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JOHANNES KEPLER UNIVERSITY OF LINZ

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by

Johann K. BRUNNER and Susanne PECH

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Johannes Kepler University of Linz
Department of Economics
Altenberger Strasse 69
A-4040 Linz - Auhof, Austria
www.econ.jku.at

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Abstract

The market for private life annuities is characterised by adverse selection, that is, contracts offer lower than fair payoffs to individuals with low life expectancy. Moreover, life expectancy and income have been found to be positively correlated. The paper shows that a linear tax on annuity payoffs, which raises more revenues from long-living individuals than from short-living, represents an appropriate instrument for redistribution, in addition to an optimally designed labour income tax. Further, we find that a nonlinear tax on annuity payoffs can be directly employed to correct the distortion of the rate of return caused by asymmetric information. These results are contrasted with theoretical findings concerning the role of a tax on capital income.

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* *Address:* Department of Economics, University of Linz, Altenberger Straße 69, A-4040 Linz, Austria. *Phone:* +43-732-2468-8248, -8593 *FAX:* -9821. *E-mail:* johann.brunner@jku.at, susanne.pech@jku.at.

I. Introduction

As in many industrialised countries social security systems are under pressure because of an aging population, governments are trying to establish a so-called third pillar of old-age provision. The idea is that individuals should compensate a possible decline of the public pensions by increased private saving, that is, by shifting part of their income in the working period to the period of retirement. Among the various ways of how savings can be invested and how wealth, available for the financing of retirement consumption, can be accumulated, the purchase of private life annuities has the advantage of providing appropriate insurance against a long life. It allows the individual to avoid running out of assets before death as well as leaving unintended bequests. By transferring wealth from those, who die early, to those, who live long, annuities provide a higher rate of return than investments in the capital market (Yaari 1965).

There is a tendency among governments to grant some way of preferential taxation to individuals who purchase private life annuities.[1] The arguments for such a preferential treatment are rarely formulated explicitly; in the public discussion there seems to prevail a merit-good view, saying that individuals are myopic and therefore save too little in their active period of life.[2] In particular, they might not have a sufficient perception of the likely reduction of the replacement rate offered by the public pension.[3] In accordance with this view, several studies have investigated the incentive effect of taxes on the demand for private life annuities and on savings in general (for an overview see Bernheim 2002).

Usually, economists have reservations against arguments based on irrationality of the individuals. If at all, these arguments might serve as a justification for public

[1] For instance, in Austria the state subsidises the premium and, in addition, guarantees tax exemption of the payoffs. A similar regulation was introduced in Germany, the so-called Riester-Rente. In OECD countries the prevailing system seems to be that contributions to a pension fund are tax exempt, up to some limit, and pension payments are taxed, see, e.g., Whitehouse 1999 or OECD 1994.

[2] For the UK, Disney et al. 2001a,b find some evidence, especially for low-wage earners, that the savings rate is too low in order to transfer sufficient income to the period of retirement.

[3] A similar argument rests on the suspicion that individuals might deliberately save too little, because they expect to receive some social assistance anyway.

programs of limited size only, which means in the case of private pensions that preferential taxation for merit-goods reasons should be restricted to a minimum provision to cover basic needs. In principle, taxation of private pensions should follow the same rules that are guiding the design of the tax system as a whole.

This leads one to the question of what is the appropriate tax treatment of annuities with regard to efficiency and equity. In particular, the distributive effect of the way of how annuities are taxed deserves a more thorough analysis than has been provided by the theoretical literature so far.[4] To consider that together with the effect on economic efficiency in a coherent model is the subject of the present contribution.

Obviously, besides providing insurance, the purchase of annuities represents an alternative to an investment on the capital market. It is therefore interesting to refer to results concerning the question of how such an investment should be taxed. On this, it seems fair to say that most theoretical studies have lead to the conclusion that capital income should be left tax-free, at least if some other instrument is applied for (redistributive) taxation in an optimal way and preferences are separable between leisure and consumption in different periods. Such a result can be derived in a static representative-consumer model of the Ramsey-type (see, e.g., Atkinson and Stiglitz 1980), but also rather generally in dynamic models with many households, which differ in their ability to earn income or in their capital endowment.[5]

The important question is then whether this result changes, if the insurance aspect of private pensions is taken into account. To answer this, one has to consider the functioning of the annuity market, in particular the problem of asymmetric information: insurance companies cannot distinguish between individuals with low and high life expectancy. This fact, together with the assumption of price competition among insurance companies,[6] implies that firms are forced to offer the same rate of return to

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[4] Some studies concentrate on simulation results concerning to distributional effects of different tax-treatments of annuities, see, e.g., Brown et al. (1999) and Burman et al. (2004).

[5] See Chamley (1986) and Judd (1985) for infinite-horizon models, and Ordover and Phelps (1979) for OLG-models. On the other hand, in both frameworks there are a few studies which establish the desirability of a tax on capital income, e.g. if credit constraints (Chamley 2001, Aiyagari 1995) or human capital accumulation (Jacobs and Bovenberg 2005) are incorporated.

[6] Price competition follows from the assumption that insurance firms cannot monitor whether costumers hold annuities also from other firms. In contrast, price and quantity competition, which

all customers, irrespective of their expected duration of life. This in turn induces an adverse-selection effect, which was estimated to make private pension more costly, to the extent of 7 - 15 percent.[7]

What we focus on in the present study are the distributive consequences of this phenomenon and whether it provides a specific rationale for taxation. As a first step of the analysis, we compare, by means of a simple example, the effects of two forms of tax exemption, namely either a wage tax, which leaves annuity payoffs tax-free, or a consumption tax, which leaves saving untaxed but taxes dissaving (i.e., annuity payoffs) fully. In particular, we show that, though both forms are equivalent as to consumption and welfare of the individuals, the latter extracts less revenue from the short-living individual than the former.

Next, we turn to a model with fixed labour income and endogenous annuity demand and demonstrate that the introduction of a proportional tax on annuity payoffs indeed redistributes income, which is due to the larger annuity demand of the longer-lived individuals. Hence, given that society wants to treat individuals with high mortality better, it can do so and use this instrument to compensate them for their disadvantage. We offer some ideas on the normative issue, which relative weights should be given to short- and long-living individuals in the social objective.

For a more accurate analysis we have to allow for incentive effects on labour supply, where we take into account the empirical finding that income and life expectancy are positively correlated (see, among others, Attanasio and Hoynes 2000, Lillard and Panis 1998). We introduce this into the simplest possible model of optimum income taxation, consisting of two types of individuals, who live for two periods and differ in their wage rate and in their probability of survival to the second period. In this framework we consider two cases, that of a linear and of a nonlinear tax on annuity

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was studied first by Rothschild and Stiglitz (1976) and Wilson (1977), requires that costumers can buy only one contract. Since this is regarded to be inapplicable for the annuity market, price competition is usually adopted for the analysis of the annuity market, see e.g. Pauly (1974), Abel (1986), Brugiavini (1993), Walliser (2000), Brunner and Pech (2005).

[7] Compared to a situation without adverse selection, i.e., where mortality of costumers is identical to that of the average population. See, e.g. Mitchell et al. (1999), Walliser (2000), Finkelstein and Poterba (2002).

income, resp. For a linear tax on annuity payoffs (in addition to the optimum nonlinear labour-income tax), we can prove that it increases social welfare, given that the weight of the low-wage (and low life expectancy) individual is sufficiently large in the social objective and that demand for annuities does not decrease too much with leisure. Hence, also in case of weak separability between leisure and consumption in both periods, taxation of annuity payoffs is optimal. This finding is in contrast to the results on capital income taxation mentioned above.

Concerning the case of an optimum tax system, which is nonlinear not only with respect to wage income, but also with respect to annuity payouts, a remarkable, new feature turns out important: Such a fully nonlinear tax system has the advantage over the one with linear taxation of annuity payouts that it not only allows redistribution, but also a correction of the market failure arising from the adverse-selection problem. In particular, we find that the annuity payout of the long-living (and high-income) individual is reduced by the marginal tax rate to her individually fair payout. For the short-living individual the analogous effect increases her payout, however, a distortion may occur – familiar from optimum labour income taxation - and impede the (full) realisation of the first-best payout according to her low survival probability. This corrective role is characteristic for nonlinear annuity taxation, it differs markedly from the corresponding results for the optimum nonlinear tax on capital income. These typically show a zero marginal rate for the high-income individual and the same, but possibly distorted, for the low-income individual (see, e.g., Ordover and Phelps 1979, Brett 1998).

The paper proceeds as follows: Section II provides an intuitive example, illustrating that taxation of annuity payoffs indeed extracts more tax revenue from the long-living individuals, compared to an equivalent wage tax. Section III contains the detailed analysis, which first considers the effect of a tax on annuity payoffs vis-à-vis a proportional tax on fixed labour income. In the second part of this section a Mirrlees-type model is formulated in order to study the optimum properties of a linear as well as of a nonlinear tax on annuity payoffs. Section IV provides concluding remarks.

II. Equivalent taxation

As is well-known from straightforward textbook analysis (see, e.g., Stiglitz 2000), a proportional tax on wages (leaving capital income untaxed) is equivalent to a proportional consumption tax (leaving saving untaxed but burdening dissaving). We study some simple examples in order to derive an intuition of how this equivalence extends to the case of annuities.

Consider a group of N identical individual who live for two periods. Each individual survives to the second period with probability $\pi = ½$. Let wage income in period 1 be $w = 300$. Suppose that after paying a wage tax with rate $t = 0.2$ each individual spends one third of net income $w^n = 240$ on annuities, that is, $a = 80$. We assume, for simplicity and in order to concentrate on the insurance aspect of annuities, that the interest rate is zero. Given the fair payoff rate $q = 1/\pi = 2$ per unit of annuity, the individual receives 160 in period 2. Altogether we have

$$w = 300,\ w^n = 240,\ a = 80,\ c^0 = 160,\ c^1 = 160, \qquad (1)$$

and total tax revenue is 60N.

On the other hand, if expenses for annuities are deductible, but payoffs are fully taxed with rate $t = 0.2$, the individual is exactly in the same situation as before, if she chooses a higher annuity demand of $a = 100$; then we have:

$$w = 300,\ w^n = 260,\ a = 100,\ c^0 = 160,\ c^1 = 160. \qquad (2)$$

Note that the government receives 40 as a wage tax from every individual and another 40 from the tax on annuity payouts, but only from the surviving individuals. Therefore, total tax revenue is $40N + 40\frac{N}{2} = 60N$, as before.

Next assume that there are two groups L,H of individuals, where each group is of equal size N and characterised by a differing survival probability: $\pi_L = \frac{1}{3}$, $\pi_H = \frac{2}{3}$, otherwise they are identical. Insurance firms cannot distinguish between the groups, hence there exist only - so-called - pooling contracts with the same payoff rate for each individual. We assume for the moment that both groups buy the same amount of annuities, then the payoff rate, which allows zero profits, is $q = 1/((\pi_L + \pi_H)/2) = 2$.

In case of a wage tax with rate $t = 0.2$ the situation for each group is as described in (1) above, and (2) continues to reflect the effect of a consumption tax. However, there is one interesting aspect to observe: tax revenues T_L from group L are 60N with the income tax, but only 40N + 40N/3 < 60N with the consumption tax. Still, consumption is the same with both kinds of taxes. This leads us to the question: Why is paying less taxes not to the advantage of group L? The answer is that with the consumption tax this group has to invest more into annuities, compared to the situation with the income tax (namely 100 instead of 80), which provide a lower than fair rate to them. If the tax payment of 60N were unchanged, this increased annuity demand $\Delta a = 20$, given $q < 1/\pi_L$, would reduce the short-living group's lifetime income by the amount of $\Delta a(1 - q\pi_L)N = 20(1 - \frac{2}{3})N$. From these considerations, it is obvious that the smaller tax payment $\Delta T_L = 20N/3$ just compensates the short-living group for the disadvantage arising from the increased demand, given the lower than fair rate of return. Moreover, one could say that, via this increased annuity purchase, group L implicitly finances the additional tax amount, which in fact the group with the higher life expectancy (for which the situation is vice versa, as $q > 1/\pi_H$) has to pay in case of consumption taxation. Altogether, both groups are as well off in either tax regime.

We can generalise this example by assuming that the share α_i of group $i = L,H$, $\alpha_L + \alpha_H = 1$, and the first-period income w_i need not be the same for each group i. In an annuity market, which is characterised by asymmetric information, a single rate of return, offered to both types of individuals, prevails in equilibrium. Under the assumption of perfect competition, this pooling rate of return is implicitly defined by the zero-profit condition that aggregate expected payoffs must equal the aggregate spending (the interest rate is still assumed to be zero), i.e.

$$q(\pi_L \alpha_L a_L + \pi_H \alpha_H a_H) = (\alpha_L a_L + \alpha_H a_H)\,. \tag{3}$$

Moreover, we can determine the lifetime budget constraint of an individual i, in case of an income tax, by combining $c_i^0 = w_i(1-t) - a_i$ and $c_i^1 = qa_i$ and, in case of a consumption tax, by combining $c_i^0 = (1-t)(w_i - a_i)$ and $c_i^1 = q(1-t)a_i$. In either tax regime one gets (by elimination of a_i)

$$c_i^0 + \frac{c_i^1}{q} = w_i(1-t)\,.$$

From this it is obvious that an individual i chooses the same consumption path over her lifetime in either tax regime (for any arbitrary utility function $u(c_i^0, c_i^1; \pi_i))$, if the rate of return is the same. In that case, $a_i^{in} = a_i^{co}(1-t)$, where the superscripts "in" and "co" indicate the respective tax regime.

As a result, total tax revenues from group i are $T_i^{in} = \alpha_i w_i t$ with an income tax and are $T_i^{co} = \alpha_i\left(tw_i + t(q\pi_i - 1)a_i^{co}\right)$ with a consumption tax. One finds $T_i^{co} \lesseqgtr T_i^{in}$ depending on $\pi_i \lesseqgtr 1/q$. Thus, as above, group L pays less with a consumption tax than with an income tax in case of a pooling payout rate, while the opposite is true for group H. However, as with the former tax group L has to buy more insurance, which offers unfavourable conditions, it is equally well-off, though it pays less taxes, and vice versa for group H.[8]

Finally, we observe that total revenue $T_L^{co} + T_H^{co}$ of both groups from the consumption tax equals total tax revenue $T_L^{in} + T_H^{in} = t(\alpha_L w_L + \alpha_H w_H)$ from the income tax, if q is the pooling payoff rate determined by (3). Obviously, if q is below that rate, then a consumption tax raises less revenue than an income tax: the taxable base is reduced because of administrative costs and profits of the insurance companies, which are not accounted for in the present model.

III. Taxation of annuity payoffs

From the example in Section II we learn that taxing payoffs from annuity contracts not only changes the time path of tax payments, but also the tax burden falling on a

[8] As above we find that the change in tax revenues $T_i^{co} - T_i^{in}$ of each group i equals exactly the change of lifetime income $\alpha_i(a_i^{co} - a_i^{in})(q\pi_i - 1)$ for the respective group, which would occur due to their increased annuity demand $a_i^{co} - a_i^{in} = ta_i^{co}$ (use the above result that $a_i^{in} = a_i^{co}(1-t))$ for unchanged T_i^{in}.

particular group, given that survival probabilities differ across groups and that there is a pooling rate of return. However, in this example, the two tax systems still were equivalent, leaving welfare of both groups unaltered, due to a compensating effect from increased annuity purchases.

In the present section we ask whether the fact that taxation of annuity payoffs reduces the revenue raised from the low life-expectancy group, compared to an income tax, can be used for redistributive purposes. The intuition is that in a tax system where a tax on annuity payoffs is employed, in addition to a tax on labour income (whose rate can then be reduced), some burden might be shifted from the short-living group to the long-living group through a related mechanism as described above. We consider two different kinds of models: the first assumes a fixed first-period income, as in Section II, and is designed to answer the question of whether a distorting tax on annuity payoffs is desirable in addition to a proportional tax on labour income. The other model allows for incentive effects on labour supply in order to deal with the role of a tax on annuities, given the optimum nonlinear income tax in the tradition of Mirrlees.

III.1 A tax on annuity payoffs in addition to a proportional income tax

As in Section II we consider an economy that consists of two groups of individuals L,H with shares α_L, α_H ($\alpha_L + \alpha_H = 1$), who live for at most two periods and differ in the probability to survive to the second period, with $\pi_L < \pi_H$. In order to concentrate on the implications of different longevity risks, the fixed first-period income w is assumed to be identical for both types of individuals. Preferences over consumption c_i^0, c_i^1, i = L,H, in both periods are described by a utility function $u(c_i^0, c_i^1; \pi_i)$, which is strictly concave with respect to c_i^0, c_i^1, and depends positively on the survival probability, i.e. $\partial u / \partial \pi_i > 0$. Note that in case of expected utility, which is typically assumed for the study of old-age provision under longevity risk (see e.g., Abel 1986, Brugiavini 1993, Walliser 2000), $u(c_i^0, c_i^1; \pi_i)$ reads

$$u(c_i^0, c_i^1; \pi_i) = \tilde{u}(c_i^0) + \pi_i \tilde{u}(c_i^1), \tag{4}$$

where $\tilde{u}$ is (strictly concave) per-period utility derived from consumption. Let furthermore t denote a proportional tax on labour income, which is lump-sum (as income is fixed) and τ be the tax rate on annuity payouts. Indirect utility $v_i(\pi_i,t,\tau)$ of an individual i is given as the solution of the optimisation problem $\max\left\{u(c_i^0,c_i^1;\pi_i)\,\middle|\, c_i^0+c_i^1\big/(q(1-\tau))\le w(1-t), c_i^0,c_i^1\ge 0\right\}$. The budget constraint results from the two separate conditions: $c_i^0+a_i\le w(1-t)$, $c_i^1\le q(1-\tau)a_i$. As in Section II, we assume for the sake of simplicity that the interest rate is zero and that individuals have no other savings instrument.

Let $S(t,\tau)$ be the weighted utilitarian social welfare function (with weights ρ_L, ρ_H),

$$S(t,\tau)\equiv\sum_{i=L,H}\rho_i\alpha_i v_i(\pi_i,t,\tau) \tag{5}$$

depending on the tax rates $t,\tau\ge 0$ that generate the required tax revenue G,

$$G = tw+\tau(\pi_L\alpha_L q a_L(\cdot)+\pi_H\alpha_H q a_H(\cdot)), \tag{6}$$

where $a_i(\cdot)$ denotes the demand function for annuities for i = L,H. Inserting $a_i = c_i^1/(q(1-\tau))$, the constraint (6) can be written as

$$G = tw+\sigma(\pi_L\alpha_L c_L^1(\cdot)+\pi_H\alpha_H c_H^1(\cdot)), \tag{6'}$$

with $\sigma\equiv\tau/(1-\tau)$.

Let throughout Section III q be the pooling rate of return in equilibrium, implicitly determined by (3), for some given τ. Note that annuity demands a_L, a_H depend on q (and on τ), therefore q cannot be computed explicitly from (3).

We start with τ = 0 and ask whether the introduction of a positive tax rate τ on annuity payoffs and a corresponding decrease in the income tax rate t, such that (6') remains fulfilled, increases social welfare. In the following we only consider the first-round effect, that is, we take q as constant. The modification, if the effect of an increase of τ on the rate of return (via changes in annuity demand) is taken into account, will be discussed later on. Differentiation of (5) with respect to τ gives

$$\frac{\partial S}{\partial \tau} = \sum_{i=L,H} \rho_i \alpha_i \left(\frac{\partial v_i}{\partial \tau} + \frac{\partial v_i}{\partial t}\frac{\partial t}{\partial \tau}\right) \tag{7}$$

where $\partial t / \partial \tau$ is obtained by implicit differentiation of (6') as

$$\frac{\partial t}{\partial \tau} = -\frac{(\pi_L \alpha_L c_L^1 + \pi_H \alpha_H c_H^1)\frac{\partial \sigma}{\partial \tau} + \sigma(\pi_L \alpha_L \frac{\partial c_L^1}{\partial \tau} + \pi_H \alpha_H \frac{\partial c_H^1}{\partial \tau})}{w} \tag{8}$$

Using the abbreviation $Q \equiv 1/(q(1-\tau))$ and Roy's Lemma, (7) can be transformed to (w^n denotes net income and $\lambda_i \equiv \partial v_i / \partial w^n$)

$$\frac{\partial S}{\partial \tau} = \sum_{i=L,H} \rho_i \alpha_i \left(-\lambda_i c_i^1 \frac{\partial Q}{\partial \tau} - \lambda_i w \frac{\partial t}{\partial \tau}\right). \tag{7'}$$

Inserting (8) into (7'), we derive for $\partial S / \partial \tau$ at $\tau = 0$ (which also means $\sigma = 0$ and $\partial \sigma / \partial \tau = 1$, $\partial Q / \partial \tau = 1/q$)

$$\left.\frac{\partial S}{\partial \tau}\right|_{\tau=0} = \sum_{i=L,H} \rho_i \alpha_i \lambda_i \left(-\frac{c_i^1}{q} + \pi_L \alpha_L c_L^1 + \pi_H \alpha_H c_H^1\right). \tag{9}$$

Substituting the pooling rate of return, determined by (3), together with $\left.a_i\right|_{\tau=0} = c_i^1 / q$ into (9) yields

$$\left.\frac{\partial S}{\partial \tau}\right|_{\tau=0} = \alpha_L \alpha_H (\rho_L \lambda_L - \rho_H \lambda_H)(a_H - a_L), \tag{10}$$

which allows us to formulate conditions under which the introduction of a distorting tax on annuity payoffs is desirable.

Proposition 1: Suppose that annuity demand increases with the survival probability. Then the introduction of a tax τ on annuity payoffs and a corresponding decrease in the income tax rate t, such that (6) remains fulfilled, improves social welfare, if the social marginal valuation of net income for individual L is larger than that for individual H, i.e. if $\rho_L\lambda_L > \rho_H\lambda_H$.

Proof: Immediate from (10) and the assumption that $\partial a_i/\partial\pi_i > 0$. QED.

First note that a larger annuity demand of individuals with high life expectancy is indeed a reasonable assumption. It holds, for instance, if preferences are of the expected-utility type (4).[9] Moreover, from many empirical studies it is well known that adverse selection in the private annuity market in fact occurs, i.e. longer-lived individuals do purchase a larger amount of annuities.[10]

An intuition for Proposition 1 can be found by observing from (8) that $-\Delta\tau q(\pi_L\alpha_L a_L + \pi_H\alpha_H a_H)/w$ represents the reduction of t in case of a marginal increase $\Delta\tau$ (at τ = 0) to keep tax revenues constant. On the other hand, from the total differential of the indirect utility function v_i one finds that utility of an individual i remains constant or increases, if t is reduced at least by $-\Delta\tau a_i/w$.[11] It follows, using equation (3) - which defines the pooling rate of return - that the short-living individuals are made better off through such a change, as clearly $-(\alpha_L a_L + \alpha_H a_H) < -a_L$, while the long-living individuals are made worse off. The relative weights in the social objective determine whether this is desirable. However, if q were sufficiently small (below the pooling rate of return), then not even the short-living individuals would benefit, because $-q(\pi_L\alpha_L a_L + \pi_H\alpha_H a_H)$ may eventually be larger than $-a_L$. With any distorting tax the deadweight loss grows at second order with the tax rate, hence there is a upper bound to the tax rate on annuity payouts, above which an additional increase makes both types of individuals worse off.

Taking into account the effect of τ on the pooling rate of return means that an additional term occurs in $\partial Q/\partial\tau$ at τ = 0, viz. $\partial Q/\partial\tau = 1/q - q'/q^2$ (where $q' \equiv \partial q/\partial\tau$

9 Follows from implicit differentiation of the first-order condition $-\tilde{u}'(w^n - a_i) + \pi_i q\tilde{u}'(qa_i) = 0$.

10 Mortality tables for voluntary annuitants in the well-developed U.S. and U.K. markets suggest that life expectancy for a typical 65-year-old male annuitant is about 20 percent longer than for a typical 65-year-old male (see, e.g., Finkelstein and Poterba 2002, Mitchell et. al. 1999).

11 Using the envelope theorem we get $Dv_i = -\Delta tw\,\partial u/\partial c_i^0 - \Delta\tau qa_i\,\partial u/\partial c_i^1 = 0$. From this and the first-order condition for annuity demand $-\partial u/\partial c_i^0 + q\partial u/\partial c_i^1 = 0$, Δt can be computed.

can in principle be determined by implicit differentiation of (3)). As a consequence, the term $(\rho_L\alpha_L\lambda_L a_L + \rho_H\alpha_H\lambda_H a_H)q'/q$ has to be added in formula (10). In general, the sign of q' is undetermined.[12] Obviously, Proposition 1 remains true as long as q' is not too negative, i. e., as long as the composition of annuity demand does not change too much in favour of the high-risk group, if τ is introduced.

Further note that the finding of Proposition 1 is not restricted to an annuity market which is characterised by asymmetric information, such that (3) determines the pooling payout rate in equilibrium. Even in a first-best world with perfect information, in which the individually fair rates of return $q_i = 1/\pi_i$, i = L,H, can be realised as an equilibrium, the introduction of a tax on annuity payouts implies redistribution from the high-demand individuals to the low-demand individuals, due to the same arguments as above. This is straightforward to show by using $q_i = 1/\pi_i$ instead of q, and $a_i\big|_{\tau=0} = c_i^1/q_i$ in (9), which yields

$$\left.\frac{\partial S}{\partial \tau}\right|_{\tau=0} = \alpha_L\alpha_H\left(\rho_L\lambda_L - \rho_H\lambda_H\right)\left(a_H(q_H) - a_L(q_L)\right). \tag{11}$$

Moreover, we find that under the assumption of expected utility (4) $a_H(q_H) - a_L(q_L) > 0$[13], which implies that, as above, it is the group with low life expectancy that benefits from the introduction of τ at the expense of the group with the high life expectancy.

In either case, it is the relative social evaluation $\rho_L\lambda_L/(\rho_H\lambda_H)$ of the risk-groups, which is decisive, whether a positive annuity tax with the corresponding income-tax decrease should be implemented. Obviously, establishing the magnitude of $\rho_L\lambda_L/(\rho_H\lambda_H)$ brings us to a difficult normative question; to tackle the issue, note first that with expected utility (4) the marginal utility of net income of the long-living individuals is higher than that of the short-living individuals, i.e. $\lambda_H > \lambda_L$: Given equal

[12] If the per-period utility function $\tilde{u}$ in expected utility (4) exhibits constant relative risk aversion, Pech (2004) has shown that a tax on payoffs aggravates adverse selection for certain parameter values, otherwise the effect is undetermined.

[13] To proof this result, substitute the individually fair rate into the first-order condition in footnote 9 to obtain $-\tilde{u}'(w^n - a_i) + \tilde{u}'(a_i/\pi_i) = 0$ and differentiate this term implicitly.

net income, individuals with a higher life expectancy value an additional unit of money more. Consequently, assuming an unweighted utilitarian social welfare function ($\rho_H = \rho_L$), the introduction of an annuity tax (with a corresponding decrease in the income tax) is not desirable. Instead a subsidy on the annuity payoffs (with a corresponding increase in the income tax) could serve as a possible instrument to increase social welfare.

On the other hand, it is a common criticism on (unweighted) utilitarianism, that it implies distribution to be driven by individual marginal utilities, which may have nothing to do with an ethical objective. Such a position is taken, e. g. by Sen (1973, p. 16f). He illustrates this criticism by considering a handicapped individual, who derives less utility from additional income, compared to a healthy person, while for ethical reasons we might prefer to favour the handicapped person. Transferring Sen's view to the present case, society may want to treat those individuals with poor health and high mortality[14] better, which means that this group is given a higher weight $\rho_L > \rho_H$ in the utilitarian welfare function (5). If ρ_L is sufficiently large, it may imply redistribution, via the implementation of a proportional tax on annuity payouts.

Finally, consider the even stronger egalitarian concept of Rawls, which aims at maximising the utility of the worst-off group. It can be shown that in case of a pooling rate of return indeed the group with low expectancy is worse off than the one with high life expectancy.[15] In the present model, we would then have $\lambda_H = 0$ and it follows that according to the maximin criterion a positive tax rate on annuity payouts increases social welfare. However, note that with individually fair payouts the comparison of both groups' utility levels is not clear-cut: On the one hand, the individual with the low life expectancy is worse off with any given consumption bundle

[14] Obviously, what one has in mind here is that higher mortality occurs largely for biological or genetic reasons respectively and not rather out of personal choice (such as smoking or other risky behaviour).

[15] Immediate from the following considerations: With an equal rate of return, the budget set is the same for both types of individuals. Due to their higher life expectancy, individual H attain a higher utility level at type's L optimal consumption bundle $(\overline{c}_L^0, \overline{c}_L^1)$ than individual L (remember that $\partial u / \partial \pi_i > 0$). Consequently, individual H's utility is obviously higher with her optimal choice $(\overline{c}_H^0, \overline{c}_H^1)$.

than the long-living individual; on the other hand, as $q_L > q_H$, the budget set of the short-living individual is larger.

Altogether, we found in the present model that the social desirability of a tax on annuity payoffs leads one to the difficult normative issue, whether life expectancy itself can be an argument for a differentiated treatment of individuals. However, one should recognise that in any discussion of social equity and justice, illness and mortality cannot be ignored. Moreover, it should be clear that value judgments of this sort are involved in any political decisions, particular in the design of the tax system.

III.2 A tax on annuity payoffs in addition to an optimum nonlinear income tax

A more realistic model of the problem of tax design has to take into account the incentive effects on labour supply. Therefore, in the present subsection, the assumption of a fixed wage income is dismissed. Instead we consider an economy with two types of individuals, who differ in their wage rate b_i, i = L,H, with $b_L < b_H$, and, as before, in their probability of survival to the second period. We again assume $\pi_L < \pi_H$ and, thus, a positive correlation between the wage rate and life expectancy, which is plausible from empirical studies, as mentioned in the Introduction. Preferences are now described by the utility function $u(l_i, c_i^0, c_i^1; \pi_i)$, where l_i denotes labour supply.

As is usual in optimum-taxation theory in the tradition of Mirrlees, we assume that the authority does not know individual abilities, but only gross income. Thus, the tax system consists of a (nonlinear) tax on gross income and, in addition, of a tax on annuity payoffs. Concerning the latter, we consider two different models: In the first one, we ask whether the introduction of a linear tax improves welfare, while in the second we analyse an optimum tax system which is fully nonlinear with respect to income from both labour and annuities.

III.2.1 Linear taxation of annuities

This case could be seen as a dual income tax system, where annuity payoffs are taxed separately from labour income, with a uniform rate τ. Let $z_i \equiv b_i l_i$ denote gross income and x_i net income. We define the indirect utility function, depending on z_i and x_i, $v_i(x_i,z_i,\tau) \equiv \max\left\{u(z_i/b_i, c_i^0, c_i^1; \pi_i) \middle| c_i^0 + c_i^1/(q(1-\tau)) \le x_i, c_i^0, c_i^1 > 0\right\}$, for any tax rate τ on annuity payoffs. As is usual in models of optimum nonlinear income taxation, we have to assume that preferences fulfill the single-crossing condition

$$\text{AM: } -(\partial v_L / \partial z_L)/(\partial v_L / \partial x_L) > -(\partial v_H / \partial z_H)/(\partial v_H / \partial x_H),$$

for any x, z, and τ.[16] Otherwise, redistribution via the tax on labour income could go from the less to the more able individual, because the latter would choose to work so little that her gross income is below that of the former.

As already mentioned, the basic element of this Mirrlees-type model is asymmetric information concerning individual abilities, i.e., wage rates. In the model, this is accounted for through the so-called "self-selection constraints": the government assigns two gross and net income positions to the individuals in such a way that each does not prefer the position assigned to the other.

Assuming again a weighted utilitarian social objective, the problem of the optimum nonlinear income tax reads, for any given tax τ on annuity payoffs:

$$\max \quad \rho_L v_L(x_L, z_L, \tau) + \rho_H v_H(x_H, z_H, \tau), \tag{12}$$

$$\text{s. t.} \quad x_L + x_H \le z_L + z_H + \sigma(\pi_L c_L^1(\cdot) + \pi_H c_H^1(\cdot)) - G, \tag{13}$$

$$v_H(x_H, z_H, \tau) \ge v_H(x_L, z_L, \tau), \tag{14}$$

$$x_i, z_i \ge 0, i = L, H. \tag{15}$$

(14) represents the self-selection constraint for the long-living and high-wage individual. In principle, an analogous restriction has to be formulated for the short-living individual. However, one can show that with appropriate weights on the latter in the objective function (compare the discussion in III.1), the self-selection constraint for the short-living individuals is not binding in the optimum, while (14) is. Equation

[16] The condition was called "agent monotonicity" by Seade 1982. It is a "single crossing condition", because it implies that indifference curves of the two individuals in (z,x)-space cross only once. See also Brunner 1989.

(13) is the resource constraint. For simplicity, we have neglected - possibly differing - group shares α_L, α_H in (12) - (13). Introducing them would not change the results.

The first-order conditions for an optimum solution of (12) - (15) with respect to x_i, z_i, i = L,H are given in Appendix A. As in Subsection III.1, we denote by $S(\tau)$ the optimum value of the objective function (12), for given τ, and derive, by some manipulations (see Appendix A)

$$\left.\frac{\partial S}{\partial \tau}\right|_{\tau=0} = \upsilon \frac{\partial v_H}{\partial x_L}(a_H[L] - a_L), \tag{16}$$

where $\upsilon > 0$ is the Lagrange multiplier associated with (14), and $a_H[L]$ denotes annuity demand, which the high-wage individual would choose in case of mimicking, i. e. if opting for gross and net income assigned to the low-wage individual. (16) follows from (A.9) in Appendix A, if the zero-profit condition (3) is used to show that the term in square brackets vanishes. With (16) we can formulate our main result, which refers to the first-round effect (holding q fixed), as before:

Proposition 2: Assume that demand for annuities increases with the survival probability and does not increase with labour time. Then a linear tax on annuity payoffs, in addition to the optimum nonlinear income tax, improves social welfare.

Proof: $a_H[L] > a_L$ follows from the assumptions that (i) demand for annuities increases with the survival probability, which is higher for individual H and that (ii) demand is not positively associated with labour time, because, in case of mimicking, individual H works less than individual L. $\upsilon > 0$ and $\partial v_H / \partial x_L > 0$ complete the proof.

QED.

Having in mind Proposition 1, it may appear surprising that the weights of the individuals do not play a role in the above proposition. The explanation for this fact is that, when formulating the model (12) - (15) we have already assumed that with the optimum solution the self-selection constraint for individual H is binding. That is, the government would like to redistribute more income, but is restricted because of asymmetric information. This assumption, which corresponds to the usual way the problem is formulated, in turn hinges on a sufficient importance of the disadvantaged individual L in the objective function. Proposition 2 shows that in such a situation, given a positive correlation between the wage rate and life expectancy, a tax on annuity payoffs allows additional redistribution, as it makes mimicking less attractive for the more able individual.

Further, remember from Section III.1 that for expected utility the first assumption required for Proposition 2, namely that annuity demand increases with life expectancy is indeed fulfilled. Moreover, the second assumption - a non-positive association of annuity demand with labour time - is clearly guaranteed, if preferences are weakly separable between labour (leisure) and consumption in both periods, i.e. if annuity demand is independent of labour time, for given net income.

It is interesting to compare the above result with a corresponding one concerning the role of a tax on capital income. As is well-known, given that preferences are weakly separable between labour (leisure) and consumption in different periods, no tax on capital income in addition to the optimum nonlinear income tax is desirable (see, e.g. Atkinson and Stiglitz 1980, Ordover and Phelps 1979). A tax on capital income is only desirable, if saving is positively associated with leisure (compare also Corlett and Hague 1953). In the present model, however, the desirability of the tax on annuity payoffs in fact results, as long as saving (that is, annuity demand) does not decrease too much with leisure, so that this effect is outweighed by the increase in annuity demand of individual H due to her higher life expectancy.

Note also that exactly the same formula (16) would arise in a first-best world, with individual fair rates of return q_L, q_H used instead of q in (A.8) and (A.9). We have argued in Section III.1 that in the expected-utility case $a_H(q_H) > a_L(q_L)$ holds, thus a result similar to Proposition 2 applies. Finally it should be mentioned that taking into account the (second-round) effect of τ on the (pooling) equilibrium rate of return q means that an additional term depending on $\partial q / \partial \tau$ occurs in (16), analogous to the situation in Section III.1.

III.2.2 Nonlinear taxation of income from labour and annuities

A fully nonlinear system is based on the idea that in fact, by choosing the appropriate bundle of gross income and net income, an individual reveals her type (see, e. g., Brett 1998, Pirttila and Tuomala 2001 in the context of capital income taxation). This means that in the second period the tax on income from annuities can be imposed separately on each type. Consequently, we consider a tax system, which consists of a (nonlinear) tax T(z), imposed on gross income, and two (nonlinear) taxes $T_L(qa_L)$, $T_H(qa_H)$, depending on annuity payoffs. Technically, this means that the self-selection constraint now has the form

$$\tilde{v}_H(c_H^0, c_H^1, z_H) \geq \tilde{v}_H(c_L^0, c_L^1, z_L)\,, \tag{17}$$

where $\tilde{v}_i(c_i^0, c_i^1, z) \equiv u(z/b_i, c_i^0, c_i^1, \pi_i)$. That is, the government has to select two complete bundles of labour time (or gross income, equivalently) and consumption in both periods, such that the more able person does not prefer the bundle assigned to the less able. (Again we assume a-priori that the government wants to redistribute income from the former to the latter type.) As before, a single-crossing condition is required, which has the same form as AM, but with $v_i(x,z,T_i) \equiv \max\left\{u(z/b_i, c_i^0, c_i^1; \pi_i) \,\middle|\, c^0 + N_i^{-1}(c_i^1)/q \leq x,\ c_i^0, c_i^1 > 0\right\}$, where N_i^{-1} is the inverse of the net income function $N_i(qa_i) \equiv qa_i - T_i(qa_i)$. AM has to hold for appropriate T_i.

With these preparations we can formulate the planner's problem as

$$\max_{c_i^0, c_i^1, z_i} \rho_L \tilde{v}_L(c_L^0, c_L^1, z_L) + \rho_H \tilde{v}_H(c_H^0, c_H^1, z_H), \tag{18}$$

$$\text{s. t.} \quad c_L^0 + c_H^0 + \pi_L c_L^1 + \pi_H c_H^1 \leq z_L + z_H - G, \tag{19}$$

$$\tilde{v}_H(c_H^0, c_H^1, z_H) \geq \tilde{v}_H(c_L^0, c_L^1, z_L), \tag{20}$$

$$c_i^0, c_i^1, z_i \geq 0\,, \quad i = L,H. \tag{21}$$

The corresponding first-order conditions can be found in Appendix B. The solution has the following properties:

Proposition 3: For type H the optimum nonlinear tax on annuity payoffs exhibits a positive marginal tax rate equal to $(q-1/\pi_H)/q$. For type L, the sign of the marginal tax rate is undetermined.

Proof: From the first-order conditions for the individual optimisation problem

$$\max u(z_i/b, c_i^0, c_i^1; \pi_i), \text{ s.t. } c_i^0 = z_i - T(z_i) - a_i \text{ and } c_i^1 = qa_i - T_i(qa_i)$$

one derives an expression for the marginal tax rate T_i' in terms of the marginal rate of substitution as

$$T_i' = \frac{1}{q}\left(q - \frac{\partial u/\partial c_i^0}{\partial u/\partial c_i^1}\right) = \frac{1}{q}\left(q - \frac{\partial \tilde{v}_i/\partial c_i^0}{\partial \tilde{v}_i/\partial c_i^1}\right), \quad i = L,H, \tag{22}$$

where the second equality is immediate from the definition of $\tilde{v}_i$. On the other hand, from (B.4) and (B.5) in Appendix B we find that in the optimum

$$\frac{\partial \tilde{v}_H/\partial c_H^0}{\partial \tilde{v}_H/c_H^1} = \frac{1}{\pi_H}$$

must hold, thus the first part of the Proposition is proved. For type-L individuals, (B.1) and (B.2) tell us that in the optimum

$$\frac{\rho_L \partial \tilde{v}_L/\partial c_L^0 - \tilde{\upsilon}\partial \tilde{v}_H/\partial c_L^0}{\rho_L \partial \tilde{v}_L/\partial c_L^1 - \tilde{\upsilon}\partial \tilde{v}_H/\partial c_L^1} = \frac{1}{\pi_L}, \tag{23}$$

where $\partial \tilde{v}_H/\partial c_L^t$, t = 0,1, describes the marginal utility of individual H in case of mimicking, i.e. opting for the type-L bundle. In general, it cannot be concluded from (23), whether $(\partial \tilde{v}_L/\partial c_L^0)/(\partial \tilde{v}_L/\partial c_L^1) \lesseqgtr 1/\pi_L$ and therefore the marginal tax rate on annuity payoffs is undetermined. QED.

One observes that the marginal tax rate for group L will be negative, if the marginal rate of substitution between present and future consumption of individual L does not differ much from that of individual H in case of mimicking, because then (23) implies that $(\partial \tilde{v}_L/\partial c_L^0)/(\partial \tilde{v}_L/\partial c_L^1)$ is close to $1/\pi_L$, while the pooling rate of return q is smaller than $1/\pi_L$. On the other hand, if individual H values future consumption more (in case of mimicking), because of her higher life expectancy, then we have $(\partial\tilde{v}_L/\partial c_L^0)/(\partial\tilde{v}_L/\partial c_L^1) > (\partial\tilde{v}_H/\partial c_L^0)/(\partial\tilde{v}_H/\partial c_L^1)$, which together with (23) implies that $(\partial\tilde{v}_L/\partial c_L^0)/(\partial\tilde{v}_L/\partial c_L^1) < 1/\pi_L$.[17] This in turn, used in (22), tells us that the marginal tax rate on annuity payoffs, for individual may also be positive.

It is important to notice the difference of the result in Proposition 3 to the previous one: while the desirability of a linear tax on annuities essentially depends on the difference in annuity demand of high- and low-risk individuals, a further motive arises with a nonlinear tax: the correction of the rate of return in a pooling situation. This is more in line with the intuition developed in Section II: the loss (benefit) from pooling of the short-living (long-living) individual, compared to individually fair payoff rates, gives rise to a differentiated treatment by the tax system. It is obvious from the proof of Proposition 3 that if q was equal to the individually fair rates q_L, q_H, resp., then the marginal tax rate for individual H is zero, while a distortion - familiar from other Mirrlees-type models - arises for individual L. The correction of this market failure arising from asymmetric information is specific for annuity taxation; in models investigating the optimum nonlinear capital income tax, the marginal tax rate for the high-wage individual is zero, while for the low-wage individual it is distorted, except when preferences are weakly separable between consumption and leisure (Ordover and Phelps 1979).

In fact, in the present model the payoff rate fixed on the annuity market does not directly enter the planner's optimisation problem, which aims at determining optimum second-best bundles. The marginal tax rate follows from a comparison with the rate of return offered by the market. Such a correction of the rate of return through the tax system becomes possible, if we follow the idea that in the retirement period the authority can indeed identify individuals by their types, because these are revealed when gross (and net) income is reported by the end of the working period. Yet, annuity demand is expressed during this period already, when neither tax authority

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[17] This follows immediately by rewriting $(\partial\tilde{v}_L/\partial c_L^0)/(\partial\tilde{v}_L/\partial c_L^1) > (\partial\tilde{v}_H/\partial c_L^0)/(\partial\tilde{v}_H/\partial c_L^1)$ as $(\partial\tilde{v}_H/\partial c_L^1)\big/(\partial\tilde{v}_L/\partial c_L^1) > (\partial\tilde{v}_H/\partial c_L^0)\big/(\partial\tilde{v}_L/\partial c_L^0)$ and by transforming equation (23) to $\left[(\partial\tilde{v}_L/\partial c_L^0)\left(\rho_L - \tilde{\upsilon}(\partial\tilde{v}_H/\partial c_L^0)/(\partial\tilde{v}_L/\partial c_L^0)\right)\right]\big/\left[(\partial\tilde{v}_L/\partial c_L^1)\left(\rho_L - \tilde{\upsilon}(\partial\tilde{v}_H/\partial c_L^1)/(\partial\tilde{v}_L/\partial c_L^1)\right)\right] = 1/\pi_L$.

nor insurance firms can distinguish the types, therefore the latter are unable to offer individually fair rates.

Finally, it should be mentioned that in both models – with linear or nonlinear taxation of annuity payoffs – the usual properties concerning the optimum tax rates on labour income can be derived: It is zero for the more able individual but positive for the less able.

IV. Concluding comment

Private life annuities are becoming a more wide-spread instrument for old-age provision, as public pension systems are expected to provide less support in the future. However, it is well-known that the annuity market is affected by an adverse-selection problem, which is a typical obstacle to many insurance markets. As a consequence, it provides only less than fair contracts for individuals with low life expectancy. In addition, empirical studies have found that life expectancy and income are positively correlated. Therefore, it appears a natural question to ask whether these facts should have an influence on the tax system, in particular on the balancing of efficiency and equity considerations.

Theoretical studies on capital income taxation have shown that in a variety of models such a tax cannot fulfil any further redistributive task, given an optimally designed tax on labour income. Intuitively, one might be willing to accept a similar statement for the taxation of annuity payoffs. On the other hand, intuition also shows that such a tax falls on long-living individuals to a larger extent than on short-living and has, thus, a different effect compared to a tax on income from labour or capital.

Under the provision that life expectancies and wages are positively correlated, we were able to find clear results in Section III.2 within the framework of optimum income taxation: A linear tax on annuity payoffs can be used for redistribution, if annuity demand increases with life expectancy, which is quite plausible. A nonlinear tax can be directly employed to correct the distortion of the rate of return caused by asymmetric information, irrespective of demand.

Obviously, the basic question underlying the above results is, whether society favours redistribution at all and, in particular, how it values the welfare of groups with differing life expectancy (and income). On this, we presented some ideas in Section III.1, but of course, economic analysis cannot provide a final answer. Its main task is to point out the distributive consequences associated with the construction of a tax system.

Appendix A

The Lagrangian to the maximization problem (12) - (15) reads

$$L = \rho_L v_L(x_L, z_L, \tau) + \rho_H v_H(x_H, z_H, \tau) - \mu\left(x_L + x_H - z_L - z_H - \sigma(\pi_L c_L^1 + \pi_H c_H^1) + G\right) +$$
$$+\upsilon\left(v_H(x_H, z_H, \tau) - v_H(x_L, z_L, \tau)\right),$$

which gives us the first-order conditions:

$$\rho_L \frac{\partial v_L}{\partial x_L} - \mu + \mu\sigma\pi_L \frac{\partial c_L^1}{\partial x_L} - \upsilon \frac{\partial v_H}{\partial x_L} = 0, \quad \text{(A.1)}$$

$$\rho_L \frac{\partial v_L}{\partial z_L} + \mu + \mu\sigma\pi_L \frac{\partial c_L^1}{\partial z_L} - \upsilon \frac{\partial v_H}{\partial z_L} = 0, \quad \text{(A.2)}$$

$$\rho_H \frac{\partial v_H}{\partial x_H} - \mu + \mu\sigma\pi_H \frac{\partial c_H^1}{\partial x_H} + \upsilon \frac{\partial v_H}{\partial x_H} = 0, \quad \text{(A.3)}$$

$$\rho_H \frac{\partial v_H}{\partial z_H} + \mu + \mu\sigma\pi_H \frac{\partial c_H^1}{\partial z_H} + \upsilon \frac{\partial v_H}{\partial z_H} = 0. \quad \text{(A.4)}$$

Moreover, using the Envelope Theorem we get

$$\frac{\partial S}{\partial \tau} = \rho_L \frac{\partial v_L}{\partial \tau} + \rho_H \frac{\partial v_H}{\partial \tau} + \mu \frac{\partial \sigma}{\partial \tau}(\pi_L c_L^1 + \pi_H c_H^1) + \mu\sigma(\pi_L \frac{\partial c_L^1}{\partial \tau} + \pi_H \frac{\partial c_H^1}{\partial \tau}) +$$
$$+ \upsilon \frac{\partial v_H}{\partial \tau} - \upsilon \frac{\partial v_H[L]}{\partial \tau}, \quad \text{(A.5)}$$

where [L] in the last term means that the derivative of v_H is computed at (x_L, z_L, τ). By Roy's Lemma we have $\partial v_i / \partial \tau = -c_i(\partial v_i / \partial x_i)(\partial Q / \partial \tau)$, where $Q \equiv 1/(q(1-\tau))$. Using this and substituting from (A.1) for $\tau = \sigma = 0$

$$\mu = \rho_L \frac{\partial v_L}{\partial x_L} - \upsilon \frac{\partial v_H}{\partial x_L} \quad \text{(A.6)}$$

and from (A.3) (again for $\tau = \sigma = 0$)

$$\mu = \rho_H \frac{\partial v_H}{\partial x_H} + \upsilon \frac{\partial v_H}{\partial x_H} \quad \text{(A.7)}$$

in turn in (A.5) we derive (note that, at $\tau = 0$, $\partial\sigma/\partial\tau = 1$, $\partial Q/\partial\tau = 1/q$)

$$\left.\frac{\partial S}{\partial \tau}\right|_{\tau=0} = \rho_L \frac{\partial v_L}{\partial x_L} c_L^1 (\pi_L - \frac{1}{q}) + \rho_H \frac{\partial v_H}{\partial x_H} c_H^1 (\pi_H - 1/q) + \\ + \upsilon \frac{\partial v_H}{\partial x_H} c_H^1 (\pi_H - \frac{1}{q}) + \upsilon \frac{\partial v_H}{\partial x_L} (\frac{c_H^1[L]}{q} - \pi_L c_L^1), \tag{A.8}$$

where $c_H^1[L]$ denotes consumption in the second period, which the high-wage individual would choose, if endowed with gross and net income of the low-wage individual.

Appropriate grouping and using (A.6) and (A.7) again gives us from (A.8)

$$\left.\frac{\partial S}{\partial \tau}\right|_{\tau=0} = \mu[c_L^1(\pi_L - \frac{1}{q}) + c_H^1(\pi_H - \frac{1}{q})] + \upsilon \frac{\partial v_H}{\partial x_L} \frac{(c_H^1[L] - c_L^1)}{q}. \tag{A.9}$$

Appendix B

We write the Lagrangian function for (18) - (21) as

$$\tilde{L} = \rho_L \tilde{v}_L(c_L^0, c_L^1, z_L) + \rho_H \tilde{v}_H(c_H^0, c_H^1, z_H) - \tilde{\mu}\left(c_L^0 + c_H^0 + \pi_L c_L^1 + \pi_H c_H^1 - z_L - z_H + G\right) + \\ + \tilde{\upsilon}\left(\tilde{v}_H(c_H^0, c_H^1, z_H) - \tilde{v}_H(c_L^0, c_L^1, z_L)\right)$$

and get the first-order conditions:

$$\rho_L \frac{\partial \tilde{v}_L}{\partial c_L^0} - \tilde{\mu} - \tilde{\upsilon} \frac{\partial \tilde{v}_H}{\partial c_L^0} = 0, \tag{B.1}$$

$$\rho_L \frac{\partial \tilde{v}_L}{\partial c_L^1} - \tilde{\mu}\pi_L - \tilde{\upsilon} \frac{\partial \tilde{v}_H}{\partial c_L^1} = 0, \tag{B.2}$$

$$\rho_L \frac{\partial \tilde{v}_L}{\partial z_L^0} + \tilde{\mu} - \tilde{\upsilon} \frac{\partial \tilde{v}_H}{\partial z_L} = 0, \tag{B.3}$$

$$\rho_H \frac{\partial \tilde{v}_H}{\partial c_H^0} - \tilde{\mu} + \tilde{\upsilon} \frac{\partial \tilde{v}_H}{\partial c_H^0} = 0, \tag{B.4}$$

$$\rho_H \frac{\partial \tilde{v}_H}{\partial c_H^1} - \tilde{\mu}\pi_H + \tilde{\upsilon} \frac{\partial \tilde{v}_H}{\partial c_H^1} = 0, \tag{B.5}$$

$$\rho_H \frac{\partial \tilde{v}_H}{\partial z_H} + \tilde{\mu} + \tilde{\upsilon} \frac{\partial \tilde{v}_H}{\partial z_H} = 0. \tag{B.6}$$

References

Abel, A. B. (1986), Capital Accumulation and Uncertain Lifetime with Adverse Selection, Econometrica 54, 1079-1097.

Aiyagari, R. (1995), Optimal capital income taxation with incomplete markets, borrowing constraints and constant discounting, Journal of Political Economy 103, 1158-1175.

Atkinson, A. B. and J. E. Stiglitz (1980), Lectures on Public Economics, London et al.: McGraw-Hill.

Attanasio, O. P. and H. W. Hoynes (2000), Differential mortality and wealth accumulation, Journal of Human Resources 35, 1-29.

Bernheim, B. D. (2002), "Taxation and Saving", in: Auerbach, A. J. and M. Feldstein (eds.), Handbook of Public Economics, Vol. 3, Chapter 18, North-Holland: Elsevier.

Brett, C. (1998), Notes on Non-linear Taxation in an OLG Model. University of Essex, Mimeo.

Brown, J. R., O. S. Mitchell, J. M. Poterba and M. J. Warshawsky (1999), Taxing Retirement Income: Nonqualified Annuities and Distributions from Qualified Accounts, National Tax Journal 52, 563-591.

Brugiavini, A. (1993), Uncertain resolution and the timing of annuity purchases, Journal of Public Economics, 31-62.

Brunner, J. K. (1989), Theory of equitable taxation, Berlin et al.: Springer.

Brunner, J. K. and S. Pech (2005), Adverse selection in the annuity market when payoffs vary over the time of retirement, Journal of Institutional and Theoretical Economics 161m 155-183.

Burman, L. E., W. G. Gale, M. Hall and P. R. Orszag (2004), Distributional Effects of Defined Contribution Plans and Individual Retirement Arrangements, National Tax Journal 57, 671-701.

Chamley, C. (1986), Optimal taxation of capital income in general equilibrium with infinite lives, Econometrica 54, 607-622.

Chamley, C. (2001), Optimal income taxation, wealth distribution and borrowing constraints, Journal of Public Economics 79, 55-69.

Chari, V.V. and P. J. Kehoe (1999), Optimal Fiscal and Monetary Policy, in: J. Taylor and M. Woodford (eds.), Handbook of Macroeconomics, Amsterdam: North Holland.

Corlett, W. J. and D. C. Hague (1953), Complementary and the excess burden of taxation, Review of Economic Studies 21, 21-30.

Disney, R., C. Emmerson and M. Wakefield (2001a), Pension reform and saving in Britain, Oxford Review of Economic Policy 17, 70-94.

Disney, R., M. Eichenbaum and S. Smith (2001b), Pension reform and economic performance in the 1980s and 19990s, mimeo.

Finkelstein, A. and J. Poterba (2002), Selection Effects in the United Kingdom Individual Annuities Market, Economic Journal 112(476), 28-50.

Jacobs, B. and A. L. Bovenberg (2005), Human Capital and Optimal Positive Taxation of Capital Income, Tinbergen Institute Discussion Paper 2005-035/3.

Judd, K. J. (1985), Redistributive taxation in a simple perfect foresight model, Journal of Public Economics 28, 59-83.

Lillard, L. A. and C. W. A. Panis (1998) Panel attrition from the panel study on income dynamics: household income, marital status and mortality, Journal of Human Resources 33, 437-457.

Mitchell, O. S., J. M. Poterba, M. J. Warshawsky and J. R. Brown (1999), New Evidence on the Money's Worth of Individual Annuities, American Economic Review 89, 1299-1318.

OECD 1994, Taxation and household saving, Paris.

Ordover, J. A. and E. S. Phelps (1979), The concept of optimal taxation in the overlapping-generations model of capital and wealth, Journal of Public Economics 12, 1-26.

Pauly, Mark V. (1974), Overinsurance and Public Provision of Insurance: The Roles of Moral Hazard and Adverse Selection, Quarterly Journal of Economics 88, 44-62.

Pech, S. (2004), Tax incentives for private life annuities and the social security reform: effects on consumption and on adverse selection, Finanzarchiv 60, 556-592.

Pirttilä, J. and M. Tuomala (2001), On optimal non-linear taxation and public good provision in an overlapping generations economy, Journal of Public Economics 79, 485-501.

Rothschild, M. and J. Stiglitz (1976), Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information, Quarterly Journal of Economic 90, 29-649.

Seade, J. (1992), On the sign of the optimum marginal income tax, Review of Economic Studies 49, 537-643.

Sen, A. (1973), On Economic Inequality, Oxford: Clarendon Press.

Stiglitz, J. E. (2000), Economics of the Public Sector, 3rd ed., W.W. Norton & Company.

Walliser, Jan (2000), Adverse Selection in the Annuities Market and the Impact of Privatizing Social Security, Scandinavian Journal of Economics 102, 373-393.

Whitehouse, E. (1999), The tax treatment of funded pensions, Pension Reform Primer Series, Social Protection Discussion Paper no. 9910, World Bank.

Wilson, Ch. (1977), A Model of Insurance Markets with Incomplete Information, Journal of Economic Theory 16, 167-207.

Yaari, M. E. (1965), Uncertain lifetime, life insurance, and the theory of the consumer, Review of Economic Studies 32, 137-150.