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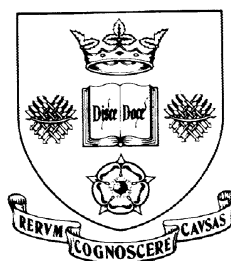
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Household's Preferences and Monetary Policy Inertia

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Key words: Optimal monetary policy; interest rate smoothing; household's preferences.

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1 Introduction

The estimation of monetary policy rules suggests that the interest rates set by the central banks (CBs) move with a certain inertia. In most empirical works aiming at estimating monetary policy reaction functions, the estimated coefficient (usually indicated with ρ) of a lagged interest rate term has been found to be fairly large (close to 1) and highly significant.

A few hypotheses, ranging from the theoretical to the empirical realms, have been suggested to explain this result. Some authors argue that it reflects the choice of optimising CBs that purposefully aim at smoothing the interest rate path. Other authors claim instead that the high estimated ρ's are the artifact of improper estimations. Thus, the debate on the ultimate origin of the estimated autoregressive term remains lively, thereby delineating this issue as a modern "puzzle" in monetary economics.

The rationales for a gradual movement of the interest rate over time could be several and complementary: i) some interest rate smoothing allows discretionary monetary policy equilibrium to approximate the superior commitment equilibrium[1]; ii) CBs prefer moving the policy rates with caution because they are uncertain about the correct model of the economy and are not endowed with accurately measured economic indicators[2]; iii) monetary authorities dislike the volatility induced into financial markets by frequent and large movements in the interest rates[3]; and iv) CBs dislike frequent reversions of interest rates since these hamper their reputation-building processes and communication strategies.[4]

All these rationales, which enter into the so-called conventional wisdom on interest rate inertia, are challenged by those who maintain that the estimated interest rate inertia is in fact the product of mis-specified monetary policy reaction functions. The estimated inertia, they claim, is an empirical illusion and implies no policy gradualism at all. Rudebusch (2002 and 2006) argues that the deviations of actual interest rates from those predicted by a Taylor rule (without any auto-regressive compo-

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[1]See Woodford (1999) and (2003) and Amato and Laubach (1999).

[2]See for instance Orphanides (1998), Sack and Wieland (2000), and Aoki (2003).

[3]See for example Goodfriend (1987) and Levin et al (1999).

[4]See Caplin and Leahy (1997) and Goodhart (1999).

nent) "represent persistent influences on CB behaviour that are not captured in a simple Taylor-type rule"(2006 p. 90). This view is also supported by Consolo and Favero (2009). These authors relate the weak instruments problem to the correct identification of the degree of monetary policy inertia; by addressing the weak instruments problem, the authors show that the estimated degree of policy inertia is significantly lower than what usually found in the empirical literature on monetary policy rules. Similarly, Lansing (2002) and Carare and Tchaidze (2005) show that the inclusion of ill-measured determinants in the estimated rule (due for instance to measurement errors and the incorrect choice of indices and time horizons), as well as the mis-specification of the rule, may lead to a high estimated coefficient for the lagged interest rate term even when there is actually no inertia in the policy-making process.

These criticisms cast a doubt on interpreting the high estimates of ρ as signals of intentional policy smoothing on the part of the CBs. However, neither theoretical nor empirical studies have so far been able to provide a definitive account of the issue.[5] Against this background, Corrado and Holly (2004) maintain that the observed interest rate inertia may in fact be due to the macroeconomic inertia in the economy. Rudebusch (2002), Castelnuovo (2006) and Driffill and Rotondi (2007) investigate how different characteristics of the private sector can generate optimal monetary inertia by means of empirical, reduced-form New Keynesian models and conclude that the aggregate demand plays a crucial role. The adoption of empirical models, however, has two shortcomings. First, it prevents from referring any findings to the deep parameters characterising agents' behaviour. Second, it abstracts from the constraints on the reduced-form coefficients that stem from the structural relations of the theoretical New Keynesian framework.

In this work we make a step forward by distinguishing the relative importance of

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[5]Söderström et al (2005), for instance, show that Rudebusch's interpretation (i.e. smoothing is the by-product of serially correlated shocks) is inconsistent with the high predictability of the output gap found in actual data. As argued by Castelnuovo (2003), Gerlach-Kristen (2004) and Alcidi et al (2009), it is most likely that both intentional policy inertia and some mis-specification problems contribute to determine the high estimated values of ρ.

the agents' preferences, while keeping into account the restrictions imposed by the structure of the theoretical model. Adopting a theoretical New Keynesian framework, we investigate the relationship between the deep parameters characterising the preferences of the household and the degree of optimal monetary policy inertia, measured by the variability of the interest rate changes[6]. We find that the elasticity of intertemporal substitution (EIS) and the household's persistence in consumption (HPC) considerably affect optimal interest rate inertia via their impact on the efficiency of the monetary policy transmission mechanism. This perspective is important given that empirical studies provide contentious values for the deep parameters underlying the household's preferences[7]. In fact, potentially different sets of parameters, and therefore different preferences of the household, could account for various degrees of inertia.

The remainder of the paper proceeds as follows. In section 2 we shall present the model and the results. Section 3 will discuss and explain the results, comparing them with those in the relevant literature. Section 4 will conclude.

2   The model

We develop our analysis in a basic New Keynesian framework where the CB determines the optimal monetary policy under discretion. We choose the discretion set up because, besides being consistent with what usually maintained by policy-makers, it leads to a less inertial policy response compared to the alternatives of i) commitment and ii) commitment in a timeless perspective. Abstracting from the link between commitment and interest rate smoothing allows a better understanding of the relationship between the preferences of the private sector and the optimal interest rate inertia.

In the stylised economy we consider, agent j belongs to a continuum of unit mass of identical households, each producing a different variety of a consumption good

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[6]It is worth noting that the paper does neither dwell on the welfare properties of interest smoothing as in Woodford (1999, 2003), nor on the predictability of interest rates, as in Rudebusch (2002, 2006).

[7]See for instance Okubo (2008a and 2008b) on the estimates of the intertemporal elasticity of substitution in consumption.

indexed by j. For the sake of simplicity, but without loss of generality, we assume preferences only on consumption. These preferences are parametrised by the elasticity of intertemporal substitution, i.e. σ, and a measure of inertia in the household's preferences that we assume to be captured by the degree of habit persistence, i.e. κ. Following Abel (1990), we write the household's utility function as

$$E_t \sum_{\tau=0}^{\infty} \delta^{\tau} U\left(C_{t+\tau}, \overline{C}_{t+\tau}\right), \qquad U\left(C_{t+\tau}, \overline{C}_{t+\tau}\right) = \frac{\left(C_t/\overline{C}_{t-1}^{\kappa}\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \qquad \sigma, \kappa \in (0,1),$$

where δ is the intertemporal discount factor, C_t is total consumption of consumer/producer j, which is a composite good given by the Dixit-Stiglitz aggregator of the continuum of varieties of the consumption good, $C_t \equiv \left[\int \left(C_t(j)\right)^{1-\frac{1}{\vartheta}} dj\right]^{\frac{1}{1-\vartheta}}$. The parameter $\vartheta > 1$ denotes the elasticity of substitution among any two differentiated varieties, and $\overline{C}_t$ is total aggregate consumption.

In order to build a realistic transmission mechanism able to incorporate a one-period lag in the transmission of monetary policy interventions to the real activity, we assume one-period ahead predetermined consumption decisions. Thus, preferences maximization subject to a budget constraint and a no-Ponzi condition leads to the following log-linearised Euler equation

$$c_{t+1} = \beta c_t + (1-\beta) c_{t+2|t} - (1-\beta) \sigma \left(i_{t+1|t} - \pi_{t+2|t}\right) + \eta_{t+1}, \quad \beta \equiv \frac{\kappa(1-\sigma)}{1+\kappa(1-\sigma)}, \quad (1)$$

where c_t is consumption, π_t is inflation, i_t is the short term nominal interest rate, and η_t is a demand shock. For any variable x, the expression $x_{t+\tau|t}$ stands for the expected value of that variable in period $t+\tau$ conditional on the information available in period t; lower case variables denote log-deviations from their respective constant steady-state values. As in Flamini (2007), equation (1) can be solved forward obtaining the following equation

$$c_{t+1} = \kappa(1-\sigma) c_t - \sigma \overline{r}_{t+1|t} + \eta_{t+1}, \qquad \overline{r}_t \equiv \sum_{\tau=0}^{\infty} \left(i_{t+\tau|t} - \pi_{t+\tau+1|t}\right), \qquad (2)$$

which will be referred to in the discussion below.

As said, we assume that each agent j produces a differentiated variety using the composite consumption good C. Letting Y^p be the total quantity of consumption C

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used in the economy for production, the aggregate demand Y_t can be written as

$$Y_t = C_t + Y_t^p. \tag{3}$$

Then, we also assume that variety j is produced according to

$$Y_t(j) = f(Y_t^p), \qquad \forall j, \tag{4}$$

where f is an increasing, concave, and isoelastic production function. Hence the total quantity of good C used in production turns out to be related to the aggregate demand by the relationship

$$Y_t^p = f^{-1}(Y_t). \tag{5}$$

Log-linearising (3) around the steady state values and using (5), one obtains

$$y_t = c_t \tag{6}$$

where y_t denotes the log-deviation of the aggregate demand from its steady-state[8]. Thus, accounting for the Euler equation (2), the aggregate demand can be appropriately rewritten as

$$y_{t+1} = \kappa(1 - \sigma) y_t - \sigma \bar{r}_{t+1|t} + \eta_{t+1}. \tag{7}$$

The supply side of the economy is summarised by the following New-Keynesian Phillips curve, which is derived in the Appendix:

$$\pi_{t+2} = \frac{1}{1+\zeta} \left[\zeta \pi_{t+1} + \pi_{t+3|t} + \frac{(1-\alpha)^2}{\alpha(1+\omega\vartheta)} \omega y_{t+2|t} \right] + \varepsilon_{t+2}, \tag{8}$$

where $(1 - \alpha)$, using the Calvo (1983) staggered prices scheme, is the probability that in any period the consumer/producer chooses an optimal price; ζ, following Christiano et al. (2005), is the indexation to previous period inflation when it is not possible to up-date optimally the price of a variety; ω is the output elasticity of the marginal input requirement function; and ε is a zero-mean i.i.d. cost-push shock. In order to have a realistic two-period lag in the transmission of monetary policy to inflation, pricing decisions are assumed to be predetermined two periods in advance.

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[8]See the Appendix for details.

We point out that predetermined decisions in consumption and production are important to generate realistic transmission lags of the monetary policy to real activity and inflation. This feature of the model is in line with the conventional wisdom at CBs.

The model is closed assuming that the CB finds a policy that maximizes its preferences under the assumption of discretion and subject to the law of motion of the economy given by the aggregate relations (7) and (8). The CB's preferences are represented as a quadratic loss function depending on the volatility of inflation and output gap around their steady-state values, and on the volatility of the interest rates changes. This loss function can be written as

$$E_t \sum_{\tau=0}^{\infty} \beta^\tau \left[\pi_{t+\tau}^2 + \lambda y_t^2 + \nu \left(i_t - i_{t-1} \right)^2 \right],$$

where λ and ν are the weights expressing, respectively, the preferences for the stabilisation of the output gap and for the smoothing of the interest rate path in terms of those for the stabilisation of inflation.

At any period, the CB minimises its loss choosing a path for the short-term nominal interest rate which turns out to be a linear function of the state variables. In this economy, the equilibrium is a set of sequences, $\left\{ i_{t+\tau|t}, \ \pi_{t+\tau|t}, \ y_{t+\tau|t}, \right\}_{\tau=0}^{\infty}$, conditional to the structural parameters of the model[9].

3 Household's and CB preferences, and interest rate inertia

Several works investigating the degree of interest rate inertia by means of empirical versions of a reduced-form new Keynesian model reached the conclusion that the aggregate demand plays a crucial role. In what follows we will make a step forward and try to distinguish the relative importance of the agent's preferences, while keeping into account the restrictions imposed by the structure of the theoretical model on the interaction of the deep parameters.

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[9]To solve the optimization problem we use the algorithm developed by Oudiz and Sachs (1985) as well as by Backus and Driffill (1986); a detailed description is provided by Soderlind (1999).

Accordingly, our analysis will focus on how two basic parameters belonging to the demand side, i.e the EIS (σ) and the persistence in the household's behavior (κ), affect the optimal degree of interest rate inertia, defined as the optimal unconditional variability of interest rate changes and measured by the standard deviation of the first difference of the interest rates, i.e. $std\left(\Delta i_t\right)$.

In this exercise we shall start considering the impact on $std\left(\Delta i_t\right)$ of changing one parameter at the time while keeping the other fixed. Notably, we take into account the limitations that fixing the values of some parameters imposes on the admissible range of variability of those we change. Specifically, the ranges of values for κ and σ we investigate are consistent with the empirical estimates on the reduced-form coefficient in the aggregate demand equation. Figure 1, panel a presents the relationship between σ, the preferences of the CB over smoothing the interest rate path (i.e. ν), and the interest rate inertia $std\left(\Delta i_t\right)$, while panel b presents the relationship between the κ, the preferences of the CB over smoothing the interest rate path (i.e. ν), and the interest rate inertia $std\left(\Delta i_t\right)^{10}$.

Panel a shows that both the EIS and the CB preferences for smoothing the interest rate contribute to determine the size of the optimal $std\left(\Delta i_t\right)$. It also reveals that, in presence of not too large smoothing preferences, σ plays an important role in determining the interest rate inertia. Furthermore, it indicates that the degree of volatility in interest rates changes remains limited as long as the EIS is large enough. Focusing on HPC, panel b reveals that κ is negatively related to the degree of interest rate inertia. It also shows that when we reduce to the minimum the role played by the CB preferences for smoothing the path of the interest rate, the amount of optimal interest rate inertia varies a lot according to the persistence in the household's behavior.

Figures 1 (based on Table 1 reported in the Appendix) reveals that high CB preferences for smoothing the interest rate, which could be due to the various ra-

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[10]The structure of the current model is similar to Svensson (2000). Thus, we follow Svensson in the choice of the structural parameters that are fixed, i.e. $\alpha = 0.5$, $\vartheta = 1.25$, $\omega = 0.8$, $\sigma^\eta = 1$, and $\sigma^\varepsilon = 1$, and $\lambda = 0.5$. The remaining parameter is ζ which we set equal to 1 as in Boivin and Giannoni (2006). In Figure 1, panel a, we also set $\kappa = 0.5$ and in panel b, $\sigma = 0.2$.
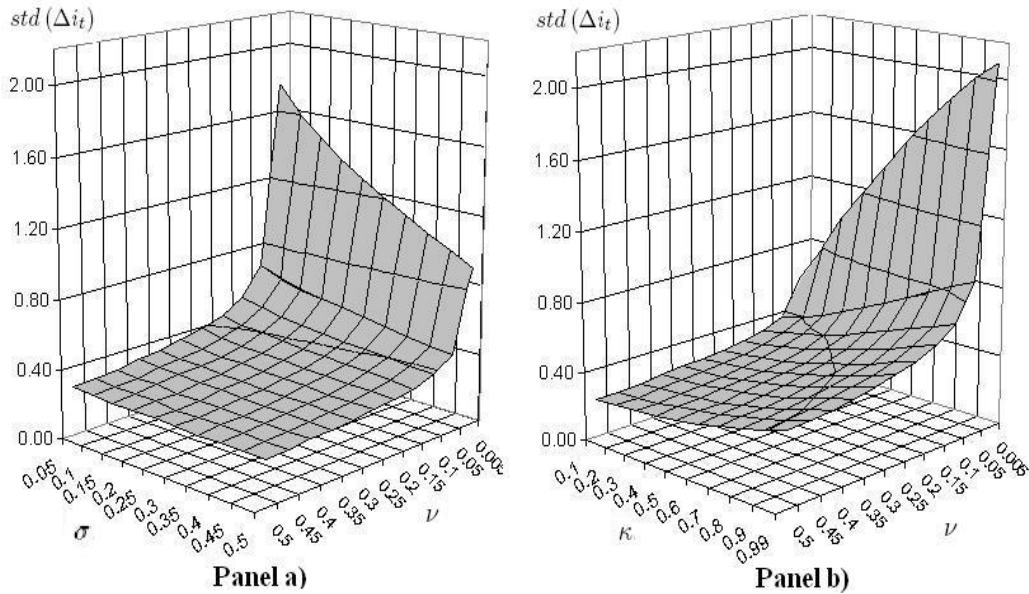
Figure 1: The relationship between σ, κ, ν and $std\left(\Delta i_t\right)$

tionales discussed in the Introduction, tend to limit the role played by other factors in determining the degree of interest rate inertia. It is thus interesting to observe what happens to the interaction of σ, κ and $std\left(\Delta i_t\right)$ when the relative importance of the smoothing component in the CB loss function becomes negligible. Notably, we do not argue to go as far as striking the concern for interest rate smoothing out of the CB loss function. In fact, even if the rationales for smoothing were not actually contemplated by the central bankers (so that they did not really have an important smoothing preference per se), it would still be reasonable to postulate a small value of ν in the loss: this guarantees that monetary policy does not become unrealistic, that is characterised by huge and frequent reversals in the interest rate path. Accordingly, in what follows we shall consider a tiny, yet positive value of the smoothing preferences.[11]

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[11]The smallest value of ν we consider, i.e. 0.005, is 20 times smaller than that usually employed to represent little CB preferences for the smoothing component (see Rudebusch 2002).

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3.1 The role of the transmission mechanism and the forward-lookingness of the agents in determining optimal policy inertia

The findings discussed above depend on how σ and κ affect the transmission mechanism of the monetary policy when the CB wishes to stabilise the economy with a realistic, i.e. not too aggressive, policy. To explain the mechanism through which σ and κ affects $std\left(\Delta i_t\right)$, it is useful to rewrite equation (2) as

$$c_{t+1} = -\sigma \bar{r}_{t+1|t} - \sigma\left[\kappa\left(1-\sigma\right)\right]\bar{r}_t - \frac{\sigma\left[\kappa\left(1-\sigma\right)\right]^2}{1-L\kappa\left(1-\sigma\right)}\bar{r}_{t-1} + \eta_{t+1}, \qquad L \equiv \text{Lag operator} \tag{9}$$

Equation (9) splits consumption in three components. The first two embed the most recent plans decided by the CB, i.e. $\left\{i_{t+\tau|t}\right\}_{\tau=0}^{\infty}$, that affect consumption in period $(t+1)$; we call these terms "recent past policy". The third term embeds the plans decided in all the periods before and including $(t-1)$, and we call it "remote past policy".

We now define the impact on c_{t+1} of each of the two components of the *recent past* monetary policy relative to the *remote past* monetary policy as the ratios

$$R^{\bar{r}_{t+1|t}} = \frac{1-L\kappa\left(1-\sigma\right)}{\left[\kappa\left(1-\sigma\right)\right]^2}, \qquad R^{\bar{r}_t} = \frac{1-L\kappa\left(1-\sigma\right)}{\kappa\left(1-\sigma\right)}.$$

These ratios can be interpreted as measures of the relative efficiency of the most recent policy actions. Clearly, the lower $R^{\bar{r}_{t+1|t}}$ and $R^{\bar{r}_t}$, the less efficient is the transmission mechanism of the recent policies with respect to the old ones. Since these ratios are positively and negatively related to σ and κ, the lower σ and/or the larger κ, the smaller is the impact that the CB can exert on the decisions of the household in any period[12]. Thus, when the EIS is small, the CB needs to move aggressively the interest rate to affect the household's decisions. Similarly, the larger the HPC, the more the past matters for the household, and the more difficult it is for the CB to offset unforeseen shocks by means of small interest rate changes.

A relation between persistence in consumption and interest rate smoothing is also found by Corrado and Holly (2004). Interestingly, in their work this relationship

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[12]The first derivative of $R^{\bar{r}_{t+1|t}}$ and $R^{\bar{r}_t}$ with respect to σ and κ are $R_\sigma^{\bar{r}_{t+1|t}} > 0$, $R_\kappa^{\bar{r}_{t+1|t}} < 0$ and $R_\sigma^{\bar{r}_t} > 0$, $R_\kappa^{\bar{r}_t} < 0$.
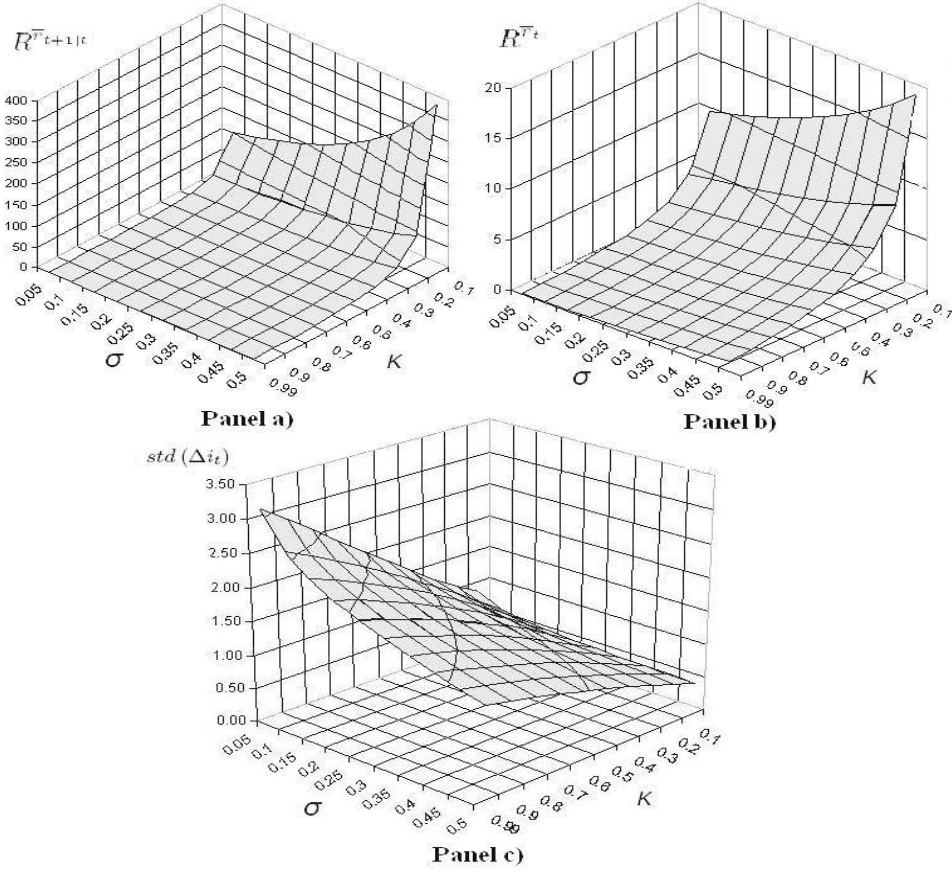
Figure 2: The relationship between σ, κ, $R^{\bar{r}_{t+1|t}}$, $R^{\bar{r}_t}$ and $std\left(\Delta i_t\right)$

is generated by the maximisation of the household welfare, which is decreasing in interest rate changes due to the presence of multiplicative habits in consumption. In our work, instead, this relationship is generated by the negative impact of habits on the efficiency of the monetary policy transmission mechanism. It is therefore independent of the household's welfare.

Our result is also in line with the empirical findings of Castelnuovo (2006), who relates the interest rate smoothness to the agents' degree of forward-lookingness. The current analysis, however, makes two steps forward by i) explaining the relation between the agents' degree of forward-lookingness, which is inversely related to κ and the policy inertia, and ii) showing that this is modulated by σ. Figure 2 (based on Table 2 reported in the Appendix) illustrates the relationship between σ, κ, $R^{\bar{r}_{t+1|t}}$, $R^{\bar{r}_t}$ and $std\left(\Delta i_t\right)$.
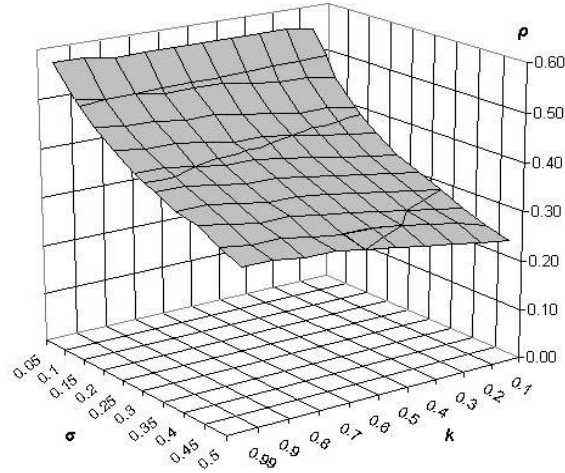
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Figure 3: The relationship between σ, κ and ρ

Summing up, these results suggest that household's preferences can have a relevant impact on optimal interest rate inertia besides the influence of the interest rate smoothing concern on the part of the CB.

This mechanism is also reflected in the optimal coefficients of the reaction function, among which that for the lagged interest rate, i.e. ρ. Indeed, ρ is the tool that allows the policy-makers to adjust the aggressiveness of the monetary policy according to the efficiency of the transmission mechanism. Figure 3 (based on Table 3 reported in the Appendix) shows, for instance, the impact of σ and κ on ρ.

As can be seen, when σ increases, the optimal ρ tends to decrease: since the transmission mechanism becomes more efficient in transmitting the most recent policy so that the bank needs not to implement large changes in the interest rate, a higher interest rate inertia (i.e. a smaller $std\left(\Delta i_t\right)$) is achieved with an optimal reaction function in which the previous interest rate enters with a smaller weight.

We note that κ has a minor impact on ρ. Clearly, this finding is not at odds with the important impact of κ on the optimal degree of policy inertia $\left(std\left(\Delta i_t\right)\right)$ discussed above. Rather, it reveals that the high degree of policy inertia due to large values of κ does not necessarily mirrors in a large ρ. Indeed, at any change in the structure of the economy, all other coefficients of the reaction function can change together with ρ. This suggests that some caution is necessary in interpreting the results of those studies on the interest rate smoothing that look exclusively at ρ.

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Our results shed some light on the role played by the degree of forward-lookingness of the agents with respect to interest rate inertia. We find that the degree of inertia in consumption contributes to a limited extent to the determination of ρ, and yet it does affect the degree of inertia in monetary policy. It could be argued that this conclusion is partially at odds with some findings in Rudebusch (2002), who claims that the degree of forward-lookingness of the agents matters a lot for ρ. The difference in the results depends on two main factors. First, as said above, our analysis only focuses on the extent of variation of the deep parameters that is consistent with the constraints imposed by the micro-founded structure of our model. Second, while Rudebusch attributes the degree of forward-lookingness of the AD in the reduced-form model to the degree of forward-lookingness of the agents, the former may depend, as we show here, on both the latter and the elasticity of intertemporal substitution.

4   Concluding remarks and policy implications

This paper investigates the relation between the household's and CB preferences and the optimal interest rate inertia. Using a theoretical new Keynesian model, allows us to draw a few interesting results which contribute to the literature on interest rate inertia.

First, the intertemporal elasticity of substitution in consumption and the degree of persistence in the household's behavior play an important role in determining the level of optimal monetary policy inertia. In particular, when the CBs do not have a special reason to smooth the interest rate beyond avoiding a roller coast policy, economies featuring different household's preferences can exhibit important differences in the degree of interest rate inertia.

Second, this contribution shows that interest rate inertia does not necessarily depend on high CB preferences for smoothing the path of the interest rate. In fact, it can also be associated to either a large elasticity of substitution, or a high degree of forward-lookingness in the household's behavior, or both.

As we argue, these results are grounded on the impact of the household's preferences on the transmission mechanism of the monetary policy. When the elasticity

of substitution is small and/or the degree of forward-lookingness of the household is low, the impact of the most recent monetary policy on the household's decisions is low relative to the impact of past policies. Thus, the CB needs a more aggressive policy to buffer unforeseen shocks. As a consequence, the volatility of interest rate changes increases.

Acknowledgements

5   Appendix

Details on the derivation of the aggregate demand

To derive equation (6) first log-linearize (3) around steady state values $\left(\overline{Y}, \overline{C}, \overline{Y^p}\right)$ to obtain

$$y_t = \phi c_t + (1 - \phi) y_t^p, \qquad \phi \equiv \frac{\overline{C}}{\overline{Y}}.$$

Then, using (5) and recalling that the function f is isoelastic, leads to

$$y_t = \phi c_t + (1 - \phi) \xi y_t,$$

where the ξ is the output elasticity of the input requirement function (5). From which we obtain

$$y_t = \frac{\phi}{1 - (1 - \phi) \xi} c_t.$$

Finally, we assume that a percentage increase of the quantity produced requires the same percentage increase of the input[13], that is $\xi = 1$, and we obtain

$$y_t = c_t.$$

Derivation of the aggregate supply

This derivation of the aggregate supply follows Flamini (2007). Considering (4), for each firm j, the input requirement function is $Y_t^p = f^{-1}(Y_t(j))$ so that the

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[13]This is a reasonable assumption in the short run model used here.

variable cost of producing the quantity $Y_t(j)$ is $P_t f^{-1}[Y_t(j)]$. Since there is a Dixit-Stiglitz aggregate of consumption goods, the demand for each variety j is $Y_t(j) = Y_t \left(\frac{P_t(j)}{P_t}\right)^{-\vartheta}$, where $P_t(j)$ is the nominal price for variety j. Then, assuming that pricing decisions are made 2 periods in advance, firm j profit maximization problem is

$$\max_{\widetilde{P}_{t+2}} E_t \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \widetilde{\lambda}_{t+\tau+2} \left\{ \frac{\widetilde{P}_{t+2}\Psi_{t+\tau+1}}{P_{t+2+\tau}} Y_{t+\tau+2} \left[\frac{\widetilde{P}_{t+2}\Psi_{t+\tau+1}}{P_{t+2+\tau}} \right]^{-\vartheta} \right.$$
$$\left. -V\left[Y_{t+\tau+2} \left(\frac{\widetilde{P}_{t+2}\Psi_{t+\tau+1}}{P_{t+2+\tau}} \right)^{-\vartheta} \right] \right\},$$

where $\Psi_{t+\tau+1} \equiv \left(\frac{P_{t+\tau+1}}{P_{t+1}}\right)^\zeta$, $V \equiv f^{-1}$, and $\widetilde{\lambda}_t$, $\widetilde{P}_{t+2}$ denote the marginal utility of consumption, and the new price chosen in period t for period $t+2$, respectively. The first-order condition is

$$E_t \left\{ \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \widetilde{\lambda}_{t+\tau+2} \left[\frac{\widetilde{P}_{t+2}\Psi_{t+\tau+1}}{P_{t+2+\tau}} \right. \right.$$
$$\left. \left. -\phi V'\left(Y_{t+\tau+2} \left(\frac{\widetilde{P}_{t+2}\Psi_{t+\tau+1}}{P_{t+2+\tau}} \right)^{-\vartheta} \right) \right] Y_{t+\tau+2} \left(\frac{\widetilde{P}_{t+2}\Psi_{t+\tau+1}}{P_{t+2+\tau}} \right)^{-\vartheta} \right\} = 0$$

where $\phi \equiv -\frac{\vartheta}{1-\vartheta}$, which can be rewritten as

$$E_t \left\{ \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \widetilde{\lambda}_{t+\tau+2} \left[\frac{X_{t+2}\prod_{s=1}^\tau \Pi_{t+1+s}^\zeta}{\prod_{s=1}^\tau \Pi_{t+2+s}} \right. \right. \tag{10}$$
$$\left. \left. -\phi V'\left(Y_{t+\tau+2} \left(\frac{X_{t+2}\prod_{s=1}^\tau \Pi_{t+1+s}^\zeta}{\prod_{s=1}^\tau \Pi_{t+2+s}} \right)^{-\vartheta} \right) \right] Y_{t+\tau+2} \left(\frac{X_{t+2}\prod_{s=1}^\tau \Pi_{t+1+s}^\zeta}{\prod_{s=1}^\tau \Pi_{t+2+s}} \right)^{-\vartheta} \right\} = 0$$

where $X_{t+2} \equiv \frac{\widetilde{P}_{t+2}}{P_{t+2}}$ and $\Pi_{t+2} \equiv \frac{P_{t+2}}{P_{t+1}}$.

In equilibrium any firm that is free to choose the price in period t will choose the same price, $\widetilde{P}_t$, and the remaining firms that are not free to choose the price in period t will keep the previous period price updated to the previous period inflation. Thus, the aggregate price is given by

$$P_t = \left[\alpha \left(P_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^\zeta \right)^{1-\vartheta} + (1-\alpha)\widetilde{P}_t^{(1-\vartheta)} \right]^{\frac{1}{1-\vartheta}}$$
$$\iff \Pi_t = \left[1 - (1-\alpha)X_t^{(1-\vartheta)} \right]^{-\frac{1}{1-\vartheta}} \alpha^{\frac{1}{1-\vartheta}} \Pi_{t-1}^\zeta.$$

Then we log-linearize the first order condition around the steady state. Let us allow bounded fluctuations in $\left(\widetilde{\lambda}_t,\ X_t,\ \Pi_t,\ Y_t\right)$ around the steady state $\left(\widetilde{\lambda},\ 1,\ 1,\ Y,\right)$ and let small letters be log-deviations from their steady state value. Then we obtain

$$v' = \omega y\left(j\right),$$

$$\pi_t = \zeta \pi_{t-1} + \frac{1-\alpha}{\alpha} x_t, \tag{11}$$

where $\omega > 0$ is the elasticity of V' with respect to $y_t\left(j\right)$. Then, log-linearizing (10) yields

$$E_t\left\{\sum_{\tau=0}^{\infty}\alpha^\tau\delta^\tau\left[x_{t+2}-\sum_{s=1}^{\tau}\left(\pi_{t+2+s}-\zeta\pi_{t+1+s}\right)\right.\right.$$

$$\left.\left.-\omega\left(y_{t+\tau+2}-\vartheta\left(x_{t+2}-\sum_{s=1}^{\tau}\left(\pi_{t+2+s}-\zeta\pi_{t+1+s}\right)\right)\right)\right]\right\}=0$$

which can be rewritten as

$$E_t\left\{\sum_{\tau=0}^{\infty}\alpha^\tau\delta^\tau\left[\left(1+\omega\vartheta\right)\left(x_{t+2}-\sum_{s=1}^{\tau}\left(\pi_{t+2+s}-\zeta\pi_{t+1+s}\right)\right)-\omega y_{t+\tau+2}\right]\right\}=0 \tag{12}$$

Now note that

$$\sum_{\tau=0}^{\infty}\alpha^\tau\delta^\tau\sum_{s=1}^{\tau}\left(\pi_{t+2+s}-\zeta\pi_{t+2+s-1}\right)=\sum_{s=1}^{\infty}\left(\pi_{t+2+s}-\zeta\pi_{t+1+s}\right)\sum_{\tau=s}^{\infty}\alpha^\tau\delta^\tau$$

$$=\sum_{s=1}^{\infty}\left(\pi_{t+2+s}-\zeta\pi_{t+1+s}\right)\frac{\alpha^s\delta^s}{1-\alpha\delta}$$

$$=\frac{1}{1-\alpha\delta}\sum_{\tau=1}^{\infty}\alpha^\tau\delta^\tau\left(\pi_{t+2+\tau}-\zeta\pi_{t+1+\tau}\right).$$

Then, equation (12) can be rewritten as

$$E_t\left\{\frac{1+\omega\vartheta}{1-\alpha\delta}x_{t+2}-\frac{1+\omega\vartheta}{1-\alpha\delta}\sum_{\tau=1}^{\infty}\alpha^\tau\delta^\tau\left(\pi_{t+2+\tau}-\zeta\pi_{t+2+\tau-1}\right)-\sum_{\tau=0}^{\infty}\alpha^\tau\delta^\tau y_{t+\tau+2}\right\}=0$$

$$\Longleftrightarrow$$

$$E_t x_{t+2}=E_t\left\{\sum_{\tau=1}^{\infty}\alpha^\tau\delta^\tau\left(\pi_{t+2+\tau}-\zeta\pi_{t+2+\tau-1}\right)+\frac{1-\alpha\delta}{1+\omega\vartheta}\sum_{\tau=0}^{\infty}\alpha^\tau\delta^\tau\omega y_{t+\tau+2}\right\}$$

$$=E_t\left\{\alpha\delta\left(\pi_{t+2+1}-\zeta\pi_{t+2}\right)+\frac{1-\alpha\delta}{1+\omega\vartheta}\omega y_{t+2}\right\}+\alpha\delta E_t x_{t+2+1}$$

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Next, combining the previous equation with (11) we obtain

$$\frac{\alpha}{1-\alpha}E_t\left(\pi_{t+2}-\zeta\pi_{t+1}\right) = E_t\left\{\alpha\delta\left(\pi_{t+2+1}-\zeta\pi_{t+2}\right)+\frac{1-\alpha\delta}{1+\omega\vartheta}\omega y_{t+2}\right\}+\alpha\delta\frac{\alpha}{1-\alpha}E_t\left(\pi_{t+2+1}-\zeta\pi_{t+2}\right).$$

Finally, approximating δ by unit in order to ensure the Natural-rate hypothesis, rearranging, and adding the cost-push shock ε_{t+2} and $\frac{\zeta}{1+\delta\zeta}\varepsilon_{t+1}$ to the previous equation we obtain

$$\pi_{t+2} = \frac{\zeta}{(1+\zeta)}\pi_{t+1}+\frac{1}{(1+\zeta)}\pi_{t+3|t}+\frac{1}{(1+\zeta)}\frac{(1-\alpha)(1-\alpha)}{\alpha(1+\omega\vartheta)}\omega y_{t+2|t}+\varepsilon_{t+2}$$

Solution of the model

The CB problem consisting in maximizing the CB preferences subject to the aggregate demand and supply is solved using the algorithm developed by Oudiz and Sachs (1985) as well as by Backus and Driffill (1986); a detailed description of this numerical method is provided by Soderlind (1999).

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