# Would you like to enter first with a low-quality good?[1]

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**Abstract**

Using a two-period duopoly model with vertical differentiation, we show that there exists a unique subgame perfect equilibrium where the first entrant supplies a lower quality and gains higher profits than the second entrant. We also prove that this entry sequence is socially efficient.

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# 1 Introduction

According to the established wisdom concerning vertically differentiated markets, earlier entrants appropriate the high-quality niches, while later entrants fill the remaining lower part of the quality spectrum (Gabszewicz and Thisse, 1979, 1980; Shaked and Sutton, 1982, 1983; Donnenfeld and Weber, 1992; Aoki and Prusa, 1997; Lehmann-Grube, 1997). This is due to two basic assumptions according to which the distribution of cumsumers' willingness to pay is uniform and the game unravels in a single period, so that earlier entrants finds it more profitable to serve high-income consumers, irrespectively of the different assumptions concerning full vs partial market coverage, or the shape of the cost function, that characterise the aforementioned contributions.[1]

Here, we want to relax the second assumption, by adopting a simple two-period setup, with sequential entry.[2] Using a model with convex costs whose original formulation is in Cremer and Thisse (1991), we show that explicitly accounting for the monopoly phase suffices to show that profit incentives drive firms toward a unique subgame perfect equilibrium where the first entrant supplies a lower quality and gains higher profits as compared to the second entrant. The straightforward intuition behind this result is that a reduction in production costs, combined with ad interim monopoly power, makes it attractive for the first entrant to offer a low-quality good. That is, the first entrant produces a low-quality product because the associated reduction in

[1]Hoppe and Lehmann-Grube (2001) show that there may exist a second-mover (low-quality) advantage in an innovation race.

[2]In the above mentioned literature, entry is euristically considered in one-shot games, without allowing any explicit role for calendar time. One exception is Dutta *et al.* (1995) where, however, a high-quality advantage obtains.

costs overcompensates the losses incurred in the second period, when the market becomes a duopoly and the second entrant supplies a superior variety. Moreover, we also prove that this entry sequence is socially efficient, in that it entails a higher average quality level than the alternative one.

The remainder of the note is structured as follows. The setup and the static benchmark cases are laid out in section 2. The entry process and the welfare performance are investigated in section 3. Section 4 contains some concluding remarks.

# 2 The model

We borrow the demand and cost setup from Cremer and Thisse (1991, 1994) and Lambertini (1996), *inter alia*. The market exists over two periods, $t \in \{0, 1\}$. Discounting of profits and consumer surplus is measured by the rate $\rho \in [0, \infty)$. In each period, a population of consumer of unit size is uniformly distributed over the interval $\left[\underline{\theta}, \overline{\theta}\right]$, with $\underline{\theta} = \overline{\theta} - 1$; $\underline{\theta} > 0$. Parameter $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ measures a consumer's marginal willingness to pay for quality, and the net surplus from consumption is:

$$U = \theta q_i - p_i \geq 0 \tag{1}$$

where $p_i$ and $q_i$ are the price and quality of the product supplied by firm $i$. We confine our attention to the case where (1) holds for all consumers in both periods, so that the market is always fully covered irrespective of the market regime. In the remainder, we will appropriately discuss the sufficient conditions for full market coverage to hold in every market regime.

On the supply side, any firm $i$ must bear total cost $C_i = cq_i^2 x_i$ per period, where $x_i$ is the market demand for her product and $c$ is a positive parameter.

Accordingly, firm $i$'s profit function is $\pi_i = (p_i - cq_i^2)\, x_i$ in each period.

In the remainder, we will consider the following game:

- Each firm irreversibly sets quality at the time of entry.
- At $t = 0$, the firm 1 enters and remains a monopolist in that period.
- At $t = 1$, firm 2 enters and the market becomes a duopoly.

Hence, the problem of the first entrant (the leader) consists in choosing whether to offer a low- or a high-quality good, correctly anticipating the optimal behaviour of the second entrant (the follower). That is, the stage describing quality choices is going to be solved *à la* Stackelberg. Once both qualities are set, simultaneous Bertrand competition takes place. The solution concept, as usual, is the subgame perfect equilibrium by backward induction.

The objective of the leader (firm 1) is:

$$\max_{p_M, p_1, q_1} \Pi_1 \equiv \pi_1^M + \delta \pi_1^D = p_M - cq_1^2 + \delta \left(p_1 - cq_1^2\right) x_1 \qquad (2)$$

where $\pi_1^M = p_M - cq_1^2$ are monopoly profits at $t = 0$, $\pi_1^D = (p_1 - cq_1^2)\, x_1$ are duopoly profits at $t = 1$, the latter being discounted by the factor $\delta \equiv 1/(1+\rho)$, with $\delta \in [0,1]$ for all $\rho \in [0,\infty)$. The objective of the follower (firm 2) consists in maximising duopoly profits $\pi_2^D = (p_2 - cq_2^2)\, x_2$ w.r.t. $p_2$ and $q_2$.

In the second period, the two firms will supply qualities $q_H \geq q_L > 0$ at duopoly prices $p_H \geq p_L$, and either $q_1 = q_L$; $q_2 = q_H$ or the opposite. In either case, at $t = 1$, the consumer indexed by

$$\widehat{\theta} = \frac{p_H - p_L}{q_H - q_L} \qquad (3)$$

will be indifferent between the two goods, so that we may define duopoly demands as follows:

$$x_H = \overline{\theta} - \frac{p_H - p_L}{q_H - q_L} \, ; \, x_L = \frac{p_H - p_L}{q_H - q_L} - \left(\overline{\theta} - 1\right) \, . \tag{4}$$

## 2.1 Optimal myopic behaviour

Here we describe the optimal static behaviour in the cases of (i) monopoly and (ii) duopoly with sequential play, to be used as a benchmark for the subsequent analysis of the dynamic game.

Optimal monopoly pricing can be quickly characterised, for any given $q_1$. Under full coverage, firm 1 sets the price driving to zero the net surplus of the poorest consumer located at $\underline{\theta} = \overline{\theta} - 1$, i.e., $p_M = \left(\overline{\theta} - 1\right) q_1$. Hence, monopoly profits are $\pi_1^M = \left(\overline{\theta} - 1\right) q_1 - cq_1^2$, and

$$q_M \equiv \arg\max_{q_1} \left(\overline{\theta} - 1\right) q_1 - cq_1^2 = \frac{\overline{\theta} - 1}{2c} \, . \tag{5}$$

Observe that $q_M$ is the quality preferred by the poorest consumer in the market.[3] This is clearly due to the monopolist's incentive to distort quality downwards (Spence, 1975; Mussa and Rosen, 1978, *inter alia*). As shown in Lambertini (1997a), full coverage emerges at the static monopoly optimum provided that $\overline{\theta} \geq 3$.

[3] Consider a consumer indexed by $\theta \in \left[\overline{\theta} - 1, \overline{\theta}\right]$. His preferred quality $q_\theta$ maximises his net surplus when he is able to purchase such quality at marginal cost, that is

$$q_\theta = \arg\max_q U = \theta q - cq^2$$

which yields $q_\theta = \theta/(2c)$ (see Cremer and Thisse, 1991, 1994). Accordingly, $q_M$ coincides with the quality that the poorest consumer indexed by $\overline{\theta} - 1$ would purchase under either social planning or perfect competition.

The simultaneous game in prices is also well known; therefore we omit the detailed exposition (see Cremer and Thisse, 1991, 1994; and Lambertini, 1996, *inter alia*). Equilibrium prices are:

$$p_H = \frac{(q_H - q_L)(\overline{\theta} + 1) + 2cq_H^2 + cq_L^2}{3};\ p_L = \frac{(q_H - q_L)(2 - \overline{\theta}) + 2cq_L^2 + cq_H^2}{3} \tag{6}$$

so that the duopoly profit functions simplify as follows:

$$\pi_H = \frac{(q_H - q_L)\left[\overline{\theta} + 1 - c\left(q_H + q_L\right)\right]^2}{9};\ \pi_L = \frac{(q_H - q_L)\left[2 - \overline{\theta} + c\left(q_H + q_L\right)\right]^2}{9}. \tag{7}$$

Hence, we have two alternative scenarios. The first, where $q_1 = q_L$ and $q_2 = q_H$, is labelled as *low-quality leadership*; the second, where $q_1 = q_H$ and $q_2 = q_L$, is labelled as *high-quality leadership*. Before proceeding to the exposition of the entry games in the two-period model, it can be useful to expose the essential features of the Stackelberg outcomes at the first stage of the static (single-period) game, based upon profit functions (7). This can be quickly done by observing that the leader (firm 1) chooses $q_1 = \left(2\overline{\theta} - 1\right)/(2c)$ which corresponds to the quality preferred by the average (or median) consumer (see fn. 4). This holds irrespectively of whether the leader is the high- or the low-quality firm: for the first entrant, it is always optimal to locate in the middle of the space of consumer preferences (see Lambertini, 1996, 1997b). Then, the follower maximises profits by choosing the best reply to $q_1$, which is either $q_{2H} = \left(4\overline{\theta} + 1\right)/(6c)$ or $q_{2L} = \left(4\overline{\theta} - 5\right)/(6c)$, depending on whether firm 2 enters above or below the leader's quality. With $q_1 = \left(2\overline{\theta} - 1\right)/(2c)$ and $q_2 = q_{2H}$, profits are:

$$\pi^*_{2H}\left(q_1, q_{2H}\right) = \frac{4\left(2 - \overline{\theta}\right)^3}{243c};\ \pi^*_{1L}\left(q_1, q_{2H}\right) = \frac{\left(2 - \overline{\theta}\right)\left(2\overline{\theta} + 5\right)^2}{243c} \tag{8}$$

both being positive for $\overline{\theta} \in (1, 2)$. Otherwise, with $q_1 = \left(2\overline{\theta} - 1\right)/(2c)$ and

$q_2 = q_{2L}$, profits are:

$$\pi^*_{1H}(q_1, q_{2L}) = \frac{\left(1+\overline{\theta}\right)\left(7-2\overline{\theta}\right)^2}{243c}; \pi^*_{2L}(q_1, q_{2L}) = \frac{4\left(1+\overline{\theta}\right)^3}{243c} \tag{9}$$

which are always positive. Then, it can be easily checked that

$$\begin{aligned} \pi^*_{1H}(q_1, q_{2L}) - \pi^*_{1L}(q_1, q_{2H}) &= \frac{\left(2\overline{\theta}-1\right)^3}{243c} > 0 \\ \pi^*_{2L}(q_1, q_{2L}) - \pi^*_{2H}(q_1, q_{2H}) &= \frac{4\left(2\overline{\theta}-1\right)\left[7+\overline{\theta}\left(\overline{\theta}-1\right)\right]}{243c} > 0 \end{aligned} \tag{10}$$

Accordingly, we may state:

**Lemma 1** *In the single-period Stackelberg game, the leader prefers to supply the high-quality good, while the follower prefers to supply the low-quality good. Therefore, the subgame perfect equilibrium is unique and involves* $q_1 = \left(2\overline{\theta}-1\right)/(2c)$; $q_2 = q_{2L}$.

This is in line with the acquired wisdom (Gabszewicz and Thisse, 1979; Shaked and Sutton, 1983). As to the social standpoint, we may evaluate the social welfare function defined as the sum of industry profits and consumer surplus:

$$SW = \pi_1 + \pi_2 + CS \tag{11}$$

$$CS = \int_{\overline{\theta}-1}^{\widehat{\theta}} (sq_i - p_i)\, ds + \int_{\widehat{\theta}}^{\overline{\theta}} (sq_j - p_j)\, ds,\ i,j = 1,2; i \neq j \tag{12}$$

in the two cases, for $\overline{\theta} \in (1,2)$. By doing so, we find:

$$SW(q_1, q_{2H}) > SW(q_1, q_{2L})\ \forall \overline{\theta} \in \left(1, \frac{1}{2}\left(1+3\sqrt{3}\right)\right) \tag{13}$$

where $\left(1+3\sqrt{3}\right)/2 \cong 3.098$. This proves:

**Lemma 2** *In the parameter range where both outcomes are admissible, i.e., for all* $\overline{\theta} \in (1,2)$, *social welfare is higher when the leader supplies the low-quality good.*

This is clearly due to the fact that social welfare increases with the average quality supplied to the market. Additionally, Lemmata 1-2 entail that there exists a conflict between private and social incentives as to the quality spectrum selected through the sequence of moves. Our aim in the remainder of the paper is precisely that of showing that a slightly more realistic setup where the entry process is explicitly sketched may indeed produce drastically different results.

## 3 A two-period game with entry

In order to ensure the attainment of full market coverage in both games, we introduce the following:

**Assumption** $\overline{\theta} \geq 3$.

In particular, as stated in the previous section, this is necessary and sufficient to ensure that all consumers be able to buy at the static monopoly equilibrium. Consequently, given that any further entry entails that prices are lower than at the monopoly optimum, it is sufficient to yield full coverage in correspondence of any duopolistic equilibrium, be that considered either in a static or in a dynamic game. The assumption is in fact sufficient but not necessary to ensure that the outcomes of the dynamic entry game exposed in the remainder are admissible. Intuitively, the condition $\overline{\theta} \geq 3$ is slack for all finite values of the discount rate $\rho$, since only in the limit case where $\rho$ is infinitely high the dynamic game replicates the static monopoly equilibrium.[4]

[4]Spelling out the specific conditions to be met in the dynamic entry games would require numerical calculations involving $\overline{\theta}$ and $\rho$ which we leave aside for the sake of simplifying the exposition.

### 3.1 Low-quality leadership

In this case, the first entrant supplies a low-quality good. Therefore, $\pi_1^D = \pi_L$ and $\pi_2^D = \pi_H$. The leader's problem consists of:[5]

$$\begin{aligned} \max_{q_L} \Pi_{1L} &= \left(\overline{\theta} - 1\right) q_L - c q_L^2 + \frac{(q_H - q_L)\left[2 - \overline{\theta} + c\,(q_H + q_L)\right]^2}{9\,(1+\rho)} \\ s.t. &: \frac{\partial \pi_H}{\partial q_H} = 0 \Leftrightarrow q_H^* = \frac{\overline{\theta} + 1 + c q_L}{3c} \end{aligned} \tag{14}$$

Plugging $q_H^*$ into $\Pi_{1L}$ and solving the first order condition (FOC) $\partial \Pi_{1L} / \partial q_L = 0$ w.r.t. $q_L$, we obtain:

$$q_L = \frac{16\overline{\theta} - 81\rho - 113 \pm 3\sqrt{9\rho\,(81\rho + 194) + 1081}}{32c}\,. \tag{15}$$

The concavity condition is:

$$\frac{\partial^2 \Pi_{1L}}{\partial q_L^2} = -\frac{2c\left(32 c q_L - 16\overline{\theta} + 81\rho + 113\right)}{81\,(1+\rho)} \leq 0. \tag{16}$$

Using (16), one finds that the leader's optimal quality choice is $q_L^+$. Moreover,

$$\lim_{\rho\to\infty} q_L^- = -\infty\,;\ \lim_{\rho\to\infty} q_L^+ = \frac{\overline{\theta} - 1}{2c} \tag{17}$$

the latter being the single-period optimal monopoly quality $q_M$, i.e., as $\rho \to \infty$ the first entrant behaves as if it were always a monopolist. Therefore, the Stackelberg equilibrium qualities are:

$$\begin{aligned} q_L^{\mathfrak{l}} &= \frac{16\overline{\theta} - 81\rho - 113 + 3\sqrt{9\rho\,(81\rho + 194) + 1081}}{32c} \\ q_H^{\mathfrak{f}} &= \frac{16\overline{\theta} - 27\,(1+\rho) + \sqrt{9\rho\,(81\rho + 194) + 1081}}{32c} \end{aligned} \tag{18}$$

where superscripts $\mathfrak{l}$ and $\mathfrak{f}$ stand for *leader* and *follower*, respectively. Comparing (18) with $q_M$, the following can be easily acertained:

[5] There exists another solution to $\partial \pi_H / \partial q_H = 0$, i.e., $q_H = \left(\overline{\theta} + 1 - c q_L\right)/c$. However, this can be excluded on the basis of concavity conditions.

**Lemma 3** $q_H^f > q_L^l > q_M$ *for all* $\rho \in [0, \infty)$.

That is, the leader chooses a quality level that, for any finite discount rate, is higher than the single-period monopoly quality. This is due to the fact that the first entrant anticipates that the follower will locate further up in the quality spectrum, and therefore raises its own quality level as compared to $q_M$, driven by the strategic complementarity characterising quality choice.

Equilibrium profits are:

$$\begin{aligned} \Pi_{1L}^* &= \left\{1152\overline{\theta}\left(\overline{\theta}-2\right)(1+\rho) - \rho\left(19683\rho^2 + 70713\rho + 84969\right) + \right. \\ &\quad \left. -33427 + \Psi^3\right\} / \left[4608c\,(1+\rho)\right] \qquad (19) \end{aligned}$$

$$\pi_H^* = \frac{\rho\,(19683\rho^2 + 82377\rho + 115641) + 54739 - [1657 + 9\rho\,(81\rho + 242)]\,\Psi}{2304c} \qquad (20)$$

where $\Psi \equiv \sqrt{9\rho\,(81\rho + 194) + 1081}$. The following result can be easily ascertained:

**Lemma 4** $\Pi_{1L}^* > \pi_H^* / (1+\rho)$ *in the admissible range of parameters* $\left\{c, \rho, \overline{\theta}\right\}$.

**Proof.** Using (19-20), we obtain:

$$\begin{aligned} \Pi_{1L}^* - \frac{\pi_H^*}{(1+\rho)} &\propto 384\overline{\theta}\left(\overline{\theta}-2\right)(1+\rho) + \Psi\,[1465 + 9\rho\,(81\rho + 226)] + \\ &\quad -\rho\left(19683\rho^2 + 78489\rho + 105417\right) - 47635. \qquad (21) \end{aligned}$$

The expression on the r.h.s. of (21) has no real roots w.r.t. $\overline{\theta}$. Hence, given that the coefficient of $\overline{\theta}$ is positive, we have that $\Pi_{1L}^* - \pi_H^* / (1+\rho) > 0$, which proves the Lemma. ■

This shows that entering first with a low-quality good ultimately matters more than supplying the high-quality good later on, when the market becomes a duopoly. The source of this result is twofold: first, the earlier entrant enjoys monopoly profits for a period; second, it does so at a lower unit cost, $c\left(q_L^{\ell}\right)^2$. This must be contrasted with the traditional claim inherited from previous literature in this field based upon static games (see, e.g., Gabszewicz and Thisse, 1979; Shaked and Sutton, 1983), whereby higher quality niches should be more profitable than inferior ones, as we have summarised in section 1.2.

Equilibrium market shares in the duopoly phase are:

$$x_L^{\ell} = \frac{\Psi - 27\rho - 19}{24}\,;\; x_H^{f} = \frac{27\rho + 43 - \Psi}{24}\,;\; x_L^{\ell} > x_H^{f} \text{ always.} \tag{22}$$

Finally, $\widehat{\theta} \in \left(\underline{\theta}, \overline{\theta}\right)$ always.

## 3.2 High-quality leadership

Now we have $\pi_1^D = \pi_H$ and $\pi_2^D = \pi_L$. The leader's problem consists of:[6]

$$\begin{aligned} \max_{q_H} \Pi_{1H} &= \left(\overline{\theta} - 1\right) q_H - c q_H^2 + \frac{\left(q_H - q_L\right)\left[\overline{\theta} + 1 - c\left(q_H + q_L\right)\right]^2}{9\left(1+\rho\right)} \\ s.t. &: \frac{\partial \pi_L}{\partial q_L} = 0 \Leftrightarrow q_L^* = \frac{\overline{\theta} - 2 + c q_H}{3c} \end{aligned} \tag{23}$$

[6] Also in this case, there exists another solution to $\partial \pi_L / \partial q_L = 0$, i.e., $q_L = \left(\overline{\theta} - 2 - c q_H\right)/c$. Again, this can be excluded on the basis of second order conditions. Moreover, leapfrogging on the part of the follower can also be excluded. The proof of the absence of any incentive to leapfrog the leader's quality is omitted for brevity, although available from the authors upon request.

Adopting the same procedure as in the previous case, we can find the optimal qualities:

$$
\begin{aligned}
q_L^{\mathfrak{f}} &= \frac{16\overline{\theta} + 27\rho + 11 - \sqrt{9\rho\,(81\rho + 226) + 1369}}{32c} \\
q_H^{\mathfrak{l}} &= \frac{16\overline{\theta} + 81\rho + 97 - 3\sqrt{9\rho\,(81\rho + 226) + 1369}}{32c}
\end{aligned}
\tag{24}
$$

As in the previous case, also here the quality of the leader converges to the single period monopoly quality as $\rho$ tends to infinity:

$$
\lim_{\rho \to \infty} q_H^{\mathfrak{l}} = \frac{\overline{\theta} - 1}{2c} \tag{25}
$$

This amounts to saying that, if the discount rate is infinitely high, the high-quality firm behaves as if she stood in the market alone in both periods. Moreover, comparing (24) with $q_M$, we obtain:

**Lemma 5** $q_H^{\mathfrak{l}} > q_M > q_L^{\mathfrak{f}}$ *for all* $\rho \in [0, \infty)$.

Here, unlike the previous case, we observe that the low-quality good lies below $q_M$ for all finite values of the discount rate. This is due to the interplay between the strategic complementarity characterising qualities and the need to differentiate products in order to soften price competition in duopoly. In choosing the optimal quality level, the leader must take into account two opposite forces: one is the incentive to raise quality in order to (i) attain a large degree of differentiation and (ii) eliminate the possibility of leapfrogging by the second entrant in the second period; the other is the incentive to keep as close as possible to $q_M$ with a view to increasing its own ability to explot monopoly power in the first period. The latter effect is to be held responsible of the fact that the follower locates its product below $q_M$.

Equilibrium profits are:

$$\begin{aligned}\Pi_{1H}^{*} &= \left\{1152\overline{\theta}\left(\overline{\theta}-2\right)(1+\rho)-\rho\left(19683\rho^{2}+82377\rho+111753\right)+\right.\\ &\left.-48547+\Psi^{3}\right\}/\left[4608c\,(1+\rho)\right] \qquad (26)\end{aligned}$$

$$\pi_L^{*}=\frac{\rho\left(19683\rho^{2}+88209\rho+130761\right)+64027-(9\rho+13)(81\rho+133)\,\Phi}{2304c} \qquad (27)$$

where $\Phi\equiv\sqrt{9\rho\,(81\rho+226)+1369}$. As in the previous case, we can prove that a first-mover advantage operates:

**Lemma 6** $\Pi_{1H}^{*}>\pi_L^{*}/(1+\rho)$ *in the admissible range of parameters* $\left\{c,\rho,\overline{\theta}\right\}$.

**Proof.** Using (26-27), we obtain:

$$\begin{aligned}\Pi_{1H}^{*}-\frac{\pi_L^{*}}{(1+\rho)} &\propto 384\overline{\theta}\left(\overline{\theta}-2\right)(1+\rho)+\Phi\left[1609+9\rho\,(81\rho+242)\right]+\\ &-\rho\left(19683\rho^{2}+86265\rho+124425\right)-58887. \qquad (28)\end{aligned}$$

The expression on the r.h.s. of (28) has no real roots w.r.t. $\overline{\theta}$. Hence, given that the coefficient of $\overline{\theta}$ is positive, we have that $\Pi_{1H}^{*}-\pi_L^{*}/(1+\rho)>0$, which proves the Lemma. ■

As in the previous case, also here the first entrant's profits are larger than the second entrant's. In this setting, the reason appears to be that the initial monopoly profits add to the fact that the leader will enjoy an advantageous position when the rival enters from below and the market becomes a duopoly.

Equilibrium market shares at $t=1$ are:

$$x_L^{l}=\frac{27\rho+43-\Phi}{24}\,;\;x_H^{f}=\frac{\Psi-27\rho-19}{24}\,;\;x_L^{l}<x_H^{f}\text{ always.} \qquad (29)$$

Again, $\widehat{\theta}\in\left(\underline{\theta},\overline{\theta}\right)$ always.

## 3.3 The subgame perfect equilibrium and welfare assessment

In order to complete the characterisation of the subgame perfect equilibrium, it suffices to compare expressions (19-20) and (26-27). Proceeding as in the proofs of Lemmata 1-2, one can prove the following inequalities:

$$\Pi_{1L}^* > \Pi_{1H}^* \text{ and } \pi_H^* > \pi_L^* \text{ for all } \rho \in [0, \infty) \, . \tag{30}$$

This holds for any admissible value of $\overline{\theta}$. Accordingly, we may state:

**Proposition 7** *In the whole admissible parameter range, the first entrant prefers to supply a low-quality good, while the second entrant prefers to supply a high-quality good. Therefore, the subgame perfect equilibrium is unique and involves* $q_1 = q_L^{\mathfrak{l}}$; $q_2 = q_H^{\mathfrak{f}}$.

This is in sharp contrast with the previous wisdom in this field, which maintained that the first entrant would find it most profitable to fill the high-quality niche (see Gabszewicz and Thisse, 1979, 1980; Shaked and Sutton, 1982, 1983; Lehmann-Grube, 1997). The novelty of our result simply comes from the fact that we have explicitly allowed for a monopoly period before the formation of a duopoly. By supplying a low-quality good, the first entrant reduces costs (which are quadratic in the quality level), and the possibility of enjoying monopoly power in the first period more than offsets the decrease in profits associated with being the low-quality supplier in the next one.

We may also compare the degrees of differentiation associated with the two equilibria, to verify that:

$$\left(q_H^{\mathfrak{f}} - q_L^{\mathfrak{l}}\right) - \left(q_H^{\mathfrak{l}} - q_L^{\mathfrak{f}}\right) = \frac{\Phi - \Psi}{16c} \tag{31}$$

is positive, increasing and concave in $\rho$, for all $\rho \in [0, \infty)$; moreover, if $\rho = 0$ then $(\Phi - \Psi) / (16c) = \left(37 - \sqrt{1081}\right) / (16c) \cong 0.257/t$ and

$$\lim_{\rho \to \infty} \frac{\Phi - \Psi}{16c} = \frac{1}{3c}. \tag{32}$$

Accordingly, we can state:

**Proposition 8** *Product differentiation is larger if the leader provides the low-quality product, than conversely.*

The reason for this result is that $q_H^{l} > q_H^{l} > q_M$, i.e., the leader comes closer to the pure monopoly quality when supplying the low quality rather than the high one. Put differently, choosing to enter first with the high quality prevents the leader from appropriately exploiting monopoly power in the first period. Since distorting quality downwards is inherent to the nature of a monopolist, by entering with the low-quality good the leader gets two eggs in one basket: it enhances surplus extraction through monopoly pricing in the first period and it prepares to the opening of the duopoly phase, where the resulting product differentiation will be large enough to keep prices well above marginal costs.

Now we pass on to examine the welfare performance of the market in the two cases, in order to verify whether the conclusion reached in the single-period game (Lemma 2) is robust to the introduction of a monopoly phase. This last step is needed to clarify what kind of social preferences there exist concerning the entry process over the entire time span, and therefore whether a planner or a policy maker should worry at all about the evolution of the industry. In general, the definition of the discounted social welfare over the two periods is:

$$SW \equiv \pi_1^M + \frac{\pi_1^D + \pi_2^D}{1 + \rho} + CS \tag{33}$$

where discounted consumer surplus is:

$$CS \equiv \int_{\overline{\theta}-1}^{\overline{\theta}} (sq_1 - p_M)\, ds + \frac{1}{1+\rho} \left( \int_{\overline{\theta}-1}^{\widehat{\theta}} (sq_L - p_L)\, ds + \int_{\widehat{\theta}}^{\overline{\theta}} (sq_H - p_H)\, ds \right). \tag{34}$$

The relevant equilibrium expressions can be calculated using the equilibrium values of prices and qualities in the two settings, to obtain:

**Proposition 9** *Discounted social welfare is higher when the leader chooses to offer the low-quality good than the high-quality one, for all admissible values of $\rho$ and $\overline{\theta}$.*

Therefore, the equivalent of Lemma 2 cannot hold in the two-period game, as here there is no conflict between private and social incentives as to the sequence of entry. The reason is that average quality is higher when the leader enters with a low quality, as it can be ascertained from the difference $\left(q_L^{\mathfrak{l}} + q_H^{\mathfrak{f}}\right)/2 - \left(q_L^{\mathfrak{f}} + q_H^{\mathfrak{l}}\right)/2$, which, on the basis of Proposition 8, is always positive in the admissible parameter range.

# 4 Concluding remarks

We have analysed a simple model of sequential entry in a market for vertically differentiated goods, showing that, if the monopoly power enjoyed *ad interim* by the first entrant is properly accounted for, then the entry game produces a unique subgame perfect equilibrium where the first and second entrants prefer to supply the low- and the high-quality good, respectively. Then, we have shown that there is no conflict between private and social incentives, since welfare is higher when the first entrant supplies the low-quality good, than in the opposite case.

We have carried out our analysis under convex variable costs of quality improvement, full market coverage and a two-period model. A desirable extension of the present model would consist in relaxing either assumption (or all of them) to test for the robustness of our conclusions.

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