Labour Market Dynamics in RBC Models

Alok Johri
Department of Economics
McMaster University
1280 Main Street West
Hamilton, Ontario
Canada L8S 4M4
E-mail: johria@mcmaster.ca

Marc-André Letendre*
Department of Economics
McMaster University
1280 Main Street West
Hamilton, Ontario
Canada L8S 4M4
E-mail: letendre@mcmaster.ca

* Corresponding author.

Labour Market Dynamics in RBC Models

July 2001

Abstract

This paper explores the ability of a large set of RBC type models to explain aggregate US data by examining how well the first-order conditions (FOCs) from each model fit the data. Typically, the residuals from the FOC for hours worked are large in magnitude (more volatile than total hours), very highly persistent, and stay away from zero for long periods of time. This pattern suggests that standard RBC models are unable to capture the dynamics in the joint behaviour of consumption, output and hours that exists in the US data. We show that models which generate dynamic terms in the FOC for hours worked are able to capture this feature of the data by exploring a RBC model augmented by learning by doing which has been shown to have such a dynamic FOC. The results are remarkable. The residuals from the hours FOC are much less volatile than total hours and display no persistence. Less conclusive results emerge from models with habit formation in preferences which also yield dynamic FOCs for the labour input. We conclude that an additional dynamic component in the FOCs is essential to better capture the dynamics in the data and future research using the RBC structure should explore models that deliver it.

JEL classification: E32, E22, C52

Keywords: real business cycles, first-order conditions, labour market dynamics.

We would like to thank seminar participants at Brock University, York University, McMaster University, University of Toronto, University of Pennsylvania, UCLA, USC, the New York Fed, University of Copenhagen and conference participants at American Economic Association, Society for Economic Dynamics and Midwest Macro meetings in 2001 as well as A. Atkeson, P. Beaudry, M. Boileau, M. Browning, H. Cole, R. Cooper, T. Crossley, M. Eichenbaum, A. Lahiri, J.-F. Lamarche, A. Melino, J. Nason, R. Rogerson, W. Scarth, G. Smith and R. Wright for helpful comments on this research. The authors thank the Social Science and Humanities Research Council of Canada.

1 Introduction

RBC models are usually evaluated by comparing the moments they predict for a set of macroeconomic variables to the moments computed in the data. Gregory and Smith (1991) as well as Christiano and Eichenbaum (1992) propose methods to test the difference between historical and predicted moments. Other authors (*e.g.* Cogley and Nason (1995)) suggest looking at impulse responses and autocorrelation functions to analyze the properties of RBC models. These diagnostics, based on moment matching, impulse responses and autocorrelation functions have demonstrated many shortcomings in the standard RBC model and sparked a significant amount of research (see Hansen (1985), Rogerson (1988), Benhabib, Rogerson and Wright (1991), Christiano and Eichenbaum (1992), Burnside, Eichenbaum and Rebelo (1993), Burnside and Eichenbaum (1996), King and Rebelo (2000) among others). For instance, the moment-matching diagnostic was used to show that in the standard model, hours are not volatile enough and are too highly correlated with average labour productivity. The impulse response and autocorrelation function diagnostics were used by Cogley and Nason to argue that standard models are unable to generate dynamics in output consistent with the data. The goal of many extensions of the baseline model has been to address these shortcomings and success has been mixed so far (see Hansen and Wright (1992) and King and Rebelo (2000) for a discussion of some of these issues).

While moment matching has proved to be extremely useful as a diagnostic tool to test the ability of the model to capture basic features of the business cycle, there always remains the possibility that focusing on a limited set of moments obscures more than it reveals. Even worse, it is possible that while we appear to be making progress on bringing the model closer to the data on one or two dimensions we may actually be moving further away in many other unexplored dimensions. One possibility is to expand the set of moments used in the matching exercise but where does one stop this process? Instead, we argue that it may be useful to ask if the first-order conditions (FOCs) generated by the model, that are supposed to explain the joint behaviour of macroeconomic aggregates, are in fact consistent with the data. This is obviously not a novel suggestion in other literatures in economics but is not currently popular in evaluating RBC models. The point is that the form and nature of the inconsistency could potentially provide clues as to the direction in which the model needs to be modified to make it more consistent with the data.

In this paper, we show that looking at the consistency of the FOCs of various RBC type models is actually helpful in revealing shortcomings of these models and in suggesting which directions to fruitfully modify the model. Specifically we compare the performance of these models by looking at the properties of the residuals from the estimated or calibrated FOCs. Since the residuals capture the extent to which the data deviates from the behaviour suggested by the model we expect a well specified model to not exhibit large and persistent residuals. Consider the FOC associated with the labour input in a standard RBC model. In theory this equation, which equates the marginal rate of substitution between consumption and leisure with the marginal product of labour, is supposed to hold exactly in every period. Any residuals are a measure of economic forces left out from the model. Since the model is a simplified representation of reality, one expects to find small amounts of random movement in the residuals away from zero due to the combined influence of all excluded forces. However, if these residuals display large and systematic patterns this suggests misspecification of the model in that some important influence is missing in the model[1]. This is precisely the kind of patterns that we find for residuals associated with a large class of RBC models popular in the literature. The potential misspecification is particularly evident in the FOC for the labour input. The residuals are highly serially correlated and they are large in the sense that they are even more volatile than total hours. In comparison, the capital FOC or Euler equation fits the joint behaviour of the aggregate capital stock, output and consumption series much better. Both the volatility and persistence of the residuals are much smaller.

While many macroeconomists would not be surprised that the RBC model does not perform well on the labour market side, it is important to point out that the persistence in

[1]We discuss measurement error below.

the residuals from the hours FOC points to problems other than those highlighted by the moment matching exercises mentioned above. The FOC requires us to look for consistency with the data not in the individual series for hours, output and consumption but in their joint behaviour. The extremely high persistence in the residuals suggests that the models are missing a dynamic element in this joint behaviour. In fact we show that incorporating elements like indivisible labour into the model lowers the volatility of the residuals somewhat but essentially leaves their dynamic patterns unchanged. An alternative approach would be to assign the entire residual series from the FOC to a sequence of preference shocks with the appropriate amount of persistence. This was the route followed in Baxter and King (1991) but has not proved to be popular. In a study of sources of fluctuations, Hall (1997) concludes that the sheer size of these residual points to mis-specification of the labour side of the RBC model. In contrast to us he recommends modifying the intratemporal aspects of the model. Our view of the large and persistent residuals as evidence of specification error rather than as evidence for large shifts in preferences is also motivated by the argument that it is unsatisfactory to leave such a large fraction of the variation in the data to be accounted for by unexplained exogenous forces on which no independent evidence exists.

In addition to aggregate preference shifts and specification error a third possible explanation of the large residuals is the possibility of large and persistent measurement error. In order to see the extent to which systematic measurement error in the hours series can account for the behaviour of the residuals we also study the variable labour effort model which is known to improve the performance of the model on a number of dimensions by increasing the variability of the "true" labour input and making it more cyclical. Our focus on specification error is justified in part by the results from the variable labour effort model. We find that compared to the standard RBC model, a model with variable labour effort reduces the size of the residuals from the labour input FOC but fails to reduce their serial correlation or their basic time series pattern. To capture the joint behaviour of consumption, output and hours, it appears to be necessary to use a model for which the labour input FOC involves dynamic terms rather than just current period variables. One such model, learning by doing, is shown to not only reduce the size of the residuals from the labour input FOC but also to make them serially uncorrelated. A formal test of the overall fit of the learning by doing model is also extremely successful.

We setup and explore the overall consistency of a number of models with aggregate US data. The parameters of these models are estimated using a generalized method of moments procedure applied to moment conditions obtained from the key FOCs associated with each model. We choose to estimate the parameters using GMM because the procedure picks parameter values to minimize the average size of the residuals thus allowing us to fairly compare models on the basis of the size and volatility of the associated residuals. In addition to this we are particularly interested in the dynamic patterns of the residuals. It may be worth emphasizing that the diagnostic procedure does not rely on estimation. Once parameter values have been picked (by estimation or calibration) the only issue is regarding the consistency of the FOCs with the data. The instruments used and estimation strategies have no impact except as a guide to the numbers to be used for the parameters. Indeed, we explore the impact of varying key parameter values on the dynamics and volatility of the residuals.

Our work is related to some early studies which estimated FOCs related to RBC type models using GMM procedures and formally tested the overall fit of the model using overidentifying restrictions tests. Two notable examples are Eichenbaum, Hansen and Singleton (1988) which we discuss in more detail later and Mankiw, Rotemberg and Summers (1985). Moreover Euler equation estimation is common in the asset-pricing literature and goes back to the work of Hansen and Singleton (1982). A number of studies use generalized method of moments procedures to estimate parameters of RBC models but evaluate the models using formal or informal moment-matching exercises. (A few examples of the former are Christiano and Eichenbaum (1992), Burnside, Eichenbaum and Rebelo (1993) and Burnside and Eichenbaum (1996)).There also exist other studies which estimate dynamic stochastic general equilibrium models using procedures other than GMM. For example, Altug (1989) and McGrattan, Rogerson and Wright (1997) estimate their model using a maximum likelihood procedure based on the linearized decision rules that emerge from a quadratic approximation procedure and DeJong, Ingram and Whiteman (2000) use a Bayesian approach to estimate a model with multiple shocks. In addition there is work on evaluating RBC type models using formal econometric studies by Diebold, Ohanian and Berkowitz (1998) and Schorfheide (2000).

Section 2 contains a discussion of the models we explore including the standard (divisible labour) RBC model, an indivisible labour model, a model with variable labour effort that tries to capture the impact of systematic measurement error in the labour input and a model with learning by doing that generates additional dynamics in the FOCs. We also show that the problem may not be entirely resolved by simply making the marginal rate of substitution between consumption and leisure dynamic. This issue is explored using models with habit formation in consumption and leisure. Section 3 offers brief concluding remarks. The appendices contain sensitivity analyses with respect to detrending as well as alternative datasets.

2 Evaluating FOCs of RBC type models

In this section we present results from evaluating the FOC of several different versions of RBC type models. Initially the key parameters of the model are estimated using a GMM procedure using the FOC for hours worked, the capital Euler equation and the law of motion for the capital stock. This is backed up with sensitivity analysis in which we report the results of varying parameters on the properties of the residuals. We begin with the basic model and discuss the results with a focus on the performance of the FOC associated with labour supply. Then we look at several variants of the basic model in the literature that were developed to improve the performance of hours worked as measured by the usual moments studied in the data. Finally we look at some models that have a dynamic FOC for labour hours.

2.1 Standard RBC model

As a starting point, we estimate the parameters of a standard RBC model and look at the residuals corresponding to the FOCs for hours and capital. In our standard RBC model, the central planner maximizes

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + B \ln(1 - L_t)\right] \tag{1}$$

where C denotes consumption and L denotes hours worked, subject to the accumulation equation for capital (K)

$$K_{t+1} = (1 - \delta)K_t + I_t \tag{2}$$

and the resource constraint

$$C_t + I_t = Y_t \tag{3}$$

where I denotes investment and Y denotes output. The production function is Cobb-Douglas with constant-returns-to-scale

$$Y_t = K_t^{1-\alpha}(L_t X_t)^{\alpha} \tag{4}$$

where the level of technology (X) evolves according to the law of motion

$$X_t = X_{t-1}\exp(\tau + \upsilon_t)$$

where υ_t is an *iid* random variable with mean zero and standard deviation σ_υ and τ is the growth rate of the economy. The exact structure of the technology shocks is unimportant for our work.

The parameters are estimated using a GMM estimator. We use the GMM code written in GAUSS by Hansen, Heaton and Ogaki. The discount rate is set equal to the average real three-month US treasury-bill rate over the sample used in our empirical work (1955:1 to 1992:4). The resulting discount factor is $\beta = 1/1.00268$. [2] A detailed description of the data used in this paper is included in a data appendix.

[2]This compares to a β of 1/1.00742 used by Burnside Eichenbaum and Rebelo (1993) and Burnside and Eichenbaum (1996).

The standard RBC model implies the following FOCs for hours and capital respectively:

$$\alpha \frac{Y_t}{C_t L_t} - \frac{B}{1 - L_t} = 0 \tag{5}$$

$$\mathrm{E}_t \left\{ \beta \frac{C_t}{C_{t+1}} \left[(1-\alpha) \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right] - 1 \right\} = 0. \tag{6}$$

Let $\xi^s_{L,t}$ denote the left-hand side of equation (5), $\xi^s_{K,t+1}$ denote the expression inside braces in equation (6) and $\xi^s_{\delta,t} \equiv 1 + (I_t - K_{t+1})/K_t - \delta$. FOCs (5) and (6) together with the accumulation equation (2) yield the following moment restrictions:

$$\mathrm{E}\{\xi^s_{L,t}\} = 0 \qquad \mathrm{E}\{\xi^s_{K,t+1}\} = 0 \qquad \mathrm{E}\{\xi^s_{\delta,t}\} = 0. \tag{7}$$

We use these three moment conditions to just-identify the parameters B, α and δ. [3]

The estimates of B, δ and α are presented in the second column of Table 1 and are quite close to those estimated in the literature. We evaluate the overall ability of the model to "explain" the data by looking at graphs of the residuals from the FOCs plotted in Figure 1. The theoretical model predicts that the residuals from the labour FOC should always be zero whereas they should be zero in expected terms from the capital FOC. Figure 1 depicts those residuals.[4] The figure shows that the residuals $\hat{\chi}^s_{L,t}$ from the labour FOC deviate away from zero for long periods of time. Moreover these deviations are quite large in magnitude. Clearly, a lot of variation in the data remains unexplained by equation (5). To get a metric for the magnitude of the residuals, we compare their standard deviation to the standard deviation of hours. According to this measure, the residuals are clearly large since their standard deviation is 1.64729 times the standard deviation of hours (see Table 2). To make this comparison meaningful we rewrite the hours FOC as

$$L_t - \frac{\alpha Y_t}{B C_t}(1 - L_t) = 0.$$

[3] The first and third moment conditions may appear unusual in that the model requires equations (2) and (5) to hold exactly in each period but we have imposed a weaker requirement that they hold true only on average. This is based on the view that models are simplifications of reality and must necessarily abstract from some influences present in the data but not central to the issues addressed by the model. An additional source of deviation may be measurement error in the data. This restriction basically allows us to obtain estimates of the parameters that appear in the FOCs. An alternative approach would be to calibrate the parameters from other studies but the GMM approach used above has the advantage that the average size of the residuals are minimized.

[4] In order to have both residuals series on the same scale in Figure 1, the residuals from the labour FOC are rescaled. The Greek letter χ is used to denote rescaled residuals. We thank Angelo Melino for pointing this out.

Figure 1 also suggests that the deviations of $\hat{\xi}^s_{L,t}$ away from zero are highly autocorrelated. This is confirmed in Table 2 which presents the first-order autocorrelation coefficients for both series. The first-order autocorrelation is 0.24 in the residuals from the capital FOC and 0.99 in the residuals from the labour input FOC.[5] The high persistence in $\hat{\xi}^s_{L,t}$ is not an artifact of the apparent downward trend since adding a linear trend to the autoregression reduces the autocorrelation coefficient to 0.96 only (reported in Table 2) while allowing for a quadratic trend reduces it to 0.93.[6] Also, if we use only the subsample 1955:1 to 1969:4 in our estimation, the apparent downward trend in $\hat{\xi}^s_{L,t}$ disappears but the residuals $\hat{\xi}^s_{L,t}$ are still much larger than $\hat{\xi}^s_{K,t+1}$ and they are still highly persistent (autocorrelation coefficient of 0.89).[7]

We also consider the impact of parameter choice on the properties of the residual by varying the values of α and B. Figure 2 shows that the size and persistence of $\hat{\xi}^s_{L,t}$ as measured by the autocorrelation coefficient and the ratio $SD(\hat{\xi}^s_{L,t})/SD(L_t)$ depend little on the values chosen for α and B.

One may think that the high persistence in the residuals is due mainly to the large and systematic deviations of the residuals away from zero which in turn are caused by the presence of systematic low frequency movements in the data associated with demographic transitions or changes in labour laws affecting say the average number of hours worked in a week. This is however not the case. The results from removing the low frequency movements from the data using the Hodrick-Prescott filter are contained in Appendix I. The serial correlation in the residuals from the hours FOC is still quite high (0.87) and so is the standard deviation of the residuals relative to hours (ratio of 1.25). [8]

Having made the case for evaluating models using the fit of the key FOCs that are supposed to describe the joint behaviour of the aggregate series, it may appear surprising

[5]In a decentralized version of the model, the labour FOC (5) would be replaced by two conditions, one equating the marginal product of labour (labour demand) to the wage rate and the other equating the marginal rate of substitution (labour supply) to the wage rate. The residuals from both of these conditions are both highly persistent with AR(1) coefficients greater than 0.9.

[6]All of the first ten autocorrelation coefficients are between 0.994 and 0.97.

[7]All results are verified on an alternative and longer dataset (see appendix).

[8]We thank Richard Rogerson for encouraging us to explore detrended data.

that we have offered no formal tests of the overall fit of the model. Indeed, it is well known that the basic RBC model is strongly rejected using the usual J test statistic based on testing over-identifying restrictions. An example of this rejection can be found in Mankiw, Rotemberg and Summers (1985) for both the hours FOC as well as the Euler equation. However both of these equations are estimated using wage and interest rate data instead of the marginal product of capital and labour used in our study. As discussed in the next paragraph, we have chosen not to report the results of over-identifying restriction tests (which also reject the model) because the results are not reliable when there is a high degree of persistence in the residuals. Since $\hat{\xi}^s_{L,t}$ has an autocorrelation coefficient close to 1 we chose to use a just-identified estimator. However we offer formal tests whenever they are appropriate.

As discussed in Andrews (1991), Altonji and Segal (1996) and Christiano and den Haan (1996), estimating the covariance matrix of the empirical moments is difficult when working with a short sample of persistent data. This often leads to noticeable bias in the estimate of the covariance matrix of the empirical moments. Since this estimate plays a central role in constructing the GMM weighting matrix, we prefer to work with a just-identified estimator so that the potential bias in the covariance matrix (and the weighting matrix) do not bias our parameter estimates. The J statistic too would be severely biased in this situation so we eschew using it whenever the residuals are persistent. The standard errors attached to the parameter estimates do depend on the weighting matrix and must be interpreted with caution. For this reason, we do not discuss the significance of individual parameter estimates but report them in the tables for completeness.

We use the quadratic spectral heteroskedasticity and autocorrelation consistent (HAC) estimator with prewhitening and automatic bandwidth selection suggested by Andrews and Monahan (1992) to estimate the covariance matrix of the empirical moments. As an indication of the impact of the persistence in $\hat{\xi}^s_{L,t}$ on the HAC estimator, the automatic bandwidth selection procedure selects a bandwidth of 120 in our estimation whereas it selects a bandwidth of 0.91 when we do not use the restriction $\mathrm{E}\ \xi^s_{L,t} = 0$ (and do not estimate B).

2.2 RBC model with indivisible labour

Early moment matching exercises indicated that the behaviour of the labour market in the standard RBC model was at odds with empirical observations. In their survey article, Hansen and Wright (1992) document that the ratio of the standard deviation of hours to the standard deviation of average labour productivity (σ_L/σ_{APL}) is 1.37 in the US data (based on the household survey) and that hours and ALP are not correlated. However, their simulation of the standard model yielded $\sigma_L/\sigma_{APL} = 0.94$ and a correlation of 0.93.

To correct for the former problem, Hansen (1985) suggested a model where labour is indivisible. His model can be formulated as a version of the standard model where utility is linear in leisure. These Hansen-Rogerson (Rogerson (1988)) preferences imply that equation (1) is simply replaced by

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + B(1 - L_t)\right] \tag{8}$$

and the FOC for hours is now

$$\alpha \frac{Y_t}{C_t L_t} - B = 0 \tag{9}$$

instead of equation (5). Denoting the left-hand side of equation (9) $\xi^i_{L,t}$ and noting that $\xi^i_{K,t+1} = \xi^s_{K,t+1}$ and $\xi^i_{\delta,t} = \xi^s_{\delta,t}$ we use the GMM moment conditions

$$\mathrm{E}\{\xi^i_{L,t}\} = 0 \qquad \mathrm{E}\{\xi^i_{K,t+1}\} = 0 \qquad \mathrm{E}\{\xi^i_{\delta,t}\} = 0. \tag{10}$$

to estimate B, α and δ. Parameter estimates are presented in the third column of Table 1. The residuals from the labour FOC are plotted in Figure 3 together with their counterparts in the standard RBC model. Our measures of size and persistence are presented in the third column of Table 2. Overall, there are very slight differences between the standard model and the indivisible model. The residuals from the labour input FOC are still large (ratio of standard deviation is 1.45 vs 1.65 in the standard model) and very highly persistent (autocorrelation coefficient of 0.99).

Interestingly, while the indivisible labour model makes little progress in improving the fit of the FOC for labour to the data, it does make progress on the dimensions captured by the two moments emphasized in the literature. Hansen and Wright's (1992) simulation of the indivisible labour model yields a ratio $\sigma_L/\sigma_{APL} = 2.63$ and a correlation of 0.76 between hours and labour productivity. This improvement is seen in a reduction in the relative volatility of the residuals compared to the baseline model. The improvement is more apparent in the H-P filtered data where the relative standard deviation measure falls from 1.25 in the baseline model to .99 in the indivisible labour model. As is clear from Table 2HP and Figure 3HP the residuals are still very large and persistent. While the indivisible labour model is able to generate more volatility in hours worked, there clearly remains misspecification in the modeling of the labour market as demonstrated by the dynamics existing in the residuals from the labour input FOC. We view this as an illustration of the usefulness of FOC residuals as a complementary diagnostic procedure in our toolkit.

One possible explanation for the large and persistent residuals is the presence of systematic measurement error in some of the aggregate series, most likely in aggregate hours. One way to explore the impact of measurement error in the hours series is to model it directly and look at the properties of the residuals that emerge from this exercise. If the data has a large amount of measurement error then we should see a substantial reduction in the size of the residuals. Moreover, if this measurement error has systematic elements it could also reduce or remove the persistence in the residuals. We turn to this issue in the next section.

2.3 RBC model with variable labour effort

In reaction to criticisms regarding the fact the standard RBC model needs highly volatile productivity shocks to generate reasonable fluctuations in output, macroeconomists have investigated the properties of RBC models in which workers can vary their level of effort. Since researchers do not account for variable effort when measuring the productivity shocks (using Solow residuals), the variance of the measured shocks overestimate the variance of the actual productivity shocks. Burnside, Eichenbaum and Rebelo (1993) propose a model with variable labour effort while Burnside and Eichenbaum (1996) propose a model with both variable effort and capital utilization. From our perspective, we can view the unobserved variation in effort as measurement error and these models as models of procyclical measurement error. The question then is to what extent do the residuals purged of "measurement error" still display large and persistent deviations from the model first-order conditions. Since models with variable labour effort change the response of hours worked in a non-trivial way, it is reasonable to expect it to have an impact on the residuals from the labour input FOC. The fundamental difference between the variable labour effort model *à la* Burnside, Eichenbaum and Rebelo and the constant labour effort model is that in the former, employment cannot respond contemporaneously to shocks. Employment is chosen at the beginning of the period, then the shocks are revealed and then effort, consumption and investment are chosen. This generates a "hump-shaped" impulse response of effective hours and is the factor underlying the hump-shaped impulse response of output documented by Burnside and Eichenbaum (1996). Given the simulation results of these studies, one would expect the size of the residuals to fall and perhaps also the persistence given the additional internal propagation ability of the model. If the model were to remove a large part of the volatility of the residuals this would be surprising because it would suggest that the measurement error component of the hours series was larger than the actual series itself.

Our RBC model with variable labour effort is essentially the model of Burnside, Eichenbaum and Rebelo (1993). In this model, a central planner seeks to maximize

$$\mathrm{E}_0 \sum_{t=0}^{\infty} [\ln C_t + BN_t \ln(T - \eta - fw_t) + B(1 - N_t)\ln(T)] \tag{11}$$

where N denotes the number of workers, T is an agent's time endowment, η is the fixed cost of going to work, f is the fixed shift length and w is effort. Output is produced according to the production function

$$Y_t = K_t^{1-\alpha}(N_t f w_t X_t)^{\alpha}. \tag{12}$$

The planner's optimization is subject to the accumulation equation (2) and the resource constraint (3).

From the planner's problem we get a FOC for employment

$$\mathrm{E}_{t-1}\left\{B\ln(T-\eta-fw_t) - B\ln(T) + \alpha\frac{Y_t}{N_t C_t}\right\} = 0, \tag{13}$$

a FOC for effort

$$\alpha \frac{Y_t}{C_t w_t} - B \frac{f N_t}{T - \eta - f w_t} = 0 \tag{14}$$

and a FOC for investment in capital given by equation (6).

Since labour effort is unobservable, we use FOC (14) to substitute w_t out of FOC (13). The result is

$$\mathrm{E}_{t-1} \left\{ \alpha \frac{Y_t}{N_t C_t} - B \ln \left(\frac{T}{T - \eta - \frac{\alpha(T-\eta)}{BC_t N_t / Y_t + \alpha}} \right) \right\} = 0 \tag{15}$$

In our estimation, we follow Burnside, Eichenbaum and Rebelo (1993) and set $T = 1369$, $f = 324.8$ and $\eta = 60$. This leaves three parameters (B, α and δ) to be estimated using three equations. The moment conditions used in the estimation are

$$\mathrm{E}\{\xi^{lh}_{L,t}\} = 0 \qquad \mathrm{E}\{\xi^{lh}_{K,t+1}\} = 0 \qquad \mathrm{E}\{\xi^{lh}_{\delta,t}\} = 0 \tag{16}$$

where $\xi^{lh}_{L,t}$ denotes the expression in braces in FOC (15),[9] $\xi^{lh}_{K,t+1} = \xi^{s}_{K,t+1}$ and $\xi^{lh}_{\delta,t} = \xi^{s}_{\delta,t}$. Parameter estimates are presented in Table 1 and the residuals from the labour FOC are plotted in Figure 4. Looking across the columns of Table 1 it is clear that the parameter estimates are remarkably stable across the models. The share of labour is close to 0.7 and the depreciation rate remains close to 0.02. The estimates of B are the most variable ranging from 4.2 in the variable effort model to 6.2 in the indivisible labour constant effort model. These differences in the estimates of B are explained by the fact that preferences differ across models. These estimates are very close to those reported in other studies. For example Burnside, Eichenbaum and Rebelo (1993) reports a labour share of 0.65 and a depreciation rate of 0.02 even though those estimates were obtained using a somewhat different set of equations.

The ratio of standard deviations of residuals to hours in Table 2 (0.96) clearly shows that the residuals are smaller than the residuals from the previous two models but removing measurement error still leaves a huge amount of variation in the residuals which are about as volatile as hours. This result is easily understood by looking at equation (13). In the

[9] In estimating the variable effort model, we make use of the fact that hours worked are equal to fN_t so that equation (15) is actually estimated using hours data, as it was the case for the previous models.

indivisible labour model, the first two terms of this equation are constant since effort is constant. Therefore, large realizations of $Y_t/(N_t C_t)$ yield large residuals and vice versa. This is not the case in the variable labour effort model since effort depends positively on $Y_t/(N_t C_t)$. Therefore, the response of effort to productivity shocks helps in amplifying these shocks and reducing the residuals from the FOC. Simulation of this model by Burnside, Eichenbaum and Rebelo (1993) yields a ratio σ_L/σ_{APL} near unity and not statistically different from its counterpart in the US data. They do not report the correlation of hours and average labour productivity however.

Even though the variable labour effort model is successful at reducing the size of the residuals from the labour input FOC, it fails to reduce the serial correlation in the residuals in an important way. The first-order autocorrelation coefficient is still at 0.99.[10] This result is not surprising because variable effort does not induce any new dynamic elements into the labour FOC equation. Equation (15) is quite similar to the FOC in the fixed effort model with the only difference appearing in the term multiplying B. This term involves an expression for the unobservable variable effort with only current period variables appearing. The effort series recovered from FOC (14), is depicted in Figure 4. The series closely follows the dynamic pattern of the residuals from the labour FOC. Looking across Figures 3, 4 and 5, it is apparent that some of the residual in the indivisible labour model in Figure 3 is being relabelled as effort in Figure 4 with a corresponding reduction in the residual in Figure 4. This relabelling does not imply that the variation in effort is spurious. After all if (15) is the true FOC but we estimate (9), then the "true" effort series will be dumped into the residual. However if (15) is mis-specified then the possibility exists that any unobservable series introduced into the FOC will at least partly be a spurious proxy for the missing element. See simulation results reported in Cooper-Johri (1999). [11]

We now turn to the issue of the missing dynamics in the FOC for labour hours and discuss some models which generate additional dynamics in the labour FOC.

[10] The serial correlation for H-P filtered data is .55 as opposed to .87 in the divisible labour model. A small movement in the right direction in line with the hump shaped response of effort.

[11] This issue is being explored in more detail in another paper by Johri and Letendre.

2.4 Learning by doing and dynamic labour supply

In standard RBC models with divisible or indivisible labour, the decision facing the representative agent regarding how many hours to work boils down to a FOC which is based solely on within period variables. Basically, as is evident from (5) and (9), hours are chosen to equate the current marginal rate of substitution between consumption and leisure to the current marginal product of labour. We can think of this equality as the equilibrium condition in a decentralized labour market with the marginal rate of substitution providing the labour supply curve and the marginal product of labour the demand curve. In the absence of other dynamic considerations, the labour supply curve is static because of the time separable nature of preferences assumed in the literature (see the discussion in King, Plosser and Rebelo (1988) for example). The labour demand curve is also static because firms merely rent inputs to maximize profits at current prices. All the endogenous dynamics appears only through the saving decision of households.

The high degree of persistence in the residuals, *i.e.* that part of the data that remains unexplained by the static labour FOC, suggests that this FOC misses the rich dynamics contained in the data.[12] This leads us to explore a model with a dynamic FOC to see if it fits the data better. An example of such a model is Cooper and Johri (1999) in which the standard RBC model is modified to allow the representative agent to learn from past production and become more productive over time.[13] Unlike learning by doing (LBD) models with externalities, if the representative agent is aware that working harder today and producing more will result in higher productivity tomorrow, then the labour supply decision will involve additional dynamic terms.[14] The agent will now choose to equate the current disutility of work with the current marginal utility of the additional goods produced today

[12] Aside the various measures we use in this paper, it has been suggested that reporting the magnitude of unexplained variation using an R^2-type number may be informative. For instance if the standard RBC model is correct, regressing $\alpha Y_t/C_t$ on $BL_t/(1-L_t)$ should yield $R^2 = 1$. The actual results is $R^2 = 0.011$. For the other models we have: indivisible labour, $R^2 = 0.010$; variable labour effort, $R^2 = 0.892$; learning by doing model, $R^2 = 0.935$.

[13] Also see Chang, Gomes and Schorfeide (2001) for an alternative specification of a learning by doing model.

[14] See Romer (1986) for external learning by doing in a growth context and Cooper-Johri (1997) in a business cycle context.

as well as the future marginal utility of the additional goods produced tomorrow due to the higher productivity induced by learning by doing. They find that the model is able to generate considerable persistence in output as reflected by hump-shaped impulse responses in output and two positive autocorrelation coefficients in output growth. Other moments look very similar to the standard RBC model. Details of the model including a discussion of the labour supply response to shocks as well as simulation results and moment matching exercises are contained in the Cooper-Johri (1999) paper.

In this section of the paper we will briefly sketch the model without any justification of the modelling assumptions so that we can explore the issue of whether a dynamic labour FOC will indeed reduce the persistence and volatility in the residuals that we highlighted in the models above.

In the Cooper-Johri model, a central planner maximizes utility (8) subject to the accumulation equation for physical capital (2) and resource constraint (3). The crucial change occurs in the production technology which is now subject to learning by doing. Learning influences productivity through the stock of organizational capital , H, with the technology being given by the following production function:

$$Y_t = K_t^{1-\alpha-\varepsilon} H_t^{\varepsilon} (L_t X_t)^{\alpha}. \tag{17}$$

The stock of organizational capital itself evolves according to a log-linear accumulation equation and depends on past production as well as past organizational capital as follows:

$$H_{t+1} = H_t^{\gamma} Y_t^{1-\gamma}. \tag{18}$$

The FOCs corresponding to the planners problem are

$$\mathrm{E}_t \left\{ \frac{\alpha Y_t}{C_t L_t} - B + \beta \left(B \left(\gamma + \varepsilon(1-\gamma) \right) \frac{L_{t+1}}{L_t} - \alpha\gamma \frac{Y_{t+1}}{C_{t+1} L_t} \right) \right\} = 0 \tag{19}$$

and

$$\mathrm{E}_t \left\{ \beta \frac{C_t}{C_{t+1}} \left[(1-\alpha-\varepsilon) \frac{B}{\alpha} \frac{L_{t+1} C_{t+1}}{K_{t+1}} + 1 - \delta \right] - 1 \right\} = 0. \tag{20}$$

We immediately see that the LBD model generates two dynamic FOCs that are different from the equations we have seen so far. Equation (20) is a somewhat different version of the Euler equation that appeared in the previous models such as equation (6). Equation (19) is the dynamic labour FOC which is very different from equation (9). Note that the first two terms in (19) are actually the two terms appearing in (9). This is the same current period comparison of the disutility of work with the utility of consumption. The discounted third term incorporates the new dynamics introduced into the labour supply decision. The additional organizational capital created by working harder today changes the decision between consuming goods and leisure tomorrow and links it to the current decision. On the one hand, the additional organizational capital implies more can be produced without working harder. On the other hand, the additional organizational capital implies the marginal product of labour is higher so leisure is more expensive.

Denoting the expressions in braces in equations (19) and (20) by $\xi^{lbd}_{L,t+1}$ and $\xi^{lbd}_{K,t+1}$ respectively, and noting that $\xi^{lbd}_{\delta,t} = \xi^{s}_{\delta,t}$ we can write the moment restrictions

$$\mathrm{E}\ \{\xi^{lbd}_{L,t+1} \cdot Q_t\} = 0 \qquad \mathrm{E}\ \{\xi^{lbd}_{K,t+1} \cdot Q_t\} = 0 \qquad \mathrm{E}\{\xi^{s}_{\delta,t} \cdot Q_t\} = 0 \tag{21}$$

where Q_t is an instrument set.

Since the parameters B and α can almost always be written in a ratio (to see this, divide all terms in (19) by α) it is difficult to identify them separately. Our short datasets do not contain enough information to allow us to identify both B and α. For this reason, we set one of the parameter and estimate the other. Given the strong evidence on total labour input share of around two-thirds, we chose to set $\alpha = 0.55$ and estimate B. We picked a lower value of α than usual because it represents the returns to raw labour excluding the effect of organizational capital.[15]

Since the LBD model has more than three parameters to be estimated, the instrument set must include more than a constant. To select the instruments, and therefore the moments

[15] The implied share of labour augmented by organizational capital at the point estimates obtained by us is slightly above two-thirds. In addition, picking B equal to the value estimated in the indivisible labour model and estimating α delivers a point estimate of 0.55.

employed in the GMM optimization, we use Andrews (1999) moment selection procedure. Since we are using a relatively small sample (152 data points), we restrict our attention to small instrument sets. Monte Carlo work by Hansen, Heaton and Yaron (1996), Kocherlakota (1990) and Smith (1999) suggests using small instruments sets in small samples.

In applying Andrews procedure, we look at instrument sets including two, three or four variables only. The variables we included in our testing are: a constant, consumption growth, output growth and output to consumption ratio.[16] We use the GMM-AIC, GMM-BIC, GMM-HQIC criteria as well as the upward and downward procedures suggested by Andrews.[17] All these methods selected the instrument set

$$Q_t = \left\{1, \frac{Y_t}{C_t}\right\}. \tag{22}$$

The GMM estimates obtained using this instrument set are presented in Table 1 and the residuals from the FOCs are plotted in Figure 6. Most parameter estimates are close to those estimated earlier. $B = 6.11$, similar to the indivisible labour case and the depreciation rate remains close to two percent per quarter. The point estimate of the share of organizational capital in the production function, $\epsilon = .24$. Interestingly this implies a learning rate of 18 percent which is very close to the benchmark rate of 20% reported in a large number of industry studies.[18] The point estimate of γ, the parameter from the accumulation equation for organizational capital is 0.95 which is somewhat high relative to earlier estimates of the model. Note that Table 1 also presents the value of the over-identifying restrictions test-statistics and its associated p-value. The test does not reject the model and the instruments at conventional significance levels. The use of an over-identified estimator is not subject to the qualification made earlier because the residual series from the LBD model, have little persistence. As a result the computation of the GMM weighting matrix is less of a problem.

The two series depicted in Figure 6 are strikingly different. The figure suggests that the residuals from the labour input FOC are much smaller in the LBD model (compared

[16] Using lagged consumption growth, lagged output growth and lagged output to consumption ratio has a trivial effect on parameter estimates but does not change the results in any way.

[17] We follow the recommendations made in Andrews (1999) and perform the tests using an optimal weighting matrix and centering the contributions to the empirical moments when constructing the weighting matrix.

[18] See Cooper-Johri for an extensive discussion of the empirical lbd literature.

to the RBC model) and are much less serially correlated. These two features are confirmed by our measures of size and persistence presented in Table 2. First, the standard deviation in the residuals from the labour input FOC in the LBD model is only 19 percent of the standard deviation of hours in the US economy. Second, the coefficient of autocorrelation is not statistically significant from zero. The results of a sensitvity analysis for the effect of the parameter values on the size and persistence of the residuals for the the labour input FOC are presented in Figure 7. We see that B, and α have a small impact while ε has negligible effects on size and persistence. The value of γ appears to be an important determinant of the serial correlation in the residuals. However, the size is less sensitive to the value of γ.

The results are similar, but less dramatic, when the data are H-P filtered (see Appendix I) but still both the relative standard deviation and autocorrelation numbers are less than half those in the baseline case.

The results from the LBD model suggest that we need to incorporate elements that generate dynamic labour FOC into RBC models in order to explain the strong dynamics displayed by the data. While learning by doing is one such mechanism, another source of introducing dynamics comes from abandoning time separability of preferences. We explore this issue in the next section.

2.5 Habit Formation in an RBC model

We begin this section by describing and estimating a model with non separabilities only in consumption. Subsequently we allow for non separabilities in leisure as advocated by Kydland and Prescott (1982). In both cases we restrict attention to just one lag of consumption and leisure respectively. While we restrict ourselves to discussing the case where past consumption and leisure raise current marginal utility (habit persistence), we do not restrict the estimation procedure in this way.

Our habit formation specification is a special case of Constantinides (1990). We assume that consumption in period $t-1$ affects (positively) the marginal utility of consumption in period t. More specifically, in our habit formation model, a central planner maximizes

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t - \lambda C_{t-1}) + B[\ln(1 - L_t)\right] \tag{23}$$

subject to the usual constraints (2), (3) and (4). With these preferences, higher consumption in period t increases the marginal utility of consumption in period $t+1$ which yields to high consumption in period $t+1$. Hence the name habit formation. There are two dynamic FOCs associated with the habit formation model. The condition for hours is

$$\mathrm{E}_t \left\{ \alpha \frac{Y_t}{L_t} \left[\frac{1}{C_t - \lambda C_{t-1}} - \frac{\beta\lambda}{C_{t+1} - \lambda C_t} \right] - \frac{B}{1 - L_t} \right\} = 0 \tag{24}$$

and the condition for capital is

$$\begin{aligned} &\mathrm{E}_t \left\{ \frac{1}{C_t - \lambda C_{t-1}} - \frac{\beta\lambda}{C_{t+1} - \lambda C_t} \right. \\ &\left. -\beta \left((1-\alpha) \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right) \left(\frac{1}{C_{t+1} - \lambda C_t} - \frac{\beta\lambda}{C_{t+2} - \lambda C_{t+1}} \right) \right\} = 0 \end{aligned} \tag{25}$$

Denoting the expressions in braces in equations (24) and (25) by $\xi^{hf}_{L,t+1}$ and $\xi^{hf}_{K,t+1}$ respectively, and noting that $\xi^{hf}_{\delta,t} = \xi^{s}_{\delta,t}$ we can write the moment restrictions

$$\mathrm{E}\ \{\xi^{hf}_{L,t+1} \cdot Q_t\} = 0 \qquad \mathrm{E}\ \{\xi^{hf}_{K,t+1} \cdot Q_t\} = 0 \qquad \mathrm{E}\{\xi^{hf}_{\delta,t} \cdot Q_t\} = 0 \tag{26}$$

where Q_t is an instrument set.

For consistency with the estimation of the learning by doing model, we use the instrument set in equation (22) when estimating the habit formation model. The estimates of the parameters were similar to earlier models with the exception of α which was somewhat lower than before. The point estimates were $\alpha = 0.51$, $\delta = 0.02$, $\lambda = 0.97$ and $B = 3.58$. With such a large estimate of λ, the persistence in the residuals is now significantly smaller than in the standard RBC model *but the size of the residuals is much larger.*[19] The size of the residuals

[19] We also look at a model where utility depends on current and lagged consumption in ratio form (rather than in differences) in the spirit of Abel (1990). Looking at a range of parameter values, we found that the ratio of standard deviations is always greater than 0.9 and persistence in the residuals is always greater than 0.6. This is also true when dealing with HP filtered data and our alternative dataset.

increases dramatically with their standard deviation being 211 times the standard deviation of hours while the autocorrelation coefficient falls to -0.39. The large value of our estimate of λ explains these results. To see this, compare Euler equations (6) and (25). Essentially, habit formation in consumption replaces $\frac{1}{C_t}$ in (6) by

$$\frac{1}{C_t - \lambda C_{t-1}} - \frac{1}{C_{t+1} - \lambda C_t}.$$

With an estimate of λ close to unity, terms like $C_t - \lambda C_{t-1}$ are very close to first-differences in consumption. Since consumption is thought to be integrated of order one at most, taking first differences removes a significant amount of persistence in the residuals. Also, dividing by (approximately) the first-difference rather than the level of consumption generates much larger residuals since it is much smaller in magnitude than the level of consumption.

In the habit formation model, the estimates and the degree of persistence left in the residuals turn out to depend on the instrument set used. When using

$$Q_t = \left\{1, \frac{Y_t}{Y_{t-1}}\right\} \quad \text{or} \quad Q_t = \left\{1, \frac{C_t}{C_{t-1}}\right\},$$

the estimates of B, α and δ are very close to the estimates in the standard RBC model and the estimates of λ are 0.30 and 0.46 respectively. These estimates of λ are not sufficiently close to unity to remove the persistence in the residuals from the labour FOC.[20] Given the sensitivity of the results to the choice of instruments we decided to use an alternative estimation strategy which involved adding a fourth moment condition which imposes that the first autocorrelation coefficient of the residuals from the hours FOC equal zero. It is possible to use this condition since the model has dynamic terms in the FOC for the labour input.[21] The point estimates for this exercise are reported in Table 1 and the residuals are depicted in Figure 8. Compared to the results with the instruments, the labour share is close to the value estimated in earlier models (0.73) and the estimate of the habit persistence parameter $\lambda = 0.77$ which generates much more reasonable results. As reported in the last column of Table 2, the autocorrelation coefficient is now basically zero while the standard

[20] The corresponding numbers for $\rho_1(\xi^{hf}_{L,t+1})$ are larger than 0.94 while those for $\rho_{1,tr}(\xi^{hf}_{L,t+1})$ are larger than 0.83.

[21] We thank Martin Browning for suggesting this additional restriction.

deviation relative to hours is now a more respectable number (3.01). Unlike the LBD model there appears to be a tradeoff between the size and the persistence of the residuals as the habit formation parameter λ is varied. These properties of the residuals are highly sensitive to varying λ between 0.4 and 0.9 as is evident in Figure 9.[22]

While we have focussed so far on non-separabilities in consumption, Kydland and Prescott (1982) proposed a utility function with non-separabilities in leisure based on the idea that fatigue would raise the marginal utility of leisure after several periods of hard work. Eichenbaum, Hansen and Singleton (1988) estimate a model with both non-separabilities using the following utility function

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[[C_t + aC_{t-1}]^{\zeta} \left[(1-L_t) + b(1-L_{t-1}) \right]^{1-\zeta} \right]^{\theta} - 1}{\theta} \tag{27}$$

They estimate FOCs for labour and capital using wage and interest data for the U.S. economy and find the following values (standard errors in parentheses): $\hat{\theta} = -0.0009$ (0.0352), $\hat{a} = 0.4405$ (0.0778), $\hat{b} = -0.8321$ (0.0216) and $\hat{\zeta} = 0.1832$ (0.0006) using an overidentified GMM estimator. While it is possible to compare their results with ours, it is worth noting some important differences between the two studies. While we estimate our model using the marginal products of capital and labour, they use data on interest rates and wages. They simultaneously allow for non separabilities in consumption and leisure and use a large instrument set including the current and first lag of the growth rates of consumption, leisure, wages and interest rates as well as a constant. As a result they impose fourteen moment conditions. They test the curvature of the utility function and cannot reject the log specification we use here (the estimate of θ is not significantly different from zero).

In order to explore the implications of adding nonseparabilities in leisure to our habit formation model, we use Eichenbaum, Hansen and Singleton's estimates. Note however that their estimate of b is negative suggesting habit formation in leisure as opposed to the fatigue effect explored by Kydland and Prescott and Hansen and Wright (1992). As before, the key issue is the size and persistence of the residuals however these are generated by calibrating

[22] This sensitivity to the choice of instruments does not arise in the estimation of the LBD model. Whenever the GMM algorithm converges to estimates that are economically meaningful, those estimates are similar to the ones presented in Table 1 and the residuals from the labour FOC always have very low persistence.

as opposed to estimating the parameters associated with the utility function given in (27).[23] We find that the residuals are extremely large but that the level of persistence in the labour FOC is much smaller than in the baseline case (0.23 vs 0.99). but still larger than in the LBD model.Finally, while Eichenbaum, Hansen and Singleton report J-tests with very small p-values (all less that 0.3 percent), it is unclear how reliable these results are if persistence is a problem.

Given the results of this exercise, we also explore a model in which habit formation occurs only in leisure. Lifetime expected utility is now

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t + B \ln[(1 - L_t) + \lambda(1 - L_{t-1})] \right\} \tag{28}$$

and the associated first-order conditions are given by Euler equation (6) and

$$\frac{\alpha Y_t}{C_t L_t} = \frac{B}{(1 - N_t) + \lambda(1 - N_{t-1})} + \beta \mathrm{E}_t \left\{ \frac{\lambda B}{(1 - N_{t+1}) + \lambda(1 - N_t)} \right\}. \tag{29}$$

The results from estimating this model with the four moment conditions used in the consumption habit formation model are in the last column of Table 1. Figure 10 shows the residuals from the hours FOC (29) (compared to the corresponding residuals in the RBC model) while Figure 11 shows the effect of varying parameter values. As Figure 11 makes clear, the persistence in the residuals is a complicated function of the parameter λ. This model generates good results in that the autocorrelation in the hours FOC residuals is essentially zero. However, the standard deviation relative to hours is 3.4.

The results from this section confirm that adding dynamics in the hours FOC can potentially help to reduce the persistence in the residuals and improve the "fit" of the FOC to the data. The habit formation model is able to achieve this as long as the habit formation effects of past consumption or leisure are highly persistent but typically there is a tradeoff between reducing persistence and greater volatility of the residuals.

As a last robustness check, we verify whether the main findings of the paper hold when we use a different dataset. The "alternative" dataset is described in the data appendix and the

[23]These estimates are in column 4 of Table II in their paper, adjusted to a quarterly frequency and setting $\alpha = 0.72994$ and $\delta = 0.01952$.

sensitivity analysis graphs are included in Appendix II. Figure II.1 (standard RBC model) confirms that the size and degree of persistence in the residuals from the labour input FOC of this model are large and do not depend on the values of α and B in any important way. Figure II.2 (learning by doing model) confirms that, whatever the parameter values chosen (within a sensible range, of course) the size and degree of persistence in the residuals are always smaller in the learning by doing model compared to the standard RBC model. This finding carries over to the case where we HP filtered the data (see Figures II.3 and II.4).

3 Concluding remarks

Our work demonstrates that the standard RBC model and many of the extensions that seek to improve its explanatory power suffer from a mis-specification of the labour market. Specifically we show that the first-order conditions for hours worked needs to have an additional dynamic element which is absent in most RBC models. The lack of dynamics is demonstrated by looking at the properties of the "residuals" from the labour input first-order conditions from the standard RBC model, the indivisible labour model, as well as the variable labour effort model which are all very highly persistent and extremely volatile.

We show that models with dynamic FOCs for the labour input have the potential to solve this problem by investigating the properties of the residuals from the first-order conditions of a learning by doing model. We find that the residuals from the labour input first-order condition in this model are not persistent. Moreover the residuals are much smaller in magnitude in comparison to the standard model. As a result, the fit of the learning by doing model to the aggregate US data is much better. A formal test using over-identifying restrictions does not reject this model. We also look at models with habit persistence in consumption and leisure which also generate two dynamic first-order conditions. We show that these models are able to reduce the degree of persistence in the residuals from the labour first-order condition but this is at the expense of a substantial increase in the size of the residuals. Also, the results from this exercise were quite sensitive to small variations in parameter values.

We argue that looking at the graphs and dynamic patterns of the residuals from the key first-order conditions of the model can be a useful tool for its evaluation, complementary to moment matching and impulse response graphs which are currently popular techniques. The advantage of these simple techniques are that they do not rely on specific assumptions about the properties of shocks or specific identifying restrictions required to carry out impulse response comparisons. They focus attention on the joint behaviour of macro aggregates as opposed to individual behaviour and they do not rely on simulation of models linearized around balanced growth paths. While in this paper we have emphasized estimation procedures, and graphs of residuals from estimated relationships, the procedure can be used profitably for calibrated models as well as is illustrated by sensitivity analysis through the paper.

Data appendix

Except for the wage series, we use the same data set as Burnside and Eichenbaum (1996). See their paper for more details. We thank Craig Burnside who provided the data. The data set referred to as the "alternative dataset" is described below.

Real Wages

Wages and Salaries from the Bureau of Economic Analysis (mnemonic wascur) divided by lhours and the GDP deflator.

Capital

Sum of the net stocks of consumer durables, producer structures and equipment, and government and private residential capital plus government nonresidential capital.

Private consumption

Sum of private-sector expenditures on nondurable goods plus services plus the imputed service flow from the stock of consumer durables.

Output

Measured as $C_t + G_t + I_t$ plus net exports and time-t inventory investment.

Hours worked

Seasonally adjusted household hours series obtained from Citibase (mnemonic LHOURS).

Gross investment

Purchases of consumer durables, gross private nonresidential investment (structures and equipment) and residential investment, as well as the change in the gross stock of government capital.

Population

Data are converted to *per capital* terms using the civilian noninstitutional population aged 16 and over.

Alternative Dataset

Capital

Net stocks of nonresidential (producer structures and equipment) and residential capital. From NIPA table 15 (Bureau of Economic Analysis).

Private consumption

Sum of private-sector expenditures on nondurable goods plus services. Data are from NIPA tables (Bureau of Economic Analysis).

Output

Gross domestic product. Data are from NIPA tables (Bureau of Economic Analysis).

Hours worked

Before 1964: U.S manhours of nonfarm employees, seasonally adjusted (BLS, NBER Macrohistory data). From 1964, the total hours series is constructed using average weekly hours of production workers (seasonally adjusted) and employees on nonfarm payrolls (seasonally adjusted)

Gross investment

Purchases of consumer durables, gross private nonresidential investment (structures and equipment) and residential investment. Data are from NIPA tables (Bureau of Economic Analysis).

Population

Data are converted to *per capital* terms using the civilian noninstitutional population aged 16 and over (Citibase).

References

Abel, A.B. (1990) Asset prices under habit formation and catching up with the Joneses. *American Economic Review* 80(2),38-42.

Altonji, J.G. and Segal, L.M. (1996) Small-sample bias in GMM estimation of covariance structures. *Journal of Business and Economic Statistics* 14, 353-366.

Altug, S. (1989) Time-to-build and aggregate fluctuations: Some new evidence. *International Economic Review* 30, 889-920.

Andrews, D.W.K. (1991) Heteroskedasticity and autocorrelation consistent covariance matrix estimation. *Econometrica* 59, 817-858.

Andrews, D.W.K. (1999) Consistent moment selection procedures for generalized method of moments estimation. *Econometrica* 67, 543-564.

Andrews, D.W.K. and Monahan, J.C. (1992) An improved heteroskedasticity and autocorrelation consistent covariance matrix estimator. *Econometrica* 60, 953-966.

Baxter, M. and King, R.G. (1991) Productive externalities and business cycles. *Institute for Empirical Macroeconomics, Federal Reserve Bank of Minneapolis*, Discussion Paper 53.

Benhabib, J., Rogerson, R. and Wright, R. (1991) Homework in macroeconomics: Household production and aggregate fluctuations. *Journal of Political Economy* 99, 1166-1187.

Burnside, C. and Eichenbaum, M. (1996) Factor-hoarding and the propagation of business-cycle shocks. *American Economic Review* 86, 1154-1174.

Burnside, C., Eichenbaum, M., and Rebelo, S. (1993) Labor hoarding and the business cycle. *Journal of Political Economy* 101, 245-273.

Chang, Y., Gomes, J. and Schorfheide, F. (2001) Learning-by-doing as propagation mechanism. Manuscript, University of Pennsylvania.

Christiano, L.J. and den Haan, W. J. (1996) Small-sample properties of GMM for business-cycle analysis. *Journal of Business and Economic Statistics* 14, 309-327.

Christiano, L.J. and Eichenbaum, M. (1992) Current real business-cycle theories and aggregate labor-market fluctuations. *American Economic Review.* 82, 430-450.

Cogley, T and Nason, J. (1995) Output dynamics in real-business-cycle models. *American Economic Review* 85, 492-511.

Constantinides, G.M., (1990) Habit formation: A resolution of the equity premium puzzle. *Journal of Political Economy* 98, 519-543.

Cooper, R. and Johri, A. (1999) Learning by doing and aggregate fluctuations. Manuscript, Boston University and McMaster University.

Cooper, R. and Johri, A. (1997) Dynamic complementarities: A quantitative analysis. *Journal of Monetary Economics* 40, 97-119.

DeJong, D.N., Ingram, B.F. and Whiteman, C.H. (2000) Keynesian impulses versus Solow residuals: identifying sources of business cycle fluctuations. *Journal of Applied Econometrics* 15(3), 311-330.

Diebold, F.X., Ohanian, L.E. and Berkowitz, J. (1998) Dynamic Equilibrium Economies: A Framework for Comparing Models and Data. *Review of Economic Studies* 65(3), 433-451.

Eichenbaum, M.S., Hansen, L.P. and Singleton, K.J. (1988) Time series analysis of representative agent models of consumption and leisure choice under uncertainty. *Quarterly Journal of Economics* 103, 51-78.

Gregory, A.W. and Smith, G.W. (1991) Calibration as testing: inference in simulated macroeconomic models. *Journal of Business and Economic Statistics*, 9, 297-303.

Hall, R.E. (1997) Macroeconomic fluctuations and the allocation of time. *Journal of Labor Economics,* 15, 223-250.

Hansen, G.D. (1985) Indivisible labor and the business cycle. *Journal of Monetary Economics* 16, 309-327.

Hansen, L.P., Heaton, J. and Yaron, A. (1996) Finite-sample properties of some alternative GMM estimators. *Journal of Business and Economic Statistics* 14, 262-280.

Hansen, L.P. and Singleton K.J. (1982) Generalized instrumental variables estimation of nonlinear rational expectations models. *Econometrica* 50, 1269-1286.

Hansen, G.D. and R. Wright (1992) The labor market in real business cycle theory. *Federal Reserve Bank of Minneapolis Quarterly Review* 16, 2-12.

King, R.G., Plosser, C.I. and Rebelo, S.T. (1988) Production, growth and business cycles: I. the basic neoclassical model. *Journal of Monetary Economics* 21, 195-232.

King, R.G., and Rebelo, S.T. (1999) Resuscitating real business cycles, in *Handbook of Macroeconomics*, edited by John B. Taylor and Michael Woodford, Amsterdam ; New York : North-Holland : Elsevier.

Kocherlakota, N.R. (1990) On tests of representative consumer asset pricing models. *Journal of Monetary Economics* 26, 285-304.

Kydland, F.E. and Prescott, E.C. (1982) Time to build and aggregate fluctuations. *Econometrica* 50, 1345-1370.

Mankiw, N.G., Rotemberg, J.J. and Summers, L.J. (1985) Intertemporal substitution in macroeconomics. *Quarterly Journal of Economics* 10, 225-251.

McGrattan, E.R., Rogerson R. and Wright, R. (1997) An equilibrium model of the business cycle with household production and fiscal policy. *International Economic Review* 38, 267-

290.

Rogerson, R. (1988) Indivisible labor, lotteries and equilibrium. *Journal of Monetary Economics* 21, 3-16.

Schorfheide, F. (2000) Loss function based evaluation of DSGE models. Forthcoming in the *Journal of Applied Econometrics*.

Romer, P. (1986) Increasing returns and long-run growth. *Journal of Political Economy* 94, 1002-1037.

Smith, D.C. (1999) Finite sample properties of tests of the Epstein-Zin asset pricing model. *Journal of Econometrics* 93, 113-148.

Table 1: Parameter Estimates

	RBC Standard	RBC Indivisible	RBC-Labour Hoarding	Learning By Doing	Habit Formation Consum.	Habit Formation Leisure
B	4.90801 (0.10639)	6.18325 (0.16057)	4.18368 (0.06460)	6.11239 (0.08887)	4.98210 (0.73906)	4.99354 (0.05422)
δ	0.01952 (0.00063)	0.01952 (0.00069)	0.01952 (0.00023)	0.02006 (0.00013)	0.01949 (0.00022)	0.01952 (0.00014)
α	0.72994 (0.00703)	0.72994 (0.00785)	0.72994 (0.00500)	0.55 -	0.73366 (0.11292)	(0.72979) (0.00518)
ε				0.23925 (0.00371)		
γ				0.94952 (0.01785)		
λ					0.77410 (0.01499)	-1.16388 (0.02294)
J-test (p-value)				0.86836 (0.64779)		

Note to Table 1: Standard errors are in parentheses.

Table 2: Size of Residuals and Persistence

	RBC Standard	RBC Indivisible	RBC-Labour Hoarding	Learning By Doing	Habit Formation Consum.	Habit Formation Leisure
Size	1.64729	1.44675	0.96492	0.19052	3.01075	3.39845
$\rho_1(\hat{\xi}_K)$	0.23592 (0.08048)	0.23592 (0.08048)	0.23592 (0.08048)	0.31981 (0.07693)	-0.59962 (0.06540)	0.23589 (0.08048)
$\rho_1(\hat{\xi}_L)$	0.99302 (0.01236)	0.99112 (0.01243)	0.99030 (0.01274)	0.04979 (0.08138)	0.00000 (0.08281)	0.00000 (0.08083)
$\rho_{1,tr}(\hat{\xi}_L)$	0.96357 (0.02160)	0.95409 (0.02452)	0.95147 (0.02525)	0.01909 (0.08204)	-0.22925 (0.08080)	-0.19327 (0.08038)

Notes to Table 2:

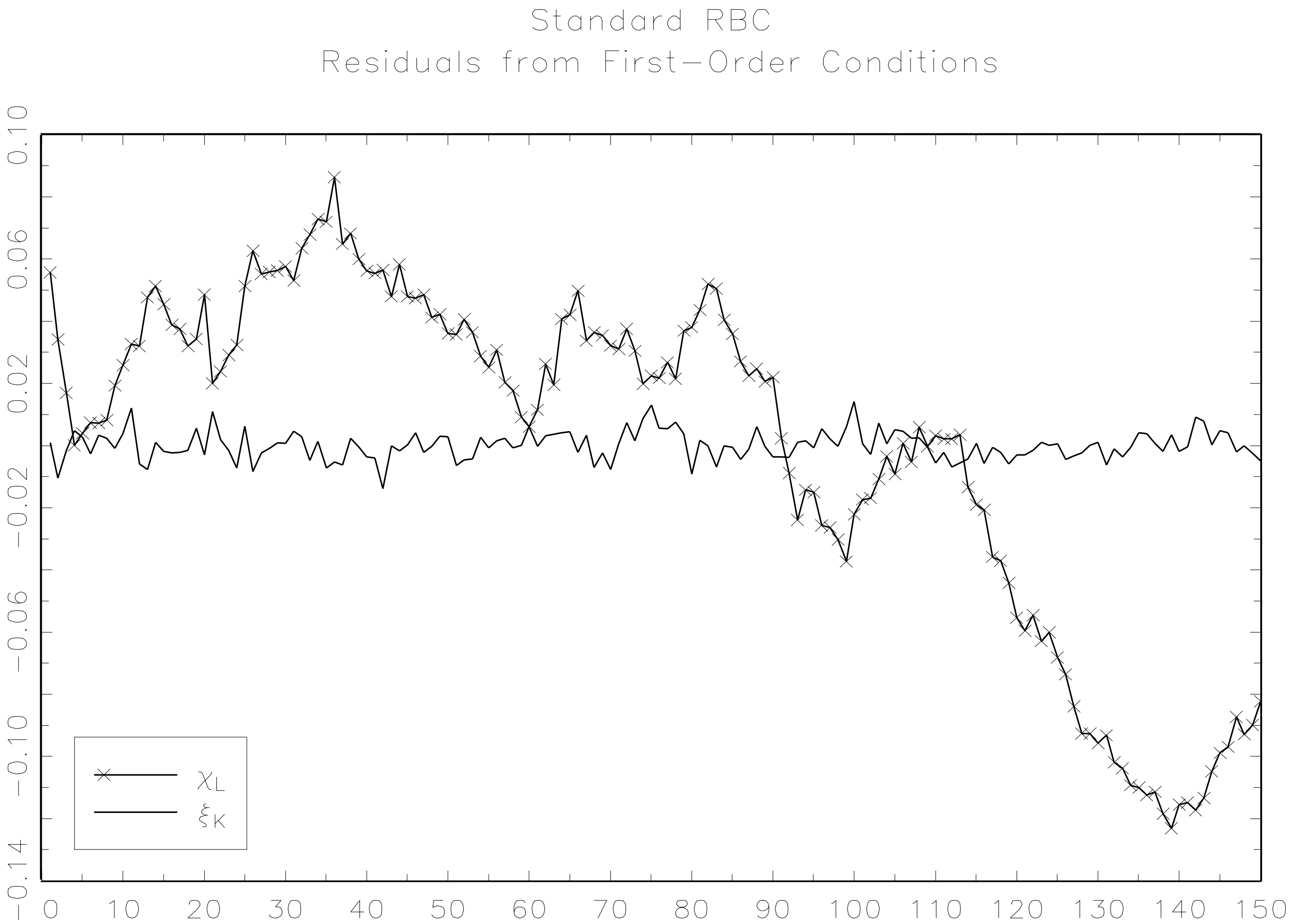
Size=SD(residuals)/SD(total hours).

Standard errors are in parentheses.

ρ_1: first-order autocorrelation coefficient.

$\rho_{1,tr}$: first-order autocorrelation coefficient, allowing for a linear trend in the regression.

Figure 1



Notes to Figure 1:

To facilitate comparisons between the two residuals, we have rewritten the residual corresponding to the hours first-order condition so that both residuals series are on the same scale. The expressions that correspond to the series depicted above are

$$\hat{\xi}^s_{K,t+1} = \beta \frac{C_t}{C_{t+1}} \left[(1-\hat{\alpha}) \frac{Y_{t+1}}{K_{t+1}} + 1 - \hat{\delta} \right] - 1$$

$$\hat{\chi}^s_{L,t} = \frac{\hat{\alpha}}{\hat{B}} \frac{Y_t}{C_t} \frac{1-L_t}{L_t} - 1.$$

Figure 2

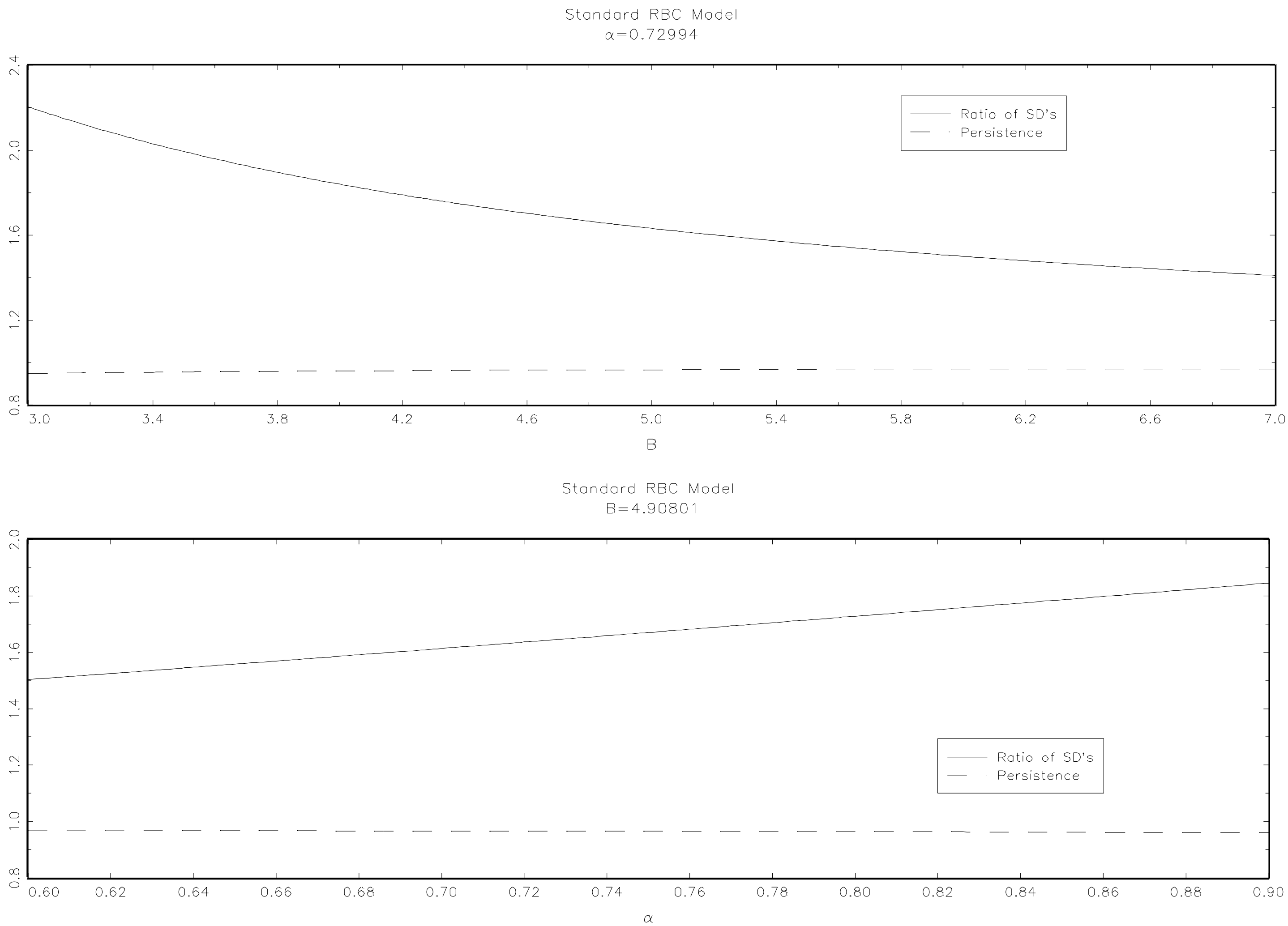

Standard RBC Model
α=0.72994
Ratio of SD's
Persistence
2.4
2.0
1.6
1.2
0.8
3.0
3.4
3.8
4.2
4.6
5.0
5.4
5.8
6.2
6.6
7.0
B
Standard RBC Model
B=4.90801
Ratio of SD's
Persistence
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.60
0.62
0.64
0.66
0.68
0.70
0.72
0.74
0.76
0.78
0.80
0.82
0.84
0.86
0.88
0.90
α

Figure 3

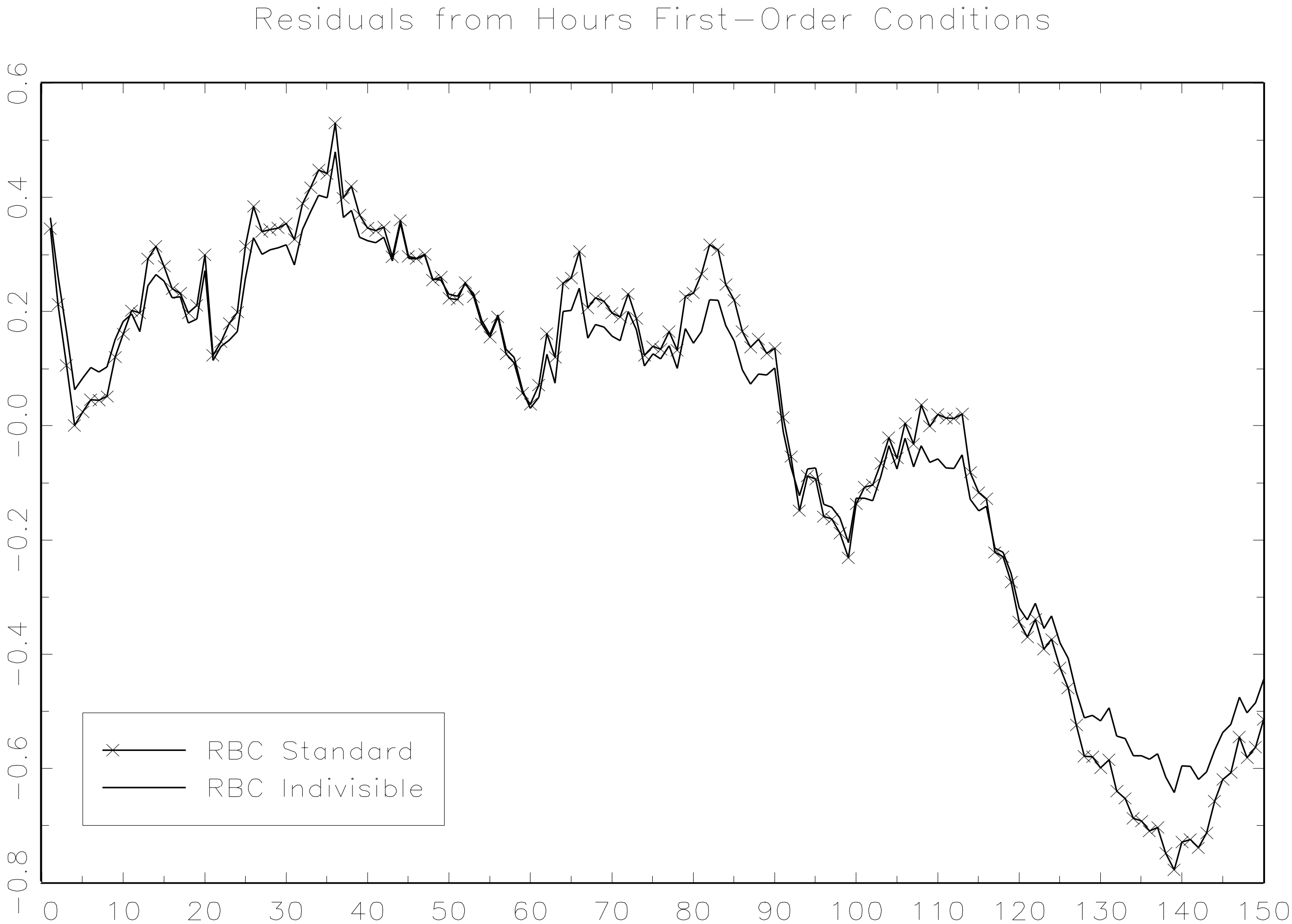
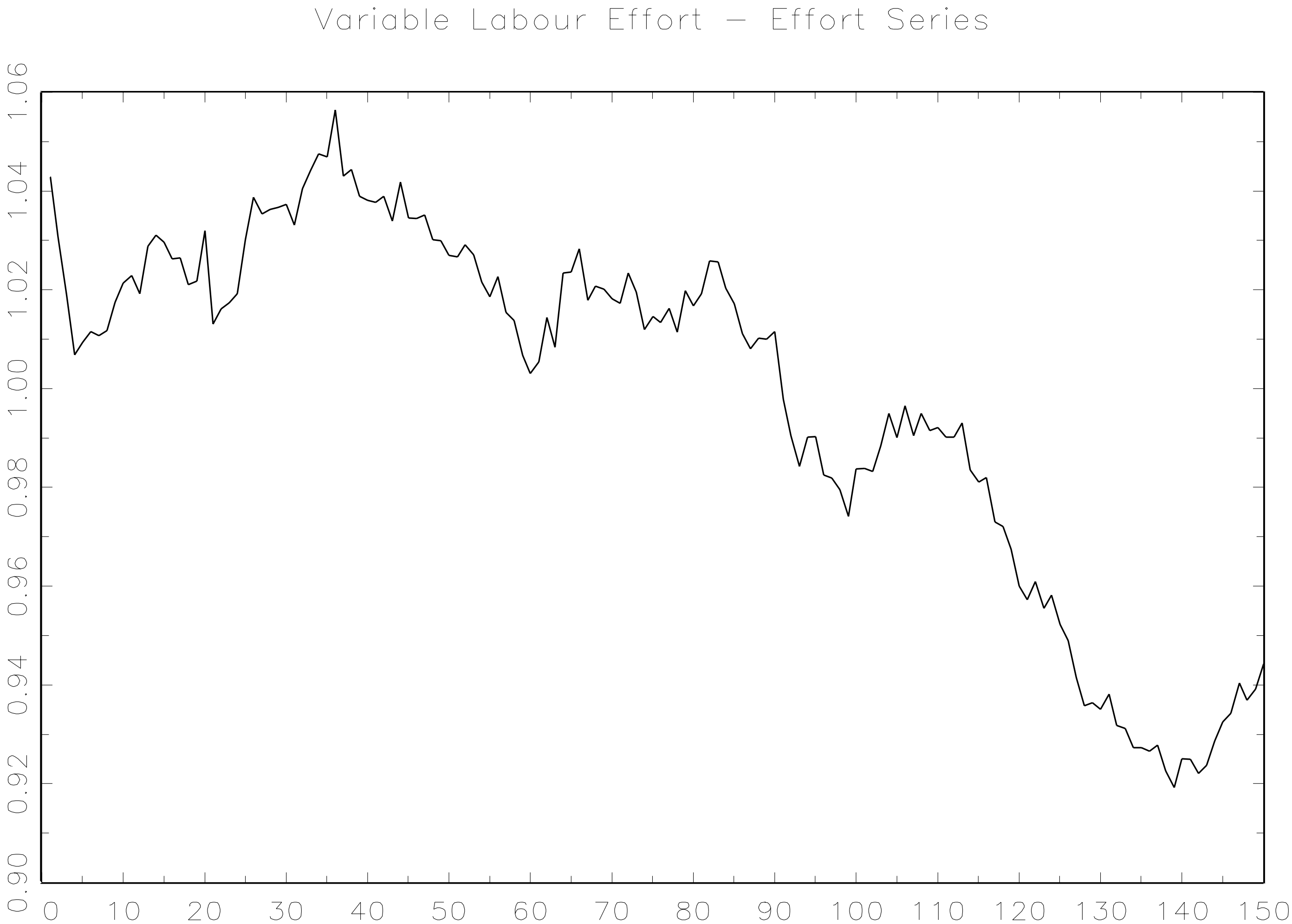

Residuals from Hours First-Order Conditions
0.6
0.4
0.2
-0.0
-0.2
-0.4
-0.6
-0.8
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
RBC Standard
RBC Indivisible

Figure 4


Residuals from Hours First-Order Conditions
Variable Labour Effort
0.03
0.02
0.01
0.00
-0.01
-0.02
-0.03
-0.04
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150

Figure 5



Variable Labour Effort – Effort Series
1.06
1.04
1.02
1.00
0.98
0.96
0.94
0.92
0.90
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150

Figure 6

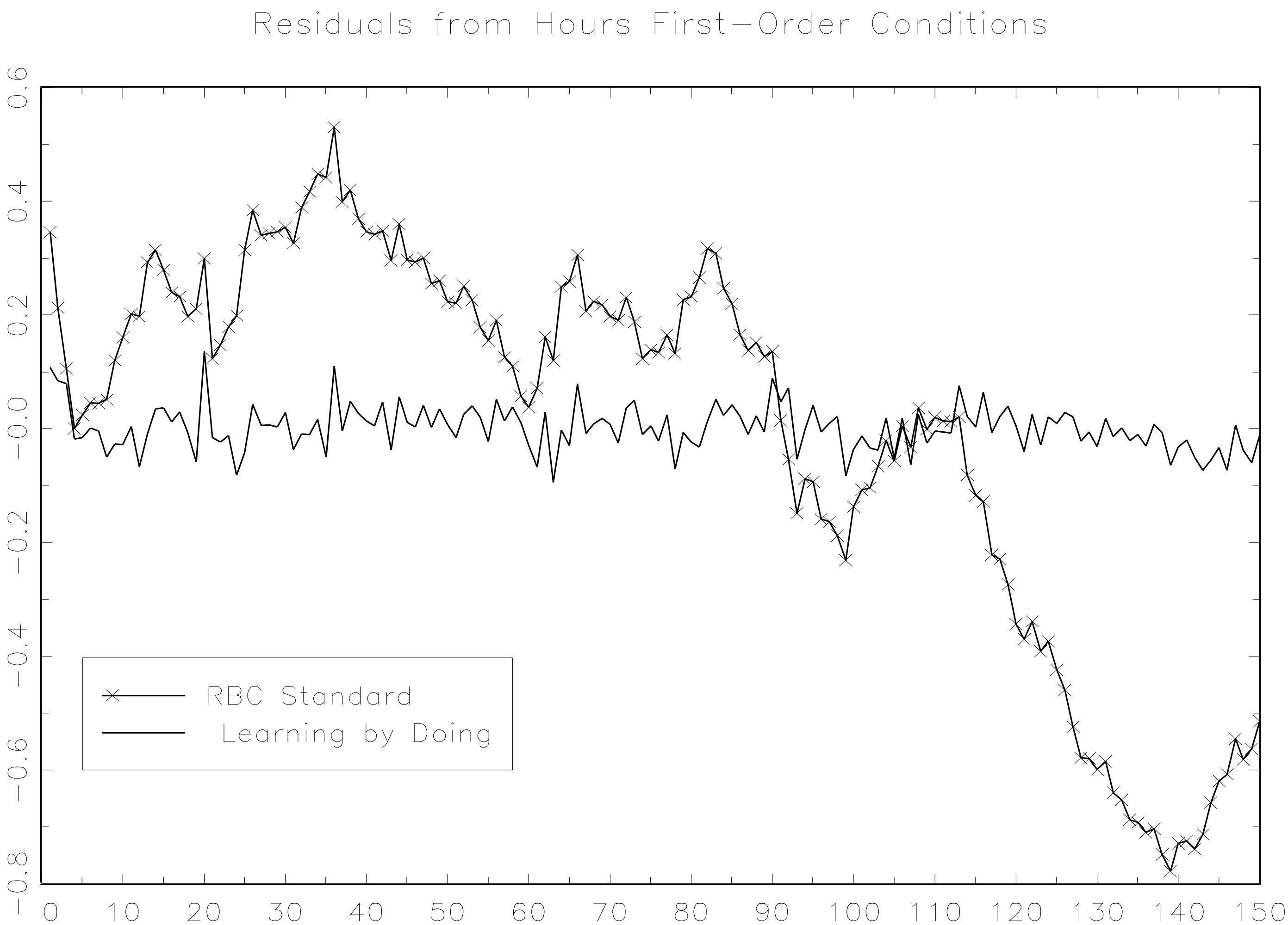
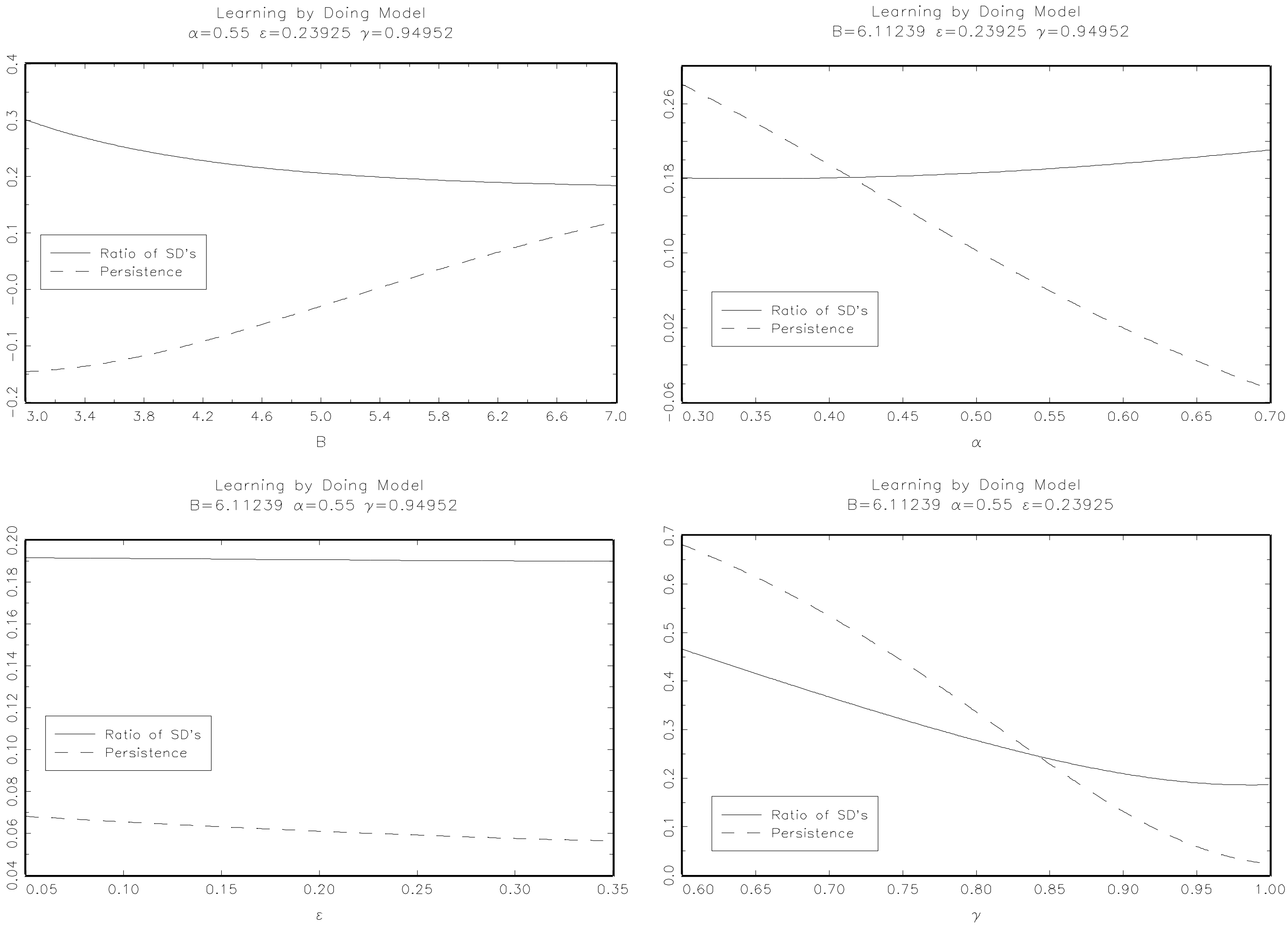

Residuals from Hours First-Order Conditions
0.6
0.4
0.2
-0.0
-0.2
-0.4
-0.6
-0.8
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
RBC Standard
Learning by Doing

Figure 7



Learning by Doing Model
α=0.55 ε=0.23925 γ=0.94952
Ratio of SD's
Persistence
B
Learning by Doing Model
B=6.11239 ε=0.23925 γ=0.94952
Ratio of SD's
Persistence
α
Learning by Doing Model
B=6.11239 α=0.55 γ=0.94952
Ratio of SD's
Persistence
ε
Learning by Doing Model
B=6.11239 α=0.55 ε=0.23925
Ratio of SD's
Persistence
γ

Figure 8

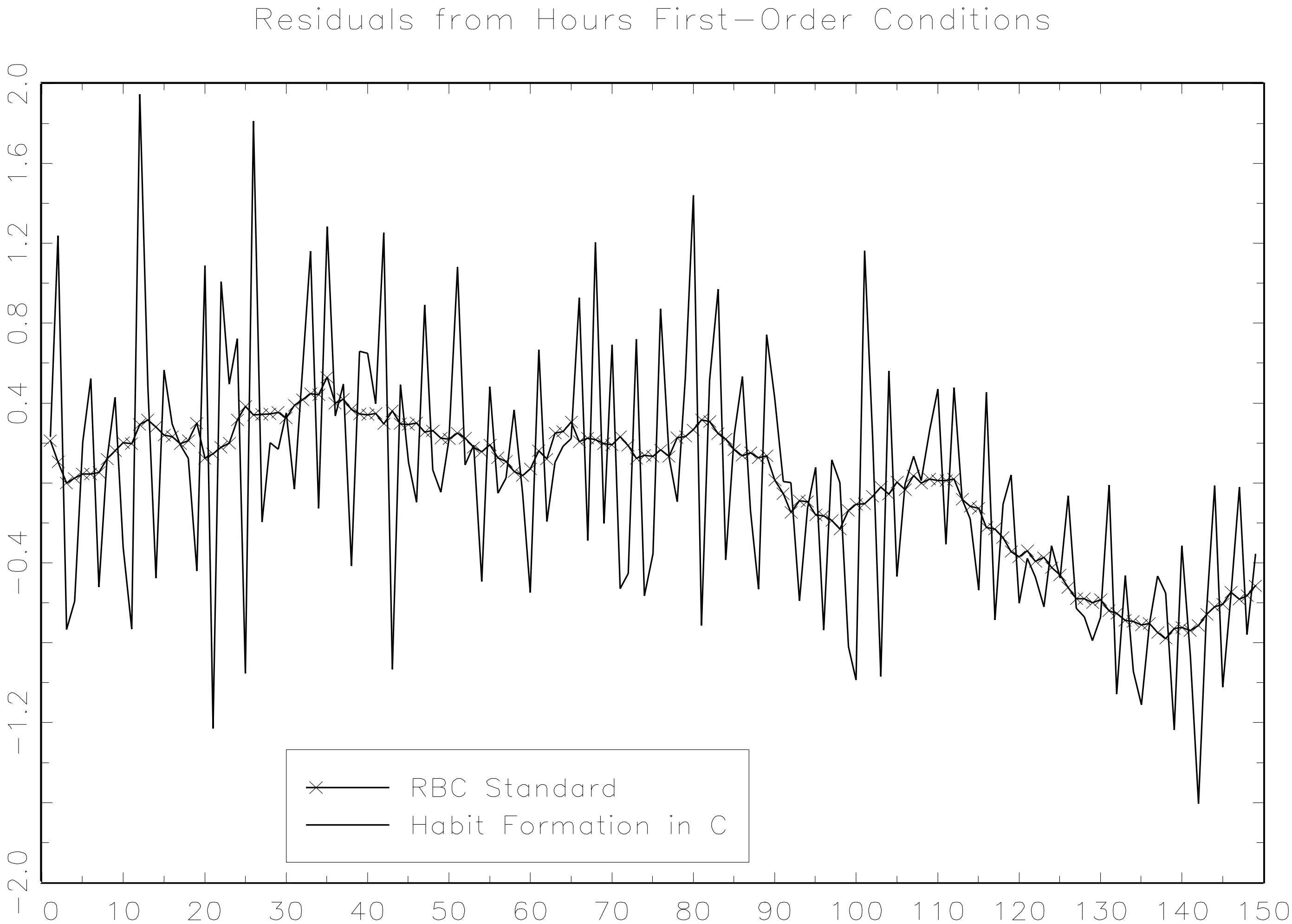

Residuals from Hours First-Order Conditions
2.0
1.6
1.2
0.8
0.4
−0.4
−1.2
−2.0
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
RBC Standard
Habit Formation in C

Figure 9

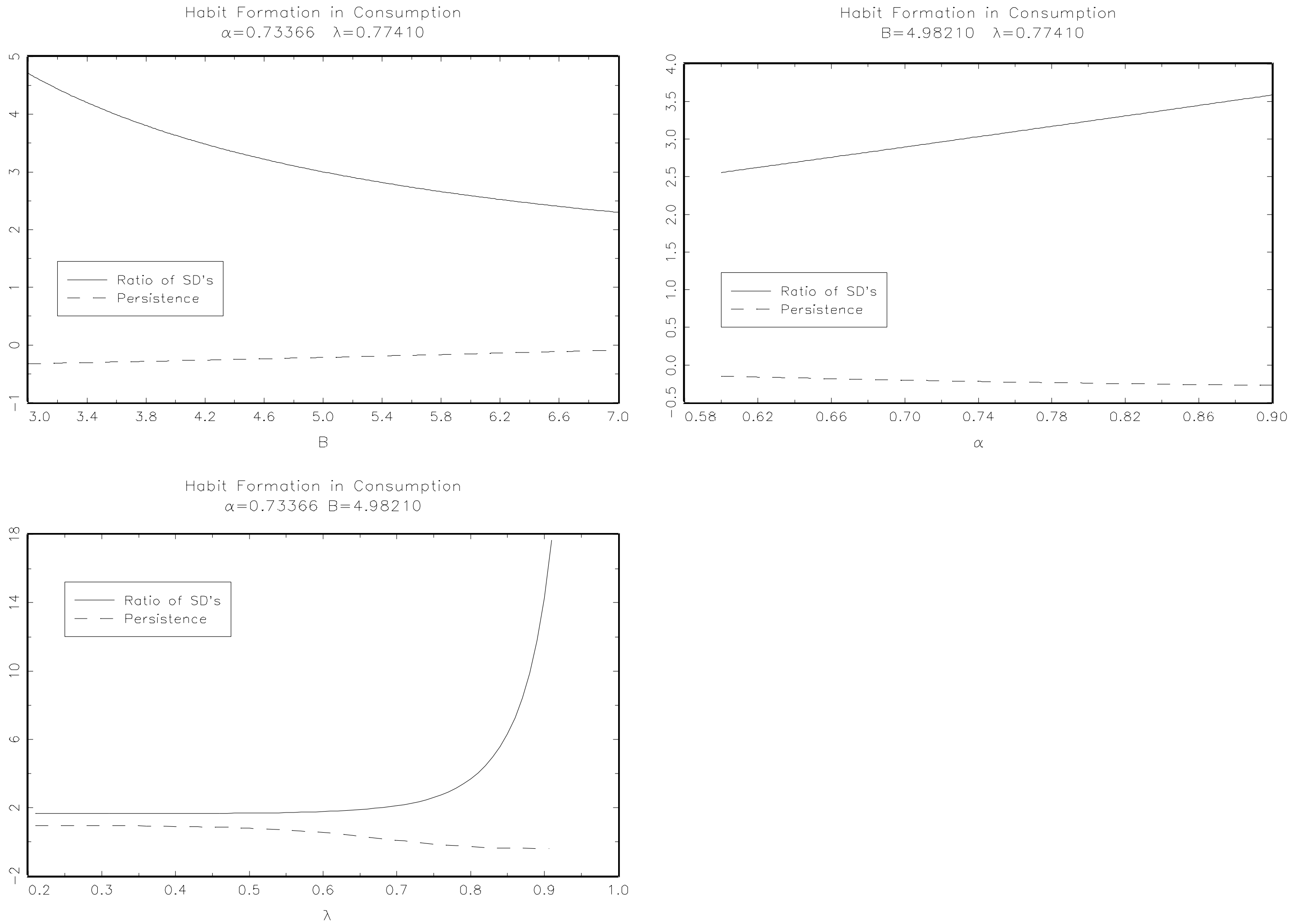

Habit Formation in Consumption
α=0.73366 λ=0.77410
Ratio of SD's
Persistence
B
Habit Formation in Consumption
B=4.98210 λ=0.77410
Ratio of SD's
Persistence
α
Habit Formation in Consumption
α=0.73366 B=4.98210
Ratio of SD's
Persistence
λ

Figure 10

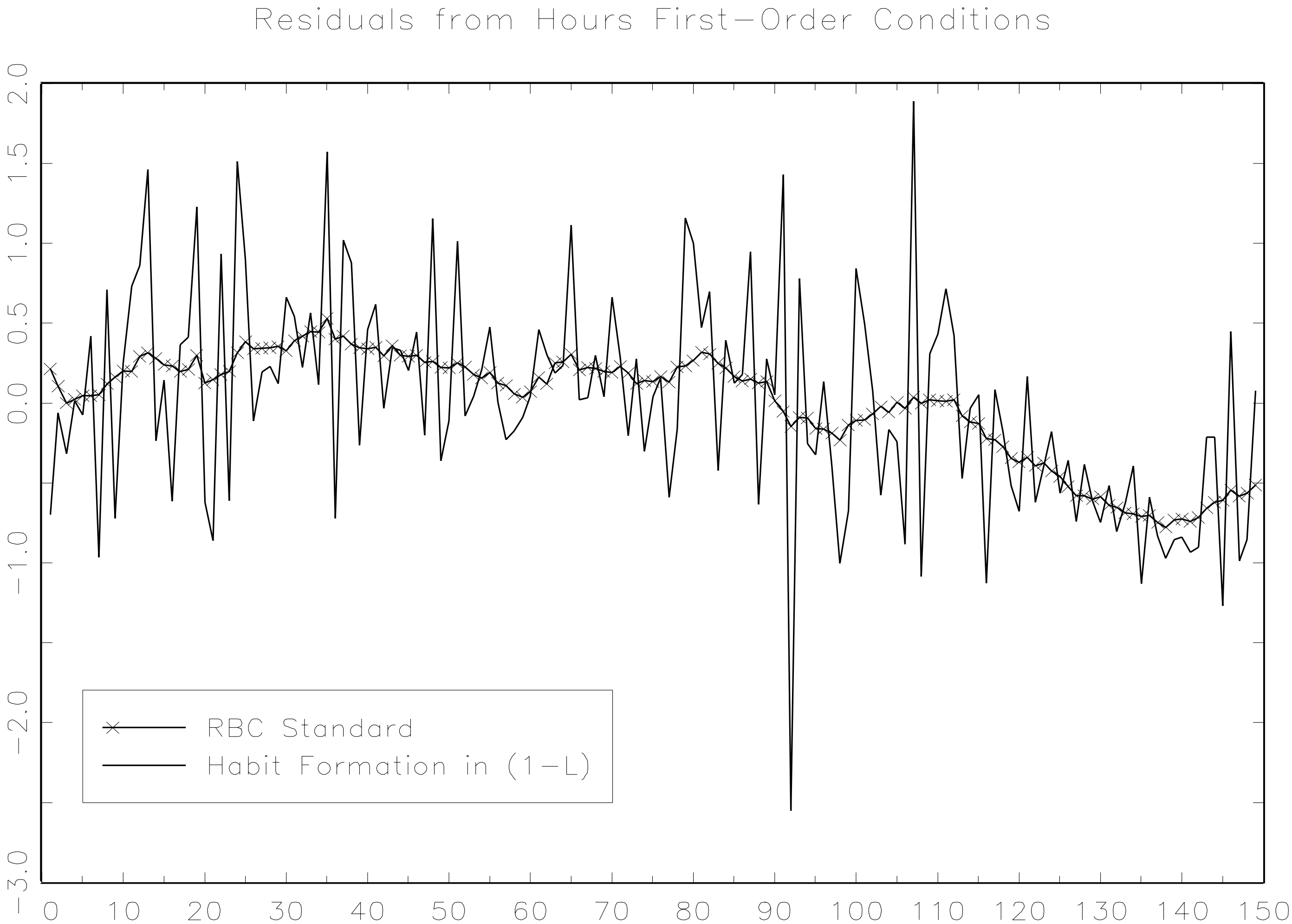
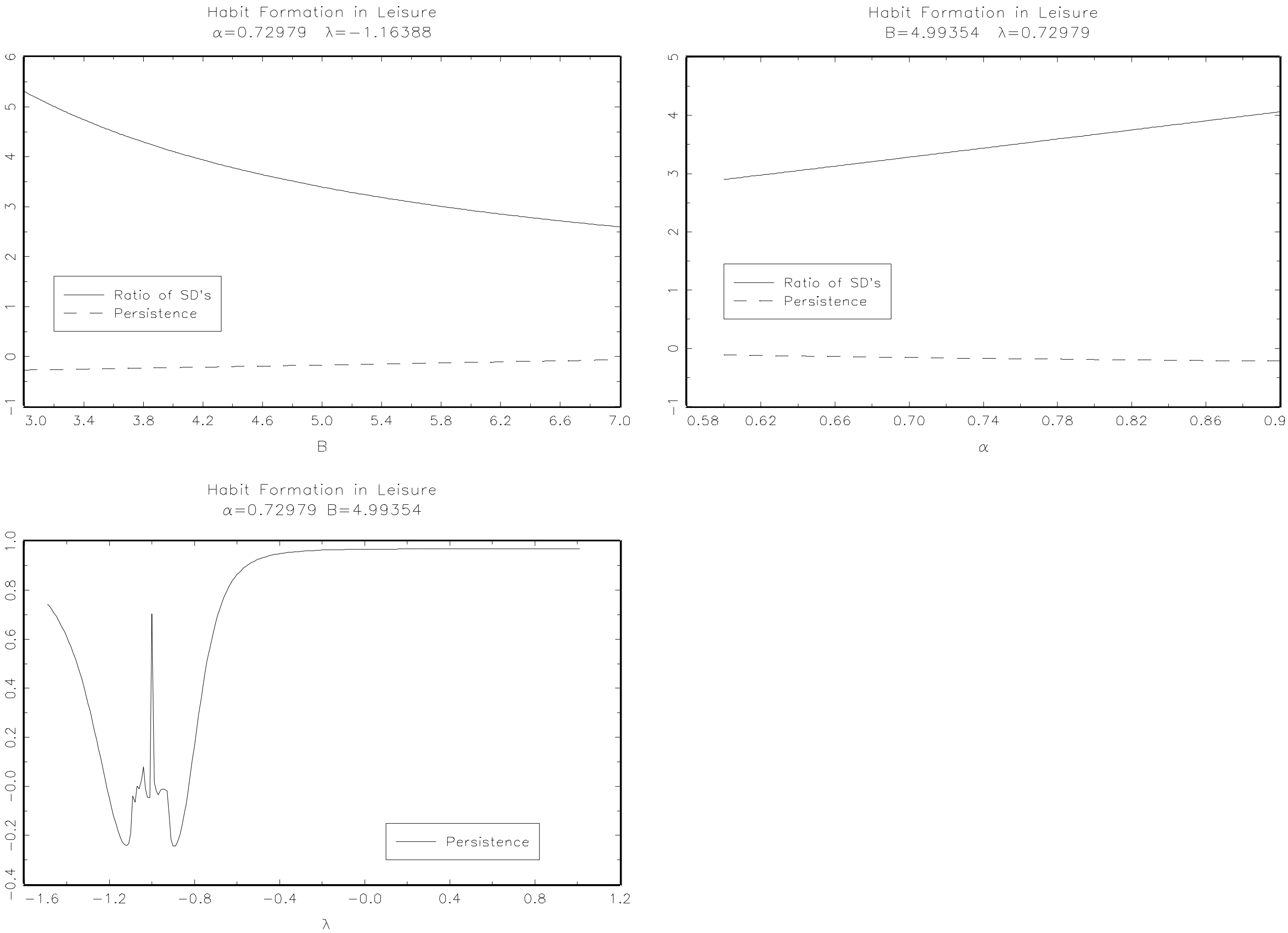

Residuals from Hours First-Order Conditions
2.0
1.5
1.0
0.5
0.0
-1.0
-2.0
-3.0
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
RBC Standard
Habit Formation in (1-L)

Figure 11



Habit Formation in Leisure
α=0.72979 λ=−1.16388
Ratio of SD's
Persistence
B
Habit Formation in Leisure
B=4.99354 λ=0.72979
Ratio of SD's
Persistence
α
Habit Formation in Leisure
α=0.72979 B=4.99354
Persistence
λ

Appendix I — Filtered Data

This appendix presents the results included in Tables 1 and 2 as well as in Figures 1 to 11 where all data series were filtered using the Hodrick-Prescott filter.

Table 1HP: Parameter Estimates

	RBC Standard	RBC Indivisible	RBC-Labour Hoarding	Learning By Doing	Habit Formation Consum.	Habit Formation leisure
B	4.87308 (0.02532)	6.14333 (0.02688)	4.13949 (0.01094)	6.11639 (0.03335)	4.88628 (0.16983)	4.82219 (0.02316)
δ	0.02372 (0.00003)	0.02372 (0.00003)	0.02372 (0.00015)	0.02373 (0.00010)	0.02370 (0.00010)	0.02374 (0.00009)
α	0.72723 (0.00034)	0.72723 (0.00034)	0.72723 (0.00199)	0.55 -	0.72880 (0.02619)	0.72675 (0.00394)
ε				0.24410 (0.00448)		
γ				0.15881 (0.47396)		
λ					0.57260 (0.02652)	-0.70954 (0.03539)
J-test (p-value)				1.12138 (0.57081)		

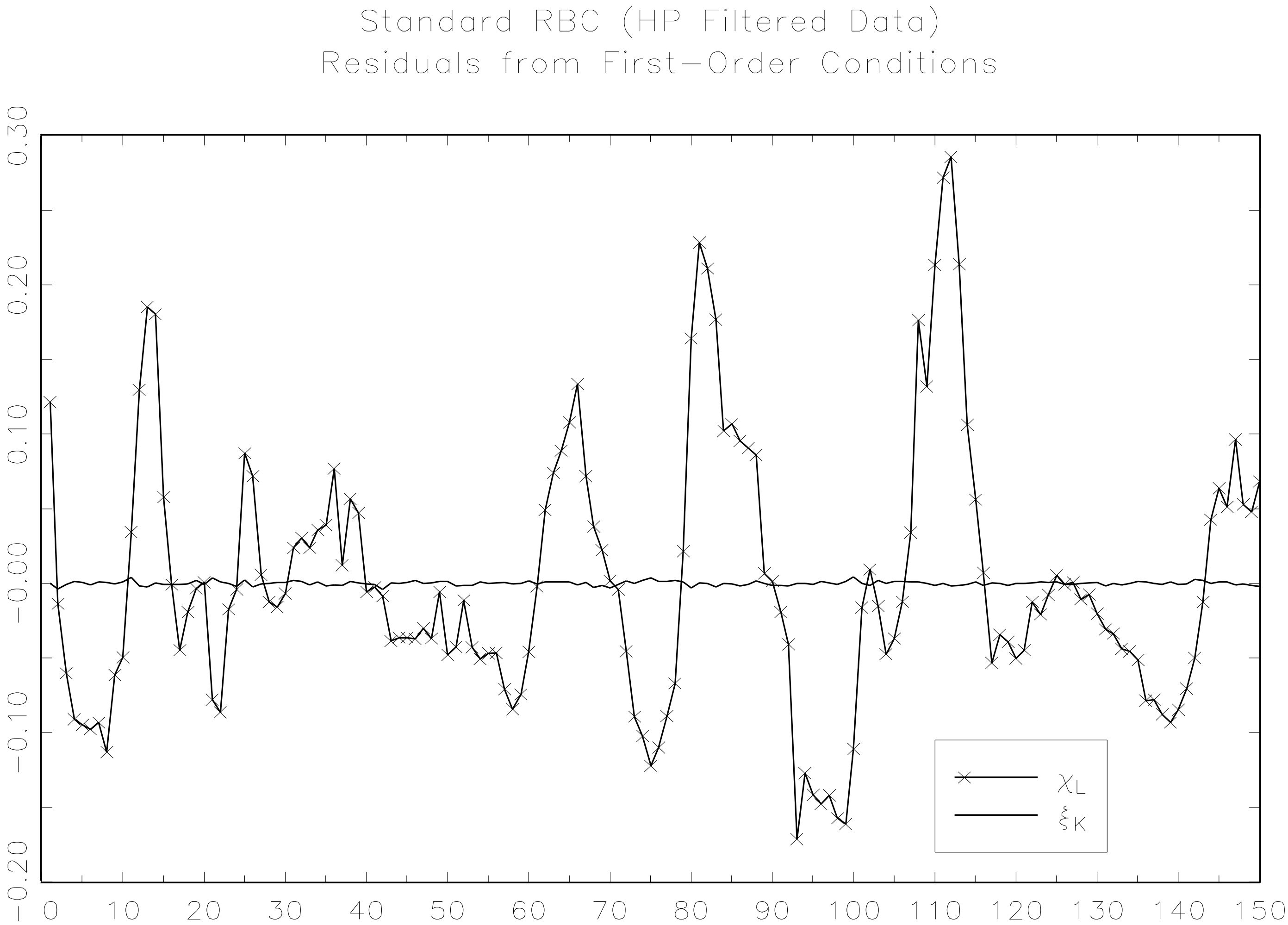
Note to Table 1HP: Standard errors are in parentheses.

Table 2HP: Persistence in Residuals

	RBC Standard	RBC Indivisible	RBC-Labour Hoarding	Learning By Doing	Habit Formation Consum.	Habit Formation Leisure
Size	1.25064	0.99167	0.89831	0.49630	1.01238	1.75159
$\rho_1(\hat{\xi}_K)$	0.13825 (0.08229)	0.13825 (0.08229)	0.13825 (0.08229)	0.18636 (0.08137)	-0.54381 (0.06922)	0.16681 (0.08232)
$\rho_1(\hat{\xi}_L)$	0.87277 (0.03952)	0.87246 (0.03955)	0.54748 (0.06851)	0.54768 (0.06972)	0.00000 (0.08262)	0.00000 (0.08064)

Notes to Table 2HP:
Standard errors are in parentheses.
Size=SD(residuals)/SD(total hours).
ρ_1: first-order autocorrelation coefficient.

Figure 1HP

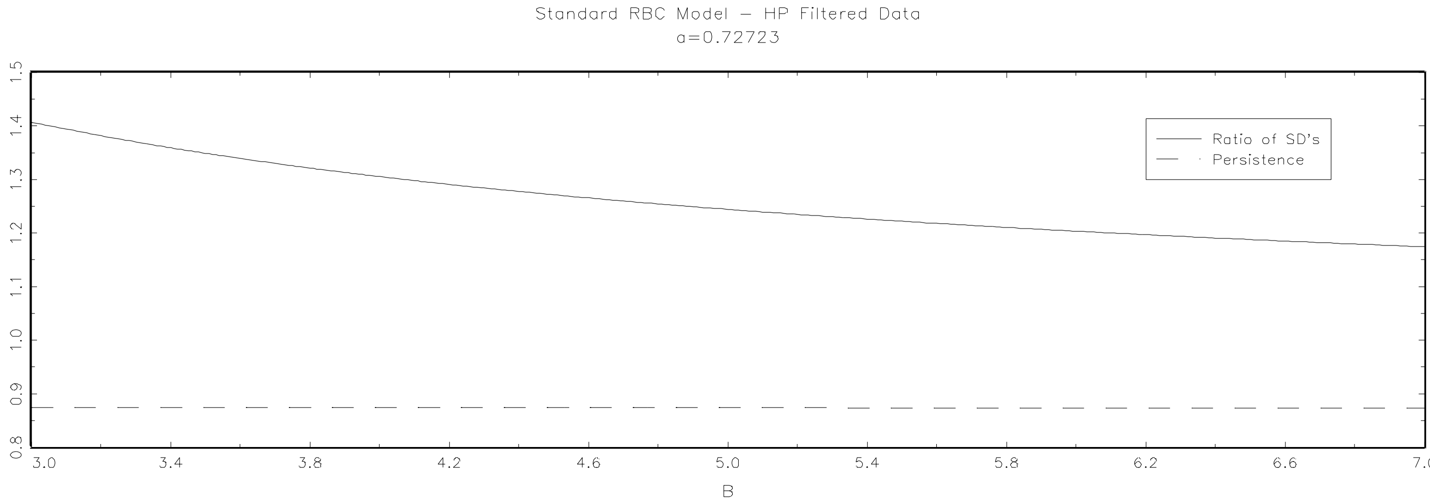


Notes to Figure 1HP:
To facilitate comparisons between the two residuals, we have rearranged their expressions relative to what appears in the body of the paper so that they are on the same scale. The expressions that correspond to the series depicted above are

$$\hat{\xi}^s_{K,t+1} = \beta \frac{C_t}{C_{t+1}} \left[(1-\hat{\alpha}) \frac{Y_{t+1}}{K_{t+1}} + 1 - \hat{\delta} \right] - 1$$

$$\hat{\chi}^s_{L,t} = \frac{\hat{\alpha}}{\hat{B}} \frac{Y_t}{C_t} \frac{1-L_t}{L_t} - 1.$$

Figure 2HP


Standard RBC Model – HP Filtered Data
a=0.72723
Ratio of SD's
Persistence
B
3.0
3.4
3.8
4.2
4.6
5.0
5.4
5.8
6.2
6.6
7.0
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5

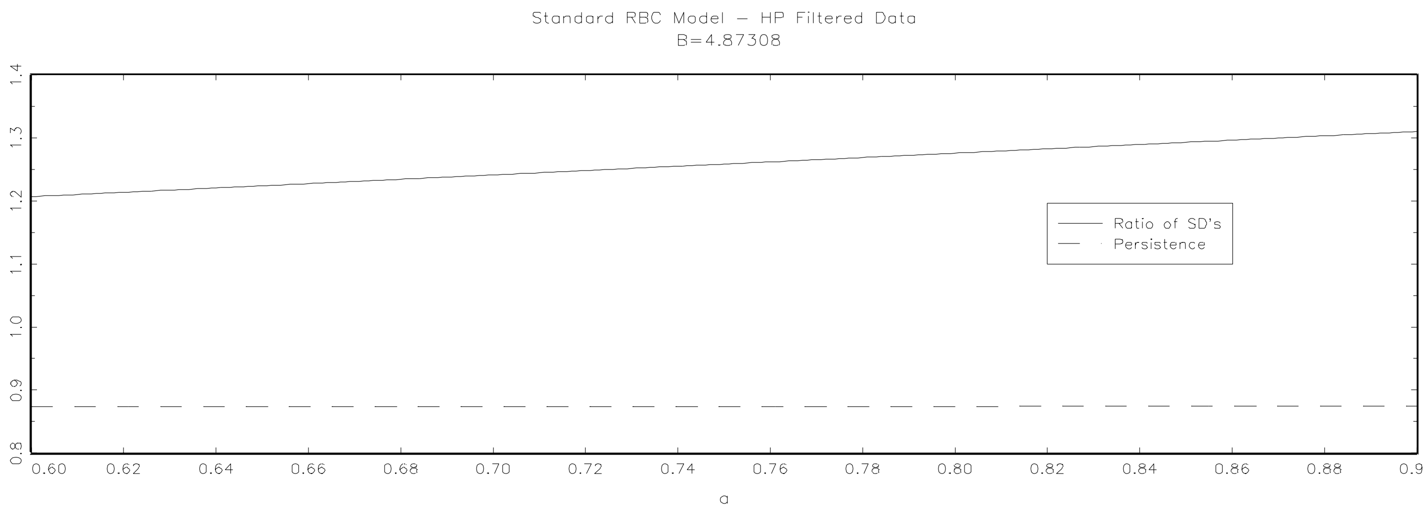
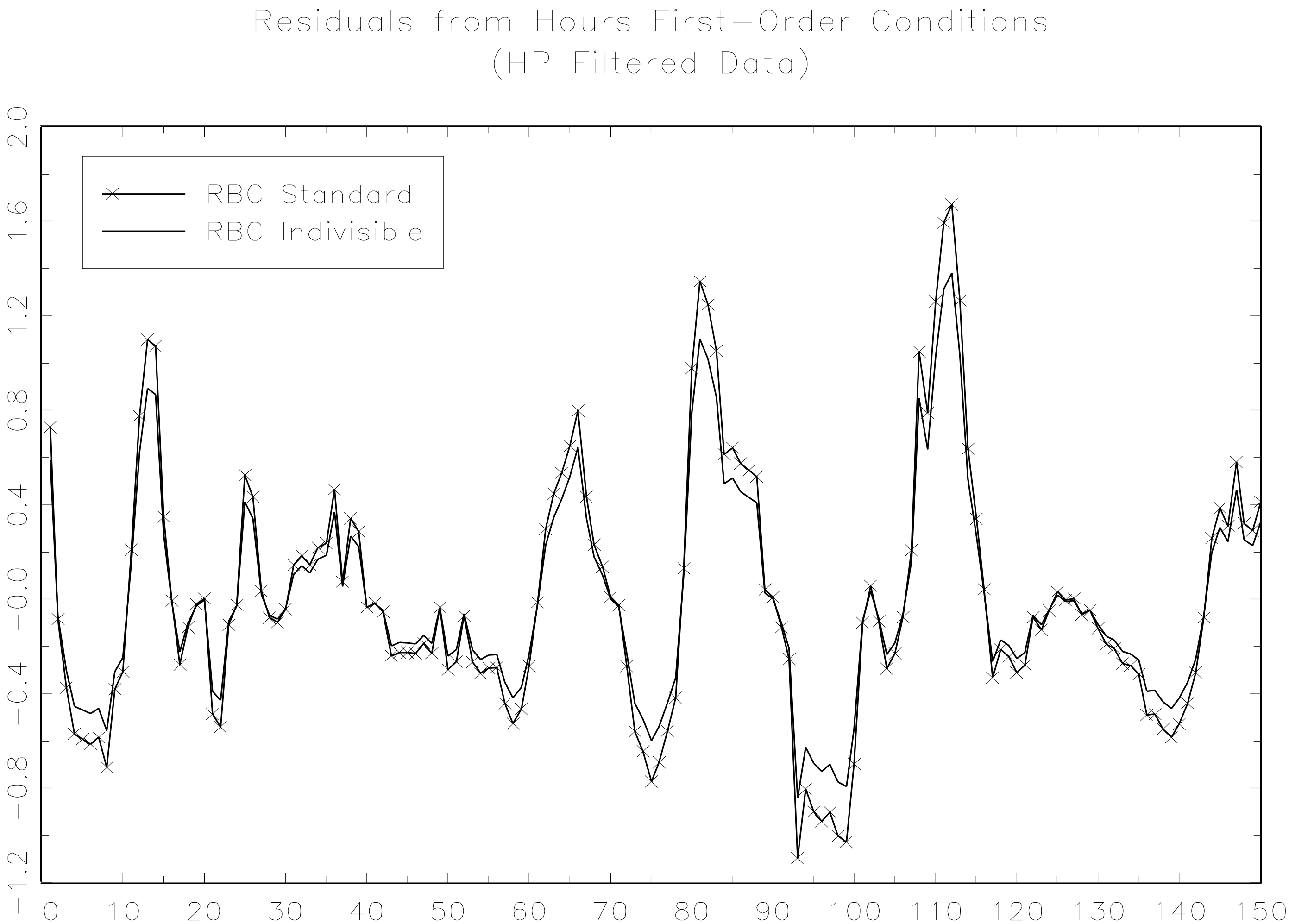

Standard RBC Model – HP Filtered Data
B=4.87308
Ratio of SD's
Persistence
a
0.60
0.62
0.64
0.66
0.68
0.70
0.72
0.74
0.76
0.78
0.80
0.82
0.84
0.86
0.88
0.90
0.8
0.9
1.0
1.1
1.2
1.3
1.4

Figure 3HP



Residuals from Hours First-Order Conditions
(HP Filtered Data)
RBC Standard
RBC Indivisible
2.0
1.6
1.2
0.8
0.4
-0.0
-0.0
-0.4
-0.8
-1.2
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150

Figure 4HP

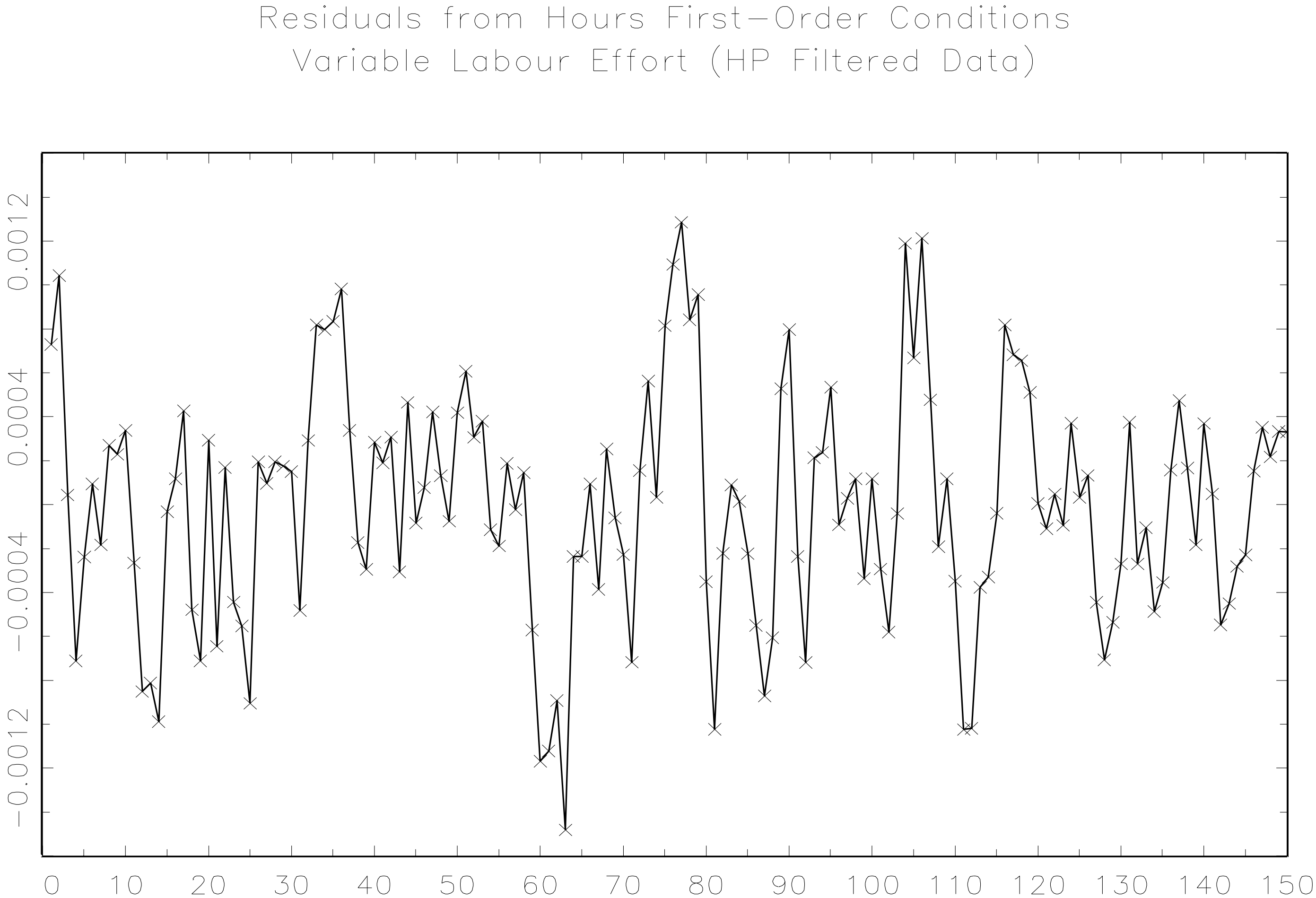

Residuals from Hours First–Order Conditions
Variable Labour Effort (HP Filtered Data)
0.0012
0.0004
−0.0004
−0.0012
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150

Figure 5HP

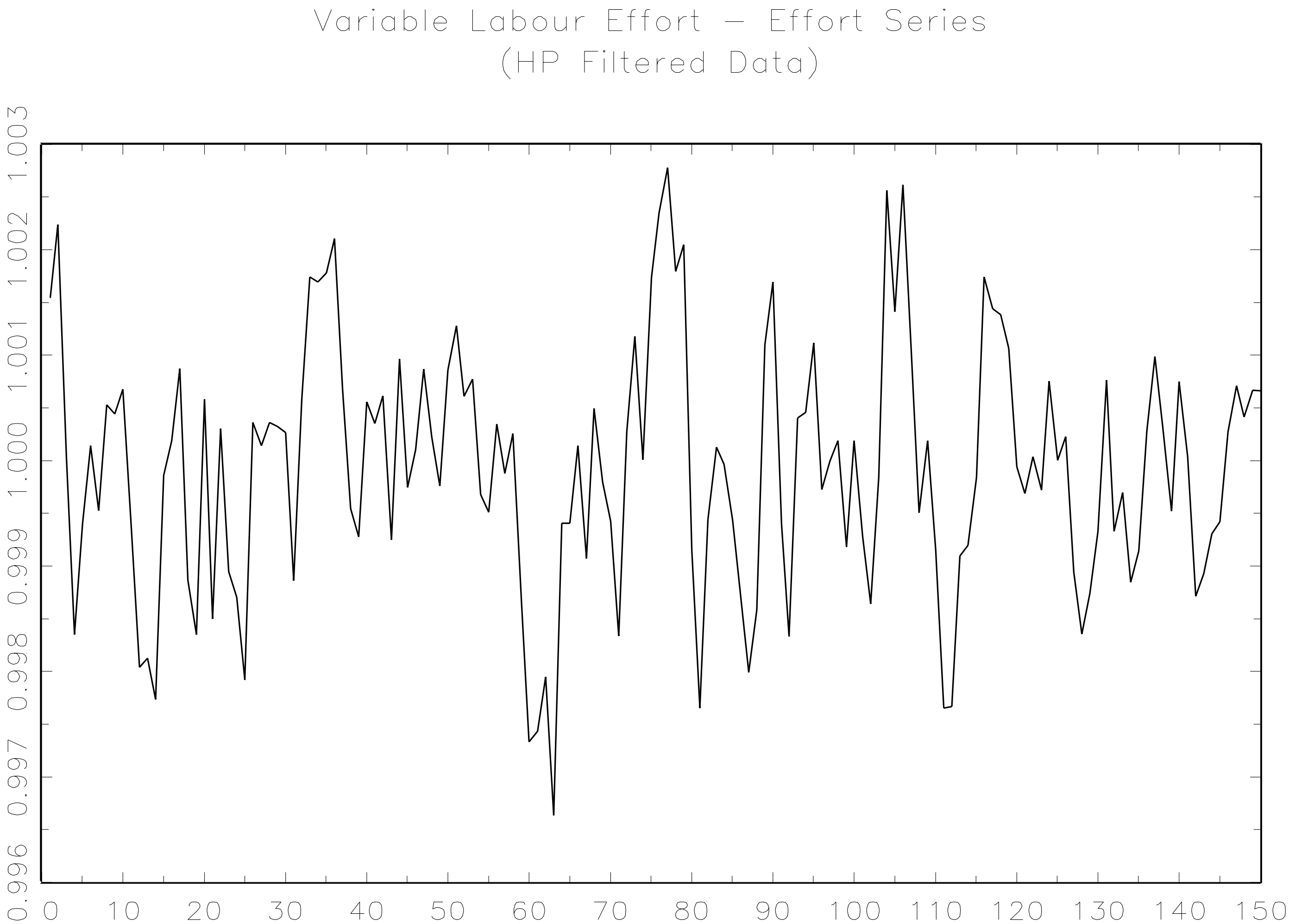
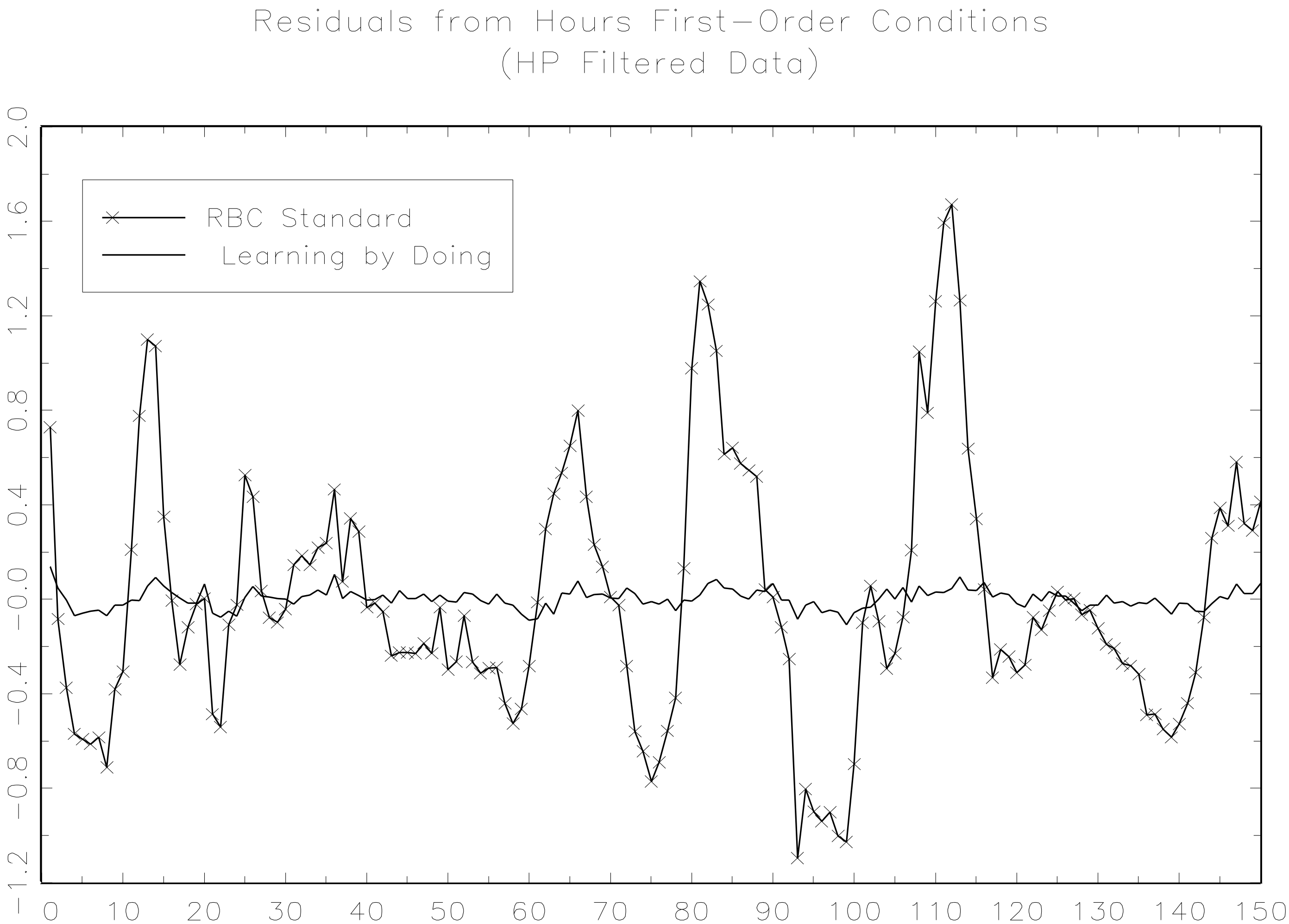
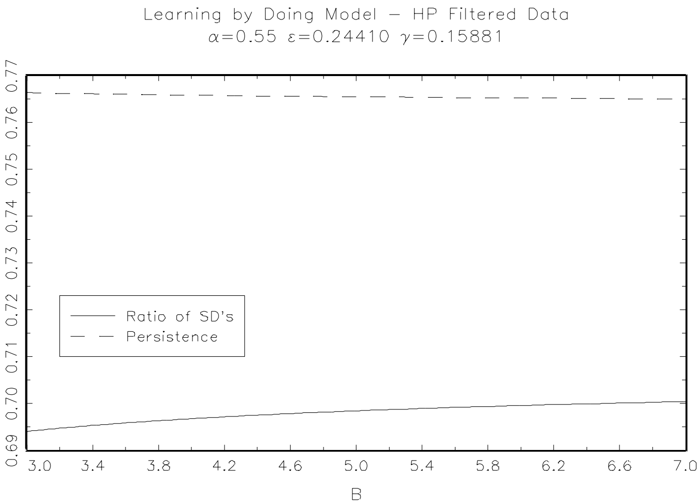

Variable Labour Effort – Effort Series
(HP Filtered Data)
0.996
0.997
0.998
0.999
1.000
1.001
1.002
1.003
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150

Figure 6HP

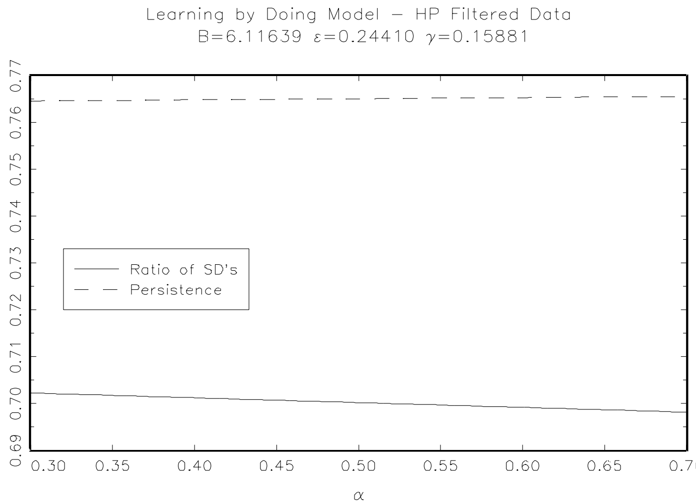


Residuals from Hours First-Order Conditions
(HP Filtered Data)
RBC Standard
Learning by Doing
2.0
1.6
1.2
0.8
0.4
-0.0
-0.4
-0.8
-1.2
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150

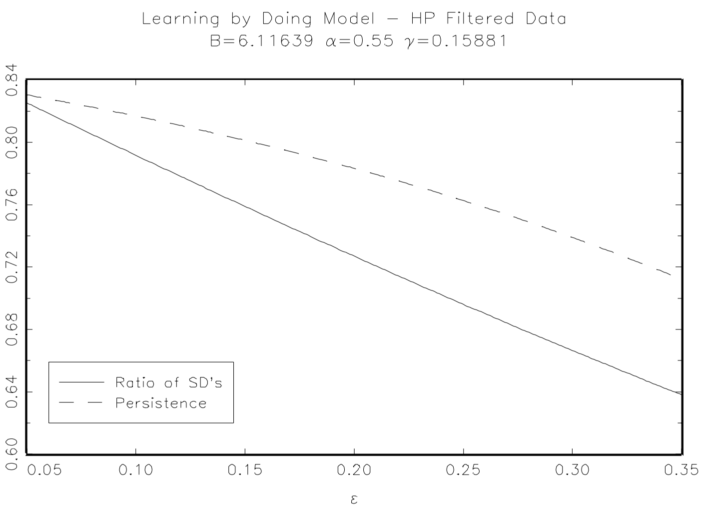
Figure 7HP



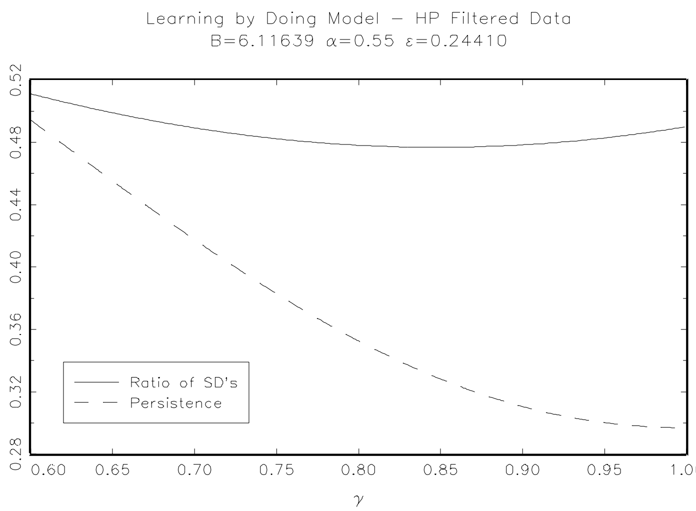
Learning by Doing Model – HP Filtered Data
α=0.55 ε=0.24410 γ=0.15881
Ratio of SD's
Persistence
0.69 0.70 0.71 0.72 0.73 0.74 0.75 0.76 0.77
3.0 3.4 3.8 4.2 4.6 5.0 5.4 5.8 6.2 6.6 7.0
B



Learning by Doing Model – HP Filtered Data
B=6.11639 ε=0.24410 γ=0.15881
Ratio of SD's
Persistence
0.69 0.70 0.71 0.72 0.73 0.74 0.75 0.76 0.77
0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70
α



Learning by Doing Model – HP Filtered Data
B=6.11639 α=0.55 γ=0.15881
Ratio of SD's
Persistence
0.60 0.64 0.68 0.72 0.76 0.80 0.84
0.05 0.10 0.15 0.20 0.25 0.30 0.35
ε



Learning by Doing Model – HP Filtered Data
B=6.11639 α=0.55 ε=0.24410
Ratio of SD's
Persistence
0.28 0.32 0.36 0.40 0.44 0.48 0.52
0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00
γ

Figure 8HP

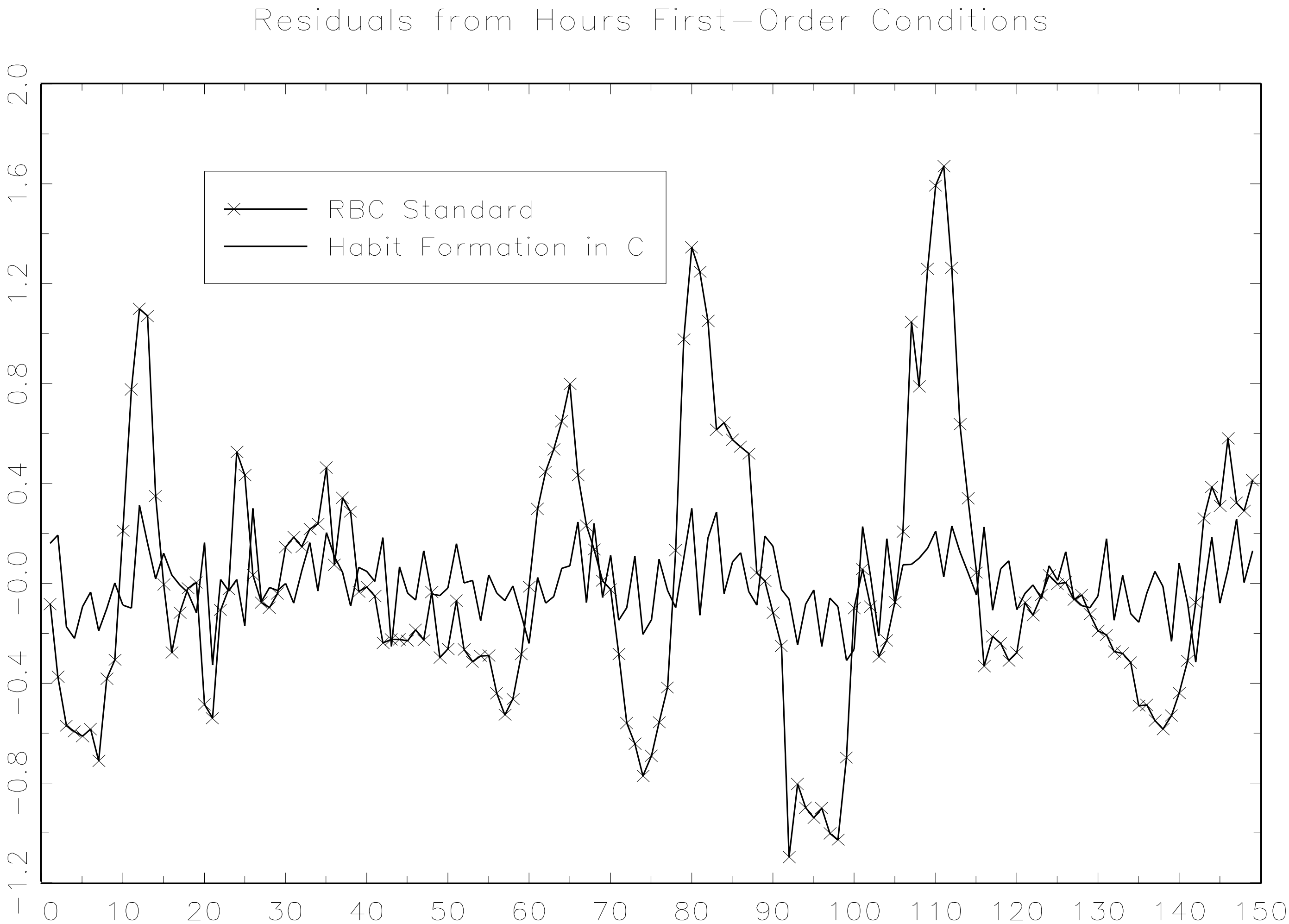

Residuals from Hours First-Order Conditions
RBC Standard
Habit Formation in C
2.0
1.6
1.2
0.8
0.4
-0.0
-0.4
-0.8
-1.2
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150

Figure 9HP

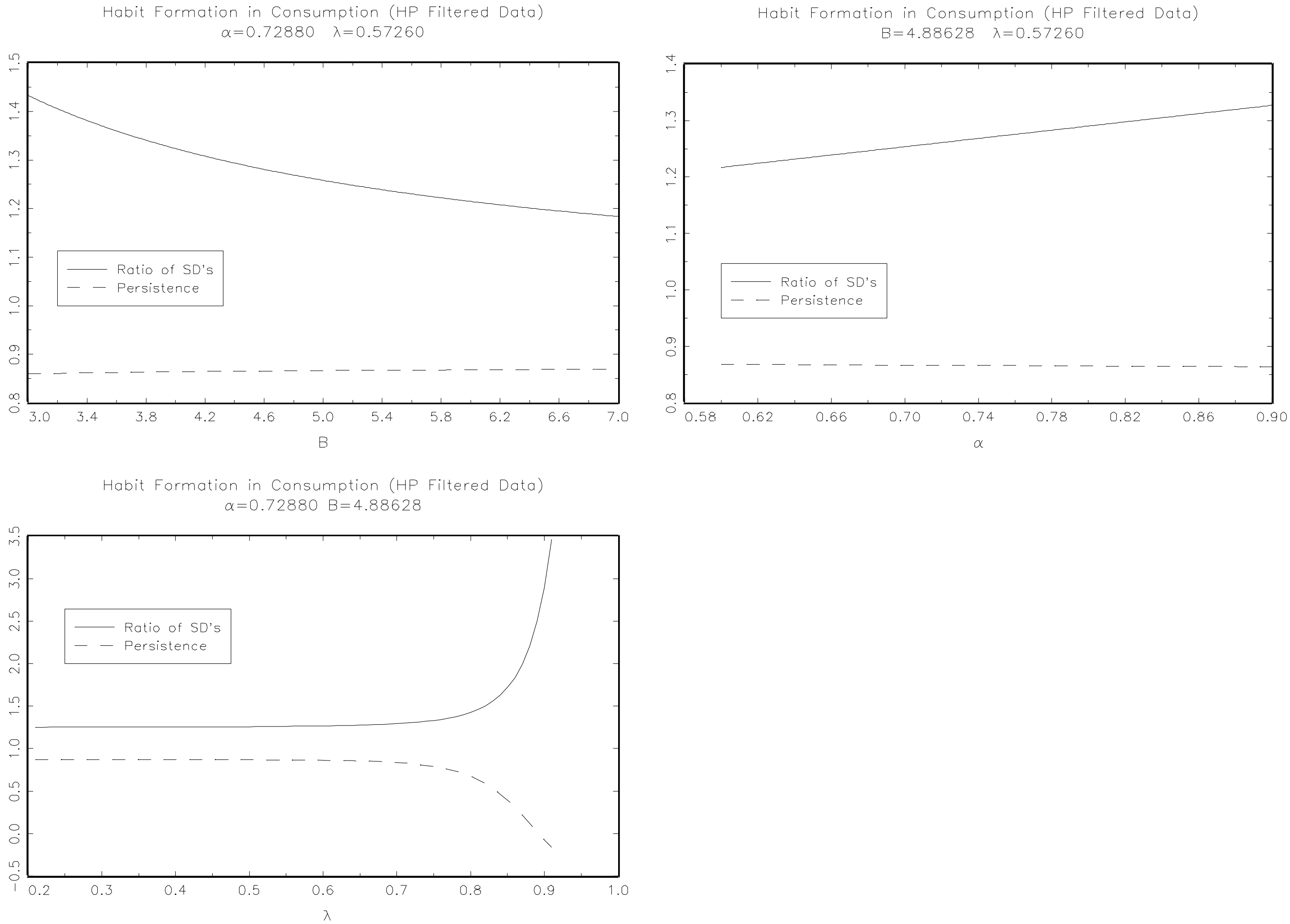

Habit Formation in Consumption (HP Filtered Data)
α=0.72880 λ=0.57260
Ratio of SD's
Persistence
B
Habit Formation in Consumption (HP Filtered Data)
B=4.88628 λ=0.57260
Ratio of SD's
Persistence
α
Habit Formation in Consumption (HP Filtered Data)
α=0.72880 B=4.88628
Ratio of SD's
Persistence
λ

Figure 10HP

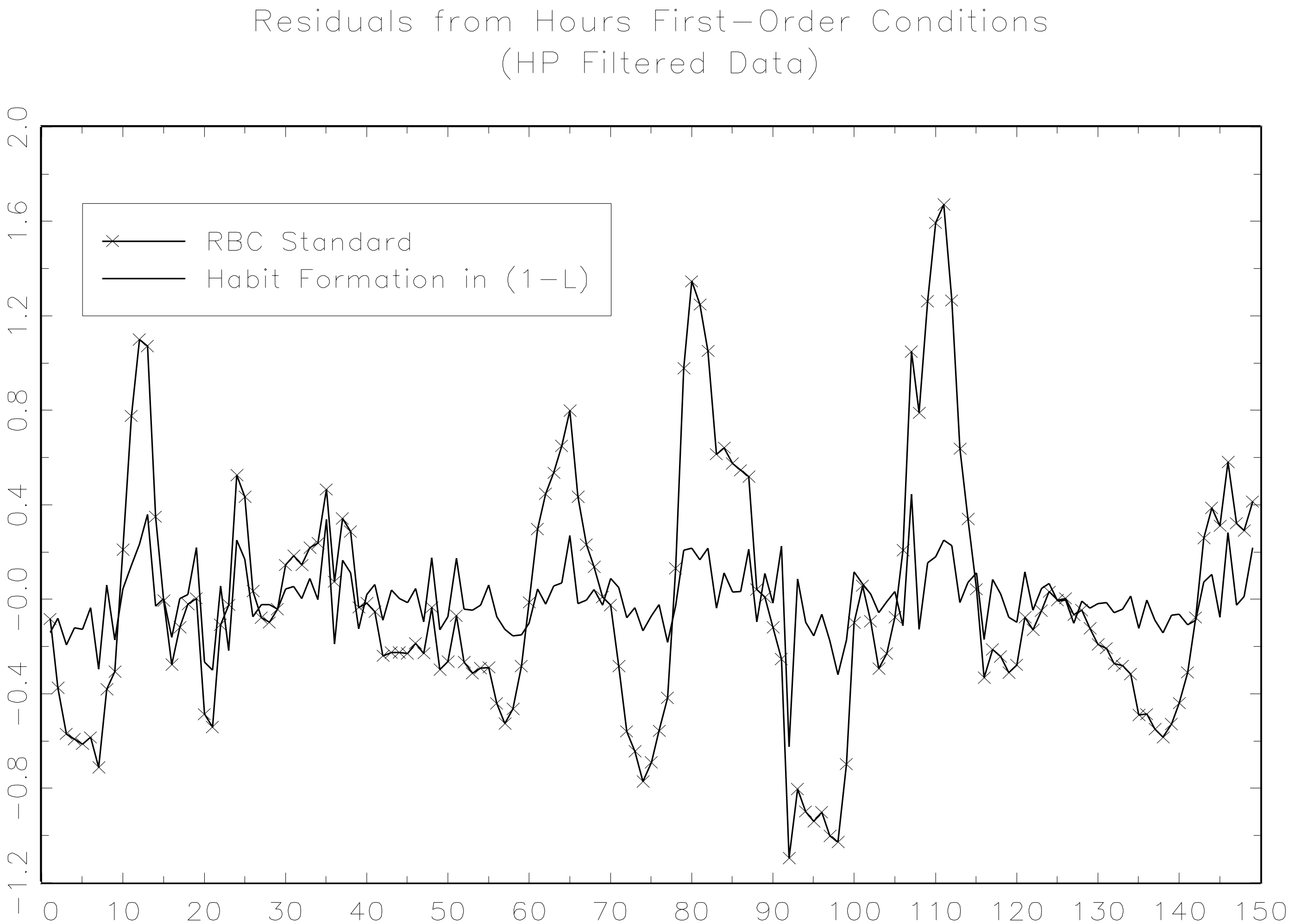
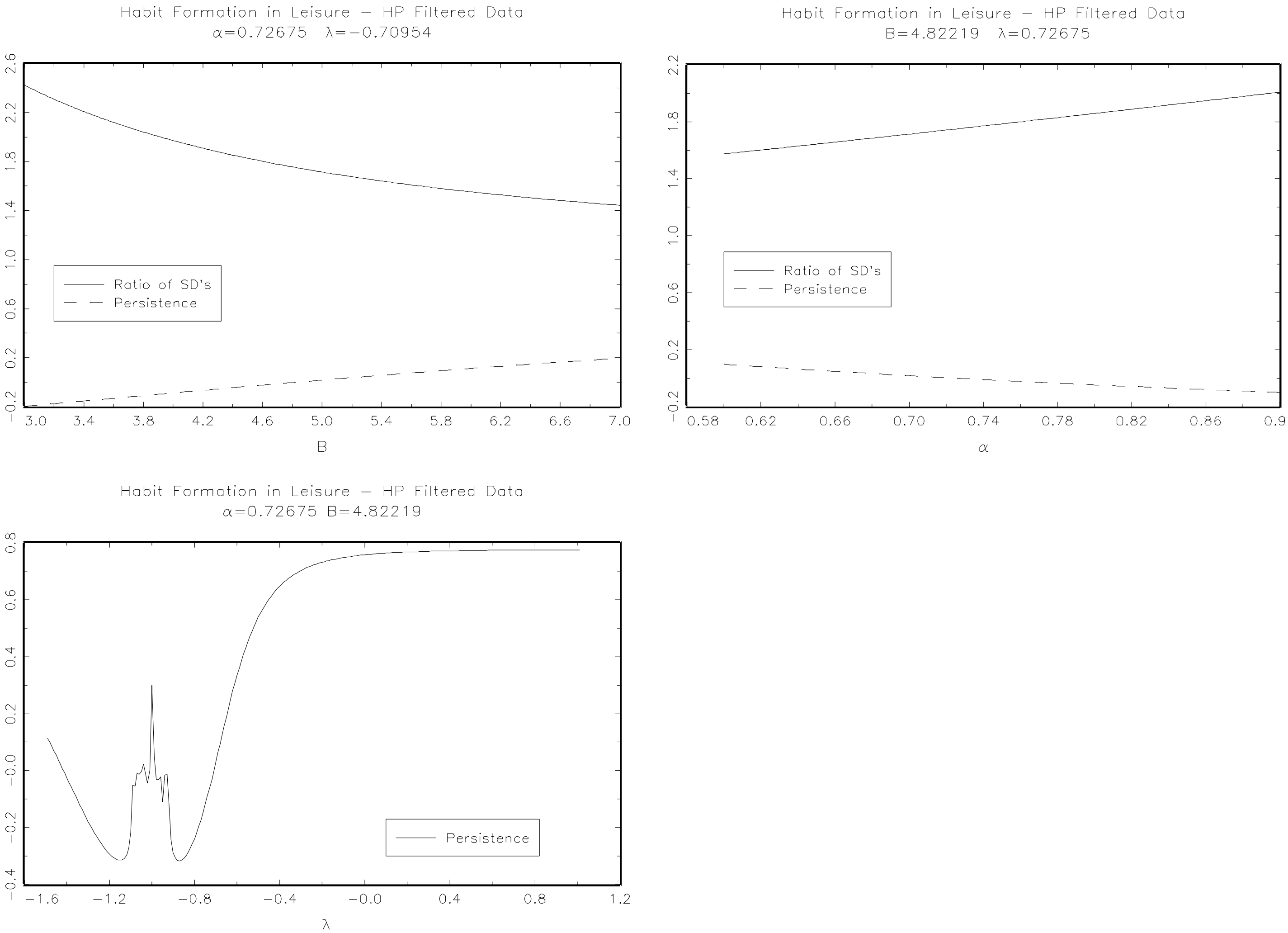

Residuals from Hours First-Order Conditions
(HP Filtered Data)
RBC Standard
Habit Formation in (1-L)
2.0
1.6
1.2
0.8
0.4
-0.0
-0.4
-0.8
-1.2
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150

Figure 11HP



Habit Formation in Leisure – HP Filtered Data
α=0.72675 λ=–0.70954
Ratio of SD's
Persistence
B
Habit Formation in Leisure – HP Filtered Data
B=4.82219 λ=0.72675
Ratio of SD's
Persistence
α
Habit Formation in Leisure – HP Filtered Data
α=0.72675 B=4.82219
Persistence
λ

Appendix II — Alternative Dataset

This appendix presents the sensitivity analysis results using the alternative data set.

Figure II.1

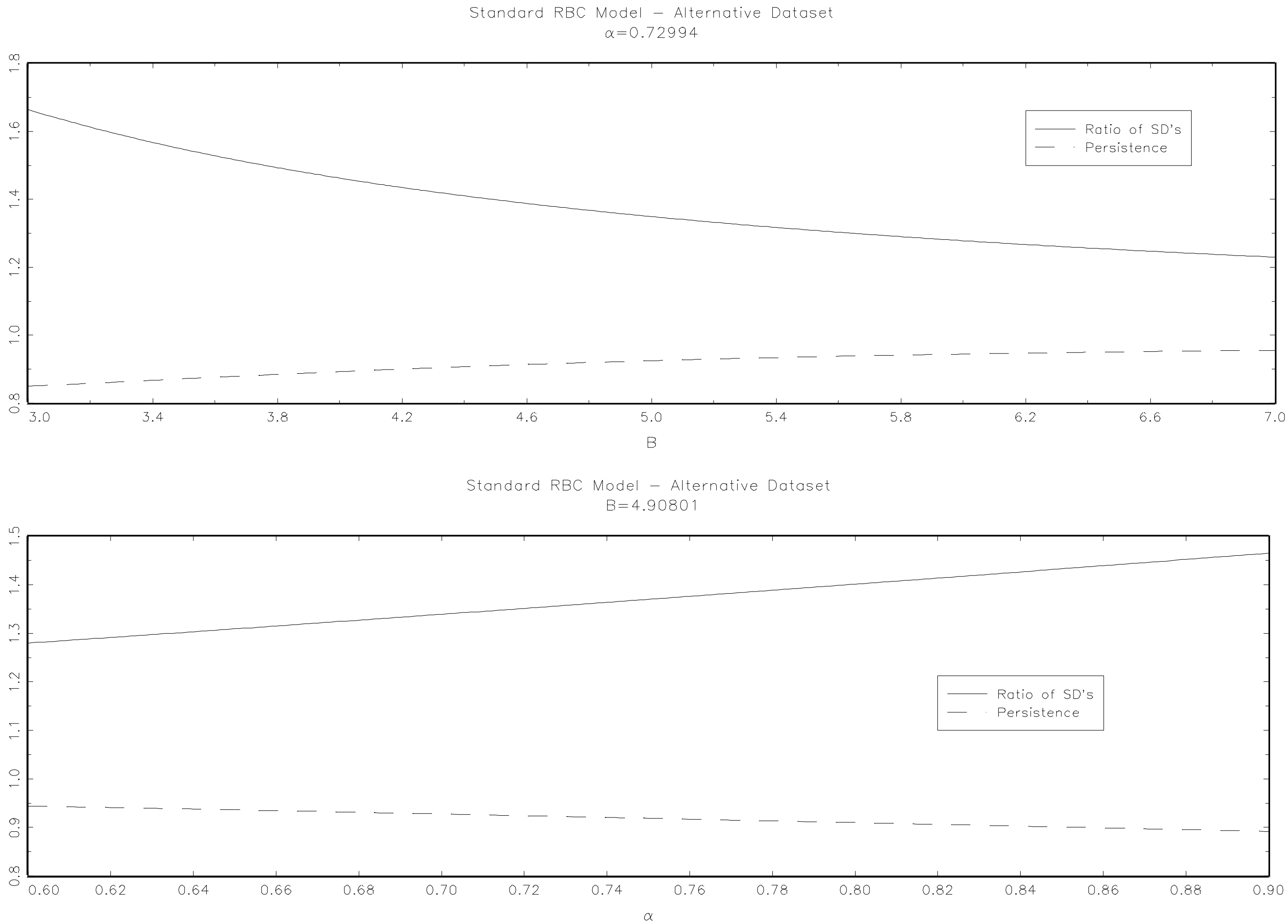


Figure II.2

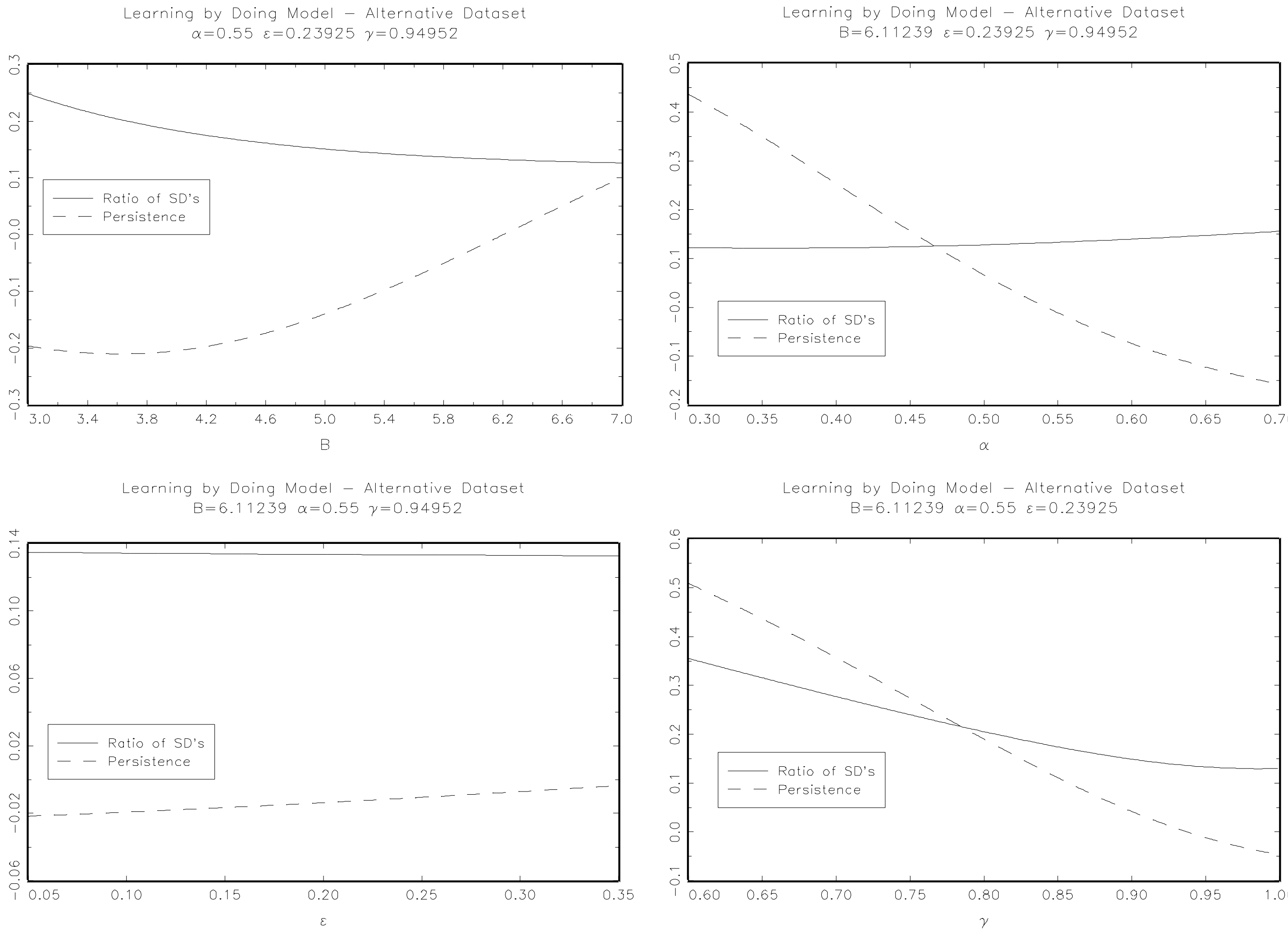
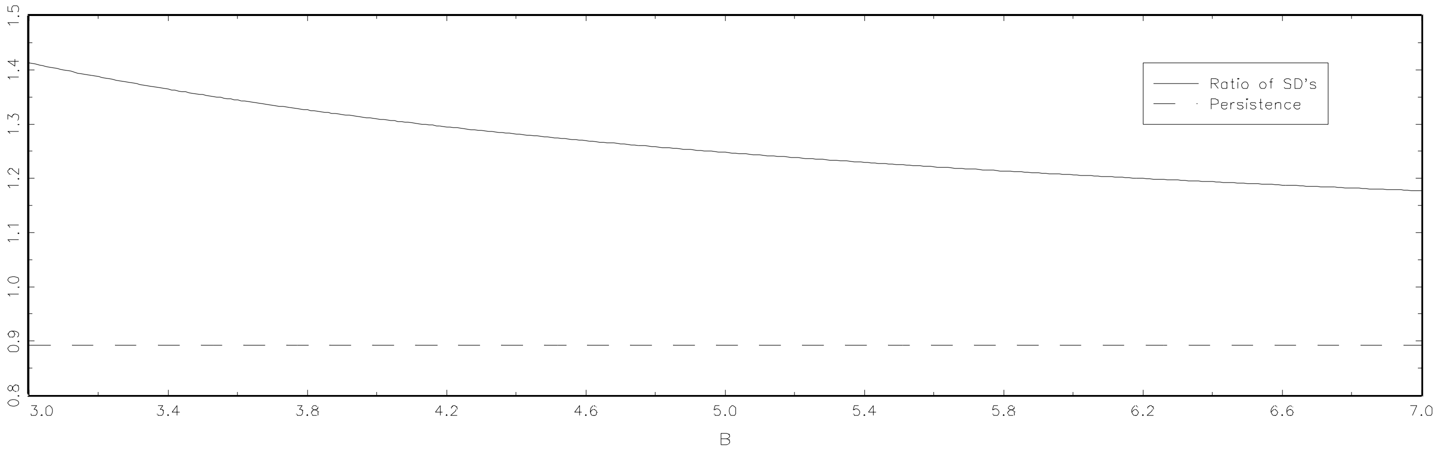

Learning by Doing Model – Alternative Dataset
α=0.55 ε=0.23925 γ=0.94952
Ratio of SD's
Persistence
B
Learning by Doing Model – Alternative Dataset
B=6.11239 ε=0.23925 γ=0.94952
Ratio of SD's
Persistence
α
Learning by Doing Model – Alternative Dataset
B=6.11239 α=0.55 γ=0.94952
Ratio of SD's
Persistence
ε
Learning by Doing Model – Alternative Dataset
B=6.11239 α=0.55 ε=0.23925
Ratio of SD's
Persistence
γ

Figure II.3

Standard RBC Model – Alternative Dataset (HP Filtered)
α=0.72994



Standard RBC Model – Alternative Dataset (HP Filtered)
B=4.90801

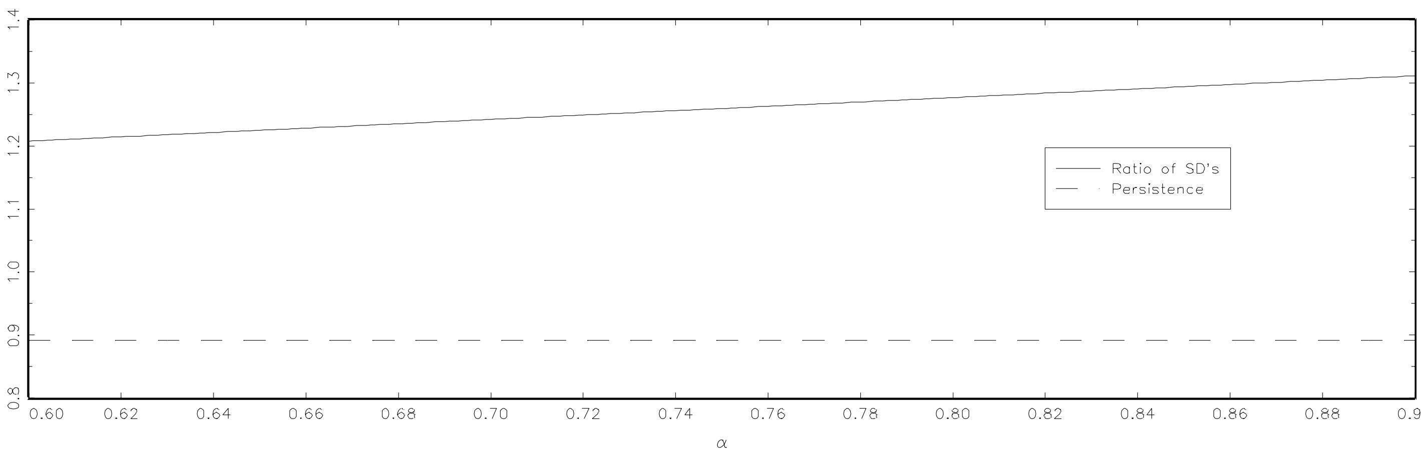
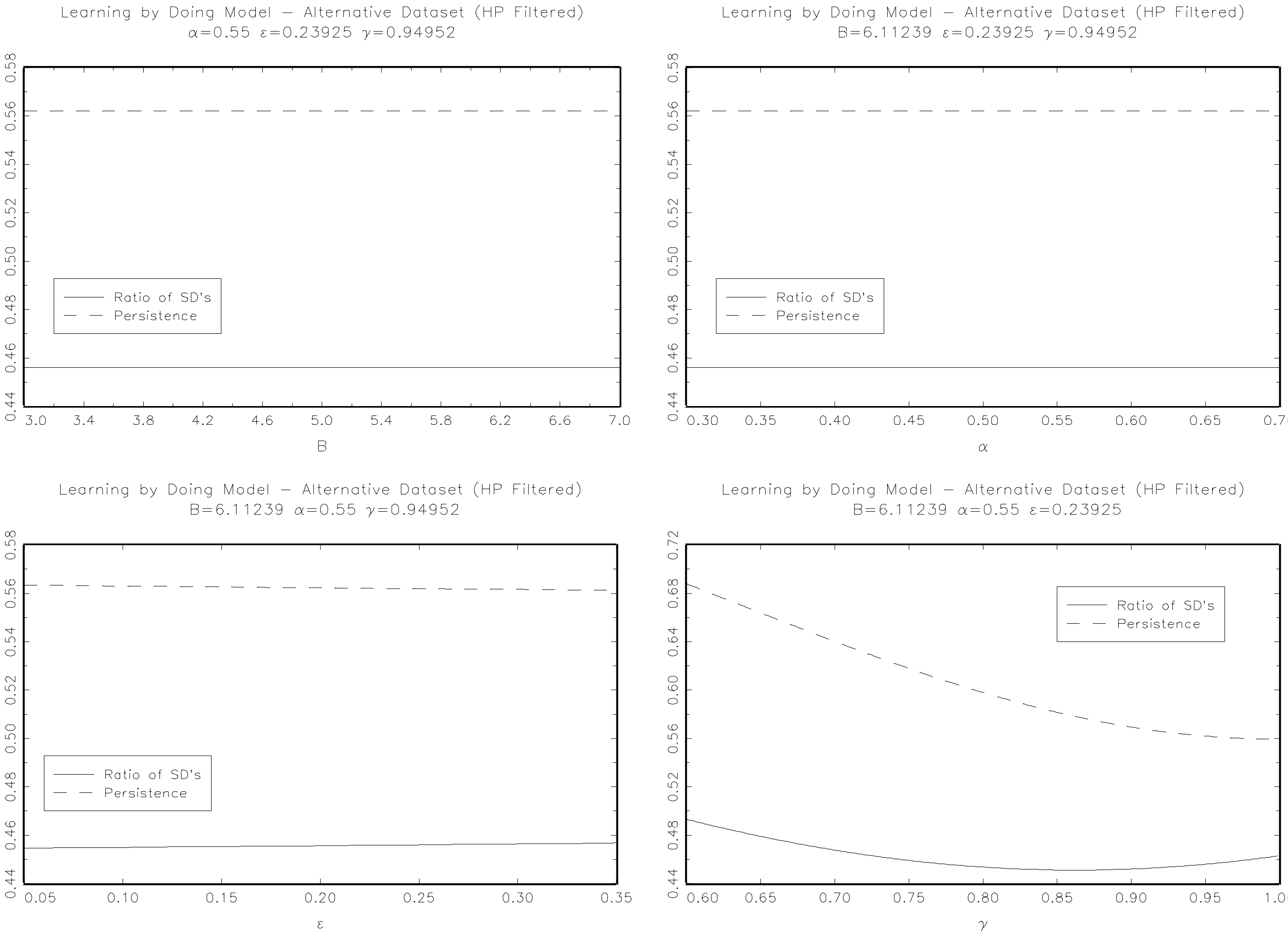


Figure II.4


Learning by Doing Model – Alternative Dataset (HP Filtered)
α=0.55 ε=0.23925 γ=0.94952
Ratio of SD's
Persistence
B
Learning by Doing Model – Alternative Dataset (HP Filtered)
B=6.11239 ε=0.23925 γ=0.94952
Ratio of SD's
Persistence
α
Learning by Doing Model – Alternative Dataset (HP Filtered)
B=6.11239 α=0.55 γ=0.94952
Ratio of SD's
Persistence
ε
Learning by Doing Model – Alternative Dataset (HP Filtered)
B=6.11239 α=0.55 ε=0.23925
Ratio of SD's
Persistence
γ