Strategic Urban Development under Uncertainty.

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(preliminary and incomplete - please do not quote)

June 19, 2006

Abstract

Aim of this paper is to analyse the equilibrium strategies of two developers in the real estate market, when demands are asymmetric. In particular, we are able to consider three key features of the real estate market. First, the cost of redevelop a building is, at least partially, irreversible. Second, the rent levels for different building vary stochastically over time. Third, demand functions for space are interrelated and may produce positive or negative externalities. Using the method of option pricing theory, we address this issue at three levels. First, we model the investment decision of a firm as a pre-assigned leader as a dynamic stochastic game. Then, we solve for the non-cooperative (decentralised) case, and for the perfectly cooperative case, in which redevelopment of an area is coordinated between firms. Finally, we analyse the efficiency/inefficiency of the equilibria of the game. We find that if one firm has a significantly large comparative advantage, the pre-emptive threat from the rival will be negligible. In this case, short burst and overbuilding phenomena as predicted by Grenadier (1996) will occur only as a limiting case.

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1 Introduction

Strategic urban development is one of the most critical determinants of success of a town. In this paper, we focus on one aspect of the (re)development decisions: the investment decisions coupled with the choice of the mix of building types, i.e. office, retail, industrial, residential.

How should a firm decide between waiting and investing at once? How should it value the different options? Which are the impacts of the mix of building types on the developer investment decisions? How can we explain the overbuilding phenomena of last years?

The present paper attempts to provide an answer to the above questions. In a real estate market comparative advantages of firms in real estate investment are differentiated by their pricing, rather than cost containment strategies. Therefore, by using different inverse demand functions for firms in the model, comparative advantages of firms and their effects on optimal timing in equilibrium can be explicitly examined. We develop a continuous time stochastic oligopoly model to analyse the sequence of events which originates a new urban area and use it to investigate the interaction of the various forces which may delay or anticipate market's creation. We find the conditions that may lead the ones or the others to prevail.

In our model we consider the investment decision of a real estate agent intending to (re)develop his fraction of building in an area. The investment will create a new market and the firm will be the market pioneer. We assume, for simplicity, that the investment is in a single new project and the investment expenditure is known and fixed but once made it is irreversible. The firm faces exogenous uncertainty about market conditions, which reflect in rent level. Moreover, the demands for space are interrelated. The use of space by one tenant may provide either positive or negative externalities for other tenants. A positive interaction between tenant types would increase the landlord's demand for a "diversified mix" of tenants (e.g. a shopping center). Conversely, a negative externality effect would occur when the use of space by one type of tenant impinges upon the efficient use of space of another. A mix of heavy industrial use with residential or commercial use would be such an example. Thus, the developer must take into account both the rent levels and the interaction effects in choosing is optimal investment policy.

We focus on two different economic settings. We first consider the case of a firm which is able to promote the development of a new area. The firm, as the pioneer of the resulting market, may choose the timing of the investment without bothering about other potential entrants (or, in other words, may act as a pre-designated leader) and enjoys furthermore an extreme first-mover advantage which forces any subsequent entrant to accept the role of follower in dynamic game[1]. The rationale behind such modelling assumption is that there

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[1] Alternatively, it can be argued that under uncertainty larger firms have a relative advantage in making commitments credibly and are inclined to move first, while smaller firms prefer to move second (see, e.g., Hay and Liu, 1998). With firms of different size, it seems therefore reasonable to model the outcome of oligopoly by a Stackelberg equilibrium.

are many economic instances in which long-run first-mover advantages arise naturally. The second economic setting we investigate is a situation where two firms may both potentially invest and thus begin development of a new area. Neither firm can now be absolutely sure to be the first to enter the market and strategic considerations may now presumably play a significant role. This second modelling strategy is intended to describe a competitive situation characterised by more limited novelty in the innovation and, correspondingly, by inferior market pioneering advantages.

The general methodology adopted in the paper is that of stochastic stopping time games (Dutta and Rustichini, 1993), whilst our basic assumptions can be contrasted with those of Dixit and Pindyck (1994), Smets (1991), Williams (1993) and Grenadier (1996 and 2002).Williams (1993) focus on the distinguishing features of a real estate market and develop a model of strategic interactions between developers. These features are summarised in the following points: i) each real asset produces goods or services that consumers demand with a finite elasticity; ii) the rate at which assets can be developed is limited by developer's capacity; iii) the supply of undeveloped assets is limited and iv) the ownership of undeveloped assets can be monopolistic, oligopolistic or competitive. The significance of these properties results in the optimal exercise policy and in the market values of real estate vs. financial assets and derives an equilibrium set of exercise strategies for real estate developers, where equilibrium development is symmetric and simultaneous. He makes a new methodological point in the real option literature applied to the real estate market. In contrast to the standard literature, Williams identifies a region of optimal exercise, replacing the single point of optimal exercise in all previous models of real options. Grenadier (1996) uses a duopolistic game theoretic approach to options exercise to explain real estate developers investment decisions. Developers are characterized by two symmetric demands and are indifferent as to who will take the role of a leader and/or a follower and, fearing the preemption by competitors, proceed into a market equilibrium in which all development occurs during a market downturn. He identifies the causes of periods of irrational overbuilding in the interaction between the fear of preemption and the time to build. Compared to Williams', in Grenadier's model, equilibrium development may arise endogenously as either simultaneous or sequential. Although this literature has made a great step toward a better understanding of investment decisions, the contribution of the real option literature to the understanding of the real estate market is still limited.

The paper is organised as follows: Section 2 is devoted to the set up of the model. The specification will serve for the subsequent analysis. In Section 3 the analysis is performed with reference to a duopolistic market in which the leader is pre-assigned, i.e an extreme first-mover advantage (which allows the pioneer to dominate the market) is considered. After deriving the value of pursuing both the leader and the follower strategy, firms' investment behaviour is derived. Section 4 presents the case of competition without pre-emption, i.e. a more limited effect in favour of the first entrant (with pioneer and follower competing under the same conditions) is analysed, and Section 5 provides the analysis of a cooperative solution that will be used to identify the efficiency/inefficiency of

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the various market structures considered. Section 6 concludes the paper.

2 The Real Estate Market Development

In this section, we present and analyse in some detail the set up of a simple continuous-time model of irreversible investment to better understand the implications of the reakestate market above described.

Let us consider two real estate developers, denoted by $i = L, F$, which own respectively a fraction w and $1 - w$ of buildings in a town. The total number of buildings is normalised at 1. Both owners have an opportunity to redevelop their properties into new, superior buildings or change their final destination. In this case they can earn potentially greater rentals. Thus, each owner holds an option to develop. The option to develop has an exercise price equal to I, the cost of construction. To keep matters as simple as possible, we assume that I is constant over time and (re)development has no operating cost. Initially, building rents, $\bar{R}_i$, are defined as

$$\bar{R}_L = \bar{R}w$$

$$\bar{R}_F = \bar{R}(1 - w)$$

The exercise of the development option will result in repercussion on both the option exerciser as well as the other building owner. Let us assume L to be the leader (the one who first exercise his development option). The leader pays an initial construction cost today and loses current rentals [on the existing buildings]. New buildings will yield potentially higher rental rate according to a demand function characterized by evolving uncertainty:

$$\pi_L = R_L = [\theta - \alpha w]\,(w)$$

It represents the leader's profits for new/redeveloped buildings, where the α's represent the own quantity effects. In this formulation uncertainty comes from the demand side once the redevelopment activity occurs and, more in particular, we assume that the demand parameter θ follows the geometric Brownian motion

$$d\theta = \theta\mu dt + \theta\sigma dw \tag{1}$$

where μ is the instantaneous expected growth rate of the market, σ is the instantaneous variance and dw is a standard normal Wiener process. It follows that the market demand curve is subject to aggregate shocks so that the developer knows current demand conditions but cannot predict future changes. This option exercise also affects the fortunes of the follower. The competitor's construction of an improved building can either improve or lessen the demand for the existing building and his profit becomes

$$\pi_F = \bar{R}_F + \eta w(1 - w)$$

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where η represents the interaction effect, e.g. positive η denotes tenant types which interact favorably[2]. This can be the case of shops, where a different mix of shop in a borough permits convenient shopping for customers and increases also the rents of the house.

Now consider the impact of the follower's exercise of the development option on both owner. The follower will pay the cost of construction, lose current rent, and begin receiving rent on the new (or improved) buildings. The leader is also affected because he can now profit from the complementarities of constructions. After the follower has invested the demand functions will be:

$$\pi_L = \left[\theta - \alpha(w)\right](w) + \varepsilon_1 w(1-w)$$

$$\pi_F = \left[\theta - \alpha(1-w)\right](1-w) + \varepsilon_2(w)(1-w)$$

where ε_1 and ε_2 indicates the complementarity of developments[3]. Negative ε_i $(i = 1, 2)$ denotes, for examples, tenant types which interact unfavorably. It is the case of a mix of heavy industrial use with residential or commercial use.

3   Equilibrium without pre-emption

As before, let us start our analysis with a model describing a situation in which there exists a pre-designed leader[4]. Let us start by assuming that the pre-designed leader, L, and follower, F, invest at different points. The expected total discounted profits of the leader and the follower are derived in what follows. As usual in dynamic games, the stopping time game is solved backwards, in a dynamic programming fashion.

The Follower's Problem   Let us first value the payoff of being a follower, denoted by $F(\theta)$. It has three different components holding over different ranges of θ. The first, $F_0(\theta)$, describes the value of investment before the leader has invested; the existing space yields a profit $\bar{R}_F$ per unit of time and its present discounted value is $\frac{\bar{R}_F}{r}$. Moreover, the follower holds the option to redevelop the existing space for the new one, conditional to the leader having already invested. The option to invest should be valued accordingly. Let us then assume that the leader has already redeveloped his property, and the follower has now to choose his redevelopment strategy to maximise his option's value. This is the second region, $F_1(\theta)$. The value of the follower can be characterised as a portfolio containing the existing properties, yielding a profit $\bar{R}_F + \eta w(1-w) > 0$ per

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[2] It can also be negative. In this case it denotes tenant type which interacts unfavourably. In both cases, η must be large enough to ensure positive π_F.

[3] This kind of complementarity has been defined "two way complementarity" by Weeds (2002). It gives rise to a temporary Second Mover Advantage: while the leader alone has invested, it does not benefit from the complementaritiy, but after the follower invests the positions of the two firms are symmetric.

[4] This can be the case where a developer owns more of the 50% of the available space normalized at 1.

unit of time and a present discounted value of $\frac{\bar{R}_F + \eta w(1-w)}{r}$, plus an option to exchange the existing properties with the new one. Finally, in the third region, $F_2(\theta)$, paying an irreversible adoption cost I, the follower can redevelop his property and obtain an instantaneous profit $[\theta - \alpha(1-w)](1-w) + \varepsilon_2(w)(1-w)$, with a present discounted value of $\frac{\theta(1-w)}{r-\mu} + \frac{[\alpha(1-w)](1-w)+\varepsilon_2(w)(1-w)}{r}$. Let us first derive the second and the third region. In order to derive the follower's optimal investment rule, notice that at each point in time the follower can either invest, and take the termination payoff, or can wait for an infinitesimal time dt and postpone the decision. The payoff of the second strategy consists of the profit flow during time period dt plus the expected discounted capital gain. Denoting by $F_{F,1}(\theta)$ the option value to invest, the Bellman equation of the problem is

$$F_{F,1}(\theta) = Max \left\{ \frac{\bar{R}_F + \eta w(1-w)}{r} - I, \frac{1}{1+rdt} E\left[F_{F,1}(\theta + d\theta \,|\theta)\right] \right\} \quad (2)$$

where E denotes the expectation operator.

Prior to investment the firm holds the opportunity to invest. It receives a profit flow $\frac{\bar{R}_F + \eta w(1-w)}{r}$, but it may experience a capital gain or loss on the value of this option, $dF_{F,1}$. Hence, in the continuation region, i.e. the RHS of equation (2), the Bellman equation for the value of the investment opportunity, $F_{F,1}(\theta)$, is given by

$$rF_{F,1}dt = E\left(dF_{F,1}\right) \quad (3)$$

Expanding $dF_{F,1}$ using Ito's lemma we can write

$$dF_{F,1} = F'_{F,1}(\theta)\,d\theta + \frac{1}{2}F''_{F,1}(\theta)\,(d\theta)^2 \quad (4)$$

Substituting from (1) and noting that $E(dw) = 0$ we can write

$$E\left(dF_{F,1}\right) = \mu\theta F'_{F,1}(\theta)\,dt + \frac{1}{2}\sigma^2\theta F''_{F,1}(\theta)dt \quad (5)$$

After some simple substitution, the Bellman equation entails the following second-order differential equation

$$\frac{1}{2}\sigma^2\theta F''_{F,1}(\theta) + \mu\theta F'_{F,1}(\theta) - rF_{F,1} = 0 \quad (6)$$

From (1) it can be seen that if θ ever goes to zero it stays there forever. Therefore the option to invest has no value when $\theta = 0$; $F_{F,1}(\theta)$ must satisfies the following boundary condition

$$F_{F,1}(0) = 0 \quad (7)$$

The general solution for the differential equation (6) is

$$F_{F,1}(\theta) = B_1\theta^\beta + B_2\theta^\lambda \quad (8)$$

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where $\beta > 1$ and $\lambda < 0$ are respectively the positive and the negative root of the fundamental characteristic equation[5] $Q(z) = \frac{1}{2}\sigma^2 z (z-1) + \mu z - r$, and B_1 and B_2 are unknown constant to be determined.

Imposing the boundary condition (7) the value of the option to invest is

$$F_{F,1}(\theta) = B_1 \theta^\beta \tag{9}$$

and the option value of waiting is $F_1(\theta) = \frac{\bar{R}_F + \eta w(1-w)}{r} + B_1 \theta^\beta$. The first part of $F_1(\theta)$ is the expected value of the firm if the firm would never invest and the second part is the option value to invest derived above. The value in the first region is derived in the same way. The value of the option to invest is $F_{F,0} = B_0 \theta^\beta$, and the expected value of the firm if it would never invest is $\frac{\bar{R}_F}{r}$. By summing up these to components gives $F_0(\theta) = \frac{\bar{R}_F}{r} + B_0 \theta^\beta$, that is the option value of waiting in the first region.

We next consider the value of the firm in the stopping region, in which the value of θ is such that it is optimal to invest at once. This is the third region, $F_2(\theta)$. Since investment is irreversible, the value of the agent in the stopping region is given by the expected value alone with no option value terms. The value of the follower adopting the new technology is given by the following expression:

$$F_2(\theta) = E \left[\int_t^{+\infty} \left([\theta - \alpha(1-w)](1-w) + \varepsilon_2(w)(1-w) \right) e^{-r(\tau-t)} d\tau - I(1-w) \right] \tag{10}$$

that is

$$F_2(\theta) = \frac{\theta(1-w)}{r-\mu} + \frac{[\alpha(1-w)](1-w) + \varepsilon_2(w)(1-w)}{r} - I(1-w) \tag{11}$$

The boundary between the continuation region and the stopping region is given by a trigger point θ_F of the stochastic process such that continued delay is optimal for $\theta < \theta_F$ and immediate investment is optimal for $\theta \geq \theta_F$. The optimal stopping time is then defined as the first time that the stochastic process θ hits the interval $[\theta_F, \infty)$ from below. Putting together the three regions gives the follower's value function $F(\theta)$:

$$F(\theta) = \begin{cases} \frac{\bar{R}_F}{r} + B_0 \theta^\beta & \theta < \theta_L \\ \frac{\bar{R}_F + \eta w(1-w)}{r} + B_1 \theta^\beta & \theta \in [\theta_L, \theta_F) \\ \frac{\theta(1-w)}{r-\mu} + \frac{[\alpha(1-w)](1-w) + \varepsilon_2(w)(1-w)}{r} - I(1-w) & \theta \geq \theta_F \end{cases} \tag{12}$$

Following Dixit and Pindyck (1994), the value matching and smooth pasting conditions are used to determine the critical value describing the boundary between the continuation and stopping regions, along with the unknown coefficient

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[5] See Dixit & Pindyck (1996), pp. 142-143, for details.

B_1. This condition requires the two components of the follower's value function to meet smoothly at θ_F with equal first derivatives, which together with the value matching condition implies

$$\begin{cases} \frac{\bar{R}_F + \eta w(1-w)}{r} + B_1\theta^\beta = \frac{\theta(1-w)}{r-\mu} + \frac{[\alpha(1-w)](1-w)+\varepsilon_2(w)(1-w)}{r} - I(1-w) \\ \beta B_1 \theta^{\beta-1} = \frac{1-w}{r-\mu} \end{cases}$$

Solving the above system, we can compute the value of the unknown B_1 and the optimal trigger point θ_F:

$$B_1 = \frac{1-w}{r-\mu} \cdot \frac{1}{\beta} \cdot \theta_F^{1-\beta} \tag{13}$$

$$\theta_F = \frac{\beta}{\beta-1} \cdot \frac{(r-\mu)}{r} \cdot \left[\bar{R} + \eta(w) + \alpha(1-w) - \varepsilon_2(w) + rI\right] \tag{14}$$

It is important to note that the optimal trigger point θ_F it is not influenced by the complementarity of developments (ε_1 and ε_2).

Proposition 1 *Conditional on the Leader having redeveloped his properties, the optimal Follower strategy is to invest the first moment that θ_t equals or exceed the trigger value θ_F, as defined in equation (14). That is, the optimal entry time of the follower, T_F, can be written as:*

$$T_F = \inf\left\{t \geq 0 : \theta \geq \frac{\beta}{\beta-1} \cdot (r-\mu) \cdot \left[\frac{\bar{R}+\eta w}{r} + \frac{[\alpha(1-w)-\varepsilon_2 w]}{r} + I\right]\right\} \tag{15}$$

The value of the unknown constant B_0 is found by considering the impact of the leader's investment on the payoff to the follower. When θ_L is first reached, the leader invests and the follower payoff is altered either positively or negatively. Since the value functions are forward-looking, $F_0(\theta)$ anticipates the effect of the leader's action and must therefore meet $F_1(\theta)$ at θ_L. Hence, a value matching condition holds at this point; however there is no optimality on the part of the follower, and so no corresponding smooth pasting condition. This implies that

$$\frac{\bar{R}_F}{r} + B_0\theta_L^\beta = \frac{\bar{R}_F + \eta w(1-w)}{r} + B_1\theta_L^\beta$$

$$B_0 = \frac{\eta(w)(1-\text{w})}{r}\theta_L^{-\beta} + \frac{1-w}{r-\mu} \cdot \frac{1}{\beta} \cdot \theta_F^{1-\beta} \tag{16}$$

Note that θ_F is independent of the point at which the leader invests: given that the firm invests second, the precise location of the leader's trigger point is irrelevant. However, it is inversely related to the magnitude of the spillover caused by the leader investment. The effect of uncertainty is standard from the real option theory. Since $\frac{\partial \beta}{\partial \sigma} > 0$ a greater uncertainty induce an higher trigger

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value. By simple substitution, the value of being the follower is thus given by the following expression:

$$F\left(\theta\right)=\begin{cases}\frac{\bar{R}_{F}}{r}+\frac{\eta(w)(1-w)}{r}\left(\frac{\theta}{\theta_{L}}\right)^{\beta}+\frac{1-w}{r-\mu}\cdot\frac{1}{\beta}\cdot\theta_{F}\left(\frac{\theta}{\theta_{F}}\right)^{\beta} & \theta<\theta_{L}\\ \frac{[\bar{R}+\eta(w)](1-w)}{r}+\frac{1-w}{r-\mu}\frac{1}{\beta}\theta_{F}\left(\frac{\theta}{\theta_{F}}\right)^{\beta} & \theta\in[\theta_{L},\theta_{F})\\ \frac{\theta(1-w)}{r-\mu}-\frac{\alpha(1-w)^{2}}{r}+\frac{\varepsilon_{2}w(1-w)}{r}-I\left(1-w\right) & \theta\geq\theta_{F}\end{cases} \quad (17)$$

The Leader's Problem. The value of the leader, denoted by $L\left(\theta\right)$, can be expressed as follows:

$$L\left(\theta\right)=\begin{cases}\frac{\overline{R}(\text{w})}{r}+B_{L0}\theta^{\beta} & \theta<\theta_{L}\\ \frac{w\theta}{r-\mu}-\frac{\alpha(w)^{2}}{r}+B_{L1}\theta^{\beta}-Iw & \theta\in[\theta_{L},\theta_{F})\\ \frac{w\theta}{r-\mu}+\frac{\varepsilon_{1}w(1-w)}{r}-\frac{\alpha(w)^{2}}{r}-Iw & \theta\geq\theta_{F}\end{cases} \quad (18)$$

where B_{L0} and B_{L1} are the coefficient of the option value to invest. Starting from equation (18) one can observe that when θ_{F} is first reached, the follower invests and the leader's expected flow payoff is altered. Since value functions are forward-looking, $L_{1}\left(\theta\right)$ anticipates the effect of the follower's action and must therefore meet $L_{2}\left(\theta\right)$ at θ_{F}. Hence, a value matching condition holds at this point; however there is no optimality on the part of the leader, and so no corresponding smooth pasting condition. This implies

$$\frac{w\theta}{r-\mu}-\frac{\alpha(w)^{2}}{r}+B_{L1}\theta^{\beta}-Iw=\frac{w\theta}{r-\mu}+\frac{\varepsilon_{1}w(1-w)}{r}-\frac{\alpha(w)^{2}}{r}-Iw$$

that gives

$$B_{L1}=\frac{\varepsilon_{1}w(1-w)}{r}\theta_{F}^{-\beta} \quad (19)$$

The usual value matching and smooth pasting conditions at the optimally-chosen θ_{L} determine the other unknown variables:

$$\begin{cases}\frac{\overline{R}(\text{w})}{r}+B_{L0}\theta^{\beta}=\frac{w\theta}{r-\mu}-\frac{\alpha(w)^{2}}{r}+B_{L1}\theta^{\beta}-Iw\\ \beta B_{L0}\theta^{\beta-1}=\frac{w\theta}{r-\mu}+\beta B_{L1}\theta^{\beta-1}\end{cases}$$

Solving the system, we can compute the value of the unknown B_{L0} and the optimal trigger point θ_{L}:

$$\theta_{L}=\frac{\beta}{\beta-1}\frac{r-\mu}{r}\left[\overline{R}+\alpha w+rI\right] \quad (20)$$

Similar to the optimal trigger point θ_{F}, also θ_{L} it is not influenced by the complementarity of developments (ε_{1} and ε_{2}).

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$$B_{L0} = \frac{1}{\beta} \left[\frac{w}{r - \mu} \right] \theta_L^{1-\beta} + \frac{\varepsilon_1 w (1 - w)}{r} \theta_F^{-\beta} \qquad (21)$$

The following proposition summarises the result.

Proposition 2 *Conditional on roles exogenously assigned, the optimal Leader strategy is to redevelop his properties the first moment that θ_t equals or exceed the trigger value θ_L, as defined in equation (20). That is, the optimal entry time of the Leader, T_L, can be written as:*

$$T_L = \inf t \geq 0 : \theta_L = \frac{\beta}{\beta - 1} \cdot \frac{r - \mu}{r} \left[\overline{R} + \alpha w + rI \right] \qquad (22)$$

Putting together the three region above derived, by simple substitution we are able to write the leader's value function:

$$L(\theta) = \begin{cases} \frac{\overline{R}(w)}{r} + \frac{1}{\beta} \left[\frac{w}{r - \mu} \right] \theta_L \left(\frac{\theta}{\theta_L} \right)^{\beta} + \frac{\varepsilon_1 w (1 - w)}{r} \left(\frac{\theta}{\theta_F} \right)^{\beta} & \theta < \theta_L \\ \frac{w\theta}{r - \mu} - \frac{\alpha(w)^2}{r} + \frac{\varepsilon_1 w (1 - w)}{r} \left(\frac{\theta}{\theta_F} \right)^{\beta} - Iw & \theta \in [\theta_L, \theta_F) \\ \frac{w\theta}{r - \mu} + \frac{\varepsilon_1 w (1 - w)}{r} - \frac{\alpha(w)^2}{r} - Iw & \theta \geq \theta_F \end{cases} \qquad (23)$$

In short, Propositions (1) and (2) define respectively the optimal entry time of the leader and of the follower. It is worth noticing that the optimal entry time of the follower is positively affected by the interaction effect η and negatively affected by the complementarily ε_2. Furthermore, in order to have $\theta_F > \theta_L$, it should be that

$$\eta > \alpha \frac{[w - (1 - w)]}{w} + \varepsilon_2$$

In particular, when each developer holds half of the space, then η (the interaction effect that the follower suffers when only the leader develops) has to be higher than ε_2 (the complementarity effect). In this case there exists a unique sequential equilibrium. Otherwise, an investment cascades might occur.

4 Equilibrium with pre-emption

Let us now assume that the role of the leader and that of the follower are determined endogenously[6]. As before, let us assume that one firm (the pre-emptor) invests strictly before the other. The follower's value function and the trigger point is the same as for the model without preemption (see equation 14). The leader's value function is as described in the previous section. However, without the ability to precommit to a defined investment strategy at the beginning of

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[6] It can be the case of a fragmented market, where, for example each developer holds half of the space.

the game, the leader's investment trigger cannot be derived as the solution to a single agent optimization problem. This means the leader can no longer choose its investment point optimally, as if the roles were preassigned. Instead, the first firm to invest does so at the point at which it prefers to lead rather than follow. Hence, the investment point, denoted in what follows θ_P, is defined by the indifference between leading and following as follows:

$$V_L(\theta_P) - I = F_F(\theta_P)$$

In order to simplify notation, let us define $A = \frac{\varepsilon_1 \beta W}{a}$, $a = \frac{r}{r-\mu}$, $B = \frac{\alpha + Ir}{a} + (\varepsilon_1 - \varepsilon_2) \frac{w(1-w)}{[w-(1-w)]}$.

Proposition 3 *If $\theta_F > B$ then there exists a unique endogenous equilibrium outcome at $\theta_P \equiv \theta_F$ with the following properties:*

$$\begin{array}{ll} V_L(\theta) - I < F_F(\theta) & for \ \theta < \theta_P \\ V_L(\theta) - I = F_F(\theta) & for \ \theta = \theta_P \\ V_L(\theta) - I > F_F(\theta) & for \ \theta_P < \theta < \theta_F \\ V_L(\theta) - I = F_F(\theta) & for \ \theta \geq \theta_F \end{array}$$

Proof. Let us define the function $\Delta(\theta) = L_1(\theta) - F_1(\theta)$, describing the gain of pre-emption, where $L_1(\theta)$ is conditional on the "pre-emptor" having invested, and $F_1(\theta)$ is the option value of the follower. By using equations (18) and (9), we get

$$\Delta(\theta) = \frac{w\theta}{r-\mu} - \frac{\alpha(w)^2}{r} - Iw + \varepsilon_1 \frac{w(1-w)}{r} \left(\frac{\theta}{\theta_F}\right)^\beta +$$

$$- \frac{[\overline{R} + \eta(w)](1-w)}{r} - \frac{(1-w)}{r-\mu} \cdot \frac{1}{\beta} \cdot \theta_F \left(\frac{\theta}{\theta_F}\right)^\beta. \quad (24)$$

First, we establish the existence of a root for $\Delta(\theta)$ in the interval $(0, \theta_F)$. Evaluating at $\theta = 0$ yields $\Delta(0) = -\frac{\alpha(w)^2}{r} - Iw - \frac{[\overline{R} + \eta(w)](1-w)}{r} < 0$. Similarly, evaluating at $\theta = \theta_F$ yields $\Delta(\theta_F) = \frac{r}{r-\mu}\theta_F - \alpha - Ir + \frac{(w)^2}{r} - Iw + (\varepsilon_1 - \varepsilon_2) \frac{w(1-w)}{[w-(1-w)]}$, i.e

$$\Delta(\theta_F) > 0 \text{ if } \theta_F > B \quad (25)$$

Therefore, $\Delta(\theta)$ must have at least one root in the interval $(0, \theta_F)$. Finally, some algebraic manipulation yields $\Delta'(0) = \frac{w}{r-\mu} > 0$ and $\lim_{\theta \to \theta_F} \Delta(\theta)' = \frac{w}{r-\mu} + \beta\varepsilon_1 \frac{w(1-w)}{r} - \frac{(1-w)}{r-\mu}\theta_F \gtrless 0$ if $\theta_F \gtrless A + \frac{w}{1-w}$. To prove uniqueness, one merely needs to demonstrate strict concavity (convexity) over the interval. Differentiating $\Delta(\theta)$ twice yields: $\Delta''(\theta) = (\beta - 1)\left[\beta\varepsilon_1 \frac{w(1-w)}{r} - \frac{(1-w)}{r-\mu}\theta_F\right] \left(\frac{\theta}{\theta_F}\right)^{\beta-1} \gtrless 0$ if $\theta_F \gtrless A$.Thus, the root is unique. The Appendix shows a graphical interpretation. ∎

Proposition 4 If $\theta_F < B$ and $\theta_F < A + \frac{w}{(1-w)}$, then $V_L(\theta) - I < F_F(\theta)$ for all $\theta < \theta_F$. It does not exist an endogenous equilibrium outcome in the interval $(0, \vartheta_F)$.

Proposition 5 If $\theta_F < B$ and $\theta_F > A + \frac{w}{(1-w)}$ then:

a) If $\Delta(\theta^*) < 0$ then it does not exist an endogenous equilibrium;

b) If $\Delta(\theta^*) = 0$ then there exists an unique endogenous equilibrium $\theta_P^* \in (0, \theta_F)$ with the following properties:

$$
\begin{aligned}
V_L(\theta) - I < F_F(\theta) &\quad \text{for } \theta < \theta_P^* \\
V_L(\theta) - I = F_F(\theta) &\quad \text{for } \theta = \theta_P^* \\
V_L(\theta) - I > F_F(\theta) &\quad \text{for } \theta_P^* < \theta < \theta_F \\
V_L(\theta) - I = F_F(\theta) &\quad \text{for } \theta \geq \theta_F
\end{aligned}
$$

.

c) If $\Delta(\theta^*) > 0$ then there exists $\theta_{P,1}^*$ and $\theta_{P,2}^* \in (0, \theta_F)$ with the following properties:

$$
\begin{aligned}
V_L(\theta) - I < F_F(\theta) &\quad \text{for } \theta < \theta_{P,1}^* \\
V_L(\theta) - I = F_F(\theta) &\quad \text{for } \theta = \theta_{P,1}^* \\
V_L(\theta) - I > F_F(\theta) &\quad \text{for } \theta_{P,1}^* < \theta < \theta_{P,2}^* \\
V_L(\theta) - I = F_F(\theta) &\quad \text{for } \theta = \theta_{P,2}^* \\
V_L(\theta) - I < F_F(\theta) &\quad \text{for } \theta_{P,2}^* < \theta < \theta_F \\
V_L(\theta) - I = F_F(\theta) &\quad \text{for } \theta \geq \theta_F
\end{aligned}
$$

Proof. Let us define the function $\Delta(\theta) = L_1(\theta) - F_1(\theta)$, describing the gain of pre-emption, where $L_1(\theta)$ is conditional on the "pre-emptor" having invested, and $F_1(\theta)$ is the option value of the follower. By using equations (18) and (9), we get

$$
\Delta(\theta) = \frac{w\theta}{r-\mu} - \frac{\alpha(w)^2}{r} - Iw + \varepsilon_1 \frac{w(1-w)}{r} \left(\frac{\theta}{\theta_F}\right)^\beta +
$$

$$
- \frac{[\overline{R} + \eta(w)](1-w)}{r} - \frac{(1-w)}{r-\mu} \cdot \frac{1}{\beta} \cdot \theta_F \left(\frac{\theta}{\theta_F}\right)^\beta. \tag{26}
$$

First, we establish the existence of a root for $\Delta(\theta)$ in the interval $(0, \theta_F)$. Evaluating at $\theta = 0$ yields $\Delta(0) = -\frac{\alpha(w)^2}{r} - Iw - \frac{[\overline{R}+\eta(w)](1-w)}{r} < 0$. Similarly, evaluating at $\theta = \theta_F$ yields $\Delta(\theta_F) = \frac{r}{r-\mu}\theta_F - \alpha - Ir + \frac{(w)^2}{r} - Iw + (\varepsilon_1 - \varepsilon_2)\frac{w(1-w)}{[w-(1-w)]}$, i.e

$$
\Delta(\theta_F) > 0 \text{ if } \theta_F < B \tag{27}
$$

Finally, some algebraic manipulation yields $\Delta'(0) = \frac{w}{r-\mu} > 0$.

It is now easy to prove that:

- if $\theta_F < A$ than $\lim_{\theta \to \theta_F} \Delta(\theta)' > 0$ and $\Delta''(\theta) > 0$; this implies that it does not exist an equilibrium;

- if $\theta_F < A + \frac{w}{1-w}$ than $\lim_{\theta \to \theta_F} \Delta(\theta)' > 0$ and $\Delta''(\theta) < 0$; this implies it does not exist an equilibrium;

- if $\theta_F > A + \frac{w}{1-w}$ than $\lim_{\theta \to \theta_F} \Delta(\theta)' < 0$ and $\Delta''(\theta) < 0$; defining $\theta^* = \arg\max \Delta(\theta)$ we get that if $\Delta(\theta^*) < 0$ then it does not exist an endogenous equilibrium; if $\Delta(\theta^*) = 0$ then there exists an unique endogenous equilibrium $\theta_P^* \in (0, \vartheta_F)$ and finally if $\Delta(\theta^*) > 0$ then there exists two endogenous equilibria $\theta_{P,1}^*, \theta_{P,2}^* \in (0, \vartheta_F)$. The Appendix shows a graphical interpretation. Q.E.D.

■

The presence of asymmetric demands implies that short burst and overbuilding phenomena as predicted by Grenadier (1996) will occur only as a limiting case. Specifically, Proposition (5b) describes the case in which Granadier (1996) is verified: it would be optimal for the leader to take the preemptive move and receive higher payoffs from the action, if and only if the trigger value of the follower falls below B. Moreover, we find a similar result in Proposition 3. In this case $\theta_P \equiv \theta_F$ and a cascade investments occurs. More important, Propositions 4, 5a and 5c describe cases not considered in Grenadier's model that reduce his findings to a particular case. Specifically, in Proposition 4 and 5a conditions for a non exsistence of equilibria are derived. These means that there are incentive to follow rather than to lead. Proposition 5c explicit the range of parameter value in which we can have either a multiplicity of equilibria or a non existetence of equilibria.

It is worth noticing that when each developer holds the 50% of the space, the above propositions can be stated as follows:

Proposition 6 *If $\varepsilon_1 + \varepsilon_2 > 0$ then there exists, somewhere in the interval $(0, \vartheta_F)$, a unique endogenous equilibrium outcome at $\theta_P \equiv \theta_F$.*

If $\varepsilon_1 + \varepsilon_2 < 0$ then two sub-cases arise:

a) When $\vartheta_F < A + 1$, then it does not exist, in the interval $(0, \vartheta_F)$, an endogenous equilibrium outcome

b) When $\vartheta_F > A + 1$ then:

b.1) If $\Delta(\theta^*) < 0$ then it does not exist an endogenous equilibrium;

b.2) If $\Delta(\theta^*) > 0$ then there exists two endogenous equilibria;

b.2) If $\Delta(\theta^*) = 0$ then there exists an unique endogenous equilibrium.

Proof. See above. ■

The complementarity relations of development (ε_1 and ε_2) are the crucial variables in determining the strategy played by the two investors. In particular, when $\varepsilon_1 + \varepsilon_2 > 0$ and when ε_1 and ε_2 have opposite sign and the magnitude of the difference is positive then the equilibrium strategy is unique. This condition arises when both ε_1 and ε_2 are positive, i.e. tenant types which interact favorably as in the case of a mix of residential use with commercial use. When $\varepsilon_1 + \varepsilon_2 < 0$ then the existence of a unique endogenous equilibrium is ensured only in one particular case. This condition arises when:

1. both ε_1 and ε_2 are negative, e.g. tenant types which interact unfavorably an in the case of a mix of heavy industrial use with residential or commercial use;

2. ε_1 and ε_2 have opposite sign and the negative one is higher then the positive one. This is the case in which one developer is affected by a negative externality that compensates the positive externality that concerns the other developer.

5 Co-operative Solution

This section analyses the co-operative solution, in which the agents' investment trigger points are chosen to maximise the sum of their two value functions. The objective is to provide a benchmark to identify inefficiencies in the next section.

Let us examine the case when investment is sequential. Two trigger points, θ_{1L} and θ_{2L}, are chosen to maximise the sum of the leader's and follower's value functions, denoted by $C_{L+F}(\theta)$. Using the same steps as before, it is given by

$$
C_{L+F}(\theta) = \begin{cases} \frac{\overline{R}}{r} + B_0\theta^\beta + B_1\theta^\beta & \theta < \theta_{2L} \\ \frac{(\overline{R}+\eta w)}{r}(1-w) + \frac{w\theta}{r-\mu} - \frac{\alpha w^2}{r} + B_2\theta^\beta - Iw + B_3\theta^\beta & \theta \in [\theta_{2L}, \theta_{2F}) \\ \frac{\theta}{r-\mu} - \frac{\alpha(w)^2}{r} - \frac{\alpha(1-w)^2}{r} - I + \frac{w(1-w)(\varepsilon_1+\varepsilon_2)}{r} & \theta \geq \theta_{2F} \end{cases}
$$
(28)

where $B_i, i = 0, 1, 2, 3$ are constants. The co-operative trigger points are determined by the value matching and smooth pasting conditions at both points. Solving the system we get the leader trigger and the follower trigger point, respectively θ_{1L} and θ_{2F}, given by

$$
\begin{cases} \frac{(\overline{R}+\eta w)}{r}(1-w) + \frac{w\theta}{r-\mu} - \frac{\alpha w^2}{r} + B_2\theta^\beta - Iw + B_3\theta^\beta = \\ \quad = \frac{\theta}{r-\mu} - \frac{\alpha(w)^2}{r} - \frac{\alpha(1-w)^2}{r} - I + \frac{w(1-w)(\varepsilon_1+\varepsilon_2)}{r} \\ \quad \frac{w}{r-\mu} + \beta(B_2 + B_3)\theta^{\beta-1} - \frac{1}{r-\mu} \end{cases}
$$

$$
\theta_{2F} = \left(\frac{\beta}{\beta-1}\right)\frac{r-\mu}{r}\left[\overline{R} + \eta w + \alpha(1-w) + rI - w(\varepsilon_1+\varepsilon_2)\right] \quad (29)
$$

$$
\begin{cases} \frac{\overline{R}}{r} + B_0\theta^\beta + B_1\theta^\beta = \frac{(\overline{R}+\eta w)}{r}(1-w) \\ \quad + \frac{w\theta}{r-\mu} - \frac{\alpha w^2}{r} + B_2\theta^\beta - Iw + B_3\theta^\beta \\ \beta(B_0 + B_1)\theta^{\beta-1} = \beta(B_2 + B_3)\theta^{\beta-1} + \frac{w}{r-\mu} \end{cases}
$$

$$
\theta_{1L} = \left(\frac{\beta}{\beta-1}\right)\frac{(r-\mu)}{r}\left[\overline{R} - \eta(1-w) + \alpha(w) + rI\right] \quad (30)
$$

Equation (30) and (29) identify the trigger values of the leader and of the follower. These values define respectively the optimal entry time of the leader and of the follower in a co-operative framework. The results arose are closed

to ones found in previous section. The strong difference between θ_{2F} and θ_F is that the optimal entry time of the follower is not only positively affected by the interaction effect η and negatively affected by the complementarily ε_2, but it is also negatively affected by ε_1 (the complementarily that the leader receive when both develop). Furthermore, comparing θ_{2L} and θ_L we find that the optimal entry time of the leader is now negatively affected by the interaction effect by η (the interaction effect that the follower receive when only the leader develops).

Comparing the trigger values of firms an important results arise. The complementarity relations of development (ε_1 and ε_2) are the crucial variables in determining the strategy played by the two investors. In particular, when $\varepsilon_1 > 0$, the complementarily that the leader receive when both develop is positive (i.e. tenant types which interact favorably as in the case of a mix of residential use with commercial use), then $\theta_{2F} < \theta_F$. According to the economic intuition, in a cooperative setting, if the decision to (re)develop of the follower implies a positive complementarity to the leader's value functions, this decision arises earlier than the non co-operative case. When $\eta > 0$, the interaction effect that the follower receive when only the leader develops (i.e. shops, where a different mix of shop in a borough permits convenient shopping for customers and increases also the rents of the house), then $\theta_{2L} < \theta_L$. According to the economic intuition, in a cooperative setting, if the decision to (re)develop of the leader implies a positive interaction effect to follower's value functions, this decision arises earlier than the non co-operative case.

6 Conclusion

In the real world, it is more realistic modeling the investment behaviors by incorporating the strategic interactions into the real option models. Williams (1993) and Granadier (1996, 2002) were among few researchers that introduce this method of modeling in the real estate applications. Williams (1993) and Granadier (2002) show that the value of waiting option is eroded when the number of developers increases. This results open the way to an important merger of the game theoretic and real options literature in the real estate field. Moreover, in contrast to the standard literature, Williams identifies a region of optimal exercise, replacing the single point of optimal exercise in all previous models of real options. Finally, Grenadier (1996) uses a duopolistic game theoretic approach to options exercise to explain real estate developers investment decisions. He identifies the causes of periods of irrational overbuilding in the interaction between the fear of preemption and the time to build and his model provide a rational equilibrium foundation to this irrational overbuilding. Compared to Williams', in Grenadier's model, equilibrium development may arise endogenously as either simultaneous or sequential.

Although this literature has made a great step toward a better understanding of investment decisions, the contribution of the real option literature to the understanding of the real estate market is still limited. In particular, in these models, firms are assumed to be identical and products are homogeneous. This

symmetric assumption can be useful in selected case but may be inadequate to describe real practices in other cases.

In this paper, we extend the symmetric model proposed by Granadier (1996) by analysing the equilibrium strategies of two developers in the real estate market, when demands are asymmetric. In particular, we are able to consider three distinguishing features of the real estate market. First, the cost of redevelop a building is, at least partially, irreversible. Second, the rent levels for different building vary stochastically over time. Third, demand functions for space are interrelated and may produce either positive or negative externalities. Furthermore, the asymmetries comes from three elements: the fraction of buildings each developer owns (w and $1 - w$), the interaction effect (η) and the complementarity of developments that can be different when the developer is the follower or the leader (ε_1 and ε_2).

In symmetric demand models, equilibrium strategies either sequential or simultaneous, are driven largely by the action of a comparatively strong leader. This result becomes a special case when we analyse an asymmetric demand. In this case, the optimal entry time of the leader and of the follower and the conditions for the existence of an endogenous equilibrium are affected by the interaction effect, by the complementarity effect and by the fraction of building each developer owns.

In our paper we analyse equilibrium strategies in asymmetric demand model by studying three different cases: i) a firm as a pre-assigned leader, ii) competition without pre-emption, and finally iii) the cooperative case, i.e. the agents' investment trigger points are chosen to maximise the sum of their two value functions. In the pre-assigned case, the theory here developed predicts that the optimal entry time of the follower is positively affected by the interaction effect η and negatively affected by the complementarily ε_2. Furthermore, an important result arises when we analyse the particular case in which each developer holds half of the space. In this case there exists a unique endogenous equilibrium only if the interaction effect that the follower receive when only the leader develops is higher than the complementarily that the follower receive when both develop. In the case with competition without pre-emption, we found that, by introducing several element of asymmetry in the model, the short burst and overbuilding phenomena as predicted by Grenadier (1996) will occur only as a limiting case. In particular, when one firm has a significantly large comparative advantage, the pre-emptive threat from the rival will be negligible. Moreovere we identified the regions of parameter value η, ε_1 and ε_2, in which we can have either a no equilibrium or a multiplicity of equilibria. The three different regions identified are the following:

(i) $\vartheta_F > B$. In this case it would be optimal for the leader to take the preemptive move and receive higher payoffs from the action, than there exists a unique endogenous equilibrium as predicted by Grenadier (1996).

(ii) $\vartheta_F < B$ and $\vartheta_F < A + \frac{w}{(1-w)}$. In this case both developers receive higher payoffs if they are the follower. Thus no equilibria arise when ϑ is below the follower's trigger point (ϑ_F). When $\vartheta = \vartheta_F$, then both developers start to build

simultaneously.

(ii) $\vartheta_F < B$ and $\vartheta_F > A + \frac{w}{(1-w)}$. In this case three sub-cases arise: (no equilibrium one equilibrium or two equilibria).

Finally, we analysed the co-operative solution, in which the agents' investment trigger points are chosen to maximise the sum of their two value functions. According to the previous cases, the optimal entry time of the leader and of the follower and the conditions for the existence of an endogenous equilibrium are affected by the interaction effect, by the complementarities and by the fraction of buildings each developer owns.

Furthermore, comparing the trigger values of both firms when there is co-operation and when there is not, another important results arose:

(i) when $\varepsilon_1 > 0$. In this case, if the decision to (re)develop of the follower implies a positive complementarity to the leader's value functions, this decision arises, in a cooperative setting, earlier than the non co-operative case

(ii) when $\eta > 0$. In this case, if the decision to (re)develop of the leader implies a positive interaction effect to follower's value functions, this decision arises, in a cooperative setting, earlier than the non co-operative case

Several extensions for future work could be explored. First, we can analyse a microfoundation of the model and its empirical implementation. Second the duopoly game theoretic framework can be extended to include multi-player dynamic game. Finally, we can analyse other elements that imply asymmetric demands.

7 Appendix

By using the pictures below, let's show the intuition of the proof.

1)   Figure (1) shows the function $\Delta(\theta)$ when $\theta_F > B$ and therefore, by (25), $\Delta(\theta_F) > 0$. It is straightforward to show that only one endogenous equilibrium arises in the interval $(0, \theta_F)$.
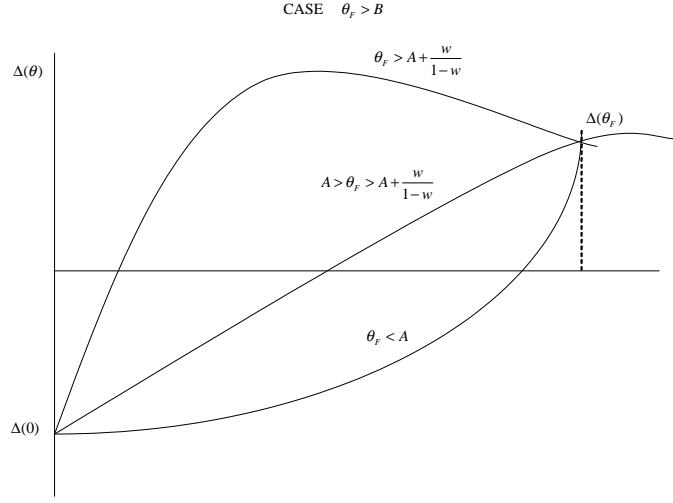

Figure 1: Existence of a unique endogenous equilibrium.

2)   When $\vartheta_F < B$, then $\Delta(\theta_F) < 0$ (see Proposition 4 and 5). In this case we have to distinguish two different situation.

a) Figure (2) shows the function $\Delta(\theta)$ when $\vartheta_F < A + \frac{w}{(1-w)}$. In this case, it does not exist an endogenous equilibrium in the interval $(0, \vartheta_F)$.

b) Figure (3) shows the function $\Delta(\theta)$ when $\vartheta_F > A + \frac{w}{(1-w)}$. In this case, we have shown that a multiplicity of equilibria might arise according with the value of θ^* that maximizes the function $\Delta(\theta)$. In particular, b.1) if $\Delta(\theta^*) < 0$ then it does not exist an endogenous equilibrium;   b.2) if $\Delta(\theta^*) > 0$ then there exist two endogenous equilibria;   b.3) if $\Delta(\theta^*) = 0$ then there exists an unique endogenous equilibrium in the interval we considered.
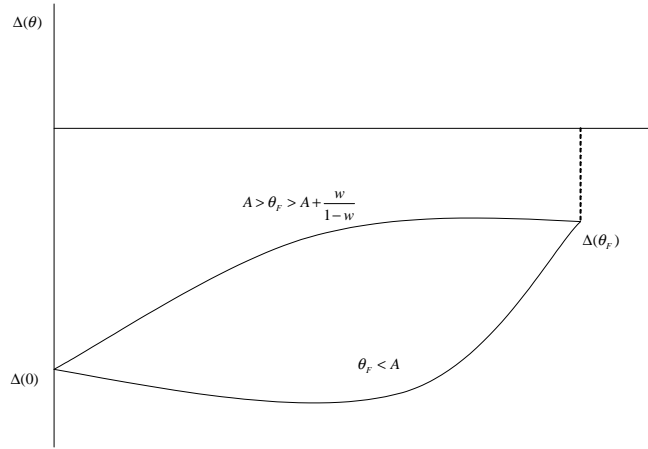
CASE $\theta_F < B$

$\Delta(\theta)$

$A > \theta_F > A + \dfrac{w}{1-w}$

$\Delta(\theta_F)$

$\theta_F < A$

$\Delta(0)$

Figure 2: Non-existence of the equilibrium.

CASE $\theta_F < B$

$\Delta(\theta)$

$\Delta(\theta^*) > 0$

$\Delta(\theta^*) = 0$

$\Delta(\theta^*) < 0$

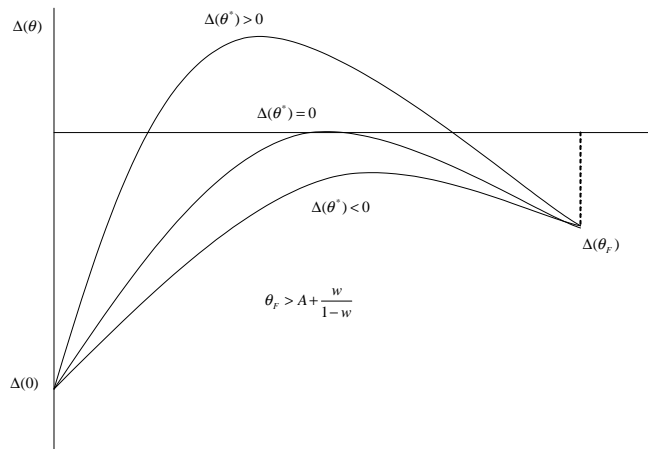$\Delta(\theta_F)$

$\theta_F > A + \dfrac{w}{1-w}$

$\Delta(0)$

Figure 3: A multiplicity of equilibria might arise.

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