Sequential Entry in a Vertically Differentiated Duopoly[1]

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Abstract

We analyse a model of vertical differentiation focusing on the trade-off between entering early and exploiting monopoly power with a low quality, versus waiting and enjoying a dominant market position with a superior product. We show that, in a relevant parameter region, there exists a unique equilibrium where the leader enters with a lower quality than the follower. This happens when the time span spent by the leader as a monopolist matters the most, i.e., in correspondence of sufficiently low discount rate values, low costs of quality improvement and high consumers' willingness to pay for quality.

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1 Introduction

An apparently well established result in the theory of vertically differentiated oligopoly states that earlier entrants supply goods of higher quality than later entrants, in that the high-quality products earn higher profits than low-quality alternatives (see, *inter alia*, Gabszewicz and Thisse, 1979, 1980; Shaked and Sutton, 1982, 1983; Donnenfeld and Weber, 1992, 1995). A general proof of this result for every convex fixed-cost function of quality improvement is provided by Lehmann-Grube (1997).[1]

Two fundamental assumptions are at the basis of this result. The first is that consumers' marginal willingness to pay for quality is uniformly distributed over a given support. Since the density of consumers (i.e., demand) is the same at any income level, the top-quality market niche is the most profitable. Therefore, in a static game, firms obviously prefer to enter with a product characterized by the highest possible quality.

The second assumption concerns the time horizon considered in the above mentioned literature. Entry in a vertically differentiated market is usually analyzed within a single-period extensive form game. However, if one models the entry problem in an explicit dynamic setup, an obvious trade-off immediately appears, even maintaining the previous assumption. In order to enter with an high quality product, the firm has to wait for the R&D activity to take place and consequently it looses monopoly profits. However, postponing entry, the firm is able to produce a higher quality good, obtaining thus higher profits. A static model does not allow to assess the possibility that there exists such a trade-off between early innovation and the attainment of a dominant position in the market.

Although it is generally asserted that quality may result from firms' R&D efforts, this aspect of vertical product differentiation has received a relatively scanty attention, the development phase being summarized by a cost function which does not account for the time elapsed before the good is produced and then marketed. To our knowledge, relevant contributions dealing explicitly with the R&D activity are Beath *et al.* (1987); Motta (1992); Rosenkranz (1995, 1997) and Dutta *et al.* (1995). These papers investigate the incentive towards R&D cooperation (Motta, 1992; Rosenkranz, 1995) and the relationship between R&D and the persistence of quality leadership (Beath *et al.*, 1987; Rosenkranz, 1997). Dutta *et al.* (1995) analyses strategic timing

[1] Aoki and Prusa (1997) adopt a specific case of the cost function analysed by Lehmann-Grube (1997), to investigate the consequences on profits, consumer surplus and social welfare of the timing of investment in product quality in a vertically differentiated duopoly where the market stage is played in the price space. To this regard, see also Lambertini (1999).

in the adoption of a new technology leading to product differentiation and quality improvements. All of these papers maintain that being the quality leader (i.e., supplying the highest quality in the market) entails higher profits than the rivals.

We present a simple model of vertical differentiation focusing upon the trade-off between entering early and exploiting monopoly power with a low quality, versus waiting and enjoying a dominant market position with a superior product. We retain the assumption of a uniform income distribution, that would make it profitable to produce a high quality good in a static game, but relax the assumption of a static extensive form game. We prove that in our model there exists a unique equilibrium where the leader enters with a lower quality than the follower, for a significant set of parameter values.[2]

This highlights that an unfavorable position in duopoly (or oligopoly), due to a lower quality than the rivals', may well be more than balanced by the monopoly rent enjoyed *ad interim* with lower development costs. Therefore, it appears that the established wisdom stating that early entry goes along with high quality (and profits) is not robust to a fully fledged investigation of the role of calendar time in shaping endogenously firms' incentives.

The intuition for the result is as follows. In our dynamic version of vertical differentiation, in equilibrium it is still optimal to differentiate. Therefore, the only possible equilibria prescribe either that the first firm enters with a high quality and the second one with a lower one or the opposite. The earlier entrant has the possibility to choose among those two equilibrium outcomes. He will prefer to enter with a low quality depending upon parameter values, namely if the cost of R&D is low, the interest rate is low and/or consumers' marginal willingness to pay for quality is high. The reason is that for these parameter values, the quality chosen by any firm is high. If the second firm wishes to enter with a higher quality than the leader, it needs to choose a very high one and therefore to engage in a very long R&D phase. This will allow the leader to enjoy the monopoly profits for a very long period, compensating the eventual loss of profits in the competitive phase.

Several real-world examples can be brought forward to support our analysis. One such example is provided by the evolution of the market for digital cameras, where earlier entrants (mainly Japanese or South Korean firms) were primarily interested in meeting the largest possible fraction of the potential demand with low- to mid-quality varieties. Even their top-notch products were (and still are) no match for their highest quality rival that decided

[2] >From a different setting, Dutta *et al.* (1995) also derive an equilibrium where the first entrant produces a lower quality than the second entrant. However, in their model the later entrant makes more profits. As it will become clear in the remainder, this conclusion rests upon the shape of the cost function.

to enter the market only a few years later, namely Leitz Wetzlar, first with the Digilux 1 and currently with the Digilux 2. It is fairly obvious that the flow of profits accruing to, say, Fuji, Nikon, Konika Minolta and the like is individually much larger than Leitz Wetzlar's. Another example of the same kind can be found in the medical industry, where the first generation disposable surgical gloves were neither anallergic nor latex-free. In particular, the second generation of disposable gloves was studied by other firms in the same industry, to meet the needs of medics, becoming hence anallergic, and only later, with the third generation, we have observed the availability of latex-free gloves apt for use with patients subject to the risk of extremely dangerous anaphylactic shocks. This chain of improvements of course has ultimately involved the supply of three types of disposable gloves to different sections of the overall surgical demand, with a monotonically increasing price schedule reflecting the increase in intrinsic product quality.

The remainder of the paper is structured as follows. The basic model of vertical differentiation is laid out in section 2. Section 3 describes the solution of all admissible subgames. The subgame perfect equilibrium of the whole game is derived in section 4. Finally, section 6 provides concluding remarks.

2 The Model

Consider a market for vertically differentiated products. Let this market exist over time t, with $t \in [0, \infty)$. Two single-product firms, labelled 1 and 2, produce goods of different qualities, q_1 and $q_2 \in [0, \infty)$, through the same technology. Without loss of generality we can assume that firms production costs are nought, while development costs are

$$C_i(q_i) = c \int_{\underline{q}}^{\underline{q}+q_i} e^{-rt} dt \tag{1}$$

with $i = 1, 2$ and $\underline{q} \geq 0$. Parameters c and r denote the instantaneous R&D cost and the instantaneous interest rate, respectively. Development costs $C_i(q_i)$ are evaluated at the beginning of the period of investment, therefore in 0 for firm 1 and in t_1 for firm 2. As usual, these costs can be interpreted as fixed cost due to the R&D effort needed to produce a certain quality. We characterize the technology represented by the above cost function as follows:

Assumption 1 *The instantaneous R&D costs are constant over time and equal to c. If firm i searches for a period of length t_i, then it can* *produce a good at most of quality t_i and any other lower quality. Once entered into the market the firm cannot invest anymore in R&D.*

The above amounts to assuming that any change in the quality level implies adjustment costs if and only if the change takes the form of a quality increase. Conversely, once firm i has borne the cost of developing a given quality, she may decide to decrease the quality of her product costlessly. For the sake of simplicity we assume that quality is strictly correlated with the time of entry. More precisely, if firm 1 enters at time t_1, its maximum feasible quality is $t_1 = q_1$. Firm 2's cost of imitation, however, are exactly equal to the costs of innovation.[3] Equivalently, we can assume that firm 2 is compelled to self-develop the innovation, since it might wish to produce a higher quality good. Therefore, firm 2's time of entry satisfies the equality $t_2 = q_1 + q_2$. In the remainder, we shall label the first entrant as the *leader*. Firm 2 enters at date $t_2 \in [t_1, \infty)$, and we shall refer to her as the *follower*.

Assumption 2 *Products are offered on a market where consumers have unit demands, and buy if and only if the net surplus derived from consumption $v_\theta(q_k, p_i(q_k)) = \theta q_k - p_i(q_k) \geq 0$, where $p_i(q_k)$ is the unit price charged by firm i on a good of quality q_k, purchased by a generic consumer whose marginal willingness to pay is $\theta \in [\underline{\theta}, \bar{\theta}]$, with $\underline{\theta} = \bar{\theta} - 1$. We assume that θ is uniformly distributed with density one over such interval, so that the total mass of consumer is one. Throughout the following analysis, we assume partial market coverage.*

The above assumption is rather common in vertically differentiated product models. Parameter θ measures consumers' marginal willingness to pay for quality. Given the previous assumption, if $\bar{\theta}$ increases, the marginal willingness to pay for quality of all consumers increases. *Therefore $\bar{\theta}$ can be thought of as a measure of dimension of the market.* Our most relevant assumption concerns the timing of the game.

Assumption 3 *Firm 1 chooses when to enter the market with the new product and simultaneously chooses the quality and the price to be offered. Then firm 2 decides whether to imitate firm 1 and when to enter the market. Once firm 2 has entered, the two firms choose simultaneously the quality levels, which become common knowledge. Finally both firms choose simultaneously the price levels.*

[3] The case for very high imitation costs is supported by empirical findings (see Mansfield *et al.*, 1981; and Levin *et al.*, 1987).

This timing can be justified as follows. Suppose that firm 1 has invented a new product, but it has to decide the quality level of that product before entry. Since nobody knows the existence of this new product, only firm 1 can enter first. Thereafter, other firms can imitate firm 1. Suppose only firm 2 has the necessary technology. However, firm 2 has to sustain the R&D costs before being able to enter and this takes time and precisely the period between t_1 and t_2.[4]

3 Solution of the Game

As usual we will solve the game backwards. However, it is useful before solving the model to introduce two definitions, concerning firms' behavior. In the remainder, we shall refer to the first entrant (firm 1) as the *leader*, and to the second entrant (firm 2) as the *follower*. We are going to examine two alternative perspectives:

A. The follower enters at t_2 with a product whose quality is lower than the leader's. We label this case as *high-quality leadership*.

B. The follower enters at t_2 with a product whose quality is higher than the leader's. We label this case as *low-quality leadership*.

3.1 The Price Game

In both cases, over $t \in [t_2, \infty)$, firms compete in prices. We borrow from Aoki and Prusa (1997) and Lehmann-Grube (1997) the assumption that downstream Bertrand competition is simultaneous. Market demands for the high- and low-quality good are, respectively:

$$x_H = \bar{\theta} - \frac{p_H - p_L}{q_H - q_L} \text{ and } x_L = \frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L} \tag{2}$$

Duopoly revenue functions are $R_H = p_H x_H$ and $R_L = p_L x_L$. Solving for the equilibrium prices, we obtain:

$$p_H = 2\bar{\theta} q_H \frac{q_H - q_L}{4q_H - q_L} \; ; \; p_L = \bar{\theta} q_L \frac{q_H - q_L}{4q_H - q_L} \tag{3}$$

[4] To solve the game we adopt subgame perfection, and we look for simultaneous Nash equilibria in each stage. Considering the Stackelberg solution would make calculations more cumbersome without affecting significantly the main results.

which allow to rewrite the revenue function of firms in terms of qualities only, as follows:[5]

$$R_H = \frac{4\bar{\theta}^2 q_H^2 (q_H - q_L)}{(4q_H - q_L)^2} \tag{4}$$

$$R_L = \frac{\bar{\theta}^2 q_H q_L (q_H - q_L)}{(4q_H - q_L)^2} \tag{5}$$

On the basis of expressions (4-5), previous literature, dealing with single-period models, establishes that the first entrant would choose to supply the high-quality good, given that $R_H > R_L$. In the remainder, we label the leader's quality as q_1 and the follower's quality as either q_H or q_L, with the understanding that $q_H \geq q_1$ and $q_1 \geq q_L$.

3.2 The Follower's Quality Choice

We characterize the optimal problem when the follower chooses to enter with a lower or a higher quality than the leader. In the final sections of the paper we will use this characterization in order to solve the game and in particular to ascertain the conditions which will induce the follower to enter either with a lower or with a higher quality. We will define the two situations entry from below and entry form above respectively and will be analyzed in a sequel.

3.2.1 Follower's entry from below

The follower's profits when entering from below are Π_{2L}, where the subscript 2 denotes the follower (firm 2) and the subscript L denotes that the firm has lower quality than the competitor. Namely, they are:

$$\begin{aligned}
\Pi_{2L} \quad &= \quad \int_{q_1+q_L}^{\infty} R_L e^{-rt} dt - c \int_{q_1}^{q_1+q_L} e^{-rt} dt = \\
& R_L \frac{e^{-(q_1+q_L)r}}{r} - \frac{c}{r} \left(e^{-q_1 r} - e^{-(q_1+q_L)r} \right) = \\
& \frac{\bar{\theta}^2}{r} \left(\frac{q_1 q_L (q_1 - q_L)}{(4q_1 - q_L)^2} e^{-rq_L} + \frac{c}{\bar{\theta}^2} e^{-rq_L} - \frac{c}{\bar{\theta}^2} \right) e^{-rq_1}
\end{aligned}$$

[5] The proof is omitted here, as it is provided by several authors (Gabszewicz and Thisse, 1979; Choi and Shin, 1992; Motta, 1993; Aoki and Prusa, 1997; Lehmann-Grube, 1997).

where the last equality uses (5). Hence the follower problem can be written as:

$$\max_{q_L} \Pi_{2L}\left(q_L, q_1\right) = \max_{q_L} \frac{\bar{\theta}^2}{r}\left(\frac{q_1 q_L (q_1 - q_L)}{(4q_1 - q_L)^2} e^{-rq_L} + \frac{c}{\bar{\theta}^2} e^{-rq_L} - \frac{c}{\bar{\theta}^2}\right) e^{-rq_1} \quad (6)$$

We are now able to state a useful result, instrumental for solving the game.

Lemma 1 *When entry occurs from below, the follower's problem can be transformed into an equivalent one, which depends only on the exogenous parameter* $\delta \equiv rc/\bar{\theta}^2$.

Proof. If we set:

$$\gamma \equiv \frac{c}{\bar{\theta}^2}, \qquad \delta \equiv \frac{rc}{\bar{\theta}^2}, \qquad \gamma \tilde{q}_L \equiv q_L \qquad \gamma \tilde{q}_1 \equiv q_1$$

and substitute in (6) we obtain:

$$\frac{r}{c} \Pi_{2L}\left(\gamma \tilde{q}_L, \gamma \tilde{q}_1\right) = \left(\left(\frac{\tilde{q}_1 \tilde{q}_L (\tilde{q}_1 - \tilde{q}_L)}{(4\tilde{q}_1 - \tilde{q}_L)^2} + 1\right) e^{-\delta \tilde{q}_L} - 1\right) e^{-\delta \tilde{q}_1} \quad (7)$$

Maximizing the right hand side of (7) with respect to $\tilde{q}_L$ is equivalent to (6). ■

Notice that c and $\bar{\theta}^2$ enter precisely in the same way into the problem. The discount rate r has an independent effect, since it enters in the definition of δ, but not in that of $\tilde{q}_L$. We are now in a position to prove that the follower's problem has a solution and to characterize it.

Proposition 2 *When entry occurs from below, the follower's problem has a solution if* $\delta \in [0, 1/16]$ *and the solution is characterized by:*

$$\arg \max_{\tilde{q}_1} \Pi_{2L}\left(\gamma \tilde{q}_L, \gamma \tilde{q}_1\right) = \frac{4 - 7x - \delta\left(4 - x\right)^3}{\delta x\left(1 - x\right)\left(4 - x\right)} \equiv \tilde{q}_L^*$$

where

$$x = \frac{\tilde{q}_L}{\tilde{q}_1} = \frac{q_L}{q_1}.$$

Proof. See the Appendix. ■

The transformation $\tilde{q}_L = x\tilde{q}_1$ allows us to express the first order condition as a linear function of $\tilde{q}_1$, simplifying considerably the calculations we will have to carry out below, when substituting the follower's equilibrium strategy into the leader's problem, as it is necessary for computing the sub-game perfect equilibria. For future reference, notice also that:

$$\frac{\partial \tilde{q}_L}{\partial x} = \frac{-8 - x - 7x\,(1-x) - \delta\,(x+2)\,(4-x)^3}{\delta\,(1-x)^2\,(4-x)^2} < 0 \tag{8}$$

hence $\tilde{q}_L$ is a monotonically decreasing function of x in the relevant range.

3.2.2 Follower's entry from above

In this paragraph we proceed in an analogous way as the previous one. The follower's profits if it enters with the high quality good are denoted by Π_{2H}, where the subscript 2 denotes the follower (firm 2) and the subscript H denotes that the firm has higher quality than the competitor. The follower profit can be written as follows:

$$\begin{aligned}
\Pi_{2H} &= R_H \int_{q_1+q_H}^{\infty} e^{-rt}dt - c\int_{q_1}^{q_1+q_H} e^{-rt}dt \\
& R_H \frac{e^{-t_2 r}}{r} - \frac{c}{r}\left(e^{-q_1 r} - e^{-(q_1+q_H)r}\right) = \\
& \frac{\overline{\theta}^2}{r}\left(\frac{4q_H^2(q_H-q_1)}{(4q_H-q_1)^2}e^{-rq_H} + \frac{c}{\overline{\theta}^2}e^{-rq_H} - \frac{c}{\overline{\theta}^2}\right)e^{-rq_1}
\end{aligned}$$

where the last equality is obtained using (4). The follower problem is therefore:

$$\max_{q_H} \Pi_{2H}\left(q_H, q_1\right) = \max_{q_H} \frac{\overline{\theta}^2}{r}\left(\frac{4q_H^2(q_H-q_1)}{(4q_H-q_1)^2}e^{-rq_H} + \frac{c}{\overline{\theta}^2}e^{-rq_H} - \frac{c}{\overline{\theta}^2}\right)e^{-rq_1} \tag{9}$$

Also in this case we can prove that the problem can be transformed in a simpler one, as stated in the following proposition.

Lemma 3 *When entry occurs from above, the follower's problem can be transformed into an equivalent one, which depends only on the exogenous parameter* $\delta \equiv rc/\overline{\theta}^2$.

Proof. If we set:

$$\gamma \equiv \frac{c}{\overline{\theta}^2}, \qquad \delta \equiv \frac{rc}{\overline{\theta}^2}, \qquad \gamma\tilde{q}_H \equiv q_H \qquad \gamma\tilde{q}_1 \equiv q_1$$

an substitute into (9) we obtain:

$$\frac{r}{c}\Pi_{2H}\left(\gamma\tilde{q}_H,\gamma\tilde{q}_1\right)=\left(\left(\frac{4\tilde{q}_H^2(\tilde{q}_H-\tilde{q}_1)}{(4\tilde{q}_H-\tilde{q}_1)^2}+1\right)e^{-\delta\tilde{q}_H}-1\right)e^{-\delta\tilde{q}_1} \tag{10}$$

Maximizing the right hand side of (7) with respect to $\tilde{q}_H$ is equivalent to solving problem (9). ■

Finally, the existence of a solution for the follower entering from above is ensured by the following proposition.

Proposition 4 *When entry occurs from above, the follower's problem has a solution, which is characterized by*

$$\arg\max_{\tilde{q}_1}\Pi_{2H}\left(\gamma\tilde{q}_H,\gamma\tilde{q}_1\right)=\frac{4\left(4-3x+2x^2\right)-\delta\left(4-x\right)^3}{4\delta\left(4-x\right)\left(1-x\right)}\equiv\tilde{q}_H^*$$

where

$$x=\frac{\tilde{q}_1}{\tilde{q}_H}=\frac{q_1}{q_H}.$$

Proof. See the Appendix. ■

Notice again that the transformation $\tilde{q}_1=x\tilde{q}_H$ allows to express the first order condition as a linear function of $\tilde{q}_1$, which simplifies considerably the solution of sub-game perfect equilibrium strategies of the previous stages of the game. Finally, notice for future reference that:

$$\frac{\partial\tilde{q}_H}{\partial x}=\frac{1}{4}\frac{4\left(8+x+7x\left(1-x\right)\right)-\left(x+2\right)\left(4-x\right)^3\delta}{\delta\left(1-x\right)^2\left(4-x\right)^2}$$

It is easy to check that:

$$\frac{\partial}{\partial x}\tilde{q}_H\left(x|\delta=4\frac{\left(4-3x+2x^2\right)}{\left(4-x\right)^3}\right)=\frac{2x\left(x+5\right)}{\left(4-x\right)^2\left(1-x\right)\delta}>0. \tag{11}$$

Hence, noticing that $\partial\tilde{q}_H/\partial x$ is decreasing in δ, we have that $\partial\tilde{q}_H/\partial x>0$ in the relevant range. Therefore, $\tilde{q}_H\left(x\right)$ is monotonically increasing in x.

3.3 The Leader's Quality Choice

The leader has to take two choices on the quality level: one when it enters as a monopolist and the other when it has to cope with the entry of the competitor. On the basis of Assumption 1, the second level of quality cannot exceed the monopoly one. As usual, we start by analyzing the last quality choice, that when the follower enters.

3.3.1 The Quality in the Last Stage Game

As for the follower, we characterize the optimal choice when entering with a higher quality than that expected from the follower and with a lower one. In the following sub-sections we will determine the conditions inducing the follower to enter either with a lower or with a higher quality than the leader's. We will define the two situations as entry from below and entry form above respectively and they will be analyzed in a sequel.

Leader's entry from above First of all notice that once firm 2 had entered, firm 1 wishes to produce at the highest quality level in the product space. It is sufficient to compute the derivative of R_H with respect to q_H and check that it is always positive. Recall that the R&D costs were already borne and therefore irrelevant in this stage. Hence:

$$\frac{\partial}{\partial q_H} R_H = 4\bar{\theta}^2 q_H \frac{4q_H^2 - 3q_H q_2 + 2q_2^2}{(4q_H - q_2)^3}$$

which is positive if $4q_H^2 - 3q_H q_2 + 2q_2^2 > 0$. However:

$$4q_H^2 - 3q_H q_2 + 2q_2^2 \geq 4q_2^2 - 3q_2 q_2 + 2q_2^2 = 3q_2^2 \geq 0$$

where the first inequality is an implication of $q_H \geq q_2$. Since $q_1 \leq q_M$, where q_M is the quality level of monopolist's product, we can summarize the result in the following lemma.

Lemma 5 *If the leader enters with the high quality good, then it will produce a good of the same quality level before and after follower's entry.*

Leader's entry from below After firm 2 had entered the market, the leader's optimal quality level is $q_1 = 4q_H/7$, if it entered with a low quality. In fact, in analogy with the entry from above, we have:

$$\frac{\partial}{\partial q_L} R_L = \bar{\theta}^2 q_H^2 \frac{4q_H - 7q_1}{(4q_H - q_1)^3} = 0$$

which indeed implies $q_1 = 4q_H/7$.

Recalling that: $q_H = \gamma \tilde{q}_H$ and $q_1 = \gamma \tilde{q}_1$, the equality $q_1 = 4q_H/7$ implies also $\tilde{q}_1 = 4\tilde{q}_H/7$. Moreover, recall that in the proof of Proposition 4 we set $x = \tilde{q}_1/\tilde{q}_H$, which for the analysis above must be: $x = 4/7$. Substituting in the follower's first order condition, we obtain:

$$\tilde{q}_H = \frac{2}{7} \cdot \frac{7 - 24\delta}{\delta}$$

which is meaningful if and only if:

$$\delta = \frac{c}{\bar{\theta}^2} r \leq \frac{7}{24}$$

Under the above condition we have:

$$\tilde{q}_1 = \frac{8}{49} \cdot \frac{7 - 24\delta}{\delta}$$

This discussion implies:

Lemma 6 *If (i) the leader has entered with the low quality good and (ii) the quality chosen after the follower's entry is lower than that chosen in the monopoly phase, then the equilibrium quality levels in the duopoly phase are:*

$$\tilde{q}_1 = \frac{8}{49} \cdot \frac{7 - 24\delta}{\delta}, \qquad \tilde{q}_2 = \frac{2}{7} \cdot \frac{7 - 24\delta}{\delta}$$

provided that: $\delta = cr/\bar{\theta}^2 \leq 7/24$.

3.3.2 Monopoly Phase

After having discussed the choices in the competition game, we have to describe what happens in the monopoly phase. As usual, we start by describing the price policy and then the choice of quality, distinguishing between entry with high and low quality, respectively.

The Monopolist's Price In the monopoly phase, revenues are $R_M = p\left(\bar{\theta} - p/q_M\right)$, where q_M is the quality level chosen by firm 1 when it is a monopolist. The first order conditions for the price is:

$$\frac{\bar{\theta} q_M - 2p}{q_M} = 0$$

and hence $p = \bar{\theta} q_M / 2$. Substituting again in the profits, it yields $R_M = \bar{\theta}^2 q_M / 4$.

Leader's entry from above The profit function of firm 1 when entering from above are denoted by Π_{1H}, where subscript 1 denotes the leader, while subscript H denotes that the leader entered with the plan to set a higher quality than the follower's. Hence:

$$\Pi_{1H}(q_L, q_M) = R_M \int_{q_M}^{q_M+q_L} e^{-rt}dt + R_H \int_{q_M+q_L}^{\infty} e^{-rt}dt - c\int_0^{q_M} e^{-rt}dt =$$

$$R_M \frac{e^{-rq_M} - e^{-(q_M+q_L)r}}{r} + R_H \frac{e^{-(q_M+q_L)r}}{r} - c\frac{(1-e^{-rq_M})}{r} =$$

$$\frac{q_M\bar{\theta}^2}{4}\frac{e^{-rq_M} - e^{-r(q_M+q_L)}}{r} + \frac{4\bar{\theta}^2 q_H^2(q_H - q_L)}{(4q_H - q_L)^2}\frac{e^{-r(q_M+q_L)}}{r} - c\frac{(1-e^{-rq_M})}{r}$$

Moreover, recall that the corresponding follower's problem is that where it enters from below. In that case, we have been able to express the follower's problem in terms of one parameter only, $\delta \equiv rc/\bar{\theta}^2$, thus simplifying it significantly. It turns out that this is possible also for the leader's problem.

Lemma 7 *When firm 1 enters from above, the leader's problem can be transformed into an equivalent one which depends only on the exogenous parameter* $\delta \equiv rc/\bar{\theta}^2$.

Proof. The profit function of the leader is equivalent to:

$$\frac{r}{\bar{\theta}^2}\Pi_{1H}(q_L, q_M) = 4\frac{q_1^2(q_1 - q_L)}{(4q_1 - q_L)^2}e^{-r(q_M+q_L)} + \\ + \frac{1}{4}\left(1 - e^{-rq_L}\right)e^{-rq_M}q_M - \frac{c}{\bar{\theta}^2}\left(1 - e^{-rq_M}\right)$$

We already know from the above analysis that, when the follower enters, the leader will produce the highest quality and hence we have $q_1 = q_M$. Therefore, the leader's problem is equivalent to maximizing:

$$\Pi_{MH}(q_L, q_1) = 4\frac{q_1^2(q_1 - q_L)}{(4q_1 - q_L)^2}e^{-r(q_1+q_L)} + \frac{1}{4}e^{-rq_1}q_1 - \frac{1}{4}e^{-r(q_1+q_L)}q_1 + \gamma e^{-rq_1} - \gamma$$

where $\Pi_{MH} = \bar{\theta}^2\Pi_{1H}/\gamma$. Setting again:

$$\gamma \equiv \frac{c}{\bar{\theta}^2}, \qquad \delta \equiv \frac{rc}{\bar{\theta}^2}, \qquad \gamma\tilde{q}_L \equiv q_L \qquad \gamma\tilde{q}_1 \equiv q_1$$

we obtain:

$$\Pi_{MH}\left(\gamma\tilde{q}_L,\gamma\tilde{q}_1\right)=\gamma\left(\frac{4\tilde{q}_1^2\left(\tilde{q}_1-\tilde{q}_L\right)}{\left(4\tilde{q}_1-\tilde{q}_L\right)^2}e^{-\delta\tilde{q}_L}-\frac{1}{4}\tilde{q}_1e^{-\delta\tilde{q}_L}+\frac{1}{4}\tilde{q}_1+1\right)e^{-\delta\tilde{q}_1}-\gamma$$

Then, defining:

$$\Pi_H\left(\tilde{q}_L,\tilde{q}_1\right)=\frac{1}{\gamma}\Pi_{MH}\left(\gamma\tilde{q}_L,\gamma\tilde{q}_1\right)+\gamma=$$
$$\left(\left(\frac{4\tilde{q}_1\left(\tilde{q}_1-\tilde{q}_L\right)}{\left(4\tilde{q}_1-\tilde{q}_L\right)^2}-\frac{1}{4}\right)\tilde{q}_1e^{-\delta\tilde{q}_L}+\frac{1}{4}\tilde{q}_1+1\right)e^{-\delta\tilde{q}_1}$$

we obtain that maximizing Π_{1H} is equivalent to maximize Π_H, which depends only on δ. ■

The above lemma allows us to simplify considerably the leader's problem, even though it remains indeed rather cumbersome. In fact, we have to optimize using backward induction, which implies that we have to introduce the follower's first order condition inside the leader's problem. However, using again the variable $x=\tilde{q}_L/\tilde{q}_1=q_L/q_1$, the profit function of the leader can be further transformed in the following:

$$\Pi_H\left(x\tilde{q}_1,\tilde{q}_1\right)=\frac{1}{4}\left(\left(1-\frac{8+x}{\left(4-x\right)^2}xe^{-\delta x\tilde{q}_1}\right)\tilde{q}_1+4\right)e^{-\delta\tilde{q}_1}$$

Finally, using $\tilde{q}_L$ and considering the fact that it is monotone in x for (8), the leader's problem is equivalent to:

$$\max_x\Pi_H\left(\frac{4-7x-\delta\left(4-x\right)^3}{\delta\left(1-x\right)\left(4-x\right)},\frac{4-7x-\delta\left(4-x\right)^3}{\delta x\left(1-x\right)\left(4-x\right)}\right)\tag{12}$$

which is still computationally a complicated problem, but in principle it is the maximization of a function of a single variable and a single parameter. Therefore the function can be easily represented graphically. Finally, recall that we need to impose $\delta\leq\left(4-7x\right)/\left(4-x\right)^3$ in order to make the follower's problem meaningful. Given the additional restrictions $x\leq 4/7$ and $\delta\leq 1/16$ (see the proof of Proposition 2), we can carry out an exploration of the monopolist profit function in Figure 1, highlighting the existence of a global maximum for any given value of δ.

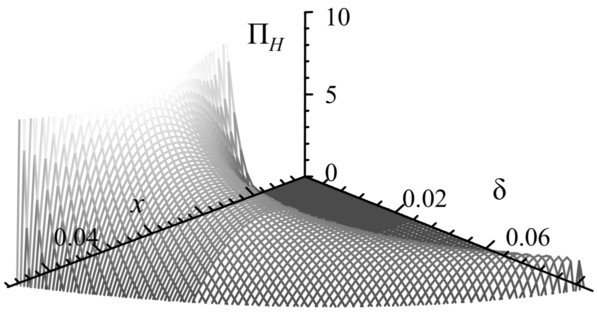

Figure 1: Profit of the leader when entering from above

The monopolist's first order condition is:

$$D_H(x,\delta) = \frac{\partial}{\partial x}\Pi_H\left(\frac{4-7x-\delta(4-x)^3}{\delta(1-x)(4-x)}, \frac{4-7x-\delta(4-x)^3}{\delta x(1-x)(4-x)}\right) = 0 \quad (13)$$

cannot be solved analytically. However, we can draw the implicit plot in Figure 2. The dotted line plots the locus $\delta = (4-7x)/(4-x)^3$. Therefore, only the area below the dotted line satisfies $\delta < (4-7x)/(4-x)^3$. The continuous line below the dotted one is the locus of the global maxima of the profit function, as established by comparing Figure 1 and 2.We can summarize the graphical analysis above in the following proposition.

Proposition 8 *The problem of the leader entering with the high quality and correctly anticipating that the follower will respond with a lower quality has a solution, characterized by $\partial\Pi_H/\partial x = 0$ for any $\delta \in [0, 1/16]$.*

Leader's entry from below The analysis of this case is very similar to that of the previous one. The profit function of firm 1 when entering from below is denoted by Π_{1L} where the subscript 1 denotes the leader, while the subscript L denotes that the leader entered with the plan to set a lower

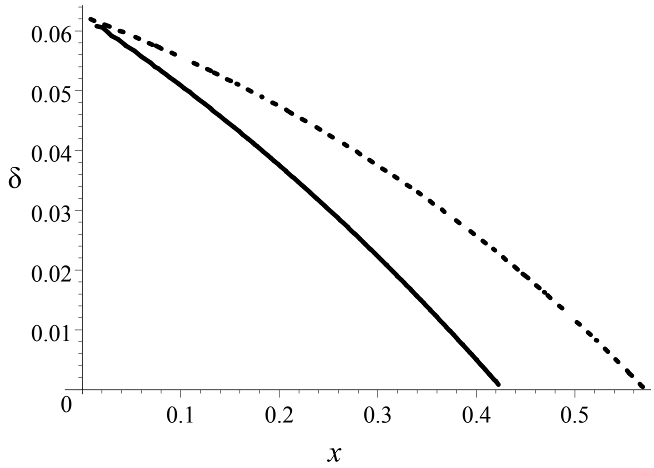

Figure 2: First order condition for the leader when entering from above

quality than the response of the follower. We have:

$$\Pi_{1L} = R_M \int_{q_M}^{q_M+q_H} e^{-rt}dt + R_L \int_{q_M+q_H}^{\infty} e^{-rt}dt - c\int_0^{q_M} e^{-rt}dt =$$

$$R_M \frac{e^{-rq_M} - e^{-(q_M+q_H)r}}{r} + R_L \frac{e^{-(q_M+q_H)r}}{r} - c\frac{1-e^{-rq_M}}{r} =$$

$$\frac{q_M\bar{\theta}^2}{4}\frac{e^{-rq_M} - e^{-(q_M+q_H)r}}{r} + \bar{\theta}^2 \frac{q_H q_L (q_H - q_1)}{(4q_H - q_1)^2}\frac{e^{-(q_M+q_H)r}}{r} - c\frac{(1-e^{-rq_M})}{r}.$$

Moreover, recall that the corresponding problem of the follower is that where it enters from below. Also in that case we were able to express the follower's problem as depending on one parameter only, $\delta \equiv rc/\bar{\theta}^2$. Likewise, for the leader's problem we can formulate the following:

Lemma 9 *When firm 1 enters from below, the leader's problem can be transformed into an equivalent one which depends only on the exogenous parameter* $\delta \equiv rc/\bar{\theta}^2$.

Proof. The leader profits can be rewritten as:

$$\frac{r}{\overline{\theta}^2}\Pi_{1L}\left(q_M, q_H, q_L\right) =$$
$$\frac{q_H q_L \left(q_H - q_L\right)}{\left(4q_H - q_L\right)^2} e^{-r(q_M+q_H)} + \frac{1}{4}\left(1 - e^{-rq_H}\right) e^{-rq_M} q_M - \frac{c}{\overline{\theta}^2}\left(1 - e^{-rq_M}\right)$$

Setting as usual:

$$\gamma \equiv \frac{c}{\overline{\theta}^2}, \qquad \delta \equiv \frac{rc}{\overline{\theta}^2}, \qquad \gamma\tilde{q}_M = q_M \qquad \gamma\tilde{q}_L \equiv q_L, \qquad \gamma\tilde{q}_H \equiv q_H$$

we have that:

$$\Pi_{ML}\left(\tilde{q}_M, \tilde{q}_H, \tilde{q}_L\right) = \frac{1}{\gamma}\Pi_{1L}\left(\gamma\tilde{q}_M, \gamma\tilde{q}_H, \gamma\tilde{q}_L\right) - \gamma = \tag{14}$$
$$\left(\left(\tilde{q}_H\tilde{q}_L \frac{\tilde{q}_H - \tilde{q}_L}{\left(\tilde{q}_L - 4\tilde{q}_H\right)^2} - \frac{1}{4}\tilde{q}_M\right) e^{-\delta\tilde{q}_H} + \frac{1}{4}\tilde{q}_M + 1\right) e^{-\delta\tilde{q}_M}$$

where it is obvious that maximizing Π_{ML} with respect to $\tilde{q}_M$ is equivalent to maximizing Π_{1L} with respect to q_M. ■

In principle here we have to consider two cases. In fact, for $\delta \leq 7/24$ it is possible that the leader offers a higher quality good when monopolist than after the follower had entered. We will prove that this is not the case.

Proposition 10 *Irrespective of whether the leader enters with the low or the high quality, the quality of the leader after the follower had entered the market coincides with the optimal monopoly quality, i.e.,* $q_1 = q_M$.

Proof. See the Appendix. ■

Given the above proposition we can analyze only the cases where the monopolist's choice is binding in the duopoly phase, that is, where $\tilde{q}_M = \tilde{q}_L$. In such a case, setting $\tilde{q}_M = \tilde{q}_L = \tilde{q}_1$ the leader's profits (14) become:

$$\begin{aligned}\Pi_L\left(\tilde{q}_H, \tilde{q}_1\right) &= \Pi_{ML}\left(\tilde{q}_1, \tilde{q}_H, \tilde{q}_1\right) = \\ &\left(\left(\tilde{q}_H\tilde{q}_1 \frac{\tilde{q}_H - \tilde{q}_1}{\left(4\tilde{q}_H - \tilde{q}_1\right)^2} - \frac{1}{4}\tilde{q}_1\right) e^{-\delta\tilde{q}_H} + \frac{1}{4}\tilde{q}_1 + 1\right) e^{-\delta\tilde{q}_1}\end{aligned}$$

Even though we have simplified also this problem, it is still computationally cumbersome. However, using again the variable $x = \tilde{q}_1/\tilde{q}_H = q_1/q_H$, the problem can be further transformed into the following:

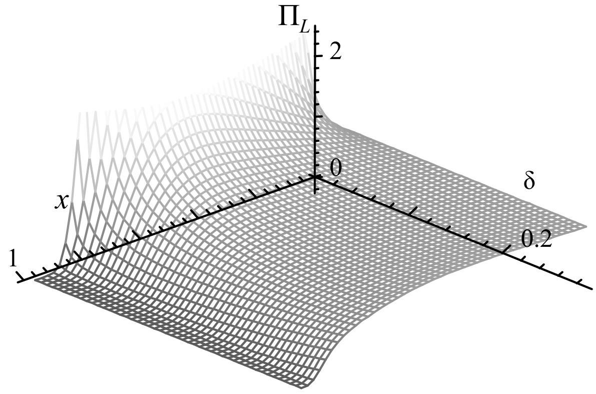

Figure 3: Leader's profit when entering from below

$$\Pi_L\left(\tilde{q}_H, x\tilde{q}_H, \delta\right) = \frac{1}{4}\left(\left(1 - \frac{12 - 4x + x^2}{(4-x)^2} e^{-\delta\tilde{q}_H}\right) x\tilde{q}_H + 4\right) e^{-\delta x\tilde{q}_H}$$

Finally, using $\tilde{q}_H$, which is monotone in x for (11), the leader's problem is equivalent to:

$$\max_x \Pi_L\left(\frac{4\left(4 - 3x + 2x^2\right) - \delta\left(4-x\right)^3}{4\delta\left(4-x\right)\left(1-x\right)}, x\frac{4\left(4 - 3x + 2x^2\right) - \delta\left(4-x\right)^3}{4\delta\left(4-x\right)\left(1-x\right)}\right) \tag{15}$$

Once again we resort to graphical analysis because it is impossible to find a closed form solution. However, we have again a function of one variable and one parameter and the graphical analysis can help us to characterize the solution. Using restriction $\delta \leq 4\left(4 - 3x + 2x^2\right) / \left(4-x\right)^3$, we can produce a graphical exploration of the problem in Figure 3. It shows that the function has a unique global maximum for each value of δ. Moreover, the first order condition is:

$$0 = D_L\left(x, \delta\right) = \\ \frac{\partial}{\partial x}\Pi_L\left(\frac{4\left(4 - 3x + 2x^2\right) - \delta\left(4-x\right)^3}{4\delta\left(4-x\right)\left(1-x\right)}, x\frac{4\left(4 - 3x + 2x^2\right) - \delta\left(4-x\right)^3}{4\delta\left(4-x\right)\left(1-x\right)}\right) \tag{16}$$

and it is not solvable analytically. However, its implicit plot is in Figure 4.We can summarize the graphical analysis above in the following proposition.

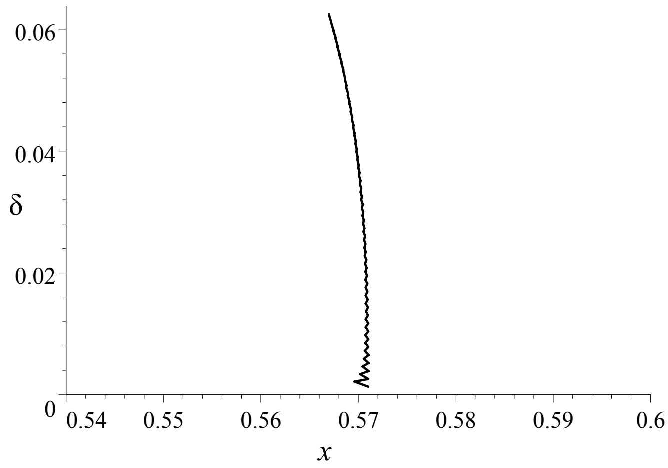

Figure 4: Leader's first order condition when entering from below

Proposition 11 *The problem of the leader entering with the low quality and correctly anticipating that the follower will respond with a higher quality has a solution, characterized by* $\partial \Pi_L / \partial x = 0$ *for any* δ*.*

4 Is it Convenient to Enter the Market with a High-quality Product?

Now we can solve for the subgame perfect equilibrium of the whole game by determining whether the leader will enter with a high or a low quality. We first prove a preliminary result.

Lemma 12 *No equilibrium with the follower entering the market with a lower quality than the leader does exist if* $\delta = rc/\bar{\theta}^2 > 1/16$*.*

Proof. It is a direct consequence of the restriction $\delta \leq 1/16$. ■

We are now in the position to prove the main lemma of this section.

Lemma 13 *There exists a* $\bar{\delta}$ *such that, for* $\delta \in \left[0, \bar{\delta}\right)$ *there is no subgame perfect equilibrium with the follower entering the market and the leader producing the high quality good, while for* $\delta \in \left(\bar{\delta}, 1/16\right] = \left(\bar{\delta}, 0.062\,5\right]$ *there exists no equilibrium with the follower entering the market and the leader producing the low quality good. The value of* $\bar{\delta}$ *is approximately:* $\delta = 0.0203125$*.*

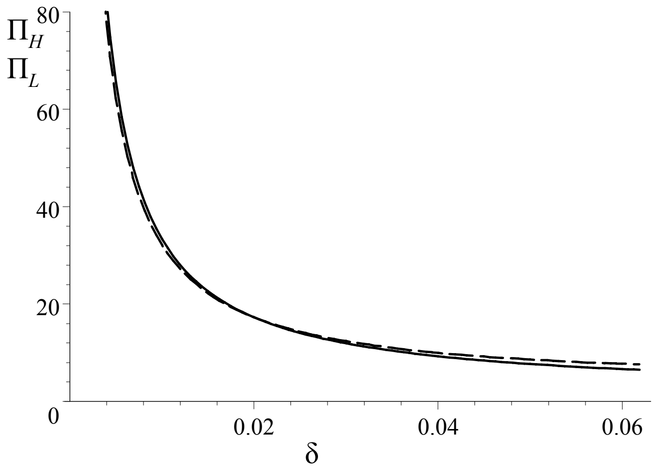

Figure 5: Profits of firm 1 when entering with high (dash) and when entering with low (solid) quality.

Proof. We solve numerically equations (13) and (16), finding the optimal x for the two problems for various values of δ. The computed values are reported in the Table 1-3 of the Appendix in columns denoted respectively by x_{HL} and x_{LH}. By using the expression for $\tilde{q}_1$, we can compute $\tilde{q}_1^*(x_{HL}(\delta), \delta)$ and $\tilde{q}_L^*(x_{HL}(\delta), \delta) = x_{HL}(\delta) \cdot \tilde{q}_1^*(x_{HL}(\delta), \delta)$, the optimal values of transformed variables replacing q_L and q_1. By using $\tilde{q}_H$ we can compute, instead, $\tilde{q}_H^*(x_{LH}(\delta), \delta)$ and $\tilde{q}_1^*(x_{LH}(\delta), \delta) = x_{LH}(\delta) \cdot \tilde{q}_H^*(x_{LH}(\delta), \delta)$. Given the various level of qualities, the profits of the monopolist entering from above and entering from below can be computed and are drawn in Figure 5. It can be seen that the profit of the low quality monopolist are higher for lower level of δ and lower thereafter. The two curves cross at $\bar{\delta}$. ■

The above proposition suggests that the equilibrium should have the following form: when $\delta < \bar{\delta}$ the leader enters with a low quality good and the follower responds with a higher quality and for $\delta > \bar{\delta}$ the opposite is true (while when $\delta = \bar{\delta}$ both equilibria are available). However, before proving such a result we need to exclude the profitability of other possible deviations. First, the leader might have an incentive to monopolize the market. Second, if the leader enters with a high quality it might be the case that the follower wishes to enter with an even higher one, that is to leap-frog the leader. Third, if the leader enters with a low quality the follower might find it convenient to undercut the leader's quality. The next three propositions take care of these three possibilities, respectively.

Proposition 14 *For δ sufficiently small, there exists no subgame perfect equilibrium where the leader succeeds in monopolizing the market. In particular, $\delta \leq 1/16$ is a sufficient condition for the leader not to be able to prevent the entry of the follower.*

Proof. In order to prove the proposition, we must check that he follower can always enter with a lower quality for any choice of the leader, making positive profits. Recall that the optimal choice of the follower is expressed by $\tilde{q}_1$, which is re-written for convenience as:

$$\tilde{q}_L = \frac{4 - 7x - \delta (4 - x)^3}{\delta (1 - x)(4 - x)}$$

We know that x must satisfy the inequality:

$$\delta \leq \frac{4 - 7x}{(4 - x)^3}$$

Recall also that profits for the follower entering with the low quality are:

$$\frac{r}{c} R_{2L} (\gamma \tilde{q}_L, \gamma \tilde{q}_1) = \left(\left(\frac{\tilde{q}_1 \tilde{q}_L (\tilde{q}_1 - \tilde{q}_L)}{(4\tilde{q}_1 - \tilde{q}_L)^2} + 1 \right) e^{-\delta \tilde{q}_L} - 1 \right) e^{-\delta \tilde{q}_1}$$

and using again the definition of $\tilde{q}_L$ and the fact that $\tilde{q}_L = x\tilde{q}_1$, profits can be re-written as:

$$\left(\frac{4 - 7x}{(4 - x)^3 \delta} e^{-\frac{4-7x-\delta(4-x)^3}{(1-x)(4-x)}} - 1 \right) e^{-\delta \tilde{q}_1}$$

Notice that if $\delta = (4 - 7x) / (4 - x)^3$, then the follower's profits are nought, otherwise they are positive for any admissible values of x and δ. ■

The only possible candidate equilibrium strategy profiles left are those with the leader entering with the low quality and the follower responding with a higher one and the other where the opposite happens, depending on the value of the composite parameter δ. However, we still have to ascertain whether it is optimal for the follower to respond with a higher (lower) quality if the leader enters with a low (high) one. This is done in the following two propositions.

Proposition 15 *If $\delta \in \left(\bar{\delta}, 1/16\right]$, the leader enters with a high quality and the follower will always respond with a lower one.*

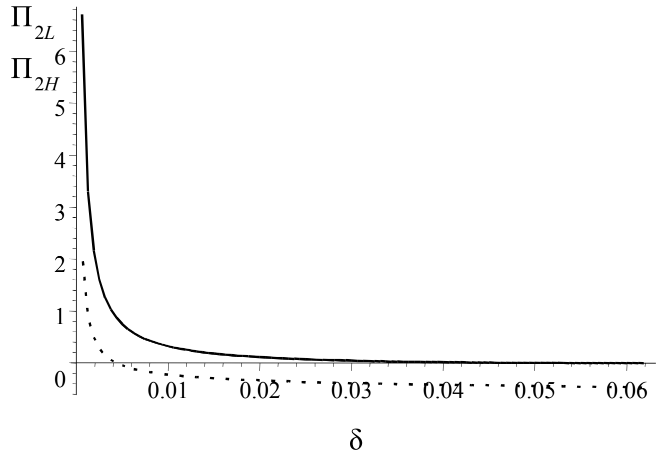

Figure 6: The leader enters with the high quality. Follower's profits when choosing the low one (solid) and the high one (dots).

Proof. The only thing we need to check is whether the follower responds with a lower quality. This proof is conceptually similar to that of Lemma 13. We can compute numerically $\tilde{q}_1^*\left(x_{HL}\left(\delta\right),\delta\right)$ and $\tilde{q}_L^*\left(x_{HL}\left(\delta\right),\delta\right)$. With the two levels of quality we can compute numerically firm's 2 profit as from equation (7). Moreover, using the first order condition of the follower when entering from above, we can compute numerically the corresponding value of x, for any given q_1 and δ. Those values of x are reported in the tables of the Appendix in the column denoted as x_{HH}. Thereafter we can compute $\tilde{q}_H^*\left(\tilde{q}_1^*,\delta\right)=\tilde{q}_1^*\left(x_{HL}\left(\delta\right),\delta\right)/\,x_{HH}$, which is the optimal response if the follower tries to leap-frog the leader. Finally, we use $\tilde{q}_1^*\left(x_{HL}\left(\delta\right),\delta\right)$ and $\tilde{q}_H^*\left(\tilde{q}_1^*,\delta\right)$ to compute the follower's profit when deviating and entering with the high quality using (10). We provide here the graphical representation of the two levels of profit of the follower showing that the follower makes higher profits when producing the low quality and therefore that she will never deviate from the low quality. ■

Proposition 16 *If $\delta\in\left[0,\bar{\delta}\right)$, the leader enters with a low quality and the follower will always respond with a higher one.*

Proof. The only thing we need to prove is that the follower responds with a higher quality and does not undercut the leader. Relying on the proof of Lemma 13, we can compute $\tilde{q}_H^*\left(x_{LH}\left(\delta\right),\delta\right)$ and $\tilde{q}_1^*\left(x_{LH}\left(\delta\right),\delta\right)$. With the

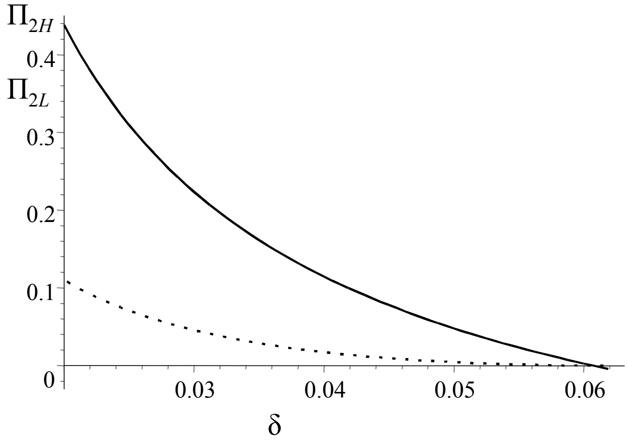

Figure 7: The leader enters with the low quality. Follower's profits when choosing the high one (solid) and the low one (dots).

two levels of quality we can compute numerically Firm's 2 profit as from equation (10). Moreover, using the first order condition of the follower when entering from below, we can compute numerically the appropriate value of x, for any given q_1 and δ. Those values of x are reported in the tables of the Appendix in the column denoted as x_{LL}. Finally we can compute and hence $\tilde{q}_L^*\left(\tilde{q}_1^*, \delta\right) = x_{LL} \cdot \tilde{q}_1^*\left(x_{LH}\left(\delta\right), \delta\right)$, the follower's optimal deviation when she tries to undercut the leader, and then use $\tilde{q}_1^*\left(x_{LH}\left(\delta\right), \delta\right)$ and $\tilde{q}_L^*\left(\tilde{q}_1^*, \delta\right)$ to compute the follower's profit when deviating and entering with the low quality using (7). We provide here the graphical representation of the two profit levels of the follower showing that the follower never deviates from the high quality for any $\delta < \bar{\delta} \approx 0.0203125$. ■

Therefore we can summarize the above analysis in the following corollary.

Corollary 17 *For $\delta \in \left[0, \bar{\delta}\right)$, there is a unique subgame perfect equilibrium with the follower entering the market and the leader producing the low quality good. For $\delta \in \left(\bar{\delta}, 1/16\right]$ there exists a unique subgame perfect equilibrium with the follower entering the market and the leader producing the high quality good. For $\delta = \bar{\delta}$ both equilibria do exist. The value of $\bar{\delta}$ is approximately $\delta =$ 0.0203125.*

A few remarks are now in order. First, a trivial one, refers to $\delta = \bar{\delta}$. For that value of δ both equilibria hold. However, while the leader is indifferent

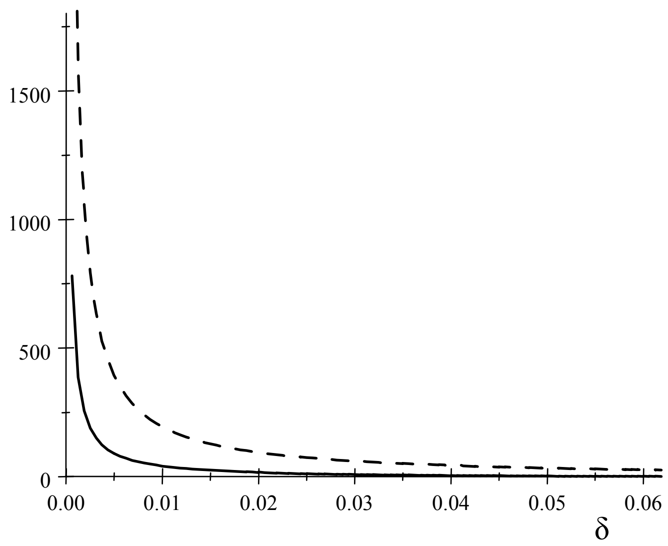

Figure 8: q_{2H} (dash) and q_{2L} (solid) for various levels of δ.

between the two equilibria, the follower makes higher profits if the leader enters with he low quality. Second, recall that $\delta = rc/\bar{\theta}^2$. The two Propositions 15 and 16 together imply that in the interval $[0, 1/16]$ there exist a subgame perfect equilibrium parameterized on δ and such that for low $\delta, \delta \in [0, \bar{\delta}]$, the leader will enter with low quality, while with high ones, $\delta \in (\bar{\delta}, 1/16]$, he will choose a high quality. That is, the leader will enter with the high quality for low levels of r and with the high quality for high levels of r, for given c and $\bar{\theta}$.

The intuition of the result is as follows. For low $\delta's$, the quality chosen by both firms is significantly higher, in both candidate equilibria, with low and high quality leadership alike. In particular the follower's quality is very high in both allocations. However, it is significantly higher when it enters with the highest quality, as can be checked in Figure 8. This implies that the leader will remain a monopolist for a significantly longer period if it enters with the low quality than with the high one. This is the driving force that makes it convenient to enter with the low quality when δ is low. It is also rather intuitive that c should have similar effects as r, while $\bar{\theta}$ should have the opposite ones. That is, any decrease in c and/or r, and any increase in $\bar{\theta}$, imply a decrease in δ.

Finally, we should like to assess our results against those of Lehmann-Grube (1997) and Dutta *et al.* (1995), so as to evaluate how different assumptions about the time horizon and the technology affect the features of the subgame perfect equilibrium. Lehmann-Grube (1997) generalizes the analysis conducted by Shaked and Sutton (1982, 1983) to account for a technology which is convex in the quality level, but remains in a single-phase model where there exist no monopoly periods. This produces the result that surplus extraction is maximized when the firm locates at the top of the available quality spectrum.

In Dutta *et al.* (1995), it is assumed that (i) per-period operative duopoly profits are proportional to relative quality and are symmetric; (ii) adoption (entry) dates are endogenous, while (iii) the growth of quality over time is not endogenously determined by firms; (iv) unit production cost is flat w.r.t. quality; and (v) innovation costs are summarized by the waiting time before the adoption. In this setup, the authors find that a later entrant obtains larger profits than an earlier entrant, and no monopoly rent is dissipated at the subgame perfect equilibrium.

In our setting, the entry timing is endogenously linked to quality improvement, and the cost borne to supply superior qualities can be high enough to offset the advantage attached to serving rich customers. The interplay of these factors may entail that, in some relevant parameter ranges, all firms would prefer to enter early with an inferior quality rather than late with a superior one.

5 Concluding Remarks

We have investigated the bearings of R&D expenditures in continuous time over the entry process in a market for vertically differentiated goods.

We have shown that entering first and enjoying an *ad interim* monopoly rent may counterbalance the incentive towards the supply of high quality goods in duopoly after the entry of a second innovator. Indeed, we have proved that this is the only subgame perfect equilibrium in a significant range of parameters, namely, the parameter region where discounting is low, quality improvements are comparatively inexpensive and the marginal willingness to pay for quality is high. all of these elements contribute to make the monopoly phase more attractive from the leader's standpoint, ultimately inducing the first entrant to supply a low-quality variety.

The foregoing analysis shows that the established wisdom produced by previous literature in this field does not properly account for the role of time and its interaction with R&D technology in determining firms' incentives.

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Appendix

Proof of Proposition 2. Differentiating (7) for $\tilde{q}_L$ we obtain the first order condition:

$$-e^{-\delta(\tilde{q}_1+\tilde{q}_L)} \cdot \frac{(7\tilde{q}_L - 4\tilde{q}_1)(\tilde{q}_1)^2 + \delta\tilde{q}_1\tilde{q}_L(\tilde{q}_1 - \tilde{q}_L)(4\tilde{q}_1 - \tilde{q}_L) + \delta(4\tilde{q}_1 - \tilde{q}_L)^3}{(4\tilde{q}_1 - \tilde{q}_L)^3} = 0$$

One could obviously solve the first and second order conditions for $\tilde{q}_L$, even though the solution of a third order polynomial is rather cumbersome. However, if we set $\tilde{q}_L = x\tilde{q}_1$, the first order condition is equivalent to:

$$\tilde{q}_1^3\left(7x - 4 + \delta\tilde{q}_1 x(1-x)(4-x) + \delta(4-x)^3\right) = 0$$

hence:

$$\tilde{q}_1 = \frac{4 - 7x - \delta(4-x)^3}{\delta x(1-x)(4-x)} \tag{a1}$$

and:

$$\tilde{q}_L = x\tilde{q}_1 = \frac{4 - 7x - \delta(4-x)^3}{\delta(1-x)(4-x)}$$

Notice that in order to have $\tilde{q}_L \geq 0$ we must impose

$$0 \leq \delta \leq \frac{4-7x}{(4-x)^3}$$

which in turn implies:

$$0 \leq x \leq \frac{4}{7}, \tag{a2}$$

and correspondingly:

$$0 \leq \delta \leq \frac{1}{16}.$$

The second order condition is:

$$-\frac{e^{-\delta(\tilde{q}_1+\tilde{q}_L)}}{(4q_1 - q_2)^4}\left[2q_1^2(4q_1 - 7q_2)(4q_1 - q_2)\delta + 2q_1^2(8q_1 + 7q_2) + \right.$$
$$\left. - \left((q_1 - q_2)q_1q_2 + (4q_1 - q_2)^2\right)(4q_1 - q_2)^2\delta^2\right] < 0$$

and setting $\tilde{q}_L = x\tilde{q}_1$ it becomes:

$$-\frac{e^{-\delta(1+x)\tilde{q}_1}}{(4-x)^4\tilde{q}_1}\left[2(7x+8) - x\delta^2(1-x)(4-x)^2 q_1^2 + \right.$$

$$-\delta(4-x)\left(14x-8+(4-x)^3\delta\right)q_1\big]<0$$

Using (a1), the condition becomes equivalent to:

$$-\frac{1}{x(1-x)}\left[(4-7x)(4-x)^3\delta+\left(14x^3-47x^2+40x-16\right)\right]>0$$

or:

$$(4-7x)(4-x)^3\delta+\left(14x^3-47x^2+40x-16\right)<0$$

Notice that (a2) implies that the polynomial is increasing in δ. Therefore if the inequality is satisfied for $\delta=(4-7x)/(4-x)^3$ is satisfied for all meaningful parameter values. For this value of δ, the second order condition becomes equivalent to:

$$2x\left(7x^2+x-8\right)<0$$

which is always satisfied.■

Proof of Proposition 4. Differentiating (10) for $\tilde{q}_H$ we obtain the first order condition:

$$-e^{-\delta(\tilde{q}_1+\tilde{q}_H)}\cdot$$

$$\frac{\left((4\tilde{q}_H-\tilde{q}_1)^3+4\tilde{q}_H^2(\tilde{q}_H-\tilde{q}_1)(4\tilde{q}_H-\tilde{q}_1)\right)\delta-4\tilde{q}_H\left(4\tilde{q}_H^2-3\tilde{q}_H\tilde{q}_1+2\tilde{q}_1^2\right)}{(4\tilde{q}_H-\tilde{q}_1)^3}=0$$

If we set $\tilde{q}_1=x\tilde{q}_H$, the numerator becomes:

$$\tilde{q}_H^3\left(4\delta(1-x)(4-x)\tilde{q}_H-4\left(4-3x+2x^2\right)+\delta(4-x)^3\right)$$

which is nought if:

$$\tilde{q}_H=\frac{4\left(4-3x+2x^2\right)-\delta(4-x)^3}{4\delta(4-x)(1-x)}. \tag{a3}$$

Notice that in order to have $\tilde{q}_H\geq 0$ we must impose:

$$0\leq\delta\leq 4\frac{\left(4-3x+2x^2\right)}{(4-x)^3}\leq\frac{4}{9} \tag{a4}$$

The second order condition is:

$$-\frac{e^{-\delta\tilde{q}_1}e^{-\delta\tilde{q}_H}}{(4\tilde{q}_H-\tilde{q}_1)^4}\left[8\tilde{q}_H(4\tilde{q}_H-\tilde{q}_1)\left(2\tilde{q}_1^2+4\tilde{q}_H^2-3\tilde{q}_1\tilde{q}_H\right)\delta+8(\tilde{q}_1+5\tilde{q}_H)\tilde{q}_1^2+\right.$$

$$\left.-\left(4(\tilde{q}_H-\tilde{q}_1)\tilde{q}_H^2+(4\tilde{q}_H-\tilde{q}_1)^2\right)(4\tilde{q}_H-\tilde{q}_1)^2\delta^2\right]<0$$

and setting again $\tilde{q}_1 = x\tilde{q}_H$, it is equivalent to:

$$\tilde{q}_H^3 \left[-4\delta^2 (1-x)(4-x)^2 (\tilde{q}_H)^2 + 8x^2 (x+5) + \right.$$

$$\left. \delta (x-4) \left(-8 \left(-3x + 2x^2 + 4 \right) + (4-x)^3 \delta \right) \tilde{q}_H \right] > 0$$

which computed in (a3) becomes:

$$\frac{-\left(-3x + 2x^2 + 4\right)(4-x)^3 \delta + 4\left(-24x + 35x^2 - 20x^3 + 2x^4 + 16\right)}{1-x} > 0.$$

Now note that the numerator is decreasing in δ. Therefore if the inequality is satisfied for the highest value of δ, then it is always satisfied. Using (a4) the numerator of the above expression becomes:

$$-8x^2 \left(x^2 + 4x - 5 \right) > 0,$$

always met except for $x \in \{0, 1\}$, where it is nought.■

Proof of Proposition 10. Notice that the proposition can be false only for the leader entering from below when $\delta \leq 7/24$. Relying on this fact, we proceed to characterise the proof by contradiction. From Proposition 6 we would have:

$$\tilde{q}_1 = \tilde{q}_L = \frac{8}{49} \frac{7 - 24\delta}{\delta}, \qquad \tilde{q}_2 = \tilde{q}_H = \frac{2}{7} \frac{7 - 24\delta}{\delta}$$

and substituting in (14) we obtain:

$$\Pi_{ML} \left(\tilde{q}_M, \tilde{q}_H, \tilde{q}_L \right) = \left(\left(\frac{1}{168} \left(\frac{7 - 24\delta}{\delta} \right) - \frac{1}{4} \tilde{q}_M \right) e^{-\frac{2}{7}(7 - 24\delta)} + \frac{1}{4} \tilde{q}_M + 1 \right) e^{-\delta q_M}$$

Differentiating the above expression:

$$-\frac{1}{168} e^{-\delta q_M} \left(168\delta - 42 + 42\delta \tilde{q}_M + \left(49 - 24\delta - 42\delta \tilde{q}_M \right) e^{\frac{48}{7}\delta - 2} \right) = 0$$

and hence the quality level is characterized by:

$$\tilde{q}_M = \frac{(49 - 24\delta)\, e^{\frac{48}{7}\delta - 2} + (4\delta - 1)\, 42}{42\delta \left(e^{\frac{48}{7}\delta - 2} - 1 \right)}$$

However, again by Proposition 6 the following inequality should hold: $\tilde{q}_M \geq$

$\tilde{q}_L$ and hence:

$$\frac{(49-24\delta)\, e^{\frac{48}{7}\delta-2}+(4\delta-1)\, 42}{42\delta\left(e^{\frac{48}{7}\delta-2}-1\right)} \geq \frac{8}{49}\frac{7-24\delta}{\delta}$$

which after some manipulations becomes:

$$\frac{1}{294}\frac{7e^{-2+\frac{48}{7}\delta}+984e^{-2+\frac{48}{7}\delta}\delta+42+24\delta}{\delta\left(e^{-2+\frac{48}{7}\delta}-1\right)} \geq 0$$

The numerator is positive. Therefore the inequality implies $\delta > 7/24$, a contradiction.■

Table 1: Numerical solutions of the firms' problems.
Low value of δ.

	Entry from below		Entry from above	
δ	x_{LH}	x_{LL}	x_{HL}	x_{HH}
0.000625	0.5710955777	0.4252061589	0.4237058956	0.2092704695
0.001250	0.5710989664	0.4215844866	0.4203553403	0.2074909267
0.001875	0.5711018893	0.4179537220	0.4169942492	0.2056948230
0.002500	0.5711043377	0.4143137629	0.4136224093	0.2038821871
0.003125	0.5711063032	0.4106645062	0.4102396054	0.2020530634
0.003750	0.5711077769	0.4070058473	0.4068456203	0.2002074979
0.004375	0.5711087500	0.4033376809	0.4034402348	0.1983455435
0.005000	0.5711092133	0.3996598999	0.4000232276	0.1964672625
0.005625	0.5711091574	0.3959723970	0.3965943754	0.1945727202
0.006250	0.5711085732	0.3922750625	0.3931534528	0.1926619948
0.006875	0.5711074507	0.3885677869	0.3897002322	0.1907351684
0.007500	0.5711057807	0.3848504580	0.3862344841	0.1887923325
0.008125	0.5711035528	0.3811229631	0.3827559768	0.1868335893
0.008750	0.5711007573	0.3773851884	0.3792644768	0.1848590436
0.009375	0.5710973838	0.3736370179	0.3757597481	0.1828688159
0.010000	0.5710934220	0.3698783352	0.3722415530	0.1808630310
0.010625	0.5710888610	0.3661090221	0.3687096516	0.1788418249
0.011250	0.5710836905	0.3623289590	0.3651638019	0.1768053412
0.011875	0.5710778990	0.3585380248	0.3616037600	0.1747537347
0.012500	0.5710714757	0.3547360970	0.3580292799	0.1726871712
0.013125	0.5710644090	0.3509230515	0.3544401135	0.1706058225
0.013750	0.5710566875	0.3470987630	0.3508360106	0.1685098810
0.014375	0.5710482994	0.3432631042	0.3472167191	0.1663995359
0.015000	0.5710392326	0.3394159465	0.3435819850	0.1642749982
0.015625	0.5710294752	0.3355571593	0.3399315520	0.1621364858
0.016250	0.5710190143	0.3316866107	0.3362651620	0.1599842274
0.016875	0.5710078378	0.3278041673	0.3325825548	0.1578184683
0.017500	0.5709959325	0.3239096932	0.3288834683	0.1556394605
0.018125	0.5709832856	0.3200030513	0.3251676384	0.1534474724
0.018750	0.5709698835	0.3160841028	0.3214347991	0.1512427825
0.019375	0.5709557124	0.3121527066	0.3176846822	0.1490256846
0.020000	0.5709407591	0.3082087198	0.3139170180	0.1467964828
0.020625	0.5709250091	0.3042519977	0.3101315343	0.1445554960

Table 2: Numerical solutions of the firms' problems.
Intermediate value of δ.

	Entry from below		Entry from above	
δ	x_{LH}	x_{LL}	x_{HL}	x_{HH}
0.021250	0.5709084481	0.3002823939	0.3063279576	0.1423030611
0.021875	0.5708910616	0.2962997593	0.3025060119	0.1400395220
0.022500	0.5708728346	0.2923039433	0.2986654198	0.1377652416
0.023125	0.5708537521	0.2882947929	0.2948059018	0.1354805970
0.023750	0.5708337985	0.2842721533	0.2909271764	0.1331859775
0.024375	0.5708129580	0.2802358667	0.2870289604	0.1308817922
0.025000	0.5707912150	0.2761857738	0.2831109687	0.1285684678
0.025625	0.5707685527	0.2721217130	0.2791729145	0.1262464329
0.026250	0.5707449547	0.2680435198	0.2752145090	0.1239161526
0.026875	0.5707204040	0.2639510277	0.2712354616	0.1215780901
0.027500	0.5706948832	0.2598440676	0.2672354799	0.1192327367
0.028125	0.5706683748	0.2557224678	0.2632142698	0.1168805932
0.028750	0.5706408607	0.2515860543	0.2591715353	0.1145221874
0.029375	0.5706123228	0.2474346499	0.2551069787	0.1121580586
0.030000	0.5705827422	0.2432680753	0.2510203005	0.1097887570
0.030625	0.5705521000	0.2390861482	0.2469111995	0.1074148657
0.031250	0.5705203767	0.2348886833	0.2427793726	0.1050369829
0.031875	0.5704875525	0.2306754927	0.2386245150	0.1026557079
0.032500	0.5704536073	0.2264463850	0.2344463204	0.1002716774
0.033125	0.5704185202	0.2222011666	0.2302444803	0.0978855505
0.033750	0.5703822707	0.2179396398	0.2260186850	0.0954979899
0.034375	0.5703448369	0.2136616045	0.2217686227	0.0931096840
0.035000	0.5703061971	0.2093668569	0.2174939800	0.0907213469
0.035625	0.5702663289	0.2050551901	0.2131944419	0.0883337068
0.036250	0.5702252095	0.2007263935	0.2088696914	0.0859475081
0.036875	0.5701828157	0.1963802531	0.2045194100	0.0835635313
0.037500	0.5701391237	0.1920165513	0.2001432774	0.0811825600
0.038125	0.5700941095	0.1876350669	0.1957409718	0.0788054078
0.038750	0.5700477480	0.1832355747	0.1913121692	0.0764329080
0.039375	0.5700000141	0.1788178457	0.1868565444	0.0740659098
0.040000	0.5699508820	0.1743816469	0.1823737701	0.0717052971
0.040625	0.5699003253	0.1699267414	0.1778635173	0.0693519571
0.041250	0.5698483171	0.1654528876	0.1733254556	0.0670068124

Table 3: Numerical solutions of the firms' problem.
High value of δ.

	Entry from below		Entry from above	
δ	x_{LH}	x_{LL}	x_{HL}	x_{HH}
0.041875	0.5697948299	0.1609598402	0.1687592523	0.0646707925
0.042500	0.5697398357	0.1564473490	0.1641645734	0.0623448659
0.043125	0.5696833057	0.1519151595	0.1595410828	0.0600300129
0.043750	0.5696252105	0.1473630125	0.1548884429	0.0577272307
0.044375	0.5695655203	0.1427906439	0.1502063139	0.0554375448
0.045000	0.5695042043	0.1381977848	0.1454943546	0.0531619999
0.045625	0.5694412313	0.1335841613	0.1407522217	0.0509016549
0.046250	0.5693765691	0.1289494940	0.1359795701	0.0486575954
0.046875	0.5693101851	0.1242934986	0.1311760528	0.0464309355
0.047500	0.5692420458	0.1196158850	0.1263413209	0.0442227972
0.048125	0.5691721169	0.1149163575	0.1214750236	0.0420343272
0.048750	0.5691003633	0.1101946149	0.1165768081	0.0398666744
0.049375	0.5690267493	0.1054503495	0.1116463198	0.0377210423
0.050000	0.5689512381	0.1006832480	0.1066832018	0.0355986197
0.050625	0.5688737922	0.0958929905	0.1016870953	0.0335006319
0.051250	0.5687943732	0.0910792507	0.0966576396	0.0314283115
0.051875	0.5687129417	0.0862416955	0.0915944716	0.0293829044
0.052500	0.5686294573	0.0813799851	0.0864972263	0.0273656750
0.053125	0.5685438792	0.0764937725	0.0813655364	0.0253779143
0.053750	0.5684561647	0.0715827035	0.0761990324	0.0234209261
0.054375	0.5683662708	0.0666464162	0.0709973427	0.0214960158
0.055000	0.5682741531	0.0616845411	0.0657600932	0.0196044799
0.055625	0.5681797662	0.0566967009	0.0604869077	0.0177476705
0.056250	0.5680830636	0.0516825096	0.0551774074	0.0159268892
0.056875	0.5679839975	0.0466415730	0.0498312113	0.0141435302
0.057500	0.5678825190	0.0415734883	0.0444479358	0.0123989271
0.058125	0.5677785781	0.0364778434	0.0390271949	0.0106944486
0.058750	0.5676721233	0.0313542170	0.0335686000	0.0090314093
0.059375	0.5675631020	0.0262021781	0.0280717598	0.0074112368
0.060000	0.5674514598	0.0210212859	0.0225362806	0.0058352248
0.060625	0.5673371415	0.0158110892	0.0169617657	0.0043047257
0.061250	0.5672200899	0.0105711263	0.0113478159	0.0028211172
0.061875	0.5671002466	0.0053009246	0.0056940290	0.0013857253