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Estimating the Inflation-Output Variability Frontier with Inflation Targeting: A VAR Approach

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Abstract: This paper (i) illustrates how a VAR model can be used to evaluate inflation targeting, (ii) derives the policy frontier available to the central bank using counterfactual experiments with real time data, and (iii) estimates how this frontier has changed over time in terms of the position and slope of the available tradeoff between output gap variability and inflation variability under inflation targeting. Various inflation targets are considered as are tolerance bands of varying width around these targets. The results indicate that over time (i) a given reduction in inflation variability is associated with a smaller rise in output variability and that (ii) a given inflation variability is achieved with smaller interest rate volatility. Consistent with the data, our results require federal funds rate persistence, though no instrument instability was observed. One interpretation of these results is that they reflect the growing credibility of the Federal Reserve.

I. Introduction

Over the past decade and a half, most major central banks around the world have adopted monetary policy frameworks that include either explicit or implicit inflation targets. An early presentation and discussion of the initial applied successes are summarized in Bernanke et al. (1998) and a recent summary of inflation targeting at a conceptual level is given in Svensson (2003).

In practice, central banks have aimed at average inflation rates over periods of two years or so rather than at the monthly inflation rate. Svensson (1997) has shown that inflation targeting is optimally characterized as inflation *forecast* targeting over the policy horizon. That is, central banks should set their policy tool so that the forecast of average inflation over the horizon is equal to the inflation target (assuming a linear model and a quadratic objective function).

In recent years, analyses of alternative approaches to monetary policy have focused on the trade-off between inflation variability and output variability implied by alternative policy rules; see, for example, Taylor (1994) and the papers in Taylor (1999). Much of this literature has concentrated on variations of the Taylor Rule or on optimal policy rules derived from an explicit loss function and a variant of a New Keynesian model. Nessén and Vestin (2005) recently derived the inflation variability-output variability frontier for the case of average inflation targeting, a useful extension since central banks appear to target longer-run average inflation rather than monthly inflation.

Under the maintained assumption that the Fed has been an implicit inflation targeter, our contribution to the monetary policy evaluation literature is estimation of the tradeoff between inflation variability and output variability under inflation targeting using a small VAR model.[1] Although the literature deriving this trade-off for alternative monetary policy rules typically employs a calibrated New Keynesian model, in light of McCallum's (1988; 1999) argument that policy rules should be evaluated in a variety of models, we use a small VAR model. Furthermore, while we use a small VAR that allows us to illustrate how inflation targeting may be evaluated using inflation forecasts that reflect all information in the model, the technique can be applied to larger structural models as well.

The model we employ comprises the variables in a small representative New Keynesian model of the U.S. economy, and we estimate the inflation variability-output variability trade-off through a series of

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[1]Goodfriend (2003) presents a persuasive argument that the Greenspan Fed can be characterized as gradually adopting and then maintaining an implicit inflation targeting policy.

counterfactual experiments. Our first experiment begins in 1983:10, using a model estimated over the period 1962:1-1983:9, the second begins in 1993:1 using a model estimated over 1980:1-1992:12, and the third begins in 2001:1 using a model estimated over 1980:1-2000:12. In an attempt to illustrate the actual tradeoffs faced by the Fed in our various experiments, we construct real-time data sets for each period under investigation. In the end, we are able to show how the policy frontier has changed through time, both in terms of position and slope of the available tradeoff.

In our model, monetary policy is conducted using the federal funds rate to target a forecast of average inflation. Consistent with Svensson (1997), we implement this assumption by allowing the funds rate to respond to all variables in the model. That is, in contrast to a simple Taylor rule, we assume the Federal Reserve adjusts the funds rate in response to all information in the model relevant to the forecast path of inflation. Support for this approach comes from Bernanke (2004), who, in comparing and contrasting use of "simple feedback policies" (instrument rules) and "forecast-based policies," indicates that the forecast-based approach "… has become increasingly dominant in the monetary policymaking of leading central banks…. [T]he Fed relies primarily on the forecast-based approach for making policy."

We compute the policy innovation needed to achieve the inflation goal exactly, so our approach can be characterized, at least roughly, as one of policy commitment. Our motivation is provided in part by Clarida, Gali and Gertler (1999), who show that policy commitment produces gains to welfare compared with discretionary policy, even when there is no inflationary bias of the type in the classic presentations of Kydland and Prescott (1977) or Barro and Gordon (1983). In particular, the Clarida-Gali-Gertler model shows that a monetary authority able to commit to a noninflationary policy can produce an improved tradeoff between inflation and output. This result arises due to the ability of a committed central bank to manage private sector expectations of inflation.

As our procedure always selects the policy innovation needed to attain the inflation goal, there is a risk of instrument instability. In sharp contrast, Woodford (1999) shows that policy inertia, the observed "sluggish" movements of the target interest rate compared with what might be suggested in a standard optimal control solution, can represent optimal (as opposed to optimizing) policy. Specifically, he argues that building and maintaining credibility requires that central bank optimization take into account not only current conditions and the bank's forecast of economic conditions in the future, but also requires validating

expectations formed in the past with policy actions that produce current conditions consistent with these expectations. He concludes (p. 8) that such an approach "… if understood by the private sector, offers the prospect of significant effects of central bank policy upon aggregate demand, without requiring excessively volatile short-term interest rates." That is, a credible, successful central bank, to paraphrase Woodford, induces the bond market to do its work for it. So, while our approach can demonstrate instrument instability, it is also possible that inertia in the interest rate will result. It turns out that there is substantial smoothing of the interest rate in our counterfactual simulations, even though we have not imposed any features that would explicitly limit the magnitude of interest rate movements.

In recent theoretical literature, a common way to analyze monetary policy is to write down a loss function subject to the constraints imposed by the economic system. We stop short of attempting an empirical implementation of this approach. Central bankers do not usually announce the loss function or the relative weights on output and inflation variabilities.[2] Our more modest goal is to estimate the policy variance frontier implied by our empirical VAR model. Note that this frontier is a natural focal point since the expected value of the output gap is zero and in a credible inflation targeting regime, the inflation rate on average is equal to the (explicit or implicit) inflation target.[3] What policymakers can influence is the variability around these values. Technically, our empirical derivation of the policy variance frontier shows the marginal rate of technical substitution, the rate at which policymakers can tradeoff inflation vs. output variability. Since we do not attempt to specify or estimate the loss function and its parameters, we do not present an estimate of the marginal rate of substitution, the rate at which policymakers want to make this tradeoff. [4]

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[2] Svensson (2003, p. 451) argues that "In practice, the loss function is not specified in this detail…The decisionmaking body of the central bank selects the combination of forecasts that 'looks best' in the sense of achieving the best compromise between the inflation gap and stabilizing the output gap, that is, that implicitly minimizes the loss function."

[3] Thus, we assume that there is no attempt to maintain output above the natural level, which eliminates inflation bias, as in Barro and Gordon (1983). Their analysis is in a model in which the central bank is not able to credibly commit today to particular policy actions in the future. So, while the bank can talk tough today about future policy, it has an incentive to cheat today to raise output above its target level. The equilibirium is where the loss due to the long-run inflation associated with cheating offsets any gains from stimulating output above target. Of course, a criticism of this analysis is that a rational central bank would understand that there is no long-run gain to output from cheating today and hence would not cheat in the first place. For a further discussion of this literature, see section 4 of Clarida, Gali and Gertler (1999).

[4] Cecchetti and Ehrmann (1999) provide evidence on the marginal rate of substitution across central banks by estimating an economic model, combining it with the actual path of interest rates, and solving for the parameters of a simple loss function consistent with the model and the interest rate actions.

In work similar in spirit to our analysis, Carlstrom and Fuerst (2005) use policy simulations to analyze the commitment by the Fed in 2003 to maintain the federal funds rate target at an historically low level for a "considerable period of time." Since the Fed was nearing the zero nominal interest bound at the time, this commitment to policy inertia may have been crucial in conditioning expectations about longer term interest rates. Successfully signaling low future longer-term interest rates by announcing an intention of a low fed funds rate for a "considerable period" almost surely counts heavily on credibility. In the extreme, a credible central bank can announce policy intentions without changing the current policy instrument at all. In illustrative simulations, Carlstrom and Fuerst find that credible commitments allow the central bank to achieve given results with smaller policy interventions than in simulations without such credibility.

II. Methodology

In this section, we present the basic methodology used to estimate the policy frontier for alternative inflation targets and associated tolerance ranges or bandwidths. We first introduce notation. Next, we provide an intuitive discussion of the approach. We conclude this section by presenting the technical details of how inflation targeting can be assessed.

We begin with a structural model

$$y_t = A_0 y_t + A_1 y_{t-1} + ... + A_p y_{t-p} + \varepsilon_t \tag{1}$$

In equation (1), y_t is an (Nx1) vector of variables, including the inflation rate and the federal funds interest rate. The elements of the A_i matrices represent the structural coefficients and the elements of ε_t are structural shocks. We assume that $E(\varepsilon\varepsilon') = \Omega$, a diagonal matrix. The reduced form of (1) is $\Pi(L)y_t = e_t$, where $\Pi(L) = I - \Pi_1 L^1 - ... - \Pi_p L^p$, with the reduced form coefficient matrices given by $\Pi_i = (I - A_0)^{-1} A_i$ and reduced form shocks by $e_t = (I - A_0)^{-1}\varepsilon_t$. The moving average matrix is defined as $C(L) = [\Pi(L)]^{-1}$, with $C_0 = I$. Define $D_s = C_s (I - A_0)^{-1}$. The moving average representation (MAR) of equation (1), expressed in terms of the structural shocks, is

$$y_t = \sum_{s=0}^{\infty} D_s \varepsilon_{t-s} \ . \tag{2}$$

Fundamental to our analysis is the historical decomposition, which in its basic form is found by advancing equation (2) by n periods:

$$y_{t+n} = \sum_{s=0}^{n-1} D_s \varepsilon_{t+n-s} + \sum_{s=n}^{\infty} D_s \varepsilon_{t+n-s} \tag{3}$$

That is, the value of y_{t+n} is decomposed into two terms. The second term on the right hand side of equation (3) is the dynamic forecast or base projection (BP) of y_{t+n} conditional on information at time t. The first term on the right hand side shows the influence on y_{t+n} of the shocks to the variables in the system between periods t+1 and t+n. Even though the expected values of these shocks are zero, policy makers know that the realizations of these shocks over any particular period are likely to be nonzero.

Policy makers are forward-looking, aiming for the forecast average inflation rate (FAIR)[5] to be within some tolerance band, consistent with current practice by inflation-targeting central banks. That is, policy makers aim for a numerical inflation target plus or minus some (possibly zero) tolerance range or "inflation band," over the next h months, the inflation horizon.[6] (In our empirical application, we use monthly data and set $h = 24$.[7]) If the forecast for inflation over the next h months is within the band, no policy intervention is undertaken. If the FAIR is outside the band, then an intervention is used to return this measure of inflation to the band.[8]

The rule we employ in our empirical work is to undertake a policy action that returns the FAIR to the closest edge of the band. There are at least three reasons to return to the edge of the band rather than the midpoint. First, although we don't model the loss function explicitly, if the underlying loss function depends on the variance of output as well as the variance of inflation, then the more aggressive policy action needed to return the FAIR to the midpoint could induce additional variability in output, raising the

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[5] The policy authority in our analysis can be either an explicit or implicit inflation targeter.

[6] Svensson (1997) suggests that non-zero bandwidths may reflect allowance for 'unavoidable' variability in inflation. In addition, he also suggests that the bandwidth may be wider the higher the weight on output stabilization in the loss function.

[7] Svensson (2003) notes that both the Bank of England and the Svierges Riksbank have used a two-year inflation forecast horizon for conducting monetary policy.

[8] While we do not do so in our simulations, it would be straightforward in practice to allow for judgment in the forecast by adding to the base projection an adjustment for factors that are outside the model but deemed by policymakers to be important for the immediate policy exercise.

overall loss.[9] In light of the "dual mandate" for the Federal Reserve, it does not seem unreasonable to us to moderate the policy response to inflation in light of concerns for output and/or employment stability. Second, if there is multiplicative uncertainty about the economy, in the sense of Brainard (1967), then the policy authority may not necessarily aim at the midpoint of the range. That is, if there is not certainty equivalence, then aiming at the midpoint no longer necessarily is optimal.[10] Given uncertainty about economic parameters, aiming for an inflation rate other than the midpoint of the inflation band can be justified. Third, if policy makers want to minimize their impact on markets, returning to the edge of the inflation band may be appropriate. That is, our rule implies that we undertake the smallest policy action needed to attain the objective. Of course, the tradeoff is that our smaller market inverventions will likely be more frequent than more aggressive actions aimed at returning to the midpoint of the band.

The policy horizon is T months (T=12 in our empirical analysis). That is, the policy authority is assumed to be concerned with longer-run inflation (24 months in our case) but plans policy on an annual basis. The inflation horizon may differ from the policy horizon.[11]

We emphasize that the objective is the FAIR over this period rather than either the current or any particular future monthly inflation rate. Current inflation is the result of past decisions by both policy makers and private agents in the economy and is presumably not directly affected by current policy actions. Reported inflation, or a forecast of a particular monthly inflation rate, may lie outside the acceptable inflation band without necessarily calling for a policy action as long as the FAIR suggests that the longer-run objective will be satisfied. However, if the FAIR lies outside the band, then a current policy action is called for. In our application, we will use the federal funds rate as the policy instrument to control the inflation rate, and a policy action in a particular month is defined as an intervention in the funds rate

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[9] This is essentially the point of opportunistic policy described in Orphanides, Small, Wieland and Wilcox (1997). For additional discussion, see also Result 12 in Clarida, Gali, and Gertler (1999).
[10] See Result 11 in Clarida, Gali, Gertler (1999). Specifically, they argue that "parameter uncertainty may reduce the response of the policy instrument to disturbances in the economy." That is, the reduction in the response may lead to aiming for the edge of the band rather than the midpoint.
[11] This setup is in the spirit of the description of policy formulation described by Blinder (1997): "First, you plan an entire hypothetical path for your policy instrument, from now until the end of the planning horizon…. Second, when next period actually comes, you must appraise the new information that has arrived and make an entirely new multiperiod plan…. Third, you must repeat this reappraisal each and every period." Our policymaker aims at a long-run inflation target and updates as new information (in the form of the random draws producing average, prospective inflation outside the band) becomes available.

equation in that month. [12] Due to interaction with other system variables via system dynamics, a policy action in a particular month will affect inflation over the remainder of the horizon. That is, even if the funds rate has a relatively small contemporaneous effect on inflation, marginal changes in this rate can still have substantial effects on long-run inflation.

Having provided the basic motivation for the approach above, we next provide an intuitive overview of the technical aspects of the computation of the policy frontier. We then conclude this section with a technical presentation of the procedure.

For each experiment, we estimate the VAR model using real-time data that ends in the period before the start of the simulation and compute the base projection at time t. This forecast of y_{t+h} is represented by the second right-hand-side term in equation (3) and is estimated from the lagged historical residuals from the VAR. Since the base projection is based on historical residuals, it does not change across the trials of a given experiment.

For each trial, we draw (with replacement) from the estimated residuals for each equation in the system a vector of residuals of length T+h, the sum of the policy horizon and the inflation horizon. [13] This vector of shocks is used to compute the first term on the right-hand-side of (3), which when combined with the base projection gives the path the economy, as represented by the system of equations, would follow under this draw.

Combining the base projection and the initial h months of the vector of draws from the residuals gives the policy maker a forecast of inflation for each of the next h months. The policy maker uses these h individual monthly inflation rates to compute the FAIR, the average h-month inflation rate conditional on the draw. If this rate is inside the band, no policy intervention is needed. On the other hand, if this inflation rate is outside the band, a "preemptive" policy action of sufficient magnitude to return the forecast inflation rate to the closest edge of the band is calculated. [14] That is, the drawn residual from the interest rate

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[12] As will be shown in the technical discussion below, the desired change in the funds rate is implemented by replacing the residual term in the funds rate expression with an appropriately-sized shock that brings the funds rate to the desired level. As detailed below, however, our policy action is not a variant of the constant interest rate rule discussed in detail by Leitemo (2003).

[13] We need T+h residuals since during the last month of the policy horizon, policy makers want to know the FAIR for the subsequent h months.

[14] We note that attempting to use the funds rate to control the inflation rate at very short horizons may lead to instrument instability. The intuition behind this statement is that the interest rate is not an

equation is replaced with one that is computed to assure that the FAIR attains the policy objective. As will

be shown technically below, this calculation is done conditional on the shocks drawn for later time periods;

the needed current policy action is "identified" using the remaining residuals from the draw. Also note that

since the residuals in equation (3) are structural, the residual drawn for the policy equation can be replaced

with the needed policy action without implications for the random shocks to the other equations since it

was assumed there is no contemporaneous correlation among the structural shocks.[15,16] If intervention is

required in a given time period, the policy action replaces that period's shock to the policy equation drawn

from the estimated residuals, and is employed along with the shocks drawn from the other equations in the

model in (3) to generate current and future values of the variables in the model. Thus, for instance, a policy

action in period t+1 will affect the inflation forecasts in periods t+2 through t+12.

This procedure is done for each month in the policy planning horizon (t+1 through t+12). For

example, whether or not an intervention is undertaken in period t+1, the next step is to compute the average

h-month forecast for the period beginning in period t+2, using the individual h-monthly inflation rates to

compute the FAIR. In period t+2, a policy action is either needed or not. Either way, the dynamic path of

the economy is computed, and so on. After passing through T months, the policy horizon (a 12 month

planning horizon in our example), we obtain at the end a path for the system of variables in which policy is

used to attain the inflation objective of maintaining the FAIR on the edge of or inside the band. Using this

counterfactual path over the course of the T month policy horizon, we then have the path of system

important component of measured prices. As such, an interest rate change would have a relatively small
near-term impact on the inflation process, requiring large interest rate movements to affect short-term
inflation. With a longer-term inflation objective, say one of several years as we employ here, a current
interest rate change has lagged effects on the inflation rate, consistent with system dynamics. This point is
recognized by central bankers, who generally implement policy via interest rate innovations which are
allowed to work their way through the dynamics of the economy.

[15] Note that an alternative approach for obtaining a desired average inflation rate would be to
employ a "constant interest rate" approach, which would take the base projection and adjust it by imposing
a constant interest rate over the h-period horizon that brings about the desired average inflation rate. This
approach thus implicitly imposes an entire path for the shocks to the interest rate equation. In our analysis,
we identify the current policy shock needed to attain the objective (given the rest of the draw) while the
constant interest rate approach implicitly identifies a vector of shocks, current and for the remainder of the
horizon, needed to maintain a constant interest rate and simultaneously attain the inflation objective. The
constant interest rate approach thus imposes more policy action than needed to attain the policy objective. It
imposes interest rate smoothness while our approach allows the path of rates to be determined by the
response of the policy maker to the forces that may drive the FAIR outside the band.

[16] It is possible to model correlations among the structural shocks, as in Bernanke and Mihov
(1998). If such modeling included contemporaneous correlation between the policy innovation and other
variables, then other structural shocks would be affected when a policy shock needed to attain the FAIR is
imposed. We do not model such contemporaneous correlations here.

variables for this particular trial which is consistent with the inflation objective of the policy authorities. We compute the variance of each element of the vector of variables over this trial.

We compute and report the average of the variances over the 1000 trials in our experiments in order to describe the tradeoff in inflation-output variability inherent with inflation targets, given the model specification. These reported results are consistent with the needed interventions to attain the inflation target.

We next provide technical detail on computation of the FAIR and how we compute the policy actions needed to maintain it on or inside a target band. Let the elements k and j in the vector y_t represent the federal funds rate and inflation, respectively. Consider the jth equation in system (3) when n=1, which is the one-period-ahead inflation equation:

$$y_{j,t+1} = \sum_{i=1}^{N} d_{0,ji}\varepsilon_{i,t+1} + BP_{j,1} \tag{4.1}$$

For periods 2 through h, the analogous equations are

$$y_{j,t+2} = \sum_{i=1}^{N} d_{0,ji}\varepsilon_{i,t+2} + \sum_{i=1}^{N} d_{1,ji}\varepsilon_{i,t+1} + BP_{j,2} \tag{4.2}$$

$$\vdots$$

$$y_{j,t+h} = \sum_{i=1}^{N} d_{0,ji}\varepsilon_{i,t+h} + \sum_{i=1}^{N} d_{1,ji}\varepsilon_{i,t+h-1} + \cdots + \sum_{i=1}^{N} d_{h-1,ji}\varepsilon_{i,t+1} + BP_{j,h} \tag{4.h}$$

Summing equations (4.1) through (4.h) and then averaging yields

$$\frac{1}{h}(y_{j,t+1} + y_{j,t+2} + \cdots + y_{j,t+h}) = \frac{1}{h}\{\sum_{i=1}^{N} d_{0,ji}\varepsilon_{i,t+1} + \sum_{i=1}^{N} d_{0,ji}\varepsilon_{i,t+2} + \sum_{i=1}^{N} d_{1,ji}\varepsilon_{i,t+1} + \cdots +$$

$$\sum_{i=1}^{N} d_{0,ji}\varepsilon_{i,t+h} + \sum_{i=1}^{N} d_{1,ji}\varepsilon_{i,t+h-1} + \cdots + \sum_{i=1}^{N} d_{h-1,ji}\varepsilon_{i,t+1} + BP_{j,1} + BP_{j,2} + \cdots + BP_{j,h}\}$$

$$= \frac{1}{h}\left\{\sum_{\substack{i=1 \\ i \neq k}}^{N} d_{0,ji}\varepsilon_{i,t+1} + d_{0,jk}\varepsilon_{k,t+1} + \sum_{i=1}^{N} d_{0,ji}\varepsilon_{i,t+2} + \sum_{\substack{i=1 \\ i \neq k}}^{N} d_{1,ji}\varepsilon_{i,t+1} + d_{1,jk}\varepsilon_{k,t+1} + \cdots + \right.$$

$$\left. \sum_{i=1}^{N} d_{1,ji}\varepsilon_{i,t+h-1} + \cdots + \sum_{i=1}^{N} d_{h-2,ji}\varepsilon_{i,t+2} + \sum_{\substack{i=1 \\ i \neq k}}^{N} d_{h-1,ji}\varepsilon_{i,t+1} + d_{h-1,jk}\varepsilon_{k,t+1} + BP_{j,1} + \cdots + BP_{j,h} \right\}.$$

We next show how to compute the current period policy shock needed to attain the FAIR. Define

$Y_{j,t+1} = \frac{1}{h}(y_{j,t+1} + y_{j,t+2} + \cdots + y_{j,t+h})$ to be the forecast inflation rate and let the targeted, average

inflation rate be $Y_{j,t+1}^{*} = \frac{1}{h}(y_{j,t+1} + y_{j,t+2} + \cdots + y_{j,t+h})^{*}$.[17] Assume for now that the goal is to achieve

this target exactly; that is, assume for now that the width of the inflation band is zero. Then conditional on

$\varepsilon_{i,t+1}, i \neq k,$ as well as on $\varepsilon_{t+2}, \varepsilon_{t+3}, \cdots \varepsilon_{t+h}$, there is a value for the current policy innovation, $\varepsilon_{k,t+1}$ that

will achieve this inflation target. Specifically, we solve the previous equation for the policy innovation

undertaken at the beginning of period t+1 designed to attain the target:

$$\overset{\wedge}{\varepsilon}_{k,t+1} = \left(\sum_{\ell=0}^{h-1} d_{\ell,jk} \right)^{-1} \{ (y_{j,t+1} + y_{j,t+2} + \cdots + y_{j,t+h})^{*} - \sum_{\substack{i=1 \\ i \neq k}}^{N} d_{0,ji}\varepsilon_{i,t+1} - \sum_{i=1}^{N} d_{0,ji}\varepsilon_{i,t+2} - \sum_{\substack{i=1 \\ i \neq k}}^{N} d_{1,ji}\varepsilon_{i,t+1} - \cdots -$$

$$\sum_{i=1}^{N} d_{0,ji}\varepsilon_{i,t+h} - \sum_{i=1}^{N} d_{1,ji}\varepsilon_{i,t+h-1} - \cdots - \sum_{\substack{i=1 \\ i \neq k}}^{N} d_{h-1,ji}\varepsilon_{i,t+1} - BP_{j,1} - BP_{j,2} - \cdots - BP_{j,h} \} \qquad (5)$$

We next relax the assumption that the average inflation rate is targeted exactly, and show how to

pursue a policy objective of constraining inflation to lie within a given, predetermined bandwidth. For

period t+1, we want the inflation rate within the pre-specified band $Y_{j,t+1}^{*} \pm \tau$ where τ is half the

bandwidth.[18] It may be that no policy intervention is needed, which will occur when the shocks to the

economic system are such that

$$Y_{j,t+1}^{*} - \tau < Y_{j,t+1} < Y_{j,t+1}^{*} + \tau.$$

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[17] If the target inflation rate is constant, then Y_{j}^{*} could be expressed more simply as the target
level, such as 2%. However, if actual inflation is above target and the target is to be approached gradually,
then the y_{t+k}, k=1,…,h, will gradually fall so that the general notation in the text is appropriate.
[18] As specified, the band is symmetric. If the policy maker were to set policy actions to return
inflation to a particualr path strictly within the band, then asymmetric bands would also be of interest. For
example, the policy maker might respond to a given upward shock to the inflation rate, but not to a
downward shock of the same absolute value, as in an opportunistic disinflation policy. It is straightforward
to allow for asymmetric bands.

If, on the other hand,

$$Y_{j,t+1} < Y^*_{j,t+1} - \tau$$

or if

$$Y_{j,t+1} > Y^*_{i,t+1} + \tau ,$$

a policy intervention is needed to return the inflation rate either to the edge of the band or to some pre-specified value interior to it. For instance, if the policy choice is to return to the edge of the band, then the policy innovation is computed by replacing the term $(y_{j,t+1} + y_{j,t+2} + \cdots + y_{j,t+h})^*$ in equation (5) with

$(y_{j,t+1} + y_{j,t+2} + \cdots + y_{j,t+h})^* \pm \tau = Y^*_{i,t+1} \pm \tau$, depending on whether the FAIR is computed to be above or below the tolerance range.

The policy action undertaken in period t+1 implies a subsequent path for the system's variables, and later evaluation of policy actions must take t+1 policy into account; again, the policy approach implies history dependence. Given this policy action, the average, prospective inflation for the h-period horizon covering periods t+2 through t+h+1 may be computed similarly to the discussion in equations (4.1) through (4.h):

$$\frac{1}{h}(y_{j,t+2} + y_{j,t+2} + \cdots + y_{j,t+h+1}) =$$

$$= \frac{1}{h}\left\{ \sum_{\substack{i=1 \\ i\neq k}}^{N} d_{0,ji}\varepsilon_{i,t+2} + d_{0,jk}\varepsilon_{k,t+2} + \sum_{\substack{i=1 \\ i\neq k}}^{N} d_{1,ji}\varepsilon_{i,t+1} + d_{1,jk}\hat{\varepsilon}_{k,t+1} + \cdots + \right.$$

$$\sum_{i=1}^{N} d_{0,ji}\varepsilon_{i,t+h} + \sum_{i=1}^{N} d_{1,ji}\varepsilon_{i,t+h-1} + \cdots + \sum_{\substack{i=1 \\ i\neq k}}^{N} d_{h-2,ji}\varepsilon_{i,t+2} + d_{h-2,jk}\varepsilon_{k,t+2} + \sum_{\substack{i=1 \\ i\neq k}}^{N} d_{h-1,ji}\varepsilon_{i,t+1} + d_{h-1,jk}\hat{\varepsilon}_{k,t+1} +$$

$$\sum_{i=1}^{N} d_{0,ji}\varepsilon_{i,t+h+1} + \cdots + \sum_{\substack{i=1 \\ i\neq k}}^{N} d_{h-1,ji}\varepsilon_{i,t+2} + d_{h-1,jk}\varepsilon_{k,t+2} + \sum_{\substack{i=1 \\ i\neq k}}^{N} d_{h,ji}\varepsilon_{i,t+1} + d_{h,kj}\hat{\varepsilon}_{k,t+1} + BP_{j,2} + BP_{j,3} + \cdots BP_{j,h+1} \left.\right\}$$

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To attain the target inflation rate exactly, solve for $\varepsilon_{k,t+2}$ conditional on $\hat{\varepsilon}_{k,t+1}$:

$$\hat{\varepsilon}_{k,t+2} = \left(\sum_{\ell=0}^{h-1} d_{\ell,jk}\right)^{-1} \{(y_{j,t+2} + y_{j,t+3} + \cdots + y_{j,t+h+1})^* - \sum_{\substack{i=1 \\ i \neq k}}^{N} d_{0,ji}\varepsilon_{i,t+2} - \sum_{\substack{i=1 \\ i \neq k}}^{N} d_{1,ji}\varepsilon_{i,t+1} - \cdots -$$

$$\sum_{i=1}^{N} d_{0,ji}\varepsilon_{i,t+h} - \sum_{i=1}^{N} d_{1,ji}\varepsilon_{i,t+h-1} - \cdots - \sum_{\substack{i=1 \\ i \neq k}}^{N} d_{h-2,ji}\varepsilon_{i,t+2} - \sum_{\substack{i=1 \\ i \neq k}}^{N} d_{h-1,ji}\varepsilon_{i,t+1} - \sum_{i=1}^{N} d_{0,ji}\varepsilon_{i,t+h+1} - \cdots -$$

$$\sum_{\substack{i=1 \\ i \neq k}}^{N} d_{h-1,ji}\varepsilon_{i,t+2} - \sum_{\substack{i=1 \\ i \neq k}}^{N} d_{h,ji}\varepsilon_{i,t+1} - \sum_{i=1}^{h} d_{i,jk}\hat{\varepsilon}_{k,t+1} - BP_{j,t+2} - BP_{j,t+3} - \cdots BP_{j,t+h+1}\} \qquad (6)$$

If the bandwidth is nonzero, then analogous to the earlier discussion, replace

$(y_{j,t+2} + y_{j,t+3} + \cdots + y_{j,t+h+1})^*$ with $(y_{j,t+2} + y_{j,t+3} + \cdots + y_{j,t+h+1})^* \pm \tau = Y^*_{i,t+2} \pm \tau.$[19]

Note that, generalizing equations like (5) or (6) to period t+j, computation of the t+j period policy shock needed to attain the FAIR for the subsequent h months would include two kinds of terms: policy interventions needed return the average inflation rate to the band and shocks from the random draw for those periods in which no intervention is needed.

In the experiments below, we specify a target path and specify a band around this path. Since we sample from the estimated residuals, we do not impose any arbitrary assumptions about the probability density generating the shocks to the economy. For each trial, computed values for the system variables are those the economy will follow using the assumed policy interventions that keep the FAIR inside the designated band, given the shocks to the other equations.[20]

Note that the iterative process by which we compute $\hat{\varepsilon}_{k,t+1}$, $\hat{\varepsilon}_{k,t+2}$, ..., incorporates the new information that has arrived in the form of the shocks affecting average, prospective inflation each period. If the realized values of these shocks are negligible, the shocks are such that the inflation rate stays within

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[19] Note that in equations (5) and (6), our policy actions generally respond to all the information in the model. In contrast, policy actions based on the well-know Taylor rule only respond to, say, information on output (relative to potential) and deviations of inflation from target.
[20] While it is possible to do so, we do not take into account the possibility that the model coefficients may be estimated imprecisely.

the band. But the policy interventions occur when realizations of shocks to any system variables move the inflation rate outside the band.

III. Empirical Model

As noted earlier, the variables in the VAR model we estimate include those in the typical New Keynesian model--the output gap, the inflation rate, and the federal funds rate. Additionally, we include the rate of change in a commodity price index for two reasons. First, we add commodity prices following earlier literature that addresses the "price puzzle" often found in VAR models (see, for example, Boivin and Giannoni (2002)). Second, since commodity price volatility is often used to represent supply shocks, as a first (and likely crude) approximation, we use this variable to help control for changes in output and inflation volatility emanating from sources outside the policy process. In order to establish the usefulness of the model for monetary policy evaluation, the macroeconomic effects of monetary policy are estimated by computing impulse response functions (IRFs) for shocks to the federal funds rate.

The model is estimated using monthly real time data over three time periods: 1962:1-1983:9, 1980:1-1992:12, and 1980:1-2000:12. Our first counterfactual inflation targeting simulation begins in 1983:10, a year after the end of reserve targeting that characterized the October 1979-October 1982 period, thus allowing for the adjustment process to the new operating procedure to be basically completed before initiating the experiments. The second counterfactual experiment begins in 1993:1, which roughly corresponds to the initial implementation of inflation targeting at central banks around the world. The third counterfactual begins in 2001:1. This starting point was chosen for two reasons. One is that there was considerable uncertainty about the macroeconomic effects of the decline in stock prices that began in 2000. The second is that it allows a year's transition from the Y2K preparations of the Federal Reserve and the subsequent volatility in the growth rate of the monetary base.[21] In estimating the VAR, twelve lags of all variables are employed.

The variables in the model are measured as follows. The output gap is the log of real GDP minus the log of Hodrick-Prescott filtered real GDP. Since real GDP is not available monthly, a quarterly output gap was first constructed and was then interpolated to monthly values using the random walk option of the

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[21]There was a big spike in total reserves in the system at the end of 1999, and the growth rate of the monetary base jumped sharply. Reserves quickly returned to the pre-Y2K level, and monetary base growth fell sharply over 2000 and even became negative toward the end of 2000, although it had begun to rise by the end of the year.

distrib.src procedure from WinRATS 6.02b. The inflation rate is measured by the year-over-year rate of the change in the personal consumption expenditure deflator, a key series in the Fed's evaluation of inflation.[22] The rate of change is the annual difference of the log of this series. The federal funds rate is the monthly average of the daily rate. The rate of change in commodity prices is calculated as the annual difference of the log of this series. The data transformations follow the specification of the typical New Keynesian model. A description of the real time data and sources of the data is provided in the appendix.

Monetary policy, represented as shocks in the federal funds rate equation, is identified using a Choleski decomposition. The ordering of the variables is: the rate of change in commodity prices, the output gap, the inflation rate and then the federal funds rate. Placing the funds rate last is based on a suggestion by Bernanke and Blinder (1992), and allows a contemporaneous response by the Fed to movements in the other three variables while simultaneously imposing a lagged effect of monetary policy on these variables.[23]

The IRFs for a shock to the federal funds rate for all three estimation periods are presented in Figure 1. In each panel, the solid line is the point estimate and the dotted lines are one standard deviation confidence intervals computed using Monte Carlo simulations employing antithetical acceleration and 10,000 draws. The general pattern of results is similar for each sample period, but the timing and magnitude of effects differs across samples. The magnitude of the one standard deviation federal funds rate

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[22]Although core inflation is measured by the personal consumption expenditure deflator minus the effects of food and energy prices, the core series was not used in the model because it was not possible to construct a real-time version of this series for use in the simulations.

[23] One concern about this ordering is that it does not allow monetary policy to have a contemporaneous effect on the commodity price index which is comprised of auction-market type variables that may well respond within the period to monetary policy shocks. Other concerns include (a) the assumption that the central bank responds contemporaneously to current period movements in output and the price level whereas data (even preliminary) on current period values of these variables is available only with a lag and (b) the constraint that output isn't allowed to respond contemporaneously to a shock to monetary policy. Imposing a lag in the effect of monetary policy on inflation is not controversial. Because of these concerns, we estimated a Bernanke (1986)-type structural VAR which differed from the Choleski described in the text by allowing a contemporaneous effect of monetary policy on commodity prices, by allowing a concurrent effect of monetary policy on output, and by imposing no contemporaneous response of the federal funds rate to output and inflation shocks. The federal funds rate was, however, allowed to respond contemporaneously to commodity price shocks. The point estimates for this structural VAR for a shock to the federal funds rate were plotted along with the confidence intervals for the Choleski decomposition. The point estimates for a monetary policy shock for all variables for all three samples were within the Choleski confidence intervals except for a few very minor departures in the very short-run for output (all three periods) and in the 1980:1-1992:12 period for the initial effect on commodity prices. Based on these results, we used the Choleski decomposition in all experiments.

shock is comparable across the three samples: 0.57 for 1962:-1983:9, 0.50 for 1980:1-1992:12, and 0.49 for 1980:1-2000:12. A positive shock to the federal funds rate persists briefly, but the confidence interval for the funds rate spans zero within 6 months, which we interpret as a return to the initial value. There is a transitory negative effect on the rate of change in commodity prices, and the effect is stronger and more persistent for the 1962:1-1983:9 sample. The output gap becomes negative after several months, but returns to its initial value over time. The magnitude of the effect is greater for the 1962:1-1983:9 sample than the other samples, but the time required for output to return to its HP trend and stay there is comparable across all three samples. There is a transitory negative effect on the rate of change in the personal consumption expenditure price index but the magnitude of the effect, the time required before the effect becomes significant, and the time that lapses until the rate of inflation returns to its initial value differs across samples. A significant negative effect is less evident in the 1980:1-1992:12 period than in the other samples.

Since the VAR models are used to assess the quantitative implications of inflation targeting, it is important that the VARs produce paths of the model variables for shocks to monetary policy that are consistent with the macro models in which monetary policy shocks can affect real variables. This appears to be the case for the VAR models used in this paper.

IV. Results

In this section, we present a variety of results from the inflation targeting experiments. Key among them is a result, consistent with a suggestion by Clarida, Gali and Gertler (1999), that policy can improve the tradeoff between inflation and output variability. Specifically, using our counterfactual methodology, we investigate the nature of the available tradeoffs between inflation and output variability and how these tradeoffs have changed in the two periods in which implicit inflation targeting is a reasonable working hypothesis. For comparison, we also investigate the period in the 1980s before implicit inflation targeting which nonetheless was a time when disinflationary policy was at the forefront of policy discussion.

In each experiment, we assume that a policy of gradualism to reduce inflation is employed. That is, rather than attempting an abrupt drop in the inflation rate, which policymakers might have viewed as risking a "hard landing" for the economy, we start with the historical inflation rate in the month prior to the experiment and then conduct policy so that the midpoint of the inflation band falls gradually to the stated

inflation target. For example, if the inflation rate at the outset of the experiment is 4% and the experiment is for a 2% target with an inflation band of ±1%, then the objective for inflation for the experiment is to lower the inflation rate linearly to 2% over a 48 month period with a ±1% band around this linear path. The benchmark policy is for the midpoint of each inflation band to approach 2% over a 48 month period with bandwidths varying between 0% and one that is arbitrarily large.

Our benchmark policy, including our choice of a 48 month transition, is based on both theoretical considerations and observation of central bank practices. Though not suggesting a specific length of the transition period, Svensson (1997) argues theoretically that a positive weight on the output gap in the loss function implies that optimal disinflationary policy will be one of gradualism. Given the "dual mandate," U.S. policymakers should then approach inflation targets gradually. In practice, according to Bernanke and Mishkin (1997), central bankers behave as suggested by Svensson. They note (p. 99): "Initial announcements of inflation targeting generally allow for a gradual transition from the current level of inflation to a desired steady state, usually the level deemed consistent with price stability." Furthermore, Bernanke and Mishkin later note that after the 1979 oil shock, the German Bundesbank "announced the 'unavoidable' inflation rate to be 4 percent, then moved its target gradually down to 2 percent over a six-year period." (p. 101). In the U.S., Goodfriend (2003) indicated that an "inflation scare" in 1987 due to the infusion of liquidity after the October 1987 stock market crash took the Greenspan Fed "… about five years to overcome" (p. 8). Our choice of 48 months as the transition period is a bit shorter than, but not at great odds with, these suggestions in the literature.

As noted in the previous section, the first counterfactual simulation begins in 1983:10. Figure 2 shows the actual inflation rate through 1983:9 and the base projection of the inflation rate along with ±1% and ±2% bands in which inflation might be tolerated if the inflation rate is to move gradually toward 2% within 48 months. While this period is prior to the emergence of the modern literature on inflation targets, we include it not only for purposes of comparison but also due to the fact that, consistent with the abrupt change in the policy regime in October 1979 (including the unusual Saturday night meeting of the FOMC), the focus was clearly on disinflationary policy. As a result of this policy shift, along with the two recessions of the early 1980s, the inflation rate as measured by the personal consumption expenditures deflator at the outset of this experiment was approximately 3.8%. Note that while the actual inflation rate was relatively

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low, the base projection suggested that inflation would quickly move outside the ±1% or ±2% bands. Thus, for policy officials using real-time data in late 1983, the need for restrictive monetary policy looked highly likely. Such a policy would likely raise the specter of another recession following on the two at the outset of the decade, making empirical estimation of the inflation variability-output variability tradeoff an important consideration.

The second counterfactual simulation begins in 1993:1. At the time, U.S. inflation was approximately 3%. As quoted in the introduction, Alan Greenspan stated that the Fed's objective was to make inflation low enough so that it would not be an important factor in economic decisions. Since other central banks were aiming at inflation targets in the 2% range, we take 2% as the midpoint of the longer-run inflation objective. Figure 3 shows the historical inflation rate policymakers would have observed, again in real time, at the outset of 1993. In addition, the plot shows the base projection of inflation from the model estimated with the then-available data, as well as the ±1% and ±2% inflation bands. Recall that the base projection is constructed under the assumption that shocks beginning in 1993:1 assume their expected values of zero. Even with this assumption, the inflation rate threatens to violate the narrower band within a year and a half and the wider band within about two and a half years. Of course, a central bank is aware that the actual path the economy follows is determined in part by the actual shocks the economy experiences, so there is a reasonable prospect for policy action of the type discussed here to the extent that one or a few "bad draws" from the distribution of shocks occurs.[24]

The third counterfactual simulation begins in 2001:1. Even though inflation was reasonably well contained at approximately 2 1/2% when our third experiment begins and the base projection in Figure 4 puts inflation within the inflation bands, uncertainty about the macroeconomic effects of the decline in stock prices that began in 2000 suggests it is worth considering the implications of inflation targeting in 2001.

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[24] It is also evident that the general path of inflation in the U.S. is downward prior to 1993, as was the case for many countries that formally adopted inflation targeting. So, any claim that low inflation resulted from explicit or implicit inflation targeting in the early 1990s is problematic. We sidestep the difficult issue of how to test whether inflation targeting was the cause of the global disinflation during the late 1980s and early 1990s. Ball and Sheridan (2005) present a provocative analysis of this question. Our focus is on the inflation-output variability tradeoffs implied by our version of inflation targeting given the data at the outset of each experiment, which would have been of interest to policymakers over and above the question of whether the adoption of explicit or implicit targets in the early 1990s caused the contemporaneous fall in inflation.

Summary statistics and basic results for the three periods are presented in Table 1.[25] As detailed

earlier, for each period these results are from 1000 draws from the estimated residuals. Note that while the

FAIR relative to the inflation band is used as the criterion of whether to intervene in a particular month, in

order to be comparable to inflation data as commonly reported, the inflation statistics from our experiments

reported in Table 1 are for the underlying inflation rates for each particular month rather than the FAIR.

Presented in Table 1, with each panel corresponding to a particular time period, are experimental

results for four different bandwidths: 0%, 1%, 2% and an infinite band. [26] Part I of each panel presents

summary statistics on the frequency of interventions aimed at maintaining the FAIR within the indicated

bands. Part II provides data on the magnitude of the policy interventions, and Part III presents results on the

variances of output, the inflation rate, and the interest rate.

Part I of each panel shows the number of interventions per trial, the average maximum number of

interventions, and the number of trials with any policy intervention. The first row of part I shows not only

the number of interventions per trial in each experiment, it also shows parenthetically the number of such

interventions that are restrictive (i.e., interventions that raise the funds rate, the first number in the

parentheses) and the number that are stimulative (i.e., interventions that lower the funds rate, the second

number). The number of interventions per trial starts at the maximum of 24 months (the inflation horizon)

when the inflation objective is to be met precisely (i.e., with bandwidth of zero) and is zero when the band

is arbitrarily wide (in which case it is not necessary to intervene). For each experiment, the number of

interventions per trial declines as the bandwidth widens from ±1% to ±2%. Furthermore, notice that over

time, as we move from Panel A to B and then from Panel B to C, the number of interventions per trial

needed to maintain the ±1% (±2%) bandwidth falls. Furthermore, given the inflationary pressures

suggested by the base projections in the first two periods, it is not surprising that for the ±1% and ±2%

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[25] We have excluded from the statistics in Table 1 those trials in which a negative (nominal) interest rate would occur. Generally, the results that include trials in which negative interest rates occur are nearly identical to those reported below. Note that negative nominal rates do show up in real-world data on occasion. For example, Cecchetti (1988) discusses negative nominal interest rates on some Treasury securities in the 1930s and, more recently, Fleming and Garbade (2004) discuss repurchase agrements with negative interest rates. Casual analysis of our trials in which negative interest rates occur suggest that they were about the same order of magnitude as those appeared in Cecchetti and Fleming and Garbade.

[26] The 0% bandwidth is the case where the average inflation target is attained precisely each month. If a strict inflation nutter is one who aims for 0% inflation with a 0% band, this band might be characterized as reflecting that of a "modified inflation nutter," focused exclusively on attaining low but nonzero inflation rate.

bands, the number of interventions needed to restrain inflation (positive policy shocks to the interest rate equation) outnumber the interventions needed to stimulate inflation in order to maintain inflation within the bands. By the early part of the current decade, however, inflation was sufficiently well restrained that this pattern disappeared. As discussed in the introduction, all these results are consistent with an increasingly credible policy.[27]

The second row of part I of each panel shows the average number of consecutive months per trial in which policy intervention is undertaken. The facts that nearly all the interventions in a given trial are undertaken consecutively and that for the ±1% and ±2% bands policy is consistently restrictive, are suggestive of Woodford's (1999) "optimal policy inertia."[28] Notice that the number of consecutive interventions, like the number of interventions in the first row of part I of each panel, declines within each experiment as the bandwidth widens from ±1% to ±2%. Also notice that the number of such interventions falls as we move from Panel A to B and then from B to C.

Our results on consecutive interventions per trial are due to our imposition of a "commitment" to the inflation target objective. That is, our analysis is designed to intervene, and by an appropriate magnitude, to maintain the FAIR within or on the edges of the bands, regardless of whatever else may happen within the economy. In our experiments, there is no option for the policymaker to deviate from this objective when computing the intervention using equations like (5) or (6). The fact that the vast majority of interventions are in consecutive months again suggests the need for commitment since once the inflation rate breaches the edge of the inflation band, several policy shocks are needed to return average, long-run inflation to an acceptable level. Also note that while there is inertia in terms of a pattern of several consecutive interventions when an intervention is needed, it is less clear that there will necessarily be inertia in the interest rate itself, since (i) the interventions are partly a function of the random draws for all the variables, which can entail consecutive interventions but not necessarily of the same sign, and (ii) since

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[27] Note that the restrictive and stimulative interventions are much more evenly distributed in the 0% bands. Once the inflation rate is brought down to the target, the random draws should produce this pattern since the OLS residuals from which we are drawing should be distributed around zero.

[28] In Woodford's analysis, policy inertia is not the result of preferences for interest rate smoothing or due to assumed serial correlation in the residuals. Rather, it is the result of the fact that agents exhibit forward-looking behavior, which the central bank takes into account in setting policy. Furthermore, a credible central bank that wishes to retain its credibility is constrained in its subsequent actions to honor its commitments made in the past. A similar argument is made in Carlstrom and Fuerst (2005), who focus on the "considerable period of time" language in recent FOMC statements.

there is an endogenous component to the funds rate equation over and above the intervention term. Additional clarification on the smoothness of interest rates will be provided below.

The third row of part I of each panel shows the number of trials in each experiment with any intervention. We again find that the number of such trials falls within each experiment; as the band widens from ±1% to ±2%, the need for interventions to maintain the FAIR declines. Furthermore, the need for interventions declines across experiments as well.

Part II of each panel provides information on the magnitudes of the computed interventions needed to maintain inflation within the indicated bandwidths (the second and fourth rows of part II) as compared with the actual maximum and minimum of the estimated residuals from that equation (the first and third rows).

Some explanation is needed to put the computed interventions into a proper perspective. For the ±0% band, a policy intervention is required every period in order to obtain the objective, so the reported maximum intervention in each panel is six to ten times as large as the largest actual residual from the estimation. (Note that since pressures in the economy, especially in the early periods studied here, tend to be inflationary rather than disinflationary, the largest intervention that lowers the funds rate does not deviate substantially from the minimum estimated residual.) For the infinitely wide band, no intervention is undertaken, so the maximum and minimum interventions correspond to the estimated residuals. For the ±1% and ±2% bands, in each month of each trial we asked whether a policy intervention was needed. In some months an intervention was needed, in which case we replaced the residual drawn from the distribution of estimated residuals with the intervention computed from equations like (5) and (6). If none was needed we retained the residual from the draw as we needed it for computation of the path of the system in later months. That is, for the ±1% and ±2% bands, not every trial required a policy intervention. For each 24-month trial, we retained the maximum and minimum values for the shock to the interest rate equation, the policy variable. Thus, sometimes these extreme values were from the random draws from the historical residuals and sometimes from the computed values if an intervention was needed. From these the extreme values across the trials, we chose to report the 95th (5th) percentile from the vector of maximum (minimum) shocks. We arbitrarily focused on the 95th (5th) percentile of the maximum (minimum) shock to the interest rate equation across the 1000 experiments to avoid placing too much weight on outliers.

In patterns similar to results in Part I, we again find that the magnitude of needed interventions declines the wider the band within a given experiment. We also again find that for a given bandwidth, as we move from Panel A to B and then from B to C, the maximum and minimum values of the needed interventions fall.

As with the results in Part I of each panel of Table 1, the results in Part II of each panel are also broadly consistent with a central bank which is gaining credibility. Specifically, the needed magnitude of policy interventions declined over time. In the theoretical literature, this is often due to the way in which credibility affects expectations. Our empirical model does not attempt to model expectations directly (though to the extent that the expectations of inflation depend at least in part upon the history of inflation, these expectations may be at least partially represented in our estimation). Nonetheless, even though Clarida, Gali, and Gertler (1999) make reference to a conceptual model that does include expectations explicitly, our results as we move across experiments in Table 1 are suggestive of their conclusion that credible policy "enables the central bank to stabilize the economy with relatively modest movements in the short rate" (p 1689-90).[29]

Part III of each panel shows the fundamental results – the variances of the key variables – for each experiment. For clarity, these variances, plotted in Figures 5 and 6, show the basic results of the paper, the estimated tradeoffs between inflation and output variability and inflation and interest rate variability, respectively, over time.

The first notable feature of Figure 5 is that the tradeoff available to the Fed since its adoption of implicit inflation targeting has improved substantially over time. Part of this improvement is the result of the shift of the tradeoff toward the origin across the time periods. Recall that commodity prices were included in an effort to control for differing economic conditions, so at least in a crude sense this inward shift in the tradeoff is due to factors other than supply shocks.[30] The other part of the shift in the tradeoff is due to the flattening of the tradeoff from 1993 to 2001 (recall that the 1983 period, while characterized as a

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[29] We emphasize the word "suggestive" in this sentence, as we do not have a formal test to support the notion that the results are due to increased credibility. Another possibility, for example, is that the results are simply due to "good luck" in terms of the non-policy shocks affecting the economy. However, while such an explanation may hold for a particular trial, it seems unlikely to hold over 1000 trials upon which the data in Table I depend.

[30] In the three years prior to the 1983 experiment, the variance of the commodity price inflation rate was .006. By the onset of the 1993 experiment, this volatility had fallen to .002. By the onset of the 2001 experiment, however, it had risen to .005, notably without an outward shift in the tradeoff.

disinflationary policy regime, is not part of the implicit inflation targeting era). That is, compared with the early 1990s, by early 2001 a given inflation variability was associated with a sizeable drop in the variability of output. We speculate below the possibility that monetary policy was responsible for this improvement.

Second, again comparing the most recent two periods, a shift occurred in the inflation-interest rate tradeoff; see Figure 6. That is, a given inflation variability is achieved with smaller interest rate volatility over time, consistent with the theoretical suggestion by Clarida, Gali and Gertler noted above.

Third, while we have not constrained the analysis to produce policy inertia, our results in the first two rows of Part I of Table I suggest that persistent (i.e., consecutive, monthly) policy applications for up to half the policy horizon are needed in order to maintain inflation within the inflation bands. While the tendency is for policy tightness (i.e., positive interventions to the interest rate equation), this data reflects policy shocks that enhance (or offset) the endogenous movement of the interest rate in response to other macroeconomic forces within the model. To obtain a perspective on the interest rate itself, and hence whether there is substantial variability in the interest rate, consider Figure 7. In this Figure, the solid line represents the actual data, the short-dashed line represents the average interest rate path for the ±1% band and the long-dashed line is the interest rate path for the ±2% band.[31] (Not pictured is the plot for the ±0% band; as might be expected, it is substantially more variable since the inflation objective is constrained to be maintained exactly.) As we compare the experiments, we notice several features regarding interest rate volatility. First, there is no obvious instrument instability in movements of the federal funds rate under our hypothetical inflation targeting scheme. Second, the volatility of rates associated with the ±1% bandwidth is higher than for the ±2% bandwidth for the 1980s disinflationary regime and the early 1990s implicit inflation targeting regime. Third, by 2001, with inflation and its expectations well contained, there was almost no difference between the average paths for the interest rate for the alternative bandwidths. Finally, the variability of the interest rate path was noticeably smaller in the most recent experiment. These results are, as with earlier results, not obviously at odds with the theoretical predictions in the literature; decreasing

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[31] While it is obviously not possible to present the interest rate path for each trial, we note that the smoothness of the average interest rate path for a given experiment does not mask substantial volatililty of interest rates with a given trial.

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interest rate movements were needed to maintain given inflation volatility, with substantial persistence in the interest rate paths needed to attain the policy objectives.[32]

Figures 8 and 9 present added detail regarding the policy process summarized in the previous two figures. Figure 8 shows the policy tradeoffs relative to other policy choices, much as real-world FOMC meetings have information for decision makers regarding alternative policies. Specifically, this figure shows the point estimates of the inflation-output variability available with 1% and 3% as the midpoints of the bands along with the 2% band presented in Figure 5 above. Note that in 1983, the 3% band dominates both the 2% and 1% bands in terms of lying closer to the origin. That is, starting at the relatively high inflation rates of the early 1980s, policy aimed at relatively low inflation objectives would entail a higher cost of output variability as the policymaker imposes increasing restraint on the inflation rate. By 1993, the inflation rate is sufficiently contained that the 2% target dominates both the 1% and 3% objectives. By 2001, however, the 1% inflation target, for all except a bandwidth of zero, dominates the other policy alternatives.

Figure 9 shows similar results with regard to the inflation-interest rate variability tradeoffs. Again in 1983, we find that the more moderate 3% inflation objective would have allowed for relatively less interest rate volatility. In 1993, the 2% inflation objective dominates in terms of interest rate fluctuations, and again in 2001, the 1% inflation target dominates. One possible interpretation of Figures 8 and 9 is that, as the inflation rate declines, for given output variability, lower inflation targets can result in less inflation variability. That is, opportunistically taking advantage of lower inflation rates to decrease the inflation target band can lead to less volatility in inflation without undue impact on the variance of output.

V. Discussion

Our focus in this paper is twofold: (i) illustration of how a VAR model can be used to evaluate inflation targeting and (ii) the derivation of the policy frontier available to the central bank and estimation of how this frontier has changed over time in terms of the position and slope of the available tradeoff between output gap variability and inflation variability under inflation targeting, controlling for variability in commodity price inflation as a proxy for supply shocks. In the spirit of McCallum's (1988; 1999)

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[32] The fact that the average interest rate in our experiments for the inflation targeting periods, 1993 and 2001, is above the actual interest rate appears to be due to the fact that the base projection in the experiments is above the actual inflation rates during 1993 and 2001.

suggestion that policies should be evaluated in a variety of types of models, we employ a small VAR, an alternative to the commonly-used calibrated DSGE model, and show how a VAR model can be used to estimate the policy frontier in a context in which inflation forecasts incorporate all information in the model. Various inflation rate targets are considered as are tolerance bands of varying widths around these inflation targets. Our results indicate a substantial improvement in the policy frontier over time, i.e. a reduction in the increase in output variability associated with a given reduction in inflation variability, and that this improvement reflects a favorable shift in the tradeoff toward the origin as well as a change in the slope of the tradeoff function. We also find a shift in the tradeoff between interest rate volatility and inflation variability over time, that is, we find a given inflation variability is achieved with smaller interest rate volatility over time. Finally, we find that persistent changes in the federal funds rate are required in order to keep the inflation rate within the tolerance bands around a given target inflation rate.

One interpretation of the results in this paper suggests that policies implicitly targeting inflation during the past decade and a half, after the disinflation policies of the Volcker Fed, improved the position of the policy frontier. To place these results in a broader context, notice that they are also consistent with the hypothesis that the Fed was gradually gaining credibility. Although we do not have formal tests, our results are consistent with the following results in the literature. First, Clarida, Gali and Gertler (1999) suggested that policy commitment can improve the tradeoff between inflation and output variability. Our results show such an occurrence for the U.S. Second, Woodford (1999) argued that inertia in the policy instrument was optimal in an economy with forward-looking agents, with the inertia serving to validate agent expectations. Our results on the experiments do seem to be consistent with relatively smooth movements in the fed funds interest rate even though we have not imposed any smoothness restrictions on the results. Without a formal presentation of expectations of inflation and how they respond to (among other things) monetary policy, however, we cannot make stronger claims that the type of policy commitment imposed here – in which the policy shock is always implemented which attains the FAIR – can statistically account for the smoothness of interest rate. Finally, Carlstrom and Fuerst (2005) presented simulations showing that central bank credibility allowed the bank to achieve given objectives with smaller policy interventions than in the case where credibility is lacking. Again, our results are consistent with

these findings. Future research might endeavor to implement statistical tests of how data can be brought directly to bear on these types of hypotheses.

Data Appendix

1. Real time real gdp data are from the routput.xls file from the qvad folder available for download from the Philadelphia Federal Reserve Bank. The relevant columns of this file are: routput83q4 for the sample that ends in 1983:9, routput93q1 for the sample that ends in 1992:12, and routput01q1 for the sample that ends in 2000:12. Data in columns routput93q1 and routput01q1 were known in the first quarter of the respective years, and we assume this data was known by the Fed at the beginning of the relevant counterfactual experiments. Data in column routput83q4 was known in the fourth quarter of 1983. Since our 1983 counterfactual begins in November 1983, the Fed may not have had all the information in this column at the beginning of the counterfactual. However, we wanted to begin the counterfactual a year after the end of reserve targeting, and this was the closest approximation to real time gdp data we could obtain for November 1983.

As noted in the text, for each sample the HP filter was used to construct a potential gdp series, and the output gap was then constructed as actual real gdp minus the HP-potential gdp. The quarterly real time output gaps were then interpolated to monthly data using the distrib.src procedure in RATS 6.02b.

2. Real time personal consumption expenditure deflator data were taken from various issues of the Survey of Current Business.
a. 1962:1-1983:9 sample. 1961-1976: November 1979 Survey of Current Business; 1977-1978: October 1982 Survey of Current Business; 1979-1983:9: July 1983 Survey of Current Business. Monthly data.
b. 1980:1-1992:12 sample. December 1992 Survey of Current Business, Table 3 and the February 1993 Survey of Current Business, Table 7.1. The data in these tables were quarterly, and were interpolated to monthly using the distrib.src procedure in RATS 6.02b.
c. 1980:1-2000:12 sample. August 2000 Survey of Current Business, Table 3 and the February 2001 Survey of Current Business, Table 7.1.   The data in these tables were quarterly, and were interpolated to monthly using the distrib.src procedure in RATS 6.02b.
d. We note that the data sets for 1980:1-1992:12 and 1980:1-2000:12 are not totally pure real time data sets since data at the very end of 1992 and 2000 were pulled from the earliest Survey of Current Business in 1993 and 2001, respectively.

3. The federal funds rate is taken from the Global Insight Basic database, series fyff, and the commodity price index is the Commodity Research Bureau spot market index for all commodities (Global Insight Basic database, series psccom). These series are not revised and hence the data pulled from the Global Insight databases were used in the real-time estimations.

Table 1

A: 1983 Experiment: Decline to 2% over 48 Months				
	0% band	1% band	2% band	∞ band
I. Summary Statistics				
Interventions per trial	24.0 (14.1 9.9)	12.8 (12.2 0.6)	9.9 (9.8 0.1)	0
Average maximum consecutive interventions	24.0	11.66	9.43	0
Trials (of 1000) with any intervention	1000	945	764	0
II. Range of Policy Interventions				
Actual maximum shock to interest rate equation	.026	.026	.026	.026
95% maximum simulated shock to interest rate equation	.150	.108	.064	.026
Actual minimum shock to interest rate equation	-.044	-.044	-.044	-.044
5% minimum simulated shock to interest rate equation	-.041	-.040	-.039	-.044
III. Fundamental Results				
Standard deviation of indicated variable around trial mean				
Output Gap	.02043	.01521	.01141	.00861
Inflation	.00662	.00975	.01300	.01655
Interest rate (Fed funds)	.04627	.02720	.01654	.01160

B: 1993 Experiment: Decline to 2% over 48 Months				
	0% band	1% band	2% band	∞ band
I. Summary Statistics				
Interventions per trial	24.0 (21.2 2.7)	10.1 (10.1 0.0)	3.6 (3.6 0.0)	0
Average maximum consecutive interventions	24.0	9.3	3.5	0
Trials (of 1000) with any intervention	1000	865	502	0
II. Range of Policy Interventions				
Actual maximum shock to interest rate equation	.018	.018	.018	.018
95% maximum simulated shock to interest rate equation	.141	.047	.041	.018
Actual minimum shock to interest rate equation	-.025	-.025	-.025	-.025
5% minimum simulated shock to interest rate equation	-.020	-.011	-.025	-.025
III. Fundamental Results				
Standard deviation of indicated variable around trial mean				
Output Gap	.01077	.00703	.00538	.00536
Inflation	.00538	.00597	.00607	.00614
Interest rate (Fed funds)	.03625	.02721	.01561	.01045

C: 2001 Experiment: Decline to 2% over 48 Months				
	0% band	1% band	2% band	∞ band
I. Summary Statistics				
Interventions per trial	24.0 (15.2 8.8)	3.3 (1.9 1.4)	0.3 (0.2 0.1)	0
Average maximum consecutive interventions	24.0	3.0	0.3	0
Trials (of 1000) with any intervention	1000	370	39	0
II. Range of Policy Interventions				
Actual maximum shock to interest rate equation	.024	.024	.024	.024
95% maximum simulated shock to interest rate equation	.117	.024	.024	.024
Actual minimum shock to interest rate equation	-.038	-.038	-.038	-.038
5% minimum simulated shock to interest rate equation	-.044	-.037	-.029	-.019
III. Fundamental Results				
Standard deviation of indicated variable around trial mean				
Output Gap	.00693	.00468	.00478	.00478
Inflation	.00424	.00564	.00631	.00630
Interest rate (Fed funds)	.02668	.01140	.01070	.01051

Figure 1: Impulse Response Functions

1962:1-1983:9
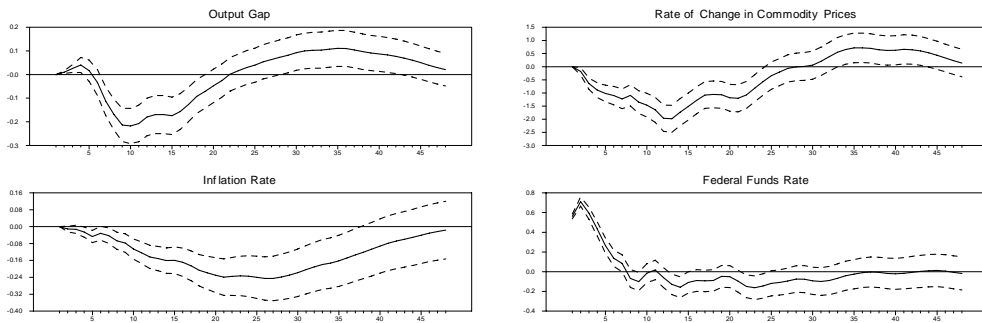
1980:1-1992:12
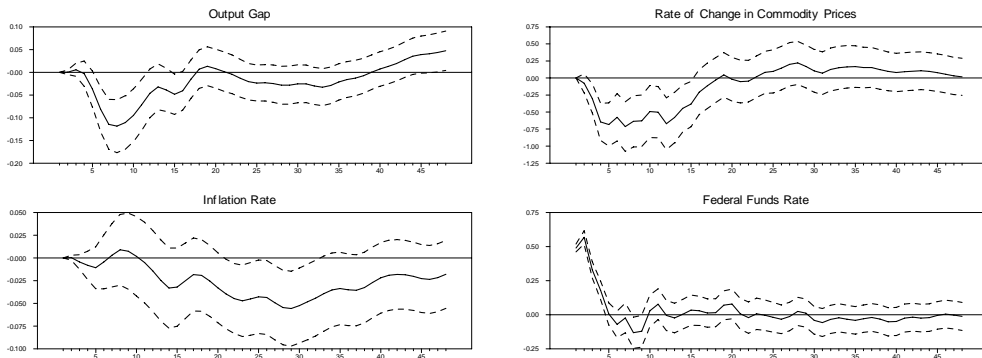
1980:1-2000:12
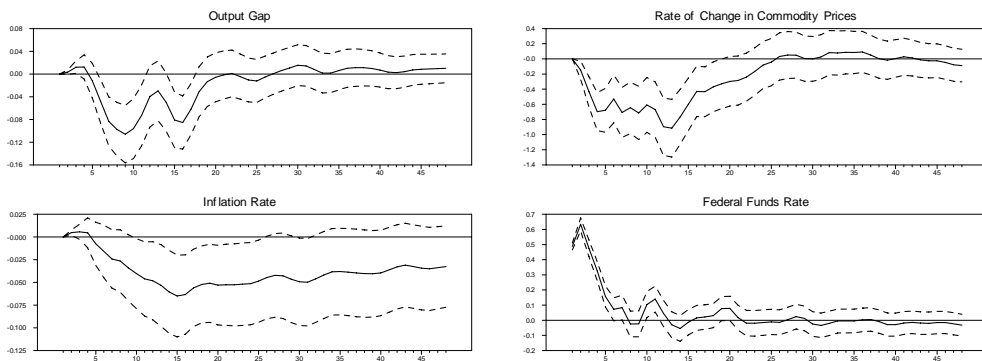
Figure 2: Actual Inflation, Base Projections, and Target Bands:1962:1-1983:9 Sample

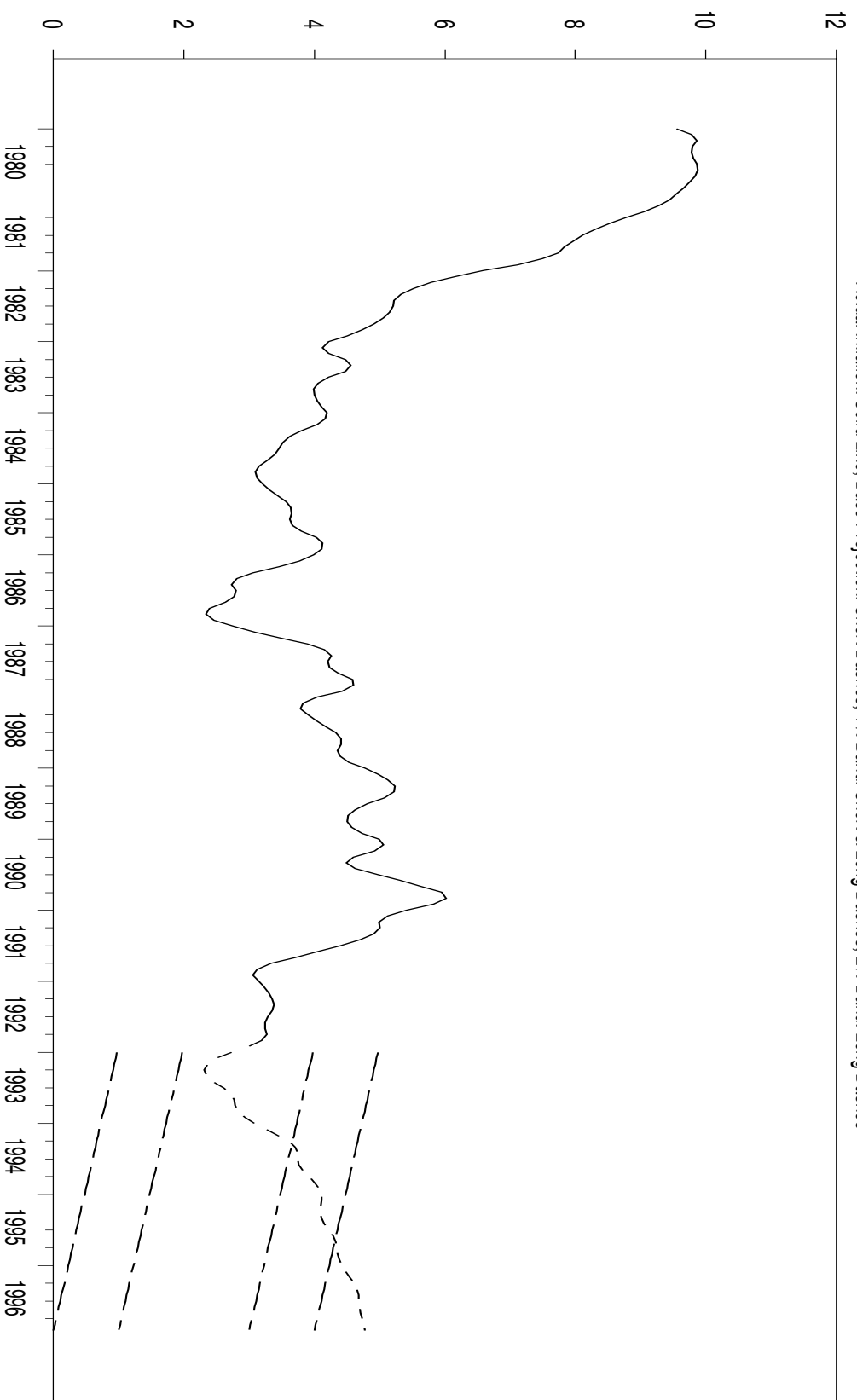Actual Inflation: Solid Line, Base Projection: Short Dashes, 1% Band: Short & Long Dashes, 2% Band: Long Dashes
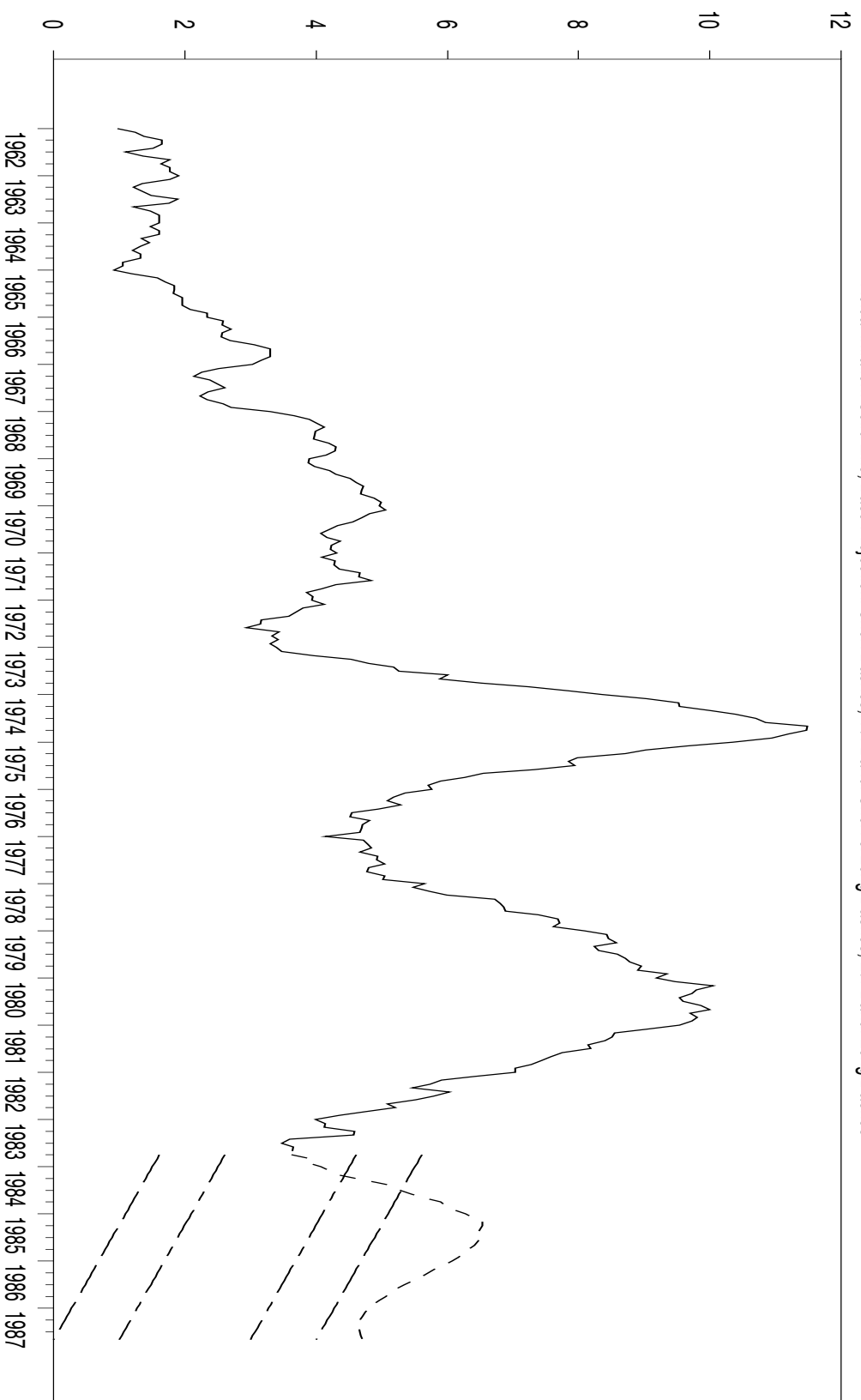
Figure 3: Actual Inflation, Base Projections, and Target Bands:1980:1-1992:9 Sample

Actual Inflation: Solid Line, Base Projection: Short Dashes, 1% Band: Short & Long Dashes, 2% Band: Long Dashes

Figure 4: Actual Inflation, Base Projections, and Target Bands: 1980:1-2000:12 Sample

Actual Inflation: Solid Line, Base Projection: Short Dashes, 1% Band: Short & Long Dashes, 2% Band: Long Dashes
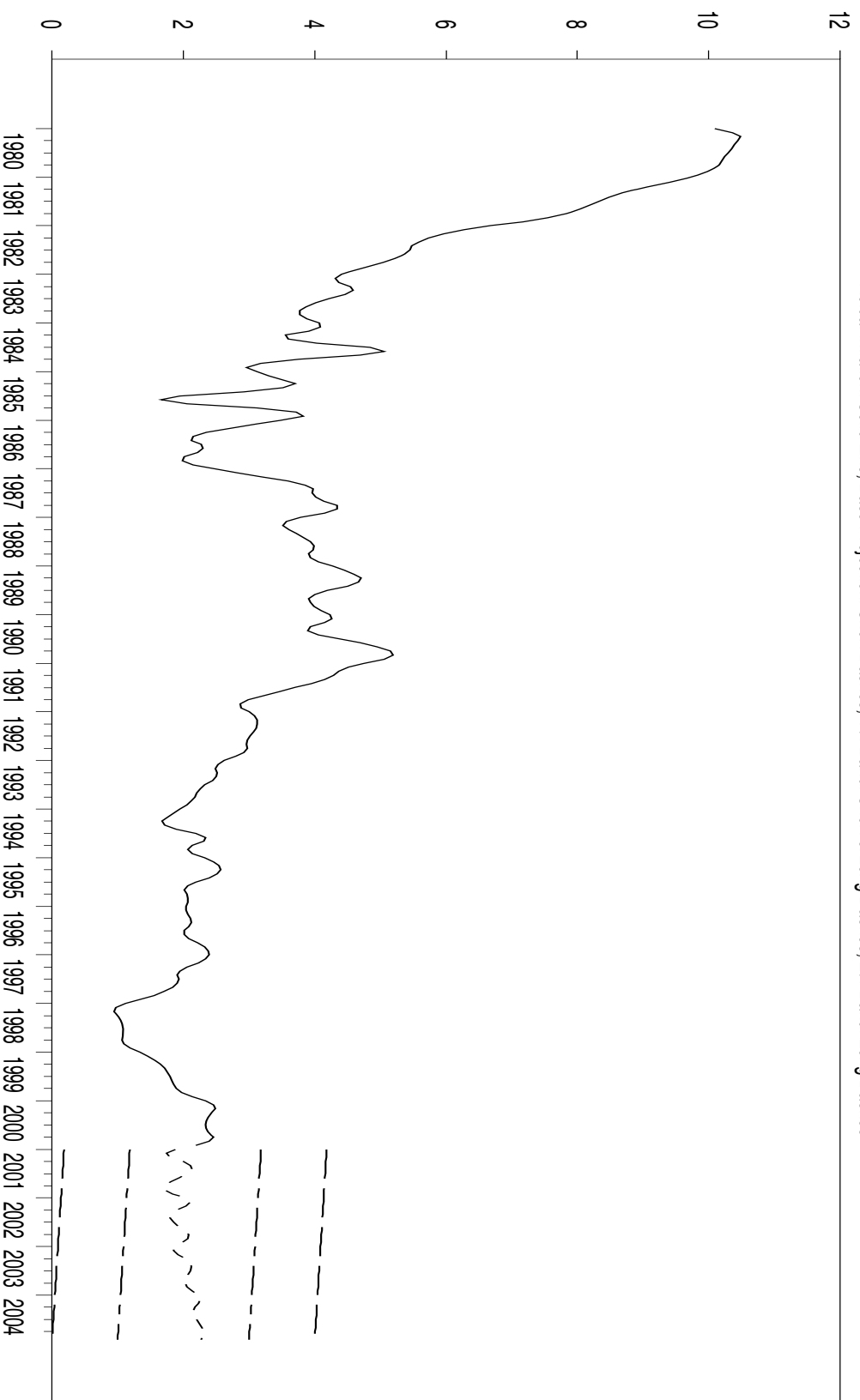
Figure 5: Inflation-Output Standard Deviation Tradeoffs Over Time: 2% Target

1983: Solid Line, 1993: Short Dashes, 2001: Long Dashes
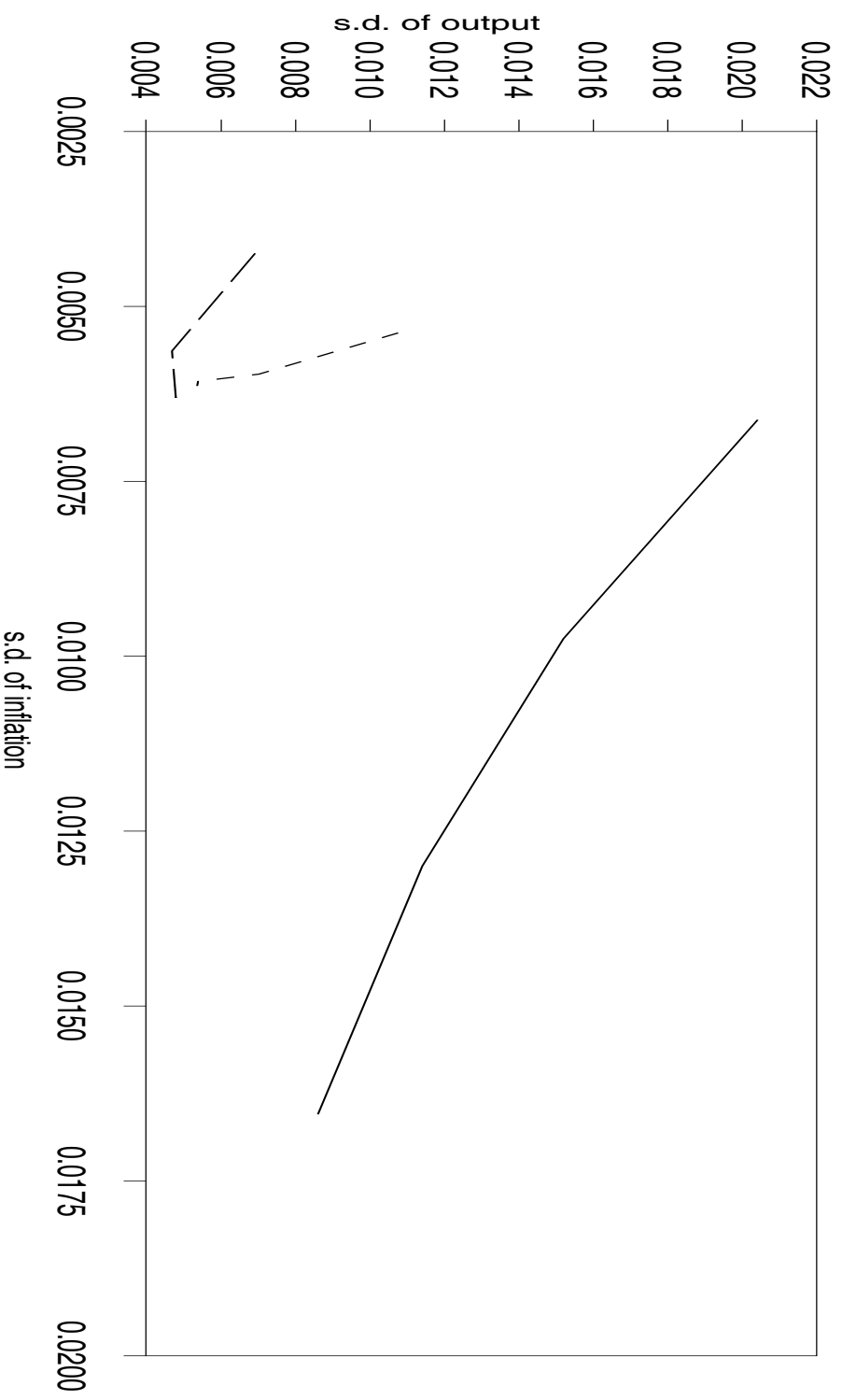
Figure 6: Inflation-Interest Rate Standard Deviation Tradeoffs Over Time: 2% Target

1983: Solid Line, 1993: Short Dashes, 2001: Long Dashes
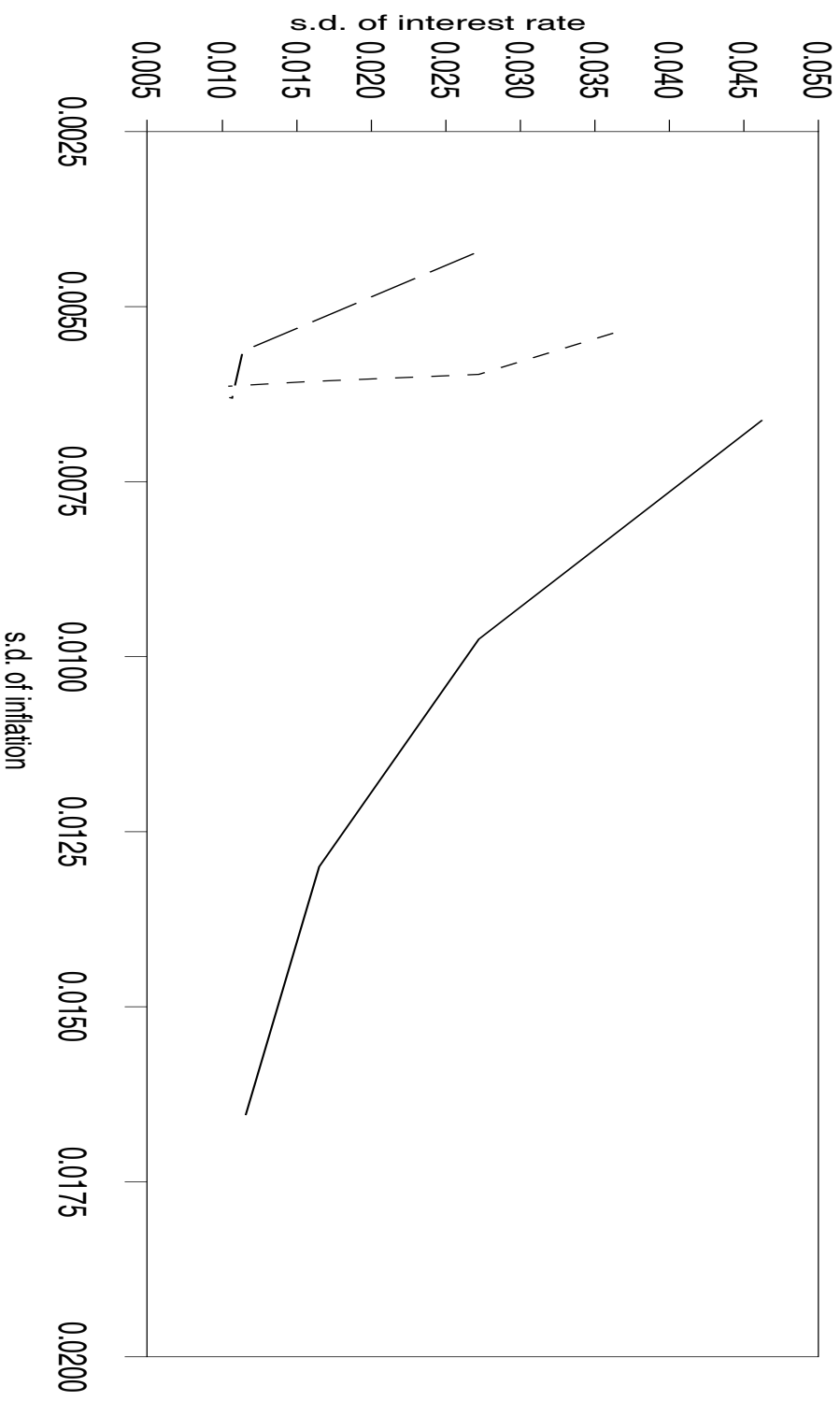
Figure 7: Actual and Average Counterfactual Interest Rates

Actual Fed Funds Rate: Solid Line, Avg Funds Rate 1% Band: Short Dashes, Avg Funds Rate 2% Band: Long Dashes
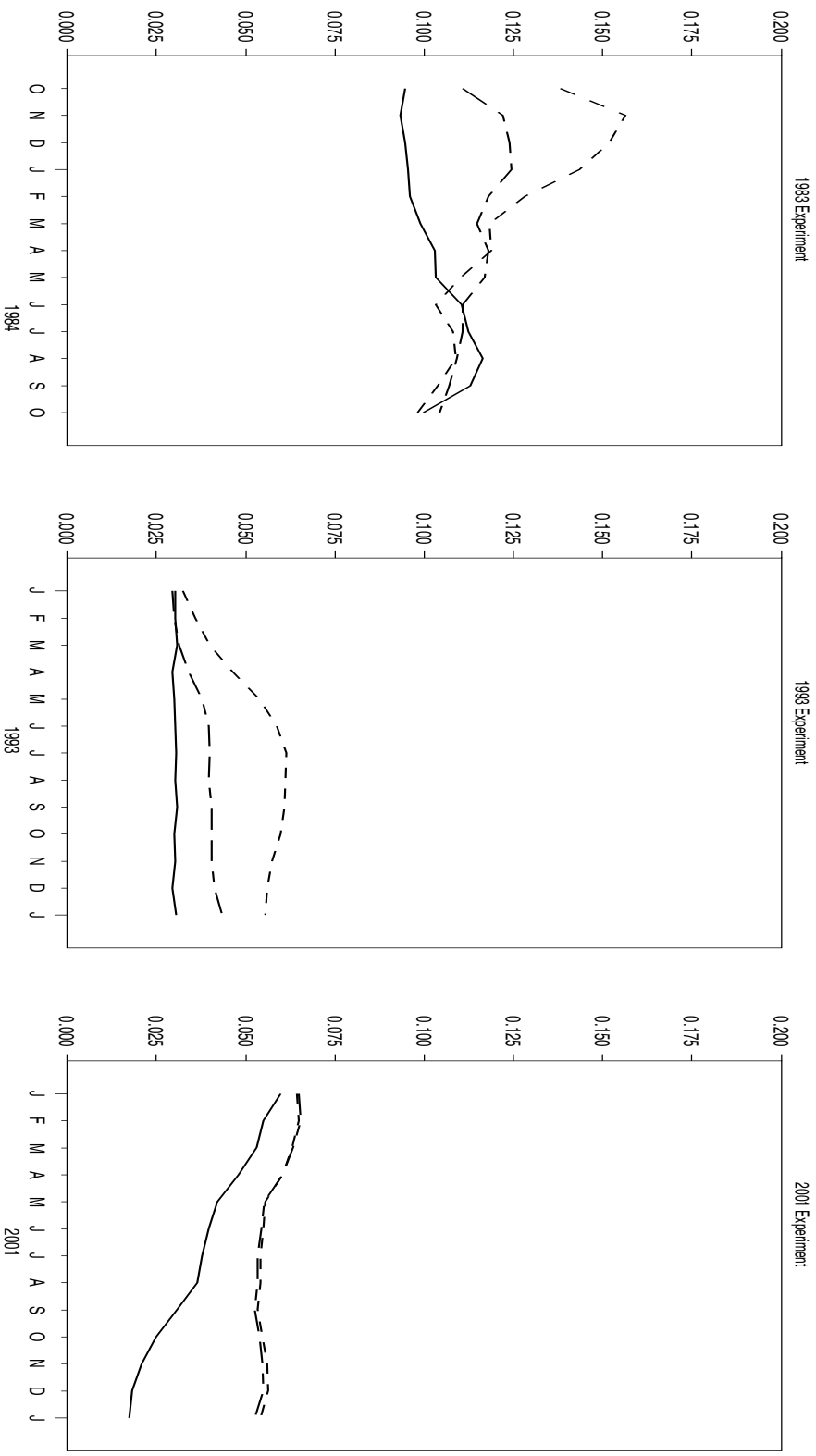
Figure 8: Inflation-Output Tradeoffs: 1%, 2%, & 3% Targets
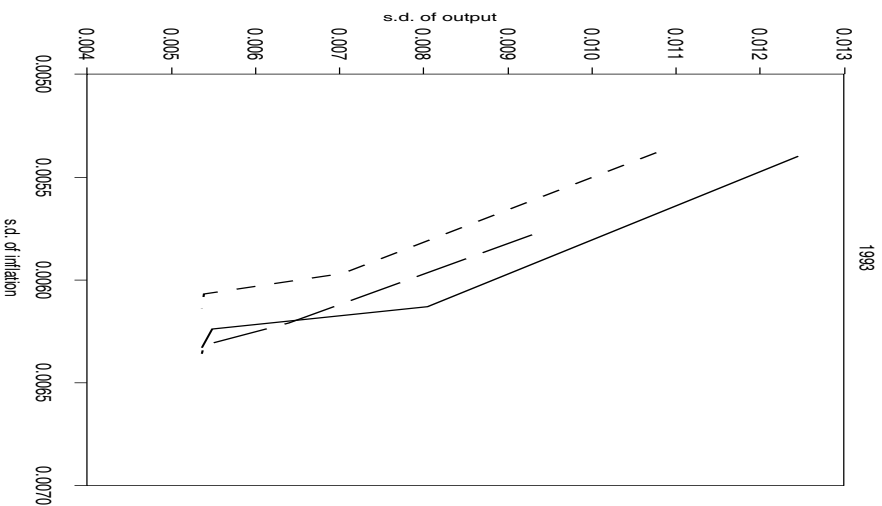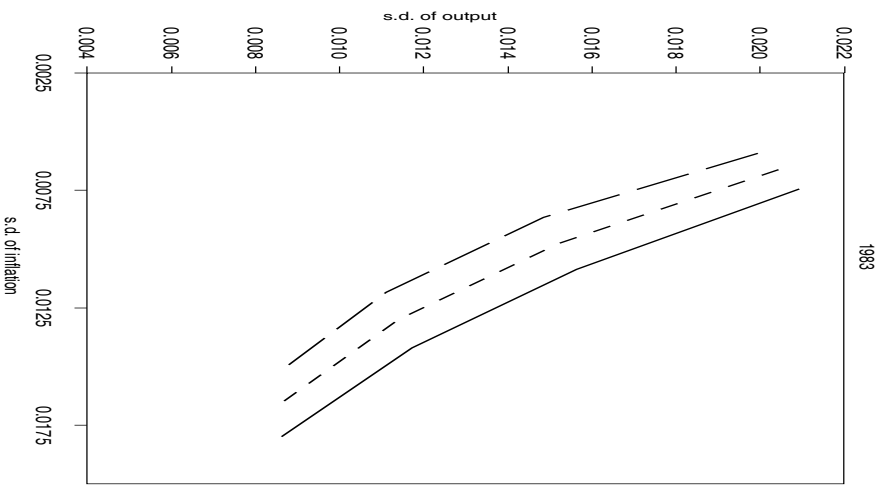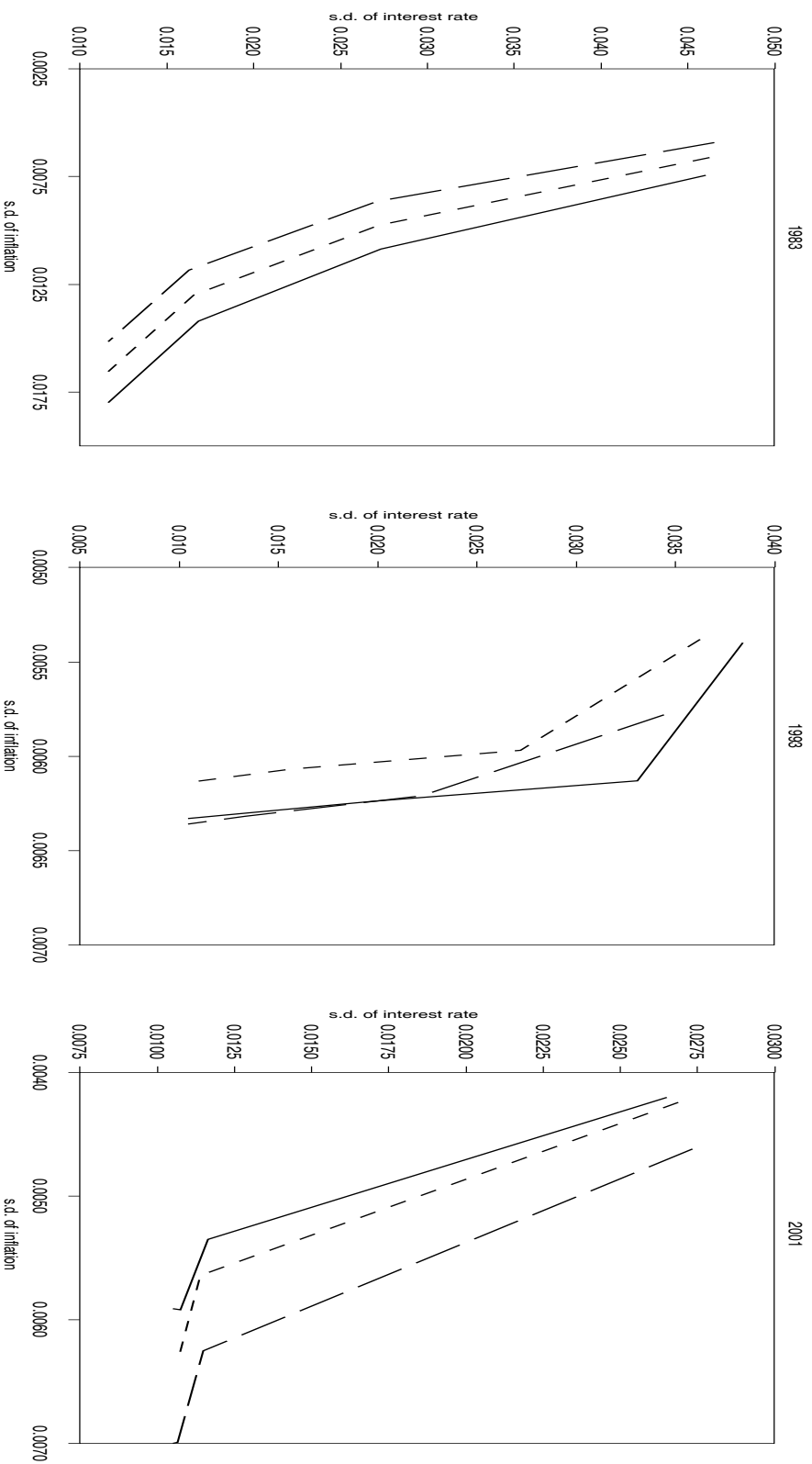1% Target: Solid Line, 2% Target: Short Dashes, 3% Target: Long Dashes
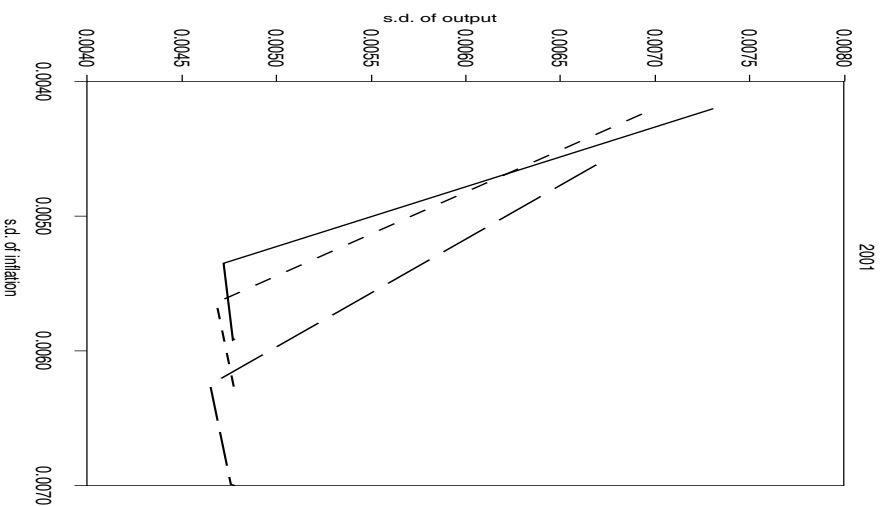
Figure 9: Inflation-Interest Rate Standard Deviation Tradeoffs: 1%, 2%, & 3% Targets
1% Target: Solid Line, 2% Target: Short Dashes, 3% Target: Long Dashes

References

Ball, Lawrence and Niamh Sheridan. "Does Inflation Targeting Matter?" In *The Inflation-Targeting Debate*. Edited by Ben S. Bernanke and Michael Woodford. Chicago: University of Chicago Press, 2005, pp. 249-76.

Barro, Robert J. and David B. Gordon. "A Positive Theory of Monetary Policy in a Natural Rate Model." *Journal of Political Economy* 91 (August 1983), 589-610.

Bernanke, Ben S. "Alternative Explanations of the Money-Income Correlation." *Carnegie-Rochester Conference Series on Public Policy* 25 (Autumn 1986), 49-100.

Bernanke, Ben S. "The Logic of Monetary Policy." Remarks Before the National Economists Club, Washington, D.C., December 2, 2004.

Bernanke, Ben S. and Alan S. Blinder. "The Federal Funds Rate and the Channels of Monetary Transmission." *American Economic Review* 82 (September 1992), 901-21.

Bernanke, Ben S., Thomas Laubach, Frederic S. Mishkin, and Adam S. Posen. *Inflation Targeting: Lessons from the International Experience*. (Princeton: Princeton University Press, 1998).

Bernanke, Ben S. and Ilian Mihov. "Measuring Monetary Policy." *Quarterly Journal of Economics* 113 (August 1998), 869-902.

Bernanke, Ben S. and Frederic S. Mishkin. "Inflation Targeting: A New Framework for Monetary Policy?" *Journal of Economic Perspectives* 11 (Spring 1997), 99-116.

Blinder, Alan S. "What Central Bankers Could Learn from Academics—and Vice Versa." *Journal of Economic Perspectives* 11 (Spring 1997), 3-19.

Boivin, Jean and Marc Giannoni. "Assessing Changes in the Monetary Transmission Mechanism: A VAR Approach." Federal Reserve Bank of New York *Economic Policy Review*, 8 (May 2002), 97-111.

Brainard, William C. "Uncertainty and the Effectiveness of Policy." *American Economic Review* 57 (May 1967), 411-25.

Carlstrom, Charles T. and Timothy S. Fuerst, "Considerable Period of Time: The Case of Signalling Future Policy." Federal Reserve Bank of Cleveland *Economic Commentary*, November 2005.

Cecchetti, Stephen G. "The Case of the Negative Nominal Interest Rates: New Estimates of the Term Structure of Interest Rates During the Great Depression." *Journal of Political Economy* 96 (December 1988), 1111-41.

Cecchetti, Stephen G. and Michael Ehrmann. "Does Inflation Targeting Increase Output Volatility? An International Comparison of Policymakers' Preferences and Outcomes." National Bureau of Economic Research, Working Paper 7426, December 1999.

Clarida, Richard, Jordi Gali, and Mark Gertler. "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature* 37 (December 1999), 1661-1707.

Fleming, Michael J. and Kenneth D. Garbade. "Repurchase Agreements with Negative Interest Rates." *Current Issues in Economics and Finance*, Volume 10, Number 5, Federal Reserve Bank of New York, April 2004.

Fuhrer, Jeffrey C., "Inflation/Output Variance Tradeoffs and Optimal Monetary Policy." *Journal of Money, Credit and Banking* 29 (May 1997), 214-34.

Goodfriend, Marvin, "Inflation Targeting in the United States?" Working Paper 9981, National Bureau of Economic Research, September 2003.

Kydland, Finn E. and Edward C. Prescott, "Rules Rather than Discretion: The Inconsistency of Optimal Plans." *Journal of Political Economy* 85 (June 1977), 473-91.

Leitemo, Kai, "Targeting Inflation by Constant-Interest-Rate Forecasts*." Journal of Money, Credit and Banking* 35 (August 2003), 609-626.

McCallum, Bennett T. "Robustness Properties of a Rule for Monetary Policy." *Carnegie-Rochester Conference Series on Public Policy* 29 (Autumn 1988), 173-203.

McCallum, Bennett T. "Issues in the Design of Monetary Policy Rules." In *Handbook of Macroeconomics*, Volume 1, Part 3. Edited by John B. Taylor and Michael Woodford. Amsterdam: North Holland, 1999, pp. 1483-530.

Nessén, Marianne and David Vestin, "Average Inflation Targeting." *Journal of Money, Credit and Banking* 37 (October 2005), 837-63.

Orphanides, Athanasios, David H. Small, Volker Wieland, and David W. Wilcox, "A Quantitative Exploration of the Opportunistic Approach to Disinflation." Finance and Economics Discussion Series Paper 1997-36, June 1997.

Svensson, Lars E.O., "Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets." *European Economic Review* 41 (June 1997), 1111-1146.

Svensson, Lars E. O., "What is Wrong with Taylor Rules? Using Judgment in Monetary Policy Through Targeting Rules." *Journal of Economic Literature*, 41 (June 2003), 426-77.

Taylor, John B., 'The Inflation/Output Variability Tradeoff Revisited." In *Goals, Guidelines, and Constraints Facing Monetary Policymakers: Proceedings of a Conference Held in June 1994*. Boston: Federal Reserve Bank of Boston, 1994).

Taylor, John B., editor. *Monetary Policy Rules*. Chicago: The University of Chicago Press, 1999.

Woodford, Michael, "Optimal Monetary Policy Inertia." *The Manchester School*, 67 (supplement June 1999), 1-35.