Revised version

Sustainability and substitution of exhaustible natural resources

How resource prices affect long-term R&D-investments

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Abstract
Traditional resource economics has been criticised for assuming too high elasticities of substitution, not observing material balance principles and relying too much on planner solutions to obtain long-term growth. By analysing a multi-sector R&D-based endogenous growth model with exhaustible natural resources, the present paper addresses this critique. We study transitional dynamics and long-term growth and identify conditions under which firms keep spending on research and development. Long-run growth can be sustained under free market conditions when the elasticity of substitution between resources and intermediate input is higher in the knowledge-using sector than in the knowledge-competing sector, even when both elasticities are below unity.

Keywords : Growth, non-renewable resources, substitution, investment incentives, endogenous technological change, sustainability

JEL-Classification : Q20, Q30, O41, O33

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1. **Introduction**

The debate on environmental sustainability and the recent upsurge in oil prices have reawakened the interest in the question whether and how natural resource scarcity impairs development prospects for future generations. The high energy prices in the 1970s pushed the world economy into a recession. Since then exploration and discovery activities have increased the known stocks of resources, in particular fossil fuels, and technological change has increased the efficiency of resource use. Accordingly, most past predictions of growth perspectives turned out to be too pessimistic. However, discoveries can only temporarily postpone the peak of oil production, and economic growth has already suffered a serious productivity slowdown in the 1980s-1990s. Therefore, in discussions among scientists and business people on future growth and well-being, the critical question is whether and how decreasing inputs of natural resources will be mitigated or offset by favourable technological developments.

The seminal literature on growth and resources from the 1970s concludes that income growth can be sustained in the long run, provided that either the "elasticity of substitution between exhaustible resources and other inputs is unity or bigger" (Solow 1974b, p.11) or that exogenous resource-augmenting technological progress occurs at a constant rate, see Dasgupta and Heal (1974) and Stiglitz (1974). Many ecological economists have argued that these assumptions are overly optimistic, e.g. Cleveland and Ruth (1997). They assume that the elasticity of substitution between natural resources and man-made inputs lies below unity and technical progress is too weak in reality. Moreover, they emphasise that material balances limit the use of physical capital in the long run, while the resource economics literature of the 1970s assumes unbounded accumulation. Finally, while sustained growth may be technically feasible under certain circumstances, it is not necessarily reached under free market conditions. Low investment incentives and externalities may result in too little investment effort in capital which substitutes for resources. Moreover, myopic behaviour of today's generations may prevent the implementation of policy measures that are needed to obtain sustainability for future generations.

The present paper reconsiders the relation between resource scarcity, substitution, and innovation. We investigate whether and when poor input substitution destroys the innovation incentives that are necessary to offset the drag on growth from resource depletion. The following issues are emphasised. First, we depart from the common assumption that elasticities of substitution between man-made inputs and natural resource inputs are unitary; we allow them to be lower. Second, we do not use the assumption of exogenous technological progress, which has been common in the resource literature up to the 1990s. We thus rule out exogenous technological change that offsets resource depletion as "manna from heaven". In our model, technological change results from investments in research and development (R&D) that is driven by profit incentives and results in blueprints for new intermediate inputs. The division of labour over a larger variety of intermediate inputs enhances not only labour productivity, as in Romer (1990) and Grossman and Helpman (1991), but also the efficiency of natural resource use.

Third, we introduce multiple sectors of production in the economy, each with different opportunities for innovation and substitution. In particular, we emphasise the distinction between knowledge-using and knowledge-competing sectors. This allows us to analyse how resource depletion drives sectoral shifts and, consequently, has an impact on the incentives for innovation. Fourth, since these sectoral shifts play a major role over time, we discuss transitional dynamics in detail, which are largely ignored in the endogenous growth resource literature cited above.

Finally, we emphasise the accumulation of knowledge rather than physical capital as the engine of growth, in order to do justice to material balance principles. We bound the total supply of man-made inputs to take into account material balances. Physical capital cannot continuously grow in the long run because it requires raw materials like metals to be built; the supply of these materials, however, is bounded. Accordingly, in our model, labour, natural resource inputs and cumulative knowledge are combined to produce a flow of services that yield utility; accumulation of purely physical capital is disregarded in the main part of the paper. We discuss the impact of additionally including physical capital on the results in a separate subsection.

Our main finding is that long-run growth can be sustained under free market conditions even when elasticities of substitution between intermediate inputs and natural resources are low, provided that the elasticity of substitution between intermediate inputs and resources in the knowledge-using sector exceeds the elasticity of substitution in the knowledge-competing sector. In this case, the long-run growth rate directly depends on the size of the two elasticities. The main mechanism underlying this result is sectoral reallocation of inputs. Given the depletion of the resource, the sector with the better substitution possibilities is less vulnerable to rising resource prices and expands at the cost of the other sector. When the relative size of the knowledge-using sector increases, innovation incentives remain intact in the long run. We also show that the reallocation process between the sectors neutralises the scale effect that is often present in endogenous growth models. We thus contribute to the discussion on "scale effects on growth" (see Jones 1995, 1999) by providing a new way to eliminate such scale effects.

We extend the older literature on growth and resources (Solow 1974a and especially Dasgupta and Heal 1979) as well as the literature on endogenous growth literature. Poor substitution was discussed in resource economics but only in combination with exogenous technological change. Moreover, these resource models focused on one-sector economies, abstracting from sectoral shifts due to depletion. The endogenous growth literature developed in the 1990s endogenised technological change by modelling R&D. Positive spillovers from R&D by profit-maximising firms to a general stock of knowledge are assumed to support subsequent R&D. The positive externalities provide sufficient incentives for entrepreneurs to keep innovating at a constant rate. For the case of a unitary elasticity of substitution between capital and resource inputs, endogenous knowledge accumulation yields sustained growth in the presence of non-renewable resources (Aghion and Howitt 1998, Scholz and Ziemes 1999, Schou 2000, Groth and Schou 2002, Grimaud and Rougé 2003). Elasticities of substitution below unity are discussed in some models of renewable resources and endogenous growth (Bovenberg and Smulders 1995, Bretschger 1998), but not in the context of non-renewable resources.

The remainder of the paper is organised as follows. In section 2, the theoretical five-sector model of the economy with two consumer goods is presented in detail.

Section 3 shows how the model can be solved. Section 4 provides results for transitional dynamics and long-run growth for different types of parameter and substitution conditions. Section 5 concludes.

2. The model

2.1 Production sector

There are two primary inputs: labour L and the exhaustible natural resource R. L produces two types of intermediate goods which are combined with R to manufacture two final goods: Y-goods, or knowledge-using "high-tech" goods, and T-goods, knowledge-competing "traditional" goods, respectively, see *figure 1*.

Fig. 1
(about here)

Y is produced with R and differentiated intermediate inputs K according to the following nested CES-function:

$$Y = \left[\bar{q} \left(\int_0^N K_j^{\beta} dj \right)^{\frac{\sigma-1}{\beta\sigma}} + (1-\bar{q}) \left(N^{\delta} \cdot R_Y \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \qquad (1)$$

where $0 < \beta, \bar{q} < 1$, and $\delta, \sigma > 0$ are given parameters and the time index has been omitted. Parameter σ represents the constant elasticity of substitution between natural resources and a CES-index of intermediate input. Each moment in time, a mass of $N(t)$ varieties of the intermediate good is available; the constant elasticity of substitution between them equals $1/(1 - \beta) > 1$.

In a symmetrical equilibrium, the quantities of intermediate goods K are equal for the different components, i.e. $K_j = K$ for all j. Production can then be written as:

$$Y = \left[\bar{q} \left(N^{\frac{1-\beta}{\beta}} \cdot I_Y \right)^{\frac{\sigma-1}{\sigma}} + (1-\bar{q}) \left(N^{\delta} \cdot R_Y \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \qquad (1')$$

where I_Y is the following aggregate measure of intermediate goods:

$$I_Y = N \cdot K. \qquad (2)$$

According to (1′), holding aggregate intermediate input I_Y constant, the production of the high-tech goods increases with the number of intermediates, N. Accordingly, the term $N^{(1-\beta)/\beta}$ captures the gains from specialization in the use of intermediates, as

introduced by Ethier (1982) and common in the endogenous growth literature (Romer, 1990 and Grossman/Helpman, 1991). The term N^{δ} extends this logic to the use of resources: producers are able to use natural resources the more efficiently, the higher is the specialization of intermediate goods a given amount of resources is combined with. As long as the former effect is stronger than the latter, i.e. $(1-\beta)/\beta > \delta$, an increase in *N* mainly affects the productivity of intermediates rather than resources.
Rearranging (1'), we find:

$$Y = N^{(1-\beta)/\beta} \cdot \left[\overline{q} I_Y^{(\sigma-1)/\sigma} + (1-\overline{q})(N^{-\nu} R_Y)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \tag{3}$$

with $\nu = (1-\beta)/\beta - \delta$. The expression in brackets on the r.h.s. of (3) corresponds to the familiar CES-approach of resource economics, see Dasgupta and Heal (1979, p. 199); it aggregates differentiated goods input I_Y and effective resource input $N^{-\nu} R_Y$ into a composite input. At a pre-determined degree of specialization, *N*, substitution is possible between man-made inputs I_Y and resources R_Y. Shifts in production possibilities arise from an increase in the degree of specialization, *N*, which has two effects. First, total factor productivity increases (see the term $N^{(1-\beta)/\beta}$), and, second, the relative productivities of resource and intermediate inputs change, which corresponds to biased technological change (see the term $N^{-\nu}$).

The market for *Y*-goods is fully competitive. Producers take prices of output, resource inputs and intermediate goods (denoted by p_Y, p_R and p_{Kj}, respectively) as given. They maximise total profits $p_Y Y - p_R R_Y - \int_0^n p_{Kj} K_j dj$, subject to the production function (1). Under symmetry ($p_{Kj} = p_K$), relative demand for intermediates and resources is given by:

$$\frac{I_Y}{R_Y} = \left(\frac{\overline{q}}{1-\overline{q}} \right)^{\sigma} \left(\frac{p_K}{p_R} \right)^{-\sigma} N^{-(1-\sigma)\nu} . \tag{4}$$

Equation (4) reveals that if $(1-\sigma)\nu < 0$ technological change (i.e. an increase in *N*) is resource-saving; if $(1-\sigma)\nu > 0$, it is resource-using. To be on the conservative side with respect to technological opportunities, from now on we assume poor substitution, $0 < \sigma < 1$, $0 < \omega < 1$, and small effects of specialisation on the resource efficiency so that technological change is resource-using, $\nu = (1-\beta)/\beta - \delta > 0$.

The production of the second type of final goods, *standard goods T*, requires homogenous intermediates I_T and resources R_T as inputs. We use again a CES-formulation:

$$T = \left[\overline{h} \cdot I_T^{\frac{\omega-1}{\omega}} + \left(1-\overline{h}\right) R_T^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \tag{5}$$

with ω being the elasticity of substitution in the *T*-sector and $0 < \overline{h} < 1$. Producers take prices as given and maximise profits $p_T T - p_{IT} I_T - p_R R_T$ subject to (5), with p_T and p_{IT} denoting the prices of T and I_T. This gives relative factor demand:

$$\frac{I_T}{R_T} = \left(\frac{\bar{h}}{1-\bar{h}} \right)^{\omega} \left(\frac{p_{IT}}{p_R} \right)^{-\omega} \tag{6}$$

In the third production sector *homogenous intermediate goods* I_T used in the *T*-sector are produced. We assume labour is the only input and perfect competition applies so that the price of the homogeneous intermediate good equals the wage rate:

$$p_{IT} = w\,. \tag{7}$$

The final type of production concerns *differentiated intermediate goods K* which are produced by monopolists, using labour *L* as an input. *K*-goods are assembled to aggregate intermediate services I_Y without further costs. The profit maximising monopolistic supplier of a *K*-variety faces a price elasticity of demand equal to $-1/(1-\beta)$. As in the standard Dixit-Stiglitz approach, this follows from the *Y*-producers demand for *K*. Thus, the monopolist optimally sets a rental rate that is a mark-up $1/\beta$ times the marginal cost of production, which equals the labour wage *w*. All monopolistic suppliers set this same price:

$$p_K = w/\beta\,. \tag{8}$$

Associated profits π for each supplier of an intermediate good can be calculated as:

$$\pi = \left(\frac{1-\beta}{\beta} \right) \frac{wI_Y}{N}\,. \tag{9}$$

Profits are used to cover the expenses for fixed costs in the production of *K*-goods, which consist of payments for the blueprint of the intermediate good. Each design contains the know-how for the production of one intermediate *K*. Thus, each *K*-firm has to acquire one design as an up-front investment before it can start production.

2.2 Innovation

Innovation expands intermediate goods variety, *N*, as in Grossman and Helpman (1991, Chapter 3). We measure innovation by the rate of innovation, defined as:

$$g = \dot{N}/N \tag{10}$$

Blueprints for new varieties $\dot{N}$ are produced in the *R&D sector*. Per blueprint, *a/N* units of labour are required. It is assumed that an increase in variety increases the stock of public knowledge on which R&D builds so that research costs decline with *N*. Knowledge capital is accumulated through positive spillovers in research and is an input into subsequent R&D; it is the driving force for long-run development.

There is free entry in R&D. Thus, whenever the cost to develop a new blueprint, $w \cdot (a/N)$, is lower than the market value of a blueprint, denoted by p_N, entry will compete away the rent. Hence we have:

$$aw / N \geq p_N \qquad \text{with equality (inequality) if } g > 0 \ (g = 0). \tag{11}$$

The market value of a blueprint follows from the condition that investors earn the market interest rate r when investing their money in blueprints, thus earning profits π and capital gains $\dot{p}_N$:

$$\pi + \dot{p}_N = r \cdot p_N \tag{12}$$

We combine (9)-(12) to get:

$$g > 0 \quad \Rightarrow \quad r - \hat{w} = \left(\frac{1-\beta}{\beta} \right) \frac{I_Y}{a} - g \tag{13}$$

Equation (13) characterises the return to innovation, which increases in the size of the knowledge-using sector, the mark-up over marginal cost in the production of differentiated intermediates, and the productivity in the research lab; it decreases with the innovation growth rate.

2.3 Factor markets

The total stock of the non-renewable resource at time t is denoted by $S(t)$. It is depleted according to:

$$\dot{S} = -(R_Y + R_T), \qquad S(0) \text{ given}, \quad S(t) \geq 0, \tag{14}$$

which reflects that any flow of resource use depletes the total resource stock proportionally, that the resource stock is predetermined, and that the stock can never become negative.

The labour market is in equilibrium if the fixed supply L equals labour demand in intermediate goods production, $I_Y + I_T$, and in research, ag:

$$L = I_Y + I_T + ag \tag{15}$$

2.4 Households

The representative household inelastically supplies L units of labour, owns the resource stock with value $p_R S$ as well as equity in intermediate goods firms with value $p_N N$. It maximises utility over an infinite horizon subject to their intertemporal budget constraint and the usual No-Ponzi-game condition. We assume a constant discount rate ρ and Cobb-Douglas preferences over the two consumption goods. Thus the household maximises:

$$U(t) = \int_t^\infty e^{-\rho(\tau - t)} \ln C(\tau) d\tau, \tag{16}$$

subject to

$$C = Y^{\phi} \cdot T^{1-\phi}, \tag{17}$$

$$\dot{V} = r(V - p_R S) + \dot{p}_R S + wL - p_T T - p_Y Y, \qquad V(0) \text{ given}, \tag{18}$$

where $V = p_N N + p_R S$ is total asset holdings. The first order conditions yield the following relations:

$$\frac{p_Y \cdot Y}{p_T \cdot T} = \frac{\phi}{1-\phi}, \tag{19}$$

$$\hat{p}_C + \hat{C} = r - \rho, \tag{20}$$

$$\hat{p}_R = r, \tag{21}$$

where hats denote growth rates and $p_C = (p_Y/\phi)^{\phi}(p_T/(1-\phi))^{1-\phi}$ is the consumer price index. The Cobb-Douglas specification in (17) implies constant expenditure shares for *T*- and *Y*-goods so that relative demand for final goods is constant, see equation (19). Equation (20) is the Ramsey rule stating that the growth rate of consumer expenditures is equal to the difference between the nominal interest rate r and the discount rate ρ. Equation (21) represents the Hotelling rule, which guarantees that resource owners are exactly indifferent between selling resources (and investing the profit with interest rate r) and preserving the stock of resources (and earning capital gains because of increases in the resource price).

The transversality condition and the No-Ponzi-game condition require:

$$\lim_{\tau \to \infty} V(\tau) e^{-\int_t^{\tau} r(s)ds} = \lim_{\tau \to \infty} [p_N(\tau)N(\tau) + p_R(\tau)S(\tau)] e^{-\int_t^{\tau} r(s)ds} = 0.$$

Together with (21) this implies that the resource stock is depleted completely and that the discounted value of household wealth approaches zero when time goes to infinity. Writing the latter condition in terms of percentage changes, we thus have:

$$\lim_{t \to \infty} \hat{N}(t) + \hat{p}_N(t) - r \le 0, \tag{22}$$

$$\lim_{t \to \infty} S(t) = 0. \tag{23}$$

3. The dynamics of the model

To characterise the dynamics in the most instructive manner, we introduce the proportional extraction rates, R_Y/S and R_T/S (to be denoted by u_Y and u_T, respectively), as well as the intermediate goods value shares in the knowledge-using sector Y and the knowledge-competing sector T (to be denoted by q and h, respectively). Thus we define:

$$q = \frac{p_K I_Y}{p_Y Y}, \qquad h = \frac{p_{IT} I_T}{p_T T}, \tag{24}$$

$$u_Y = R_Y / S, \qquad u_T = R_T / S. \tag{25}$$

Using these definitions we can write the firm's first order conditions (4) and (6) as:

$$\frac{q}{1-q} = \frac{\bar{q}}{1-\bar{q}} \left(\frac{u_Y S N^{-\nu}}{I_Y} \right)^{(1-\sigma)/\sigma}, \tag{26}$$

$$\frac{h}{1-h} = \frac{\bar{h}}{1-\bar{h}} \left(\frac{u_T S}{I_T} \right)^{(1-\omega)/\omega}. \tag{27}$$

The value shares characterise the static cross-sectional allocation of labour and extraction. First, from (7), (8), (15), (19), and (24) we find for labour allocation:

$$I_Y = \frac{\phi\beta q}{\phi\beta q + (1-\phi)h}(L - ag), \tag{28}$$

$$I_T = \frac{(1-\phi)h}{\phi\beta q + (1-\phi)h}(L - ag). \tag{29}$$

Second, from (24) and (25), the relative sectoral resource use proves to be:

$$\frac{u_Y}{u_T} = \frac{\phi(1-q)}{(1-\phi)(1-h)}. \tag{30}$$

We next determine the dynamics of sectoral shares, innovation, and extraction rates in the model. Combining demand equations (4) and (6) with supply equations (7), (8), and (21) and the definitions in (24), we derive the following differential equations to characterise how factor shares change with factor prices and innovation:

$$\hat{q} = -(1-q)(1-\sigma)(r - \hat{w} + \nu g), \tag{31}$$

$$\hat{h} = -(1-h)(1-\omega)(r - \hat{w}). \tag{32}$$

Using (19), (24), (31), and (32) we rewrite the Ramsey rule (20) in two ways:

$$\hat{I}_Y = [1 - (1-\sigma)(1-q)](r - \hat{w} + \nu g) - (\rho + \nu g), \tag{33}$$

$$\hat{I}_T = [1 - (1-\omega)(1-h)](r - \hat{w}) - \rho. \tag{34}$$

Equations (33) and (34) reflect the households' savings decisions and show how the interest rate, innovation, and the discount rate affect the growth rate of employment in both sectors.

In the above equations, the term $r-\hat{w}$ prominently governs the dynamics. Since from (7), (8), and (21) we may write $r-\hat{w}=\hat{p}_R-\hat{p}_{IT}=\hat{p}_R-\hat{p}_K$, the term represents the change in the relative input price for final goods producers and hence determines factor share dynamics. Also, from (8) we may write $r-\hat{w}=r-\hat{p}_K$, so that the term represents the real interest rate from the point of view of the innovation sector and hence naturally governs the innovation dynamics. From now on we will label the term $r-\hat{w}$ as the "real interest rate" for convenience. Using (28) we can rewrite (13) to express the real interest rate in terms of our key variables:

$$g>0 \quad \Rightarrow \quad r-\hat{w}=(1-\beta)\left(\frac{L/a-g}{\beta+[(1-\phi)/\phi](h/q)}\right)-g\,. \tag{35}$$

In a steady state without innovation, in which $\lim_{t\to\infty} g(t)=0$, discounted stock prices change at rate $\hat{p}_N-r=-\pi/p_N<0$, see (12). The transversality condition (22) then always holds for a positive firm value, i.e. when $\lim_{t\to\infty} p_N(t)>0$. In a steady state with innovation, in which $\lim_{t\to\infty} g(t)>0$, the firm value equals the cost of innovation, $p_N=aw/N$, see (11), and the transversality condition (22) boils down to:

$$\lim_{t\to\infty} r(t)-\hat{w}(t)\geq 0 \qquad \text{for } \lim_{t\to\infty} g(t)>0\,. \tag{36}$$

Differentiating (15) with respect to time and using (33) and (34), we can characterise the dynamics of the innovation rate *g* by the following expression:

$$\dot{g}=\frac{L-ag}{a}\rho+\frac{I_Y}{a}(1-\sigma)(1-q)\nu g-\left[\frac{L-ag}{a}-(1-\sigma)(1-q)\frac{I_Y}{a}-(1-\omega)(1-h)\frac{I_T}{a}\right](r-\hat{w})\,. \tag{37}$$

This key equation can be reduced to an equation in *g*, *h*, and *q* by substituting (28), (29) and (35) to eliminate I_Y, I_T and $r-\hat{w}$; we will do this stepwise when we discuss the different cases in section 4.

Resource dynamics are given by differential equations in the proportional extraction rates. First we differentiate (26) with respect to time and substitute (14), (25), (10), (31), and (33) to arrive at:

$$\hat{u}_Y=u_Y\left(1+\frac{(1-\phi)(1-h)}{\phi(1-q)}\right)-\rho+q(1-\sigma)(r-\hat{w}+\nu g)\,. \tag{38}$$

Second, we differentiate (27) with respect to time and substitute (14), (25), (32), and (34) to get:

$$\hat{u}_T=u_T\left(1+\frac{\phi(1-q)}{(1-\phi)(1-h)}\right)-\rho+h(1-\omega)(r-\hat{w})\,. \tag{39}$$

4. Solutions for different substitution conditions

To see the different mechanisms in the model most clearly, it is useful to first consider the case of unitary elasticities in both sectors. Then, intermediate cases with poor substitution in one of the sectors are discussed. Finally, the general case for poor input substitution and extensions are evaluated.

4.1 Cobb-Douglas Case

In our benchmark case of unitary elasticities both in the knowledge-using and the knowledge-competing industry, we will show the following:

Proposition 1: *If* $\sigma = \omega = 1$*, there are no transitional dynamics and the innovation rate equals:*

$$g = \max\left\{0, \frac{(1-\beta)L/a - \beta\rho - \rho(\bar{h}/\bar{q})(1-\phi)/\phi}{1+(\bar{h}/\bar{q})(1-\phi)/\phi}\right\}.$$

Proof: From (26) and (27) we see that factor shares are constant: $q = \bar{q}$ and $h = \bar{h}$. After substitution of (35) and $\sigma = \omega = 1$, the dynamics of the innovation rate in (37) simplify to the following differential equation:

$$\dot{g} = \left(\frac{L}{a} - g\right)\left[g + \rho - \frac{1-\beta}{\beta + (\bar{h}/\bar{q})(1-\phi)/\phi}\left(\frac{L}{a} - g\right)\right] \qquad \text{if } \frac{(1-\beta)L/a}{\beta + (\bar{h}/\bar{q})(1-\phi)/\phi} \geq \rho$$

$$g = 0 \qquad \text{if } \frac{(1-\beta)L/a}{\beta + (\bar{h}/\bar{q})(1-\phi)/\phi} < \rho$$

Any path converging to a negative growth rate must be ruled out. The same holds true for a path converging to $g = L/a$, since it violates the transversality condition (36). Hence, the equilibrium growth rate jumps to the value for which $\dot{g} = 0$, i.e. the value stated in proposition 1, and remains there. □

The rate of innovation is stimulated by a higher supply of labour *L*, a lower unit input coefficient in research *a*, higher mark-up rates $1/\beta$, and a lower discount rate ρ. This corresponds to the findings in other R&D-models. Novel in our multi-sector model is how resource dependence in the different sectors affects innovation incentives. In particular, the rate of innovation decreases with $\left(\bar{h}/\bar{q}\right)(1-\phi)/\phi$, which captures three effects. First, since innovation takes place in the *Y*-sector only, a lower expenditure share on *Y*-goods (lower ϕ) reduces innovation. Second, since innovation is embodied in intermediate goods in the *Y*-sector, a smaller role for intermediates, as measured by a smaller intermediate goods share $\bar{q}$, decreases the market for innovations, which makes research less profitable. Conversely, a high value for $\bar{q}$ implies a low share of non-renewable resources in *Y*-production: the sector is less dependent on non-man-made inputs and this stimulates innovation. Finally, innovation is low when the share of non-renewable resources in the *T*-sector is low (high $\bar{h}$). If the *T*-sector relies heavily on resources rather than labour input, less labour is allocated to this sector, and more becomes available for the research sector.

Hence, greater natural-resource dependence in the knowledge-competing sector reduces output in this sector, but raises innovation.

Fig. 2
(about here)

To study how resource dependence affects growth of consumption rather than innovation, we need to calculate output growth in both final goods sectors. Besides innovation, only depletion of resource inputs drives growth, since labour and materials inputs are constant. From (25), (38), and (39), we derive that with $\sigma = \omega = 1$, resource dynamics are characterised by $\hat{u} = u - \rho$ where $u \equiv (R_Y + R_T)/S$ is the aggregate proportionate extraction rate. In order to satisfy the transversality condition (23), both the stock of resources and the amount of resources used in both sectors must decline at rate $-\hat{S} = -\hat{R}_Y = -\hat{R}_T = \rho$. Differentiating consumption and production functions (17), (3) and (5) with respect to time we obtain the consumption growth rate according to:

$$\hat{C} = \left[q(1-\beta)/\beta + (1-q)\delta\right]\phi g - \left[\phi(1-q) + (1-\phi)(1-h)\right]\rho . \tag{40}$$

Consumption grows at a positive rate only if innovation (at rate g, see first term at right-hand side) is sufficiently large to offset the decline in resource inputs (at rate ρ, see second term). Consumption growth is bigger, the higher are the gains from specialisation (low β) and the larger are productivity spillovers (δ). A lower discount rate (ρ) reduces resource depletion and implies a smaller drag on growth from the scarcity of non-renewable resources. For a *given* rate of innovation g, a higher intermediate goods share in both sectors ($q = \overline{q}$ and $h = \overline{h}$) implies smaller dependence of production on natural resource inputs, which is beneficial for growth. Overall, resource dependence in the knowledge-competing sector (as measured by $1-h$) has an ambiguous impact on growth. Initial resource and knowledge stocks have no impact on dynamics in this case – they affect output levels only.

4.2 Poor substitution in the traditional sector

We now analyse the case in which substitution is more difficult in the knowledge-competing sector than in the knowledge-using sector, which – as above – has a unitary elasticity of substitution. We then arrive at the next proposition:

Proposition 2*: If $1 = \sigma > \omega > 0$, the rate of innovation is non-decreasing over time; its long-run value is determined by $g(\infty) = \max\{0, (1-\beta)(L/a) - \beta\rho\}$. At any time in an equilibrium with innovation, an increase in the resource stock reduces innovation.*

Proof: With $\sigma = 1$, we have $q = \overline{q}$, see (26). The dynamics of h and g are shown in figure 2a. From (34) and (35) we derive the $\dot{h} = 0$ locus, which by (32) corresponds to the $r - \hat{w} = 0$ line. The $\dot{g} = 0$ locus follows from (37) and (29). Accordingly we have:

$$\dot{h} \geq 0 \quad \Leftrightarrow \quad r - \hat{w} \leq 0$$

$$\dot{g} \geq 0 \quad \Leftrightarrow \quad r - \hat{w} \leq \frac{\rho}{1-(1-\omega)\left[\dfrac{(1-h)h}{\phi\beta\overline{q}/(1-\phi)+h}\right]}$$

At h = 0 and h = 1, the $\dot{g}=0$ line intersects the $r-\hat{w}=\rho$ line, while otherwise the former lies below the latter. Moreover, both lines are downward sloping for $h \in [0,1]$. If $(1-\beta)L/a > \beta\rho$, there is a unique saddlepath, which lies between the $\dot{h}=0$ and $\dot{g}=0$ loci, along which the intermediate goods share h declines to zero and the innovation rate g increases to $g=(1-\beta)(L/a)-\beta\rho$. Any other path must be ruled out since it violates the transversality condition. A path converging to h = 1 implies that the real interest rate is negative, which violates (36). A path converging to g = 0 and h = 0 implies a negative firm value. If $(1-\beta)L/a \leq \beta\rho$, the $\dot{g}=0$ locus lies outside the positive quadrant and only a path without innovation can arise in equilibrium.

To determine the initial value of h, we use (38) to plot the $\dot{u}_Y=0$ locus in figure 2b. Moreover we derive from (27), (29), and (30) the following relationship between the state variable S at t = 0, S(0), and initial values of endogenous variables:

$$u_Y(0) = \left(\frac{1-\overline{h}}{\overline{h}}\right)^{\omega/(1-\omega)} \left(\frac{h(0)}{1-h(0)}\right)^{1/(1-\omega)} \left(\frac{\phi(1-\overline{q})}{\phi\beta\overline{q}+(1-\phi)h(0)}\right)\frac{L-ag(0)}{S(0)}$$

Since for every h the unique equilibrium value of g can be determined in the upper panel, the above equation defines a relationship between the initial stock S(0) and initial values of the endogenous variables $u_Y(0)$ and h(0). In the h, u_Y plane, this relationship is a monotonically increasing curve OS, starting at 0,0 and with a vertical asymptote at h = 1. Given the development of h derived in the upper panel, there is a unique saddlepath converging to a constant positive proportionate extraction rate. The intersection between this saddlepath and the OS curve determines the initial value of h. A higher resource stock shifts the OS curve down and implies a higher intermediate goods share h. From the upper part of the figure it can be seen that a higher h implies a lower growth rate, provided that $0 < g < (1-\beta)(L/a)-\beta\rho$. □

Fig. 3
(about here)

Because of relatively poor substitution in the knowledge-competing sector, the sectoral output becomes relatively more expensive and labour moves out of this sector when the resource stock gets depleted (see (29) with h gradually declining to zero). As a result the knowledge-competing sector vanishes; only conditions in the knowledge-using sector determine innovation in the long run. Indeed, the steady-state innovation rate is the same as the one derived by Grossman and Helpman (1991, chapter 3) in their one-sector model without resources. Another implication is the "resource curse"-effect, stated in the second part of the proposition. A high resource stock benefits mainly the knowledge-competing sector if this sector has the lowest substitution possibilities. This sector expands at the cost of the innovating sector in response to a higher resource stock, and the smaller size of the innovating sector makes innovation less profitable.

The steady-state growth rate of aggregate consumption reads:

$$\hat{C}(\infty)=\left[\bar{q}\frac{1-\beta}{\beta}+(1-\bar{q})\delta\right]\phi g(\infty)-[\phi(1-\bar{q})+(1-\phi)]\rho$$

where we write $x(\infty)\equiv\lim_{t\to\infty}x(t)$ for any variable *x*. Comparing this result with the Cobb-Douglas case in (40), we see that poor substitution implies a larger drag on growth from depletion in the long run (see the second term in brackets). However, innovation turns out to be much stronger with poor substitution so that, overall, consumption grows faster than under the Cobb-Douglas assumption.

4.3 Poor substitution in the high-tech sector

For the case of a unitary elasticity in the knowledge-competing sector and poor substitution in the knowledge-using sector we state:

Proposition 3: *If* $1=\omega>\sigma>0$, *the innovation rate is non-increasing over time, becomes zero in finite time and remains zero. At any time in an equilibrium with innovation, an increase in the resource (knowledge) stock increases (reduces) innovation.*

Proof: The relevant phase planes are shown in figure 3. From (33) and (35) we get the $\dot{q}=0$ locus which by (31) corresponds to the $r-\hat{w}=0$ line. The $\dot{g}=0$ locus follows again from (37). Accordingly we obtain:

$$\dot{q}\geq 0 \quad\Leftrightarrow\quad r-\hat{w}+\nu g\leq 0$$

$$\dot{g}\geq 0 \quad\Leftrightarrow\quad r-\hat{w}+\nu g\leq \frac{\rho+\nu g}{1-(1-\sigma)\left[\dfrac{(1-q)q}{\bar{h}(1-\phi)/\beta\phi+q}\right]}$$

At q = 0 and q = 1, the $\dot{g}=0$ locus intersects the $r-\hat{w}=\rho$ line; for all other values of *q*, the former lies below the latter. The intercept of the $\dot{g}=0$ locus is negative, the locus slopes upward; only if $(1-\beta)L/a-\beta\rho>\rho\bar{h}(1-\phi)/\phi$, it cuts the horizontal axis (since then $r-\hat{w}=\rho$ requires g > 0). The result is that the innovation rate can be positive at most for a finite amount of time, as shown in figure 3a.

To determine the initial value of *q*, we use (39) to plot the $\dot{u}_T=0$ locus in figure 3b. We also derive from (26), (28), and (30) the following relationship between *S* at *t* = 0, *S*(0), and initial values of endogenous variables:

$$u_T(0)=\left(\frac{1-\bar{q}}{\bar{q}}\right)^{\sigma/(1-\sigma)}\left(\frac{q(0)}{1-q(0)}\right)^{1/(1-\sigma)}\left(\frac{\beta(1-\phi)(1-\bar{h})}{(1-\phi)\bar{h}\ +\ \phi\beta q(0)}\right)\frac{L-ag(0)}{N(0)^{-\nu}S(0)}$$

This equation is depicted as the OS line in figure 3b. The intersection of OS with the saddlepath determines the initial value *q*(0).

Changes in the initial stock variables shift the OS curve. First, a larger resource stock moves the OS curve down, increases *q*(0) and raises innovation growth, provided that *g* > 0. Second, an increase in the knowledge stock has the opposite result: given that g > 0, it shifts the OS curve up and decreases both $q(0)$ and innovation growth. □

Relatively poor substitution in the knowledge-using sector has opposite effects compared to poor substitution in the other sector: now resource depletion makes the innovating sector relatively more expensive and shifts labour to the traditional sector (see (28)-(29) with q declining to zero). As a result, research incentives fade away with the depletion of the resource stock and consumption steadily declines in the steady state. Furthermore, a higher *resource* stock now expands the knowledge-using sector and this may spur innovation in the short run. With resource-using technological change, a higher *knowledge* stock increases the demand for scarce resources in the knowledge-using sector, which reduces its size and thus innovation incentives.

4.4 Poor substitution in both consumer sectors

We now turn to the case with substitution elasticities smaller than unity in both sectors. Our key result is stated in the following proposition:

Proposition 4: *With poor input substitution in both sectors, the steady state innovation rate is given by:*

$$g(\infty) = 0 \qquad \text{if } 0 < \sigma \le \omega < 1 \text{ or if } 0 < \omega < \sigma < 1 \text{ and } L < \underline{L}, \tag{41a}$$

$$g(\infty) = \frac{(1-\beta)\sigma L/a - \beta\rho}{(1-\sigma)\beta\nu + \sigma} \equiv \bar{g}_{scale} \qquad \text{if } 0 < \omega < \sigma < 1 \text{ and } \underline{L} \le L \le \bar{L}, \tag{41b}$$

$$g(\infty) = \frac{\rho(\sigma-\omega)}{\nu(1-\sigma)\omega} \equiv \bar{g}_{nonscale} \qquad \text{if } 0 < \omega < \sigma < 1 \text{ and } L > \bar{L}, \tag{41c}$$

where $\underline{L} \equiv \left(\frac{\beta}{1-\beta}\right)\frac{a\rho}{\sigma}$ and $\bar{L} \equiv \left(\frac{\beta(1-\sigma)\nu + \sigma - \omega}{(1-\sigma)\nu(1-\beta)}\right)\frac{a\rho}{\omega}$.

Proof: From (31), (32) and the transversality condition (36) we see that both value shares q and h continue to fall in the steady state, so that they must eventually approach zero. Hence, in the long run we have:

$$h(\infty) = q(\infty) = 0. \tag{42}$$

Substituting (41) into (39) and (40) and taking into account (14) and (22), we find the long-run proportional extraction rates:

$$u_Y(\infty) = \phi\rho; \quad u_T(\infty) = (1-\phi)\rho. \tag{43}$$

Substituting (43) into (34) and (35) we find:

$$\hat{I}_Y(\infty) = \sigma(r(\infty) - \hat{w}(\infty)) - (1-\sigma)\nu g(\infty) - \rho \tag{44}$$

$$\hat{I}_T(\infty) = \omega(r(\infty) - \hat{w}(\infty)) - \rho \tag{45}$$

Together with (36), (13) and (15) these two equations determine the long-run values of the labour allocation, innovation growth and the real interest rate. There are three possible steady states: (i) an interior steady state with g, I_T and I_Y strictly positive and constant, (ii) a corner solution with $I_T = 0$ zero, and (iii) a corner solution with g = 0. We discuss each of these equilibria in turn.

(i) In the interior steady state with $\hat{I}_Y = \hat{I}_T = 0$, we derive from (45) that the real interest rate equals $r - \hat{w} = \rho / \omega$ and from (44) that the growth rate equals $g(\infty) = \overline{g}_{nonscale}$. Since innovation cannot be negative, this equilibrium is only feasible if $\overline{g}_{nonscale} \geq 0$ which requires $\sigma \geq \omega$. Moreover, feasibility requires $I_T(\infty) \geq 0$. Using (13) to solve for I_Y and plugging this solution into (15) to obtain I_T we find that in this equilibrium $I_T(\infty) \geq 0$ requires $L \geq \overline{L}$.

(ii) In a corner solution with $I_T(\infty) = 0$ and $\hat{I}_Y(\infty) = 0$, we find from (13), (15) and (44) that the innovation rate equals $g(\infty) = \overline{g}_{scale}$. Since innovation cannot be negative, feasibility requires $\overline{g}_{scale} \geq 0$, which requires $L \geq \underline{L}$. Moreover, in this equilibrium we must have $\hat{I}_T(\infty) < 0$ in order to not violate $I_T(\infty) = 0$. Substituting $g = \overline{g}_{scale}$ into (13) and (15) to solve for the real interest rate and substituting this solution into (45), we see that $\hat{I}_T(\infty) < 0$ requires $L < \overline{L}$.

(iii) In the other corner solution with $I_Y(\infty) = 0$, $\hat{I}_Y(\infty) < 0$, and $\hat{I}_T(\infty) = 0$ results in $r - \hat{w} = \rho / \omega$, $\hat{I}_Y = \rho(\sigma - \omega) / \omega$ and hence requires $\sigma < \omega$. From (13) and (36) we see that this implies g = 0. □

The proposition exhibits that there is a unique steady state for given parameters. The appendix shows that for given initial state variables, $N(0)$ and $S(0)$, a unique trajectory leads to the unique steady state. Proposition 4 implies that the steady-state innovation rate:

(i) can be positive even with poor input substitution in all production sectors,

(ii) is non-decreasing in substitution possibilities in the innovative sector, σ,

(iii) is non-increasing in the substitution possibilities of the non-innovative sector, ω,

(iv) is non-decreasing in labour supply L and, for a large enough labour supply, is independent of labour supply,

(v) is non-monotonous in the discount rate ρ if $\sigma > \omega$.

The remainder of the section discusses these results and the implications for consumption growth. A necessary condition for innovation to remain active in the long run is that substitution possibilities in the knowledge-using sector are better than in the knowledge-competing sector. As the resource stock is depleted, the sector with poorest substitution possibilities is hurt most, i.e. demand shifts away from it because the sector faces the steepest increase in costs. Hence, if the Y-sector suffers from poorest substitution ($\sigma < \omega$), it shrinks over time, which sooner or later makes innovation unprofitable. In contrast, if the T-sector has poorest substitution possibilities ($\sigma > \omega$), labour moves towards the knowledge-using sector which increases the incentives to innovate. If, in addition, the labour force is large relative to the discount rate ($L > \underline{L}$), the long-run size of the Y-sector is large enough to sustain incentives to invest in new firms. If the labour supply is still relatively small

($\underline{L} < L < \overline{L}$), the growing scarcity of resources drives labour out of the T-sector. An exogenous increase in the labour force eventually ends up in the Y-sector and in innovation. This implies a scale effect: growth is increasing in the size of the economy as measured by the labour force.

However, this scale effect only applies for small L. As soon as $L > \overline{L}$, an exogenous increase in the labour force will raise production in both sectors, but will not increase innovation. To see why, we note that two opposing – but inseparable – forces from depletion and technological change, respectively, determine labour allocation. On the one hand, we already saw what we call the "differential substitution effect": as the resource stock is depleted, labour tends to move to the sector with good substitution, which is the Y-sector provided that $\sigma > \omega$. On the other hand, there is an "innovation effect": innovation causes resource-using technological change (since $\nu > 0$) and increases the demand for resources in the Y-sector. Resource inputs per unit of labour rise which makes it harder to employ workers in the Y-sector when resources get depleted. This counteracts the differential substitution effect when $\sigma > \omega$. In a steady state with small labour supply ($\underline{L} < L < \overline{L}$), the innovation effect is too weak to offset the differential substitution effect. However, in a steady state with large labour supply ($L > \overline{L}$), the rate of innovation could be potentially so large that labour would, on balance, move from the Y-sector to the T-sector. This is self-defeating, however, since smaller employment in the Y-sector reduces innovation. Hence, in equilibrium the two forces exactly cancel each other out and both sectors keep employing labour. If $L > \overline{L}$, no increase in the labour force can increase the innovation rate, as it would make the innovation effect dominate the depletion effect (which is fixed at ρ) but, again, this is self-defeating.

We are now able to explain why the innovation rate does not always fall with the discount rate. If $\underline{L} < L < \overline{L}$ as well as in the Cobb-Douglas case (section 4.1) and in most endogenous growth models, discounting disfavours investment in general and investment in R&D in particular. However, if $L > \overline{L}$, the two types of investments, resource conservation and innovation, do not necessarily move in the same direction. An increase in the discount rate makes investors less patient so that the resource stock is depleted faster (see (43)). This implies that the differential substitution effect becomes stronger and there is room for a stronger innovation effect to counteract it in the steady state. Although discounting reduces investment in the resource by speeding up depletion, it makes investment in innovation more attractive if depletion expands the Y-sector at the cost of the T-sector (which requires $\sigma > \omega$).

In the long run, with $q = h = 0$, growth of consumption is (cf. (40)):

$$\hat{C}(\infty) = \phi\delta g(\infty) - \rho . \qquad (46)$$

Whenever endogenous knowledge accumulation affects the productivity of resource use in Y-production ($\delta > 0$), innovation implies resource-augmenting technological change that makes long-run consumption growth technically feasible. However, in the market equilibrium, consumption grows in the long run only if incentives to innovate are large enough relative to the incentive to deplete the resource stock. From (45) and (46) we derive that this requires,

(i) sufficiently large spillovers, $\delta > [1+\phi(\sigma-\omega)/(1-\sigma)]^{-1}(1-\beta)/\beta$,

(ii) better substitution in the knowledge-using sector than in the knowledge-competing sector ($\sigma > \omega$), and

(iii) a sufficiently low discount rate, $\rho < [\beta/\phi\delta + \sigma + (1-\sigma)\beta v]^{-1}(1-\beta)\delta L/a$.

Under these conditions, the discount rate affects consumption growth in an ambiguous way: substituting (41) into (46), we see that as long as $\underline{L} < L < \overline{L}$ an increase in the discount rate lowers both innovation and consumption growth; however, as long as $L > \overline{L}$ (which requires ceteris paribus a low discount rate), an increase in the discount rate raises both innovation and consumption growth. As a result, there is an inverted-V shaped relationship between the discount rate and long-run consumption growth.

4.5 Model extension: physical capital

The long-run results of the previous subsections continue to hold when we make different assumptions regarding the production technology for intermediate inputs. In particular, we may allow for the use of materials (distinct from the resource *R*, e.g. metals), which is bounded because of materials balance principles, or the use of physical man-made capital, which may accumulate over a certain period of time.

When we assume that both intermediate goods are produced with such an additional input, we find that the price of this input does not play a role in determining the relative size of the knowledge-using and the knowledge-competing sector, which – according to our discussion in section 4.4 – ultimately determines the innovation rate. Hence, material and physical capital inputs have no impact on the technical change in this model. The growth rate of consumption is still given by (45). The reason is that the share of intermediates in production approaches zero, so that the growth rate of the input does not matter for the growth rate of final production.

5. Conclusions

In this paper we have used a multi-sector framework in which differences in sectoral substitution opportunities cause labour reallocation when the resource stock is depleted. Endogenous innovation generates labour and resource augmenting technological change and, as a by-product, public knowledge, on which further innovation can build. Combined with a sufficiently low discount rate, knowledge spillovers would be sufficient to keep growth going in a model without natural resources (like the standard endogenous growth model) or with natural resources and good input substitution (our Cobb Douglas case). However, we have shown that with poor input substitution, the knowledge spillovers can only sustain growth if substitution in the sector for which innovation is developed is larger than in the sector without innovation opportunities. In this case, the increasing scarcity-price of resources makes the sector without innovation opportunities relatively expensive, shifting consumer demand towards the innovating sector and increasing the incentives for innovation.

The model has some new interesting implications. With relatively poor substitution in the sector without innovation opportunities, long-run consumption growth may be higher with poorer substitution and, during transition, resource abundance may reduce innovation incentives. Furthermore, the size of the elasticities of substitution, rather than resource and labour endowments, bound the rate of growth. As a result, beyond a certain threshold, the scale of the economy has no effect on long-run growth.

We have made some simplifying assumptions that may be relaxed in future research. First, we allowed for only one type of R&D activity. The model could be extended with separate R&D activities for capital augmenting and energy augmenting innovations (cf. Acemoglu, 2002), possibly in multiple sectors. Differences in substitution across sectors will still determine sectoral allocation in response to resource depletion and thus market size for innovation, but more complex patterns of innovation over time can arise. Second, we have abstracted from resource extraction costs and pollution from resource use, which may be taxed by the government. These features may change the price profile of the resource but they hit both consumer sectors in the same way. As the effects of price changes in the two sectors work in opposite directions, as seen in sections 4.2 and 4.3, the quality of our results is not expected to change substantially when enlarging the general model set-up in this way. Third, as the paper focuses on market solutions, the issue of optimal policies has not been discussed. Resource use produces no negative externalities in this model, only R&D generates positive spillovers which lead, as in the original "Romer-type" approach to R&D, to positive subsidies for innovations in the social optimum.

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Figures

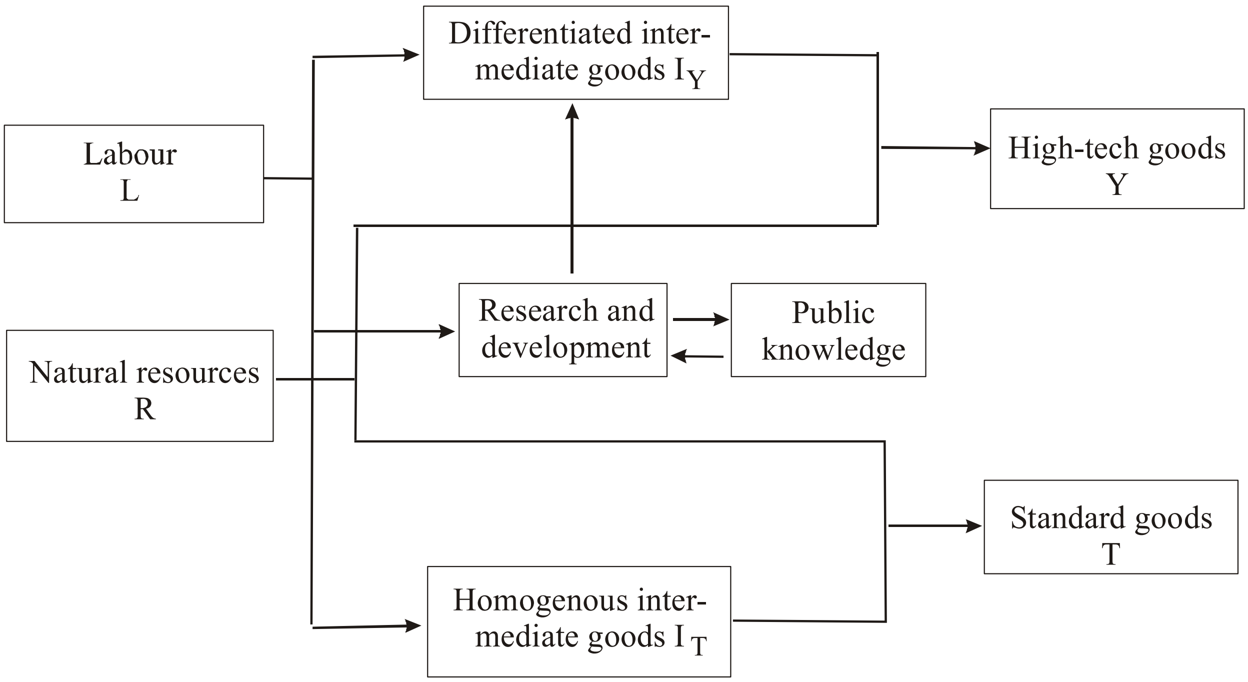

Fig. 1

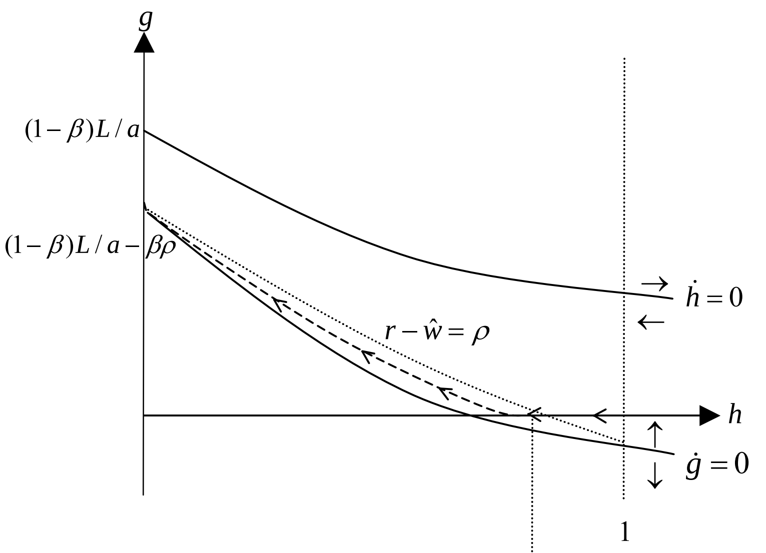

Fig. 2a

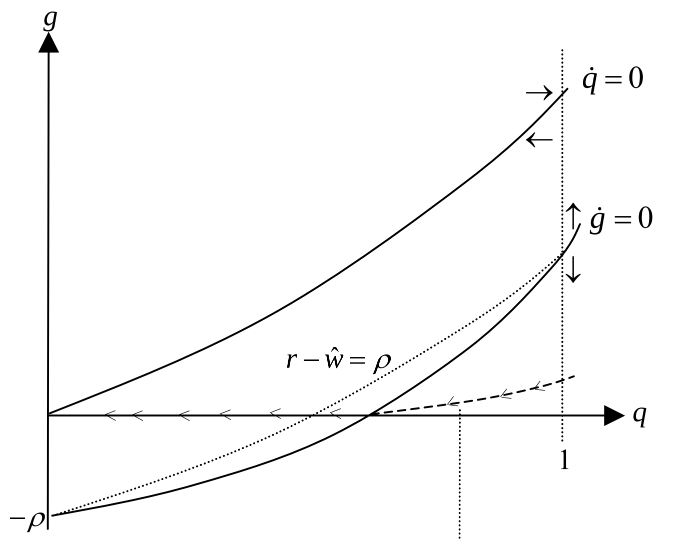

Fig. 3a

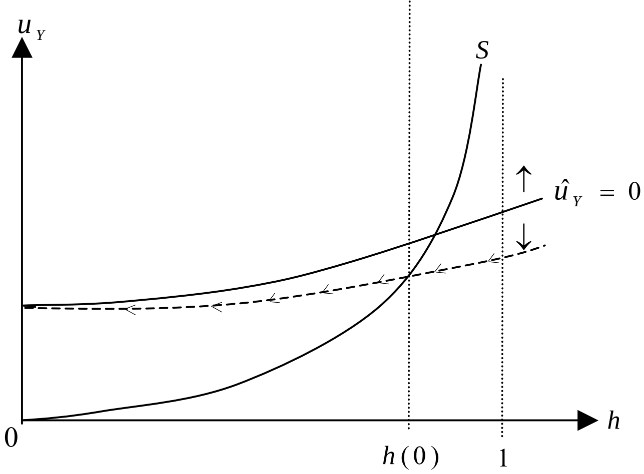

Fig. 2b

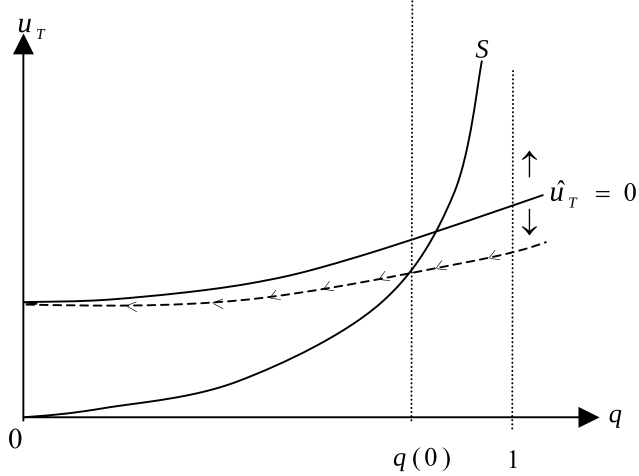

Fig. 3b

Appendix

We can show the stability of the steady state in the general case as follows.

Steady state with innovation
We first discuss the stability of steady state solutions (45b) and (45c) of the proposition in section 4.4. These are solutions with $g > 0$ so that we can use the equality in (13). Evaluating partial derivatives in differential equations (33), (34), (35), (40), taking into account how $r - \hat{w}$ and g depend on h, I_Y and I_T through (13) and (15), respectively, taking into account that $q = [(1-\phi)/\phi\beta] \cdot hI_Y / I_T$ (from (28)-(29), and evaluating the partial derivatives for $q = h = 0$, we find the following Jacobian:

$$J(h, I_Y, I_T, u_T) \equiv \begin{bmatrix} \partial\dot{h}/\partial h & \partial\dot{h}/\partial I_Y & \partial\dot{h}/\partial I_T & \partial\dot{h}/\partial u_T \\ \partial\dot{I}_Y/\partial h & \partial\dot{I}_Y/\partial I_Y & \partial\dot{I}_Y/\partial I_T & \partial\dot{I}_Y/\partial u_T \\ \partial\dot{I}_T/\partial h & \partial\dot{I}_T/\partial I_Y & \partial\dot{I}_T/\partial I_T & \partial\dot{I}_T/\partial u_T \\ \partial\dot{u}_T/\partial h & \partial\dot{u}_T/\partial I_Y & \partial\dot{u}_T/\partial I_T & \partial\dot{u}_T/\partial u_T \end{bmatrix} =$$

$$\begin{bmatrix} -(1-\omega)\dfrac{\rho}{\omega} & 0 & 0 & 0 \\ J_{21} & \left(\dfrac{\sigma+(1-\sigma)\nu\beta}{1-\beta}\right)\dfrac{I_Y}{a} & [\sigma+(1-\sigma)\nu]\left(\dfrac{1-\phi}{\phi\beta}\right)\dfrac{I_Y}{a} & 0 \\ (1-\omega)\dfrac{\rho}{\omega}\left(\dfrac{\phi\beta}{1-\phi}\right)\dfrac{I_T}{a} & \dfrac{\omega}{1-\beta}\left(\dfrac{\phi\beta}{1-\phi}\right)\dfrac{I_T}{a} & \omega\dfrac{I_T}{a} & 0 \\ (1-\phi)\rho[(1-\omega)(r-\hat{w})-\phi\rho] & 0 & 0 & \rho \end{bmatrix}$$

where $J_{21} = (1-\sigma)(r - \hat{w} + \nu g)\dfrac{\beta}{1-\beta}\left(\dfrac{\beta\phi}{1-\phi}\right)\dfrac{I_T}{a}$.

Using Laplace expansion, we can write the characteristic equation of this system as $(J_{11}-\lambda)[(J_{22}-\lambda)(J_{33}-\lambda) - J_{23}J_{32}](J_{44}-\lambda)$, where J_{ij} are the elements of the above matrix. Hence we find:

$$\lambda_1 = J_{11} < 0, \lambda_4 = J_{44} > 0, \lambda_2\lambda_3 = J_{22}J_{33} - J_{23}J_{32} = -(1-\sigma)\nu\omega I_Y I_T / a^2 < 0$$

This implies we have two positive and two negative eigenvalues. Since we also have two initial conditions, (26) and (27) (to be evaluated at time zero), there is a unique path leading to the steady state.

Steady state without innovation
Now we discuss the equilibrium without innovation. From (37) and $g = \dot{g} = 0$, we can solve for the real interest rate:

$$r - \hat{w} = \left[1 - (1-\sigma)(1-q)(I_Y/L) - (1-\omega)(1-h)(I_T/L)\right]^{-1}\rho > 0 \qquad (37')$$

With g = 0, we have $L_T = L - L_Y$ from (15) and $L_Y/(L - L_Y) = [\phi\beta/(1-\phi)]q/h$ from (28)-(29). Hence, we can use (31), (32) and (39) to reduce the model to three differential equations in h, $z_q \equiv q/h$, and u_T.

$$\dot{h} = -h(1-h)(1-\omega)(r-\hat{w}), \tag{32'}$$

$$\dot{z}_q = z_q\left[(1-\sigma)hz_q - (1-\omega)h - (\omega-\sigma)\right](r-\hat{w}), \tag{31'}$$

$$\dot{u}_T = u_T\left[u_T\left(1 + \frac{\phi(1-z_q h)}{(1-\phi)(1-h)}\right) - \rho - h(1-\omega)(r-\hat{w})\right]. \tag{39'}$$

First, we consider the case with $\sigma < \omega$. Since $r - \hat{w} > 0$, see (37′), (32′) implies $h(\infty) = 0$. Then (31′) implies $z_q(\infty) = 0$, (39′) implies $u_T(\infty) = (1-\phi)\rho$, and (37′) implies $r - \hat{w} = \rho/\omega$. Evaluating the Jacobian of the system in (32′), (31′) and (39′) around this steady state, we find:

$$J_0(h, z_q, u_T) \equiv \begin{bmatrix} \partial\dot{h}/\partial h & \partial\dot{h}/\partial z_q & \partial\dot{h}/\partial u_T \\ \partial\dot{z}_q/\partial h & \partial\dot{z}_q/\partial z_q & \partial\dot{z}_q/\partial u_T \\ \partial\dot{u}_T/\partial h & \partial\dot{u}_T/\partial z_q & \partial\dot{u}_T/\partial u_T \end{bmatrix} = \begin{bmatrix} -\frac{\omega-\sigma}{\omega}\rho & 0 & 0 \\ 0 & -\frac{1-\omega}{\omega}\rho & 0 \\ 0 & J_{0,32} & \rho \end{bmatrix}$$

where $J_{0,32}$ is a finite number. The eigenvalues are the elements on the diagonal. Hence there are two negative and one positive eigenvalues. Given our two initial conditions this implies uniqueness of the transition to the steady state.

Second, the case $\sigma > \omega$ is completely symmetric: rewriting (31), (32), and (38) as a system of three differential equations in q, $z_h \equiv h/q$, and u_Y and calculating the Jacobian, we find the eigenvalues $\{-(\sigma-\omega)\rho/\sigma, -(1-\sigma)\rho/\sigma, \rho\}$ so that again uniqueness applies.