job reallocation. Specifically, our analysis has indicated that raising  $\mu$  causes both the OC destruction rate and SUM to fall. In contrast, recall that raising  $\tau_d$  caused the OC destruction to increase, but it caused SUM to fall. Finally, it should be noted that the actual trend growth rate in the stock of organizational capital is jointly determined by the (exogenous) trend growth rate of firm-specific organizational capital, given by  $\mu$ , and by the (endogenous) destruction of organizational capital associated with employment changes, given by the OC destruction rate. The foregoing discussion implies that an increase in  $\mu$  does raise the actual trend rate of growth in the stock of organizational capital, both directly through the increase in  $\mu$  and indirectly through the reduction in the OC destruction rate.

#### 5 Conclusion

This paper has illustrated the potential importance of the distinction between adjustment costs that are associated with a loss of output (*output-costs of labor adjustment*) and those associated with a loss of organizational capital (*OC-costs of labor adjustment*). In particular, our analysis has illustrated how the OC-costs associated with labor adjustments may influence the behavior of job flows in the aggregate when the output-costs of labor adjustment may be unable to do so.

We have studied the aggregate implications of the OC-costs of labor adjustment in the presence of heterogeneity across firms and in the absence of aggregate shocks. This is precisely the case where one is most likely to find that the effect of microeconomic rigidities can be undone by aggregation, as discussed by Caballero (1992). We have illustrated how the destruction of organizational capital associated with job destruction can influence the behavior of net job creation, whereas the changes in the stock of organizational capital associated with the creation of jobs may have no significant impact on net job creation. We have also shown how, in the presence of the OC-costs of adjustment, asymmetries in the cross-sectional distribution of firms associated with trend growth in the stock of organizational capital can induce a negative relation between net job creation and gross job reallocation.

Our results concerning the influence of the OC-costs of labor adjustment on the behavior of aggregate job flows are better understood in comparison with our analysis of the benchmark model with output-costs of adjustment. In this respect, our analysis has extended Caballero's (1992) work to indicate that neither asymmetries in the output-costs of hiring and firing nor asymmetries in the cross-sectional distribution of firms alone may be able to explain a relation between net job creation and gross job reallocation. Of special interest is the result that productivity trends do not have a significant influence on gross job reallocation in this benchmark model. Confronting our comparative analysis of the influence of output-costs and OC-costs of labor adjustment with industry data would be an important avenue of further research as would investigating the behavior of the intensities and the frequencies of job creation and destruction across industries. This would provide useful information about the sources and the structure of the costs of labor adjustment, and may help understand the behavior of job flows across different industries. One possibility that is suggested by our analysis is that differences in the OC-costs of labor adjustment and in the accumulation of organizational capital that leads to productivity growth may underlie the observed significant differences in total factor productivity, net employment growth and gross job reallocation across industries. Investigation of this possibility is, however, beyond the scope of this paper.

#### Appendix

#### **Optimal Labor Demand Policy**

We show that the solution to the problem of the firm depends on employment and organizational capital only through their ratio. Because V(z,n) is homogeneous of degree one in z and n, we can simplify the problem further. Letting  $x(n) \equiv \frac{z}{n}$  and defining v and  $\pi$  by  $V(z,n) \equiv nv(x(n))$  and  $\Pi(z,n) \equiv n\pi(x(n))$ , we can then rewrite the Bellman equation for the firm's problem, equation (3.1), as an ordinary differential equation for v(x(n))

$$rv(x(n)) = \pi(x(n)) + \mu x(n)v'(x(n)) + \frac{1}{2}\sigma^2 x(n)^2 v''(x(n)).$$
(A.1)

Letting  $\delta_1 < 0 < 1 < \delta_2$  be the roots of the quadratic equation

$$\psi(\eta) = r - \mu\eta - \frac{1}{2}\sigma^2\eta \left(\eta - 1\right),\tag{A.2}$$

the general solution to equation (A.1) is

$$v(x(n)) = \frac{x(n)^{\alpha}}{\psi(\alpha)} - \frac{w}{r} + A(n)x(n)^{\delta_1} + B(n)x(n)^{\delta_2},$$
(A.3)

where A(n) and B(n) are constants, for fixed size n, to be determined together with the optimal labor demand strategy.

Similarly, letting  $x_d(n) \equiv z_d(n)/n$ ,  $X_d(n) \equiv Z_d(n)/N_d(n)$ ,  $X_c(n) \equiv Z_c(n)/N_c(n)$  and  $x_c(n) \equiv z_c(n)/n$ , the value matching conditions given by (3.2) and (3.3) can be rewritten as

$$\left(\frac{N_c(n)}{n}\right) v\left(X_c(n)\right) = v\left(x_c(n)\right) + c \tag{A.4}$$

and

$$\left(\frac{N_d(n)}{n}\right) v\left(X_d(n)\right) = v\left(x_d(n)\right) + c \tag{A.5}$$

and the smooth pasting conditions given by (3.4)–(3.7) can be rewritten as

$$v(x_c(n)) = (x_c(n) + \tau_c) v'(x_c(n)) - c$$
(A.6)

$$v(X_c(n)) = (X_c(n) + \tau_c) v'(X_c(n))$$
(A.7)

$$v(x_d(n)) = (x_d(n) - \tau_d) v'(x_d(n)) - c$$
(A.8)

$$v(X_d(n)) = (X_d(n) - \tau_d) v'(X_d(n)).$$
(A.9)

Inspection of the Bellman equation (A.3) and the boundary conditions given by (A.4)–(A.9), together with the fact that

$$\frac{N_c(n)}{n} = 1 + \frac{x_c(n) - X_c(n)}{X_c(n) + \tau_c}$$
(A.10)

and

$$\frac{N_d(n)}{n} = 1 - \frac{X_d(n) - x_d(n)}{X_d(n) - \tau_d},$$
(A.11)

reveals that the four boundaries  $x_c(n)$ ,  $X_c(n)$ ,  $X_d(n)$  and  $x_d(n)$  and the constants A(n) and B(n) are independent of n. Differentiating (A.3) and substituting v and v' in (A.4)–(A.9), we obtain a system of six non-linear equations which can be easily solved numerically for the four boundaries  $x_c$ ,  $X_c$ ,  $X_d$  and  $x_d$ , and the two constants A and B.

#### Long-Run Distributions

The process  $\{x_t\}$  never leaves the interval  $[x_d, x_c]$  and reaches every point in that interval with probability one, hence possesses an invariant, ergodic distribution. In order to derive it, we first calculate the ergodic distribution for the case of linear Brownian motion and then use Ito's lemma to obtain the corresponding distribution for the case of geometric Brownian motion. Then we use a change of variable in order to obtain the ergodic distribution of the proportional distance of the firm from the destruction margin.

In order to obtain the ergodic distribution for the case of linear Brownian motion one can approximate Brownian motion by a discrete random walk and calculate its unique invariant probability distribution using standard results in the theory of Markov chains. Just as the discrete random walk converges to Brownian motion as the length of a time period becomes negligible in a certain way, its invariant distribution converges to the invariant distribution of the continuous-time process. Bertola and Caballero (1990) provide details of the derivation of the ergodic density function for linear Brownian motion  $\{y_t\}$  with drift  $\Theta$  and standard deviation  $\Sigma$  regulated at a and b with return points A and B, with a < A < B < b. In the end, calculation of the ergodic density amounts to straightforward but tedious manipulation of a system of linear equations. Letting  $P_y(a, b)$  denote the probability of hitting a before b, starting at  $y \in [a, b]$ ,

$$P_y(a,b) \equiv \frac{e^{-\gamma y} - e^{-\gamma b}}{e^{-\gamma a} - e^{-\gamma b}}, \quad \text{with} \quad \gamma \equiv \frac{2\Theta}{\Sigma^2}$$
(A.12)

and defining

$$Q \equiv \frac{P_B(a,b)}{1 - P_A(a,b)},\tag{A.13}$$

we obtain

$$f_{y}(y) = \begin{cases} \frac{Q(e^{\gamma(y-a)}-1)}{(b-B)-Q(A-a)} & \text{if } y \in [a, A] \\ \frac{e^{-\gamma(B-y)}-e^{-\gamma(b-y)}}{(b-B)-Q(A-a)} & \text{if } y \in [A, B] \\ \frac{1-e^{-\gamma(b-y)}}{(b-B)-Q(A-a)} & \text{if } y \in [B, b] \end{cases}$$
(A.14)

for  $\Theta \neq 0$ , and

$$f_{y}(y) = \begin{cases} \frac{y-a}{(A-a)\left[\frac{1}{2}(b-a)+\frac{1}{2}(B-A)\right]} & \text{if } y \in [a, A] \\\\ \frac{1}{\frac{1}{2}(b-a)+\frac{1}{2}(B-A)} & \text{if } y \in [A, B] \\\\ \frac{b-y}{(b-B)\left[\frac{1}{2}(b-a)+\frac{1}{2}(B-A)\right]} & \text{if } y \in [B, b] \end{cases}$$
(A.15)

for  $\Theta = 0$ .

The corresponding cumulative distribution follows by integration.

If  $\Theta \neq 0$ , we get

$$F_{y}(y) = \begin{cases} \frac{Q[\gamma^{-1}(e^{\gamma(y-a)}-1)-(y-a)]}{(b-B)-Q(A-a)} & \text{if } y \in [a, A] \\ F_{y}(A) + \frac{\gamma^{-1}(e^{\gamma y}-e^{\gamma A})(e^{-\gamma B}-e^{-\gamma b})}{(b-B)-Q(A-a)} & \text{if } y \in (A, B] \\ F_{y}(B) + \frac{(y-B)-\gamma^{-1}(e^{-\gamma(b-y)}-e^{-\gamma(b-B)})}{(b-B)-Q(A-a)} & \text{if } y \in (B, b] \end{cases}$$
(A.16)

If  $\Theta = 0$ , we obtain

$$F_{y}(y) = \begin{cases} \frac{(y-a)^{2}}{(A-a)[(b-a)+(B-A)]} & \text{if } y \in [a, A] \\ F_{y}(A) + \frac{y-A}{\frac{1}{2}(b-a)+\frac{1}{2}(B-A)} & \text{if } y \in (A, B] \\ F_{y}(B) + \frac{(y-B)-\frac{1}{2}\left(\frac{(y-B)^{2}}{(b-B)}\right)}{\frac{1}{2}(b-a)+\frac{1}{2}(B-A)} & \text{if } y \in (B, b] \end{cases}$$
(A.17)

By Ito's lemma,  $\{y_t\} \equiv \{\log(x_t)\}\$  is a linear Brownian motion with drift  $\Theta = \mu - \sigma^2/2$  and standard deviation  $\Sigma = \sigma$ , regulated at  $\log(x_d)$  and  $\log(x_c)$ , with return points  $\log(X_d)$  and  $\log(X_c)$ , respectively. The long-run cumulative distribution of  $\{x_t\}$  is then easily derived from (A.20) and (A.21), noting that  $\Pr[x_t \leq x] = \Pr[\log(x_t) \leq \log(x)]$ .

If  $\mu \neq \sigma^2/2$ , we obtain

$$F_{x}(x) = \begin{cases} \frac{Q[\gamma^{-1}((x/x_{d})-1)^{\gamma}-\log(x/x_{d})]}{\log(x_{c}/X_{c}) - Q\log(X_{d}/x_{d})} & \text{if } x \in [x_{d}, X_{d}] \\ F_{x}(X_{d}) + \frac{\gamma^{-1}(x^{\gamma}-X_{d}^{\gamma})(X_{c}^{-\gamma}-x_{c}^{-\gamma})}{\log(x_{c}/X_{c}) - Q\log(X_{d}/x_{d})} & \text{if } x \in (X_{d}, X_{c}] \\ F_{x}(X_{c}) + \frac{\log(x/X_{c}) - \gamma^{-1}[(x/x_{c})^{\gamma} - (X_{c}/x_{c})^{\gamma}]}{\log(x_{c}/X_{c}) - Q\log(X_{d}/x_{d})} & \text{if } x \in (X_{c}, x_{c}] \end{cases}$$
(A.18)

where

$$Q = \frac{X_c^{-\gamma} - x_c^{-\gamma}}{x_d^{-\gamma} - X_d^{-\gamma}}$$
(A.19)

and

$$\gamma = \frac{2\mu}{\sigma^2} - 1. \tag{A.20}$$

If  $\mu = \sigma^2/2$ , then

$$F_{x}(x) = \begin{cases} \frac{\left[\log(x/x_{d})\right]^{2}}{\log(X_{d}/x_{d})\left[\log(x_{c}/x_{d}) + \log(X_{c}/X_{d})\right]} & \text{if } x \in [x_{d}, X_{d}] \\ F_{x}(X_{d}) + \frac{\log(x/X_{d})}{\frac{1}{2}\log(x_{c}/x_{d}) + \frac{1}{2}\log(X_{c}/X_{d})} & \text{if } x \in (X_{d}, X_{c}] \\ F_{x}(X_{c}) + \frac{\log(x/X_{c}) - \frac{1}{2}\left(\frac{\left[\log(x/X_{c})\right]^{2}}{\log(x_{c}/X_{c})}\right)}{\frac{1}{2}\log(x_{c}/x_{d}) + \frac{1}{2}\log(X_{c}/X_{d})} & \text{if } x \in (X_{c}, x_{c}] \end{cases}$$
(A.21)

Next, a change of variable gives the steady-state cumulative distribution of the proportional distance of  $x_t$  from the destruction margin,  $x_d$ . Defining

$$u \equiv \frac{(x - x_d)}{(x_c - x_d)},\tag{A.22}$$

then, for  $\mu \neq \sigma^2/2$ ,

$$F_{u}(u) = \begin{cases} \frac{Q\left[\gamma^{-1}\left(\frac{x_{d}+(x_{c}-x_{d})u}{x_{d}}-1\right)^{\gamma}-\log\left(\frac{x_{d}+(x_{c}-x_{d})u}{x_{d}}\right)\right]}{\log(x_{c}/X_{c})-Q\log(X_{d}/x_{d})} & \text{if } u \in \left[0,\frac{X_{d}-x_{d}}{x_{c}-x_{d}}\right] \\ F_{u}\left(\frac{X_{d}-x_{d}}{x_{c}-x_{d}}\right)+\frac{\gamma^{-1}\left(\left(x_{d}+(x_{c}-x_{d})u\right)^{\gamma}-X_{d}^{\gamma}\right)\left(X_{c}^{-\gamma}-x_{c}^{-\gamma}\right)}{\log(x_{c}/X_{c})-Q\log(X_{d}/x_{d})} & \text{if } u \in \left(\frac{X_{d}-x_{d}}{x_{c}-x_{d}},\frac{X_{c}-x_{d}}{x_{c}-x_{d}}\right] \\ F_{u}\left(\frac{X_{c}-x_{d}}{x_{c}-x_{d}}\right) \\ +\frac{\log\left(\frac{x_{d}+(x_{c}-x_{d})u}{X_{c}}\right)-\gamma^{-1}\left(\left(\frac{x_{d}+(x_{c}-x_{d})u}{x_{c}}\right)^{\gamma}-\left(\frac{X_{c}}{x_{c}}\right)^{\gamma}\right)}{\log(x_{c}/X_{c})-Q\log(X_{d}/x_{d})} & \text{if } u \in \left(\frac{X_{c}-x_{d}}{x_{c}-x_{d}},1\right] \end{cases}$$
(A.23)

whereas for  $\mu = \sigma^2/2$  we have

$$F_{u}(u) = \begin{cases} \frac{\left[\log((x_{d}+(x_{c}-x_{d})u)/x_{d})\right]^{2}}{\log(X_{d}/x_{d})\left[\log(x_{c}/x_{d})+\log(X_{c}/X_{d})\right]} & \text{if } u \in \left[0, \frac{X_{d}-x_{d}}{x_{c}-x_{d}}\right] \\ F_{u}\left(\frac{X_{d}-x_{d}}{x_{c}-x_{d}}\right) + \frac{\log((x_{d}+(x_{c}-x_{d})u)/X_{d})}{\frac{1}{2}\log(x_{c}/x_{d})+\frac{1}{2}\log(X_{c}/X_{d})} & \text{if } u \in \left(\frac{X_{d}-x_{d}}{x_{c}-x_{d}}, \frac{X_{c}-x_{d}}{x_{c}-x_{d}}\right] \\ F_{u}\left(\frac{X_{c}-x_{d}}{x_{c}-x_{d}}\right) & \\ + \frac{\log((x_{d}+(x_{c}-x_{d})u)/X_{c})-\frac{1}{2}\left(\frac{\left[\log((x_{d}+(x_{c}-x_{d})u)/X_{c})\right]^{2}}{\log(x_{c}/X_{d})}\right)}{\frac{1}{2}\log(x_{c}/x_{d})+\frac{1}{2}\log(X_{c}/X_{d})} & \text{if } u \in \left(\frac{X_{c}-x_{d}}{x_{c}-x_{d}}, 1\right] \end{cases}$$
(A.24)

#### References

- Arrow, Kenneth J. (1962): "The Economic Implications of Learning by Doing." Review of Economic Studies 29(3) (June), 155–173.
- Atkeson, Andrew and Patrick J. Kehoe (2001): "Measuring Organizational Capital." Federal Reserve Bank of Minneapolis, Research Department Staff Report 291 (July).
- Benkard, C. Lanier (2000): "Learning and Forgetting: The Dynamics of Aircraft Production." American Economic Review 90(4) (September), 1034–1054.
- Bentolila, Samuel and Giuseppe Bertola (1990): "Firing Costs and Labour Demand: How Bad is Eurosclerosis." *Review of Economic Studies* 57, 381–402.
- Bertola, Giuseppe and Ricardo J. Caballero (1990): "Kinked Adjustment Costs and Aggregate Dynamics." In NBER Macroeconomics Annual, Olivier J. Blanchard and Stanley Fischer (eds.).
- Caballero, Ricardo J. (1992): "A Fallacy of Composition." American Economic Review 82(5) (December), 1279–1292.
- Caballero, Ricardo J. and Eduardo M.R.A. Engel (1993): "Microeconomic Adjustment Hazards and Aggregate Dynamics." *Quarterly Journal of Economics* 108(2) (May), 313–358.
- Caballero, Ricardo J., Eduardo M.R.A. Engel and John Haltiwanger (1997): "Aggregate Employment Dynamics: Building from Microeconomic Evidence." American Economic Review 87(1) (March), 115–137.
- Campbell, Jeffrey R. and Jonas D.M. Fisher (2000): "Aggregate Employment Fluctuations with Microeconomic Asymmetries." *American Economic Review* 90(5) (December), 1323–1345.
- Cooper, Russell and Jonathan L. Willis (2001): "The Economics of Labor Adjustment: Minding the GAP." NBER Working Paper No. 8527.

- Cooper, Russell and Alok Johri (2001): "Learning by Doing and Aggregate Fluctuations." (Manuscript).
- Davis, Steven J. and John Haltiwanger (1990): "Gross Job Creation and Destruction: Microeconomic Evidence and Macroeconomic Implications." In NBER Macroeconomics Annual, Olivier J. Blanchard and Stanley Fischer (eds.).
- Davis, Steven J., John Haltiwanger and Scott Schuh (1996): Job Creation and Destruction. Cambridge, MA: MIT Press.
- Dixit, Avinash (1991): "A simplified treatment of the theory of optimal regulation of Brownian motion." Journal of Economic Dynamics and Control 15, 657–673.
- Dumas, Bernard (1991): "Super contact and related optimality conditions." Journal of Economic Dynamics and Control 15, 675–685.
- Ericson, Richard and Ariel Pakes (1995): "Markov-Perfect Industry Dynamics: A Framework for Empirical Work." *Review of Economic Studies* 61(1) (January), 53–82.
- Foote, Christopher L. (1998): "Trend Employment Growth and the Bunching of Job Creation and Destruction." *Quarterly Journal of Economics* 113(3) (August), 809–834.
- Hamermesh, Daniel S. (1993): Labor Demand. New Jersey: Princeton University Press.
- Hamermesh, Daniel S. (1995): "Labor Demand and the Source of Adjustment Costs." Economic Journal 105 (May), 620–634.
- Hamermesh, Daniel S. and Gerard A. Pfann (1996): "Adjustment Costs in Factor Demand." Journal of Economic Literature 34 (September), 1264–1292.
- Jovanovic, Boyan (1979): "Job Matching and the Theory of Job Turnover." Journal of Political Economy 87(5) (October, Part 1). 972–990.

- Jovanovic, Boyan and Peter L. Rousseau (2001): "Vintage Organizational Capital." NBER Working Paper No. 8166 (March).
- Prescott, Edward C. and Michael Visscher (1980): "Organizational Capital." Journal of Political Economy 88(3) (June), 446–461.
- Rosen, Serwin (1972): "Learning by Experience as Joint Production." Quarterly Journal of Economics 86(3) (August), 366–382.
- Taylor, Howard M. and Samuel Karlin (1998): An Introduction To Stochastic Modeling (Third Edition), MA: Academic Press.

	$\mu=0$		$\mu = \sigma^2/$	2 = 0.045
(H,F)	<b>NET</b> (%)	$\mathbf{SUM}\ (\%)$	<b>NET</b> (%)	<b>SUM</b> (%)
(0.1, 0.1)	-0.25	1.62	0.28	1.62
(1.2, 0.1)	-0.26	0.86	0.24	0.84
(0.1,  1.2)	-0.27	0.84	0.24	0.82
(1.2, 1.2)	-0.27	0.68	0.23	0.66

TABLE 1: Output-Costs of Adj. and Job Flows <sup>a,b</sup>

<sup>*a*</sup> Standard deviation of approximation error is 0.02 in all cases. <sup>*b*</sup> Parameters: c = 0.0005,  $\alpha = 0.7$ ,  $\sigma = 0.3$ , w = 1, r = 0.1.

(H,F)	(0.1,  0.1)	(1.2, 0.1)	(0.1, 1.2)	(1.2,  1.2)
creation intensity $(\%)$	11.36	8.83	9.04	8.24
destruction intensity $(\%)$	11.28	9.86	10.16	9.83
# firms expanding	1,566	797	741	584
# firms contracting	2,779	2,250	2,189	2,209
<b>POS</b> (%)	0.68	0.30	0.29	0.20
<b>NEG</b> (%)	0.94	0.56	0.55	0.47
<b>NET</b> (%)	-0.25	-0.26	-0.27	-0.27
<b>SUM</b> (%)	1.62	0.86	0.84	0.68

 TABLE 2: Influence of Output-Costs of Adjustment <sup>a,b</sup>

<sup>*a*</sup> Maximum standard deviation of approximation error for all measures of job flows is 0.02. <sup>*b*</sup> Parameters: c = 0.0005,  $\mu = 0$ ,  $\alpha = 0.7$ ,  $\sigma = 0.3$ , w = 1, r = 0.1.

(H,F)	(0.1,  0.1)	(1.2, 0.1)	(0.1, 1.2)	(1.2,1.2)
creation intensity (%)	11.74	9.38	9.63	8.88
destruction intensity (%)	10.99	9.40	9.68	9.28
# firms expanding	2,038	1,357	1,273	1,083
# firms contracting	1,986	1,281	1,217	1,009
<b>POS</b> (%)	0.95	0.54	0.53	0.44
<b>NEG</b> (%)	0.67	0.30	0.29	0.22
<b>NET</b> (%)	0.28	0.24	0.24	0.23
<b>SUM</b> (%)	1.62	0.84	0.82	0.66

 TABLE 3: Productivity Trends in Model with Output-Costs of Adjustment <sup>a,b</sup>

<sup>*a*</sup> Maximum standard deviation of approximation error for all measures of job flows is 0.02. <sup>*b*</sup> Parameters: c = 0.0005,  $\mu = \sigma^2/2 = 0.045$ ,  $\alpha = 0.7$ ,  $\sigma = 0.3$ , w = 1, r = 0.1.

	$\mu$ =	= 0	$\mu = \sigma^2/$	2 = 0.045
$( au_c, au_d)$	<b>NET</b> (%)	$\mathbf{SUM}\ (\%)$	<b>NET</b> (%)	<b>SUM</b> (%)
(0.1, 0.1)	-0.28	1.01	0.21	0.79
(0.6, 0.1)	-0.29	0.67	0.18	0.51
(0.1,  0.6)	-0.37	0.71	0.15	0.49
(0.6, 0.6)	-0.36	0.61	0.13	0.40

TABLE 4: OC-Costs of Adj. and Job Flows <sup>a,b</sup>

<sup>*a*</sup> Standard deviation of approximation error is 0.02 in all cases. <sup>*b*</sup> Parameters: c = 0.0005,  $\alpha = 0.7$ ,  $\sigma = 0.3$ , w = 1, r = 0.1.

$( au_c, au_d)$	(0.1, 0.1)	(0.6, 0.1)	(0.1,  0.6)	(0.6,  0.6)
creation intensity (%)	9.38	7.63	8.08	7.21
destruction intensity (%)	10.51	9.85	13.94	13.85
# firms expanding	906	504	360	274
# firms contracting	2,226	1,906	1,731	1,635
<b>POS</b> (%)	0.36	0.19	0.17	0.12
<b>NEG</b> (%)	0.65	0.48	0.54	0.49
<b>NET</b> (%)	-0.28	-0.29	-0.37	-0.36
<b>SUM</b> (%)	1.01	0.67	0.71	0.61
OC destruction from POS (%)	0.02	0.06	0.01	0.05
OC destruction from NEG $(\%)$	0.04	0.03	0.22	0.19
OC destruction rate (%)	0.06	0.09	0.23	0.23

 TABLE 5: Influence of OC-Costs of Adjustment <sup>a,b</sup>

 $^{a}$  Maximum standard deviation of approximation error for all measures of job flows and OC destruction is 10% of the corresponding mean values. <sup>b</sup> Parameters: c = 0.0005,  $\mu = 0$ ,  $\alpha = 0.7$ ,  $\sigma = 0.3$ , w = 1, r = 0.1.

$( au_c, au_d)$	(0.1,  0.1)	(0.6,  0.1)	(0.1,0.6)	(0.6,  0.6)
$-(x_c+ au_c)/x_c$	-1.01	-1.04	-1.01	-1.04
$-(x_d- au_d)/x_d$	-0.97	-0.96	-0.70	-0.65
$ au_c  V_z  (z_c(n), n)$	0.40	2.21	0.39	2.19
$ au_d  V_z  (z_d(n), n)$	0.42	0.41	2.74	2.72

 TABLE 6: Destruction of Organizational Capital <sup>a</sup>

 $^{a}$  Parameters:  $c=0.0005,\,\mu=0,\,\alpha=0.7,\,\sigma=0.3,\,w=1,\,r=0.1.$ 

$( au_c, au_d)$	(0.1,  0.1)	(0.6, 0.1)	(0.1,  0.6)	(0.6,  0.6)
creation intensity (%)	9.27	7.68	7.63	7.46
destruction intensity (%)	9.78	9.16	40.08	36.99
# firms expanding	1,252	925	617	562
# firms contracting	1,203	838	513	461
<b>POS</b> (%)	0.50	0.34	0.32	0.27
<b>NEG</b> (%)	0.29	0.16	0.17	0.14
<b>NET</b> (%)	0.21	0.18	0.15	0.13
<b>SUM</b> (%)	0.79	0.51	0.49	0.40
OC destruction from POS (%)	0.03	0.10	0.02	0.09
OC destruction from NEG $(\%)$	0.02	0.01	0.07	0.05
OC destruction rate (%)	0.05	0.11	0.09	0.14

 TABLE 7: Productivity Trends in Model with OC-Costs of Adjustment <sup>a,b</sup>

<sup>*a*</sup> Maximum standard deviation of approximation error for all measures of job flows and OC destruction is 10% of the corresponding mean values. <sup>*b*</sup> Parameters: c = 0.0005,  $\mu = \sigma^2/2 = 0.045$ ,  $\alpha = 0.7$ ,  $\sigma = 0.3$ , w = 1, r = 0.1.

FIGURE 1: Optimal Labor Demand Policy



















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