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Adverse Selection with individual- and joint-life annuities
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#### Abstract

This paper includes couples on the demand side and analyses their implications on the problem of adverse selection in the annuity market. First, we examine the pooling equilibrium for individual-life annuities and show that in the presence of couples the rate of return on individuallife annuities is lower in case that couples do not have the advantage of joint consumption of "family public goods" as well as in case of a logarithmic utility function. Second, we examine the market for joint-life annuities. Due to their higher chance that only one partner survives to the retirement, couples with short-lived partners put more weight on the survivor benefit than couples with at least one longer-lived partner. This fact is used by annuity companies to separate couples according to their partners' life-expectancies. Hence, we find that only a separating equilibrium may exist. These results are obtained in a framework where couples are mandated to buy joint-life annuities and only single persons buy individual-life annuities. When relaxing this assumption by allowing couples to choose between individual- and joint-life annuities, we find that in equilibrium couples with long-lived partners buy individual-life annuities, while couples with short-lived partners buy joint-life annuities. However, couples with one long-lived and one short-lived partner may decide for either type of annuities, depending on the exogenous parameters. Accordingly, we identify two different types of equilibria.


Keywords: annuity market, uncertain lifetime, adverse selection, equilibrium.
JEL codes: D13, D82, D91, G22.

## 1. Introduction

Governments of many developed countries are now looking to reform their social security system in order to cope with the expected financial problems arising due the demographic changes. In the policy debate on the reform options, much attention has been concentrated on the functioning of the private life annuity markets in order to clarify the question whether the cut back of public pensions and more reliance on the private annuity market can be regarded as an appropriate measure to maintain the long-run solvency of the social security system. In this context, the treatment of families in the social security system is one point at issue, raising the question whether it makes sense to continue to mandate protection for family members, especially for surviving spouses, or to leave it up in the responsibility of the families (see e.g. Diamond 2004).

However, although $75 \%$ of men and $44 \%$ of women of age 65 and older are married (see Ameriks and Yakoboski 2003) and, hence, their decision over lifetime consumption may have an impact on a large portion of the annuity market, research on the determination of old-age provision within a family is still in an early stage of development. Theoretical analysis has not progressed much beyond the identification of the fact that the allocation of resources within a family does not conform to that of a single-person household.

The present contribution fits in the rare literature that focuses on consumption behaviour of a family over the uncertain lifetime of its members and their provision for retirement. In particular, two previous studies, namely those by Kotlikoff and Spivak (1981) and by Brown and Poterba (2000), explore the demand of a couple for life annuities. ${ }^{1}$ Kotlikoff and Spivak (1981) find that the benefits of buying actuarially fair individual annuities are lower for a married person than for a single person. This is due to the fact that couples are able to self-insure against the uncertain date of death by consumption- and bequest-sharing arrangements, which is obvious from the following consideration: If one member of a couple lives to be very old, there is a high probability that his or her spouse has already died leaving him or her a bequest. This provides some insurance against the risk of a long life, even without a formal annuity contract. Hence, marriage can serve as a substitute for life annuities. While Kotlikoff and Spivak consider individual-life annuities (or single-life annuities), Brown and Poterba (2000) investigate joint-life annuities. They describe the structure of joint-life annuity products that are available to married couples, and calculate the potential utility gain that couples derive from the purchase of joint-life annuities. In particular, they estimate the amount of wealth that a couple would require in the

[^0]absence of actuarially fair joint-life annuities in order to achieve the same utility level that they receive when actuarially fair joint-life annuities are available. Equivalently to Kotlikoff and Spivak, Brown and Poterba find that for a married person the utility gain from joint-life annuities is smaller than the utility gain from an individual-life annuity for a single person. Hence both papers draw the conclusion that, as most potential annuity buyers are married, their findings may help to explain the limited size of the private annuity market.

Although these papers have recognised the importance of couples for the private annuity market, none of them determined the effect of their consumption behaviour on the equilibrium outcome. On the other hand, various contributions, including Abel (1986), Brunner and Pech (2002, 2005), Eckstein et. al. (1985), Pech (2004), Townley and Boadway (1988) and Walliser (2000), have investigated the functioning of the market for individual-life annuities, in view of the problem of adverse selection. However all of these studies have assumed individuals rather than couples as the decision-making units. The present contribution attempts to close the gap which was left by previous research, mentioned above, by providing a theoretical analysis of the problem of adverse selection in the private annuity market, when couples are included on the demand side. This allows an assessment of the implications on the equilibrium outcome, when turning away from the assumption that the consumption path of a married person over his/her uncertain life time conforms to that of a single person.

Adverse selection arises due to asymmetric information between the market participants: The fact that the annuity companies cannot distinguish individuals according to their life expectancy induces a higher annuity demand of individuals with reason to expect long lives than of those expecting short lives. This leads to the well-known result that lower payouts than the actuarially fair ones, based on the average life expectancy of the population, are necessary to reflect the over-representation of annuities bought by the long-living individuals. ${ }^{2}$

This paper examines individual- as well as on joint-life annuities. First, we investigate the market for individual-life annuities and compare the equilibrium outcome of an economy consisting of single persons only with an economy in which single persons and couples coexist. For this, we consider the standard model with one working and one retirement period and assume price competition among the annuity companies, which means that they fix the price (i.e. the payoff per unit of annuity) and consumers can buy as many annuities as they want. In this framework only a pooling equilibrium can exist, where all persons (irrespective whether single or married) receive the same rate of return, which - as mentioned above - is below the fair one. Obviously,

[^1]the consequences of couples as market participants on the equilibrium rate of return depend on how their demand affects the composition of aggregate annuity demand regarding to the highrisk and the low-risk types. Whenever the demand share of high-risk types is increased (at the expense of that of the low-risk types), the equilibrium rate of return will fall. We obtain this result of a lower rate of return in case that couples do not have the advantage of joint consumption of "family public goods" as well as in case of a logarithmic utility function. Otherwise the effect on the equilibrium rate of return on individual-life annuities is ambiguous.

Secondly, we focus on joint-life annuities, which guarantee a certain payout provided that both partners are alive, as well as a certain ratio of this payout after the death of one partner. Hence, in contrast to individual-life annuities, contracts for joint-life annuities are characterised by two payoffs, which are fixed by the annuity companies. In the U.S. annuity market it is common practice that the ratio of the survivor benefit (to the payoff that a couple receives in case that both partners survive) varies from $50 \%$, two-thirds, $75 \%$ up to $100 \%$. The present contribution offers an explanation for the fact that annuity companies offer different survivor benefit options. Separation according to the survival probabilities of both couple-members takes place: Due to their higher chance that only one partner survives to the retirement, couples with both partners having a low survival probability put more weight on the survivor benefit than a couple with partners having a higher life expectancy. This fact can be used by annuity companies to separate couples according to their life expectancy. Hence, we find that no pooling NashCournot equilibrium for joint-life exists; if a Nash-Cournot equilibrium exists, it is a separating one. ${ }^{3}$ These results are obtained in a model where couples are mandated to provide for old age through joint-life annuities and only single persons buy a pooling contract for individual-life annuities.

However, in real world mandating couples to buy joint-life annuities is not common practice, instead couples can choose between individual- and joint-life annuities. Hence, in a final step, we combine the previous analysis of equilibria for each type of annuities and consider a framework, in which single persons and couples are included on the demand side and the latter are free to choose between individual- and joint-life annuities. Note that there is empirical evidence that some couples indeed buy the latter type of annuities. For instance, a recent survey, conducted by the American Council of Life Insurers, reports that among married owners

[^2]of life annuities, only $63 \%$ indeed choose joint-life annuities, the remaining $37 \%$ choose individual-life annuities(see Ameriks and Yakoboski 2003). The present contribution offers an explanation for this empirical observation: We find that couples with both partners having a high life expectancy will decide for individual-life annuities. This is due to the fact that this type of annuities can offer higher expected returns to them than their separate contract for joint-life annuities, because in the former case they are pooled with single persons with a low life expectancy. Whether equivalent considerations apply for couples with one long-lived and one short-lived partner depends on the constellations of the exogenous parameters. These findings explain why we identify two different types of equilibria: For sufficiently large shares of high-risk types, an equilibrium may exist where couples with both partners having a high life expectancy, together with the single persons, buy a pooling contract for individual-life annuities and couples with at least one short-lived partner buy a separate contract for joint-life annuities. On the other hand, for lower shares of high-risk types, an equilibrium may exist where also mixed-risk couples buy the pooling contract for individual-life annuities and only couples with both partners having a low survival probability buy their separate contract for joint-life annuities.

The rest of the present paper proceeds as follows: In Section 2 we focus on individual-life annuities. First, the basic model of consumption behaviour of single persons as well as off couples is introduced. Then, the implications of couples as participants in the market for individual-life annuities on the equilibrium outcome are investigated. In Section 3 we concentrate on joint-life annuities. First, the demand of couples for this type of life annuities is analysed. Then, we turn to the investigation of equilibria. First, we derive all results concerning the existence and characterisation of equilibria in the market for joint-life annuities under the assumption that couples are mandated to provide for old-age via joint-life annuities. This serves as a benchmark, which is then compared to the more relevant situation, where couples are free to choose between individual-life annuities and joint-life annuities. Section 4 summarises and concludes the paper.

## 2. Individual-life annuities

Most previous research has focused on individual-life annuities with single persons as the decision-making units. An individual-life annuity (or single-life annuity) guarantees the annuitant regular payouts conditional on the annuitant's survival. In Section 2.1 we describe the model used to analyse the demand for this prevalent type of annuity, where we pay special attention to couples as the decision-making unit. We refer to two individuals as a couple, if they have agreed to pool their income and wealth, which is then used to support lifetime consumption of both partners. This will be commonly the case for married couples as well as for most long-term relationships. Otherwise we talk about single persons. The results obtained are then used in

Section 2.2 to examine the problem of adverse selection in the market of individual-life annuities, when beside single persons also couples are included on the demand side. This allows an assessment of the implications on the equilibrium outcome, when turning away from the assumption that the decision problem of a couple conforms to that of a single person.

### 2.1 Demand of a single person and of a couple for individual-life annuities

Consider an economy with N individuals with a share $\beta$ of single persons; the remaining individuals live with their partner. Hence, the economy consists of $(1-\beta) N / 2$ couple-households and $\beta N$ single-person households. Each individual (regardless of whether married or not) lives for sure in the working period; however survival to the retirement period 1 is uncertain and occurs with probability $\pi, \quad 0<\pi<1$. During the working period, a single person earns a fixed labour income $\omega$, a couple earns $2 \omega$. We assume that no public pension system exists. Thus, at the end of the working period, when the individuals retire, they will provide for the period of retirement by the purchase of individual-life annuities. For each unit of life annuity, bought by an individual in the working period, he/she receives $q$ units of payouts in the retirement period, if he/she survives.

Preferences for consumption over the uncertain lifetime can be represented by expected utility, where the assumption is made that neither a single person nor a couple derives utility from leaving bequests to other persons. This means that there would be unintended bequests, if either the single person or both spouses do not survive to the retirement period. Note that, under the assumption of competitive firms, the rate of return on life annuities is necessarily greater than that on bonds. ${ }^{4}$ Therefore a single person, who has no bequest motive, always decides for annuities against bonds.

Taking this result into account, we formulate the decision problem of a single person: A single person i (with survival probability $\pi_{\mathrm{i}}$ ) chooses his/her consumption plan over the uncertain lifetime by maximising his/her expected utility

$$
\begin{equation*}
\mathrm{U}_{\mathrm{i}}=\mathrm{u}\left(\mathrm{c}_{0 \mathrm{i}}\right)+\pi_{\mathrm{i}} \alpha \mathrm{u}\left(\mathrm{c}_{1 \mathrm{i}}\right), \tag{1}
\end{equation*}
$$

subject to the budget equations

[^3]\[

$$
\begin{align*}
& c_{0 i}=\omega-B_{i},  \tag{2}\\
& c_{i j}=q B_{i} . \tag{3}
\end{align*}
$$
\]

in the working period $t=0$ and the retirement period $t=1$. In equation (1), $u\left(c_{t i}\right)$ is the perperiod utility depending on consumption $\mathrm{c}_{\mathrm{t}}$, with $\mathrm{u}^{\prime}>0, \mathrm{u}^{\prime \prime}<0$ and $\lim _{\mathrm{c} \rightarrow 0} \mathrm{u}^{\prime}(\mathrm{c})=\infty$, and $\alpha$ denotes the discount factor of utility due to time preference, $0<\alpha \leq 1$. Substituting (2) and (3) into (1) and differentiating with respect to $\mathrm{B}_{\mathrm{i}}$, we obtain the first-order condition of this maximization problem as

$$
\begin{equation*}
-\mathrm{u}^{\prime}\left(\omega-\mathrm{B}_{\mathrm{i}}\right)+\mathrm{q} \pi_{\mathrm{i}} \alpha \mathrm{u}^{\prime}\left(\mathrm{qB}_{\mathrm{i}}\right)=0 \tag{4}
\end{equation*}
$$

which determines annuity demand $B_{i}(q)$ for single person $i$.

A couple, on the other hand, share their combined income $2 \omega$ to choose consumption over the uncertain lifetime of both partners. In doing so, a couple has to distinguish between four distinct risk-states: Either both partners are alive in the retirement period, only one partner has survived or both are deceased. This is in contrast to the case of a single person, who has to be concerned solely, whether he/she is alive or not. We follow Brown and Poterba (2000) as well as Kotlikoff and Spivak (1981) and assume that the per-period utility function of a couple is the unweighted sum of the two partners' utilities. Thus, expected utility $\mathrm{U}_{\mathrm{ij}}$ of a couple with partner i (with survival probability $\pi_{\mathrm{i}}$ ) and partner j (with survival probability $\pi_{\mathrm{j}}$ ) is given by

$$
\begin{equation*}
\mathrm{U}_{\mathrm{ij}}=\mathrm{u}\left(\mathrm{c}_{0 \mathrm{ij}}^{\mathrm{s}}\right)+\mathrm{u}\left(\mathrm{c}_{0 \mathrm{ij}}^{\mathrm{s}}\right)+\alpha\left(\pi_{\mathrm{i}} \pi_{\mathrm{j}}\left(\mathrm{u}\left(\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{s}}\right)+\mathrm{u}\left(\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{s}}\right)\right)+\pi_{\mathrm{i}}\left(1-\pi_{\mathrm{j}}\right) \mathrm{u}\left(\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{w}}\right)+\pi_{\mathrm{j}}\left(1-\pi_{\mathrm{i}}\right) \mathrm{u}\left(\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{j}}\right)\right),^{\mathrm{w}}, \tag{5}
\end{equation*}
$$

where $c_{1 i j}^{i w}$ indicates consumption of partner $i$ in the retirement period 1 , in case that he/she is a widow/widower, represented by the superscript $w . \mathrm{c}_{\mathrm{tij}}^{\mathrm{s}}, \mathrm{t}=0,1$, indicates consumption of partner $i$, when the other partner $j$ is alive, thus partner $i$ is (and has) a spouse, represented by the superscript $s$. Equivalently, $c_{1 i j}^{j w}$ and $c_{t i j}^{j s}$ denote consumption of partner $j$, when partner $i$ is deceased or has survived, respectively. Obviously, the consumption level and annuity demand of a couple-member will depend on his/her survival probability as well as on that of his/her partner; to indicate this dependency we make use of the subscript ij.

By the same argument as above, namely a higher rate of return on annuities than on bonds, both members of a couple will always decide to buy positive amounts of individual-life annuities to provide for old-age. However note that bonds, although offering a lower return than individual-life annuities, have one advantage over the latter: The purchase of bonds by a couple member, who does not survive to the retirement period, provides bequests for his/her surviving

[^4]partner, while an individual-life annuity pays nothing in this case. This is the reason why it may be attractive for a couple to supplement annuities by some bonds, which is the case for sufficiently low returns on annuities only. Otherwise, couples will invest solely in individual-life annuities. For sake of simplicity of the analysis, we neglect a situation, where bonds may serve as supplement for annuities by excluding the purchase of bonds as a possible strategy for couples. ${ }^{6}$

We take up the idea of Brown and Poterba (2000) and consider the possibility of goods which can be consumed jointly by both members of the couple. For instance, books and furniture can be regarded as such "public goods" for the couple. By this, the couple's budget equations in the working period 0 and in the retirement period 1 , when both partners $i$ and $j$ are alive, are given by

$$
\begin{align*}
& c_{0 i j}^{i^{s}}+c_{0 i j}^{j^{s}}=\sigma\left(2 \omega-B_{i j}^{i}-B_{i j}^{j}\right),  \tag{6}\\
& c_{1 i j}^{i^{s}}+c_{1 i j}^{j s}=\sigma q\left(B_{i j}^{i}+B_{i j}^{j}\right), \tag{7}
\end{align*}
$$

where $\sigma$ indicates the degree of joint consumption with $1 \leq \sigma \leq 2$. For $\sigma=1$ there is no joint consumption, for $\sigma=2$ all consumption outlays of the couple are for public goods. In case that only one partner i or j survives to the retirement period, the budget equations read

$$
\begin{align*}
& c_{1 \mathrm{ij}}^{\mathrm{i}^{w}}=\mathrm{qB} \mathrm{~B}_{\mathrm{ij}}^{i},  \tag{8}\\
& c_{1 \mathrm{ij}}^{\mathrm{c}^{\mathrm{w}}}=\mathrm{qB} B_{\mathrm{ij}}^{j} . \tag{9}
\end{align*}
$$

The couple's decision problem is to maximise its expected utility (5) subject to the budget constraints (6) - (9). Eliminating $c_{0 i j}^{\mathrm{j}}$, $\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{i}}$, $\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{j}}$ and $\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{j}}$ in (5) by use of (6) - (9) and differentiating with respect to $\mathrm{c}_{0 \mathrm{oj}}^{\mathrm{j}}, \mathrm{C}_{1 \mathrm{ij}}^{\mathrm{s}}, \mathrm{B}_{\mathrm{ij}}^{\mathrm{i}}$ and $\mathrm{B}_{\mathrm{ij}}^{\mathrm{j}}$, respectively, we obtain the first-order conditions of this maximization problem,

$$
\begin{align*}
& -\mathrm{u}^{\prime}\left(\mathrm{c}_{0 \mathrm{ij}}^{\mathrm{s}}\right)+\mathrm{u}^{\prime}\left(\mathrm{c}_{0 \mathrm{jij}}^{\mathrm{s}}\right)=0,  \tag{10}\\
& \pi_{\mathrm{i}} \pi_{\mathrm{j}} \alpha\left(-\mathrm{u}^{\prime}\left(\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{s}}\right)+\mathrm{u}^{\prime}\left(\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{s}}\right)\right)=0 \text {, }  \tag{11}\\
& -\sigma u^{\prime}\left(\mathrm{c}_{0 \mathrm{ij}}^{\mathrm{s}}\right)+\mathrm{q} \sigma \pi_{\mathrm{i}} \pi_{\mathrm{j}} \alpha \mathrm{u}^{\prime}\left(\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{s}}\right)+\mathrm{q} \pi_{\mathrm{i}}\left(1-\pi_{\mathrm{j}}\right) \alpha \mathrm{u}^{\prime}\left(\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{iw}}\right)=0 \text {, }  \tag{12}\\
& -\sigma u^{\prime}\left(c_{0 i j}^{s}\right)+q \sigma \pi_{i} \pi_{j} \alpha u^{\prime}\left(c_{1 i j}^{\mathrm{s}}\right)+q \pi_{j}\left(1-\pi_{\mathrm{i}}\right) \alpha \mathrm{u}^{\prime}\left(\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{w}}\right)=0 \text {. } \tag{13}
\end{align*}
$$

From (10) and (11) it follows immediately that $c_{t i j}^{i s}=c_{t i j}^{j}$ for $t=0,1$. Thus, whenever both partners are alive, they decide to consume the same amounts, thus - use (6) and (7) -

[^5]$\mathrm{c}_{0 \mathrm{ij}}^{\mathrm{s}}=\frac{\sigma}{2}\left(2 \omega-\mathrm{B}_{\mathrm{ij}}^{\mathrm{i}}-\mathrm{B}_{\mathrm{ij}}^{\mathrm{j}}\right)$ and $\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{i}}=\frac{\sigma \mathrm{q}}{2}\left(\mathrm{~B}_{\mathrm{ij}}^{\mathrm{i}}+\mathrm{B}_{\mathrm{ij}}^{\mathrm{j}}\right)$. Substituting these terms together with (8) and (9) into (12) and (13) we get
\[

$$
\begin{align*}
& -\sigma \mathrm{u}^{\prime}\left(\frac{\sigma}{2}\left(2 \omega-\mathrm{B}_{\mathrm{ij}}^{\mathrm{i}}-\mathrm{B}_{\mathrm{ij}}^{\mathrm{j}}\right)\right)+\mathrm{q} \sigma \pi_{\mathrm{i}} \pi_{\mathrm{j}} \alpha \mathrm{u}^{\prime}\left(\frac{\sigma \mathrm{q}}{2}\left(\mathrm{~B}_{\mathrm{ij}}^{\mathrm{i}}+\mathrm{B}_{\mathrm{ij}}^{\mathrm{j}}\right)\right)+\mathrm{q} \pi_{\mathrm{i}}\left(1-\pi_{\mathrm{j}}\right) \alpha \mathrm{u}^{\prime}\left(\mathrm{qB}_{\mathrm{ij}}^{\mathrm{i}}\right)=0,  \tag{12'}\\
& -\sigma \mathrm{u}^{\prime}\left(\frac{\sigma}{2}\left(2 \omega-\mathrm{B}_{\mathrm{ij}}^{\mathrm{i}}-\mathrm{B}_{\mathrm{ij}}^{\mathrm{j}}\right)\right)+\mathrm{q} \sigma \pi_{\mathrm{i}} \pi_{\mathrm{j}} \alpha \mathrm{u}^{\prime}\left(\frac{\sigma \mathrm{q}}{2}\left(\mathrm{~B}_{\mathrm{ij}}^{\mathrm{i}}+\mathrm{B}_{\mathrm{ij}}^{\mathrm{j}}\right)\right)+\mathrm{q} \pi_{\mathrm{j}}\left(1-\pi_{\mathrm{i}}\right) \alpha \mathrm{u}^{\prime}\left(\mathrm{qB}_{\mathrm{ij}}^{\mathrm{j}}\right)=0 . \tag{13'}
\end{align*}
$$
\]

which determine demand $B_{i j}^{i}(q)$ and $B_{i j}^{j}(q)$ of each partner $i$ and $j$ for individual-life annuities for any given rate of return $q$.

Lemma 1: For any rate of return $q$, it is optimal for a couple whose members differ in their survival probability that the partner with the higher survival probability demands a larger amount of individual-life annuities; in case of identical survival probabilities both partners demand the same amount, i.e. $\mathrm{B}_{\mathrm{ij}}^{\mathrm{i}}(\mathrm{q}) \gtreqless \mathrm{B}_{\mathrm{ij}}^{\mathrm{i}}(\mathrm{q})$, if $\pi_{\mathrm{i}} \gtreqless \pi_{\mathrm{j}}$. In the latter case, where $\pi_{\mathrm{i}}=\pi_{\mathrm{j}}$, the demand of a married person coincides with that of a single person with the same survival probability $\pi_{\mathrm{i}}$, i.e., $B_{i j}^{i}(q)=B_{i}(q)$, if there is no joint consumption of goods within the couple $(\sigma=1)$ or in case of logarithmic per-period utility

$$
\begin{equation*}
u\left(c_{t i j}^{i}\right)=\ln \left[c_{t i j}^{i}\right] . \tag{14}
\end{equation*}
$$

Proof: Combining (12') and (13') yields

$$
\begin{equation*}
\pi_{\mathrm{i}}\left(1-\pi_{\mathrm{j}}\right) \mathrm{u}^{\prime}\left(\mathrm{qB}_{\mathrm{ij}}^{\mathrm{i}}\right)=\pi_{\mathrm{j}}\left(1-\pi_{\mathrm{i}}\right) \mathrm{u}^{\prime}\left(\mathrm{qB}_{\mathrm{ij}}^{\mathrm{j}}\right) . \tag{15}
\end{equation*}
$$

Thus, if $\pi_{\mathrm{i}} \gtrless \pi_{\mathrm{j}}, \mathrm{u}^{\prime}\left(\mathrm{qB}_{\mathrm{ij}}^{\mathrm{i}}\right) \lesseqgtr \mathrm{u}^{\prime}\left(\mathrm{qB}_{\mathrm{ij}}^{\mathrm{j}}\right)$ and, as $\mathrm{u}^{\prime \prime}<0, \mathrm{~B}_{\mathrm{ij}}^{\mathrm{i}}(\mathrm{q}) \gtreqless \mathrm{B}_{\mathrm{ij}}^{\mathrm{j}}(\mathrm{q})$.

For $\pi_{i}=\pi_{j}$, and hence $B_{i j}^{i}(q)=B_{i j}^{j}(q),\left(12^{\prime}\right)$ equals

$$
\begin{equation*}
-\sigma u^{\prime}\left(\sigma\left(\omega-\mathrm{B}_{\mathrm{ij}}^{\mathrm{i}}\right)\right)+\mathrm{q} \pi_{\mathrm{i}} \alpha\left(\sigma \pi_{\mathrm{i}} \mathrm{u}^{\prime}\left(\sigma \mathrm{q} \mathrm{~B}_{\mathrm{ij}}^{\mathrm{i}}\right)+\left(1-\pi_{\mathrm{i}}\right) \mathrm{u}^{\prime}\left(\mathrm{q} \mathrm{~B}_{\mathrm{ij}}^{\mathrm{i}}\right)\right)=0 \tag{16}
\end{equation*}
$$

which coincides with (4) for $\sigma=1$; thus $B_{i j}^{i}(q)=B_{i}(q)$. The same holds for the case of $a$ logarithmic per-period utility function (14) for any $1 \leq \sigma \leq 2$. This follows from the fact that for log-utility the first-order condition (16), which reads

$$
-1 /\left(\omega-B_{i j}^{i}\right)+q \pi_{i} \alpha\left(\pi_{i} / q B_{i j}^{i}+\left(1-\pi_{i}\right) / q B_{i j}^{i}\right)=0
$$

is independent of $\sigma$.
Q.E.D.

The result, that within a couple the longer-lived partner expresses a larger demand for individual-life annuities, corresponds with intuition and can be explained as follows: Any shift of individual-life annuities from the shorter-lived partner to the longer-lived partner leaves the
aggregate amount $B_{i j}^{i}+B_{i j}^{j}$ and, hence, aggregate consumption levels $c_{0 i j}^{j}+c_{0 i j}^{j s}, c_{1 i j}^{j s}+c_{1 i j}^{j}$, when both are alive, unchanged. However such a shift provides a higher consumption level $\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{w}}$ of the longer-lived partner at the expense of that of the shorter-lived partner, when only one partner survives to retirement. As the former has a higher weight in the couple's utility (due to his/her higher chance to be the only survivor), the couple benefits from such a shift. Obviously, in case of identical life expectancy, the couple puts equal weights on consumption of both partners, thus $B_{i j}^{i}=B_{i j}^{j}$. Furthermore, in case that a couple with identical survival probabilities does not have the advantage of joint consumption compared to a single person, the demand of each of them coincides with that of a single person with the same survival probability (due to identical perperiod utility function of married and single persons and equal weights in couple's utility for both partners). Note, however, that this result also holds for logarithmic per-period utility (14), irrespective of the value of $\sigma{ }^{7}$

From now on we assume that all N individuals, irrespective of whether they live together with a partner or live as a single person, are characterized either by a low or a high survival probability. Thus, there are two types of a single person $\mathrm{i}=\mathrm{L}, \mathrm{H}$, while we distinguish between three types of couple $\mathrm{ij}=\mathrm{LL}, \mathrm{LH}, \mathrm{HH}$, where LL, HH, resp. denotes a couple with both partners having a low or high, resp., survival probability and LH denotes a couple consisting of a type-L and a type-H partner. Next, we investigate annuity demand of these different types.

Lemma 2: For any rate of return q,
(i) a single person of type H demands a higher amount of individual-life annuities than a single person of type $L$, i.e. $B_{H}(q)>B_{L}(q)$.
(ii) each member of a type- HH couple demands more individual-life annuities than each member of a type-LL couple, i.e. $B_{H H}^{H}(q)>B_{L L}^{L}(q)$.
(iii) a couple member of type $\mathrm{i}=\mathrm{L}, \mathrm{H}$ demands less individual-life annuities, in case that partner j is of type H compared to the case where partner j is of type L , i.e. $\mathrm{B}_{\mathrm{LH}}^{\mathrm{L}}<\mathrm{B}_{\mathrm{LL}}^{\mathrm{L}}$ and $B_{H H}^{H}<B_{L H}^{H}$, if the following necessary and sufficient condition holds:

$$
\begin{align*}
& -\frac{\sigma^{2}}{2}\left(\mathrm{u}^{\prime \prime}\left(\frac{\sigma}{2}\left(2 \omega-\mathrm{B}_{\mathrm{ij}}^{\mathrm{i}}-\mathrm{B}_{\mathrm{ij}}^{\mathrm{j}}\right)\right)+\mathrm{q}^{2} \pi_{\mathrm{i}} \pi_{\mathrm{j}} \alpha \mathrm{u}^{\prime \prime}\left(\frac{\sigma \mathrm{q}}{2}\left(\mathrm{~B}_{\mathrm{ij}}^{\mathrm{i}}+\mathrm{B}_{\mathrm{ij}}^{\mathrm{j}}\right)\right)\right)\left(\left(1-\pi_{\mathrm{i}}\right) \mathrm{u}^{\prime}\left(\mathrm{qB}_{\mathrm{ij}}^{\mathrm{j}}\right)+\pi_{\mathrm{i}} \mathrm{u}^{\prime}\left(\mathrm{q} \mathrm{~B}_{\mathrm{ij}}^{\mathrm{i}}\right)\right)> \\
& \mathrm{q}^{2} \pi_{\mathrm{i}} \pi_{\mathrm{j}} \alpha\left(1-\pi_{\mathrm{i}}\right) \mathrm{u}^{\prime \prime}\left(\mathrm{qB}_{\mathrm{ij}}^{\mathrm{j}}\right)\left(\mathrm{u}^{\prime}\left(\mathrm{qB}_{\mathrm{ij}}^{\mathrm{i}}\right)-\sigma \mathrm{u}^{\prime}\left(\frac{\sigma \mathrm{q}}{2}\left(\mathrm{~B}_{\mathrm{ij}}^{\mathrm{i}}+\mathrm{B}_{\mathrm{ij}}^{\mathrm{j}}\right)\right)\right) \tag{17}
\end{align*}
$$

Hence, in this case $B_{L H}^{L}(q)<B_{L L}^{L}(q)<B_{H H}^{H}(q)<B_{L H}^{H}(q)$.

Proof: See the Appendix.

[^6]The result (i) of a higher annuity demand of individuals with high life-expectancy was shown in many contributions, which examined the problem of adverse selection in the market of individual-life annuities; see e.g. Abel (1986), Walliser (2000), Pech (2004). Result (ii) offers its counterpart for couples: A couple with both partners having a high survival probability demands a larger amount of individual-life annuities than a couple with both partners having a low survival probability. The intuition for the result (iii) is related to Lemma 1: A type-L spouse has a higher relative chance to be the only surviving member within a type-LL couple than within a type-LH couple (obviously the opposite applies to his/her respective partner). Due to this, a type-LL couple will shift a higher amount of annuities from the considered type-L spouse to his/her partner than a type-LH couple. Such a behaviour is found, whenever condition (17) holds, for which we observe that the sign of LHS is positive, while the sign of the RHS depends on the difference $u^{\prime}\left(q B_{i j}^{i}\right)-\sigma u^{\prime}\left(\frac{\sigma q}{2}\left(B_{i j}^{i}+B_{i j}^{j}\right)\right)$. Hence, condition (17) is fulfilled whenever this difference is non-negative (as multiplied by $u^{\prime \prime}<0$ ), otherwise the difference must be sufficiently small. Whether this is indeed the case, actually depends on the specifics of the per-period function $u$ and on the values of the exogenous parameters. However, it is straightforward to see that for $\sigma=1$ the difference is non-negative, if $\mathrm{B}_{\mathrm{ij}}^{\mathrm{i}}(\mathrm{q}) \leq \mathrm{B}_{\mathrm{ij}}^{\mathrm{j}}(\mathrm{q})$, which is fulfilled if $\pi_{\mathrm{i}} \leq \pi_{\mathrm{j}}$ (compare Lemma 1). Further, it can be shown that (17) is fulfilled for the logarithmic per-period utility (14).

### 2.2 Pooling equilibrium for individual annuities

In this section we examine the equilibrium outcome, when there is asymmetric information in the market for individual-life annuities, which leads to the problem of adverse selection: Because annuity companies cannot distinguish individuals according to their survival probabilities, the first-best outcome, in which each risk type buys annuities at his/her individually fair rate of return according to his/her survival probability, cannot be realized. In the following, we pay special attention to couples as decision-making units and analyse how their annuity demand affects the equilibrium rate of return.

In the model usually employed to study adverse selection in the market for individual-life annuities, it is typically assumed that competition takes place via the price (i.e. via the rate of return), which is fixed by the annuity companies. Individuals can buy as many annuities as they want. In this framework only a pooling equilibrium is possible, where all individuals receive the same rate of return. ${ }^{8}$ However, for any given rate of return, individuals with high life expectancy demand a larger amount of annuities (compare Lemma 2). This over-representation of annuities, bought by high-risk individuals, accounts for the well-known result that annuity

[^7]companies, in order to avoid losses, offer a rate of return which is lower than the actuarially fair rate of return based on the average survival probability of the population. This consequence of adverse selection in the market of individual-life annuities has been shown in various contributions, however, as far as we know, they all made implicitly the assumption that the decision problem of any household conforms to that of a single person, described above, see e.g. Abel (1986), Walliser (2000), Pech (2004).

We expand these previous studies by including couples as decision-making units. To do so, we make the following assumptions: Let $\gamma$ be the share of type-H individuals in population and $\beta$ the share of single persons (as before). These shares as well as the survival probabilities $\pi_{\mathrm{H}}$ and $\pi_{\mathrm{L}}$ are public information, known by the annuity companies. But it is the private information for each single person to know his/her risk-type and for each couple to know their risk-type, i.e. the probabilities of survival of both partners. By this, we have introduced asymmetric information into the model in the usual way.

We neglect any correlation between the survival probability and the marital status. ${ }^{9}$ Then, the fraction of single persons of type $H$ is given $\beta \gamma$ and that of single persons of type $L$ is $\beta(1-\gamma)$. Further, we exclude any dependencies between the survival probability of both couple members ${ }^{10}$. By this, $(1-\beta) \gamma^{2}$ is the fraction of type-H partners within a type-HH couple, $(1-\beta) \gamma(1-\gamma)$ is the share of type-H individuals as well as the share of type-L individuals within a type-LH couple and $(1-\beta)(1-\gamma)^{2}$ are type-L individuals within a type-LL couple. Given these group shares, expected profits are given by

$$
\begin{align*}
\mathrm{P}(\mathrm{q})= & \beta\left(\gamma \mathrm{B}_{\mathrm{H}}+(1-\gamma) \mathrm{B}_{\mathrm{L}}\right)+(1-\beta)\left(\gamma^{2} \mathrm{~B}_{H H}^{H}+\gamma(1-\gamma)\left(\mathrm{B}_{\mathrm{LH}}^{\mathrm{H}}+\mathrm{B}_{\mathrm{LH}}^{\mathrm{L}}\right)+(1-\gamma)^{2} \mathrm{~B}_{\mathrm{LL}}^{\mathrm{L}}\right)- \\
& \mathrm{q}\left(\beta\left(\gamma \pi_{H} \mathrm{~B}_{\mathrm{H}}+(1-\gamma) \pi_{\mathrm{L}} \mathrm{~B}_{\mathrm{L}}\right)+(1-\beta)\left(\gamma^{2} \pi_{H} \mathrm{~B}_{\mathrm{HH}}^{\mathrm{H}}+\gamma(1-\gamma)\left(\pi_{H} \mathrm{~B}_{\mathrm{LH}}^{H}+\pi_{\mathrm{L}} \mathrm{~B}_{\mathrm{LH}}^{\mathrm{L}}\right)+(1-\gamma)^{2} \pi_{\mathrm{L}} \mathrm{~B}_{\mathrm{LL}}^{\mathrm{L}}\right)\right) \tag{18}
\end{align*}
$$

where, for shortness, we suppress the dependency of annuity demand $B_{i}, B_{i j}^{i}$ on $q$. Note that in (18) a zero interest rate is assumed, which is chosen for sake of simplicity only; a positive interest rate would not affect the qualitative results. Since the annuity companies are assumed to behave perfectly competitive, the equilibrium rate of return $\tilde{q}$ is implicitly defined by the zeroprofit condition

$$
\begin{equation*}
P(q)=0 . \tag{19}
\end{equation*}
$$

[^8]Of course, $\tilde{q}$ must be a weighted average of $1 / \pi_{H}$ and $1 / \pi_{L}$, the individually fair rate of returns for each risk-type H and $\mathrm{L} .{ }^{11}$ This implies that for low-risk individuals expected returns are lower than required for individual fairness, while for high-risk individuals they are higher.

Next we will compare the equilibrium outcome of an economy consisting of single persons only ( $\beta=1$ ) with an economy where single and couples coexist $(\beta<1)$. For this, we assume that (18), a continuous function of $q$, is strictly decreasing, i.e. $P^{\prime}(q)<0 .{ }^{12}$

Proposition 1: Assume that condition (17) holds, which ensures that $\mathrm{B}_{\mathrm{LH}}^{\mathrm{H}} / \mathrm{B}_{\mathrm{HH}}^{\mathrm{H}}>1$ and $B_{L H}^{L} / B_{L L}^{L}<1$. Then the equilibrium rate of return on individual-life annuities is higher in an economy with single persons only $(\beta=1)$ than in a mixed economy with singles and couples ( $\beta<1$ ), if there is no joint consumption of goods within the couple ( $\sigma=1$ ) or in case of logarithmic per-period utility.

Proof: Let $\tilde{\mathrm{q}}_{\mathrm{s}}$ be the equilibrium rate of return, determined by the zero-profit condition (19) for for $\beta=1$, which is

$$
\begin{equation*}
\tilde{\mathrm{q}}_{\mathrm{S}} \equiv \frac{\gamma \mathrm{~B}_{\mathrm{H}}+(1-\gamma) \mathrm{B}_{\mathrm{L}}}{\gamma \pi_{H} B_{H}+(1-\gamma) \pi_{\mathrm{L}} \mathrm{~B}_{\mathrm{L}}} . \tag{20}
\end{equation*}
$$

Substituting (20) into expected profits (18) for $\beta<1$ yields

$$
\begin{aligned}
\mathrm{P}\left(\tilde{q}_{\mathrm{S}}\right)= & (1-\beta)\left[\gamma^{2} \mathrm{~B}_{\mathrm{HH}}^{H}+\gamma(1-\gamma)\left(\mathrm{B}_{\mathrm{LH}}^{H}+\mathrm{B}_{\mathrm{LH}}^{\mathrm{L}}\right)(1-\gamma)^{2} \mathrm{~B}_{\mathrm{LL}}^{\mathrm{L}}-\right. \\
& \left.\frac{\gamma \mathrm{B}_{H}+(1-\gamma) \mathrm{B}_{\mathrm{L}}}{\gamma \pi_{H} \mathrm{~B}_{H}+(1-\gamma) \pi_{\mathrm{L}} \mathrm{~B}_{\mathrm{L}}}\left(\gamma^{2} \pi_{H} \mathrm{~B}_{H H}^{H}+\gamma(1-\gamma)\left(\pi_{H} \mathrm{~B}_{\mathrm{LH}}^{\mathrm{H}}+\pi_{\mathrm{L}} \mathrm{~B}_{\mathrm{LH}}^{\mathrm{L}}\right)(1-\gamma)^{2} \pi_{\mathrm{L}} \mathrm{~B}_{\mathrm{LL}}^{\mathrm{L}}\right)\right]
\end{aligned}
$$

and further, by some steps of transformation,

$$
\begin{equation*}
\mathrm{P}\left(\tilde{\mathrm{q}}_{\mathrm{S}}\right)=\frac{(1-\beta) \gamma(1-\gamma)}{\gamma \pi_{H} \mathrm{~B}_{H}+(1-\gamma) \pi_{\mathrm{L}} \mathrm{~B}_{\mathrm{L}}}\left(\pi_{H}-\pi_{\mathrm{L}}\right)\left[\gamma\left(\mathrm{B}_{\mathrm{LH}}^{\mathrm{L}} \mathrm{~B}_{\mathrm{H}}-\mathrm{B}_{\mathrm{L}} \mathrm{~B}_{H H}^{H}\right)+(1-\gamma)\left(\mathrm{B}_{\mathrm{LL}}^{\mathrm{L}} \mathrm{~B}_{H}-\mathrm{B}_{\mathrm{L}} \mathrm{~B}_{\mathrm{LH}}^{\mathrm{H}}\right)\right] . \tag{21}
\end{equation*}
$$

Note that, as $\pi_{H}>\pi_{\mathrm{L}}$, the sign of $\mathrm{P}\left(\tilde{q}_{\mathrm{S}}\right)$ depends on the last term in the squared brackets on the RHS of (21), which can be rearranged to

$$
\begin{equation*}
\mathrm{P}\left(\tilde{\mathrm{q}}_{\mathrm{S}}\right)=\frac{(1-\beta) \gamma(1-\gamma) \mathrm{B}_{\mathrm{L}} \mathrm{~B}_{H}}{\gamma \pi_{H} \mathrm{~B}_{H}+(1-\gamma) \pi_{\mathrm{L}} \mathrm{~B}_{\mathrm{L}}}\left(\pi_{\mathrm{H}}-\pi_{\mathrm{L}}\right)\left(\gamma\left(\frac{\mathrm{B}_{\mathrm{LH}}^{L}}{\mathrm{~B}_{\mathrm{LL}}^{\mathrm{L}}} \frac{\mathrm{~B}_{\mathrm{LL}}^{\mathrm{L}}}{\mathrm{~B}_{\mathrm{L}}}-\frac{\mathrm{B}_{\mathrm{HH}}^{H}}{\mathrm{~B}_{H}}\right)+(1-\gamma)\left(\frac{\mathrm{B}_{\mathrm{LL}}^{\mathrm{L}}}{\mathrm{~B}_{\mathrm{L}}}-\frac{\mathrm{B}_{H H}^{H}}{\mathrm{~B}_{H}} \frac{\mathrm{~B}_{\mathrm{LH}}^{H}}{\mathrm{~B}_{H H}^{H}}\right)\right) \tag{21'}
\end{equation*}
$$

[^9]We know from Lemma 1 that for $\sigma=1$ or for logarithmic per-period utility $B_{L L}^{L} / B_{L}=1$ and $B_{H H}^{H} / B_{H}=1$. By use of these identities and (21') it is immediate that $P\left(\tilde{q}_{S}\right)<0$, if condition (17) holds. As $\mathrm{P}^{\prime}(\mathrm{q})<0$, the equilibrium rate of return for $\beta<1$ must be lower than $\tilde{\mathrm{q}}_{\mathrm{s}}$.
Q.E.D.

Obviously, the consequences of the existence of couples as market participants on the equilibrium rate of return depend on how they affect the composition of aggregate annuity demand between the high-risk and the low-risk types. Whenever the share of annuities bought by high-risk types (at the expense of the low-risk types) in overall demand is increased, the equilibrium rate of return will fall. As for $\sigma=1$ or for logarithmic utility, $B_{L L}^{L} / B_{L}=B_{H H}^{H} / B_{H}$ (compare Lemma 1), the same-risk couples LL and HH leave the demand shares of the highand low-risk types unchanged. But as $B_{L H}^{H} / B_{H H}^{H}>1$ and $B_{L H}^{L} / B_{L L}^{L}<1$ (guaranteed by condition (8)) there is a shift in the composition of aggregate annuity demand away from the low-risk "profitable" types towards the high-risk "unprofitable" types. As a consequence, annuity companies would make a loss, if they paid a rate of return $\tilde{\mathrm{q}}_{\mathrm{S}}$, which allows zero-profits in the case of $\beta=1$. In order to restore zero profits, the rate of return must fall.

Note however that this result of a lower rate of return is not confined to $\sigma=1$ or logarithmic utility, where we have coincidence of annuity demand of a single person $i=L, H$ with that of a member within a same-risk couple $\mathrm{ij}=\mathrm{LL}, \mathrm{HH}$. A lower rate of return is also obtained for any arbitrary per-period utility function with $\sigma>1$, whenever $B_{L L}^{L} / B_{L} \leq B_{H H}^{H} / B_{H}$ (compare (21')), which means that demand expressed by individuals who live together with a partner of the same risk-type do not decrease the overrepresentation of annuities bought by the high-risk types. Finally note that even in case that $B_{L L}^{L} / B_{L}>B_{H H}^{H} / B_{H}$, we observe a lower equilibrium rate of return in the presence of couples, if this respective decrease in the overrepresentation of annuities bought by the high-risk types due to demand behaviour of the same-risk couples LL and HH does not outweigh the increase in the overrepresentation of annuities bought by the high-risk types due to demand behaviour of the mixed-risk couples LH.

## 3. Joint-life annuities

A joint-life annuity guarantees some regular payout as long as either of the two annuitants is alive. Basically two different types are available, where both types guarantee a main payout in case that both partners are alive in retirement, but they offer different survivorship options. The first specifies a payout-fraction which is paid to the survivor after the death of one couple member, regardless of which one. The second specifies a payout fraction which is paid in case that the primary annuitant predeceases the second annuitant, however if the second annuitant predeceases the first, there will be no change in the payout. In this section we focus on the first
more popular type ${ }^{13}$ with "the last survivor payout rule", as it was called by Brown and Poterba (2000). First, we analyse the demand of couples for this type of joint-life annuities. Then we introduce the supply side in order to study the existence and the characteristics of equilibria, when the information about the survival probability is asymmetrically held by the market participants. Observe that in contrast to individual-life annuities, where all annuitants receive a single payout, joint-life annuities are characterised by two payoffs, namely the main payoff, when both couple members survive, and the last survivor payoff. This fact can be used by the annuity companies to offer different contracts for joint-life annuities to couples with differing survival probabilities, who in turn will choose them, as they prefer different payoff-ratios. These considerations give a first intuition for the results shown in Section 3.3: In contrast to individuallife annuities, no pooling equilibrium for joint-life annuities exists; a separating equilibrium may but need not exist.

### 3.1 Demand of a couple for joint-life annuities

We consider the same framework as in section 2.1 to study now the demand of a couple for joint-life annuities. In the working period a couple ij with income $2 \omega$ buys an amount $A_{i j}$ of jointlife annuities to provide for the retirement: Per unit of joint-life annuity the couple receives a payout $r$ in case that both partners survive and a payout $k$ in case that only one partner survives. Hence, the budget equations for a couple in the working period as well as in the retirement periods are given by

$$
\begin{align*}
& c_{0 i j}^{i^{s}}+c_{0 i j}^{j s}=\sigma\left(2 \omega-A_{i j}\right),  \tag{22}\\
& c_{1 j}^{\mathrm{i}}+\mathrm{C}_{1 \mathrm{ij}}^{\mathrm{s}}=\sigma r \mathrm{~A}_{\mathrm{ij}} \text {, }  \tag{23}\\
& c_{1 \mathrm{ij}}^{\mathrm{w}}=k \mathrm{~A}_{\mathrm{ij}} \text {, }  \tag{24}\\
& c_{1 i j}^{j w}=k A_{i j} . \tag{25}
\end{align*}
$$

The budget equations (22) - (25) are built on the assumption that a couple does not save and buy riskless bonds, in addition to joint-life annuities. As for individual-life annuities, a couple, who derives no utility from leaving a bequest to other persons, always decides to buy positive amounts of joint-life annuities. This is due to the fact that the returns $r+k$ on joint-life annuities are necessarily greater than those on bonds, given perfect competition among the annuity companies. ${ }^{14}$ However note that for sufficiently small or sufficiently large ratios r/k of payouts, a couple would supplement joint-life annuities by bonds in order to smooth consumption appropriately over the risk-states, in which either both partners survive or only one partner

[^10]survives to old-age. ${ }^{15}$ Note, however, that for adequate payout-ratios $\mathrm{r} / \mathrm{k}$ in between, the optimal strategy for a couple is to buy joint-life annuities only.

The decision problem of a couple ij is to maximise their expected utility (5) subject to the budget constraints (22) - (25). Again we eliminate ${c_{0 i j}^{i s}}_{i s}^{j_{1 j}^{s}}, c_{1 i j}^{i w}$ and $c_{1 i j}^{j w}$ in (5) by use of (22) - (25) and differentiate with respect to $\mathrm{A}_{\mathrm{ij}}, \mathrm{C}_{\mathrm{oj}}^{\mathrm{s}}$ and $\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{j}}$, respectively, to obtain the first-order conditions

$$
\begin{equation*}
-\sigma u^{\prime}\left(\mathrm{c}_{0 \mathrm{ij}}^{\mathrm{s}}\right)+\mathrm{r} \sigma \pi_{i} \pi_{\mathrm{j}} \alpha u^{\prime}\left(\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{s}}\right)+\mathrm{k} \pi_{\mathrm{i}}\left(1-\pi_{\mathrm{j}}\right) \alpha \mathrm{u}^{\prime}\left(\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{i}}\right)+\mathrm{k} \pi_{\mathrm{j}}\left(1-\pi_{\mathrm{i}}\right) \alpha \mathrm{u}^{\prime}\left(\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{j}}\right)=0, \tag{26}
\end{equation*}
$$

(10) and (11), where the latter two imply that $c_{t i j}^{s}=c_{t i j}^{j s}$ for $t=0,1$. By use of these identities together with (22) - (25), (26) yields

$$
\begin{equation*}
-\sigma u^{\prime}\left(\frac{\sigma}{2}\left(2 \omega-\mathrm{A}_{\mathrm{ij}}\right)\right)+\mathrm{r} \sigma \pi_{\mathrm{i}} \pi_{\mathrm{j}} \alpha \mathrm{u}^{\prime}\left(\frac{\sigma r}{2} \mathrm{~A}_{\mathrm{ij}}\right)+\mathrm{k}\left(\pi_{\mathrm{i}}\left(1-\pi_{\mathrm{j}}\right)+\pi_{\mathrm{j}}\left(1-\pi_{\mathrm{i}}\right)\right) \alpha \mathrm{u}^{\prime}\left(\mathrm{kA} \mathrm{~A}_{\mathrm{ij}}\right)=0, \tag{26'}
\end{equation*}
$$

which determines demand $\mathrm{A}_{\mathrm{i}}(\mathrm{r}, \mathrm{k})$ of a couple ij for joint-life annuities for any given payouts (r,k). Next we investigate a joint-life annuity contract which specifies a payoff-ratio $\mathrm{r} / \mathrm{k}=2$, which we call a "half to last survivor" rule.

Lemma 3: If joint-life annuities are specified according to the "half to last survivor" rule ( $r / k=2$ ) and offer the same return $r=q$ as individual-life annuities, the demand of a same-risk couple ij = LL,HH for joint-life annuities coincides with their aggregate demand for individual-life annuities, i.e. $A_{i j}(r, k) / 2=B_{i j}^{i}(q)$, if $r=2 k=q$ and $\pi_{i}=\pi_{j}$.

Proof: Assume that $\pi_{\mathrm{i}}=\pi_{\mathrm{j}}$, that $\mathrm{r}=2 \mathrm{k}=\mathrm{q}$ and that $\mathrm{A}_{\mathrm{ij}}(\mathrm{r}, \mathrm{k}) / 2=\mathrm{B}_{\mathrm{ij}}^{\mathrm{i}}(\mathrm{q})$. Then $\left(26^{\prime}\right)$ coincides with (16).
Q.E.D.

This result is driven by the fact that the same-risk couples $\mathrm{ij}=\mathrm{LL}, \mathrm{HH}$ prefer the same consumption level $\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{iw}}=\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{jw}}$, provided that only either partner i or j survives. Hence, in case of individual-life annuities they adapt annuity demands accordingly, i.e., $\mathrm{qB}_{\mathrm{ij}}^{\mathrm{i}}=\mathrm{qB}_{\mathrm{ij}}^{\mathrm{j}}$, while joint-life annuities restrict the couples to equal consumption levels $\mathrm{kA}_{\mathrm{ij}}$. Further, it is optimal for couples (regardless of the type of annuities) to share consumption and, hence, the annuity payouts to equal parts, provided that both partners survive (due to equal weights of $c_{\mathrm{tij}}^{\mathrm{s}}$ and $\mathrm{c}_{\mathrm{tij}}^{\mathrm{j}}$ in the couples' utility function). From these considerations it follows that same-risk couples have the same consumption behaviour over the partners' lifetimes, if $r=2 k=q$, by demanding the same amount $A_{i j}(r, k) / 2=B_{i j}^{i}(q)$.

[^11]Corollary 1: Lemma 3 implies that any same risk-type couple $\mathrm{ij}=\mathrm{LL}, \mathrm{HH}$ is indifferent between individual-life annuities and joint-life annuities with "a half to last survivor" rule, if $q=r$, while they are better off at any $q>r$ and worse off at any $q<r$. This is in contrast to mixed-risk couples LH, who are better off with individual-life annuities than with joint-life annuities with "a half to last survivor" rule, if $q=r$. The intuition, which follows from the comments below Lemma 1 , is that individual-life annuities allow for adjustment to the higher weight of $\mathrm{c}_{1 \mathrm{LH}}^{\mathrm{H}^{\mathrm{W}}}$ of the longerlived partner in the couple's expected utility than that of $c_{1 L H}^{L^{W}}$ of the shorter-lived partner $L$, while joint-life annuities restricts demand to $c_{1 L H}^{H^{w}}=C_{1 L H}^{L^{W}}$, which implies lower expected utility given that $\mathrm{r}=\mathrm{q}$.

### 3.2 Separate and pooling contracts for joint-life annuities

Joint-life annuities are characterised by two payoffs ( $\mathrm{r}, \mathrm{k}$ ). First of all, this allows annuity companies to offer contracts which differ in their payoff ratio r/k. Second, observe that a typeHH couple has a higher probability that both partners survive to retirement (relative to the probability that only one partner survives) than a type-LH couple, who has in turn a relatively higher chance that both survive than a type-LL couple, i.e.

$$
\begin{equation*}
\pi_{\mathrm{H}}^{2} /\left(2 \pi_{\mathrm{H}}\left(1-\pi_{\mathrm{H}}\right)\right)>\pi_{\mathrm{L}} \pi_{\mathrm{H}} /\left(\pi_{\mathrm{L}}\left(1-\pi_{\mathrm{H}}\right)+\pi_{\mathrm{H}}\left(1-\pi_{\mathrm{L}}\right)\right)>\pi_{\mathrm{L}}^{2} /\left(2 \pi_{\mathrm{L}}\left(1-\pi_{\mathrm{L}}\right)\right), \tag{27}
\end{equation*}
$$

which is due to the assumption that $\pi_{\mathrm{H}}>\pi_{\mathrm{L}}$. Hence, for any given r , a type-LL couple will put most weight on the last-survivor payoff $k$, while a type-HH couple will put least weight on $k$. Due to these considerations it is obvious that annuity companies may have an incentive to separate couples according to their survival probabilities by varying the relative payoff. Before we turn (in Section 3.3) to investigate the implications of these considerations on the equilibrium outcome, we introduce contracts for joint-life annuities which produce zero-profits, when either bought by one type of couples only ("separate contract") or bought by all three types of couples ("pooling contract'). These are of special interest, because under the assumption of perfect competition in the market for joint-life annuities, only zero-profit contracts (whether separate or pooling) can persist.

A contract $\left(r_{i j}, k_{i j}\right)$, which is bought solely by couples of one type ij , is called a separate contract. It produces zero-profits, if

$$
\begin{equation*}
1-\pi_{i} \pi_{\mathrm{j}} \mathrm{r}_{\mathrm{ij}}-\left(\pi_{\mathrm{i}}\left(1-\pi_{\mathrm{j}}\right)+\pi_{\mathrm{j}}\left(1-\pi_{\mathrm{i}}\right)\right) \mathrm{k}_{\mathrm{ij}}=0 \quad \mathrm{i}, \mathrm{j}=\mathrm{L}, \mathrm{H} \tag{28}
\end{equation*}
$$

given the assumption of a zero interest rate. Note that the zero-profit condition (28) implies that the joint-life annuity is fair for a couple ij , as expected payoffs equal its price. However, as many
contracts ( $\mathrm{r}_{\mathrm{ij}}, \mathrm{k}_{\mathrm{ij}}$ ) fulfil (28), the next Lemma investigates which of them is the most preferred one by a couple of type ij .

Lemma 4: Among all separate contracts ( $\mathrm{r}_{\mathrm{ij}}, \mathrm{k}_{\mathrm{ij}}$ ), which fulfil (28) for a couple of type ij , the most preferred is characterised by

$$
\begin{equation*}
\sigma \mathrm{u}^{\prime}\left(\frac{\sigma r}{2} \mathrm{~A}_{\mathrm{ij}}\right)=\mathrm{u}^{\prime}\left(\mathrm{kA}_{\mathrm{ij}}\right) \tag{29}
\end{equation*}
$$

which implies a payoff-ratio $r_{\mathrm{ij}} / \mathrm{k}_{\mathrm{ij}}=2$ for $\sigma=1$ or for logarithmic utility, with $\mathrm{r}_{\mathrm{ij}}=2 /\left(\pi_{\mathrm{i}}+\pi_{\mathrm{j}}\right)$.

Proof: See the Appendix.

From (29) it is immediate that in case that there is no joint consumption ( $\sigma=1$ ), the "half to last survivor" rule is optimal for a couple ij, given their respective zero-profit separate contract. By use of $\mathrm{r}_{\mathrm{ij}} / \mathrm{k}_{\mathrm{ij}}=2$ and (28) the optimal contract $\left(\mathrm{r}_{\mathrm{ij}}, \mathrm{k}_{\mathrm{ij}}\right)$ for couple ij is given by $\left(2 /\left(\pi_{\mathrm{i}}+\pi_{\mathrm{j}}\right), 1 /\left(\pi_{\mathrm{i}}+\pi_{\mathrm{j}}\right)\right)$. For $\sigma>1$, however, the optimum ratio $\mathrm{r}_{\mathrm{ij}} / \mathrm{k}_{\mathrm{ij}}$ depends on the specifics of the per-period function. One checks easily that for any utility function which exhibits a constant relative risk aversion R , the most preferred ratio $\mathrm{r}_{\mathrm{ij}} / \mathrm{k}_{\mathrm{ij}}$ determined by (29) and is larger (smaller) than 2 , if $R$ is smaller (larger) than 1 , specifically for logarithmic utility $(R=1)$, the "half to last survivor" rule is optimal. ${ }^{16}$

On the other hand, a contract $(r, k)$, which is bought by at least two types of couples $\mathrm{ij}=$ $\mathrm{LL}, \mathrm{LH}, \mathrm{HH}$, is called a pooling contract. In order that a pooling contract produces zero profits, it must fulfil the condition, together with annuity demand $\mathrm{A}_{\mathrm{ij}}>0$ for at least two $\mathrm{ij}=\mathrm{LL}, \mathrm{LH}, \mathrm{HH}$, (for brevity, we use $A_{i j}$ instead of $\left.A_{i j}(r, k)\right)$

$$
\begin{align*}
& \frac{\gamma^{2}}{2} \mathrm{~A}_{\mathrm{HH}}\left(1-\pi_{\mathrm{H}}^{2} \mathrm{r}-2 \pi_{\mathrm{H}}\left(1-\pi_{\mathrm{H}}\right) \mathrm{k}\right)+\gamma(1-\gamma) \mathrm{A}_{\mathrm{LH}}\left(1-\pi_{\mathrm{H}} \pi_{\mathrm{L}} \mathrm{r}-\left(\pi_{\mathrm{H}}\left(1-\pi_{\mathrm{L}}\right)+\pi_{\mathrm{L}}\left(1-\pi_{\mathrm{H}}\right) \mathrm{k}\right)+\right.  \tag{30}\\
& +\frac{(1-\gamma)^{2}}{2} \mathrm{~A}_{\mathrm{LL}}\left(1-\pi_{\mathrm{L}}^{2} r-2 \pi_{\mathrm{L}}\left(1-\pi_{\mathrm{L}}\right) \mathrm{k}\right)=0,
\end{align*}
$$

where $\gamma^{2} / 2, \gamma(1-\gamma)$ and $(1-\gamma)^{2} / 2$ are the respective shares of type-HH couples, type-LH couples and type-LL couples, resp., according to the assumptions made in Section 2.2.

[^12]Further note that our assumptions on the survival probabilities implies that a pooling contract $(r, k)$ specified according to the "half to last survivor" rule $(r / k=2)$ offers the highest expected payoffs to couple HH and the lowest to couples of type LL, as $\pi_{\mathrm{H}}^{2} r+\pi_{\mathrm{H}}\left(1-\pi_{\mathrm{H}}\right) r>$ $\pi_{\mathrm{L}} \pi_{\mathrm{H}} \mathrm{r}+\frac{1}{2}\left(\pi_{\mathrm{H}}\left(1-\pi_{\mathrm{L}}\right)+\pi_{\mathrm{L}}\left(1-\pi_{\mathrm{H}}\right)\right) \mathrm{r}>\pi_{\mathrm{L}}^{2} \mathrm{r}+\pi_{\mathrm{L}}\left(1-\pi_{\mathrm{L}}\right) \mathrm{r}$. Obviously, this relation reveals that given this contract, couples of type LL are the most "profitable" ones (the good risks) and couples of type HH are the least "profitable" ones (the bad risks). This means that on the one hand, a zeroprofit pooling contract with $\mathrm{r} / \mathrm{k}=2$ would produce positive profits, if it were chosen only by couple LL, or that, on the other hand, a zero-profit separate contract with $r_{H H} / \mathrm{k}_{\mathrm{HH}}=2$ for couple HH would produce positive profits, if it were chosen by couple LH and/or couple LL additionally. However, it will be shown in the next Lemma that the above relation of expected payoffs for different couple-types and hence their profitability for the annuity companies does not hold for all payoff-ratios r/k. Note that this is in contrast to many other models with asymmetric information (e.g. like in the market for individual life annuities, see Section 2.2), where lowestrisk types are always the most profitable costumers for firms, while the highest-risk types are always the least profitable ones.

Lemma 5: Consider a pooling contract ( $r, k$ ) for joint-life annuities and let $\mathrm{p}_{\mathrm{ij}} \equiv \pi_{\mathrm{i}} \pi_{\mathrm{j}} \mathrm{r}+\left(\pi_{\mathrm{i}}\left(1-\pi_{\mathrm{j}}\right)+\pi_{\mathrm{i}}\left(1-\pi_{\mathrm{j}}\right)\right) \mathrm{k}$ be expected payoffs for a couple $\mathrm{ij}=\mathrm{LL}, \mathrm{LH}, \mathrm{HH}$. Then the relation of expected payoffs of $(r, k)$ for each couple $i j$ depends on the payoff-ration $r / k$ in the following way:
(L5a) $\mathrm{p}_{\mathrm{LL}}<\mathrm{p}_{\mathrm{LH}} \leq \mathrm{p}_{\mathrm{HH}}$, if $\frac{\mathrm{r}}{\mathrm{k}} \geq 2-\frac{1}{\pi_{\mathrm{H}}}$,
(L5b) $\mathrm{p}_{\mathrm{LL}} \leq \mathrm{p}_{\mathrm{HH}}<\mathrm{p}_{\mathrm{LH}}$, if $2-\frac{1}{\pi_{\mathrm{H}}}>\frac{\mathrm{r}}{\mathrm{k}} \geq 2-\frac{2}{\pi_{\mathrm{H}}+\pi_{\mathrm{L}}}$, given that $\pi_{\mathrm{H}}>\frac{1}{2}$,
(L5c) $\mathrm{p}_{\mathrm{HH}}<\mathrm{p}_{\mathrm{LL}}<\mathrm{p}_{\mathrm{LH}}$, if $2-\frac{2}{\pi_{\mathrm{H}}+\pi_{\mathrm{L}}}>\frac{\mathrm{r}}{\mathrm{k}}>2-\frac{1}{\pi_{\mathrm{L}}}$, given that $\pi_{\mathrm{H}}+\pi_{\mathrm{L}}>1$,
(L5d) $\quad \mathrm{p}_{\mathrm{HH}}<\mathrm{p}_{\mathrm{LH}} \leq \mathrm{p}_{\mathrm{LL}}$, if $2-\frac{1}{\pi_{\mathrm{L}}} \geq \frac{\mathrm{r}}{\mathrm{k}}$, given that $\pi_{\mathrm{L}}>\frac{1}{2}$.

Proof: We determine the signs of the differences $p_{L H}-p_{L L}, p_{H H}-p_{L L}$ and $p_{H H}-p_{L H}$ and find that

$$
\begin{equation*}
\mathrm{p}_{\mathrm{LH}} \gtrless \mathrm{p}_{\mathrm{LL}} \text { if } \frac{\mathrm{r}}{\mathrm{k}} \geqslant 2-\frac{1}{\pi_{\mathrm{L}}}, \quad \mathrm{p}_{\mathrm{HH}} \gtrless \mathrm{p}_{\mathrm{LL}} \text { if } \frac{\mathrm{r}}{\mathrm{k}} \geqslant 2-\frac{2}{\pi_{\mathrm{H}}+\pi_{\mathrm{L}}}, \quad \mathrm{p}_{\mathrm{HH}} \geqslant \mathrm{p}_{\mathrm{LH}} \text { if } \frac{\mathrm{r}}{\mathrm{k}} \geqslant 2-\frac{1}{\pi_{\mathrm{H}}}, \tag{31}
\end{equation*}
$$

where, as $\pi_{\mathrm{L}}<\pi_{\mathrm{H}}$,

$$
\begin{equation*}
2-\frac{1}{\pi_{\mathrm{L}}}<2-\frac{2}{\pi_{\mathrm{H}}+\pi_{\mathrm{L}}}<2-\frac{1}{\pi_{\mathrm{H}}} . \tag{32}
\end{equation*}
$$

From (31) together with (32) it follows (L5a) - (L5d) of Lemma 5.

The reasoning for Lemma 5 is the following: Among all three types of couples $\mathrm{ij}=\mathrm{LL}, \mathrm{LH}, \mathrm{HH}$, couples LL have the lowest chance that both partners survive, while couples HH have the highest chance that both partners survive (as $\pi_{\mathrm{L}}^{2}<\pi_{\mathrm{L}} \pi_{\mathrm{H}}<\pi_{\mathrm{H}}^{2}$ ). However, this ordering does
generally not hold for their risk that only one partner survives [as $2 \pi_{\mathrm{L}}\left(1-\pi_{\mathrm{L}}\right) \lesseqgtr \pi_{\mathrm{L}}\left(1-\pi_{\mathrm{H}}\right)+\pi_{\mathrm{H}}(1-$ $\left.\pi_{\mathrm{L}}\right) \lesseqgtr 2 \pi_{\mathrm{H}}\left(1-\pi_{\mathrm{H}}\right)$ ]. This gives us an intuitive argument why only for sufficiently high payoff-ratios $r / k$, namely those defined by (L5a) of Lemma 5 (among them $r / k=2$, considered above), couples LL are the best risks and couples HH are the worst risks. For lower payoff-ratios, however, the ordering changes, where it even reverses for sufficiently low payoff-ratios r/k, namely those defined by (L5d). Then, couples LL are the worst risks and couples HH are the best risks.

Corollary 2: Note that the results in Lemma 5 found for pooling contracts has its counterpart for a separate contract: When drawing the zero-profit conditions (28) for a separate contract $\left(\mathrm{r}_{\mathrm{ij}}, \mathrm{k}_{\mathrm{ij}}\right)$ in the ( $\mathrm{r}, \mathrm{k}$ )-space, denoted by $\mathrm{ZP}_{\mathrm{ij}}, \mathrm{ij}=\mathrm{LL}, \mathrm{LH}, \mathrm{HH}$ in Figure 1, one observes that the zero-profit lines $Z P_{i j}$ intersect at different payoff-ratios indicated by the straight lines $\mathrm{S} 1-\mathrm{S} 3$. Hence, the position of $\mathrm{ZP}_{\mathrm{HH}}, \mathrm{ZP}_{\mathrm{LH}}$ and $\mathrm{ZP}_{\mathrm{LL}}$ to each other also depends on the payoff-ratio r/k. For an explanation, we note that any contract on $\mathrm{ZP}_{\mathrm{ij}}$ offers the same expected payoffs, equal to 1 , to couple ij . By this, the following correlation becomes obvious: Given a payoff-ratio r/k, at which the expected payoffs of a pooling contract is lower, e.g., for couple LL than for couple LH, the separate contract for couple LL has to lie above the separate contract for couple LH in order that each of the contracts offers the same expected payoffs (equal to one) to the respective couple. This explains why the payoff-ratios below and on S1 are equal to those defined in (L5a) of Lemma 5, consequently $\mathrm{ZP}_{\mathrm{LL}}$ lies above $\mathrm{ZP}_{\mathrm{LH}}$, which in turn lies above (and on) $\mathrm{ZP}_{\mathrm{HH}}$.

Figure 1: Zero-profit conditions for a separate contract $\left(\mathrm{r}_{\mathrm{i}}, \mathrm{k}_{\mathrm{ij}}\right)$


Analogously, payoff-ratios between S1 and S2 (as well as on S2) coincide with those defined in (L5b), those between S2 and S3 coincide with those defined in (L5c) and those above and on S 3 coincide with those defined in (L5d). In the latter case, $\mathrm{ZP}_{\mathrm{HH}}$ lies above $\mathrm{ZP}_{\mathrm{LH}}$, which in turn lies above $Z_{L L}$, which means that a zero-profit separate contract ( $r_{H H}, \mathrm{k}_{\mathrm{HH}}$ ) for couples HH offers
higher payoffs than a zero-profit separate contract for couple LH or couple LL. This implies that ( $\mathrm{r}_{\mathrm{H}}, \mathrm{K}_{\mathrm{HH}}$ ) would produce negative profits, if chosen by couple LH and/or couple HH . As its counterpart, any zero-profit pooling contract would produce positive profits, if it were chosen only by couple HH . Obviously, these results on the profitability of contracts are essential for the incentives for the annuity companies, which contracts they will offer in equilibrium and, hence, for the upcoming analysis of the equilibria.

### 3.3 Equilibria for joint-life annuities

To investigate the equilibrium outcome in the market for joint-life annuities we make use of the well-known concept of a Nash-Cournot equilibrium, which was studied by Rothschild and Stiglitz (1976) in the context of insurance markets. First we show that no pooling equilibrium for joint-life annuities exists. Then, in subsection 3.3.2 we derive all results concerning the existence and characterisation of a separating equilibrium, when couples are mandated to buy joint-life annuities. This serves as a benchmark, which is then compared to a situation, where couples can choose between individual- and joint-life annuities.

### 3.3.1 The non-existence of a pooling equilibrium for joint-life annuities

We call a contract $(r, k)$ a pooling equilibrium, if, together with $A_{i j}(r, k)>0, i j=L L, L H, H H$, the zeroprofit condition (30) is fulfilled and if no other contract exists, which is preferred to ( $\mathrm{r}, \mathrm{k}$ ) by at least one type of couple $\mathrm{ij} \in\{L \mathrm{~L}, \mathrm{LH}, \mathrm{HH}\}$ and which allows a nonnegative profit. We find that in general no pooling equilibrium exists. As a preparation we show:

Lemma 6: Let $(r, k)$ be a pooling contract that, together with $A_{i j}(r, k)>0, i j=L L, L H, H H$, fulfils the zero-profit condition (30).
(i) If $(r, k)$ is characterised by $r / k \geq 2-2 /\left(\pi_{\mathrm{L}}+\pi_{\mathrm{H}}\right)$ any contract ( $\mathrm{r}+\delta \mathrm{r}, \mathrm{k}+\delta \mathrm{k}$ ), which is close enough to ( $r, k$ ) and which is chosen only by couples of type LL, allows a nonnegative profit.
(ii) If $\pi_{\mathrm{L}}+\pi_{\mathrm{H}}>1$ and $(\mathrm{r}, \mathrm{k})$ is characterised by $\mathrm{r} / \mathrm{k}<2-2 /\left(\pi_{\mathrm{L}}+\pi_{\mathrm{H}}\right)$, any contract ( $r+\delta r, k+\delta k$ ), which is close enough to $(r, k)$ and which chosen only by couples of type HH , allows a nonnegative profit.

Proof: See Appendix.

This result follows from Lemma 5, from which we know that the expected payoffs of any zeroprofit pooling contract $(r, k)$, characterised by $r / k \geq 2-2 /\left(\pi_{L}+\pi_{H}\right)$, is lower for couple LL than for couple LH and HH. This in turn implies positive profits, if only type-LL couples buy this zero-
profit pooling contract or one close to it. Equivalent considerations apply for any zero-profit pooling contract, characterised by $r / k<2-2 /\left(\pi_{\mathrm{L}}+\pi_{\mathrm{H}}\right)$, given that $\pi_{\mathrm{L}}+\pi_{\mathrm{H}}>1$ : Then, $(\mathrm{r}, \mathrm{k})$ produces positive profits, if only type- HH couples buy this contract or one close to it, because, due to Lemma 5, it offers smaller expected payoffs to a type-HH couple than to a type-LL couple as well as to a type-LH couple.

We now introduce a further assumption on expected utility $\mathrm{U}_{\mathrm{ij}}$, in addition to strict concavity of the per-period utility function $u$ : Indifference curves in the $(r, k)$-space satisfy the single-crossing condition,

$$
\begin{equation*}
-\frac{\partial \mathrm{V}_{\mathrm{HH}} / \partial \mathrm{r}}{\partial \mathrm{~V}_{\mathrm{HH}} / \partial \mathrm{k}}<-\frac{\partial \mathrm{V}_{\mathrm{LH}} / \partial \mathrm{r}}{\partial \mathrm{~V}_{\mathrm{LH}} / \partial \mathrm{k}}<-\frac{\partial \mathrm{V}_{\mathrm{LL}} / \partial \mathrm{r}}{\partial \mathrm{~V}_{\mathrm{LL}} / \partial \mathrm{k}} \tag{33}
\end{equation*}
$$

where $V_{\mathrm{ij}}(\mathrm{r}, \mathrm{k})$ indicates indirect (expected) utility of couple $\mathrm{ij}=\mathrm{LL}, \mathrm{LH}, \mathrm{HH}$ for any contract $(\mathrm{r}, \mathrm{k})$, defined in the usual way as utility attained with $\mathrm{A}_{\mathrm{ij}}(\mathrm{r}, \mathrm{k})$. Condition (33) implies that the indifference curves of couples of different types can cross only once, as the slope of an indifference curve of a type-HH couple is always steeper than that of a type-LH couple, which in turn is steeper than that of a type-LL couple. Using the Envelope Theorem, (33) reduces to

$$
\begin{equation*}
\frac{\pi_{H}^{2} \sigma u^{\prime}\left(\frac{\sigma r}{2} \mathrm{~A}_{\mathrm{HH}}\right)}{2 \pi_{\mathrm{H}}\left(1-\pi_{\mathrm{H}}\right) \mathrm{u}^{\prime}\left(\mathrm{k} A_{\mathrm{HH}}\right)}>\frac{\pi_{\mathrm{L}} \pi_{\mathrm{H}} \sigma \mathrm{u}^{\prime}\left(\frac{\sigma r}{2} \mathrm{~A}_{\mathrm{LH}}\right)}{\left(\pi_{\mathrm{L}}\left(1-\pi_{\mathrm{H}}\right)+\pi_{\mathrm{H}}\left(1-\pi_{\mathrm{L}}\right)\right) \mathrm{u}^{\prime}(\mathrm{kA} \mathrm{LH})}>\frac{\pi_{\mathrm{L}}^{2} \sigma \mathrm{u}^{\prime}\left(\frac{\sigma r}{2} \mathrm{~A}_{\mathrm{LL}}\right)}{2 \pi_{\mathrm{L}}\left(1-\pi_{\mathrm{L}}\right) \mathrm{u}^{\prime}(\mathrm{kA} \mathrm{LL})}, \tag{33'}
\end{equation*}
$$

which, together with (27), implies that the single-crossing condition is certainly fulfilled for any per-period utility function which exhibits a constant coefficient of relative risk aversion, in particular for logarithmic utility (14).

Proposition 2: No pooling equilibrium exists, given the single-crossing condition (33).

We demonstrate the result graphically (see Figure 3) in a diagram where the payoffs $r$ and $k$ are drawn on the axis. ${ }^{17}$ The dashed line $Z P_{P}$ represents the zero-profit condition (30) for a pooling contract, the slope $\lambda$ of the straight line $S 2$ shows the payoff ratio $r / k=2-2 /\left(\pi_{\mathrm{L}}+\pi_{\mathrm{H}}\right)$. First, consider any contract $(r, k)$ on $Z P_{P}$, which lies below or on the straight line $S 2$. Due to the singlecrossing condition, the indifference curve $\mathrm{V}_{\mathrm{LL}}$ of a type-LL couple is the flattest. Hence, one can find another contract ( $r+\delta r, k+\delta k$ ), close to $(r, k)$, which is preferred only by couple LL. As we know from Lemma 6, such a contract only chosen by couple LL is profitable for the annuity companies. Hence any zero-profit pooling contract $(r, k)$, which fulfils $r / k \geq 2-2 /\left(\pi_{L}+\pi_{H}\right)$, does not represent a pooling equilibrium. The same applies for any contract contract ( r ', $\mathrm{k}^{\prime}$ ) on zero-

[^13]profit line $\mathrm{ZP}_{\mathrm{P}}$ and above the straight line S 2 for analogous reasons: Due to the single-crossing condition, the indifference curve $\mathrm{V}_{\mathrm{H}}$ of a type-HH couple is the steepest. Therefore, another contract ( $r^{\prime}+\delta r, k^{\prime}+\delta k$ ) close to ( $r^{\prime}, k^{\prime}$ ) exists, which is only preferred by couple $\mathrm{HH}-$ and is therefore profitable, as Lemma 6 tells us. Altogether, no pooling equilibrium exists. ${ }^{18}$

Figure 2: The non-existence of a pooling equilibrium


By means of Figure 2 the significance of the single-crossing condition can be discussed. One observes immediately that the result of Proposition 2 certainly holds whenever the slopes of $\mathrm{V}_{\mathrm{LL}}$ and $\mathrm{V}_{\mathrm{HH}}$ differ, independently of which one is steeper, as long as the indifference curve $\mathrm{V}_{\mathrm{LH}}$ of type LH-couple turns out not to be steeper or flatter than $\mathrm{V}_{\mathrm{LL}}$ and $\mathrm{V}_{\mathrm{HH}}$. Even when the slopes of all three indifference curves are the same, the result holds, given that the slope of the zero-profit line $Z P_{P}$ is different. In this case one can find another pooling contract close to the original zeroprofit pooling contract which is preferred by the couples of all three types and produces a nonnegative profit. Only if a point on $\mathrm{ZP}_{\mathrm{P}}$ exists in which the slopes of $\mathrm{ZP}_{\mathrm{P}}, \mathrm{V}_{\mathrm{LL}}, \mathrm{V}_{\mathrm{LH}}$ and $\mathrm{V}_{\mathrm{HH}}$ are identical this represents a pooling equilibrium. Clearly, this can occur only for very specific parameter constellations. Further, one cannot exclude the possibility that a pooling equilibrium exists, in case that the slope of $\mathrm{V}_{\mathrm{LH}}$ is either steeper or flatter than the slope of $\mathrm{V}_{\mathrm{LL}}$ and $\mathrm{V}_{\mathrm{HH}}$. In this case, it can occur that any contract, close to the original zero-profit pooling contract, which is preferred either by couple HH or by couple LL , produces a negative profit, as it will be chosen

[^14]also by couple LH. If this is the case, then the original zero-profit contract constitutes a pooling equilibrium.

### 3.3.2 Separating equilibrium with mandatory joint-life annuities for couples

In this section we will show the possibility of a separating equilibrium for joint-life annuities in a framework in which couples are mandated to buy joint-life annuities. Under this assumption individual-life annuities are bought by single persons only, who receive a pooling contract $\tilde{\mathrm{q}}_{\text {s }}$ defined as in (20). In Section 3.3 .3 we relax the assumption of mandatory joint-life annuities for couples and allow couples to choose between individual- and joint-life annuities.

We call a set of three contracts ( $\left.r_{L L}, k_{L L}\right)$, ( $\left.r_{L H}, k_{L H}\right)$, ( $r_{H H}, \mathrm{k}_{\mathrm{HH}}$ ) a separating Nash-Cournot equilibrium, if each fulfils the respective zero-profit condition (28) and the self-selection constraint for the respective couple $\mathrm{ij}=\mathrm{LL}, \mathrm{LH}, \mathrm{HH}$, i.e.

$$
\begin{align*}
& \mathrm{V}_{\mathrm{HH}}\left(\mathrm{r}_{\mathrm{HH}}, \mathrm{k}_{\mathrm{HH}}\right) \geq \mathrm{V}_{\mathrm{HH}}\left(\mathrm{r}_{\mathrm{LH}}, \mathrm{k}_{\mathrm{LH}}\right),  \tag{34a}\\
& \mathrm{V}_{\mathrm{LH}}\left(\mathrm{r}_{\mathrm{LH}}, \mathrm{k}_{\mathrm{LH}}\right) \geq \mathrm{V}_{\mathrm{LH}}\left(r_{\mathrm{HH}}, \mathrm{k}_{\mathrm{HH}}\right),  \tag{35a}\\
& \mathrm{V}_{\mathrm{LL}}\left(r_{\mathrm{LL}}, \mathrm{k}_{\mathrm{LL}}\right) \geq \mathrm{V}_{\mathrm{LL}}\left(r_{\mathrm{LH}}, \mathrm{k}_{\mathrm{LH}}\right), \tag{36a}
\end{align*}
$$

$$
\begin{align*}
& \mathrm{V}_{\mathrm{HH}}\left(\mathrm{r}_{\mathrm{HH}}, \mathrm{k}_{\mathrm{HH}}\right) \geq \mathrm{V}_{\mathrm{HH}}\left(\mathrm{r}_{\mathrm{LL}}, \mathrm{k}_{\mathrm{LL}}\right),  \tag{34b}\\
& \mathrm{V}_{\mathrm{LH}}\left(\mathrm{r}_{\mathrm{LH}}, \mathrm{k}_{\mathrm{LH}}\right) \geq \mathrm{V}_{\mathrm{LH}}\left(\mathrm{r}_{\mathrm{LL}}, \mathrm{k}_{\mathrm{LL}}\right),  \tag{35b}\\
& \mathrm{V}_{\mathrm{LL}}\left(\mathrm{r}_{\mathrm{LL}}, \mathrm{k}_{\mathrm{LL}}\right) \geq \mathrm{V}_{\mathrm{LL}}\left(\mathrm{r}_{\mathrm{HH}}, \mathrm{k}_{\mathrm{HH}}\right), \tag{36b}
\end{align*}
$$

and if no other contract exists, which is preferred by at least one couple $\mathrm{ij} \in\{\mathrm{LL}, \mathrm{LH}, \mathrm{HH}\}$ and which allows a nonnegative profit. Note that this definition of a separating equilibrium implies that if a couple of type ij is indifferent between their separate contract and another one (or two), they indeed choose the separate contract which is designed for them. Moreover, it is assumed that each couple of type ij is restricted to buy only one type of contract, i.e. no mix of ( $r_{L L}, \mathrm{k}_{\mathrm{LL}}$ ), ( $r_{L H}, \mathrm{k}_{\mathrm{LH}}$ ) and $\left(\mathrm{r}_{\mathrm{HH}}, \mathrm{k}_{H H}\right){ }^{19}$ However, couples may purchase as many contracts of the chosen type as they want. Further, we refer to a logarithmic per-period function (14), which has two convenient properties helping to keep the analytical and graphical analysis simple: First, the single-crossing condition (33) is fulfilled, as mentioned above. Second, the demand for joint-life annuities does not depend on the payoffs $(r, k)$ and on the parameter $\sigma$ for the degree of joint consumption.

Let the three contracts $\left(\hat{r}_{H H}, \hat{\mathrm{k}}_{\mathrm{HH}}\right),\left(\hat{\mathrm{r}}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}\right)$ and $\left(\hat{r}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}\right)$ be defined as follows:

- $\quad\left(\hat{r}_{H H}, \hat{\mathrm{k}}_{\mathrm{HH}}\right)$ is the most preferred separate contract for couple HH , which fulfils the zero-profit condition (28) for couple HH; therefore $\hat{r}_{H H}=2 \hat{\mathrm{k}}_{\mathrm{HH}}$, as shown in Lemma 4.

[^15]- ( $\hat{r}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) is implicitly defined by the zero-profit condition (28) for couple LH, by the property that a couple HH is indifferent between ( $\hat{r}_{H H}, \hat{\mathrm{k}}_{\mathrm{HH}}$ ) and ( $\hat{\mathrm{r}}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) and by $\hat{\mathrm{r}}_{\mathrm{LH}}<2 \hat{\mathrm{k}}_{\mathrm{LH}}$.
- ( $\hat{r}_{L L}, \hat{k}_{\mathrm{LL}}$ ) is implicitly defined by the zero-profit condition (28) for couple LL, by the property that a couple LH is indifferent between ( $\hat{r}_{L H}, \hat{k}_{L H}$ ) and ( $\hat{r}_{L L}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ) and by $\hat{\mathrm{r}}_{\mathrm{LL}}<2 \hat{\mathrm{k}}_{\mathrm{LL}}$.

In the following we will show that the set of these three contracts may but need not constitute an equilibrium. We begin the analysis by illustrating graphically how the three contracts are defined. In Figure 3, analogously to Figure 1, we have drawn the zero-profit lines $Z P_{i j}$ for a separate contract ( $\mathrm{r}_{\mathrm{i},}, \mathrm{k}_{\mathrm{ij}}$ ) and their respective points of intersection S1 and S2. Note that in order to simplify the graphics we now assume a value of $\pi_{\mathrm{L}}<0.5$, hence in Figure $3 \mathrm{ZP}_{\mathrm{LL}}$ and $Z P_{\mathrm{LH}}$ do not cross. Given logarithmic utility, the indifference curves $\mathrm{V}_{\mathrm{ij}}$ are strictly convex, with respective slope $-2 \pi_{\mathrm{i}} \pi_{\mathrm{j}} \mathrm{k} /\left(\left(\pi_{\mathrm{i}}\left(1-\pi_{\mathrm{j}}\right)+\pi_{\mathrm{j}}\left(1-\pi_{\mathrm{i}}\right)\right) r\right)$; the respective slope of ZP ij is equal to $-\pi_{i} \pi_{\mathrm{j}} /\left(\pi_{\mathrm{i}}\left(1-\pi_{\mathrm{j}}\right)+\pi_{\mathrm{j}}\left(1-\pi_{\mathrm{i}}\right)\right)$. Hence, at $\left(\hat{r}_{H H}, \hat{\mathrm{k}}_{H H}\right)$ the indifference curve $\mathrm{V}_{H H}$ is tangent to the zero-profit line $\mathrm{ZP}_{\mathrm{HH}}$, while the indifferences curves, $\mathrm{V}_{\mathrm{HH}}$ going through ( $\hat{r}_{\mathrm{HH}}, \hat{\mathrm{k}}_{\mathrm{HH}}$ ) and $\mathrm{V}_{\mathrm{LH}}$ going through ( $\hat{r}_{L H}, \hat{k}_{L H}$ ), have exactly two points of intersection with the zero-profit line $Z P_{L H}, Z P_{L L}$ resp. $\hat{r}_{L H}<2 \hat{k}_{\mathrm{LH}}$ and $\hat{r}_{\mathrm{LL}}<2 \hat{\mathrm{k}}_{\mathrm{LL}}$ uniquely define the respective points of intersection above the straight line $S^{\prime}$ where $r=2 k$.

Figure 3: The possibility of a separating equilibrium


By definition, ( $\hat{r}_{\mathrm{HH}}, \hat{\mathrm{k}}_{\mathrm{HH}}$ ), ( $\hat{r}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) and ( $\hat{r}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ) produce zero profits, if each is chosen only by the respective couple $\mathrm{ij}=\mathrm{LL}, \mathrm{LH}, \mathrm{HH}$. That this is indeed the case is shown in the next Lemma.

Lemma 7: $\left(\hat{r}_{H H}, \hat{\mathrm{k}}_{\mathrm{HH}}\right),\left(\hat{r}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}\right)$ and $\left(\hat{r}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}\right)$ fulfil the self-selection constraints (34a) - (36b).

Proof: See the Appendix.

The self-selection constraint (34a) for couple HH is fulfilled by the condition that $\mathrm{V}_{\mathrm{HH}}\left(\hat{r}_{\mathrm{HH}}, \hat{\mathrm{k}}_{\mathrm{HH}}\right)=\mathrm{V}_{\mathrm{HH}}\left(\hat{r}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}\right)$. Further it turns out that the self-selection constraint (34b) holds as well, which is due to the single-crossing condition: It is evident from Figure 3 that couples of type HH are better off at $\left(\hat{r}_{\mathrm{HH}}, \hat{\mathrm{k}}_{\mathrm{HH}}\right)$ than at the separate ( $\hat{r}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ) for couple LL. Altogether, couples of type HH buy their separate contract ( $\hat{\mathrm{r}}_{\mathrm{HH}}, \hat{\mathrm{k}}_{\mathrm{HH}}$ ). Equivalent considerations apply to couples of type LH: They choose the contract $\left(\hat{r}_{L H}, \hat{\mathrm{k}}_{\mathrm{LH}}\right)$, as, by definition, $V_{L H}\left(\hat{r}_{L H}, \hat{k}_{L H}\right)=V_{L H}\left(\hat{r}_{L L}, \hat{k}_{\mathrm{LL}}\right)$, and as, due to the single-crossing condition, $V_{L H}\left(\hat{r}_{L H}, \hat{k}_{L H}\right)>V_{L H}\left(\hat{r}_{H H}, \hat{k}_{H H}\right)$. Finally observe from Figure 3 that (again due to the single-crossing condition) couples of type LL prefer their contract ( $\hat{r}_{L L}, \hat{k}_{L L}$ ) to the other two contracts, designed for couples of type LH and HH; hence the self-selection constraints (36a) and (36b) hold with inequality.

Lemma 7 ensures that no couple of type ij has an incentive to deviate from their respective separate contract, if only ( $\hat{r}_{H H}, \hat{\mathrm{k}}_{\mathrm{HH}}$ ), ( $\hat{\mathrm{r}}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) and ( $\hat{\mathrm{rL}}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ) are offered to them, which in turn imply zero profits for the annuity companies. Next, we will show that annuity companies have no incentive to offer any other separate contract than ( $\hat{r}_{H H}, \hat{\mathrm{k}}_{\mathrm{HH}}$ ), ( $\hat{\mathrm{r}}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) and ( $\hat{r}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ) to the respective couple $\mathrm{ij}=\mathrm{LL}, \mathrm{LH}, \mathrm{HH}$. As a preparation we show:

Lemma 8: Any separate contract which fulfils the zero-profit condition (28)
(i) for couple LH and which is preferred to ( $\hat{\mathrm{L}}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) by couple LH, would be chosen also by couple HH and then produce a loss.
(ii) for couple $L L$ and which is preferred to ( $\hat{r}_{L L}, \hat{k}_{L L}$ ) by couple $L L$, would be also chosen by couple LH or by couple LH and HH and then produce a loss.

Proof: See the Appendix.

The arguments behind (i) of Lemma 8 are the following: First observe that ( $\hat{r}_{L H}, \hat{k}_{\mathrm{LH}}$ ) as the point of intersection of $\mathrm{V}_{\mathrm{HH}}$ and $\mathrm{ZP}_{\mathrm{LH}}$ must lie below the point of intersection of $\mathrm{Z}_{\mathrm{HH}}$ and $\mathrm{ZP}_{\mathrm{LH}}$, which is due to the fact that at $\left(\hat{r}_{L H}, \hat{k}_{L H}\right)$ the slope of $V_{H H}$ is steeper than $Z P_{H H}$. Consequently, $\left(\hat{r}_{L H}, \hat{k}_{L H}\right)$ as well as any contract on $\mathrm{ZP}_{\mathrm{LH}}$ below ( $\hat{r}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) are characterised by a payoff-ratio below S1 where $\mathrm{ZP}_{\mathrm{LH}}$ lies above $\mathrm{ZP}_{\mathrm{HH}}$ (compare Corollary 2). Second, observe that couple LH and HH prefer any contract on $Z P_{\mathrm{LH}}$ below ( $\hat{r}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) to their respective separate contract ( $\hat{r}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) and ( $\hat{r}_{H H}, \hat{\mathrm{k}}_{\mathrm{HH}}$ ), while couple LL is better off at ( $\hat{r}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ) (note the single-crossing property of $\mathrm{V}_{\mathrm{ij}}$ ). Hence, any such contract would produce a loss, if it were offered by the annuity companies (as it would be chosen also by couple HH). By analogous consideration we find that ( $\hat{r}_{L L}, \hat{k}_{L L}$ ) is characterised by a payoff-ratio, where $\mathrm{ZP}_{\mathrm{LL}}$ lies above $\mathrm{ZP}_{\mathrm{LH}}$, and that the contract, at which $\mathrm{V}_{\mathrm{HH}}$
and $\mathrm{ZP}_{\mathrm{LH}}$ cross, denoted as D in Figure 3, is characterised by a payoff-ratio below S 2 , where $Z P_{\mathrm{LL}}$ lies above $Z \mathrm{P}_{\mathrm{HH}}$. Hence, any contract on $\mathrm{ZP}_{\mathrm{LL}}$ which is preferred to ( $\hat{\mathrm{LL}}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ) by couple LL would produce a loss, because it would be chosen also by couple LH or by couple LH and HH .

Proposition 3: If a separating equilibrium exists, it consists of ( $\hat{r}_{H H}, \hat{\mathrm{k}}_{\mathrm{HH}}$ ), ( $\hat{r}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) and $\left(\hat{r}_{L L}, \hat{k}_{L L}\right)$.

Proof: See the Appendix.

As an intuition for this result, note first that $\left(\hat{r}_{H H}, \hat{\mathrm{k}}_{\mathrm{HH}}\right)$, which is most preferred by couple HH among all contracts on $\mathrm{ZP}_{\mathrm{HH}}$, must be part of the equilibrium: Any other contract is dominated by ( $\hat{r}_{H H}, \hat{\mathrm{k}}_{\mathrm{HH}}$ ), and firms need not care whether couples of type LH and/or LL might buy this contract too, because this would only increase profits (as ( $\hat{r}_{H H}, \hat{\mathrm{k}}_{\mathrm{HH}}$ ) lies below S1). However, when offering a specific contract to couple LH, annuity companies have to be concerned that this contract is not chosen by a couple of type HH , because then they would make a loss, as Lemma 8 tells us. This implies that the self-selection constraint (34a) is essential: Among all contracts on $Z P_{L H}$, only ( $\hat{L}_{L H}, \hat{k}_{L H}$ ), which is the most preferred by couple LH , subject to the selfselection constraint (34a), can be part of a separating equilibrium. Note, in particular that due to this argument the contract ( $r_{L H}^{b}, \mathrm{k}_{\mathrm{LH}}^{\mathrm{b}}$ ), which is most preferred by couple LH among all contracts on $\mathrm{ZP}_{\mathrm{LH}}$ cannot be part of the equilibrium. However, annuity companies need not to care whether couples of type LL buy ( $\hat{\mathrm{L}}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ), because this would imply positive profits. Analogous considerations apply to the separate contract for couple LL, where only ( $\hat{r}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ), which provides the maximum utility to couple LL subject to the self-selection constraint (35b), can be part of the separating equilibrium. Any other contract on $\mathrm{ZP}_{\mathrm{LL}}$, at which couple LL would be better off than at ( $\hat{r}_{L L}, \hat{k}_{L L}$ ), would be chosen by couple LH and/or couple HH and then produce a loss (compare Lemma 8), and hence, will not be offered by the annuity companies.

Proposition 3 says that the contract set $\left(\hat{r}_{H H}, \hat{k}_{H H}\right),\left(\hat{r}_{L H}, \hat{k}_{L H}\right)$ and ( $\hat{\mathrm{r}}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ) is indeed the only candidate for a separating equilibrium: No other separate contract ( $r_{i j}, \mathrm{k}_{\mathrm{ij}}$ ) exists which is preferred by the respective couple $\mathrm{ij}=\mathrm{LL}, \mathrm{LH}, \mathrm{HH}$ only, and produces a nonnegative profit. However, to prove that this contract set indeed constitutes an equilibrium, one has to show that no other contract exists, which is preferred by at least two couples $\mathrm{ij} \in\{\mathrm{LL}, \mathrm{LH}, \mathrm{HH}\}$ and which allows a nonnegative profit.

Proposition 4: For sufficiently high values $\gamma$ of the share of the high-risk types, a separating equilibrium exists.

Proof: See the Appendix.

We demonstrate the intuition behind this Proposition graphically by means of Figure 4. First, consider a contract, which lies above the indifference curves $\mathrm{V}_{\mathrm{LL}}, \mathrm{V}_{\mathrm{LH}}$ and $\mathrm{V}_{\text {HН }}$, such as E in Figure 4. If $E$ is offered, couples of all three types will purchase it. If moreover $E$ allows a nonnegative profit, when bought by couples of all three types, it will be offered by the annuity companies, and consequently it will upset the potential separating equilibrium ( $\hat{r}_{H H}, \hat{\mathrm{k}}_{H H}$ ), ( $\hat{r}_{L H}, \hat{k}_{L H}$ ) and ( $\hat{L}_{L L}, \hat{k}_{L L}$ ). We note that the profitability of $E$ and hence the position of the dashed zero-profit line $\mathrm{ZP}_{\mathrm{P}_{1}}{ }^{20}$, which represents all pooling contracts which fulfil the zero-profit condition with positive annuity demand $\mathrm{A}_{\mathrm{ij}}>0$ for $\mathrm{ij}=\mathrm{LL}, \mathrm{LH}, \mathrm{HH}$, depends on the composition of aggregate annuity demand. If there are relatively many married persons with a high survival probability, i.e a high value of $\gamma$, then E will lose money. Such a case is drawn in Figure 4, where $\mathrm{ZP}_{\mathrm{P}_{1}}$ does not cross $\mathrm{V}_{\mathrm{LL}}$; hence no pooling contract exists, which is chosen by couples of all three types and allows a non-negative profit. However, for lower values $\gamma$ of high-risk types, $\mathrm{ZP}_{\mathrm{P}_{1}}$ lies closer to $Z P_{L L}$, if E lies on or below $\mathrm{ZP}_{\mathrm{P}_{1}}, \mathrm{E}$ will make a profit and no separating equilibrium exists.

Figure 4: The existence of a separating equilibrium


Second, consider a contract which lies above the indifference curves $\mathrm{V}_{\mathrm{LH}}$ and $\mathrm{V}_{H H}$, but below $V_{L L}$, like contract $F$ in Figure 4. If $F$ is offered, it is purchased by couples of type LH and HH, but not by couples of type LL. As above, the profitability of $F$ and hence the position of the zeroprofit line $\mathrm{ZP}_{\mathrm{P} 2}$ for a pooling contract with $\mathrm{A}_{H H}>0, A_{L H}>0, A_{L L}=0$, depends on the level of the share $\gamma$ of the high-risk types. For sufficiently large values of $\gamma, \mathrm{ZP}_{\mathrm{P} 2}$ lies below $\mathrm{V}_{\text {LH }}$. Such a situation is presented in Figure 4, which implies that any pooling contract, preferred by couple LH and HH , would produce negative profits and thus, will not be offered by the annuity

[^16]companies. However, for sufficiently low values of $\gamma, \mathrm{F}$ makes a non-negative profit, when bought by couples of type LH and HH , (which means that F will lie on or below $\mathrm{ZP}_{\mathrm{P}_{2}}$ ), and consequently the contract set $\left(\hat{r}_{H H}, \hat{\mathrm{k}}_{\mathrm{HH}}\right),\left(\hat{r}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}\right)$ and ( $\left.\hat{r}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}\right)$ does not constitute a separating equilibrium. Analogous arguments apply to a contract like $G$, which lies above $\mathrm{V}_{\mathrm{LH}}$ and $\mathrm{V}_{\mathrm{LL}}$ but below $\mathrm{V}_{\mathrm{HH}}$, and hence would be chosen by couples of type LH and LL , but not by couple HH . If G lies on or below the zero-profit line $\mathrm{ZP}_{\mathrm{P} 3}$, which represents all pooling contracts which fulfil the zero-profit condition with $A_{L H}>0, A_{L L}>0, A_{H H}=0$, no separating equilibrium exists. However, for a sufficiently large $\gamma$, as drawn in Figure $4, \mathrm{ZP}_{\mathrm{P} 3}$ lies below $\mathrm{V}_{\mathrm{LL}}$; hence no dominating pooling contract, which allow non-negative profits, exists. Finally note that there does not exist a contract, which - due to the single-crossing condition - lies above $\mathrm{V}_{\mathrm{LL}}$ and $\mathrm{V}_{\mathrm{HH}}$, but below $\mathrm{V}_{\mathrm{LH}}$. Altogether, we can conclude that for sufficiently large shares $\gamma$ of high-risk types, no pooling contract exists, which is chosen by at least two types of couples, and then produces a non-negative profit. Such a situation is drawn in Figure 4. In this case ( $\hat{r}_{H H}, \hat{\mathrm{k}}_{\mathrm{HH}}$ ), ( $\hat{r}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) and ( $\hat{r}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ) indeed constitute a separating equilibrium.

However, for lower shares $\gamma$, where at least one pooling contract like E, F or G allows a nonnegative profit, no separating equilibrium exists. ${ }^{21}$ In case that $E$ allows non-negative profits, $E$ does not constitute an equilibrium either, as we know from Proposition 2. From this it follows that the market for joint-life annuities will have no equilibrium at all. Due to analogous arguments, there cannot exist an equilibrium which consists of a contract set with one pooling contract for couples of two different types and one separate contract for the couple of the remaining type, like $F$ and $\left(\hat{r}_{L L}, \hat{k}_{\mathrm{LL}}\right)$ or $G$ and $\left(\hat{r}_{H H}, \hat{\mathrm{k}}_{\mathrm{HH}}\right)$ : For this note first that any contract F which is preferred by couples of type LH and HH to their separate contracts must lie below S1, where $\mathrm{ZP}_{\mathrm{LH}}$ lies above $\mathrm{ZP}_{\mathrm{HH}}$ (immediate from similar arguments as used to prove Lemma 8). Thus in case that there exists a contract $F$, which lies on the zero-profit line $Z P_{P 2}$, annuity companies can offer another contract close to F, which is preferred by couples of type LH only, and then produces a positive profit (compare Lemma 5). Thus, a zero-profit pooling contract F cannot be part of an equilibrium, and due to similar considerations, also a zero-profit pooling contract $G$ cannot be part of an equilibrium either.

Finally, it should be mentioned that the basic arguments remain valid in case of a general perperiod utility function, not just for a logarithmic one, as long as the single-crossing condition holds. It is straightforward to see that the zero-profit lines $\mathrm{ZP}_{\mathrm{ij}}$ for separate contracts for couples of each type, are unaffected by the type of the utility function. Hence the characterization of the

[^17]separating equilibrium remains valid. The main difference is that with a general utility function, the curves $\mathrm{ZP}_{\mathrm{P} 1}, \mathrm{ZP}_{\mathrm{P} 2}$ and $\mathrm{ZP}_{\mathrm{P} 3}$, defined by the zero-profit condition (30) for pooling contracts, will no longer be a straight line, because annuity demand depends on the payoff rates. The shape of these zero-profit curves in turn has an influence on the existence of the equilibrium.

Altogether, we can conclude that in case that couples are mandated to buy joint-life annuities, only a separating equilibrium can exist, where couples with both partners having a high survival probability can buy their "first-best" contract with a payoff-ratio r/k = 2, while couples of the other types (with at least one partner having a low survival probability) can only buy a "distorted" contract with a lower payoff-ratio r/k. This result fits to the empirical observation that annuity companies offer joint-life annuities which differ in their payoff-fraction. For instance, TIAA-CREF offers joint-life annuities with a payoff ratio r/k of 2, 1.5 and 1 (see Amerik, 2002). The results found in this framework suggest that these different options are offered by the annuity companies in order to select couples according to the partners' life-expectancies.

However, among those TIAA-CREF participants, who bought a joint-life annuity, about twothirds of women and men chose that option with the lowest payoff-ratio of one (Ameriks, 2002). In the framework with mandatory joint-life annuities for all couples, this means that the share $\gamma$ of high-risk types is relatively low, which in turn would imply that the existence of a separating equilibrium is less likely. However, in reality, couples are free to choose between individual-life annuities and joint-life annuities. Accounting for this fact, a plausible explanation for the high majority of the lowest payoff-ratio r/k = 1 among those selecting joint-life annuities can be offered: Mainly couples with partners having a low survival probability indeed choose joint-life annuities, while couples with partners expecting a longer life rather decide for individual-life annuities. The next Section 3.3.3, where free choice between individual- and joint-life annuities is considered, investigates whether this guess turns out to be correct.

### 3.3.3 Equilibria with free choice between individual or joint-life annuities

In the presence of mandatory joint-life annuities for couples, only single persons of either type i $=\mathrm{L}, \mathrm{H}$ buy individual-life annuities, which are offered to them at an equilibrium rate of return $\tilde{\mathrm{q}}_{\mathrm{S}}$, as defined by (20) in Section 2.2. In this section we extend the analysis of equilibria by allowing couples to choose between individual- and joint-life annuities within the same framework as above. Specifically, note that maintaining the assumption about restricting individuals to the purchase of one contract-type, now also excludes the purchase of a mix between contracts for individual-life annuities and for joint-life annuities (which will be abbreviated by IA- and JAcontracts from now on). However, as above, individuals (regardless of whether married or single) may purchase any amount of the chosen contract-type as they want. First of all note that
this, together with the above considerations concerning the characterisation and existence of equilibria for both contract-types, implies that IA-contracts, traded in equilibrium, continue to be pooling contracts, which are bought at least by the single persons $i=L, H$. On the other hand, if JA-contracts are traded in equilibrium, then they continue to be separate contracts, designed for the respective couple ij.

The second important observation, relevant for the analysis of equilibria in this extended framework, is that couples of type HH prefer any zero-profit pooling contract for individual-life annuities to any zero-profit separate contract for joint-life annuities. The reason for this result is that with the former contract-type, couples of type HH are in a pool with low-risk individuals (at least with those being unmarried), hence it can offer higher expected payoffs than the latter contract-type, which payoffs are determined according to the high survival probability of both couple members. Specifically, recall that the most preferred zero-profit separate contract $\left(\hat{r}_{H H}, \hat{k}_{H H}\right)$ for couple HH is given by $\left(1 / \pi_{H}, 1 /\left(2 \pi_{H}\right)\right)$, while a zero-profit IA-contract provides a payoff $q>1 / \pi_{H}$. This together with the fact, explored in Corollary 1 , that couples of type HH are better off with individual-life annuities than with joint-life annuities specified according to the "half to last survivor" rule $r / k=2$, if $q>r$, explains the next Lemma.

Lemma 9: Couple HH prefers any pooling contract q for individual-life annuities, which, together with annuity demand $\mathrm{B}_{\mathrm{i}}(\mathrm{q})>0$ and $\mathrm{B}_{\mathrm{ij}}^{\mathrm{i}}(\mathrm{q}) \geq 0$ for $\mathrm{i}, \mathrm{j}=\mathrm{L}, \mathrm{H}$, fulfils the zero-profit condition (19), to their most preferred separate contract ( $\hat{r}_{H H}, \hat{\mathrm{k}}_{\mathrm{HH}}$ ) which fulfils the zero-profit condition (28) for couple HH.

Proof: Immediate by use of Lemma 3, Corollary 1, Lemma 4 and the fact, explored in Section 2.2, that any zero-profit pooling contract q must lie between $1 / \pi_{\mathrm{H}}$ and $1 / \pi_{\mathrm{L}}$.
Q.E.D.

It follows from Lemma 9 that in equilibrium no separate JA-contract ( $\mathrm{r}_{\mathrm{HH}}, \mathrm{k}_{\mathrm{HH}}$ ) for couple HH will be traded. The third important observation is that it is ambiguous whether couples of type LH are better off with a zero-profit pooling contract $q$ for individual-life annuities or with a zero-profit separate contract ( $r_{L H}, \mathrm{~K}_{\mathrm{LH}}$ ) for joint-life annuities. For this note that the zero-profit pooling contract q may but need not offer a higher expected payoff than a zero-profit separate contract for couple LH, as the latter depends on $\pi_{\mathrm{L}}$ and $\pi_{\mathrm{H}}$ only, while the former depends additionally on the share $\gamma$ of the high-risk types (and on the share $\beta$ of single persons). The lower $\gamma$, the higher is $q$, which makes it less likely that couples LH prefer any zero-profit separate contract ( $r_{\text {LH }}, \mathrm{k}_{\mathrm{LH}}$ ) to the zero-profit pooling contract $q$.

These observations give us a first intuition for the results shown in the following. We identify two different types of equilibria, denoted by E1 and E2: If $\gamma$ is sufficiently high for any given $\beta$ and $\pi_{\mathrm{i}}$,
$\mathrm{i}=\mathrm{L}, \mathrm{H}$, an E1 equilibrium may exist where couples of type HH , together with the single persons, buy a pooling contract for individual-life annuities and couples of type LH and LL buy a separate contract for joint-life annuities. Otherwise, an E2 equilibrium may exist where also couples of type LH (together with the single persons and couples HH ) buy the pooling contract for individual-life annuities and only couples of type LL buy their separate contract for joint-life annuities.

Our analysis of possible equilibria proceeds along the same lines as before, where we first investigate the existence and characterisation of the E1 equilibrium, before we turn to the analysis of the E2 equilibrium. We call a set of three contracts $q$, ( $\left.r_{L H}, k_{L H}\right)$, ( $r_{L L}, k_{L L}$ ) an E1 equilibrium, if the pooling contract $q$ for individual-life annuities together with annuity demand $\mathrm{B}_{\mathrm{H}}(\mathrm{q})>0, \mathrm{~B}_{\mathrm{L}}(\mathrm{q})>0, \mathrm{~B}_{\mathrm{HH}}^{\mathrm{H}}(\mathrm{q})>0$ and $\mathrm{B}_{\mathrm{LH}}^{\mathrm{H}}(\mathrm{q})=0, \mathrm{~B}_{\mathrm{LH}}^{\mathrm{L}}(\mathrm{q})=0, \mathrm{~B}_{\mathrm{LL}}^{\mathrm{L}}(\mathrm{q})=0$, fulfils the zero-profit condition (19), and each of the separate contracts ( $r_{L H}, \mathrm{k}_{\mathrm{LH}}$ ) and ( $\mathrm{r}_{\mathrm{LL}}, \mathrm{k}_{\mathrm{LL}}$ ) fulfils the respective zero-profit condition (28), if each contract fulfils the self-selection constraints for the respective couple $\mathrm{ij}=\mathrm{LL}, \mathrm{LH}, \mathrm{HH}$, i.e.

$$
\begin{align*}
& \mathrm{V}_{\mathrm{HH}}(\mathrm{q}) \geq \mathrm{V}_{\mathrm{HH}}\left(r_{\mathrm{LH}}, \mathrm{k}_{\mathrm{LH}}\right),  \tag{37a}\\
& \mathrm{V}_{\mathrm{LH}}\left(\mathrm{r}_{\mathrm{LH}}, \mathrm{k}_{\mathrm{LH}}\right) \geq \mathrm{V}_{\mathrm{LH}}(\mathrm{q}),  \tag{38a}\\
& \mathrm{V}_{\mathrm{LL}}\left(r_{\mathrm{LL}}, \mathrm{k}_{\mathrm{LL}}\right) \geq \mathrm{V}_{\mathrm{LL}}\left(r_{\mathrm{LH}}, \mathrm{k}_{\mathrm{LH}}\right), \tag{39a}
\end{align*}
$$

$$
\begin{align*}
& \mathrm{V}_{\mathrm{HH}}(\mathrm{q}) \geq \mathrm{V}_{\mathrm{HH}}\left(\mathrm{r}_{\mathrm{LL}}, \mathrm{k}_{\mathrm{LL}}\right),  \tag{37b}\\
& \mathrm{V}_{\mathrm{LH}}\left(\mathrm{r}_{\mathrm{LH}}, \mathrm{k}_{\mathrm{LH}}\right) \geq \mathrm{V}_{\mathrm{LH}}\left(\mathrm{r}_{\mathrm{LL}}, \mathrm{k}_{\mathrm{LL}}\right),  \tag{38b}\\
& \mathrm{V}_{\mathrm{LL}}\left(r_{\mathrm{LL}}, \mathrm{k}_{\mathrm{LL}}\right) \geq \mathrm{V}_{\mathrm{LL}}(\mathrm{q}), \tag{39b}
\end{align*}
$$

and if no other contract $q$ or $(r, k)$ exists, which if preferred to by at least one couple $\mathrm{ij} \in$ $\{\mathrm{LL}, \mathrm{LH}, \mathrm{HH}\}$ and which allows a nonnegative profit.

Let $\bar{q}$ denote the zero-profit pooling contract for individual-life annuities, as defined for an E1 equilibrium ${ }^{22}$, and note that due to Corollary 1 the mixed risk-couple LH is better off at $\bar{q}$ than at any JA-contract $(r, k)$ with $r=2 k$, if $\bar{q}=r$. From this it follows that the self-selection constraint (38a) cannot be fulfilled, if $\bar{q} \geq r_{L H}^{b}$, where ( $r_{L H}^{b}, k_{L H}^{b}$ ) indicates the zero-profit separate contract, which is most preferred by couple $L H$, with $r_{L H}^{b}=2 k_{L H}^{b}$. Hence, an E1 equilibrium can only exists, if $\bar{q}<r_{L H}^{b}$. In that case the other two parts of a potential $E 1$ equilibrium, denoted by ( $\bar{r}_{L H}, \overline{\mathrm{k}}_{\mathrm{LH}}$ ) and ( $\bar{r}_{\mathrm{LL}}, \overline{\mathrm{k}}_{\mathrm{LL}}$ ), have the following properties:

- ( $\bar{r}_{L H}, \bar{k}_{L H}$ ) is implicitly defined by the zero-profit condition (28) for couple LH, by the property that a couple HH is indifferent between $\overline{\mathrm{q}}$ and ( $\overline{\mathrm{r}}_{\mathrm{LH}}, \overline{\mathrm{k}}_{\mathrm{LH}}$ ) and by $\overline{\mathrm{r}}_{\mathrm{LH}}<2 \overline{\mathrm{k}}_{\mathrm{LH}}$.
- ( $\bar{r}_{L L}, \overline{\mathrm{k}}_{\mathrm{LL}}$ ) is implicitly defined by the zero-profit condition (28) for couple LL, by the property that a couple $L H$ is indifferent between ( $\bar{r}_{L H}, \overline{\mathrm{k}}_{\mathrm{LH}}$ ) and ( $\overline{\mathrm{r}}_{\mathrm{LL}}, \overline{\mathrm{k}}_{\mathrm{LL}}$ ) and by $\bar{r}_{L L}<2 \overline{\mathrm{k}}_{\mathrm{LL}}$.

[^18]Obviously, the definition of ( $\bar{r}_{L H}, \overline{\mathrm{k}}_{\mathrm{LH}}$ ) and ( $\overline{\mathrm{r}}_{\mathrm{LL}}, \overline{\mathrm{k}}_{\mathrm{LL}}$ ) imply that the self-selection constraints (37a) and (38b) are fulfilled with equality, which is illustrated in Figure 5 which replicates Figure 3 for the case where couples are free to choose between IA- and JS-contracts. In Figure 5 the indifference curve $\bar{V}_{H H}$ of couple HH represents all contracts $(r, k)$ at which couple HH is as well off as at the pooling IA-contract $\overline{\mathrm{q}}$. Because couple HH is indifferent between $\overline{\mathrm{q}}$ and any JScontract with $r=2 \mathrm{k}=\overline{\mathrm{q}}$ (as mentioned above and explored in Corollary 1), the intersection of $\overline{\mathrm{V}}_{\mathrm{HH}}$ with the straight line $\mathrm{S}^{\prime}$ (where $\mathrm{r}=2 \mathrm{k}$ ) shows the level of $\overline{\mathrm{q}}$. As in Figure $3, \overline{\mathrm{~V}}_{\mathrm{HH}}$ has exactly two points of intersection with the zero-profit line $Z P_{L H} ; \bar{r}_{L H}<2 \overline{\mathrm{k}}_{\mathrm{LH}}$ uniquely defines the respective point of intersection above $S^{\prime}$. In the same way, ( $\bar{r}_{L L}, \bar{k}_{L L}$ ) is found as the point of intersection of $Z P_{\mathrm{LL}}$ and $\overline{\mathrm{V}}_{\mathrm{LH}}$, going through ( $\overline{\mathrm{r}}_{\mathrm{LH}}, \overline{\mathrm{k}}_{\mathrm{LH}}$ ), above $\mathrm{S}^{\prime}$.

Figure 5: The possibility of an E1 equilibrium ( $\left.\overline{\mathrm{q}},\left(\overline{\mathrm{L}}_{\mathrm{LH}}, \overline{\mathrm{k}}_{\mathrm{LH}}\right),\left(\bar{r}_{\mathrm{LL}}, \overline{\mathrm{k}}_{\mathrm{LL}}\right)\right)$


Due to the single-crossing property of the indifference curves the self-selection constraint (37b) for couple HH and the self-selection constraints (39a) and (39b) for couple LL (note that $\overline{\mathrm{V}}_{\mathrm{LL}}$ going through ( $\bar{r}_{L L}, \bar{k}_{L L}$ ) lies above the contract with $\left.r=\bar{q}\right)^{23}$ hold with inequality. However, it depends on the constellations of the parameters $\pi_{\mathrm{H}}, \pi_{\mathrm{L}}, \gamma$ and $\beta$ whether the self-selection constraint (38a) is fulfilled. Because a JS-contract with $r=2 \mathrm{k}$ does not allow to adapt consumption according to the relatively higher chance that only partner H survives than that only partner $L$ survives (as mentioned above and explored in Corollary 1), a couple LH needs a JS-contract with $\mathrm{r}=2 \mathrm{k}>\overline{\mathrm{q}}$ in order to be indifferent to $\overline{\mathrm{q}}$, which means that the dashed indifference curve $\overline{\mathrm{V}}_{\mathrm{LH}}^{\prime}$, which represents all contracts $(\mathrm{r}, \mathrm{k})$ at which couple LH is as well off as

[^19]at $\bar{q}$, must lie above $\bar{q}$. Only if $\overline{\mathrm{V}}_{\mathrm{LH}}^{\prime}$ lies below the indifference curve $\overline{\mathrm{V}}_{\mathrm{LH}}$ (or coincide with $\overline{\mathrm{V}}_{\mathrm{LH}}$ ), which goes through ( $\overline{\mathrm{L}}_{\mathrm{LH}}, \overline{\mathrm{k}}_{\mathrm{LH}}$ ), the self-selection constraint (38a) holds. Such a case is drawn in Figure 5, which is found for appropriate parameter constellations.

Further, it follows from analogous arguments as in the Proof of Lemma 8, together with the fact that $\overline{\mathrm{V}}_{\mathrm{HH}}$ lies above $\mathrm{ZP}_{\mathrm{HH}}$ (and thus above $\mathrm{V}_{\mathrm{HH}}$ going through $\left(\hat{r}_{\mathrm{HH}}, \hat{\mathrm{k}}_{\mathrm{HH}}\right)$ in Figure 3), ( $\overline{\mathrm{r}}_{\mathrm{LH}}, \overline{\mathrm{k}}_{\mathrm{LH}}$ ) is characterised by a payoff-ratio below S 1 , at which $\mathrm{ZP}_{\mathrm{LH}}$ lies above $\mathrm{ZP}_{\mathrm{HH}}$, that ( $\bar{r}_{\mathrm{LL}}, \overline{\mathrm{k}}_{\mathrm{LL}}$ ) is characterised by a payoff-ratio, at which $\mathrm{ZP}_{\mathrm{LL}}$ lies above $Z \mathrm{P}_{\mathrm{LH}}$, and D is characterised by a payoff-ratio below S 2 , at which $\mathrm{ZP}_{\mathrm{LL}}$ lies above $\mathrm{ZP}_{\mathrm{HH}}$. This implies that, as for the separating equilibrium with mandated joint-life annuities for couples, the self-selection constraints (37a) and (38b) are essential: Annuity companies have to care that ( $\bar{r}_{\mathrm{LH}}, \overline{\mathrm{k}}_{\mathrm{LH}}$ ) is not chosen by couple HH as well as that ( $\overline{\mathrm{r}}_{\mathrm{LL}}, \overline{\mathrm{k}}_{\mathrm{LL}}$ ) is not chosen by couple LH and/or by couple HH , because in either case they would make a loss. Thus, any separate contract on $Z P_{L H}$ above $\overline{\mathrm{V}}_{\mathrm{LH}}$ as well as any separate contract on $Z P_{L L}$ above $\bar{V}_{\mathrm{LL}}$ cannot be part of an E1 equilibrium. On the other hand, all other contracts on $\mathrm{ZP}_{\mathrm{LH}}, \mathrm{ZP}_{\mathrm{LL}}$, resp. are dominated by ( $\bar{r}_{L H}, \overline{\mathrm{k}}_{\mathrm{LH}}$ ), ( $\bar{r}_{\mathrm{LL}}, \overline{\mathrm{k}}_{\mathrm{LL}}$ ), resp., and, hence, cannot be part of the equilibrium either. Moreover, it follows from Lemma 9 that no separate contract for couple HH can be part of the E1 equilibrium, which is obvious also from Figure 5; there $\overline{\mathrm{V}}_{\mathrm{HH}}$ lies above $\mathrm{ZP}_{\mathrm{HH}}$. From these considerations we can conclude that no other separate contract exists which is preferred by a couple $\mathrm{ij}=\mathrm{LL}, \mathrm{LH}, \mathrm{HH}$ and then allows a non-negative profit. Therefore, if an E1 equilibrium exists, it consists of $\bar{q}$, ( $\bar{r}_{L H}, \overline{\mathrm{k}}_{\mathrm{LH}}$ ) and ( $\overline{\mathrm{r}}_{\mathrm{LL}}, \overline{\mathrm{k}}_{\mathrm{LL}}$ ).

However, this contract set does not constitute an equilibrium, if the self-selection constraint (38a) is not fulfilled, i.e. if $\mathrm{V}_{\mathrm{LH}}\left(\bar{r}_{\mathrm{LH}}, \mathrm{k}_{\mathrm{LH}}\right)<\mathrm{V}_{\mathrm{LH}}(\overline{\mathrm{q}})$. Moreover, the potential E 1 equilibrium may be upset by (i) a pooling IA-contract $q$, which is preferred also by couple LH and/or by couple LL, and produces a non-negative profit, and/or by (ii) a pooling JS-contract ( $\mathrm{r}, \mathrm{k}$ ), which is preferred by at least two couple $\mathrm{ij} \in\{\mathrm{LL}, \mathrm{LH}, \mathrm{HH}\}$ and which produces a non-negative profit. Only if neither of these pooling contracts exists and if condition (38a) holds, then $\overline{\mathrm{q}}$, ( $\bar{r}_{\mathrm{LH}}, \overline{\mathrm{k}}_{\mathrm{LH}}$ ) and ( $\overline{\mathrm{r}}_{\mathrm{LL}}, \overline{\mathrm{k}}_{\mathrm{LL}}$ ) indeed constitute an equilibrium.

For (i) note first that the share of individual-life annuities bought by low-risk individuals (in aggregate demand for IA) increases, if couples of type LL would buy individual-life annuities instead of their separate JA-contract ( $\bar{r}_{L L}, \overline{\mathrm{k}}_{\mathrm{LL}}$ ). Hence, any contract, which is bought additionally by couple LL, must offer a payoff larger than $\bar{q}$ in order to restore zero profits. This means that $\bar{q}<\bar{q}^{\prime}$ with $\bar{q}^{\prime}$ denoting the root of the zero-profit condition (19) with positive annuity demand of all types of single and married persons except of those of couples LH. It follows that $\mathrm{V}_{\mathrm{LL}}\left(\bar{r}_{\mathrm{LL}}, \overline{\mathrm{k}}_{\mathrm{LL}}\right)>\mathrm{V}_{\mathrm{LL}}(\overline{\mathrm{q}})$ does not exclude the possibility that $\mathrm{V}_{\mathrm{LL}}\left(\overline{\mathrm{r}}_{\mathrm{LL}}, \overline{\mathrm{k}}_{\mathrm{LL}}\right)<\mathrm{V}_{\mathrm{LL}}\left(\overline{\mathrm{q}}^{\prime}\right)$. Analogous considerations apply for a pooling contract $\overline{\bar{q}}$, which together with positive annuity
demand of single persons of both types $\mathrm{i}=\mathrm{L}, \mathrm{H},{ }^{24}$ as well as of couples of type HH and LH , fulfils the zero-profit condition (19) and for a pooling contract $\tilde{q}$, which together with positive annuity demand of all types of single and married persons, fulfils (19). We test for the very existence of an E1 equilibrium by numerical calculations, which are summarised in Table 1. ${ }^{25}$ These computations for parameter constellations of $\pi_{\mathrm{L}}, \pi_{\mathrm{H}}, \beta$ and $\gamma$, where all of the four parameters vary from $0.8,0.6,0.4$ to 0.2 , show the following: For those parameter constellations, at which the self-selection constraint (38a) for couple LH holds, couple LH prefers also ( $\bar{r}_{\text {LH }}, \overline{\mathrm{k}}_{\mathrm{LH}}$ ) to $\overline{\overline{\mathrm{q}}}$ and couple LL prefers ( $\bar{r}_{L L}, \overline{\mathrm{k}}_{\mathrm{LL}}$ ) to $\overline{\mathrm{q}}^{\prime}$ and $\tilde{\mathrm{q}}$. Hence, no other pooling contract q exists, which is chosen by couple LH and/or couple LL, and produces a non-negative profit.

For (ii) the same considerations as in Section 3.3.2 apply where couples are assumed to be mandated to buy joint-life annuities: One has to show whether a contract E, as drawn in Figure 4, which lies above $\overline{\mathrm{V}}_{\mathrm{HH}}, \overline{\mathrm{V}}_{\mathrm{LH}}$ and $\overline{\mathrm{V}}_{\mathrm{LL}}$, a contract F (above $\overline{\mathrm{V}}_{\mathrm{HH}}$ and $\overline{\mathrm{V}}_{\mathrm{LH}}$, but below $\overline{\mathrm{V}}_{\mathrm{LL}}$ ) and a contract $G$ (above $\overline{\mathrm{V}}_{\mathrm{LH}}$ and $\overline{\mathrm{V}}_{\mathrm{LL}}$, but below $\overline{\mathrm{V}}_{\mathrm{HH}}$ ), allows non-negative profits. If such a contract exists, it upsets the potential equilibrium $\overline{\mathrm{q}},\left(\overline{\mathrm{r}}_{\mathrm{LH}}, \overline{\mathrm{k}}_{\mathrm{LH}}\right)$, ( $\left.\overline{\mathrm{r}}_{\mathrm{LL}}, \overline{\mathrm{k}}_{\mathrm{LL}}\right)$. However note that $\overline{\mathrm{q}}$, ( $\bar{r}_{\mathrm{LH}}, \overline{\mathrm{k}}_{\mathrm{LH}}$ ), ( $\bar{r}_{\mathrm{LL}}, \overline{\mathrm{k}}_{\mathrm{LL}}$ ) can constitute an equilibrium only, if the self-selection constraint (38a) holds. As argued above, (38a) holds for a sufficiently large share $\gamma$ of high-risk types only. However, a large share $\gamma$ makes it less likely that any pooling contract like E, F and/or G produces a non-negative profit (compare Proposition 4). This correlation is also reflected in the numerical simulations: For those parameter constellations, for which the self-selection constraint (38a) holds, no dominating zero-profit pooling contract ( $r, k$ ) exists, which would upset the potential E1 equilibrium. Altogether, one can conclude that for sufficiently great shares $\gamma$ of highrisk types the self-selection constraint (38a) holds. Then an E1 equilibrium exists, indicated by "E1" in Table 1.

However, what we did find indeed are parameter constellations, at which $\bar{q}<r_{\text {LH }}^{b}$ and the selfselection constraint (38a) is not fulfilled, i.e. $V_{\mathrm{LH}}\left(\bar{r}_{\mathrm{LH}}, \overline{\mathrm{k}}_{\mathrm{LH}}\right)<\mathrm{V}_{\mathrm{LH}}(\overline{\mathrm{q}})$ indicated by *) in Table $1 .{ }^{26}$ For these, as explored above, no E1 equilibrium exists and two cases have to be distinguished: If $\mathrm{V}_{\mathrm{LH}}\left(\overline{\mathrm{r}}_{\mathrm{LH}}, \overline{\mathrm{k}}_{\mathrm{LH}}\right)<\mathrm{V}_{\mathrm{LH}}(\overline{\overline{\mathrm{q}}})$, then, obviously, there exists a pooling contract for individual-life annuities which is preferred also by couple LH and produces zero-profits, which imply that an E2 equilibrium may exist. However, if $\mathrm{V}_{\mathrm{LH}}(\overline{\bar{q}}) \leq \mathrm{V}_{\mathrm{LH}}\left(\overline{\mathrm{r}}_{\mathrm{LH}}, \overline{\mathrm{k}}_{\mathrm{LH}}\right)$, then no such IA-contract exists. It follows that neither an E1 nor an E2 equilibrium exists: Such a situation is found for some parameter constellations and is denoted in Table 1 by NE*).

[^20]Table 1: The (non-)existence of an E1- and of an E2-equilibrium
Numerical calculations for logarithmic utility with $\omega=100$

|  | $\pi_{\mathrm{H}}=0.8, \pi_{\mathrm{L}}=0.6$ |  |  |  | $\pi_{\mathrm{H}}=0.8, \pi_{\mathrm{L}}=0.4$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\gamma=0.8$ | $\gamma=0.6$ | $\gamma=0.4$ | $\gamma=0.2$ | $\gamma=0.8$ | $\gamma=0.6$ | $\gamma=0.4$ | $\gamma=0.2$ |
| $\beta=0.8$ | E1 | E1 | E2 | E2 | E1 | E2 *) | E2 *) | E2 |
| $\beta=0.6$ | E1 | E1 | E2 *) | E2 | E1 | E1 | E2 *) | E2 |
| $\beta=0.4$ | E1 | E1 | E2 *) | E2 | E1 | E1 | E2 *) | E2 |
| $\beta=0.2$ | E1 | E1 | E1 | NE**) $^{*}$ | E1 | E1 | NE*) | E2 |


|  | $\pi_{\mathrm{H}}=0.6, \pi_{\mathrm{L}}=0.4$ |  |  |  | $\pi_{\mathrm{H}}=0.6, \pi_{\mathrm{L}}=0.2$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\gamma=0.8$ | $\gamma=0.6$ | $\gamma=0.4$ | $\gamma=0.2$ | $\gamma=0.8$ | $\gamma=0.6$ | $\gamma=0.4$ | $\gamma=0.2$ |
| $\beta=0.8$ | E1 | E1 | E2 | E2 | E1 | E1 | E2 *) | E2 |
| $\beta=0.6$ | E1 | E1 | E2 ${ }^{*}$ ) | E2 | E1 | E1 | E2 *) | E2 |
| $\beta=0.4$ | E1 | E1 | NE $\left.^{*}\right)$ | E2 | E1 | E1 | NE*) $^{\text {E1 }}$ | E2 |
| $\beta=0.2$ | E1 | E1 | E1 | NE** $^{*}$ | E1 | E1 | E1 | E2 *) |


|  | $\pi_{\mathrm{H}}=0.4, \pi_{\mathrm{L}}=0.2$ |  |  |  | $\pi_{\mathrm{H}}=0.8, \pi_{\mathrm{L}}=0.2$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\gamma=0.8$ | $\gamma=0.6$ | $\gamma=0.4$ | $\gamma=0.2$ | $\gamma=0.8$ | $\gamma=0.6$ | $\gamma=0.4$ | $\gamma=0.2$ |
| $\beta=0.8$ | E1 | E1 | E2 *) | E2 | E1 | E2 *) | E2 *) | E2 |
| $\beta=0.6$ | E1 | E1 | E2 *) | E2 | E1 | NE*) | E2 *) | E2 |
| $\beta=0.4$ | E1 | E1 | NE*) | E2 | E1 | E1 | E2 *) | E2 |
| $\beta=0.2$ | E1 | E1 | E1 | NE**) | E1 | E1 | NE*) | E2 *) |

E1: existence of an E1 equilibrium
E2: existence of an E2 equilbrium
*) $\quad V_{L H}\left(\bar{r}_{\mathrm{LH}}, \overline{\mathrm{k}}_{\mathrm{LH}}\right)<\mathrm{V}_{\mathrm{LH}}(\overline{\mathrm{q}})$
$\left.N E^{*}\right) \quad V_{L H}(\overline{\bar{q}})<V_{L H}\left(\bar{r}_{L H}, \overline{\mathrm{k}}_{\mathrm{LH}}\right)<\mathrm{V}_{\mathrm{LH}}(\overline{\mathrm{q}})$
$\mathrm{NE}^{\star *}$ ) zero-profit pooling contract (r,k) exists, which is preferred by couples of all three types.

Next, we concentrate our attention on the characterisation and existence of an E2 equilibrium. A contract set consisting of $q$ and ( $r_{L L}, k_{L L}$ ) constitute an E2 equilbrium, if the pooling contract $q$ for individual-life annuities, together with annuity demand $\mathrm{B}_{\mathrm{H}}(\mathrm{q})>0, \mathrm{~B}_{\mathrm{L}}(\mathrm{q})>0, \mathrm{~B}_{\mathrm{HH}}^{\mathrm{H}}(\mathrm{q})>0$, $\mathrm{B}_{\mathrm{LH}}^{\mathrm{H}}(\mathrm{q})>0, \quad \mathrm{~B}_{\mathrm{LH}}^{\mathrm{L}}(\mathrm{q})>0, \quad \mathrm{~B}_{\mathrm{LL}}^{\mathrm{L}}(\mathrm{q})=0$, fulfils the zero-profit condition (19), and the separate contract ( $r_{L L}, \mathrm{k}_{\mathrm{LL}}$ ) fulfils the zero-profit condition (28) for $\mathrm{ij}=\mathrm{LL}$, if each contract fulfils the selfselection constraint for the respective couple $\mathrm{ij}=\mathrm{LL}, \mathrm{LH}, \mathrm{HH}$, i.e.

$$
\begin{align*}
& \mathrm{V}_{\mathrm{HH}}(\mathrm{q}) \geq \mathrm{V}_{\mathrm{HH}}\left(\mathrm{r}_{\mathrm{LL}}, \mathrm{k}_{\mathrm{LL}}\right),  \tag{40}\\
& \mathrm{V}_{\mathrm{LH}}(\mathrm{q}) \geq \mathrm{V}_{\mathrm{LH}}\left(\mathrm{r}_{\mathrm{LL}}, \mathrm{k}_{\mathrm{LL}}\right),  \tag{41}\\
& \mathrm{V}_{\mathrm{LL}}\left(r_{\mathrm{LL}}, \mathrm{k}_{\mathrm{LL}}\right) \geq \mathrm{V}_{\mathrm{LL}}(\mathrm{q}), \tag{42}
\end{align*}
$$

and if no other contract $q$ or ( $\mathrm{r}, \mathrm{k}$ ) exists, which if preferred to by at least one couple $\mathrm{ij} \in$ $\{\mathrm{LL}, \mathrm{LH}, \mathrm{HH}\}$ and which allows a nonnegative profit.

Obviously, $\overline{\bar{q}}$ must be one part of the potential E2 equilibrium, however we have to distinguish between two cases, depending on the constellations of parameters, how the second part of the

E2 equilibrium, the separate contract ( $r_{L L}, k_{L L}$ ) for couple $L L$, is defined. We demonstrate this via geometric arguments: In both Figures 6 a and 6 b , the indifference curve $\overline{\bar{V}}_{\mathrm{HH}}$ represents all contracts $(\mathrm{r}, \mathrm{k})$ at which couple HH is as well off as with the pooling contract $\overline{\overline{\mathrm{q}}}$. In the same way $\overline{\bar{V}}_{\text {LH }}$ represents all contracts ( $r, k$ ) at which couple LH is as well off as with $\overline{\bar{q}}$ (as above, we know that $\overline{\bar{V}}_{\mathrm{HH}}$ goes through the point where $\mathrm{r}=2 \mathrm{k}=\overline{\overline{\mathrm{q}}}$, while $\overline{\overline{\mathrm{V}}}_{\mathrm{LH}}$ has to lie above $\mathrm{r}=2 \mathrm{k}=\overline{\overline{\mathrm{q}}}$ ). Observe first that in Figure 6a, $\overline{\bar{V}}_{H H}$ and $\overline{\bar{V}}_{\text {LH }}$ intersect below $Z P_{\text {LL }}$. Hence, only the contract $\left(\overline{\bar{r}}_{\mathrm{LL}}, \overline{\overline{\mathrm{k}}}_{\mathrm{LL}}\right)$, which is implicitly defined by the zero-profit condition (28) for couple LL, by the property that a couple LH is indifferent between $\overline{\bar{q}}$ and ( $\overline{\bar{r}}_{L L}, \overline{\bar{k}}_{L L}$ ) and by $\overline{\bar{r}}_{L L}<2 \overline{\bar{k}}_{L L}$, can constitute, together with $\overline{\overline{\mathrm{q}}}$, the potential E2 equilibrium. In that case the self-selection constraint (41) holds with equality and (40) holds with inequality. However note that $\gamma$ may be that small and hence $\overline{\bar{q}}$ may be that large that $\overline{\bar{V}}_{H H}$ and $\overline{\bar{V}}_{\text {LH }}$ intersect above $Z P_{L L}$, as illustrated graphically in Figure 6b. In that case the second part of the potential E2 equilibrium, denoted by ( $\overline{\bar{r}}_{\mathrm{LL}}^{\prime}, \overline{\overline{\mathrm{k}}}_{\mathrm{LL}}^{\prime}$ ), is found as the point of intersection of $\overline{\overline{\mathrm{V}}}_{\mathrm{HH}}$ and $Z P_{\mathrm{LL}}$ above the straight line $\mathrm{S}^{\prime} .{ }^{27}$ There (40) holds with equality and (41) holds with inequality. However irrespective of whether $\left(\overline{\bar{r}}_{\mathrm{LL}}, \overline{\overline{\mathrm{k}}}_{\mathrm{LL}}\right)$ or ( $\overline{\bar{r}}_{\mathrm{LL}}^{\prime}, \overline{\overline{\mathrm{k}}}_{\mathrm{LL}}^{\prime}$ ) is part of the potential equilibrium, in either case the self-selection constraint (42) is fulfilled due to the single-crossing property of the indifference curves.

Figure 6: The possibility of an E2 equilibrium


Due to equivalent arguments as explored above, given a potential E2 equilibrium no other separate contract ( $\left(\mathrm{r}_{\mathrm{i}}, \mathrm{k}_{\mathrm{ij}}\right)$ exists, which is preferred by the respective couple ij and produces a non-negative profit: First, as in both Figures 6 a and 6 b , $\overline{\overline{\mathrm{V}}}_{\mathrm{HH}}$ lies above $\mathrm{ZP} \mathrm{PH}_{\mathrm{HH}}$ and $\overline{\overline{\mathrm{V}}}_{\mathrm{LH}}$ lies above $\mathrm{ZP}_{\mathrm{LH}}$, any separate contract for couple LH and HH , which is preferred by the couple of

[^21]the respective type, would produce a loss. Second, any other contract on $\mathrm{ZP}_{\mathrm{LL}}$ than ( $\overline{\bar{r}}_{\mathrm{LL}}, \overline{\bar{k}}_{\mathrm{LL}}$ ), ( ${\overline{r_{L L}}}^{\prime}, \overline{\bar{k}}_{\mathrm{LL}}^{\prime}$ ), resp., which is preferred by couple LL, would be chosen by couple LH and/or couple HH , and hence produce a loss. From this, we can conclude that, if an E2 equilibrium exists, it consists either of the contract set $\overline{\overline{\mathrm{q}}},\left(\overline{\bar{r}}_{L L}, \overline{\overline{\mathrm{k}}}_{\mathrm{LL}}\right)$ or of the contract set $\overline{\overline{\mathrm{q}}},\left(\overline{\bar{r}}_{\mathrm{T}}^{\prime}, \overline{\overline{\mathrm{k}}}_{\mathrm{LL}}^{\prime}\right)$.

However, as above, no E2 equilibrium exists, if there exists (i) a pooling contract $q$, which is preferred also by couple LL and produces a non-negative profit, and/or (ii) a pooling contract (r,k), which if preferred to by at least two couple $\mathrm{ij} \in\{\mathrm{LL}, \mathrm{LH}, \mathrm{HH}\}$ and which produces a nonnegative profit. In order to assess the prevalence of such dominating zero-profit pooling contracts, we refer to the numerical computations in Table 1, where we proceed along the same lines, as when testing the existence of an E1 equilibrium: (i) As $\overline{\bar{q}}<\tilde{\mathrm{q}}$, we check whether $\mathrm{V}_{\mathrm{LL}}(\tilde{\mathrm{q}}) \leq \mathrm{V}_{\mathrm{LL}}\left(\overline{\bar{r}}_{\mathrm{LL}}, \overline{\bar{k}}_{\mathrm{LL}}\right)$ or $\mathrm{V}_{\mathrm{LL}}(\tilde{\mathrm{q}}) \leq \mathrm{V}_{\mathrm{LL}}\left(\overline{\bar{r}}_{\mathrm{LL}}, \overline{\overline{\mathrm{k}}}_{\mathrm{LL}}^{\prime}\right)$, resp. If the respective condition holds, a pooling contract q , which is preferred by couples LL and produces a non-negative profit, does not exist. The numerical calculations show that for those parameter constellations, for which we found a situation as drawn in Figure 6a or 6b, couple LL prefer their respective separate contract to $\tilde{q}$. (ii) Note that for a potential E2 equilibrium to exist the share $\gamma$ of high-risk types must be sufficiently low. Consequently, a JA-contract, which is preferred by at least two couples-types and produces a non-negative profit, is more likely to exist. Numerical calculations show that given $\pi_{H}-\pi_{L}=0.2, \gamma=0.2$ and $\beta=0.2$, a pooling contract $(r, k)$, like E in Figure 4, exists, which is preferred by couples of all three types and produces a nonnegative profit. Consequently, no E2 equilibrium exists, which is indicated by $\mathrm{NE}^{* *) ~ i n ~ T a b l e ~ 1 . ~ H o w e v e r, ~ f o r ~ a l l ~ o t h e r ~ p a r a m e t e r ~}$ constellations with sufficiently low shares $\gamma$ of high-risk types, any pooling contract which is chosen by at least two types of couples, produce negative profits. Therefore, an E2 equilibrium exists, which is indicated by "E2" in Table 1.

Altogether, we can conclude that the equilibria found in this Section can serve as a plausible explanation for the empirical observation (mentioned at the end of Section 3.3.2) that two-thirds of TIAA-CREF participants, who purchased joint-life annuities, chose the contract that specifies the lowest payoff-ratio $\mathrm{r} / \mathrm{k}=1$ : Mainly couples with partners having a low survival probability choose their separate contract for joint-life annuities, while couples with at least one longer-lived partner rather decide for a pooling contract for individual-life annuities, because such a contract can offer them higher expected payoffs, as they are pooled with the short-lived single persons.

## 4. Summary and Conclusions

This paper has demonstrated that the consumption behaviour of couples, who have agreed to pool resources, has important implications on the private annuity market. First, we have focused on the market for individual life annuities and have assumed the standard model of asymmetric
information with two periods of life and with price competition. We have shown that the influence of couples as market participants on the pooling equilibrium depends on how their annuity demand affects the overrepresentation of annuities bought by the individuals with a high lifeexpectancy. We find that the extent of adverse selection is increased, in case that couples do not have the advantage of joint consumption of "family public goods" as well as in case of a logarithmic utility function. Then the equilibrium rate of return on individual-life annuities is smaller in an economy where couples and singles coexist compared to an economy with single persons only, otherwise the effect is ambiguous. As the majority of individuals are married or live in a long-term relationship, this result may contribute to explain the limited size of the private annuity market.

Second, considering couples as the decision-making units allows us to model the market for joint-life annuities. Due to their higher chance that only one partner survives to the retirement, couples with partners having a low survival probability put more weight on the survivor benefit than a couple with partners having a higher life expectancy. We have shown that this provides an incentive for the annuity companies to offer different survivor benefit options in order to separate couples according to the partners' life expectancies. Hence, in a framework where couples are mandated to provide for retirement through joint-life annuities, only a separating equilibrium for joint-life annuities can exist. This result is taken as a benchmark, which is compared to the more relevant situation in which couples are free to choose between individualand joint-life annuities. In this framework we identify two possible equilibria, which occurrence depends on the exogenous parameters: In the first equilibrium couples with two long-lived partners, together with the single persons, buy a pooling contract for individual-life annuities and couples with at least one short-lived partner buy their separate contracts for joint-life annuities. In the other equilibrium also mixed-risk couples buy the pooling contract for individual-life annuities and only couples with two short-lived partners buy their separate contract for joint-life annuities.

These results can explain the empirical observations, already mentioned in the Introduction, that that couples choose indeed joint-life annuities with differing survivor benefit options and that a certain fraction of married annuitants indeed choose individual-life annuities. Further, in a recent empirical study, Finkelstein and Poterba (2004) tested for selection effects in the U.K. annuity market, and they found systematic relationships between ex-post mortality and certain characteristics of contracts for individual-life annuities. Estimating a hazard model regarding the annuitants' life-spans, they found clear evidence that annuity contracts that provide higher payoffs in later years are selected by the longer-lived individuals; annuity contracts that specify payments to the annuitant's estate in the event of an early death are selected by shorter-lived individuals. These findings makes it plausible to assume that further annuitants' self-selection
may exist, namely those suggested by this paper: Couples with short-lived partners should choose joint-life annuities that offer a higher survivor benefit (relative to the payout in case that both partners survive) than couples with at least one longer-lived partner. Couples with partners having an even higher life-expectancy should choose individual-life annuities. To test for these selection effects appears to be an interesting task for future empirical research.

## Appendix

## Proof of Lemma 2:

Implicit differentiation of the first-order condition (4) with respect to $\pi_{\mathrm{i}}$ yields

$$
\begin{equation*}
\frac{\partial \mathrm{B}_{\mathrm{i}}}{\partial \pi_{\mathrm{i}}}=-\frac{\mathrm{q} \alpha \mathrm{u}^{\prime}\left(\mathrm{qB}_{\mathrm{i}}\right)}{\mathrm{u}^{\prime \prime}\left(\omega-\mathrm{B}_{\mathrm{i}}\right)+\mathrm{q}^{2} \pi_{i} \alpha \mathrm{u}^{\prime}\left(\mathrm{qB}_{\mathrm{i}}\right)} \tag{A1}
\end{equation*}
$$

which is positive. Hence $B_{H}(q)>B_{L}(q)$.

We denote the LHS of (12') by $\mathrm{V}^{i}$ and the LHS of (13') by $\mathrm{V}^{j}$. Implicit differentiation gives

$$
\left(\begin{array}{ll}
\frac{\partial B_{i j}^{i}}{\partial \pi_{i}} & \frac{\partial B_{i j}^{i}}{\partial \pi_{j}}  \tag{A2}\\
\frac{\partial B_{i j}^{j}}{\partial \pi_{i}} & \frac{\partial B_{i j}^{j}}{\partial \pi_{j}}
\end{array}\right)=-\left(\begin{array}{ll}
\frac{\partial \mathbf{V}^{i}}{\partial B_{i j}^{i}} & \frac{\partial \mathbf{V}^{i}}{\partial B_{i j}^{j}} \\
\frac{\partial \mathbf{V}^{j}}{\partial B_{i j}^{i}} & \frac{\partial \mathbf{V}^{j}}{\partial B_{i j}^{i}}
\end{array}\right)^{-1}\left(\begin{array}{ll}
\frac{\partial \mathbf{V}^{i}}{\partial \pi_{i}} & \frac{\partial \mathbf{V}^{i}}{\partial \pi_{\mathrm{j}}} \\
\frac{\partial \mathbf{V}^{j}}{\partial \pi_{i}} & \frac{\partial \mathbf{V}^{j}}{\partial \pi_{\mathrm{j}}}
\end{array}\right),
$$

where

$$
\begin{array}{ll}
\frac{\partial V^{i}}{\partial B_{i j}^{j}}=\frac{\sigma^{2}}{2}\left(u^{\prime \prime}\left(\frac{\sigma}{2}\left(2 \omega-B_{i j}^{i}-B_{i j}^{j}\right)\right)+q^{2} \pi_{i} \pi_{j} \alpha u^{\prime \prime}\left(\frac{\sigma q}{2}\left(B_{i j}^{i}+B_{i j}^{j}\right)\right)\right)<0, & \frac{\partial V^{j}}{\partial B_{i j}^{i}}=\frac{\partial V^{i}}{\partial B_{i j}^{j}}, \\
\frac{\partial V^{i}}{\partial B_{i j}^{i}}=\frac{\partial V^{i}}{\partial B_{i j}^{j}}+q^{2} \pi_{i}\left(1-\pi_{j}\right) \alpha u^{\prime \prime}\left(q B_{i j}^{i}\right)<0, & \frac{\partial V^{j}}{\partial B_{i j}^{j}}=\frac{\partial V^{j}}{\partial B_{i j}^{i}}+q^{2} \pi_{j}\left(1-\pi_{i}\right) \alpha u^{\prime \prime}\left(q B_{i j}^{j}\right)<0, \\
\frac{\partial V^{i}}{\partial \pi_{i}}=q \alpha\left(\sigma \pi_{j} u^{\prime}\left(\frac{\sigma q}{2}\left(B_{i j}^{i}+B_{i j}^{j}\right)\right)+\left(1-\pi_{j}\right) u^{\prime}\left(q B_{i j}^{i}\right)\right)>0, & \frac{\partial V^{j}}{\partial \pi_{i}}=q \pi_{j} \alpha\left(u^{\prime}\left(\frac{\sigma q}{2}\left(B_{i j}^{i}+B_{i j}^{j}\right)\right)-u^{\prime}\left(q B_{i j}^{j}\right)\right), \\
\frac{\partial V^{j}}{\partial \pi_{j}}=q \alpha\left(\sigma \pi_{i} u^{\prime}\left(\frac{\sigma q}{2}\left(B_{i j}^{i}+B_{i j}^{j}\right)\right)+\left(1-\pi_{i}\right) u^{\prime}\left(q B_{i j}^{j}\right)\right)>0, & \frac{\partial V^{i}}{\partial \pi_{j}}=q \pi_{i} \alpha\left(u^{\prime}\left(\frac{\sigma q}{2}\left(B_{i j}^{i}+B_{i j}^{j}\right)\right)-u^{\prime}\left(q B_{i j}^{i}\right)\right) .
\end{array}
$$

Let

$$
\begin{array}{ll}
x^{i} \equiv \frac{\partial V^{i}}{\partial \pi_{i}}-\frac{\partial V^{j}}{\partial \pi_{i}}=q \alpha\left(\left(1-\pi_{j}\right) u^{\prime}\left(q B_{i j}^{i}\right)+\pi_{j} u^{\prime}\left(q B_{i j}^{j}\right)\right)>0, & z^{i} \equiv q^{2} \pi_{i}\left(1-\pi_{j}\right) \alpha u^{\prime \prime}\left(q B_{i j}^{i}\right)<0, \\
x^{j} \equiv \frac{\partial V^{j}}{\partial \pi_{j}}-\frac{\partial V^{i}}{\partial \pi_{j}}=q \alpha\left(\left(1-\pi_{i}\right) u^{\prime}\left(q B_{i j}^{j}\right)+\pi_{i} u^{\prime}\left(q B_{i j}^{i}\right)\right)>0, & z^{j} \equiv q^{2} \pi_{j}\left(1-\pi_{i}\right) \alpha u^{\prime \prime}\left(q B_{i j}^{j}\right)<0,
\end{array}
$$

$$
M \equiv \frac{\partial V^{i}}{\partial B_{i j}^{i}} \frac{\partial V^{j}}{\partial B_{i j}^{j}}-\frac{\partial V^{i}}{\partial B_{i j}^{j}} \frac{\partial V^{j}}{\partial B_{i j}^{i}}=\left(z^{i}+z^{j}\right) \frac{\partial V^{i}}{\partial B_{i j}^{j}}+z^{i} z^{j}>0 .
$$

Inverting the first matrix on the RHS of (A2) and multiplying yields:

$$
\begin{align*}
& \frac{\partial B_{i j}^{i}}{\partial \pi_{i}}=-\frac{1}{M}\left(\frac{\partial V^{j}}{\partial B_{i j}^{j}} \frac{\partial V^{i}}{\partial \pi_{i}}-\frac{\partial V^{i}}{\partial B_{i j}^{i}} \frac{\partial V^{j}}{\partial \pi_{i}}\right)=-\frac{1}{M}\left(\frac{\partial V^{i}}{\partial B_{i j}^{j}} x_{i}+z_{j} \frac{\partial V^{i}}{\partial \pi_{i}}\right),  \tag{A3}\\
& \frac{\partial B_{i j}^{i}}{\partial \pi_{j}}=-\frac{1}{M}\left(\frac{\partial V^{j}}{\partial B_{i j}^{j}} \frac{\partial V^{i}}{\partial \pi_{j}}-\frac{\partial V^{i}}{\partial B_{i j}^{j}} \frac{\partial V^{j}}{\partial \pi_{i}}\right)=-\frac{1}{M}\left(-\frac{\partial V^{j}}{\partial B_{i j}^{i}} x_{j}+z_{i} \frac{\partial V^{j}}{\partial \pi_{i}}\right), \tag{A4}
\end{align*}
$$

From the above inequalities and the RHS of (A3), (A4) resp., it follows that $\partial \mathrm{B}_{\mathrm{ij}}^{\mathrm{i}} / \partial \pi_{\mathrm{i}}>0$, while the sign of $\partial \mathrm{B}_{\mathrm{ij}}^{\mathrm{i}} / \partial \pi_{\mathrm{j}}$ is ambiguous: $\partial \mathrm{B}_{\mathrm{ij}}^{\mathrm{i}} / \partial \pi_{\mathrm{j}}<0$, if $-\mathrm{x}_{\mathrm{j}} \partial \mathrm{V}^{j} / \partial \mathrm{B}_{\mathrm{ij}}^{\mathrm{i}}>-\mathrm{z}_{\mathrm{i}} \partial \mathrm{V}^{\mathrm{j}} / \partial \pi_{\mathrm{i}}$ (which is equivalent to condition (17) in the text), where the LHS of this inequality is positive, while the RHS is indeterminate. Due to symmetry the same considerations apply to $\partial \mathrm{B}_{\mathrm{ij}}^{\mathrm{j}} / \partial \pi_{\mathrm{j}}$ and $\partial \mathrm{B}_{\mathrm{ij}}^{\mathrm{j}} / \partial \pi_{\mathrm{i}}$. Hence, $\mathrm{B}_{\mathrm{LH}}^{\mathrm{L}}(\mathrm{q})<\mathrm{B}_{\mathrm{LL}}^{\mathrm{L}}(\mathrm{q})$ and $\mathrm{B}_{\mathrm{HH}}^{\mathrm{H}}(\mathrm{q})<\mathrm{B}_{\mathrm{LH}}^{\mathrm{H}}(\mathrm{q})$, if condition (17) holds.

We use (A3) and (A4) to determine

$$
\begin{equation*}
\frac{\partial \mathrm{B}_{\mathrm{ij}}^{\mathrm{i}}}{\partial \pi_{\mathrm{i}}}+\left.\frac{\partial \mathrm{B}_{\mathrm{ij}}^{\mathrm{i}}}{\partial \pi_{\mathrm{j}}}\right|_{\pi_{\mathrm{i}}=\pi_{\mathrm{j}}}=-\frac{1}{\mathrm{M}} \mathrm{z}_{\mathrm{j}}\left(\frac{\partial \mathrm{~V}^{\mathrm{i}}}{\partial \pi_{\mathrm{i}}}-\frac{\partial \mathrm{V}^{\mathrm{j}}}{\partial \pi_{\mathrm{i}}}\right)=-\frac{1}{\mathrm{M}} \mathrm{z}_{\mathrm{j}} q \alpha \mathrm{u}^{\prime}\left(\mathrm{qB}_{\mathrm{ij}}^{\mathrm{j}}\right)>0, \tag{A5}
\end{equation*}
$$

where the positive sign follows from the inequalities from above; hence, $B_{H H}^{H}(q)>B_{L L}^{L}(q) . Q . E . D$.

## Proof of Lemma 4:

We maximise expected utility (5) with respect to $r$ and $q$ subject to (28). By use of (23) - (25), the first-order conditions of this maximisation problem are

$$
\begin{align*}
& \mathrm{A}_{\mathrm{ij}} \alpha \pi_{\mathrm{i}} \pi_{\mathrm{j}} \sigma\left(\mathrm{u}^{\prime}\left(\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{s}}\right)+\mathrm{u}^{\prime}\left(\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{s}}\right)\right)+\lambda \pi_{\mathrm{i}} \pi_{\mathrm{j}}=0  \tag{A6}\\
& \mathrm{~A}_{\mathrm{ij}} \alpha\left(\pi_{\mathrm{i}}\left(1-\pi_{\mathrm{j}}\right) \mathrm{u}^{\prime}\left(\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{w}}\right)+\pi_{\mathrm{j}}\left(1-\pi_{\mathrm{i}}\right) \mathrm{u}^{\prime}\left(\mathrm{c}_{1 \mathrm{ij}}^{\mathrm{w}}\right)\right)-\lambda\left(\pi_{\mathrm{i}}\left(1-\pi_{\mathrm{j}}\right)+\pi_{\mathrm{j}}\left(1-\pi_{\mathrm{i}}\right)\right)=0, \tag{A7}
\end{align*}
$$

where $\lambda$ is the Lagrange multiplier associated with constraint (28). From (A6) and (A7) together with (24), (25) and (11) we find that maximisation requires (29). From (29) one concludes that for any arbitrarily given $A_{i j}, r_{i j} / k_{i j}=2$ for $\sigma=1$. The same applies for logarithmic utility (14), as (29) reduces to $\mathrm{r} /\left(2 \mathrm{~A}_{\mathrm{ij}}\right)=\mathrm{k} / \mathrm{A}_{\mathrm{ij}}$ by use of (14).
Q.E.D.

## Proof of Lemma 6:

Consider any pooling contract ( $\mathrm{r}, \mathrm{k}$ ), which fulfils the zero-profit condition (30) together with $r / k \geq 2-2 /\left(\pi_{\mathrm{H}}+\pi_{\mathrm{L}}\right)$. Due to Lemma 5 , we know that for any such contract $(r, k), p_{\mathrm{LL}}<p_{\mathrm{LH}}$ and $\mathrm{p}_{\mathrm{LL}}<\mathrm{p}_{\mathrm{HH}}$. It follows that $\mathrm{p}_{\mathrm{LL}}$ is negative and $\left(\mathrm{p}_{\mathrm{LH}}+\mathrm{p}_{\mathrm{HH}}\right)$ is positive, otherwise the LHS of (30)
would be non-zero. This implies that given that only type-LL couples buy the contract $(r, k)$, it would make positive profits. By continuity, this holds for any contract ( $r+\delta r, k+\delta k$ ) in the neighbourhood of (r,k). 9

Equivalently, for any pooling contract, which fulfils (30) together with $r / k<2-2 /\left(\pi_{\mathrm{H}}+\pi_{\mathrm{L}}\right)$, it follows from Lemma 5 that $\mathrm{p}_{\mathrm{HH}}<\mathrm{p}_{\mathrm{LH}}$ and $\mathrm{p}_{\mathrm{HH}}<\mathrm{p}_{\mathrm{LL}}$. Therefore, in order that the LHS of (30) is equal to zero, $\mathrm{p}_{\mathrm{HH}}<0$ and $\left(\mathrm{p}_{\mathrm{LH}}+\mathrm{p}_{\mathrm{LL}}\right)>0$. This implies that annuity companies make a positive profit, given that only couples of type HH buy this pooling contract $(r, k)$ or one close to it. Q.E.D.

## Proof of Lemma 7

(34a) and (35b) are fulfilled by definition. That (35a) is fulfilled, follows from $\hat{r}_{\mathrm{LH}} / \hat{\mathrm{k}}_{\mathrm{LH}}<\hat{\mathrm{r}}_{\mathrm{HH}} / \hat{\mathrm{k}}_{\mathrm{HH}}$, from the fact that ( $\hat{r}_{H H}, \hat{k}_{H H}$ ) and ( $\hat{r}_{L H}, \hat{k}_{L H}$ ) are on the same indifference curve $V_{H H}$ for couple $H H$ and that the slope of the indifference curve going through ( $\hat{r}_{L H}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) is flatter for couple LH than for couple HH. Equivalently, (36a) holds, because $\hat{\mathrm{r}}_{\mathrm{LL}} / \hat{\mathrm{k}}_{\mathrm{LL}}<\hat{\mathrm{r}}_{\mathrm{LH}} / \hat{\mathrm{k}}_{\mathrm{LH}}$, ( $\hat{\mathrm{r}}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) and ( $\hat{\mathrm{rLL}}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ) are on the same indifference curve $\mathrm{V}_{\mathrm{LH}}$ for couple LH and that the slope of the indifference curve going through ( $\hat{r}_{L L}, \hat{k}_{\mathrm{LL}}$ ) is flatter for couple LL than for couple LH . The conjunction of the arguments, given to show that (35a) hold, with those, given to show that (36a), implies that also (34b) and (36b) hold.
Q.E.D.

## Proof of Lemma 8

1) Consider the indifference curve $V_{H H}$ of couple $H H$. By definition, at ( $\hat{r}_{H H}, \hat{k}_{H H}$ ) $V_{H H}$ is tangent to zero-profit line $Z P_{H H}$, while due to strict convexity of $\mathrm{V}_{\mathrm{HH}}$, at any payoff-ratio $\mathrm{r} / \mathrm{k}<2$, the slope of $\mathrm{V}_{\mathrm{HH}}$ is steeper than that of $\mathrm{ZP}_{\mathrm{H}}$. From these observations it follows that
(a) $\left(\hat{r}_{L H}, \hat{k}_{L H}\right)$ as the point of intersection of $V_{H H}$ and $Z P_{L H}$, where $\hat{r}_{L H}<2 \hat{k}_{L H}$, lies below the straight line S 1 , which goes through the point of intersection of $Z P_{H H}$ and $Z P_{\text {LH }}$. It follows that any contract on $\mathrm{ZP}_{\mathrm{LH}}$, which lies above $\mathrm{V}_{\mathrm{HH}}$, is characterised by a payoff-ratio, where $\mathrm{ZP}_{\mathrm{HH}}$ lies below $\mathrm{ZP}_{\mathrm{LH}}$. By this, any such contract would be chosen by couple HH and hence produce a loss.
(b) the point of intersection of $\mathrm{V}_{\mathrm{HH}}$ and $\mathrm{ZP}_{\mathrm{LL}}$, where $\mathrm{r}<2 \mathrm{k}$, denoted as D in Figure 3, lies below the straight line S 2 , which goes through the point of intersection of $\mathrm{ZP}_{\mathrm{HH}}$ and $Z P_{\text {LL }}$. It follows that any contract on $\mathrm{ZP}_{\mathrm{LL}}$, which lies above $\mathrm{V}_{\mathrm{HH}}$, is characterised by a payoff-ratio, where $\mathrm{ZP}_{\mathrm{HH}}$ lies below $\mathrm{ZP}_{\mathrm{LL}}$. By this, any such contract would be chosen by couple HH and hence produce a loss.
2) Given a value $\pi_{\mathrm{L}} \leq 0.5, \mathrm{ZP}_{\mathrm{LL}}$ lies above $\mathrm{ZP}_{\mathrm{LH}}$ for any payoff-ratio $\mathrm{r} / \mathrm{k}$ (compare Corollary 2 ). Hence, to prove (ii) of Lemma 8, it suffice to show that given a value $\pi_{\mathrm{L}}>0.5$, as drawn in Figure 1, ( $\hat{r}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ) is characterised by a payoff-ratio below the straight line S 3 , at which
$Z P_{\text {LH }}$ and $\mathrm{ZP}_{\mathrm{LL}}$ intersect. Consider the indifference $\mathrm{V}_{\mathrm{LH}}$ of couple LH , which crosses $\mathrm{ZP}_{\mathrm{LH}}$ at $\left(\hat{r}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}\right)$ and $\mathrm{ZP}_{\mathrm{LL}}$ at $\left(\hat{r}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}\right)$. We know from 1a) of this Proof that ( $\hat{r}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) lies below S 1 and hence also below S3, which is immediate from Corollary 2. From this, together with the fact that $\mathrm{V}_{\mathrm{LH}}$ is strictly convex, that at ( $\hat{\mathrm{r}}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) the slope of $\mathrm{V}_{\mathrm{LH}}$ is steeper than that of $\mathrm{ZP} \mathrm{P}_{\mathrm{LH}}$, and that $\hat{r}_{\mathrm{LL}} / \hat{\mathrm{k}}_{\mathrm{LL}}<\hat{\mathrm{r}}_{\mathrm{LH}} / \hat{\mathrm{k}}_{\mathrm{LH}}$, it follows that ( $\hat{\mathrm{L}}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ) also lies below S3. Altogether, any contract on $\mathrm{ZP}_{\mathrm{LL}}$ and above $\mathrm{V}_{\mathrm{LL}}$ is characterised by a payoff-ratio, where $\mathrm{ZP}_{\mathrm{LL}}$ lies above $\mathrm{ZP}_{\text {LH }}$. By this, any such contract would be chosen by couple LH and hence produce a loss.

Finally note that due to the single crossing-property of the indifference curves $\mathrm{V}_{\mathrm{ij}}$,

- any contract on $Z P_{L H}$, which is preferred to ( $\hat{\mathrm{r}}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) by couple LH , is also chosen by couple HH (but not by couple LL), and then - due to 1a) of this Proof - produces a loss.
- any contract on $Z P_{\mathrm{LL}}$, which is preferred to ( $\hat{r}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ) by couple LL , is chosen also by couple LH or by couple LH and HH, and then - due to 2 ) and 1 b ) of this Proof - produces a loss.
Q.E.D.


## Proof of Proposition 3

Note first that if ( $\hat{r}_{H H}, \hat{\mathrm{k}}_{\mathrm{HH}}$ ) is part of the separating equilibrium, then ( $\hat{\mathrm{r}}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) and ( $\hat{\mathrm{r}}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ) must be the other parts: ( $\hat{r}_{\text {LH }}, \hat{k}_{\text {LH }}$ ) provides maximum utility for couple LH, subject to the self-selection constraint (34a) for couple HH and the zero-profit condition (28) for $\mathrm{ij}=\mathrm{LH}$. With ( $\hat{r}_{\mathrm{HH}}, \hat{\mathrm{k}}_{\mathrm{HH}}$ ) and $\left(\hat{r}_{L H}, \hat{k}_{L H}\right),(34 a)$ is fulfilled with equality. On the one hand, note from Figure 3 that the second point of intersection of $\mathrm{V}_{H H}$ and $\mathrm{ZP}_{\text {LH }}$, as well as all contracts on $\mathrm{ZP}_{\text {LH }}$ below this point of intersection and also those above ( $\hat{r}_{L H}, \hat{k}_{L H}$ ) fulfil (34a). However, they all provide lower utility for couple LH, remember the single-crossing property. On the other hand, as explored in the text and shown in Lemma 8, all contracts on $\mathrm{ZP}_{\mathrm{LH}}$, which provide a higher utility than ( $\hat{r}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) for couple LH, would be also chosen by couple HH and then produce a loss. Equivalently, ( $\hat{r}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ) provides maximum utility for couple LL, subject to the self-selection constraint (35b) for couple LH and the zero-profit condition (28) for $\mathrm{ij}=\mathrm{LL}$. With ( $\hat{\mathrm{L}}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) and ( $\hat{\mathrm{r}}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ), (35b) is fulfilled with equality. Again we have: All contracts on $\mathrm{ZP}_{\mathrm{LL}}$ above ( $\hat{r}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ) and below the second the second point of intersection of $\mathrm{V}_{\mathrm{LH}}$ with $\mathrm{ZP}_{\mathrm{LL}}$ as well as this point of intersection fulfil (35b), but provide lower utility for couple LL. All other contracts on $\mathrm{ZP}_{\mathrm{LL}}$, which lie above $\mathrm{V}_{\mathrm{LL}}$, can be excluded as part of the equilibrium due to Lemma 8.

Further, one observes that, for the same reason, if any other contract ( $r_{H H}^{\prime}, \mathrm{k}_{\mathrm{HH}}^{\prime}$ ) on $\mathrm{ZP}_{\mathrm{HH}}$ is part of the separating equilibrium, then the second part ( $\mathrm{r}_{\mathrm{LH}}^{\prime}, \mathrm{k}_{\mathrm{LH}}^{\prime}$ ) must be found as the point of intersection of $Z P_{L H}$ with the indifference curve of couple HH trough ( $\mathrm{r}_{\mathrm{HH}}^{\prime}, \mathrm{k}_{\mathrm{HH}}^{\prime}$ ), where $r_{L H}^{\prime}<2 \mathrm{k}_{\mathrm{LH}}^{\prime}$, and the third part ( $\mathrm{r}_{\mathrm{LL}}^{\prime}, \mathrm{k}_{\mathrm{LL}}^{\prime}$ ) must be found as the point of intersection of $Z P_{\mathrm{LL}}$ with the indifference curve of couple LH trough ( $\mathrm{r}_{\mathrm{LH}}^{\prime}, \mathrm{k}_{\mathrm{LH}}^{\prime}$ ), where $\mathrm{r}_{\mathrm{LL}}^{\prime}<2 \mathrm{k}_{\mathrm{LL}}^{\prime}$. Obviously, couples of
type HH prefer ( $\hat{\mathrm{r}}_{\mathrm{HH}}, \hat{\mathrm{k}}_{\mathrm{HH}}$ ) to any other ( $\mathrm{r}_{\mathrm{HH}}^{\prime}, \mathrm{k}_{\mathrm{HH}}^{\prime}$ ), couples of type LH prefer ( $\hat{\mathrm{LH}}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) to any other ( $r_{L H}^{\prime}, \mathrm{k}_{\mathrm{LH}}^{\prime}$ ) and couples of type LL prefer ( $\hat{\mathrm{r}}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ) to any other ( $\mathrm{r}_{\mathrm{LL}}^{\prime}, \mathrm{k}_{\mathrm{LL}}^{\prime}$ ).
Q.E.D.

## Proof of Proposition 4

A variation of the share $\gamma$ of the high-risk types only influences the zero-profit condition (30) for a pooling contract with positive annuity demand $A_{i j}>0$ for at least two types $i j \in\{L L, L H, H H\}$, while leaving the zero-profit condition (28) for a separate contract (and the indifference curves, of course) unchanged. Comparison of (30) and (28) makes the following obvious: If $\gamma$ approaches to one, the zero-profit condition (30) for any pooling contract with $A_{H H}>0, A_{L H}>0$ and $A_{L L} \geq 0$ converges to the zero-profit condition (28) for couple HH and the zero-profit condition (30) for a pooling contract with $A_{H H}=0, A_{L H}>0$ and $A_{L L}>0$ converges to the zero-profit condition (28) for couple LH.

Further note from Figure 4 and Lemma 7 the following: Couple LH prefers ( $\hat{\mathrm{L}}_{\mathrm{LH}}, \hat{\mathrm{k}}_{\mathrm{LH}}$ ) to any contract on $Z P_{H H}$ where $r / k>\hat{r}_{\mathrm{LH}} / \hat{\mathrm{k}}_{\mathrm{LH}}$ and that couple LL prefers ( $\hat{r}_{\mathrm{LL}}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ) to any contract on $Z P_{L H}$, where $r / k>\hat{r}_{L L} / \hat{k}_{L L}$. Hence, any contract $(r, k)$, which is preferred by couple $L H$ to $\left(\hat{r}_{L H}, \hat{k}_{L H}\right)$, lies above $Z P_{H H}$, if $r / k>\hat{r}_{L H} / \hat{k}_{L H}$, which is then also preferred by couple $H H$, and may or may not be preferred by couple LL. Equivalently, any contract ( $r, k$ ), which is preferred by couple $L L$ to ( $\hat{r}_{L L}, \hat{k}_{L L}$ ), lies above $Z P_{L H}$, if $r / k>\hat{r}_{L L} / \hat{k}_{L L}$, which is then also preferred by couple LH, and may or may not be preferred by couple HH.

From the above considerations it follows that for or $\gamma \rightarrow 1$
(i) any contract which is preferred by couples of type LH and HH to their separate contract ( $\hat{r}_{L H}, \hat{k}_{L H}$ ) and ( $\hat{r}_{H H}, \hat{k}_{H H}$ ), resp., but is not preferred by couples of type LL to ( $\hat{r}_{L L}, \hat{k}_{L L}$ ), must lie above $\mathrm{ZP}_{\mathrm{HH}}$ and, by continuity, also above the zero-profit condition $Z P_{\mathrm{P} 2}$ for a pooling contract with $A_{H H}>0, A_{L H}>0, A_{L L}=0$, as long as $\gamma$ is sufficiently close to one. Thus any such pooling contract makes negative profits.
(ii) any contract which is preferred by couples LL and LH to their separate contract ( $\hat{r}_{L L}, \hat{\mathrm{k}}_{\mathrm{LL}}$ ) and ( $\hat{r}_{L H}, \hat{k}_{L H}$ ), resp., but not by couples HH to ( $\hat{r}_{H H}, \hat{\mathrm{k}}_{\mathrm{HH}}$ ), must lie above $\mathrm{ZP}_{\mathrm{LH}}$ and by continuity also above the zero-profit condition $\mathrm{ZP}_{\mathrm{P}_{3}}$ for a pooling contract with $\mathrm{A}_{H H}=0, \mathrm{~A}_{L H}>$ $0, A_{L L}>0$, as long as $\gamma$ is sufficiently close to one. Thus any such pooling contract makes negative profits.
(iii) any contract which is preferred by couples of all three types $\mathrm{ij}=\mathrm{LL}, \mathrm{LH}, \mathrm{HH}$ to their separate contract $\left(\hat{r}_{L L}, \hat{k}_{L L}\right),\left(\hat{r}_{L H}, \hat{\mathrm{k}}_{\mathrm{LH}}\right)$ and $\left(\hat{r}_{H H}, \hat{\mathrm{k}}_{\mathrm{HH}}\right)$, resp., must lie above $\mathrm{ZP} \mathrm{P}_{\mathrm{LH}}$ and by continuity also above the zero-profit condition $Z P_{P 1}$ for a pooling contract with $A_{H H}>0, A_{L H}>0, A_{L L}>0$, as long as $\gamma$ is sufficiently close to one. Thus any such pooling contract makes negative profits.

Finally we note that for each case (i) - (iii) there exists an infimum of all $\gamma$, for which a pooling contract which is preferred by the respective couples ij , makes a negative profit. We define $\gamma^{*}$ as the largest value of all three infima. It follows that for any $\gamma \geq \gamma^{*}$ a separating equilibrium exists. On the other hand, analogous considerations show that for sufficiently small $\gamma$, a profitable pooling contract, which is preferred by couples of at least two types $\mathrm{ij} \in\{\mathrm{LL}, \mathrm{LH}, \mathrm{HH}\}$, always exists.
Q.E.D.

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[^0]:    1 Another contribution that deals with couples as the decision-making units is that of Hurd (1999). It investigates a life-cycle model of consumption by couples to determine their optimal consumption path over the uncertain life-span of both partners. However, this analysis excludes the possibility of the purchase of life annuities in the private annuity market, but considers public pensions, which are assumed to be given exogenously.

[^1]:    2 Empirical studies for the well developed US annuity market give evidence that prices are about 7 $15 \%$ above the fair price due to adverse selection (Walliser, 2000; Mitchell et al., 1999; Friedman and Warshawsky, 1988, 1990). Finkelstein and Poterba (2002) find that adverse selection exists to some similar extent in the voluntary annuity market of the United Kingdom.

[^2]:    3 The Nash-Cournot equilibrium in insurance markets was studied first by Rothschild and Stiglitz (1976) and Wilson (1977). In their framework firms offer a number of different contracts which specify both a price and a quantity. Individuals who prefer a higher quantity are willing to pay a higher price for it. A prerequisite for the existence of price and quantity competition is that individuals can buy at most one contract, which may be a reasonable assumption for some insurance markets, e.g. insurance against accidents, but seems difficult to apply to the annuity market. Consequently, in our model individuals are free to buy as many annuities as they want. Separation becomes possible because firms can fix two prices instead of a price and a quantity.

[^3]:    4 The general intuition for this result, which goes back to Yaari (1965), is the following: In case that an individual provides for old-age consumption through bonds, he/she leaves unintended bequests if dying prematurely. In this case, the deceased's wealth is distributed to the heirs. If, in contrast, the individual puts her wealth into life annuities and dies prematurely, this unconsumed wealth is distributed as annuity payouts to the surviving annuitants. These considerations apply equivalently for couples, when both partners decease prematurely. The result of higher returns on annuities holds as long as an annuity company does not have the market power to collect all of the consumer surplus generated in the annuity market.

[^4]:    5 Note that we assume that the survival probabilities $\pi_{\mathrm{i}}$ and $\pi_{\mathrm{j}}$ are constant over the different risk-states. By this we exclude the so-called "broken-heart" effect, which means that the survival probability would be lower in case of the partner's death than in case of his/her survival to retirement.

[^5]:    6 Alternatively, one could introduce a condition which ensures that it is attractive for couples to buy solely annuities.

[^6]:    7 It can be shown (by implicit differentiation of (12') and (13')) that $\partial \mathrm{B}_{\mathrm{ij}}^{\mathrm{i}}$ (q) $/ \partial \sigma \gtreqless 0$, if $\mathrm{R} \gtreqless 1$, where $R \equiv-c_{t i j}^{i}{ }^{\prime \prime}\left(c_{t i j}^{i}\right) / u^{\prime}\left(c_{t i j}^{i}\right)$ denotes the coefficient of relative risk aversion. As logarithmic utility exhibits a constant $R=1, \sigma$ has no influence on annuity demand.

[^7]:    8 Price competition appears to be a more plausible assumption for annuity markets than price and quantity competition, which requires that individuals can buy only one insurance contract (as firms fix both both a price and a quantity), but generates the possibility of a separating equilibrium (see Rothschild and Stiglitz, 1976; Wilson, 1977).

[^8]:    9 One could presume that marriage has a positive (negative) influence on the individuals' life expectancy or that individuals who expect a long life have a higher preference for being married.
    ${ }^{10}$ One could think that an individual prefers a partner of the same risk type.

[^9]:    ${ }^{11}$ As Abel (1986) has argued, for any rate of return equal or lower than $1 / \pi_{\mathrm{H}}$, an annuity company can slightly increase the rate of return and profitably attract both risk-types; for any rate of return equal or greater than $1 / \pi\llcorner$ annuity companies would suffer losses on the annuities sold to both risk-types; see also Pech (2004).
    12 Note that this assumption implies that there is a unique equilibrium. In general, however, multiple equilibria cannot be excluded; see e.g. Abel (1986), Pech (2004).

[^10]:    ${ }^{13}$ Among TIAA-CREF annuitants who choose a joint-life annuity about $83 \%$ of women and $93 \%$ of men choose this option (Ameriks, 2002).
    ${ }^{14}$ The intuition is equivalent to that given in footnote 4 for individual life annuities.

[^11]:    ${ }^{15}$ For a sufficiently low relative payoff $r / k$, the couple substitutes annuities partly by bonds to increase consumption for the case that both of them survive. For a sufficiently high relative payoff $r / k$, they will do so to increase consumption for the case that only one partner survives.

[^12]:    16 Note, moreover, that relation (29) which characterises the optimum division of consumption between the different risk-states in retirement, also determines the optimal division of consumption between the working and the retirement period, namely $u^{\prime}\left(\frac{\sigma}{2}\left(2 \omega-A_{i j}\right)\right)=\alpha u^{\prime}\left(\frac{\sigma r}{2} A_{i j}\right)$, which can be seen by elimination of $\mathrm{u}^{\prime}\left(\mathrm{kA}_{\mathrm{ij}}\right)$ in (26') by use of (29) and (28). It follows that a couple of type ij , who does not discount future consumption due to time preference ( $\alpha=1$ ), consumes the same amounts in the working period and in the retirement period, when both partners survive. Otherwise $(\alpha<1)$, the couple chooses a greater consumption level in the working period, i.e. $c_{0 i j}^{i s}>c_{1 i j}^{i s}$.

[^13]:    17 As the formal arguments to prove Proposition 2 are similar to those given by Brunner and Pech (2005), although in the context of individual-life annuities, which payoffs may vary over two periods of retirement, we refer to Brunner and Pech (2005).

[^14]:    18 Obviously, all arguments remain valid, when considering a zero-profit pooling contract which is chosen only by couples of type LL and HH (but not by couple LH). Analogous arguments apply for any zeroprofit pooling contract which is chosen by two types of couples only (i.e. either by couples LH and LL or couples LH and HH ): For any payoff-ratio one can find another contract close to the original pooling contract, which is preferred by one type of couples only and then produces a nonnegative profit (which follows from Lemma 5 and the single-crossing property (33)). Therefore, as we note already at this stage of analysis, there cannot exist an equilibrium which consists of one pooling contract, chosen by two types of couples, and one separate contract for the remaining type of couple. This will be discussed in detail in Section 3.3.2.

[^15]:    19 Note that this assumption is usually made in this type of asymmetric-information models, see e.g. Brunner and Pech (2005), Townley and Boadway (1988). Further, it is related to the analogous assumption needed to model price and quanitity competition, as applied by Rothschild and Stiglitz (1976) and Eckstein, Eichenbaum and Peled (1985) in the context of life annuities. There, individuals are restricted to buy only one insurance contract, which specifies a quantity and a price. Thus the assumption of excluding couples from buying a mix of contract types is less demanding; in reality, however, it requires a system of information exchange among annuity companies.

[^16]:    ${ }^{20}$ Note that in case of logarithmic per-period utility the zero-profit conditions for a pooling contract is indeed a straight line, as annuity demand $\mathrm{A}_{\mathrm{ij}}$ does not depend on $(\mathrm{r}, \mathrm{k})$.

[^17]:    21 We ran numerical simulations which show that for sufficiently low $\gamma$, at which a pooling contract G allows non-negative profits, also the pooling contracts E and F allow non-negative profits. On the other hand, for higher shares $\gamma$, only a pooling contract $\mathcal{F}$ produces non-negative profits, while the contracts G and E do not.

[^18]:    22 Note that in case of logarithmic utility (14) the profit function (18) is strictly decreasing in q, which implies that there exists a unique root of the zero-profit condition (19).

[^19]:    23 Obviously, the indifference curve for the same-risk couple LL (as for the same-risk couple HH ) which represents all contracts $(r, k)$ at which they are as well off as at the pooling IA-contract $\bar{q}$, must go through the point on $S^{\prime}$ where $r=\bar{q}$.

[^20]:    ${ }^{24}$ Note however that $\overline{\bar{q}} \gtrless \bar{q}$, as in case that couple LH (with one low-risk and one high-risk partner) additionally would buy the pooling IA-contract, the share of individual life annuities bought by low-risk individuals (in aggregate demand for IA) may increase or decrease.
    25 Detailed calculations are provided on request.
    ${ }^{26}$ Remember that, if $\bar{q} \geq r_{\text {LH }}^{b}$, (38a) cannot be fulfilled (as it was argued at the beginning of Section 3.3.3).

[^21]:    27 More formally, ( $\overline{\bar{r}_{\mathrm{LL}}}, \overline{\overline{\mathrm{k}}_{\mathrm{LL}}^{\prime}}$ ) is implicitly defined by the zero-profit condition (28) for couple LL, by the property that a couple HH is indifferent between $\overline{\overline{\mathrm{q}}}$ and ( $\overline{\bar{r}_{\mathrm{LL}}}, \overline{\overline{\mathrm{k}}_{\mathrm{LL}}^{\prime}}$ ) and by $\overline{\overline{\mathrm{r}}_{\mathrm{LL}}}<2 \overline{\overline{\mathrm{k}}_{\mathrm{LL}}^{\prime}}$.

