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Abstract. Green accounting theories have shown that negative genuine savings at some point in time imply unsustainability. Consequently, recent studies advocate the use of the genuine savings measure for empirical testing: a negative index implies sustainability be rejected. This criterion is not forward-looking: positive current genuine savings do not rule out 'genuine dissaving' in the future. This paper derives a one-to-one relationship between the sign of long-run genuine savings and the limiting sustainability condition in the capital-resource model: if the sum of the rates of resource regeneration and augmentation exceeds (falls short of) the discount rate, long-run genuine savings are positive (negative). Testing this limiting condition allows to reveal whether current genuine savings are delivering a false message.

Keywords: Genuine Saving, Green Accounting, Renewable Resources, Sustainable Development, Technological Progress.

JEL codes: Q01, O47, D90.

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1. Introduction

Defining suitable criteria for testing sustainability is a major goal for theoretical research on economic growth and resource economics. In recent literature, sustainable development is defined as a path characterized by non-declining utility. Building on this notion, a number of studies advocate the use of the genuine saving criterion for testing sustainable development empirically (*e.g.* Hamilton and Clemens, 1999; Neumayer, 1999; Pezzey, 2004). This criterion consists of evaluating, at a given point in time, an environmentally-adjusted measure of savings which represents the difference between aggregate investment in produced assets and the value of net depletion of natural resources (Pearce and Atkinson, 1993; Pearce *et al.*, 1996): a negative value of genuine savings is held to imply unsustainability.

The rationale for this criterion is provided by the literature on green accounting. Several authors studied the properties of the genuine saving measure in capital-resource models - *i.e.* optimal growth models where aggregate output is produced by means of natural resources, and the consumption time-path is determined by the Keynes-Ramsey rule (Asheim and Weitzman, 2001; Hartwick *et al.* 2003). In this framework, Pezzey (2004) has shown negative genuine savings at any point in time imply unsustainability, under very general conditions.

A problem with this method is that the genuine saving criterion cannot ascertain sustainable development. Positive current genuine savings are not sufficient for sustainability, as firstly noted by Asheim (1994) and extensively discussed in Vellinga and Withagen (1996), and Aronsson and Lofgren (1998). In particular, genuine savings may be positive even for long intervals, in economies where consumption is bound to decrease in the long run. Consequently, the sign of *current* genuine savings might deliver a false message at the empirical level: even if genuine savings are positive in the present, there is no guarantee that they will in the future. The *false-message problem* - the possibility that genuine savings turn negative in the long run - is a critical issue with respect to empirical testing, because a major aim of applied methods is to check whether present economies satisfy the conditions for obtaining non-declining welfare in the future. Since the sign of current genuine savings may be misleading in this regard, alternative criteria are needed in order to test sustainability in a forward-looking manner.

This paper addresses the problem at the theoretical level, and studies necessary and sufficient conditions for obtaining positive genuine savings in the long run. The analysis employs an extended version of the capital-resource growth model, which includes positive rates of technical progress and natural regeneration. In this setup, long-run utility is non-declining if a precise limiting condition is satisfied, *i.e.* if the sum of the rates of resource regeneration and technical augmentation are at least equal to the utility discount rate (Valente, 2005). The main result of this paper is that there exists a one-to-one relationship between the limiting condition and the sign of long-run genuine savings: if the sum of the rates of resource regeneration and augmentation exceeds (falls short of) the utility discount rate, long-run genuine savings are positive (negative). From the theoretical standpoint, this result extends previous findings of the literature on green accounting, providing an analytical characterization of genuine savings long-run dynamics. From an empirical perspective, testing the limiting condition for 'genuine dissaving' allows to reveal whether current genuine savings are delivering a false message, and more generally, provides a forward-looking criteria upon which sustainability tests can be built.

The structure of the paper is as follows: section 2 lists basic definitions used in the analysis and briefly summarizes the state of the art; section 3 presents the theoretical model and derives the main results, and section 4 concludes.

2. Sustainability and genuine savings

2.1. Basic definitions

Sustainable development. In this paper, sustainable development is defined as a path characterized by non-declining welfare over time. Instantaneous social welfare is represented by $u(c)$, where c is consumption and $u : \Re \rightarrow \Re$ is twice continuously differentiable, strictly increasing, strictly concave, and satisfies $\lim_{c\rightarrow 0} \partial u/\partial c = \infty$. Considering an infinite time-horizon, sustainability can be expressed as follows:

$$\text{S.D.} \quad \Leftrightarrow \quad \frac{du\left(c\left(t\right)\right)}{dt} \geq 0 \text{ for each } t \in [0, \infty)\,. \tag{1}$$

It follows from (1) that a necessary condition for sustainable development is that the variation in social welfare must be non-negative as times goes to infinity:

$$\lim_{z\rightarrow\infty} \left.\frac{du\left(c\left(t\right)\right)}{dt}\right|_{t=z} \geq 0. \tag{2}$$

Condition (2) will be called the limiting condition for sustainability.

Genuine savings. In the original formulation proposed by Pearce and Atkinson (1993) and Pearce *et al.* (1996), aggregate genuine savings - also called 'green net investments' - are defined as the difference between conventional savings and the value of net resources depletion. However, when the economy exhibits a positive rate of technical progress, the genuine savings measure must be augmented to include a *time-premium* term, as shown by the recent literature on green accounting (Asheim and Weitzman, 2001). Formally, considering a closed economy producing a single good, which can be either consumed or accumulated in the form of man-made capital, genuine savings at time t equal

$$\theta\left(t\right) = \dot{k}\left(t\right) + p_r\left(t\right)\dot{s}\left(t\right) + \pi\left(t\right), \tag{3}$$

where k and s are the stocks of man-made and natural capital, p_r the gross marginal rent on natural capital, and π is the time premium. When technical progress is absent, $\pi = 0$ and (3) reduces to the non-augmented measure of Pearce and Atkinson (1993). If there is technical progress, the time premium at time t equals the present discounted value of all future improvements in production possibilities: denoting by r the flow of natural resources extracted from the resource stock s and used in production, and assuming that aggregate output is represented by the production function $\Psi(k(t), r(t), t)$, the time premium can be defined as[1]

$$\pi(v) = \int_v^\infty \Psi_t(t) \cdot e^{-\int_v^t i(w)dw} dt, \tag{4}$$

where $\Psi_t = \partial\Psi/\partial t$ and $i(t)$ is the prevailing interest rate. Expression (4) implies

$$\dot{\pi}(t) = i(t)\,\pi(t) - \Psi_t(t). \tag{5}$$

The role of π in determining the properties of the genuine savings measure is briefly explained in sect.2.2 below, and the expression for the time-premium term in the optimal growth model will be given after suitable technology specifications.

The genuine saving criterion. The genuine saving criterion of sustainability testing consists of checking that the condition

$$\theta(\bar{t}) > 0 \tag{6}$$

is satisfied at a given point in time $\bar{t}$. This method is widely used and employed for a large number of countries (see *e.g.* Neumayer, 1999; Hamilton and Clemens, 1999). The link between sustainability and the genuine saving criterion has been investigated by the theory of green national accounting: some results of this literature are briefly summarized below.

2.2. The false-message problem

The link between sustainability and genuine savings has been subject to extensive analysis after the original definition given by Pearce and Atkinson (1993) and Pearce *et al.* (1996). A first connection between genuine savings and the time-path of utility is provided by the Hartwick rule: assuming stationary technology, consumption is kept constant over time if net rents from natural resources are entirely invested in the accumulation of man-made capital at each point in time (Hartwick, 1977), *i.e.* a situation where genuine savings equal zero in every instant (Dixit *et al.* 1980). As shown by Withagen and Asheim (1998), also the converse is true: $\dot{c}(t) = 0$ for all t implies zero genuine savings for all t.

These results assume static technology and thus make reference to the original, non-augmented measure of Pearce and Atkinson (1993). A similar principle applies with respect to the augmented measure (3) when a positive rate of technical progress is assumed: zero augmented genuine savings for all t imply $\dot{c}(t) = 0$ for all t (Asheim and Weitzman, 2001). The economic reason for this result is that when production possibilities are improved over time also by means of technological progress, consumption can be kept constant over time by re-investing *less* than total rents in each instant. Indeed, the role of the time-premium term is to incorporate in the genuine savings measure the 'value of time', as determined by exogenous improvements in production possibilities.

In the last decade, the properties of the genuine saving measure have been increasingly analysed in standard models of optimal growth, where the consumption time-path is determined by the Keynes-Ramsey rule (Hamilton and Clemens, 1999; Hartwick *et al.* 2003). In this framework, Pezzey (2004) has shown that $\theta(\bar{t}) \leq 0$ at some $\bar{t} \in [0, \infty)$ implies unsustainability, under very general conditions. This result legitimates the use of the genuine saving criterion as a one-sided test of *un*sustainability: if current estimated genuine savings are negative, sustainability is rejected.[2]

However, the genuine saving criterion cannot ascertain sustainable development, since $\theta(\bar{t}) > 0$ at some $\bar{t}$ is necessary but not sufficient for sustainability (Vellinga and Withagen, 1996; Aronsson and Lofgren, 1998). In particular, genuine savings may be positive even for long intervals, in economies where consumption is bound to decrease in the long run (Asheim, 1994). This is the root of the 'false-message problem': if one can observe current genuine savings and show that $\theta > 0$ in the present, there is no guarantee that θ will be positive in the future.

The false-message problem raises major questions for economic analysis, and this paper tackles the following issue: under what circumstances genuine savings *will* surely be negative? Formally, the analysis focuses on those conditions implying

$$\lim_{t \to \infty} \theta(t) < 0. \tag{7}$$

The situation described by (7) can be labelled as 'long-run genuine dissaving'. Since a full analytical characterization of the genuine savings time-path in optimal growth models is lacking in the literature so far, deriving necessary and sufficient conditions for long-run genuine dissaving would shed more light on the dynamic properties of $\theta(t)$. This point is nonetheless relevant from an empirical perspective. A major aim of applied methods is to check whether present economies satisfy the conditions for obtaining non-declining welfare in the *future*, which requires to test sustainability in a forward-looking manner: testing the sign of *current* genuine savings can be misleading in this regard, due to the false-message problem. Deriving conditions for long-run genuine dissaving yields the possibility of testing inequality (7), which brings additional information about sustainability and allows to reveal whether positive current genuine savings are delivering a false message.

3. Genuine savings and optimal growth

This section studies the link between the sign of long-run genuine savings and the limiting condition (2) for non-declining long-run utility. The analysis employs an extended version of the capital-resource model pioneered by Dasgupta and Heal (1974) and Stiglitz (1974), where aggregate output is produced by means of man-made capital and natural resources, and consumption dynamics are governed by the Keynes-Ramsey rule. This setup is widely recognized as the theoretical benchmark for analyzing the properties of the genuine saving measure (Asheim, 1994; Hamilton and Clemens, 1999; Hartwick *et al.*, 2003). The model presented is a specialized version of the general framework developed by Asheim and Weitzman (2001), and employed by Pezzey (2004) to derive the genuine saving criterion for testing unsustainability.

3.1. The model

Gross output equals $y = f(k, mr)$, where r is the flow of extracted resources used in production and m is resource-augmenting technical progress, representing the development of resource-saving techniques that improve the productivity of r over time. The rate of resource augmentation $\nu \equiv \dot{m}/m > 0$ is exogenous and assumed constant. Both k and r are essential inputs ($f(k,0) = f(0,mr) = 0$) and $f : \Re^2 \to \Re$ is homogeneous of degree one, twice continuously differentiable, strictly increasing, strictly concave, and satisfies $\lim_{r\to 0} \partial f(k,r)/\partial r = \infty$. The natural resource is renewable, with constant rate of regeneration $g > 0$. The evolution of the stocks is described by

$$\dot{s}(t) = gs(t) - r(t), \tag{8}$$
$$\dot{k}(t) = f(k,r) - c(t). \tag{9}$$

Population is constant and normalized to unity, and instantaneous utility is represented by

$$u(c) = \frac{c^{1-\sigma^{-1}} - 1}{1-\sigma^{-1}}. \tag{10}$$

An optimal path is defined as a sequence $\{c(t), r(t), s(t), k(t)\}_0^\infty$ that solves

$$\max_{\{c(t),r(t)\}_0^\infty} \int_0^\infty u(c(t)) \exp[-\delta t]\, dt \tag{11}$$

subject to (8), (9), $\dot{m} = m\nu$, and to $s(t) \geq 0$, $k(t) > 0$, $c(t) \geq 0$, $r(t) > 0$ for each $t \in [0,\infty)$; the social discount rate $\delta > 0$ is assumed constant and initial amounts $s(0)$ and $k(0)$ are taken as given. Assuming an interior solution, first order conditions for problem (11) imply

$$\dot{c} = c\sigma(f_1 - \delta) \tag{12}$$
$$\dot{f}_2 = f_2(f_1 - g - \nu), \tag{13}$$

where $f_1 = \partial f/\partial k$ and $f_2 = \partial f/\partial(mr)$. Equation (12) is the standard Keynes-Ramsey rule, whereas equation (13) is a modified Hotelling rule: since the marginal rent from extracted natural resource equals $p_r = df/dr = f_2 m$, it follows from (13) that[3]

$$\dot{p}_r = p_r(f_1 - g). \tag{14}$$

Denoting by λ_k, λ_s, and λ_m the dynamic multipliers associated to constraints (8), (9) and $\dot{m} = m\nu$ respectively, an optimal path must also satisfy the transversality conditions

$$\lim_{t\to\infty} \lambda_k k e^{-\delta t} = \lim_{t\to\infty} \lambda_s s e^{-\delta t} = \lim_{t\to\infty} \lambda_m m e^{-\delta t} = 0. \tag{15}$$

The analysis of long-run dynamics focuses on two key variables: the input ratio $x = k/mr$, and the consumption-capital ratio, $z = c/k$. Having assumed constant returns to scale, the ratio between output and augmented resource-flow $f(k, mr)/(mr)$ can be represented by the intensive production function $\varphi = \varphi(x)$ with

$$\varphi_x = f_1 > 0, \qquad \varphi_{xx} < 0, \qquad f_2 = \varphi - \varphi_x x > 0. \tag{16}$$

Using (16), the dynamics of x and z are described by

$$-x\dot{x}\varphi_{xx} = (\varphi - x\varphi_x)(\varphi_x - g - \nu), \tag{17}$$

$$\dot{z}/z = z + \sigma(\varphi_x - \delta) - (\varphi/x), \tag{18}$$

where (17) is the Hotelling rule in terms of the input ratio, and (18) is obtained as the difference between the growth rates of consumption and man-made capital. On the basis of (17)-(18), the long run equilibrium of the economy can be characterized as follows.

PROPOSITION 1. *Along the optimal path, the input ratio, the consumption-capital ratio and the consumption rate of return converge in the long run to finite steady state values:*

$$\lim_{t\to\infty} x(t) = x^{ss} > 0, \tag{19}$$

$$\lim_{t\to\infty} z(t) = z^{ss} = \frac{\varphi(x^{ss})}{x^{ss}} - \sigma(\varphi_x^{ss} - \delta) > 0, \tag{20}$$

$$\lim_{t\to\infty} f_1(t) = \varphi_x^{ss} = g + \nu. \tag{21}$$

Proof. See Appendix. ||

As shown in the Appendix, the joint dynamics of x and z display the typical saddle-point property, and the optimal path is the one converging to the simultaneous steady state (x^{ss}, z^{ss}) along the stable arm of the saddle. Hence, in the long run, consumption c, man-made capital k, and augmented natural capital mr all grow at the same rate (denoting $\lim_{t\to\infty} w$ as w^∞)

$$(\dot{c}/c)^\infty = \left(\dot{k}/k\right)^\infty = (\dot{r}/r)^\infty + \nu = \sigma(g + \nu - \delta), \tag{22}$$

which is obtained by substituting (21) in the Keynes-Ramsey rule (12). Expression (22) immediately yields a necessary and sufficient condition for non-declining long-run utility: satisfying the limiting sustainability condition (2) clearly requires $(\dot{c}/c)^{\infty}$ be positive. Hence, utility is non-decreasing in the long run if and only if

$$g + \nu \geq \delta, \tag{23}$$

which implies the following necessary condition for sustainable development: the sum of the rates of natural regeneration and technical progress must be at least equal to the utility discount rate. Valente (2005) shows that satisfying (23) is indeed necessary for sustainability, because $\delta > g+\nu$ implies the consumption time-path be single-peaked - *i.e.* strictly declining from some point in time onwards. In the present context, condition (23) provides the basis for deriving the key result of this paper, the one-to-one relationship between long-run sustained utility and the positiveness of long-run genuine savings.

3.2. Dynamics of genuine savings

The main result is derived in two steps: first, it is shown that along the optimal path, the genuine savings-capital ratio converges to a steady-state equilibrium in the long run; second, in such a steady state there is a one-to-one relationship between the sign of θ and sustainability condition (23). In order to compute genuine savings in the present model, the expression for the time-premium term (4) is readapted to our technology specification: with resource-augmenting technical progress, the present value of future improvements takes the form[4]

$$\pi(t) = \nu \int_t^{\infty} p_r(t)\, r(t)\, e^{-\int_t^s f_1(v)dv} ds, \tag{24}$$

where the interest rate $i(t)$ has been replaced by $f_1(t)$. Differentiating (24) gives

$$\dot{\pi}(t) = f_1(t)\,\pi(t) - \nu p_r(t)\, r(t). \tag{25}$$

Differentiating (3) and substituting (8), (9), and (25) in the resulting expression, the dynamics of genuine savings are described by[5]

$$\dot{\theta}(t) = f_1(t)\,\theta(t) - \dot{c}(t). \tag{26}$$

In order study the sign of θ in the long-run along the optimal path, it is useful to define the genuine savings-capital ratio $q = \theta/k$. The growth rate of $q(t)$ thus equals the difference between growth rates of $\theta(t)$ and $k(t)$: using (9), (18) and (26), the genuine savings-capital ratio evolves according to

$$\dot{q} = q\left[f_1 + \frac{\dot{z}}{z} - \sigma(f_1 - \delta)\right] - z\sigma(f_1 - \delta). \tag{27}$$

The following proposition ensures the dynamic stability of $q(t)$ towards a steady-state equilibrium:

PROPOSITION 2. *Along the optimal path, the genuine savings-capital ratio converges to a finite steady-state equilibrium in the long run:*

$$\lim_{t\to\infty} \dot{q}(t) = 0, \quad \lim_{t\to\infty} q(t) = q_{ss} < \infty. \tag{28}$$

Proof. See Appendix. ||

The proof of Proposition 2 builds on the fact that explosive dynamics of the genuine savings-capital ratio would violate transversality conditions (15). Convergence of $q(t)$ towards a finite steady-state value implies, setting $\dot{q}(t) = 0$ in (27),

$$\lim_{t\to\infty} q(t) = q^{ss} = z^{ss} \cdot \frac{\sigma(f_1^{ss} - \delta)}{f_1^{ss} - \sigma(f_1^{ss} - \delta)}, \tag{29}$$

Expression (29) allows to prove the following

PROPOSITION 3. *If the utility discount rate is greater (less) than the sum of rates of resource regeneration and augmentation, long-run genuine savings are negative (positive):*

$$sign \lim_{t\to\infty} \theta(t) = sign(g + \nu - \delta) \tag{30}$$

Proof. The transversality condition on k requires $f_1^{ss} > \sigma(f_1^{ss} - \delta)$, so that the denominator in (29) is strictly positive: being $z^{ss} > 0$ and $\sigma > 0$, the sign of q^{ss} is determined by the sign of $(f_1^{ss} - \delta)$. Since $f_1^{ss} = g + \nu$ by Proposition 1,

$$g + \nu \geq \delta \Rightarrow q^{ss} \geq 0, \qquad g + \nu < \delta \Rightarrow q^{ss} < 0. \tag{31}$$

Since $q = \theta/k$ and k is constrained to be non-negative, result (31) implies (30). ‖

Proposition 3 establishes the one-to-one relationship between long-run genuine savings and the sustainability condition (23). With respect to previous literature, this result provides an analytical characterization of the long-run dynamics of genuine savings. For example, consider the standard Dasgupta and Heal (1974) setup with exhaustible resources and no technical progress, which is obtained setting $g = \nu = 0$ in the present model. In this environment, Asheim (1994) showed that it is possible to have positive genuine savings for relatively long time-intervals, although utility is bound to decrease in the long run along the optimal path. Proposition 3 adds a precise conclusion: since the sustainability condition (23) is violated in the Dasgupta-Heal economy,[6] genuine savings eventually turn negative in the long run. Moreover, since the analysis is not restricted to the case $g = \nu = 0$, we have the following general statement:

COROLLARY 4. *If genuine savings are positive at some t but condition (23) is violated, genuine savings eventually turn negative in the long run.*

From the theoretical viewpoint, this result complements Proposition 3 in Pezzey (2004), which states that if genuine savings are initially positive and utility is single-peaked, there is a finite time-period with genuine savings positive but utility unsustainable (*ibid.*, p.620). Corollary 4 then establishes that such an economy[7] actually experiences genuine dissaving in the long run. At the same time, the result is relevant from the perspective of applied methods for sustainability testing. Corollary 4 implies that testing the sustainability condition (23) would overcome the false-message problem: if (23) is empirically rejected, sustainability is violated regardless of the sign of current estimated genuine savings.

More generally, our results suggests that applied research on sustainable development should include both approaches - *i.e.* estimates of current genuine savings as well as empirical tests of long-run sustainability conditions - since they provide different information about sustainable development: the 'forward-looking criterion' suggested by Proposition 3 and the 'instantaneous criterion' implicit in the genuine savings method are mutually independent.

3.3. Further remarks

Some remarks on the assumptions of the model, and possible empirical applications of the above results are as follows.

(i) Alternative specifications of technical progress. Technical progress of the resource-augmenting type rules out disembodied forms, like Hicks-neutral progress. Disembodied forms generally complicate the analysis, in that long-run dynamics are modified and might not yield the same results. In the Cobb-Douglas case, however, all previous results remain valid: assuming $y = k^{1-\alpha} r^{\alpha} e^{\varpi t}$, where ϖ is the exogenous rate of Hicks-neutral technical progress, we can redefine $\nu = \varpi/\alpha$ in order to obtain $y = k^{1-\alpha} \left(e^{\nu t} r\right)^{\alpha}$, which is perfectly consistent with the technology specification assumed.

(ii) Population growth. Dealing with sustainability issues, it is reasonable to object that sustaining utility *per capita* is what really matters. In this regard, results are essentially the same if we assume a constant population growth rate $n > 0$, define θ as genuine savings in per capita terms, and define instantaneous social welfare as the sum of individual instantaneous utilities.[8] This framework is consistent with the view that environmentally-adjusted measures of wealth per capita should be used while assessing sustainability at the empirical level, as argued and recently done by Hamilton (2000).

(iii) Labor inputs and decreasing returns. If labor ℓ is supplied inelastically for production purposes, a condition for non-declining long-run utility similar to (23) can be explicitly derived by specifying $y = k^{\alpha_1} r^{\alpha_2} \ell^{\alpha_3} e^{\varpi t}$ with $\alpha_1 + \alpha_2 + \alpha_3 \leq 1$, and following the dynamic analysis of Stiglitz (1974). In this case, the long-run condition allows to avoid the restrictive assumption of constant returns to scale when estimating the relevant parameters of the production function - in particular, the derived rate of resource-augmenting technical progress.

(iv) Forward-looking indicators. The model suggests a simple procedure for obtaining forward-looking indicators of sustainability. For the sake of clarity, assume that output is given by $y = k^{1-\alpha} r^{\alpha} e^{\varpi t}$. Estimated parameters for the input share ($\hat{\alpha}$) and time-trend (ϖ) allow to obtain the estimated rate of resource-augmenting technical progress as $\hat{\nu} = (\hat{\varpi}/\hat{\alpha})$. Denoting by $\hat{g}$ the estimated rate of natural regeneration, it is possible to define the maximum sustainable discount rate $\delta^* = \hat{g} + \hat{\nu}$, to be either compared with an estimated 'true discount rate', or even used as a measure of future-consumption sustain, since the higher is δ^* the less likely is long-run consumption declining.

4. Conclusions

Green accounting theories have shown that negative genuine savings at some point in time imply unsustainability. Building on this point, recent studies advocate the use of the genuine savings criterion for testing sustainable development empirically: negative (estimated) genuine savings in the present imply sustainability be rejected. However, positive genuine savings in the present are not sufficient for sustainability, and may deliver a false message at the empirical level, as pointed out by previous literature. It derives that alternative criteria are needed in order to test sustainability in a forward-looking manner.

The main result of this paper is that there exists a one-to-one relationship between the sign of long-run genuine savings and the limiting sustainability condition in the capital-resource model: if the sum of the rates of resource regeneration and augmentation exceeds (falls short of) the utility discount rate, long-run genuine savings are positive (negative). On the one hand, this result provides an analytical characterization of genuine savings long-run dynamics. On the other hand, the analysis suggests that limiting conditions may be profitably used at the empirical level: testing for long-run 'genuine dissaving' allows to reveal whether current genuine savings are delivering a false message and, more generally, provides a forward-looking criterion upon which sustainability tests can be built.

Appendix

Proof of Proposition 1. Equation (17) exhibits dynamic stability in the global sense: since $\varphi - x\varphi_x = f_2 > 0$ and $\varphi_{xx} < 0$, the sign of $\dot{x}$ is the same as $\varphi_x - g - \nu$, where it must be recalled that φ_x equals the marginal product of man-made capital f_1 and is therefore a decreasing function of x. Hence, if φ_x is initially greater than $g+\nu$, we have $\dot{x} > 0$, implying φ_x be decreasing thereafter; conversely, if φ_x is initially below $g+\nu$, we have $\dot{x} < 0$, implying φ_x be increasing thereafter. Eq.(17) therefore ensures dynamic stability of x and $\varphi_x = g + \nu$ in the long run, which proves expressions (19) and (21). Linearizing system (17)-(18) around the simultaneous steady-state equilibrium (x^{ss}, z^{ss}) gives

$$\begin{pmatrix} \frac{d}{dt}(x - x^{ss}) \\ \\ \frac{d}{dt}(z - z^{ss}) \end{pmatrix} = \begin{bmatrix} -\frac{f_2^{ss}}{x^{ss}} & 0 \\ \\ z^{ss}\left[\frac{f_2^{ss}}{(x^{ss})^2} + \sigma\varphi_{xx}^{ss}\right] & z^{ss} \end{bmatrix} \begin{pmatrix} (x - x^{ss}) \\ \\ (z - z^{ss}) \end{pmatrix}. \tag{32}$$

The roots of (32) are z^{ss} and $-\left(f_2^{ss}/x^{ss}\right) < 0$. Hence, any steady state equilibrium with positive consumption $z^{ss} > 0$ is a saddle-point equilibrium. Since explosive paths would violate optimality conditions, $\lim_{t\to\infty} z = z^{ss}$ along the optimal path.[9] The steady-state value for z^{ss} in (20) is obtained by setting $\dot{z} = 0$ in (18). $\|$

Proof of Proposition 2. Equation (14) implies that

$$\lim_{t\to\infty}\frac{k(t)}{p_r(t)\,s(t)}=\lim_{t\to\infty}\frac{k(t)\,e^{-\int_0^t f_1(v)dv}}{s(t)\,e^{-gt}}. \tag{33}$$

Since $\dot{\lambda}_k=\lambda_k(\delta-f_1)$ and $\dot{\lambda}_s=\lambda_s(\delta-g)$, transversality conditions (15) imply limit (33) be indeterminate (0/0). Applying l'Hopital, this limit equals

$$\lim_{t\to\infty}\frac{\left[\dot{k}(t)-f_1(t)k(t)\right]e^{-\int_0^t f_1(v)dv}}{[\dot{s}(t)-gs(t)]e^{-gt}}=-\lim_{t\to\infty}\frac{p_r(t)r(t)e^{-\int_0^t f_1(v)dv}-c(t)e^{-\int_0^t f_1(v)dv}}{r(t)e^{-gt}}$$

where we have substituted dynamic laws (9)-(8) and used $f(k,mr)=f_1k+p_rr$. Substituting (14) in the above expression and recalling that $c/r=zx$, we obtain

$$\lim_{t\to\infty}\frac{k(t)}{p_r(t)\,s(t)}=-\lim_{t\to\infty}\left[1-\frac{z(t)\,x(t)}{p_r(t)}\right]=\frac{z^{ss}x^{ss}-p_r^{ss}}{p_r^{ss}}, \tag{34}$$

where the last term is constant from Proposition 1 since $p_r^{ss}=\varphi^{ss}-x^{ss}\varphi_x^{ss}$. Result (34) allows to prove Proposition 3 as follows. By definition (3), the genuine saving-capital ratio equals

$$q(t)=\frac{\pi(t)}{k(t)}+p_r(t)\frac{\dot{s}(t)}{k(t)}+\frac{\dot{k}(t)}{k(t)}. \tag{35}$$

Take limits of the three terms in (35) separately: the first-term limit is

$$\lim_{t\to\infty}\frac{\pi(t)}{k(t)}=\nu\lim_{t\to\infty}\frac{p_r(t)\,s(t)}{k(t)}=\nu\frac{p_r^{ss}}{z^{ss}x^{ss}-p_r^{ss}}. \tag{36}$$

where the last term is ν times the inverse of (34). From (14) and (8), the second-term limit equals

$$\lim_{t\to\infty}p_r(t)\frac{\dot{s}(t)}{k(t)}=g\lim_{t\to\infty}\frac{p_r(t)\,s(t)}{k(t)}-\lim_{t\to\infty}\frac{p_r(t)}{x(t)}=g\frac{p_r^{ss}}{z^{ss}x^{ss}-p_r^{ss}}-\frac{p_r^{ss}}{x^{ss}}, \tag{37}$$

and the third-term limit is, from (22),

$$\lim_{t\to\infty}\frac{\dot{k}(t)}{k(t)}=\sigma(g+\nu-\delta). \tag{38}$$

From (35), results (36)-(37)-(38) imply that $q(t)$ converges to a finite steady state in the long-run. ‖

Notes

[1] Equation (4) corresponds to the formal definition of 'value of time' used in Pezzey (2004: equation n.23).

[2] Many previous studies justified the genuine saving method by invoking the 'weak sustainability criterion', according to which an economy is sustainable if the value of the total aggregate stock in the economy is preserved over time (see *e.g.* Neumayer, 1999): ruling out technical progress ($\pi = 0$), equation (3) shows that positive genuine savings imply that the value of net resource depletion ($-\dot{s} \cdot p_r$) is more than offset by capital accumulation.

[3] Natural regeneration modifies the standard Hotelling rule ($\dot{p}_r = f_1 p_r$) into (14): resource renewability reduces the depletion of natural capital, contrasting the growth of marginal rents.

[4] With the assumed technology, the instantaneous exogenous shift in production possibilities - which is represented by $\Psi_t(t)$ in equation (4) - is given by $\frac{df}{dm} \cdot \frac{\partial m}{\partial t} = f_2 r\dot{m}$. Since $p_r = f_2 m$, we can substitute $f_2 r\dot{m} = \nu p_r r$ inside the integral to obtain (24). It can be easily verified that using definition (24) in the genuine-savings expression (3) implies the augmented-Hartwick rule be satisfied: $\theta(t) = 0$ for all t implies $\dot{c}(t) = 0$ for all t (see eq.(26) below).

[5] Differentiating (3) and substituting (8) and (25) yields

$$\dot{\theta} = \ddot{k} + p_r \left(f_1 \dot{s} - \dot{r} - \nu r\right) + f_1 \pi.$$

Substituting $\ddot{k} = f_1 \dot{k} + f_2 \left(\dot{m} r + m \dot{r}\right) - \dot{c}$ from (9) and using $f_2 = p_r/m$ yields eq.(26).

[6] Sustainability condition (23) is necessarily violated in violated the Dasgupta and Heal (1974) because $\delta > g + \nu = 0$.

[7] In the present model, the economy is single-peaked because violating (23) implies consumption be declining from some point in time onwards (Valente, 2005, Proposition 2).

[8] Since individual utility takes the isoelastic functional form (10), instantaneous social welfare is isoelastic as well: denoting by δ the individual pure rate of time preference, and solving the optimization problem using the implicit social discount rate $\delta' = \delta - n\sigma^{-1}$, the model yields exactly the same results, the only difference being the focus on genuine savings per capita.

[9] Explosive paths are ruled out as follows: if z diverges to $-\infty$, consumption will become negative, which is not allowed along the optimal path; if z diverges to plus infinity, the budget constraint (9) implies that the stock of man-made capital will become negative in finite time, violating the non-negativity constraint.

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