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Abstract

The paper develops a model with non-exponential population growth, non-renewable natural resources, and endogenous knowledge creation to analyse economic development in the medium and long run. We further assume poor substitution between primary inputs and an essential use of resources in the innovation sectors, which is generally considered as most unfavourable for growth. We show that population growth and poor input substitution are not detrimental but even needed to obtain sustainable consumption. A permanent increase in living standards can be achieved under free market conditions. With a backstop technology, the system converges to a balanced growth path with classical properties.

Keywords: Population growth, non-renewable resources, poor input substitution, technical change, sustainability

JEL-Classification: Q32, Q55, Q56, O41

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1 Introduction

Population growth and natural resource scarcity are often perceived as severe threats to sustainable development. World population is currently growing fast and will continue to grow in the future. It is confronted with a natural resource supply that is ultimately limited. Declining oil production in several regions such as the North Sea and reports about proven reserves which are lower than previously estimated are clear indications of the boundaries. Even when other energy and raw material deposits have been less exploited so far, total use of natural resources and energies will have to shrink in future centuries. This is a fundamental change in economic history because up to now, the expanding world economy has relied on growing resource input.

The Malthusian perception of population growth is well represented in the literature, see e.g. Meadows et al. (1972) and Ehrlich and Ehrlich (1990). The neo-classical growth model and capital resource models based on neo-classical assumptions give rise to similar predictions, see the seminal contributions of Solow (1974), Stiglitz (1974), Dasgupta and Heal (1974, 1979) and in particular Dasgupta (1995) for an extensive treatment. Regarding resource constraints, finance ministers and central bank heads from the world's seven largest economies said that the high oil prices were a threat to global prosperity, see Financial Times (2008). However, recent growth theory emphasises that labour not only uses resources but also has the capacity to build resource substitutes, notably knowledge. Moreover, the size of the labour force may determine the intensity of dynamic scale effects. But are these new elements in theory powerful enough to change the general perception of population growth and resource scarcity?

The present paper asks whether and how it is possible to obtain positive innovation and consumption growth under free market conditions even when population is growing and resource stocks are bounded. The model has the following features, which build on empirical regularities. First, unlike the majority of existing literature, the model does not postulate that population grows at a constant exponential rate. Instead, we assume a dynamic law reflecting demographic transition, which extends the standard framework in resource models. Endogenous population growth and knowledge are also treated in Kremer (1993), but there natural resources and the demographic transition are disregarded. In the resource context, population dynamics have recently been analysed in Asheim et al. (2007) who assume (exogenous) quasi-arithmetic population growth. Second, non-renewable resources are assumed to be an essential input in all sectors of the economy, including the innovation sector. This is usually not considered, with the exception of Groth and Schou (2002, 2007), who argue that resources are an important element in the technologies of present-day economies. Endogenous innovations drive the growth process but are severely constrained by natural resources; resource use restricts the emergence of productive knowledge and plays a similar role as the scarce investment funds in the theory of recombinant growth, see Weitzman (1998) and Tsur and Zemel (2007). The innovation sector supplements the rest of the economy by producing intermediates and final consumer goods, see especially Romer (1990) and Grossman and Helpman (1991) for the theoretical foundations and Bovenberg and Smulders (1995), Scholz and Ziemes (1999), Smulders (2000), Grimaud and Rougé (2003), and Xepapadeas (2006) for the combination with resource economics.

Third, the model assumes poor substitution between inputs in intermediates production. This reflects that, in empirical studies, the elasticity of substitution between natural resources and other inputs, specifically labour and capital, is estimated to be less than unity, see e.g. Christopoulos and Tsionas (2002) and Kemfert (1998). Poor input substitution is often disregarded in resource models because of its complexity; it has been used in Bretschger (1998) for renewable resources and in Bretschger and Smulders (2006) for exhaustible resources and a constant population. Fourth, sectoral change, which impacts resource use, will be reproduced by the model. Economies are undergoing a substantial structural change during long-run development. Between 1979 and 2002, the share of total employment in manufacturing decreased by 30 % in Europe and 34 % in the US, while employment in the research sector rose by 28 % in Europe and 40 % in the US, see GGDC (2004). In López, Anriquez and Gulati (2007) structural change is identified as a major topic in the sustainability debate. Fifth, physical capital has no impact on the growth rate in this model, because the scope for physical capital build-up is limited because of material balance constraints, as emphasised by Cleveland and Ruth (1997). Population growth, non-renewable resources, and most of the model's assumptions might be called "unfavourable" conditions for development: they seem to limit both the scope for input substitution and the capacity to accumulate capital as a compensation for lower resource use. Nevertheless, the present paper shows that sustainable growth is feasible under these conditions.

We find that issues, which have been described as critical (or even lethal) before, turn out to be superable, neutral, or even positive under the assumptions of the model, which explains the qualification "seemingly unfavourable" in the title of the paper. In particular, it will turn out that population growth is not detrimental for growth but even needed to ensure enough innovation. This helps the economy during the transition phase and increases the chance of developing a backstop technology, which is favourable in the long run. Specifically, the capital-producing effect of labour is highly useful to compensate for fading resource use in research. In addition, poor input substitution fosters sectoral change, which turns out to be a central mechanism sustaining economic growth.

Our general results are in line with earlier contributions, mainly Boserup (1965) who found a positive impact of population density on development, Simon (1981) who labelled labour, i.e. imagination coupled to the human spirit, as "ultimate resource" and Johnson (2001) who emphasised the role of knowledge for development with a growing population. The present paper provides a coherent model-based foundation of their reasoning. When introducing non-renewable resources, we primarily think of fossil fuels and, in a somewhat broader sense, of energy supplies. However, one can interpret the resource input in a broader fashion, as the world as a materially closed economy is confronted with a fixed supply of raw materials needed for physical capital, housing etc. In addition, basic needs like food have an essential material component.

The model has three peculiar features that differentiate it from most of the existing literature. It (i) uses a specific law of motion for population, (ii) introduces the essential use of a non-renewable resource in all sectors of the economy, and (iii) assumes poor input substitution in the intermediate goods sector, which determines consumer goods production. The combination of these assumptions entails that structural change becomes an important ingredient of development. Accordingly, we first focus extensively on the transition phase before turning to the long run steady state. This is in contrast to most growth models, but it is the appropriate procedure here as the adjustment may take more than a century. Interestingly, the nature of the steady state depends on the characteristics of the transition phase, so that development becomes path-dependent. A crucial element of long-run development is the possible emergence of a so-called backstop technology, i.e. a (perfect) substitute for non-renewable resources, see Tsur and Zemel (2005). We will also include this technology in the analysis, although in a very basic fashion, similar to Dasgupta, Heal and Majumdar (1977).

The remainder of the paper is organised as follows. Section 2 develops the model with natural resource use and endogenous innovations. Section 3 presents the results for transitional dynamics and for different scenarios regarding population growth. In section 4, the nature of the long-run equilibrium is analysed. Finally, section 5 concludes.

2 The model

The framework uses a standard expansion-in-varieties approach to model growth through innovations. Labour and non-renewable natural resources, which depict material inputs, are introduced as primary input factors. Differentiated intermediate services are the inputs for final goods production and knowledge capital is accumulated by endogenous R&D-activities through positive spillovers. Innovations are embodied in new intermediate goods varieties. They increase the productivity of the aggregate intermediate input. For the long run, a possible switch in technologies is evaluated to consider the effects of backstop technologies. Through this setting, the simplest case of a sectoral economy with endogenous innovations can be depicted in a very basic yet general way. The simultaneous motion of the three stocks knowledge, resources, and population drives the final results.

2.1 Firms

The model economy consists of three different sectors, which are R&D, intermediate services, and final goods, each with a different type of operating firm. R&D firms use labour L and non-renewable resources R as rival inputs and public knowledge κ as non-rival input to produce incremental technical change. Specifically, they generate the know-how for new intermediate goods in the form of designs. n denotes the number of intermediate goods at each point in time. With $\dot{n}$ denoting the derivative of n with respect to time and L_g and R_g the labour and resource inputs into R&D, the production of new designs $\dot{n}$ is given by:

$$\dot{n} = L_g{}^{\alpha} \cdot R_g{}^{1-\alpha} \cdot \kappa \qquad (0 < \alpha < 1) \qquad . \qquad (1)$$

Time indices are omitted whenever there is no ambiguity. According to (1), R and L are both essential inputs into research. This reflects the observations that research institutions use, besides labour, fossil fuels for heating and transportation or mineral products or other materials for machines and experiments, and that the cost shares in research do not change much (i.e. are constant) over time. With positive spillovers from R&D to public knowledge, we get $\kappa = n^{\eta}$ where η denotes the intensity of the externalities; with proportional spillovers (see Romer 1990 and Grossman and Helpman 1991) we have $\eta = 1$ so that $\kappa = n$, which will be used below. Consequently, the growth rate of the number of designs g becomes:

$$g = \frac{\dot{n}}{n} = L_g^{\alpha} \cdot R_g^{1-\alpha} \quad . \tag{1'}$$

With perfect competition in the research sector, the market value of an innovation p_n equals the per-unit costs of designs, which depend on the labour wage w, the resource price p_R and n:

$$p_n = \left(w/\alpha\right)^{\alpha} \cdot \left(p_R/(1-\alpha)\right)^{1-\alpha} / n \quad . \tag{2}$$

The backstop technology to substitute for natural resources is generated in the same research sector. It is generally perceived that a backstop technology can only emerge when a lot of knowledge is accumulated and the research efforts are highly intensive. Accordingly, in the model, the successful development of the backstop is tied to two conditions. The first is that the accumulated knowledge in the economy has to exceed a critical level, i.e. $\kappa(t) \geq \overline{\kappa}$, where t is the time index. Second, it requires a critical research intensity in the economy, i.e. $g(t) \geq \overline{g}$. As soon as the backstop is available at time $\overline{t}$, it fully substitutes for the resource at current market prices. If it is never available, we have $\overline{t} = \overline{\kappa} = \infty$.

Y-firms assemble intermediate goods x_i on fully competitive markets to final output Y under a CES-production function restriction; i is used as an index with $i \in [0, n]$. Provided that the costs to produce x_i-goods are equal for all x-firms, we obtain $x_i = x \; (\forall x_i)$ so that Y is determined by:

$$Y = \left(\int_0^n x_i^{\beta} di \right)^{\frac{1}{\beta}} = n^{\frac{1-\beta}{\beta}} X \tag{3}$$

$$(X = n \cdot x; \quad 0 < \beta < 1)$$

In (3), the gains from diversification, given by $(1-\beta)/\beta$, determine the impact of additional varieties n on output Y (and the effect of the innovation rate g on consumption growth). n has to be interpreted as a productivity index for total input of intermediates X in Y-production; it emerges from the symmetry assumption in the CES function so that (3) is clearly distinct from a Cobb-Douglas function. Intermediate goods firms use L and R as inputs to produce intermediate goods under the restriction of a CES production function:

$$X = \left[\lambda \cdot L_X^{(\sigma-1)/\sigma} + (1-\lambda) \cdot R_X^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \tag{4}$$

$$(0 < \lambda, \sigma < 1)$$

with σ being the elasticity of substitution between *L* and *R*, assumed to be lower than unity. (4) reflects the relevant input substitution process governing the dynamic behaviour of the economy, while the relationship between *n* and *X* in (3) determines the efficiency of final goods production. Input substitution is supplemented by intersectoral substitution when inputs move from intermediates to research, which is the structural change modelled within this framework.

2.2 Inputs

The total stock of resource *R* at time *t* is denoted by *S(t)*; its depletion occurs according to:

$$\dot{S} = -R, \qquad \text{with } S(0) \text{ given and } S(t) \geq 0 \text{ for all } t. \tag{5}$$

To ensure that *S* is exactly depleted in the long run, total extraction must equal total resource stock in equilibrium. This can be achieved by setting the optimum price at the beginning, which requires agents to form rational expectations. When a backstop technology is available at some point in time, the initial resource price has to take this into consideration, so that the stock is depleted when the backstop becomes available.

The population growth rate g_L is determined by an exogenous trend as in the neoclassical capital resource models and an endogenous term reflecting demographic transition, which postulates that population growth decreases with rising living standards, see e.g. Tamura (2000) and de la Croix and Doepke (2003). The living standard is measured by the (real) wage as in classical economics, see Samuelson (1978), and by consumer goods variety, which is the crucial dynamic ingredient of the model, according to (3) and (1). Using $\omega = w / p_Y$ for the real wage and $\tilde{\beta} = (1-\beta)/\beta$ for the variety effect we obtain the general form:

$$g_L = \xi\left[\phi - \hat{\omega} - \tilde{\beta} g\right] \tag{6}$$

$$(\xi, \phi > 0)$$

where ϕ is a constant trend, the hat denotes a differential in logarithms, and ξ serves as a "population response" parameter which will be useful for the discussion below. Equilibrium on labour and resource markets is given by:

$$L = L_X + L_g \tag{7}$$
$$R = R_X + R_g \tag{8}$$

2.3 Individuals

Individual agents have a lifetime utility function with instantaneous utility depending on $y \equiv Y / L$:

$$U(t)=\int_t^{\infty} e^{-\rho(\tau-t)}\log y(\tau)d\tau \qquad (9)$$

(9) is maximised by each agent subject to the individual budget constraint: $\dot{z}=(r-g_L)p_n n/L+w+p_R R/L-p_Y y$, where z is individual wealth, r the interest rate, np_n/L firm asset holdings of the agent, and g_L is taken as given, as the arguments in (6) are outside the scope of the individual agent. The usual no-Ponzi game restriction limits the amount of individual debt; R is restricted by (5), which reads in per capita terms $\dot{s}=-\left(g_L S+R\right)/L$ with $s=S/L$. Including the optimisation for resource ownership in the individual agent's optimisation problem, the current-value Hamiltonian reads:

$$H=\log y+\nu_1\left[(r-g_L)p_n n/L+w+p_R R/L-p_Y y\right]-\nu_2\left(g_L S+R\right)/L \qquad (10)$$

where ν_1,ν_2 denote the costate variables. Necessary conditions for an interior solution are given by the following first order and transversality conditions:

$$1/y=p_Y\nu_1 \qquad (11)$$
$$\nu_1 p_R=\nu_2 \qquad (12)$$
$$\dot{\nu}_1=\rho\nu_1-(r-g_L)\nu_1 \qquad (13)$$
$$\dot{\nu}_2=(\rho+g_L)\nu_2 \qquad (14)$$
$$\lim_{t\to\infty}\left[n(t)\cdot p_n(t)\right]\cdot e^{-\rho t}=0 \qquad (15)$$
$$\lim_{t\to\infty}\left[S(t)\cdot p_R(t)\right]\cdot e^{-\rho t}=0 \qquad (16)$$

The transversality conditions (15) and (16) require that total firm and resource wealth each approaches a value of zero in the long run. Differentiating (11) logarithmically with respect to time and using (13) yields the Keynes-Ramsey rule:

$$g_{pY}+g_y=r-\rho-g_L \qquad (17)$$

while differentiating (12) logarithmically with respect to time and using (13) and (14) gives the Hotelling rule:

$$g_{pR}=r \qquad (18)$$

which holds for any $t<\bar{t}$; again g denotes growth rates. As no resources are used to assemble differentiated goods to final output, expenditures can be expressed in terms of Y or X. Nothing pins down the price level of the considered economy, so that the price path of one nominal variable can be freely chosen while, at any point in time, all prices are measured against the chosen numeraire. For convenience, prices are normalised such that household expenditures are constant and unity at every point in time:

$$p_Y\cdot Y=p_x\cdot X\equiv 1 \qquad (19)$$

which yields by (17) that $r = \rho$ and by (18) that $\hat{p}_R = \rho$. This means that the *nominal* interest rate always corresponds to the discount rate, which itself equals the percentage change in resource prices. This holds true for any population growth rate, as at any point in time total expenditures are equal to expenditures per capita times population size. The evolution of the real interest rate, which is crucial for the development of the economy, is not predetermined by (19). As aggregate consumer expenditures are normalised to unity, the present value of consumption from any point in time onward is equal to $1/\rho$, so that the intertemporal budget constraint is well-defined in this economy.

The market form in the intermediate sector is monopolistic competition. The demand for an intermediate good can be derived from (3), see the appendix. Accordingly, the mark-up over marginal costs for the optimal price of an intermediate good is $1/\beta$, so that, together with (19), we get the per-period profit flow to each design holder:

$$\pi = (1-\beta)/n \tag{20}$$

On capital markets, the return on innovative investments (consisting of the direct profit flow π and the change in value of the design) is equalised to the return on a riskless bond investment of size p_n (with interest rate $r = \rho$):

$$\pi + \dot{p}_n = \rho \cdot p_n \tag{21}$$

3 The transition phase

3.1 Systems dynamics

We label the cost share of labour in intermediate goods production with *d*; observing (19) and the fact that the mark-up factor in intermediates production is $1/\beta$ we have:

$$d \equiv \frac{w \cdot L_X}{\beta} \tag{22}$$

while 1- *d* denotes the resource share in intermediates production. Calculating relative factor demands of profit-maximising *x*-firms, for the relative share size we obtain from (4):

$$\frac{d}{1-d} = \left(\frac{\lambda}{1-\lambda}\right)^{\sigma} \left(\frac{w}{p_R}\right)^{1-\sigma} \tag{23}$$

Furthermore, the sectoral depletion rates are defined as:

$$v_X \equiv \frac{R_X}{S} \quad \text{and} \quad v_g \equiv \frac{R_g}{S} \tag{24}$$

We now arrive at:

Lemma 1: *The dynamics of the system are fully given by the differential equations for d, g,* v_X *, and* v_g*, which read:*

$$\dot{d} = \frac{(1-d)(1-\sigma)}{\alpha\beta}\left\{ g\left[\alpha(1-\beta)+\beta d\right] - \left(\frac{\alpha}{1-\alpha}\right)^{\alpha} d^{\frac{1-\alpha}{1-\sigma}-\xi}\left(1-d\right)^{-\frac{1-\alpha}{1-\sigma}}\left(1-\alpha\right)\left(1-\beta\right)\mu \right\} \tag{25}$$

$$\begin{aligned}\dot{g} = \frac{1}{\alpha^2\beta d}\Big\{ & d^{2(\frac{1-\alpha}{1-\sigma}-\xi)}\left(1-d\right)^{-2\frac{1-\alpha}{1-\sigma}}\delta_6(d+\delta_3+\sigma(1-d)) + \alpha g(-d\delta_7 + g(\delta_4 - \beta d) \\ & (\delta_8 + d(1-\sigma)) - \alpha\mu d^{\frac{1-\alpha}{1-\sigma}-\xi}\left(1-d\right)^{-\frac{1-\alpha}{1-\sigma}}(\delta_2)^{\alpha-1}(-d\delta_7 + g(\alpha\delta_4 + \beta d \\ & (\xi + d(\sigma-1) - \delta_5) - \delta_4(1+\xi+2d(\sigma-1)-\delta_5-\sigma))) \Big\}\end{aligned} \tag{26}$$

$$\dot{v}_X = v_X\hat{d}\left[1-1/(1-\alpha)\right]+\rho \tag{27}$$

$$\dot{v}_g = v_g\alpha\left[\hat{g}+\hat{d}/(1-\alpha)(1-\sigma)\right] \tag{28}$$

where: $\delta_1 = \left(\alpha-1\right)^2(\beta-1); \delta_2 = \alpha/(1-\alpha); \delta_3 = \xi(\sigma-1); \delta_4 = \alpha(\beta-1); \delta_5 = (1+\xi)\sigma;$
$\delta_6 = \delta_1\left(\delta_2\right)^{2\alpha}\mu^2; \delta_7 = \alpha\beta\rho; \delta_8 = \alpha+\sigma-1$.

Proof: See the appendix. ▪

We use the fact that the system is decomposable: (25) and (26) constitute a system alone. The term $d^{(1-\alpha)/(1-\sigma)-\xi}$ is decisive for the dynamics of this subsystem, especially in the long run. To see this more clearly, we use phase diagrams for different assumptions regarding the response of population to its determinants in the following. In each case we describe the transition phase and the associated long-term equilibrium. When needed, we will refer to the threshold values for the emergence of a backstop technology, which are $\kappa(t) > \bar{\kappa}$ and $g(t) > \bar{g}$.

We use the assumption of poor substitution in the production of intermediate goods (σ < 1) throughout, avoiding the knife-edge assumption σ = 1 often used in literature. Under all scenarios regarding population growth, this entails a crowding out of labour from the intermediate sector, which supports economic dynamics through lower (nominal) wages. Goods prices decrease as well while the number of varieties increases during adjustment; thus, decreasing *nominal* wages are indeed compatible with constant or increasing well-being in this model. We will focus on income and consumption growth in section 4.

3.2 Low population growth

When population responds weakly to its determinants given in (6), i.e. when ξ takes a low value, population growth is low and innovation during transition is moderate because the fading resource input cannot be fully replaced by additional labour input. Indeed, we are ready to show:

Proposition 1 *With low population growth the economy converges to a state without innovation and production in the long run. Specifically, this happens when we have* $\xi < (1-\alpha)/(1-\sigma)$.

*** Figure 1 ****
about here

Proof In figure 1, the dynamics are depicted in the *d*-*g*- space for $\xi < (1-\alpha)/(1-\sigma)$. As becomes clear from the phase diagram, innovation ceases in the long run. This can also be seen in (25) where the term $d^{(1-\alpha)/(1-\sigma)-\xi}$ approaches zero in the long run so that for $\dot{d} = 0$ we must have $g = 0$. With given parameters for the production technology (σ and α), it is definitely weak population growth (a low ξ) that leads to this outcome. A constant population unambiguously falls into this category. ▪

This is a challenging first result: in a knowledge-driven economy, positive population growth is not detrimental but needed to sustain economic growth. In fact, this holds true for the case without backstop technology. However, we state:

Corollary 1 *With low population growth, i.e. when* $\xi < (1-\alpha)(1-\sigma)$, *the backstop technology is never developed and the decline of economic activities becomes inevitable.*

Proof Figure 1 shows that for low population growth the innovation rate decreases over time. Accordingly, the condition $g(t) > \bar{g}$ for any *t* is either fulfilled at the beginning of the optimisation or never. ▪

In the figures, point *B* shows where $g(t) = \bar{g}$, assuming $\kappa(t) \geq \bar{\kappa}$ is fulfilled. According to the result, low population growth is not a blessing but a curse because it limits innovation growth. The model suggests that labour is indeed the ultimate resource, as it is highly productive in the accumulation of knowledge capital. Moreover, it is highly efficient in the development of the backstop. The critical level for knowledge, i.e. $\kappa(t) \geq \bar{\kappa}$, prevents the policy option of investing heavily in innovations during a short period of time. Moreover, as the backstop comes as an externality, there is no incentive for an agent to promote the backstop in any way. To confirm these statements, let us now analyse the cases with higher population growth in more detail.

3.3 Intermediate population growth

Provided that population growth is higher than under 3.2, i.e. that it is governed by $\xi = (1-\alpha)/(1-\sigma)$ the steady decline in the innovation rate can be avoided. We arrive at:

Proposition 2 *With intermediate population growth, i.e. when* $\xi = (1-\alpha)/(1-\sigma)$ *, the economy approaches a long-term equilibrium with constant positive innovation growth on a saddle path. Provided that* $\kappa(t) < \bar{\kappa}$ *and* $g(t) < \bar{g}$ *for all t, the long-run innovation rate is given by:*

$$g = \left(\frac{1}{1/[\xi(1-\sigma)]-1} \right)^{\xi(1-\sigma)} \cdot \mu \tag{29}$$

*** Figure 2 ****
about here

Proof In figure 2, the dynamics are depicted in the *d*-*g*-space. The innovation rate approaches a constant on the *Y* axis following a saddle path, which lies between the two isoclines for $\dot{d} = 0$ and $\dot{g} = 0$. The equilibrium satisfies the transversality conditions. Using logarithmic differentials, (15) reads $\lim_{t\to\infty} \hat{n}(t) + \hat{p}_n(t) - \rho \leq 0$ which becomes with (2) $\lim_{t\to\infty} \hat{w}(t) - \rho \leq 0$; this is satisfied for $\hat{d} < 0$, see (23) and (A.7) in the appendix. Any path converging to *d*=1 must be ruled out since $\hat{d} > 0$ would imply $\rho - \hat{w}(t) > 0$. Any path converging to $g = d = 0$ must also be ruled out as it violates (26). With logarithmic differentials and using (19), (16) becomes $\lim_{t\to\infty} \hat{S}(t) \leq 0$ which is always satisfied with $R(t) > 0$. Thus, the economy jumps on the saddle path and asymptotically approaches the equilibrium given by (29). ▪

In the long-run steady state, all labour is used in R&D, where the drag of decreasing resource input is exactly compensated by increasing labour input due to population growth. Using realistic parameter values shows that the adjustment process is very long, i.e. it takes several centuries. In the long run, *g* only depends on technical parameters, that is on the elasticity of substitution in the intermediate goods sector σ and the population growth parameters ξ and μ, see (A.9) in the appendix, but not on preferences, i.e. on ρ. Using the intermediate growth condition, the innovation rate can alternatively be expressed in terms of the output elasticity α , a technical parameter as well.

High values of population response ξ and μ are positive for innovation growth. This clearly exhibits the importance of sufficient labour supply to support R&D-activities in the long run. A low σ means there is a strong intersectoral substitution effect, which leads to high innovation growth in the long run. The discount rate has two opposing effects on innovation: on the one hand, a high discount rate discourages investments; but on the other, it accelerates the price increase of natural resources and therefore sectoral reallocation of labour. According to (29), the two opposing effects are of the same size so that the impact of ρ becomes exactly zero.

Without a backstop technology, the steady state is never entirely reached. But as soon as we arrive at $\kappa > \overline{\kappa}$ and $g > \overline{g}$, a different scenario emerges, as analysed in the following.

3.4 Backstop technology

Assuming that population growth is the same as under 3.2, the economy follows a saddle path with increasing innovation growth. Provided that $\kappa(t) > \overline{\kappa}$, and $g(t) > \overline{g}$ for all $t > \overline{t}$, the economy switches in $t = \overline{t}$ to a new regime with a constant supply of a backstop resource, denoted by *B,* fully replacing *R* by assumption.

*** Figure 3 ****
about here

In figure 3, the dynamics in the case of backstop are depicted in the *d*-*g*-space. On the saddle path the innovation rate and the labour share reach point *C,* where we assume $\kappa(t) \geq \overline{\kappa}$ so that both variables *d* and *g* remain in *C* forever, see section 4. As we focus on market outcomes and the backstop comes as a pure externality, there is no specific pre-arrival activity by any agent in the economy.

3.5 High population growth

Following the discussion up to now, high population growth accelerates the innovation rate; we arrive at:

Proposition 4 *With high population growth, i.e. when* $\xi > (1-\alpha)/(1-\sigma)$, *the economy follows a path with an increasing innovation rate as long as the backstop technology is not available. Higher population growth causes faster adjustment to the equilibrium with a backstop technology.*

Proof Figure 4 shows the corresponding dynamics in the *d*-*g*-space. During transition the innovation rate increases because the inflow of labour in the research sector, determined by the population growth parameter ξ, overcompensates the increasing scarcity of the resource input in the research sector. The economy switches to a constant supply of *B,* fully replacing *R,* as soon as $\kappa(t) > \overline{\kappa}$ and $g(t) > \overline{g}$, which will happen if not $\overline{t} = \overline{\kappa} = \infty$. ■

*** Figure 4 ****
about here

4 Long-run development

In the long run, the growth rates of innovation and consumption depend on whether a backstop technology is available or not. This has been shown to be related to the properties of the transition path. The long-term income level also depends on the transition period, as it is a function of the number of varieties which result from cumulated research efforts in the past. A different impact on income may arise in the long run if the economy operates under a minimum resource constraint. This constraint says that a minimal resource input is needed to keep production running. Finally, in a world of structural change adjustment costs affect the final results. These topics are treated in the following, with a focus on consumption growth.

4.1 No backstop technology

Assume that $\overline{t} = \overline{\kappa} = \infty$ such that no backstop technology enters the economy in finite time. Following (3), aggregate consumption growth g_Y is determined by:

$$g_Y = \tilde{\beta} g + g_X \tag{30}$$

recalling that $\tilde{\beta} = (1-\beta)/\beta$. g_X is negative because of the decreasing input of *R* into intermediate goods production. Labour gradually moves from the intermediate to the innovation sector, which increases R&D activities. In order to have positive consumption growth, the equilibrium innovation growth rate *g* must be big enough to compensate for the drag of *R* in the *X*-sector. In the (very) long run, labour is fully employed in research and the growth rate of resource use approximates $g_{RX} = -\rho$ so that $g_X = -\rho$. Inserting (29) into (30) we obtain:

$$g_Y = \left(\frac{1}{1/\left[\xi(1-\sigma)\right]-1} \right)^{\xi(1-\sigma)} \tilde{\beta} \cdot \mu - \rho \tag{31}$$

Whether consumption growth is positive in the long run depends on the parameters; a positive $\hat{Y}$ is a possible outcome, and with realistic parameter values it is the likely result. High innovation growth (from 29), large gains from diversification and monopoly power (low β), and a large effect of labour shares on population size (high μ) favour positive (aggregate) consumption growth, whereas a high discount rate has a negative effect on consumption dynamics. Note that the negative effect of the discount rate stems from the negative effect of resource use on intermediates production and not from investment behaviour.

In order to get long-term per capita consumption growth we need to calculate $g_Y - g_L$. Dividing both sides of (15) by p_n yields that a constant innovation growth rate requires the quotient π / p_n to be constant, which means using (2), (19) and (20) that $\alpha\hat{w} + (1-\alpha)\rho = 0$. Without production of intermediates (in the limit), with a constant "output" of the research sector (a constant innovation growth rate), and a constant design price in the long run, factor incomes are fixed due to the Cobb-Douglas production technique in research (a share α of income goes to labour, 1-α to resources). Combining these results leads to the asymptotic population growth rate:

$$g_L = \frac{1-\alpha}{\alpha}\rho \tag{32}$$

which equals the negative wage change rate, so that we get for per capita consumption growth g_y:

$$g_y = \left(\frac{1}{1/\left[\xi(1-\sigma)\right]-1}\right)^{\xi(1-\sigma)} \tilde{\beta}\cdot\mu - \frac{1}{\alpha}\rho \ . \tag{33}$$

Again, high innovation growth, large gains from diversification, and a large labour force are the best means to compensate for a positive discount rate, which is now weighted by $1/\alpha$ due to positive population growth. For positive (sustainable) growth in the long run, the discount rate must be bounded from above according to:

$$g_y \geq 0 \quad \Leftrightarrow \quad \alpha\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} \tilde{\beta}\mu \geq \rho \tag{34}$$

where we made use of $\xi = (1-\alpha)/(1-\sigma)$. This can be met, assuming realistic parameter values. A special case applies when the economy needs a minimum resource input to operate, see section 4.3.

4.2 With backstop technology

When interpreting *R* as fossil fuels, it is likely that a backstop technology will become available at some point in the future. This new technology could build on resources like solar, wind, and/or tidal power or similar energies, all of them being renewable (as long as the sun is shining). In the model, the adoption happens at time $\bar{t}$ when $\kappa(\bar{t}) = \bar{\kappa}$ and $g(\bar{t}) = \bar{g}$. As a consequence, the backstop resource *B* replaces *R* in (1), (4), and (8), where we then postulate a fixed supply of *B* to be equal to the quantity demanded in the two sectors. The price is equal to a given c_B, which is the constant unit production cost of the backstop resource. A constant c_B results when the opposing effects of learning (causing a decreasing c_B) and of increasing scarcities (causing an increasing c_B) have the same size. In order to satisfy the first order conditions of optimisation, initial prices of the exhaustible resource are on a level that guarantees all resources R are depleted before $\bar{t}$. Following (4) and (22), the share of the backstop resource in intermediates production can be expressed as:

$$c_B \cdot B_X = \beta(1-d) \tag{35}$$

which shows that, for a given d, the backstop input in X-production becomes constant with given c_B. From the profit maximisation of the research labs we have:

$$\frac{B_g}{L_g} = \frac{w(1-\alpha)}{c_B \alpha} \tag{36}$$

The labour share in intermediates production now evolves according to:

$$\hat{d} = (1-\lambda)(1-\sigma)\hat{w} \tag{37}$$

To find the equilibrium of the system with the backstop resource, hypothetically suppose that (nominal) wages decrease over time. With poor input substitution, this would imply that, following (37), d falls so that $1-d$ increases and B_X rises, following (35). With a given B this would decrease B_g, which harms research and growth. Obviously, this is not an optimum. On the other hand, a constant wage implies a constant d, a constant allocation of energy to the two sectors and a constant population, according to (A.8), see the appendix. The constant input of labour and energy in research yields constant innovation and consumption growth rates which is the optimum outcome in the case of a backstop technology. Individuals with rational expectation choose this development path. With a backstop technology, the model resembles the approach of Grossman and Helpman (1991, ch. 5) which provides constant growth rates due to constant returns to research. Summarising, we thus find that for any point in time after $\bar{t}$:

(i) d becomes constant, i.e. sectoral change stops,
(ii) the innovation rate becomes constant,
(iii) the population growth rate becomes zero, and
(iv) per-capita consumption is constantly increased in the long run.

Result (iii) corresponds to the prediction that world population will be stable in the distant future. Implication (iv) is the consequence of a constant aggregate X-production and a positive innovation growth rate, as is the case in basic endogenous growth models.

Adopting a material interpretation of the resource R, recycling has a function which is similar to that of the backstop technology for energy. Regarding the minimum level, it is often assumed that a certain amount of material throughput is necessary to sustain economic activities in the long run. Recycling is the key to increasing the quantity of raw materials like metals etc. Assuming that a recycling technology is ready in $\bar{t}$ basically the same analysis as above applies. This holds true provided that it is possible to completely recycle the required (constant) quantity of material at a constant speed. If, however, it is not possible to recycle one hundred percent of the material, the minimum material requirement will not be met at some point in time and production in the model has to stop. Note that not all materials are non-renewable or predicted to be critical with regard to the minimum condition. For instance in food production, we primarily turn to the field of renewable natural resources. Here, limited regeneration and complementary inputs like land and water are possible bottlenecks for production. Regarding housing, natural supplies of materials seem to be (relatively) more abundant and partly renewable (e.g. timber).

4.3 Minimum resource use

In the present model, the progressive exhaustion of the resource stock decreases the labour income share in the intermediate goods sector, while the labour share in the research sector remains constant. As a consequence, the relative value of labour decreases and workers move from the intermediate goods to the R&D sector with a parallel increase in total labour force. Without a backstop technology, the economy evolves toward a steady state where the knowledge stock grows to infinity, whereas natural resource use and the production of intermediate goods approach zero. The economy becomes "immaterial" in the long run because growth depends on increasing knowledge with an ever-decreasing input of intermediate goods and resources. In the long-run steady state, costs of innovations are (approximately) constant, that is the decreasing wages compensate for increasing resource prices.

During transition, resource use becomes very low and even converges to zero in the very long run. Note that final goods production in (3) states that a sufficiently increasing knowledge stock can compensate for fading intermediate services, which is independent of material use, so that (3) remains to be valid in the long run. But is this realistic? If a minimum resource input is needed for intermediates production we must write, instead of (4):

$$X = \left[\lambda \cdot L_X{}^{(\sigma-1)/\sigma} + (1-\lambda) \cdot R_X{}^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} - \bar{R} \tag{4'}$$

Now a path leading to $R_X(t) < \bar{R}$ causes consumption to fall back to zero as soon as the minimum resource input is reached. In principle, any path analysed in section 3 is a candidate for such a development. One might think about optimal strategies for agents anticipating this development, but this is beyond the scope of this paper. Just note that an unfavourable scenario including $R_X(t) < \bar{R}$ is not the consequence of excessive growth during convergence. In the model, growth results from research which is less resource intensive than intermediates production in the longer run so that moderation in the growth rate does not help the economy in any way. (Sufficiently) Increasing resource prices are the best way to get a smooth transition to backstop technologies. Zero production in the long run emerges as the model outcome from the combination of a minimum resource requirement and a lack of a backstop technology and/or incomplete recycling.

4.4 Adjustment costs

Two further issues could prevent the system from following the saddle paths depicted in figures 1-4. First, as structural change is the main mechanism driving the result, any deviation from zero adjustment cost can become critical for the outcome. Indeed, many causes for slow sectoral adjustments of labour, such as wage-setting procedures and efficiency wages, can be found in reality. Even more important, the research sector might require special skills which are not readily available in the economy. It becomes immediately clear from the results that, once we have too slow an inflow of labour into the R&D-sector, innovation growth rates will decrease. Specifically, equation (A.10) in the appendix gives the percentage change of labour input into R&D as a function of the change of the labour wage and the labour share in X-production. Provided that wages do not adjust as indicated on an equilibrium convergence path, the percentage change of labour input in R&D becomes smaller, which entails a lower innovation growth rate according to (1b). The same holds true for the world economy, where sectoral shifts are associated with international changes in the division of labour.

Also, several equations postulate perfect foresight of the agents that is we abstract from information costs. In addition to the usual assumptions regarding capital markets and the intertemporal budget constraint, this model includes optimisation of resource owners. When deviating from perfect information in the resource sector, it might be that price levels are too low or price increases are too slow (at least in a first phase), for instance due to myopia. As a consequence, too little knowledge is accumulated and, combined with adjustment costs on labour markets, the increase of labour in the innovative sector becomes too sluggish compared to the model solution.

Turning to the issue of optimal economic growth, the market equilibrium reached in the present economy does not correspond to a first best-solution. Due to the positive spillovers in R&D, research efforts are too weak in equilibrium. Activities in the intermediate goods sector are also too low compared to the optimum because of monopolistic competition. This would lead to a static distortion in consumer expenditures if there were another consumer sector with goods priced at marginal costs. However, there is only one consumer sector in this economy. Regarding the intermediate goods sector, relative prices between goods reflect relative marginal cost, so that no static distortion arises. Thus, depending on the size of positive spillovers, policy could restore optimum sector size and provide optimal growth by subsidising research. According to the assumption, this would also have an impact on population growth.

5 Conclusions

The paper presents a model in which population growth supports sustainable consumption in an economy with non-renewable resource constraints. An increasing labour force is positive for growth because it fosters knowledge capital substituting for natural resources. In the present model, the knowledge creation effect of labour prevails because knowledge is a public good, which can be equally used by all the agents even in case of population growth. It is also shown that increasing resource prices cause structural change, which helps innovation. The lower the elasticity of substitution between inputs is, the faster the sectoral change and labour inflow into research. This effect can be so strong as to overcome the negative effect of the essential use of resources in R&D. Thus even a combination of several seemingly unfavourable conditions is not necessarily detrimental for long-run growth. The model suggests that labour and backstop technologies rather than natural inputs are the ultimate resources for an economy.

The non-Malthusian results of this study do not suggest a laisser-faire policy; rather, by emphasising central mechanisms for development, the model indicates that the debate on population growth and the substitution of non-renewable resources should focus on issues like sectoral adjustment costs and the formation of long-term expectations. The results show that facilitating labour reallocation from knowledge-extensive to knowledge-intensive sectors is the best means to support sustainable development. The removal of subsidies to energy production (like the ones for coal in certain countries) and to shrinking and lagging sectors emerges as being desirable. The steady increase of resource prices is not seen as detrimental, quite to the contrary, it helps the economy to adjust in continuous small steps to a sustainable equilibrium. However, policies targeting at the population size cannot be advocated.

In a richer model, learning effects in the intermediates sector could sustain the incentive for a part of the labour force to remain in the intermediates sector in the long run, a straightforward extension of the present approach. The framework could also include that reallocating labour needs education efforts, which seems to be another possible direction for future research. A further extension of the model would be to introduce stock pollution, which represents an exhaustibility constraint similar to non-renewable resources.

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Appendix

This appendix explains the derivations of the equations in the main text in detail. If needed, more information is available from the author upon request.

Profits

To obtain (20), use the price index of final goods *Y*, which is written as:

$$p_Y = \left[\int_0^n \left(p_{xj} \right)^{1-\varepsilon} dj \right]^{1/(1-\varepsilon)} \tag{A.1}$$

With perfect competition in the *Y*-sector, this price equals the per-unit costs, so that differentiating (A.1) with respect to the price of intermediate good *i* yields, according to Shephard's Lemma, the per-unit input coefficient x_i / Y. Using this coefficient and (19), the demand for intermediate good *i* becomes:

$$x_i = \frac{\left(p_{xi} \right)^{-\varepsilon}}{\int_0^n \left(p_{xj} \right)^{1-\varepsilon} dj} \tag{A.2}$$

For the case of many *x*-firms (large group case of Chamberlin), the denominator of (A.2) is given for the single firm so that the elasticity of demand for x_i is ε, and the optimum mark-up over marginal costs is indeed $1/\beta$, with $\beta = \left(\varepsilon - 1\right)/\varepsilon$. Hence, profits of *x*-firms used to compensate research are a share $1-\beta$ of total sales.

Solving the model

In this subsection, we reduce the system in order to determine $\hat{g}, \hat{L}_g, \hat{d},$ and $\hat{w}$ for which we need also expressions for *w* and *L*. (1b) can be rewritten as:

$$g = L_g \cdot \left(R_g / L_g \right)^{1-\alpha} \tag{A.3}$$

The innovation growth rate depends on the labour input in R&D L_g and the relation of resource and labour input in the innovative sector. Cost minimisation in the R&D sector yields:

$$\frac{R_g}{L_g} = \frac{w \cdot (1-\alpha)}{p_R \cdot \alpha} \tag{A.4}$$

From (A.3) and (A.4) we derive, using differentials in logarithms:

$$\hat{g} = \hat{L}_g + (1-\alpha)\hat{w} - (1-\alpha)\rho \tag{A.5}$$

To derive $\hat{L}_g$, use (7) to get:

$$\hat{L}_g = \frac{L}{L-L_X} g_L - \frac{L_X}{L-L_X} \hat{L}_X \quad . \tag{A.6}$$

To reduce (A.6) we need several steps. From (23) we derive:

$$(\hat{w}-\rho) = \hat{d}/(1-d)(1-\sigma) \quad . \tag{A.7}$$

According to (6) we have $g_L = \xi\left[\phi - (\hat{w}-\hat{p}_y) - \tilde{\beta}g\right]$; with (3) and (19) we can write $\hat{p}_y = d\cdot w + (1-d)\rho - \tilde{\beta}g$. Without loosing generality we set $\phi = \xi\sigma$ to get with (A.7):

$$g_L = -\xi(1-\sigma)(1-d)(\hat{w}-\rho) = -\xi\hat{d} \tag{A.8}$$

and:

$$L = \mu \cdot d^{-\xi} \tag{A.9}$$

where $\mu, \xi > 0$ are the parameters linking population to the sectoral structure given by *d*. Using (A.6), (A.8), and (22) to calculate L_X and $\hat{L}_X$ yields:

$$\hat{L}_g = \left(1 - wL/\beta d\right)^{-1}\left[-\hat{w} + \left(1 + \frac{\xi w L}{\beta d}\right)\hat{d}\right] \tag{A.10}$$

The percentage change of the wage rate $\hat{w}$ in (A.10) is obtained by first dividing (21) by p_n

$$\frac{\pi}{p_n} + \hat{p}_n = \rho \quad , \tag{A.11}$$

and calculating *w* as value marginal product from (1'):

$$w = \alpha \cdot L_g^{\alpha-1} \cdot R_g^{1-\alpha} \cdot p_n \cdot n = \alpha \cdot p_n \cdot g \cdot n / L_g \quad . \tag{A.12}$$

Solve (A.12) for p_n, insert in (A.11) and use (20) to have:

$$\frac{(1-\beta)\alpha g}{wL_g} + \hat{p}_n = \rho \quad . \tag{A.13}$$

To proceed with (A.13), use (7) and (22) to eliminate L_g and (2) to eliminate $\hat{p}_n$ to get:

$$\hat{w} = \frac{g}{\alpha} + \rho - \frac{(1-\beta)g}{wL - d\beta} \tag{A.14}$$

Finally, to calculate the wage rate *w*, note that (A.3) can be rewritten, using (7) and (22) as:

$$g = (L - d\beta / w)\left(R_g / L_g\right)^{1-\alpha} \tag{A. 15}$$

As can be seen from (A. 15), with a given resource/labour input ratio in research, the innovation growth rate is high when labour supply is large, wages are low, the labour share in intermediates is low and monopoly power in intermediates (yielding profits for innovations) is high (low β). From (A.4) we know that the input ratio R_g / L_g in (A.15) depends on relative input prices according to (A.4). (23) says that relative input prices relate to $d/(1-d)$ representing relative sector shares. So we use (22) and (23) and solve (A.15) to get an equation for the wage rate, assuming $\lambda = 0.5$ for simplicity:

$$w = -d\beta\left[\frac{g}{u} - L\right]^{-1} \quad \text{with} \quad u = \left(\frac{d}{1-d}\right)^{\frac{1-\alpha}{1-\sigma}} \cdot \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} \tag{A. 16}$$

(A.16) relates sector shares, given by $\left(d/(1-d)\right)^{\frac{1-\alpha}{1-\sigma}}$, to wages and, by this, to the incentives for research activities.

Equation of motion for d

To find (25), insert (A.14) into (A.7) to get:

$$\hat{d} = (1-d)(1-\sigma)g\left[\frac{1}{\alpha} - \frac{(1-\beta)}{wL - d\beta}\right] \tag{A.17}$$

From (A.9) and (A.16) we obtain:

$$wL = -d\beta\left[g \Big/ \left\{d^{\frac{1-\alpha}{1-\sigma}-\xi}(1-d)^{-\frac{1-\alpha}{1-\sigma}} \cdot \left((1-\alpha)/\alpha\right)^{1-\alpha}\right\} - 1\right]^{-1} \tag{A.18}$$

This says that labour income wL is directly associated to $d^{\frac{1-\alpha}{1-\sigma}-\xi}$ providing intuition for the equation of motion for *d* in (25), which is obtained by inserting (A.18) into (A.17). Using (25), the equation for the $\dot{d} = 0$ locus reads:

$$g = \frac{d^{\frac{1-\alpha}{1-\sigma}-\xi}(1-d)^{\frac{1-\alpha}{1-\sigma}}c}{d\beta + \alpha(1-\beta)} \quad \text{with} \quad c = \mu(1-\alpha)(1-\beta)\left(\frac{\alpha}{1-\alpha}\right)^{\alpha} > 0$$

from where it can be seen why the different cases of population growth yield the results in the main text.

Further equations of motion

To obtain (26) we take (A.5), use (A.10) to replace $\hat{L}_g$, (A.14) for $\hat{w}$, (A.16) for *w*, as well as (25) to get, after rearranging, the equation of motion for the innovation rate *g*. The shape of the $\dot{g} = 0$ locus in the *d*-*g*-plane is best seen by comparing it to the $\dot{d} = 0$ locus, which is easier, see above. The two expressions yield the same innovation rate *g* for $d = 0$; moreover, for $d > 0$, the $\dot{g} = 0$ locus always lies below the $\dot{d} = 0$ locus, as depicted in the figures.

To get (27) we take the first order condition from profit maximisation in *X*-production:

$$\frac{L_X}{R_X} = \left(\frac{\lambda}{1-\lambda}\right)^{\sigma} \left(\frac{w}{p_R}\right)^{-\sigma} \tag{A.19}$$

to obtain

$$\hat{d} = (1-d)\left(\frac{1-\sigma}{\sigma}\right)(\hat{v}_X - \hat{L}_X) \tag{A.20}$$

and with (23) we arrive at the equation of motion for v_X. To obtain (28) we use

$$\hat{g} = \alpha \hat{L}_g + (1-\alpha) R_g \tag{A.21}$$

as well as (A.7), which gives, after rearranging, the equation of motion for v_g.

Innovation rate

To get the innovation rate in (29) use (25) as well as $\xi = (1-\alpha)/(1-\sigma)$ and set $\dot{d} = d = 0$.

Figures

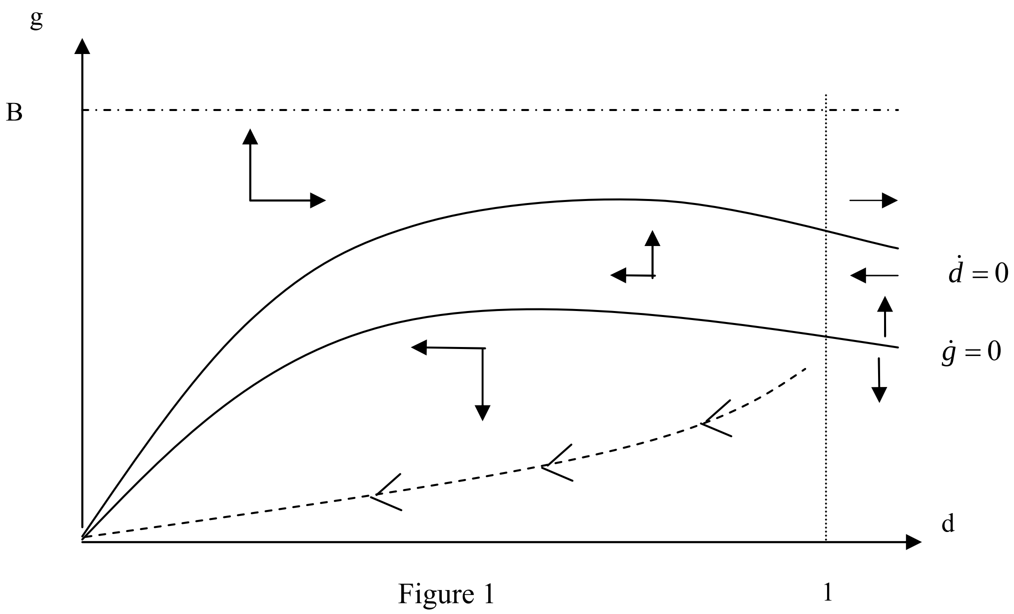

Figure 1

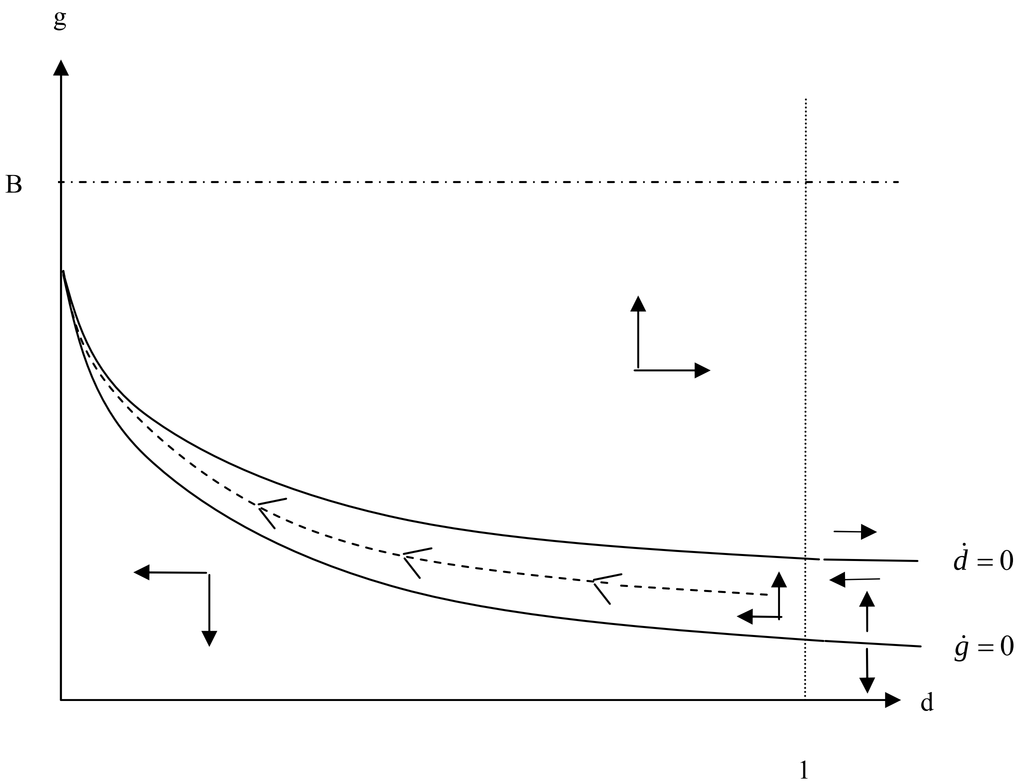

Figure 2

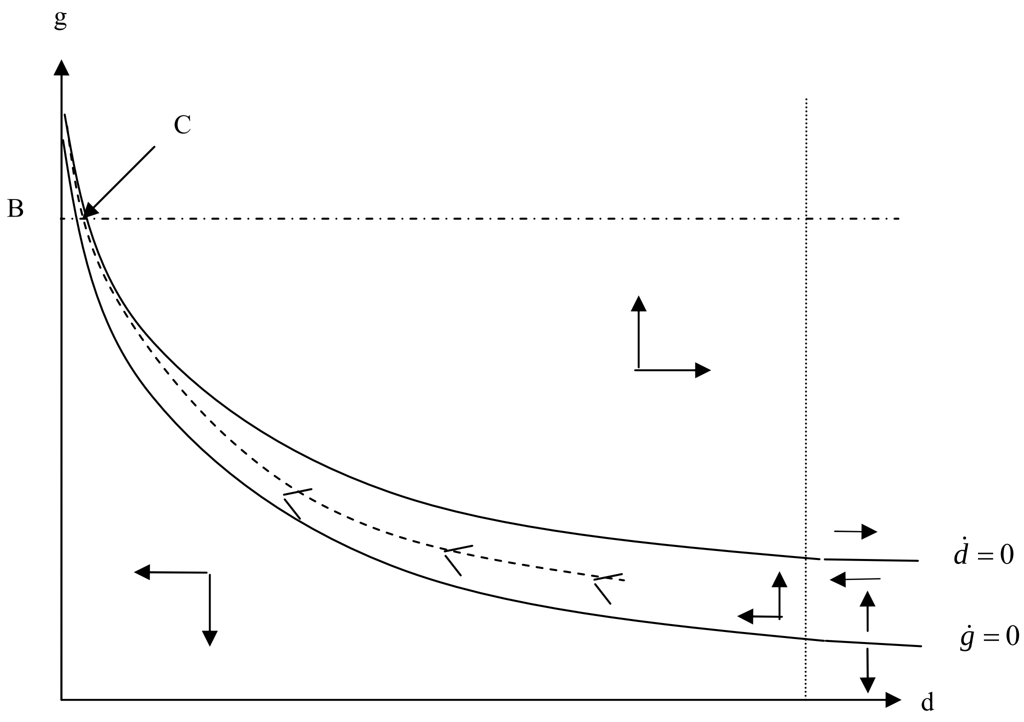

Figure 3

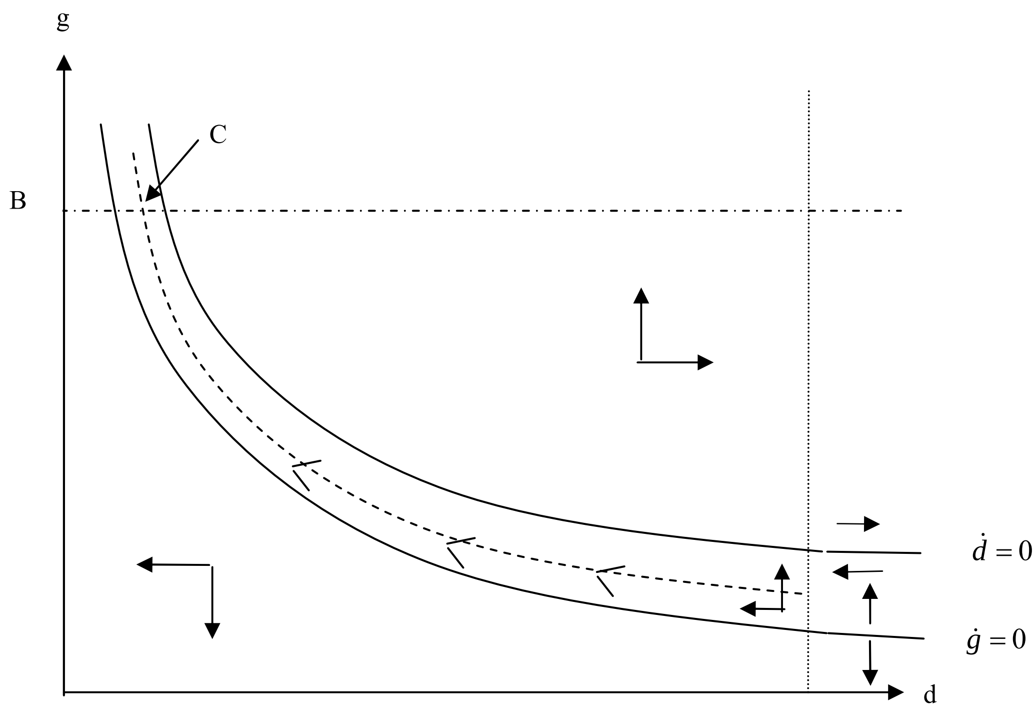

Figure 4

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