Transport Cost Sharing[1]

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May 10, 2002

[1]We are grateful to Hans Haller and Amoz Kats for helpful suggestions. We thank Robert Gilles, Cathy Johnson, Tito Cordella, Rickard Wall, Gayneh Kyureghian, Masha Plotnikova, Urooj Amjad and seminar participants at Virginia Tech, Louisiana State University and Xavier Institute for Management for their comments. The paper has also benefitted from comments at Southeastern Economic Theory Meetings 1999, Games 2000 and Sothern Economics Association Meetings 2000. Sudipta Sarangi gratefully acknowldges the hospitality of the American University of Armenia where a part of this research was carried out. The usual disclaimer applies.

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Abstract

We consider a linear city model where both firms and consumers have to incur transport costs. Following a standard Hotelling (1929) type framework we analyze a duopoly where firms choose locations and prices, with the transportation rate being linear in distance. We model these two different transport costs by assuming one transport cost which is then shared by the buyers and sellers according to an exogenously given rule. From a theoretical point of view such a model is interesting since mill pricing and uniform delivery pricing arise as special cases. We first obtain the profit function for the two stage game. Given the complex nature of the profit function for the two-stage game, we invoke simplifying assumptions and solve for two different games. We provide a complete characterization for the equilibrium of a location game between the duopolists by removing the price choice from the strategy space through an exogenously given price. We also find that when the two firms are constrained to locate at the same spot, the resulting price competition leads to a mixed strategy equilibrium which always yields positive expected profits. This allows us to obtain some insights into the two stage game.

Keywords: Spatial Competition, Cost sharing, Rationing

JEL Classification: R1, L13, D43, C72

1 Introduction

The spatial competition literature in the Hotelling tradition has two main strands. One concerns itself with models of *mill pricing* in which firms choose location and prices, while the spatially dispersed consumers pay the cost of travelling to the firm to buy the product. The other strand of the literature assumes that firms absorb the transport cost of shipping the item to the consumers and is called *uniform delivery pricing* since all consumers pay the same price.[1] In this paper we analyze a model of a linear city that incorporates features of both mill pricing and uniform delivery pricing. We assume that firms charge the same price to all consumers, but have a cost of delivering to all those who purchase from it, just as in the models of uniform delivery pricing. Buyers on the other hand pay the price and also incur a transport cost which, for instance, captures the delivery time of the good. The delivery time increases with the consumer's distance from the firm and is a source of disutility. It captures the opportunity cost of being able to consume sooner than later.[2] The transport cost for consumers can be interpreted broadly to include time, effort and other transaction costs, apart from the costs of travel. This feature is shared by the models of mill pricing. Thus, our model is a hybrid of the standard mill price and uniform delivery price models.

The economic relevance of location games does not stem exclusively from their initial geographical set-up. This idea can be extended to competition among firms selling differentiated products, where each firm's product is viewed as a point in the characteristic space. This product differentiation aspect of location theory dates back to Hotelling's (1929) seminal work. He recognized that while location was a source of market power in itself, it could also be a proxy for other characteristics of the product. The following quote serves to illustrate this point quite well: "*... distance, as we have used it for illustration, is only a figurative term for a great congeries of qualities. Instead of sellers of an identical commodity separated geographically we might consider two competing cider merchants side by side, one selling a sweeter liquid than the other.*"

Asides from the purely theoretical aspects of the model, one encounters many examples of this sort in the real world. Retailers bear the cost of bringing the commodity over to the shopping center, while the buyers must drive there to actually inspect and purchase the items. Buying furniture usually involves a trip to the furniture store and selecting the desired items, and the furniture store usually delivers the items to the consumer location free of charge. The labor market also has similar features. The commute time to work has to be borne by the employees. Hence, one consideration for firms in choosing to locate in the suburbs is the desire to avoid traffic congestion thereby making the job attractive to workers. The large numbers of hi-tech firms located in sub-urban Washington D.C. provide ample testimony to this fact. This phenomenon can also be observed in certain types of differentiated

[1] A third concept, less frequently encountered is that of *spatial price discrimination* (Hoover, (1937)). For an insightful exposition of this issue see Anderson, de Palma and Thisse (1989).

[2] One need look no further than the wide array of shipping options provided to consumers by FedEx, UPS and the United States Postal Service to be convinced of the value of consuming earlier.

product markets. In particular, it is quite common in certain segments of the software industry. Often each firm produces its own standard product and then customizes it to suit the needs of individual buyers, while buyers have to learn the intricacies of the software. The cost of learning new software or customizing it to suit the individual client's needs can be treated as transport cost in our framework.

For the purpose of modelling these issues one might imagine that there is a total cost for moving a commodity from the store to the consumer's location. We then assume that the total pecuniary burden of shipping a commodity from the firm to a consumer is shared by both buyers and sellers. So, consumers in our model pay an exogenously given part of the transport cost while firms pay the remainder. As in most of the examples, assuming an exogenously set transport cost sharing rule is reasonable since the consumers have their own transport cost, while firms have incur to transport costs which are particular to them. Notice that when the consumers' share of costs goes to zero we have the uniform delivery price model and when they bear the entire cost we have a mill pricing situation. In the subsequent section we develop a model to analyze the two stage game. The profit function of the two stage game is found to have an intricate and complex expression rendering it difficult to proceed further without making simplifying assumptions. The problem is then analyzed for a pure location game and its counter-part where firms locate at the same spot (thereby removing location choice from the strategy space) and compete in prices. The insights from these two games are then used to develop conjectures about existence of pure strategy location-price equilibria in the two-stage game.

The next section provides a brief overview of the related literature. Section 3 derives the profit function of the two stage game and solves for location equilibria, assuming fixed prices. We then analyze a price game where both firms are located at the same spot. Section 4 summarizes the results to provide further insights.

2 Review of Literature

Given the plethora of work both on models of mill pricing and uniform delivery pricing an exhaustive survey of all aspects of the literature would be a considerable digression. We limit the scope of our review only to those results which are pertinent to the model under consideration. Graitson (1982) is an early survey of the literature. A more up-to date and comprehensive survey can be found in Anderson, de Palma and Thisse (1992). The literature on mill price is more abundant and we will start by discussing those.

The mill price models trace their heritage from the original Hotelling (1929) model.[3] Typically in these models firms choose locations and then prices and consumers incur the transportation cost. Hotelling ((1929), pg. 53) claimed that under mill pricing the two firms in the market would "...*crowd together as closely as possible*." , while he noted the possibility of Bertrand competition for the extreme con-

[3] Note however that Ferreira and Thisse (1996) provide evidence of the fact Launhardt had already proposed such a model of a spatial duopoly in 1885.

centration case only.[4] Fifty years later d'Aspremont, Gabszewicz, and Thisse (1979) (henceforth DGT) revisited the model and formally characterized the flawed nature of Hotelling's solution.[5] They found that the price equilibrium found by Hotelling holds, but only if the two firms are sufficiently far apart. If the two firms were located close to each other, undercutting the opponent is profitable. Higher profits destroy the pure strategy equilibrium in prices. Consequently, finding the location equilibrium for the two stage game is also jeopardized. Of course as pointed out by Hotelling, when the two firms are exogenously located at the same spot, the game reduces to pure Bertrand competition. What he missed though, and was pointed out by DGT, was that price undercutting or Bertrand competition would arise 'earlier', long before the firms 'arrived' at the same location. The tendency to undercut which allows the successful firm to capture the entire market would arise as soon as the firms are not too far apart since it allows them to capture the entire market. DGT suggest one way out of the nonexistence problem − by introducing quadratic transport. The game now exhibits a 'centrifugal' location tendency rather than central location tendency. The firms would like to locate outside of the linear city, and hence in equilibrium the two firms charge the same price and locate at the endpoints of the line segment.

There are also some other approaches to deal with the non-existence problem. One of the more ingenious ones by de Palma *et al.* (1985) shows the existence of Nash equilibrium in pure strategies by introducing sufficiently heterogenous products. A different solution has been provided by Kats (1995) where the linear city was replaced by a one dimensional bounded space without boundary, i.e., a circle. Another approach is to characterize the mixed strategy equilibrium. This line of research stems from the two Dasgupta and Maskin (1986) papers on games with discontinuous payoffs guaranteeing the existence of an equilibrium in Hotelling type models. Osborne and Pitchik (1987) undertake the task of actually identifying the equilibrium mixed strategy price distribution functions of Hotelling's original model. They identify a support for mixed strategies in prices when the firms locate close to each other. They find a unique pure strategy equilibrium in locations, in which the firms are located at about 0.27 from the respective endpoints. For those locations, the equilibrium support for prices consists of two distinct line segments. As an explanation Osborne and Pitchik invoke the intuitive parallel with the phenomenon of 'sales'. It is worth emphasizing that during this analysis they encounter highly nonlinear equations and resort to computational methods to come up with approximate numbers.

For the uniform delivery pricing models, where each firm quotes a single delivered price to all its customers, the non-existence problem is even more severe. It arises because the rationing of some consumers by one firm allows its rival to service

[4] In the industrial organization literature this result is also referred to as the *principle of minimum differentiation*. The term was coined by Boulding (1955) who used it among other things to explain the existence of similarities between Methodists, Quakers and Baptists.

[5] As noted in Osborne and Pitchik (1987), Vickrey (1964) had already identified the problem with Hotelling's analysis.

this segment of the market at a high price. This gives the first firm an incentive to undercut, thereby destroying the equilibrium (see Beckmann and Thisse (1986)).[6] A holistic analysis for the circular space scenario can be found in Kats and Thisse (1993). After showing the nonexistence of a pure strategy equilibrium in prices, they invoke Dasgupta and Maskin (1986) and characterize the mixed strategy equilibrium in prices. The location equilibrium in the first stage of the game is in pure strategies. The second part of their paper is devoted to the endogenous choice of the pricing policy by the firms. For the monopoly case, uniform delivery pricing is the optimal policy, partly because it allows the monopolist to extract all the surplus from the consumers. In the duopoly case, the consumer's reservation price r is the crucial parameter. For low $r < \frac{5}{8}$, both firms choosing uniform delivery pricing is the unique equilibrium of the pricing policy game. For higher r, the competitive region (the overlapping market area) for the two firms becomes larger, intensifying price competition between the firms, making mill pricing quite attractive. Hence both price policies can be sustained as equilibria for the duopoly, with mill pricing resulting in higher profits. Another solution to the nonexistence problem also using heterogenous products can be found in de Palma, Labbé and Thisse (1986). The interested reader may also refer to Anderson, de Palma and Thisse (1989) for an excellent comparison the above two pricing policies, as well spatial price discrimination using a heterogeneous product formulation.

3 The Model

Consider a linear city of length l with a continuum of consumers distributed uniformly on this line. Each consumer derives a surplus from consumption (gross of price and transportation costs) denoted by V. In keeping with the terminology used in the spatial competition literature we will refer to this as the consumer's reservation price. Consumers are assumed to have unit demands when their reservation value exceeds the price plus the transport cost they incur. Otherwise, they do not purchase the commodity. The transportation rate t is assumed to be linear in distance. Consumers pay a proportion α and firms pay a proportion $(1 - \alpha)$ of the transport cost.[7] Consequently, a consumer who travels a distance of d pays αtd as transport cost and the firm pays the remaining $(1-\alpha)td$ of the cost. For notational convenience we set $(1 - \alpha)t = s$ and $\alpha t = t - s$. Due to the sharing of transport costs by firms and consumers, consumers face horizontal product differentiation and firms engage in some price discrimination in our model. There are two firms in the market called A and B. The firms are located at respective distances a and b from the ends of the line ($a + b \leq l$, $a \geq 0$, $b \geq 0$), and charge prices of p_1 and p_2 respectively. In order to focus on the transport cost issue, we assume that there are zero marginal costs. Consumers buy from the firm that quotes the smallest effective price, i.e., the

[6]For more on uniform delivery pricing models also see Greenhut and Greenhut (1975) and de Palma, Portes and Thisse (1987).

[7]While assuming an exogenous α is problematic for the second interpretation of our model, it goes quite naturally with the first story.

mill price plus their share of the transport cost, in order to maximize utility. The location of the indifferent customer is denoted by $z = \frac{p_2 - p_1}{2(t-s)} + \frac{1}{2}(l - b + a)$. Firms in the model first choose a location and then quote a price. Based on the price and the transport cost consumers make their purchase decision. *Figure 1* (all figures have been attached at the end) represents the most general situation, i.e., the two stage location-price game and provides a graphical depiction of the notation developed here.

We are now in a position to obtain the profit function of the two stage game. Note that the expression below is derived for firm A, and we require that $a \leq l - b$, or that firm A is located to the left of firm B. For the firm on the right, a symmetric expression applies with only relevant change in notation. Set

$$
\begin{aligned}
\Delta &= \max\{0, \min\{a, \max\{0, l - b - \frac{p_2}{s}\}\} - \max\{0, a - \frac{V - p_1}{t - s}, a - \frac{p_1}{s}\}\},\\
\Phi &= \min\{\max\{a, l - b - \frac{p_2}{s}\}, a + \frac{V - p_1}{t - s}\} - a,\\
\Gamma &= \max\{0, \min\{l, a + \frac{p_1}{s}, a + \frac{V - p_1}{t - s}\} - \max\{a, (b + \frac{p_2}{s})\}\},\\
H &= \min\left[\left\{\frac{p_2 - p_1}{2(t - s)} + \frac{1}{2}(l - b + a) - a\right\}, \frac{p_1}{s}, \frac{V - p_1}{t - s}\right],\\
K &= a - \max\{0, a - \frac{V - p_1}{t - s}, a - \frac{p_1}{s}\},\\
M &= \min\{\frac{p_1}{s}, \frac{V - p_1}{t - s}, l - a\},\\
P &= (t - s)(l - b - a).
\end{aligned}
$$

Then the general expression for the profit function is as follows:

$$
\Pi_1(p_1, p_2, a, b) = \begin{cases}
(i) & \Delta \cdot \{p_1 - s \cdot \max[0, a - (b - \frac{p_2}{s})]\} \\
 & -\frac{s}{2}\Delta^2 + \Phi \cdot p_1 - \frac{s}{2}\Phi^2 + \Gamma \cdot p_2 - \frac{s}{2}\Gamma^2 \quad \text{if } p_1 > p_2 + P; \\
(ii) & H \cdot p_1 - \frac{s}{2}H^2 + K \cdot p_1 - \frac{s}{2}K^2 \quad \text{if } |p_1 - p_2| \leq P; \\
(iii) & M \cdot p_1 - \frac{s}{2}M^2 + K \cdot p_1 - \frac{s}{2}K^2 \quad \text{if } p_1 < p_2 - P.
\end{cases}
$$

Notice that the expression depends on the relationship between the price difference $p_1 - p_2$ and $P = (t - s)(l - b - a)$, the cost to a consumer to go the extra way from a to b. Further, observe that if the firm serves an adjacent market area (an interval immediately to its left or to its right) of length N, then the revenue from these customers is $N \cdot p_1$ and the cost of serving then is $\frac{s}{2} \cdot N^2$. It remains to determine that $\Delta, \Phi, \ldots$ are the correct market sizes. We now briefly discuss each of the three possible cases separately.

(i) $p_1 > p_2 + (t - s)(l - b - a)$. This case occurs when firm A is being undercut by firm B. Here Δ is the size of the market area to the left of its location and Φ is

the size of the market area to the right of its location. The last two terms represent the possible case of 'leapfrogging' a far away market: Γ is a possible market area to the right of the opponent's territory. It occurs since the opponent is charging a lower price and may not be willing to serve all the customers on its right, thus making it feasible for the left-hand side firm to serve that chunk of the market by sufficiently raising its price.

(ii) $|p_1 - p_2| \leq (t-s)(l-b-a)$. In this case no firm is able to undercut its rival. The first two terms here signify profits from the right hand side and the last two terms signify profits from the left hand side. H is the minimum of the three following possibilities: either the line segment between a and location of the indifferent consumer, or $\frac{p_1}{s}$ which is all the market the firm A would like to serve (in this case, on its right hand side), or only the line segment representing the locations of those consumers (located to the right of a) who would like to buy from firm A. Since this is the no undercutting case, the (left-hand side) extent of the market area to which firm B is willing to sell play no role here. The expression K just depicts the firm A's captive market on the left hand side.

(iii) $p_1 < p_2 - (t-s)(l-b-a)$. This case occurs when firm A undercuts firm B. The first two terms are profits from the market area, of size M, on the right and the last two are profits from the market area, of size K, on the left. Consider M. Firm A can sell to the market segment it wishes to, given by $\frac{p_1}{s}$, unless of course some of those customers themselves do not want to purchase from it ($\frac{V-p_1}{t-s}$). Finally, it allows for the fact that firm A sells to the entire line segment, from its own location a up to the right hand side boundary of the linear space, l. The last two terms of this part of the profit function represent the market area of the firm to the left of its location.

The general expression for the profit function given below indicates a host of possibilities from which one may surmise that multiple equilibria can exist in our setting. Clearly, it will not be possible to analyze the model without making some simplifying assumptions. Any equilibrium outcome of the model will be determined by the interplay of the consumer's reservation value and the firm's choice of location and prices. Since our model combines elements from both the mill pricing and the uniform delivery pricing models, absence of sales can occur for two reasons. Under certain parameter conditions, because of their reservation utility, the consumers will not want to purchase the product at the price offered by the firm. On the other hand, for certain location-price pairs, it is also possible that a firm may not want to sell to some consumers who are willing to buy from it. Keeping these in mind we analyze two different games to gain some insight into the problem. We first study a pure location game. Here V does not play any role and we are able to focus on the interaction between price and location choice. We then look at the situation where the firms are located at the same spot. Here location does not play a role and allows us to concentrate on the interaction between V and the prices set by the firm.

3.1 A Location Game

In this section we assume that the price p of the commodity is exogenously given, for example as in the regulator's world. This can also happen if the firms have previously agreed to fix prices. Another possibility is that prices have been chosen earlier in the distribution channel by manufacturers or wholesalers and retailers are subject to resale-price maintenance. For the sake of simplicity, in this section we assume that $p + (t - s)l < V$.[8] In this model the location of the consumer who is indifferent between buying from A and buying from B simplifies to $z = \frac{l-b+a}{2}$. Suppose the regulator sets prices high enough to ensure that both firms can sell to all the consumers. Then from the previous section we can write each firm's profit function as

$$\Pi_1(a,b) = pz - \frac{(1-\alpha)t}{2}\{a^2 + (z-a)^2\}$$

and

$$\Pi_2(a,b) = p(l-z) - \frac{(1-\alpha)t}{2}\{b^2 + (l-b-z)^2\}$$

As before, in each of these profit functions the first term denotes revenues and the second term is the share of the transport cost that each firm has to incur to sell to all of the customers who wish to purchase from it.

In order to look for equilibria we define three ranges for the exogenously given price since this is critical in deciding whether the firms can cover the cost of transporting to all the consumers who wish to purchase. Case (i) $p < \frac{1}{4}sl$: Prices are so low in this range that the firms are not able to serve the entire market area. See *Figure 2* for this case. At best each can only sell to a market size of $\frac{l}{2}$. Case (ii) $\frac{1}{4}sl \leq p \leq sl$: In this price range *together* the two firms can cover the entire market area. The above profit functions are appropriate for this range. Notice that the total costs are triangles whose areas the firms try to minimize in this instance. Case (iii) $p > sl$: This is the range of very high prices and each firm can cover the entire market by itself.

Proposition 1. *If $\frac{1}{4}sl \leq p \leq sl$, there exists a unique equilibrium in locations. The equilibrium locations are symmetric and are given by*

$$a^* = b^* = \frac{2p + sl}{6s}$$

Equilibrium profits for each seller are identical and given by

$$\Pi_i(a^*, b^*) = \frac{1}{72s}[40pls - 8p^2 - 5(sl)^2]$$

[8] We analyze the consequences of reservation prices in the next section where firms play a pure price game with fixed locations.

Proof. The proof consists of taking the derivative of each firm's profit function with respect to its location and solving the following system of two equations obtained from the first order conditions.

$$a = \frac{4}{5}\frac{\frac{1}{2}p + \frac{1}{4}ls - \frac{1}{4}sb}{s}$$
$$b = \frac{4}{5}\frac{\frac{1}{2}p + \frac{1}{4}ls - \frac{1}{4}sa}{s}$$

We also verify that the second order conditions are satisfied. Substituting the optimal locations in the profit functions yields the equilibrium profits. Furthermore, we can check that firm A does not wish to locate to the right of firm B. Given b^*, we know that $l - b^*$ is less than $\frac{l}{2}$ which denotes the location of the indifferent consumer in the above equilibrium. Consequently, in the above equilibrium firm A has half the market. If firm A locates to the right of firm B then the new indifferent consumer will lie in the interval $l - b^*$ and firm A's market area will be strictly less than half the market area $\frac{l}{2}$.[9] Hence, firm A will not gain by selecting a location to the right of firm B. By symmetry firm B will never locate to the left of firm A. So, (a^*, b^*) constitutes an equilibrium. ■

In contrast to the original Hotelling model, here transport cost considerations in maximizing profits prevent the firms from locating at the center in all instances. Notice that the optimal location can be rewritten as $a^* = \frac{l}{6} + \frac{p}{3s}$. In fact $a^* \in [\frac{l}{4}, \frac{l}{2}]$ with the firm never locating to the left of $\frac{l}{4}$ to ensure that costs are minimized. The corresponding profits lie in the range $[\frac{2}{8}pl, \frac{3}{8}pl]$ with profits increasing as the firm A moves to the right. Observe that in equilibrium the indifferent consumer is always located at $\frac{l}{2}$ irrespective of the location of the two firms.

When $p \in [\frac{1}{4}sl, sl]$ as assumed in the proposition, the optimal location varies inversely with s and directly with p. Comparative statics results suggest that firms in our model also have a central location tendency. Clearly, $\frac{da^*}{dp} > 0$, suggesting that both firms want to locate closer to the center as the exogenously given price grows. As the regulator raises the price, each firm can sell to a larger segment of the market and in order to minimize costs moves towards the centre. Since this is true for both firms, equilibrium behavior ensures that the position of z remains unchanged. Finally, using the fact that $s = (1 - \alpha)t$, it is also possible to show that $\frac{da^*}{d\alpha} > 0$. Thus as the consumers bear a greater proportion of the transport cost, we get an outcome closer to Hotelling's mill pricing model. This is intuitive since the closer the situation is to Hotelling's case, the stronger is the central location tendency.

Equilibrium location for the other two cases cannot be obtained using standard first order conditions and is discussed next.

Fact 1. Case (i) $p < \frac{1}{4}sl$: In this case the combined market area for both firms together is no longer l (see *Figure 2*). The regulator's price is so low that firms do

[9] Note that firm A could have chosen the symmetric (about Firm B's location) location on the left hand side and earned at least as much profit. However, we find that optimal profits from choosing a location to the left of Firm B are obtained at a^*.

not serve all the consumers in the city. To see this we equate $a^* + \frac{p}{s} = (l - b^*) - \frac{p}{s}$ and solve for p. This equation enables us to find the price at which the market areas of the two firms just touch each other without overlapping. Since this occurs at $p = \frac{1}{4}sl$, for prices below this the firms can have isolated markets, provided locations are chosen optimally. Firms will locate such that their market segments do not overlap while maximizing the market area served. They choose locations so as (a) not to have overlapping market areas ($\frac{p}{s} < \frac{l-b-a}{2}$), and (b) to ensure that $(p - as)$ is nonpositive. Thus, it is possible to have a whole range of locations as equilibria in this instance.

Fact 2. Case (iii) $p > sl$: Both firms locate at $\frac{l}{2}$. From the previous proposition we have already seen that no seller wants to choose a location to the left of their original location with increases in price. By symmetry of the profit functions rightward movements are ruled out, as this amounts to relabelling the firms and therefore both firms choose $\frac{l}{2}$.

Thus, with a slight modification of the location game we find that Boulding's ubiquitous principle of minimum differentiation is no longer so pervasive. The implications of this for the regulator are also fairly obvious. If the regulator decides to lower prices after the firms have chosen locations, the firms' locations will be sub-optimal and in order to maximize profits the firms will not serve all the customers they were selling to before the price reduction. Of course when prices are raised locations will still be sub-optimal but the earlier customers will not be left out. We next solve for a price game where both firms are at the same location by decree. The insights from these two games and their implications for the two-stage game are discussed in the concluding section.

3.2 A Special Price Game

In this section we analyze a price game when both players are constrained to being at the same location. Admittedly, this is an extreme assumption, but it is also the analogue of the problem in the previous section where the firms faced exogenously given prices and were allowed to compete only in locations.[10] All other assumptions of the previous section are assumed to hold. As argued earlier, for low V some consumers will not want to purchase the product at the price offered by the firm. These consumers may then be left out of the market. Another possibility is the fact that a firm may not want to sell to some willing consumers. This opens up an interesting possibility. Suppose firm A does not wish to sell to a particular segment of the market. Then given a sufficiently high reservation price for consumers, firm B can alter its price and sell to the excluded section of the market. Thus, transactions between the agents depends on mutual consent between buyers and sellers regarding the trade. Clearly, rationing of certain consumers is a distinct possibility in this model. Furthermore, this rationing will be of a discriminatory nature, as each additional consumer will pay a higher effective price based on the distance from the seller's location.

Since both firms are located at the same place, there is no horizontal product differentiation and we have a case of pure price competition. As shown by DGT, in

[10] One might think of city planners who will only let firms set up shop at a particular location!

the Hotelling model there exists a pure strategy equilibrium for this price subgame where prices are equal to the marginal cost, i.e., zero. Given positive transport costs for firms, our results differ from the Hotelling model since there is no pure strategy equilibrium in the price subgame. Instead, we show that a mixed strategy equilibrium exists. In this equilibrium prices always exceed the marginal cost of production. This is similar to the results in models of Bertrand-Edgeworth competition (see for example Allen and Hellwig (1986), Dasgupta and Maskin (1986) and Kats and Thisse (1993) in the context of spatial models.

We will first establish a result about the upper (p^u) and lower (p^l) bounds on prices. Without loss of generality, consider a realization of the mixed strategy where firm A is charging a low price and firm B sets a high price. In the rest of this section while we often refer to firm A as the low price firm and firm B as the high price firm, we have in mind only a particular realization of the mixed strategy. We obtain p^u by computing the monopoly price while taking V into account. When the reservation price is below a certain threshold (say $\widehat{V}$), at the price upper bound denoted by p_r^u some consumers in the market will not wish to purchase from firm B.[11] When $V \geq \widehat{V}$, the upper bound is given by the highest price at which firm B can sell to consumers located furthest from it, and is denoted by p_a^u ($> p_r^u$)since the firm sells to all residual consumers, i.e., consumers who could not purchase from the low price firm A. At that point the gains from increased price to firm B clearly outweigh the losses from forgoing *any* market share. Denote the markets segments of firm for these two cases by x_r and x_a respectively. This will then be used to prove a proposition about mixed strategy equilibria in the price game.

Lemma. *The support of any equilibrium in the price game is a strict subset of* $[0, V]$*.*

Proof. We will demonstrate that the interval $[p^l, p^u] \subset [0, V]$ for each of the two possible cases. We show that when firm A now raises its price starting from zero, firm B will not raise its price beyond p^u. Similarly, firm A will not reduce its price below a lower bound denoted by p^l.

(I) For $V < \widehat{V}$, p_r^u is the candidate upper bound. By definition we know that p_r^u dominates all prices in the interval $(p^h, V]$. When firm A raises its price beyond zero, firm B loses customers from the center of its market. The optimal response for firm B is to either lower its price and sell to the consumers previously left out or to undercut firm A. Thus prices do not exceed p_r^u.

(II) For $V \geq \widehat{V}$, the candidate upper bound is p_a^u. When firm A now raises its prices, firm B can continue to sell to the residual market at p_a^u or undercut firm A. Thus in either case there is an upper bound on prices.

Now consider the existence of the lower bound for the two possible cases.

[11]The precise value of the threshold reservation price is not relevant to the argument here. Exact computations are shown in the next result.

(III) Again, consider $V < \widehat{V}$. Since firm A's profits are increasing in its price, it will prefer to raise its price up to a point. The lower bound on prices can be obtained from equating $\Pi_2(p_1, p_2^{'}(p_1))$ (where $p_2^{'}(p_1)$ is the best response price of firm B) with the profit firm A from raising its price. Prices in the range $[0, p_r^l)$ are dominated by p_r^l and when the firm raises its price above p_r^l the rival firm will have an incentive to undercut.

(IV) For $V \geq \widehat{V}$, the candidate lower bound is p_a^l. A similar argument establishes the lower bound on prices for this case. Thus prices will not fall below p^l.

This allows us to conclude that prices above p^h are dominated by it. Also, choosing p^l gives higher profits compared to prices less than p^l. So there cannot be a pure strategy equilibrium outside this interval, and hence there cannot be one in mixed strategies either as it would involve the play of dominated strategies. ■

Having isolated the bounds within which an equilibrium exists we will now argue that the price game does not have a pure strategy equilibrium in this range. However, a mixed strategy equilibrium exists. The next proposition shows the support of this equilibrium when the firms locate at the center. We then argue that this can be generalized to asymmetric location choices of the firms.

Proposition 2 *For $a = b = \frac{l}{2}$, the price game has no pure strategy equilibrium. A mixed strategy equilibrium does exist for this price game. For $V < \widehat{V}$, the support is given by $[sV\left(\frac{t-\sqrt{2ts-s^2}}{(t-s)^2}\right), \frac{sV}{\sqrt{2ts-s^2}}]$ and for $V \geq \widehat{V}$, the support is given by $[s(l-a)\left(1+\frac{s(l-a)}{2(V-(l-a)(t-s))}\right), V-(l-a)(t-s)]$.*

Proof. From the previous lemma we know that any equilibrium must lie in the interval $[p^l, p^u]$. Assuming $a = \frac{l}{2}$ we will now compute the critical V and the two associated intervals. The threshold V is found by taking the derivative of firm B's profit function at the price when the furthest consumer is indifferent between buying and not buying the commodity from firm B, and occurs at $p = V - (t-s)\frac{l}{2}$.[12] For a low reservation price i.e., $V < \frac{l}{2}(2t-s) = \widehat{V}$, the upper bound is obtained by solving the monopolist's profit maximization problem. Assuming that firm B behaves as a monopolist on the residual demand area left by firm A, its profits can be written as $\Pi_2 = \frac{1}{2}(\frac{V-p_2}{t-s} - \frac{p_1}{s})(2p_2 - s(\frac{V-p_2}{t-s} + \frac{p_1}{s}))$. The best response price then is $p_2^{'}(p_1) = \frac{stV-p_1(t-s)^2}{(2t-s)s}$. Using this we compute profits of firm B in terms of p_1

$$\Pi_2(p_1, p_2^{'}(p_1)) = \frac{1}{2}\frac{(sV-p_1t)^2}{(2t-s)\,s^2}$$

Clearly $\frac{\partial \Pi_2(p_1, p_2^{'}(p_1))}{\partial p_1} < 0$. To find the lower bound on price we next equate the profits of the two firms using the fact that $\Pi_1(p_1,p_2) = \frac{p_1^2}{2s}$. Note that since firm A is the low

[12]Evaluating the derivative of the profit function with respect to the price at $p = V - (l-a)(t-s)$ we obtain $\frac{\partial \pi}{\partial p} = \frac{tV-[V-(l-a)(t-s)](2t-s)}{(t-s)^2}$. A monopoly solution is possible only when the numerator is positive, i.e., when $V < (l-a)(2t-s)$.

price firm its profit expression does not contain a p_2 term. Equating the two profit expressions gives us $p_1 = sV\left(\frac{t\pm\sqrt{2ts-s^2}}{(t-s)^2}\right)$. The root with the positive discriminant yields a negative p_2 and hence is eliminated. The optimal value of p_1 which is the lower bound is then given by

$$p_r^l = sV\left(\frac{t-\sqrt{2ts-s^2}}{(t-s)^2}\right)$$

This also makes intuitive sense as a high p_1 implies that the low-price firm is selling to a large section of consumers leaving very little behind for the other firm. Using the root with the negative discriminant we find that

$$p_r^u = \frac{sV}{\sqrt{2ts-s^2}}$$

It can be checked that $p_r^u > p_r^l$ which is when such a situation will occur. Also, using these prices we find that profits are

$$\Pi_1 = \Pi_2 = s\left(\frac{V}{(t-s)^2}\right)^2\left(t-\sqrt{2ts-s^2}\right)^2$$

Suppose $V \geq \frac{l}{2}(2t-s) = \widehat{V}$. A similar argument establishes the lower bound on prices for this case. The only difference is that the monopoly profit of firm B is now different. So, $p_a^u = V-(l-a)(t-s)$ and $p_a^l = s(l-a)\left(1+\frac{s(l-a)}{2(V-(l-a)(t-s))}\right)$. Now let us consider what happens when prices are in the range $[p^l, p^u]$. Suppose a firm is charging the price p^u. Then by charging a price $p^u-\varepsilon$ (where $\varepsilon > 0$, and is small) its rival can undercut the firm completely, leaving the firm with zero profits. This phenomenon of successive undercutting will occur for any price above p^l. Once prices reach p^l, one of the firms is better off selling to the remaining consumers at a price of p^h instead of undercutting its rival further. However, the firm charging p^l would now prefer to undercut the high price firm. Hence there are no pure strategy equilibria.

It can be shown that the two stage location-price game satisfies *Theorem 5* of Dasgupta and Maskin (1986, pg. 14). Hence it is satisfied for this price subgame. This theorem proves that games with discontinuous payoff of the type seen here have mixed strategy equilibria. Hence we assert that a mixed strategy equilibrium identified here exists. ■

We now argue that the same result is also true for any other location of the two firms. The problem becomes asymmetric in this case and the critical value of V on the right hand side segment of the firm's location can differ from the critical value on the left hand side. This alters the profits of the high price firm and consequently the value of the bounds. Given that the problem is computationally intensive, we just provide the rationale for the argument without explicitly computing the bounds.

Fact 3. *For $a+b=l$, the price game has no pure strategy equilibrium. A mixed strategy equilibrium however, does exist in this price game.* To deal with the case of any location, we will consider situations where $a < \frac{l}{2}$ and $a+b=l$, using the rationale suggested above. While the method for computing the upper and lower bound remains the same, three distinct possibilities can arise in this situation. When the reservation value is high as described above firm B will charge a price such that after paying the transport cost the entire surplus from the consumer at l is exhausted. This alters the value of p_a^u and consequently of p_a^l. When reservation price is low in the sense described above, firm B charges the monopoly price at which some customers to the left and right of a are not served. It is possible to observe from the geometry of the situation (see *Figure 3*) that this alters the value of the monopoly price and hence of p_r^l as well. Finally, there is also an intermediate value of the reservation price at which some consumers to the right of a will not be able to purchase at the monopoly price. This case will also yield different values of p^h and p^l. Thus we will have three different inequalities which have to be solved using a technique similar to the one used for $a = \frac{l}{2}$. The main difference with the previous case therefore stems from the fact that the computation of monopoly profits changes. This affects the high price firm's best response and consequently the lower bound without altering the logic of the calculation. Since Theorem 5 of Dasgupta and Maskin (1986) holds in this case as well, the mixed strategy equilibrium exists.

It is worth pointing out that while the upper bound can be the monopoly price, the lower bound differs from zero and from s as well. Since s may be thought of as the marginal cost to the firm of (delivering) an additional unit, this is different from the usual lower bound of the support of the mixed strategy in rationing models. These differences arise because the rationing mechanism in our model can be described as *discriminatory rationing*. Consumers not served by the low price firm are served by the high price firm, but each additional consumer pays a higher effective price which is proportional to the distance from the firm's location. It is precisely this reason which also prevents the price from going down to zero due to price undercutting, as it becomes worthwhile for one firm to sell to the market segment that is left out instead of lowering prices further.

The mixed strategy equilibrium in our formulation has another attractive feature. Equilibrium profits in the DGT model are always zero when $a+b=l$ and involves the play of a pure strategy with both firms choosing zero prices. In general, a mixed strategy equilibrium is often considered unattractive as players are indifferent between all the pure strategies involved and it does not give any reason to select between these strategies (see Osborne and Rubinstein, (1994) for more on interpretations and criticisms of mixed strategies, including points on which even the authors of the book disagree). The redeeming feature of the equilibrium mixed strategy in our model is that fact that expected profits are always positive, whereas in the DGT framework they are always zero. In fact, profits are positive for any realization of the mixed strategies since p^l is always positive in our model.

4 Discussion

This paper analyzes a model where both firms and consumers have transportation costs. In the standard mill pricing model the pure strategy equilibrium breaks down since firms have and incentive to move to the center and this makes it easier for the rival to undercut. The firm that undercuts successfully gains the entire market. In our model while choosing locations the firms also have to ensure that they minimize their of transport cost bill. So, while there is a central location tendency, in our model as well there is also a countervailing force. The firm that undercuts its rival may not be able to sell to the entire market. Similarly, note that in the uniform delivery price models one firm may charge a high price and sell to customers who its rival may be unwilling to service. In our model the 'rationed' consumers may be not be willing to buy from a high price firm since the price inclusive of transport costs may exceed their reservation price.

Based on these facts we conjecture that a pure strategy equilibrium can exist in the two stage game. However, given the complex nature of the profit function, it becomes difficult to solve for this game. One way out might be to resort to numerical methods and try to simulate the equilibrium. This is also not necessarily an easy task as can be seen from the Osborne and Pitchik (1994) analysis.

Here we follow a different approach. We solve two different games to develop some insights for the two stage game. In the first of these we consider parametric prices, thereby restricting firms to chose a location strategy only. We find that when the share of transport costs borne by the consumers increase firms move closer to the center. Yet, the comforting feature of the model is that for any given price there is a symmetric location equilibrium where the firms chose their location keeping their transport cost in mind. It turns out that the firms never locate at the end points. In fact there is a threshold location ($\frac{l}{4}$) which the firms will never cross. In the second game firms are assumed to locate at the same spot. Thus location choice is no longer an element of the strategy space. Here we find the existence of a mixed strategy equilibrium. This is useful since the existence problem is resolved. The two stage game will have an equilibrium if not in pure strategies, at least in mixed strategies. This game also tells us that when the firms locate too close to each other there cannot be an equilibrium in pure strategies, i.e., the incentive to undercut is too strong. The importance of the reservation price is also shown by the this game. We next show how all of this intuition can play a role in simplifying the analysis.

Now consider the following situation. Assume that no firm is able to undercut its rival, i.e., the firms are located far enough. Then, it is possible to argue that a pure strategy equilibrium of the price-location game does not exist when a firm is losing market share, i.e., when it is charging a price so high that consumers cannot afford to buy from it. This can be shown by considering the following three cases separately:
Case (i) $p_1 - as < 0$. We see that the firm A has market loss on its left hand side. Also assume that $\alpha > 1 - \alpha$. Then it is easy to check that by charging a price $p_1 + \varepsilon$ ($\varepsilon > 0$), firm A can increase its profits. Suppose now $\alpha \leq 1 - \alpha$. This is shown in *Figure 4*. In this case there will exist a price pair (p_1, p_2) which will constitute an equilibrium in pure strategies. However, this is not robust to the choice

of firm location. It is easy to check that firm A can always do better by moving to its left. Since we identify conditions for the existence of an equilibrium in the two stage game, unlike DGT we require that the candidate equilibrium must also survive the next stage of subgame perfection - the choice of optimal locations. Note that the tendency to move to the left for firm A is present irrespective of the relationship between α and $1-\alpha$. Hence it is not possible for a pure strategy equilibrium to exist in this case.

Case (ii) $p_1 - (z-a)s < 0$. In this instance firm A has market loss from the right hand side. One can check that by charging a price $p_1 + \varepsilon$, firm A is better off. It gains market share and sells to all consumers at a higher price.

Case (iii) $p_1 - as < 0$, and $p_1 - (z-a)s < 0$. Under this scenario firm A is losing market areas on both sides. Clearly, by raising its price firm A will increase market areas on both sides and sell to all customers at the higher price eventually leading to one of the two situations described above.

Symmetric conditions for firm B can be explained using the above arguments.

Arguments like these based on the intuition gained from the two games solved here can be used to develop a new strategy for solving the two stage game. We suggest a constructive approach to finding the pure strategy equilibrium. The significance of the above line of reasoning showing the interaction between prices and locations lies in the fact that it allows us to eliminate some of the components of the profit function for the two stage game. We believe that by reducing the profit function into one that is less complicated through a series of simple results like the one above and focusing only on the relevant range, an easy solution to the two stage game can be found.

Finally, the paper also raises another interesting question − the issue of endogenizing the transport cost sharing decision. We believe that this will provide an alternative approach to modeling the choice between uniform delivery pricing or mill pricing for firms.

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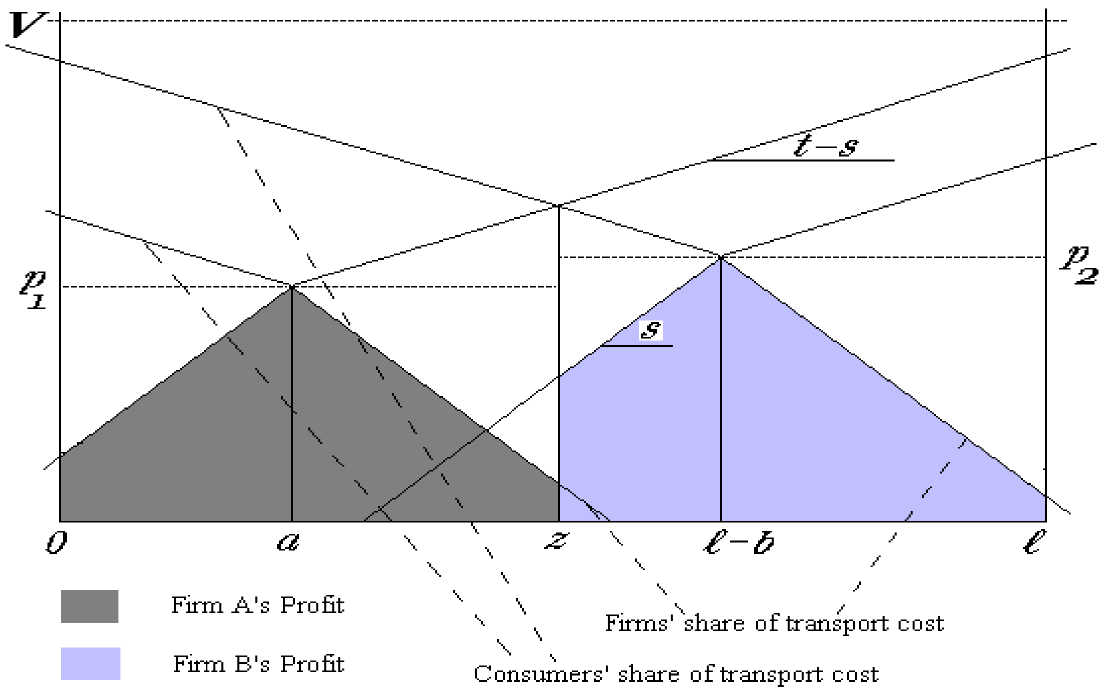

Figure 1

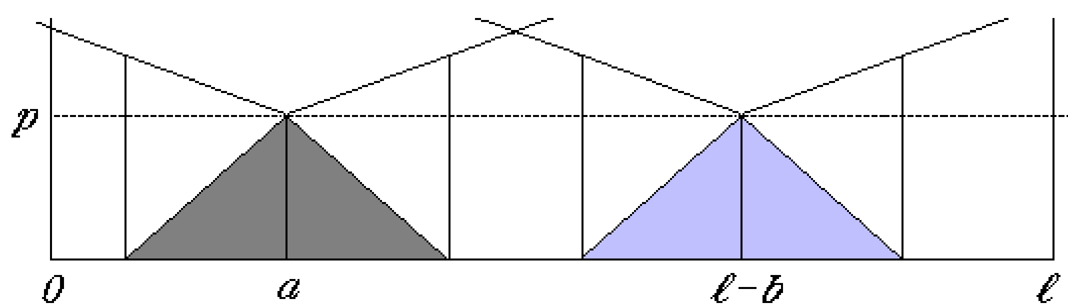

Figure 2

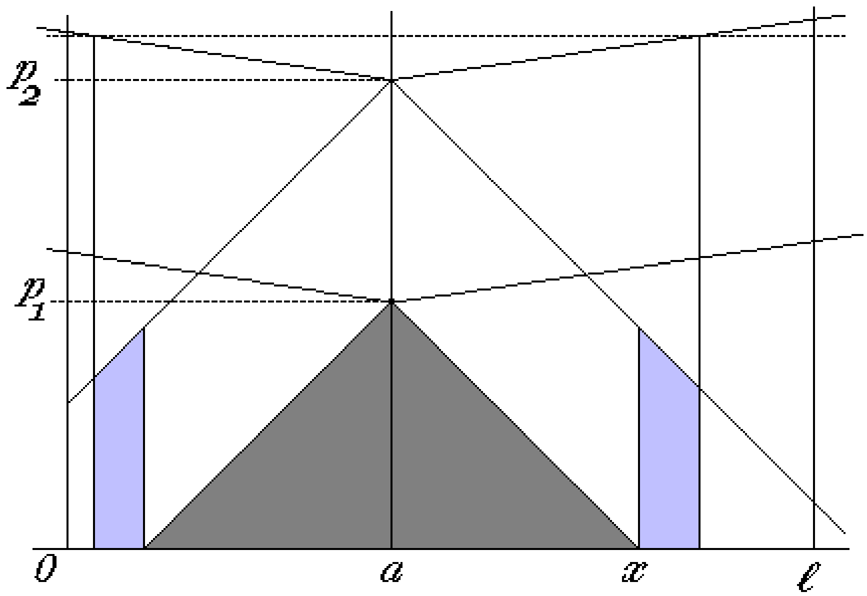

Figure 3

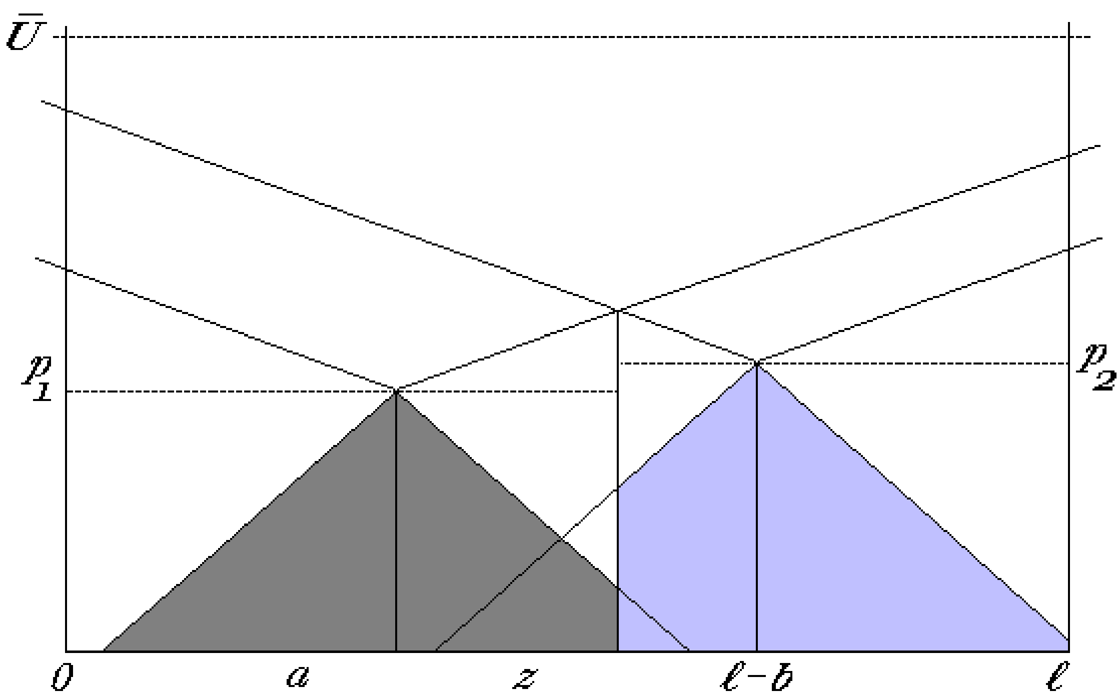

Figure 4