Human Capital and Convergence in a Non-Scale R&D Growth Model*

Chris Papageorgiou
Department of Economics
Louisiana State University
Baton Rouge, LA 70803
cpapa@lsu.edu

Fidel Perez-Sebastian
Dpto. F. del Análisis Económico
Universidad de Alicante
03071 Alicante, Spain
fidel@merlin.fae.ua.es

May 2002

Abstract

This paper extends an R&D non-scale growth model to include endogenous human capital. The goal is to study the model predictions once the complementarity between technology and human capital commonly found in the empirical literature is taken into account. To do this, a human capital accumulation technology is proposed that preserves the non-scale nature of the model. Our model suggests that cross-sector labor movements induced by the complementarity between human capital and technology can be a key factor in replicating and explaining growth miracles such as Japan and South Korea. It is shown that the speed of convergence and the adjustment paths of output growth, investment rates, interest rates, and labor shares implied by the proposed model are consistent with empirical evidence. Finally, it is argued that focusing only on the asymptotic speed of convergence may not be very informative about the overall performance of a model to explain the convergence phenomenon.

JEL Classification: O33, O41, O47.

Key words: Growth, R&D, human capital, input complementarity, cross-sector labor movement

*We thank Craig Burnside, John Duffy, Jordi Caballe, Theo Eicher, Lutz Hendricks, Pete Klenow, Kazuo Mino, Pietro Peretto, Tomoya Sakagami, Chang Yonsung, John Williams, and seminar participants in many universities and conferences for their comments and suggestions. All remaining errors are our own.

1 Introduction

Surprisingly, there have been few attempts in the theoretical literature to explore growth models with endogenous human capital *and* technical progress. In light of surging evidence that these two engines are indeed complementary, it is the argument of this paper that they ought to be incorporated and studied within a unified growth model.[1] As such, this paper extends Jones' (1995) *hybrid* R&D-based framework – admittedly one of various candidates – to include a formal schooling sector.[2] In particular, we present a model in which technical progress is enhanced through innovation and imitation, and human capital through formal schooling. Even though formal schooling is not the only source of human capital, we choose a schooling-based human capital technology because the model will ultimately be taken to the data following the approach suggested by Klenow and Rodriguez-Clare (1997). Our choice of schooling technology is based on the Mincerian approach (Mincer (1974)) that has recently been revived by Bils and Klenow (2000).[3]

The paper evaluates the model in three dimensions. First, we study its performance at steady-state. By construction the steady-state properties of the model are consistent with Jones (1995). Second, we obtain the asymptotic speed of convergence predicted by the model. We find that the speed of convergence of output per worker is consistent with the evidence. However, it is shown that this result alone may not be very informative about the overall performance of the model for reproducing convergence episodes. In particular, we show that small variations in the asymptotic speed of convergence implied by different models can produce substantial changes in the initial periods of the adjustment path. This implies that models delivering empirically-supported speeds of convergence may perform poorly at matching the whole convergence path. It is also shown that the introduction of human capital makes the asymptotic speed of convergence much less sensitive to external shocks such as policy actions. This finding gives theoretical support to Barro and Sala-i-Martin's (1995) result that convergence speed estimates do not vary substantially across different countries or regions. However, this result can not be interpreted as necessarily implying that policy

[1]For a review of empirical studies supporting that human capital is complementary to technology innovation and imitation see Nelson and Pack (1999), Bils and Klenow (2000), and Caselli and Coleman (2001), just to name a few.

[2]Our choice of Jones (1995) as the benchmark in our investigation was based on the fact that the model succeeded in reconciling important regularities in the data such as the increasing R&D intensity with constant output growth rates. Admittedly, a number of recent non-scale R&D growth models could be extended to include a schooling sector. These include Segerstrom (1998), Eicher and Turnovsky (1999a), Young (1998), Dinopoulos and Thompson (1998), and Howitt (1999). The last three papers also permit sustained output growth in the absence of population growth.

[3]For recent discussions on the advantages of the Mincerian approach in growth modeling and estimation, see Bils and Klenow (2000), and Krueger and Lindahl (2001). Other papers that employ the Mincerian approach to model schooling include Jones (1997, 2001), Jovanovic and Rob (1999), and Hall and Jones (1999). For an alternative method used to produce a human capital index, see Mulligan and Sala-i-Martin (2000).

actions have a negligible impact on the convergence speed because non-scale growth frameworks deliver a time-varying speed of convergence.

Third, we examine the capacity of the model dynamics to reproduce fast output-convergence episodes such as those of Japan and South Korea. Using standard technologies and parameterization, we show that our calibrated model is fairly successful in replicating rapid growth paths, including the hump-shaped output growth adjustment paths associated with these experiences. It is also found that the model can generate adjustment paths for interest and investment rates that follow the patterns in the data. This is in sharp contrast to the counterfactual implications of the standard one-sector neoclassical growth framework pointed out by King and Rebelo (1993). A key factor contributing to these results is the complementarity between human capital and technology adoption, which induces reallocation of labor across sectors along the adjustment path.

The implications of the Jones hybrid growth model have been extensively explored by Eicher and Turnovsky (1999a, 1999b, 2001), and Perez-Sebastian (2000). Unlike us, they do not consider human capital. There is however a small but rapidly growing literature that investigates the relationship between human capital accumulation and technological progress, and their combined effect on economic growth. Eicher (1996) and Lloyd-Ellis and Roberts (2000) develop models in which both human capital and technological innovation are endogenous, but they are only concerned with steady-state predictions. Restuccia (2001) presents a dynamic general equilibrium model with schooling and technology adoption. He focuses on how schooling and technology adoption may be amplifying the effects of productivity/policy differences on income disparity. Like us, Keller (1996) and Funke and Strulik (2000) study transitional dynamics in a model of human capital and blueprints. However, they do not take the predictions of their models to the data.

The remainder of the paper is organized as follows. Section 2 presents the basic model and examines its steady-state properties. Attention is focused on the schooling sector which is the main innovation of the model. Section 3 presents the transitional dynamics analysis. This section obtains the asymptotic speed of convergence and transition paths implied by our model and compares these results to existing models in the literature. Section 4 concludes.

2 The Basic Model

This section presents an economic growth model with endogenous human capital and technical progress. Our exposition is focused on aggregate technologies. The main reason is that the human capital technology incorporated in this paper can not be easily derived from a decentralized setup due to aggregation problems.[4] Another important reason is that previous papers that have analyzed the type of non-scale framework that we incorporate in this paper have focused on the central planner's solution.

We start by describing the model economy's environment. We then set up and solve the central planner's problem. Finally, we derive and discuss the steady-state implications of the model.

2.1 Economic environment

The population in this economy consists of identical infinitely-lived agents, and grows exogenously at rate n. Agents are involved in three types of activities: consumption-good production, R&D effort, and human capital attainment.[5] Each period, consumers are endowed with one unit of time that is allocated between working and studying. We abstract from labor/leisure decisions and assume that agents have preference only over consumption.

Assume that at period t, output (Y_t) is produced using human capital (H_{Yt}) and physical capital (K_t) according to the following aggregate Cobb-Douglass technology:

$$Y_t = A_t^{\xi} H_{Yt}^{1-\alpha} K_t^{\alpha} \quad , \quad 0 < \alpha < 1 \, , \;\; \xi > 0; \tag{1}$$

where A_t is the economy's technology level, ξ is the technology-output elasticity, and α is the share of capital.

The R&D technology incorporates the only link between economies in our model. Ideas created anywhere in the world can be copied by local researchers at a cost that diminishes with the country's technological gap. The economy's technology level evolves according to the following motion equation:

$$A_{t+1} - A_t = \mu A_t^{\phi} H_{At}^{\lambda} \left(\frac{A_t^*}{A_t} \right)^{\psi} - \delta_A A_t, \quad \phi < 1, \;\; 0 < \lambda \leq 1, \;\; \psi \geq 0, \;\; A_t^* \geq A_t; \tag{2}$$

where δ_A represents the technology depreciation rate; H_{At} is the portion of human capital employed in the R&D sector at time t; A_t^* is the worldwide technology frontier that grows exogenously at rate g_{A^*}; μ is a technology parameter; ϕ weights the effect of the stock of existing technology on

[4] See footnote 3 for papers that have also used the Mincerian approach to human capital and footnote 9 for a discussion on the aggregation problem of this approach.

[5] Schooling is assumed to be the only source of human capital attainment in this model. Allowing for other types of human capital attainment such as learning-by-doing (i.e. see Stokey (1988) and Lucas (1993)) would be an interesting extension of the model and worthy of future research.

R&D productivity; and λ captures decreasing returns to R&D effort.[6] R&D equation (2) is a modification of Jones (1995, 2001) R&D equation to allow for a *catch-up* term, $\left(\frac{A_t^*}{A_t}\right)^{\psi}$, where ψ is a technology-gap parameter. The catch-up term captures the idea that the greater the technology gap between a leader and a follower, the higher the potential of the follower to catch up through imitation of existing technologies.[7]

The production function given by (1) and the R&D equation given by (2) reflect the complementarity between technology and human capital. We consider that a higher human capital level allows workers to use ideas more efficiently, and speeds up technology acquisition. Agents increase their human capital through formal education provided by a schooling sector. The human capital technology is of particular interest in our model and deserves careful consideration. Since our aim is to take the model to the data then our specification ought to map the available data on average years of education to the stock of human capital. Using the Mincerian interpretation seems to deliver such a specification. This representation follows Bils and Klenow (2000), who suggest that the Mincerian specification of human capital is the appropriate way to incorporate years of schooling to the aggregate production function. Following their approach, aggregate human capital is given by

$$H_{jt} = e^{f(S_t)} L_{jt}\,, \quad j \in \{Y, A\}\,, \tag{3}$$

where L_{jt} is the total amount of labor allocated to sector j; and S_t is the average educational attainment of labor in period t. The derivative $f'(S)$ represents the return to schooling estimated in a Mincerian wage regression: an additional year of schooling raises a worker's efficiency by

[6] A decentralized setup behind these aggregate equations is, for example, that of Romer (1990). We can think of technology as the mass of intermediate-good varieties, x_{it}, used in production. Under this interpretation, the term $A_t^{\xi} K_t^{\alpha}$ in expression (1) is a reduced form for $[\int_0^{A_t} x_{it}^{\alpha\gamma}\, di]^{1/\gamma}$; where $\gamma > 0$ is a complementarity parameter. The two production technologies are equal in the symmetric equilibrium case in which $x_{it} = \bar{x}_t$, $K_t = A_t \bar{x}_t$, and $\xi = 1/\gamma - \alpha$. In Romer (1990), R&D effort results in new designs for use in new types of producer durables. There are incentives to carry out R&D because when a new design is produced, an intermediate-good producer acquires a perpetual patent over the design. This allows the firm to manufacture the new variety and practice monopoly pricing.

[7] Nelson and Phelps (1966) are the first to construct a formal model based on the catch-up term. Parente and Prescott (1994) notice that this formulation implies that development rates increase over time (with A_t^*), and provide empirical evidence that is consistent with this implication. Coe and Helpman (1995), and Coe, Helpman, and Hoffmaister (1997), among others, find evidence supporting the role of foreign-technology adoption in economic growth.

$f'(S)$.[8,9]

Next, we are concerned with the behavior of S_t. Suppose that at each date agents allocate time to schooling only after supplying labor services to firms. L_t denotes the population size and L_{Ht} the total amount of time allocated to schooling in period t. Assume that at the beginning of period 1 the average educational attainment equals *zero*. This implies that at the beginning of period 2, $S_2 = \frac{L_{H1}}{L_1}$. Next period, given that consumers live for ever, the average years of schooling will be $S_3 = \frac{L_{H1}+L_{H2}}{L_2}$, and so on. Hence, the average educational attainment can be written as

$$S_t = \frac{\sum_{j=1}^{t-1} L_{Hj}}{L_t}. \tag{4}$$

From equation (4), we can derive the law of motion of the average educational attainment as follows:

$$S_{t+1} = \frac{S_t\, L_t + L_{Ht}}{L_{t+1}}. \tag{5}$$

which in turn implies

$$S_{t+1} - S_t = \left(\frac{1}{1+n}\right)\left(\frac{L_{Ht}}{L_t} - n\, S_t\right). \tag{6}$$

The evolution of S across time depends on the share of people in education $\frac{L_H}{L}$ and the growth rate of population, with the latter inducing a dilution effect.

2.2 Central planner's problem

As we have mentioned previously, we focus on a centrally planned economy. A central planner chooses the sequence $\{C_t,\, S_t,\, A_t,\, K_t,\, L_{Yt},\, L_{At},\, L_{Ht}\}_{t=0}^{\infty}$ so as to maximize the lifetime utility of

[8] Mincer (1974) estimates the following wage regression equation:

$$\omega_i = \beta_0 + \beta_1(SCH)_i + \beta_2(EXP)_i + \beta_3(EXP)_i^2 + \varepsilon_i,$$

where ω_i is the log wage for individual i, SCH is the number of years in school, EXP is the number of years of work experience, and ε is a random disturbance term. Based on this micro-Mincer regression, Bils and Klenow (2000) present a more extensive formulation of expression (3) that includes schooling quality, and work experience.

[9] To be fully consistent with the Mincerian interpretation, $H_{jt} = \sum_{i=1}^{L_{jt}} e^{f(s_{it})}$; where s_{it} is the educational attainment of worker i at date t. The mapping between this expression and equation (3) is not straightforward, and has not been addressed by the literature, with the exception of Lloyd-Ellis and Roberts (2000) who perform only balanced-growth path analysis in a finitely-lived agent framework. The difficulty arises because different cohorts can possess different schooling levels. To make both expressions consistent, we could assume that the first generation of agents pins down the workers' educational attainment, and that posterior cohorts are forced to stay in school until they accumulate this educational level. In this way, all workers would have the same years of education (i.e., $s_{it} = S_t$ for all i) and then $\sum_{i=1}^{L_{jt}} e^{f(s_{it})} = L_{jt}\, e^{f(S_t)}$. However, introducing these microfoundations into the model would require to keep track of the different cohorts' years of education across time, thus making the transitional dynamics analysis much more cumbersome, if not impossible. We leave this important issue to future research.

the representative consumer subject to the feasibility constraints of the economy, and the initial values L_0, S_0, K_0, and A_0. The problem is characterized by the following set of equations:

$$\max_{\{C_t,S_t,A_t,K_t,L_{Yt},L_{At},L_{Ht}\}} \sum_{t=0}^{\infty} \rho^t \left[\frac{\left(\frac{C_t}{L_t}\right)^{1-\theta} - 1}{1-\theta} \right], \tag{7}$$

subject to,

$$Y_t = A_t^{\xi} \left[e^{f(S_t)} L_{Yt} \right]^{1-\alpha} K_t^{\alpha}\,, \tag{8}$$

$$I_t = K_{t+1} - (1-\delta_K)\, K_t = Y_t - C_t, \tag{9}$$

$$A_{t+1} - A_t = \mu A_t^{\phi} \left[e^{f(S_t)} L_{At} \right]^{\lambda} \left(\frac{A_t^*}{A_t} \right)^{\psi} - \delta_A\, A_t, \tag{10}$$

$$S_{t+1} - S_t = \left(\frac{1}{1+n} \right) \left(\frac{L_{Ht}}{L_t} - n\, S_t \right), \tag{11}$$

$$L_t = L_{Yt} + L_{At} + L_{Ht}, \tag{12}$$

$$\frac{L_{t+1}}{L_t} = 1+n, \quad \text{for all } t, \tag{13}$$

$$\frac{A_{t+1}^*}{A_t^*} = 1 + g_{A^*}, \tag{14}$$

$$L_0,\ S_0,\ K_0,\ A_0 \ \text{ given,}$$

where θ is the inverse of the intertemporal elasticity of substitution; and ρ is the discount factor. Equation (9) is the economy's feasibility constraint combined with the law of motion of the stock of physical capital; it states that, at the aggregate level, domestic output must equal consumption, C_t, plus physical capital investment, I_t. Equation (12) is the population constraint; *labor force* – the number of people employed in the output and the R&D sectors – plus the number of individuals in school must equal total *population*.

The optimal control problem can be stated as follows:

$$\begin{aligned} V\left(A_t, K_t, S_t\right) \quad &= \max_{\{L_{Ht},L_{At},I_t\}} \frac{\left[\frac{A_t^{\xi}\left[e^{f(S_t)}(L_t - L_{Ht} - L_{At})\right]^{1-\alpha} K_t^{\alpha} - I_t}{L_t} \right]^{1-\theta} - 1}{1-\theta} + \\ +\rho V &\left[A_t(1-\delta_A) + \mu A_t^{\phi} \left(e^{f(S_t)} L_{At} \right)^{\lambda} \left(\frac{A_t^*}{A_t} \right)^{\psi} ; K_t(1-\delta_K) + I_t\,;\ S_t + \frac{1}{1+n} \left(\frac{L_{Ht}}{L_t} - nS_t \right) \right], \end{aligned} \tag{15}$$

where $V(\cdot)$ is a value function; L_{Ht}, L_{At}, I_t are the control variables; and A_t, K_t, S_t are the state variables. Solving the optimal control problem obtains the Euler equations that characterize the optimal allocation of population in human capital investment, in R&D investment, and in consumption/physical capital investment as follows:

$$\left(\frac{C_t}{L_t}\right)^{-\theta}\frac{(1-\alpha)Y_t}{L_{Yt}}=\frac{\rho}{1+n}\left(\frac{C_{t+1}}{L_{t+1}}\right)^{-\theta}\frac{(1-\alpha)Y_{t+1}}{L_{Y,t+1}}\left[1+f'(S_{t+1})\left(\frac{L_{Y,t+1}+L_{A,t+1}}{L_{t+1}}\right)\right], \tag{16}$$

$$\begin{aligned}\left(\frac{C_t}{L_t}\right)^{-\theta}\frac{(1-\alpha)Y_t}{L_{Yt}} &= \frac{\rho}{1+n}\left(\frac{C_{t+1}}{L_{t+1}}\right)^{-\theta}\frac{\lambda\left[A_{t+1}-(1-\delta_A)A_t\right]}{L_{At}} * \\ &* \left\{\frac{\xi Y_{t+1}}{A_{t+1}}+\left[1-\delta_A+(\phi-\psi)\left(\frac{A_{t+2}-(1-\delta_A)A_{t+1}}{A_{t+1}}\right)\right]\left[\frac{\frac{(1-\alpha)Y_{t+1}}{L_{Y,t+1}}}{\frac{\lambda(A_{t+2}-(1-\delta_A)A_{t+1})}{L_{A,t+1}}}\right]\right\}, \end{aligned}\tag{17}$$

$$\left(\frac{C_t}{L_t}\right)^{-\theta}=\frac{\rho}{1+n}\left(\frac{C_{t+1}}{L_{t+1}}\right)^{-\theta}\left[\frac{\alpha Y_{t+1}}{K_{t+1}}+(1-\delta_K)\right]. \tag{18}$$

At the optimum, the central planner must be indifferent between investing one additional unit of labor in schooling, R&D, and final output production. The LHS of equations (16) and (17) represent the return from allocating an additional unit of labor to output production. The RHS of equation (16) is the discounted marginal return to schooling, taking into account population growth. The RHS term in brackets obtains because human capital determines the effectiveness of labor employed in output production as well as in R&D. The RHS of equation (17) is the return to R&D investment. An additional unit of R&D labor generates $\frac{\lambda[A_{t+1}-(1-\delta_A)]A_t}{L_{At}}$ new ideas for new types of producer durables. Every new design increases next period's output by $\frac{\xi Y_{t+1}}{A_{t+1}}$ and R&D production by $\frac{dA_{t+2}}{dA_{t+1}}$ *times* $\frac{(1-\alpha)Y_{t+1}}{L_{Y,t+1}}\left[\frac{\lambda(A_{t+2}-(1-\delta_A)A_{t+1})}{L_{A,t+1}}\right]^{-1}$; where $\frac{(1-\alpha)Y_{t+1}}{L_{Y,t+1}}\left[\frac{\lambda(A_{t+2}-(1-\delta_A)A_{t+1})}{L_{A,t+1}}\right]^{-1}$ denotes the value of an additional design that equalizes labor wages across sectors. Euler equation (18) states that the planner is indifferent between consuming one additional unit of output today and converting it into capital, thus consuming the proceeds tomorrow.

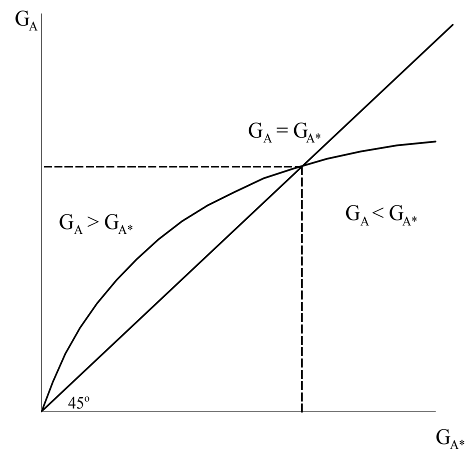
2.3 Steady-state growth

We next derive the model's balanced-growth path. Solving for the interior solution, equation (12) implies that in order for labor allocations to grow at constant rates, L_{Ht}, L_{Yt} and L_{At} must all increase at the same rate as L_t. This means that the ratio $\frac{L_{Ht}}{L_t}$ is invariant along the balanced-growth path. Hence, equation (11) implies that, at steady-state (ss), S_{ss} is constant and equal to

$$S_{ss}=\frac{u_{H,ss}}{n}, \tag{19}$$

where $u_{H,ss}=\left.\frac{L_H}{L}\right|_{ss}$. Equation (19) shows that along the balanced growth path, the economy invests in human capital just to provide new generations with the steady-state level of schooling.

Figure 1: Relationship between $G_{A,ss}$ and $G_{A^*,ss}$



This is consistent with Jones (1997), where growth regressions are developed from steady-state predictions; data on S_{ss} acts as a proxy for $u_{H,ss}$ and the estimated coefficient on S_{ss} partly reflects the parameter $\frac{1}{n}$ in our framework.

The aggregate production function, given by equation (8), combined with the steady-state condition $g_{Y,ss} = g_{K,ss}$ delivers the gross growth rate of output as a function of the gross growth rate of technology as

$$G_{Y,ss} = (G_{A,ss})^{\frac{\xi}{1-\alpha}} (1+n), \tag{20}$$

where $G_{xt} = 1 + g_{xt}$. Since $G_{A,ss}$ is constant, it follows from equation (2) that

$$G_{A,ss} = \left[(1+n)^{\lambda} G^{\psi}_{A^*,ss}\right]^{\frac{1}{1+\psi-\phi}}. \tag{21}$$

Equation (21) presents the relationship between the technology growth rate of the model economy and the technology frontier growth rate. This relationship is illustrated in Figure 1. Notice that since the ratio $\frac{\psi}{1+\psi-\phi} < 1$, the function is concave with a unique point at which

$$G_{A,ss} = G_{A^*,ss} = (1+n)^{\frac{\lambda}{1-\phi}}. \tag{22}$$

$G_{A,ss}$ cannot be larger than $G_{A^*,ss}$ otherwise A_t will eventually become bigger than A^*_t, and this has been ruled out by assumption. But $G_{A,ss}$ can be smaller than $G_{A^*,ss}$. For simplicity, we focus on the special case in which all countries grow at the same rate at steady state; that is, we assume that $G_{A^*,ss}$ is given by expression (22) and so is $G_{A,ss}$.[10] This in turn implies that

$$G_{Y,ss} = G_{C,ss} = G_{K,ss} = (1+n)^{\frac{\lambda\xi}{(1-\alpha)(1-\phi)}}. \tag{23}$$

Consistent with Jones (1995) our balanced-growth path is free of "scale effects", and policy has no effect on long-run growth. The reason why our model's long-run growth is equivalent to that of Jones even in the presence of a schooling sector, is that at steady state the mean years of education, S_t, reaches a constant level S_{ss}.

2.4 Population shares in output, R&D, and schooling

Next, we derive the steady-state shares of labor in the three sectors of the economy. Euler equation (16) combined with the balanced-growth equation (23) gives

$$u_{H,ss} = 1 - \frac{1}{f'(S_{ss})}\left[\frac{G_{y,ss}^{\theta-1}(1+n)}{\rho} - 1\right], \tag{24}$$

where $u_{H,ss} = \left.\frac{L_H}{L}\right|_{ss}$. Expectedly, the steady-state share of students in total population ($u_{H,ss}$) is positively related to the returns to education ($f'(S_{ss})$), and the preference parameters ($\rho, 1/\theta$).

Euler equation (17) combined with balanced-growth condition (23) delivers the steady-state labor share in R&D as

$$u_{A,ss} = \frac{u_{Y,ss}}{\left(\frac{1-\alpha}{\lambda\xi(g_{A,ss}+\delta_A)}\right)\left[G_{y,ss}^{\theta-1}\left(\frac{G_{A,ss}}{\rho}\right) - (\phi-\psi)(g_{A,ss}+\delta_A) - (1-\delta_A)\right]}. \tag{25}$$

As expected, R&D effort increases with the elasticities of technological change ($\phi - \psi$) and final output (ξ) with respect to the current stock of knowledge. R&D investment also increases as the degree of diminishing returns to R&D effort decreases (i.e, as λ increases). Dividing equation (12) by L gives the labor share in the output sector

$$u_{Y,ss} = 1 - u_{h,ss} - u_{A,ss}. \tag{26}$$

Equations (24), (25) and (26) represent the three steady-state shares of labor.

[10] Alternatively, we could assume that a technological leader moves the world technology frontier according to equation (2) which now reduces to

$$A_{t+1}^* - A_t^* = \mu A_t^{*\phi}\left(h_{At}^* L_{At}^*\right)^{\lambda} - \delta_A A_t^*.$$

Notice that for the leader imitation is not possible since at the frontier $\frac{A_t^*}{A_t} = 1$. In such case $G_A^* = 1 + g_A^* = (1+n^*)^{\frac{\lambda}{1-\phi}}$ as in Jones (1995). Assuming that $n = n^*$, and substituting G_A^* into equation (21) delivers equation (22). As discuss in footnote 11, had g_A^* taken on any other value, the transitional dynamics numerical analysis would become much more tedious.

3 Transitional Dynamics

This section is concerned with the transitional dynamics of the model economy. First, we redefine variables so that their values remain constant at steady state. Second, we analyze the asymptotic stability of the balanced growth equilibrium, and the asymptotic speed of convergence. Finally, we assess the capacity of the model to reproduce two distinct miraculous growth experiences: Japan and South Korea.

One of the main goals of the paper is to try to understand how the predictions of non-scale growth models change when human capital is introduced. For this reason, an important aspect of this work is to compare the predictions of our R&D model with human capital to those delivered by the *hybrid* non-scale R&D growth model without human capital.

3.1 The normalized system

We start by redefining variables so that they take constant values along the balanced-growth path. The aggregate production function, equation (8), suggests that we normalize variables by the term $A_t^{\frac{\xi}{1-\alpha}} L_t$. We can then rewrite consumption, physical capital and output as $\hat{c}_t = \frac{C_t}{A_t^{\frac{\xi}{1-\alpha}} L_t}$, $\hat{k}_t = \frac{K_t}{A_t^{\frac{\xi}{1-\alpha}} L_t}$ and $\hat{y}_t = \frac{Y_t}{A_t^{\frac{\xi}{1-\alpha}} L_t}$, respectively. Using equation (16) gives

$$\left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right)^{\theta} \left(\frac{u_{Y,t+1}}{u_{Yt}}\right) (G_{At})^{\frac{(\theta-1)\xi}{1-\alpha}} \left(\frac{\hat{y}_t}{\hat{y}_{t+1}}\right) = \left(\frac{\rho}{1+n}\right) \left[f'(S_{t+1}) \left(u_{Y,t+1} + u_{A,t+1}\right) + 1\right]. \tag{27}$$

¿From the R&D equation (2), we derive G_{At} as

$$G_{At} = \frac{A_{t+1}}{A_t} = 1 - \delta_A + \upsilon \left[e^{f(S_t)} u_{At}\right]^{\lambda} T^{(1+\psi-\phi)}, \tag{28}$$

where $T = \frac{A_t^*}{A_t}$; and $\upsilon = \mu \left(A_t^*\right)^{\phi-1} L_t^{\lambda}$, which is a constant.[11] From equation (17) we obtain

$$\begin{aligned} \left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right)^{\theta} \left(\frac{\hat{y}_t}{\hat{y}_{t+1}}\right) \left(\frac{u_{Y,t+1}}{u_{Yt}}\right) \quad &= \quad \frac{\rho \left(g_{At} + \delta_A\right)}{G_{At}^{\frac{\xi}{1-\alpha}(\theta-1)+1}} \left(\frac{u_{A,t+1}}{u_{At}}\right) * \\ &* \left[\left(\frac{\lambda \xi}{1-\alpha}\right) \left(\frac{u_{Y,t+1}}{u_{A,t+1}}\right) + \left(\frac{1-\delta_A}{\left(g_{A,t+1} + \delta_A\right)}\right) + (\phi - \psi)\right]. \end{aligned} \tag{29}$$

Finally, from equation (18) we obtain

$$\frac{1+n}{\rho} \left[\left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right) (G_{At})^{\frac{\xi}{1-\alpha}}\right]^{\theta} = \alpha \frac{\hat{y}_{t+1}}{\hat{k}_{t+1}} + (1 - \delta_K). \tag{30}$$

[11] To show that υ is constant requires some algebra. Rewriting the equality in its gross growth form, $\frac{\upsilon_{t+1}}{\upsilon_t} = G_{A^*t}^{\phi-1} (1+n)^{\lambda}$, and given that $G_{A^*t} = G_{A,ss} = (1+n)^{\frac{\lambda}{1-\phi}}$, it follows that $\frac{\upsilon_{t+1}}{\upsilon_t} = 1$. Notice that if A_t^* did not grow according to equation (22), υ could not be constant, making the simulation exercise more tedious.

The system that determines the dynamic equilibrium normalized allocations is formed by the conditions associated with three control and three state variables as follows:

Control Variables:

1. Euler equation for population share in schooling, u_{ht}: Eq. (27).
2. Euler equation for population share in R&D, u_{At}: Eq. (29).
3. Euler equation for normalized consumption, $\hat{c}_t$: Eq. (30).

Subject to the population constraint $u_{Yt} = 1 - u_{At} - u_{ht}$.

State Variables:

1. Law of motion of human capital, S_t: Eq. (6).
2. Law of motion of the technology gap, T_t:

$$T_{t+1} = T_t \left(\frac{G_{A^*t}}{G_{At}} \right). \tag{31}$$

3. Law of motion of normalized physical capital, $\hat{k}_t$:

$$(1+n)\hat{k}_{t+1} \left(G_{At}\right)^{\frac{\xi}{1-\alpha}} = (1-\delta_K)\hat{k}_t + \hat{y}_t - \hat{c}_t, \tag{32}$$

where G_{At} is given by expression (28), $G_{A^*t} = G_{A,ss}$ for all t, and

$$\hat{y}_t = \hat{k}_t^{\alpha} \left[e^{f_Y(S_t)} \, u_{Yt} \right]^{1-\alpha}. \tag{33}$$

3.2 Asymptotic speed of convergence

The literature on transitional dynamics has devoted considerable time and effort in analyzing the asymptotic speed of convergence predicted by growth models – that is, the rate at which a country's output approaches its balanced growth path once the country is sufficiently close to its long-run equilibrium. Ortigueira and Santos (1997) and Eicher and Turnovsky (1999b, 2000), among others, suggest that a desirable property for growth models is to deliver asymptotic speeds around 2 percent that is consistent with most cross-country empirical studies.[12] In addition, this analysis is crucial to establishing the asymptotic stability of the model's long-run equilibrium.

To compute the asymptotic speed of convergence we first linearize the normalized system of Euler and motion equations around the steady state, and express the resulting system as follows:

$$\vec{x}_{t+1} = H \, \vec{x}_t;$$

[12] For example, Barro and Sala-i-Martin (1995) report convergence speeds that vary from 0.4%–3% in Japan, 0.4%–6% in the U.S. and 0.7%–3.4% in Europe. Temple (1998) reports estimates for OECD nations between 1.5% and 3.6%.

where $\vec{x}$ is the vector consisting of the state and control variables; and H is the matrix of first derivatives $(\partial x_{i,t+1}/\partial x_{jt})\ \forall i,j$ evaluated at the steady state, with x_i being the i^{th} component of vector $\vec{x}$. In our case, the transpose of this vector is $\vec{x}'_t = (\hat{c}_t, u_{At}, u_{Ht}, \hat{k}_t, T_t, S_t)$. Second, we compute the eigenvalues associated with the matrix H. Convergence speed is obtained by the largest eigenvalue (denoted as *eigen*) among those contained in the unit circle. In particular, the asymptotic speed of convergence (denoted as *asc*) of normalized variable $\hat{y}$ can be written as

$$asc(\hat{y}) = -\frac{(\hat{y}_{t+1} - \hat{y}_t) - (\hat{y}_{t+1,ss} - \hat{y}_{t,ss})}{\hat{y}_t - \hat{y}_{t,ss}} = 1 - eigen.$$

Given that we are primarily interested in output *per worker*, $\frac{Y}{L_A+L_Y} = \hat{y}A^{\frac{\xi}{1-\alpha}}(u_A+u_Y)^{-1}$, we show that its asymptotic speed of convergence equals

$$asc\left(\frac{Y}{L_A+L_Y}\right) = (1-eigen)G_{y,ss} - g_{y,ss}. \tag{34}$$

To highlight the changes brought by the introduction of human capital into the model, we first present results for a benchmark economy in which the schooling sector is closed. This corresponds to the type of *two-sector non-scale growth model* studied, for example, by Eicher and Turnovsky (1999a) and Perez-Sebastian (2000). The model is then characterized by two control variables (consumption and R&D-labor) and two state variables (physical capital and technology gap), hence $\vec{x}'_t = (\hat{c}_t, u_{At}, \hat{k}_t, T_t)$. It is straightforward to show that the system of equations that determine the dynamics in this economy consists of Euler conditions (29) and (30), and motion equations (31) and (32); subject to $f(S) = 0$, the population constraint $u_{Yt} = 1 - u_{At}$, $G_{A^*t} = G_{A,ss}$, and equations (28) and (33). Eicher and Turnovsky (1999b, 2000) show that for relevant parameter values the stable manifold is two dimensional. The adjustment path is then asymptotically stable and unique; furthermore, growth rates and convergence speeds can, as a consequence, vary across time and variables.

Closed-form solutions for matrix H do not exist and therefore we resort to numerical methods.[13] We first choose parameter values (see table 1) for a benchmark economy that is very similar to those considered by Eicher and Turnovsky (1999b, 2000). As such, our basic economy is without a schooling sector and without imitation technology. There are only two sectors in this economy: a final good sector that displays constant returns in labor and capital, but increasing returns in knowledge; and an R&D sector that exhibits constant returns in knowledge and labor. Our numerical methods obtain the asymptotic speed of convergence ($asc(y)$) to be equal to 0.0179 but

[13] All numerical results were obtained using MATHEMATICA. Programs are available by the authors upon request.

Table 1: Parameter values for the benchmark model with no human capital and no imitation

α	0.36	ξ	0.1	ρ	0.96	ψ	0
δ_K	0.06	λ	0.5	θ	1	T_{ss}	1
δ_A	0.01	ϕ	0.5	n	0.015		

the implied steady-state growth rate of output per worker ($g_{y,ss}$) to be equal to 0.0023.[14] On the positive side, the model generates an asymptotic speed close to the 2 percent, consistent with most empirical evidence. Indeed, this is the major finding of Eicher and Turnovsky (1999b, 2000) that going from the neoclassical one-sector growth model to a two sector non-scale growth model reduces the asymptotic speed of convergence from about 7 percent to more reasonable values.[15] On the negative side, the model generates output per capita growth rate equal to 0.23 percent which is clearly inconsistent with evidence. For example, the average g_y in Bils and Klenow's (2000) 91-country sample is 1.6 percent. Imposing the larger value of output growth ($g_{y,ss} = 0.016$) requires a substantially stronger increasing returns in the R&D sector ($\phi = 0.931$) which in turn implies an implausibly low asymptotic speed, $asc(y) = -0.0042$.[16]

Using the fact that asymptotic speed is positively related to the parameters λ, δ_A and ξ we investigate whether admissible values for these parameters can deliver more reasonable results. The parameter $\xi = 0.1$ is already at its upper bound of empirical estimates, but we can increase the values of λ and δ_A to their upper bounds ($\lambda = 0.75$ and $\delta_A = 0.1$).[17] This exercise obtains a reasonable value of $asc(y) = 0.0156$. An alternative and possibly more attractive solution to obtaining reasonable values for both $g_{y,ss}$ and $asc(y)$ is by introducing imitation in the model along the lines of Parente and Prescott (1994) and Perez-Sebastian (2000). In our benchmark economy when we assume $\phi = 0.931$ (that results from imposing $g_{y,ss} = 0.016$) and $\psi = 0.16$ (within the calibrated values that we obtain in the following section), $asc(y)$ becomes 0.0196 which is close to

[14]Notice that in the absence of a schooling sector output per worker equals output per capita.

[15]These authors employ a continuous-time version of the model that provides slightly larger speeds than our discrete-time approach. In particular, for the benchmark economy, the continuous-time analog would imply $asc(y) = 0.0184$. The slightly larger speed implied by continuous-time holds across all the models considered in our paper.

[16]It is well known that the empirical literature does not offer much guidance about the value of ϕ. In our model, this parameter is pinned down by the selected steady-state growth rate of the economy through equation (23), for given values of λ, n, ξ, and α.

[17]Estimates of λ found in the literature vary from 0.2 (Kortum (1993)) to 0.75 (Jones and Williams (2000)). Griliches (1988) reports estimates of the elasticity of output with respect to technology ξ between 0.06 and 0.1. If we consider that δ_A includes the creative destruction effect of new technology on old designs, a value of 0.1 would imply that new ideas possess a life-span of 10 years, very close to the lower bound found by Caballero and Jaffe (1993).

Table 2: Parameter values for the proposed model with human capital and imitation

α	0.36	ξ	0.1	ρ	0.96	ψ	0.16	T_{ss}	1
δ_K	0.06	λ	0.5	θ	1	η	0.69	S_{ss}	12.03
δ_A	0.01	ϕ	0.931	n	0.015	β	0.43	$g_{y,ss}$	0.016

most empirical estimates.

Next, we analyze the asymptotic speed of convergence in the model with schooling and imitation. To do this, we need to calibrate the human capital technology. Following Bils and Klenow (2000), we assume that

$$f(S) = \eta S^{\beta}, \;\; \eta > 0, \; \beta > 0. \tag{35}$$

Then using Psacharopoulos' (1994) cross-country sample on average educational attainment and Mincerian coefficients we estimate η and β. Given equation (35), we can construct the loglinear regression equation

$$\ln\left(Mincer_i\right) = a + b\,\ln S_i + \varepsilon_i, \tag{36}$$

where $Mincer_i = f'(S_i)$ is the estimated Mincerian coefficient for country i; a and b equal $ln(\eta\beta)$ and $(\beta - 1)$, respectively; and ε_i is a random disturbance term. We obtain estimates of $\eta = 0.69$ and $\beta = 0.43$, both significantly different from zero at the 1 percent level, that are very similar to those obtained by Bils and Klenow (2000). Table 2 presents the parameter values used in our numerical exercise. It includes the parameters used in the benchmark economy when we take $\phi = 0.931$ so as to impose $g_{y,ss} = 0.016$, allow for imitation ($\psi = 0.16$) and the human capital technology parameters ($\eta = 0.69$, $\beta = 0.43$). Given the above values, equations (19) and (24) imply that the steady-state average educational attainment is 12.03 years, close to the 2000 U.S. figure of 12.05 obtained by Barro and Lee (2000).

For the proposed model economy, the stable manifold is pinned down by three eigenvalues that are contained within the unit circle. That is, the transition is characterized by a three-dimensional stable saddle-path which in turn implies that the adjustment path is asymptotically stable and unique.[18]

Our R&D model with human capital predicts an asymptotic speed of convergence for output per worker equal to 0.0132. Note that even though this convergence speed is slightly lower than

[18] This result is robust to reasonable changes in the parameter values.

that of the model with no human capital and no imitation ($asc(y) = 0.0179$) or the model with no human capital but with imitation ($asc(y) = 0.0196$), it is still well within empirical estimates. This reduction in the convergence speed occurs because of the additional schooling sector present in our model. A new sector implies that the same amount of available labor must now be allocated among three (rather than two) sectors, and that state variables move slower towards the balanced-growth path.

Another point worth noting here is that the asymptotic speed of convergence becomes much less responsive to changes in parameter values when we introduce human capital (a new state variable) in the model with imitation. For example, if δ_A increases from 0.01 to 0.1 our model with human capital and imitation predicts a small increases in $asc(y)$ from 0.0132 to 0.0172, whereas the model with no human capital but with imitation predicts a very large increase from 0.0196 to 0.0410. In addition, if λ increases from 0.5 to 0.75, then $asc(y)$ increase from 0.0132 to 0.0145 in the former model, and from 0.0196 to 0.0340 in the latter model. Finally, let us think about policy actions that affect the technology-gap parameter ψ. This could occur, for example, because of changes in the degree of barriers to technology adoption along the lines of Parente and Prescott (1994). Suppose that a successful policy to enhance technological adoption causes ψ to increase from 0.16 to 0.25. Then our model with human capital predicts an $asc(y)$ that increases from 0.0132 to 0.0168 whereas the model without human capital predicts an increase from 0.0196 to 0.0330. The reason for the different response of the asymptotic speed in the two models is once again the additional sector introduced in our model. That is, the presence of an additional sector into the model implies a lower allocation of resources to each of the different sectors, thus reducing the impact of external shocks.

This is consistent with Barro and Sala-i-Martin's (1995) finding that estimated convergence speeds do not vary much across different countries or regions. However, our result does not necessarily imply that policy actions have a small impact on the convergence speed, as Barro and Sala-i-Martin's result has been interpreted. Far away from the balanced-growth path, policy may have a larger effect on the speed of convergence over subsequent periods because the model allows the convergence speed to vary across time. It is also important to notice that Barro and Sala-i-Martin's finding obtains for a fairly homogenous group of wealthy regions – namely, U.S. states, European regions, and Japanese prefectures – which are probably close to their steady states.

In summary, we find that our non-scale R&D growth model with human capital obtains asymptotic speed of convergence consistent with the evidence. More importantly, our model's implied

Table 3: Output, Capital and Schooling in Japan and S. Korea

Country		1960	1963	1990
Japan	Y per worker (%)**	20.6		60.3
	K per worker (%)**	16.9		104.6
	S (years)	10.2		11.0*
S. Korea	Y per worker (%)**		11.0	42.2
	K per worker (%)**		11.6	50.2
	S (years)		3.2	7.7*

* 1987 figures.

** Levels relative to their U.S. counteparts.

asymptotic speed of convergence is much less responsive to policy actions compared to existing models in the literature.

3.3 Adjustment paths of Japan and South Korea

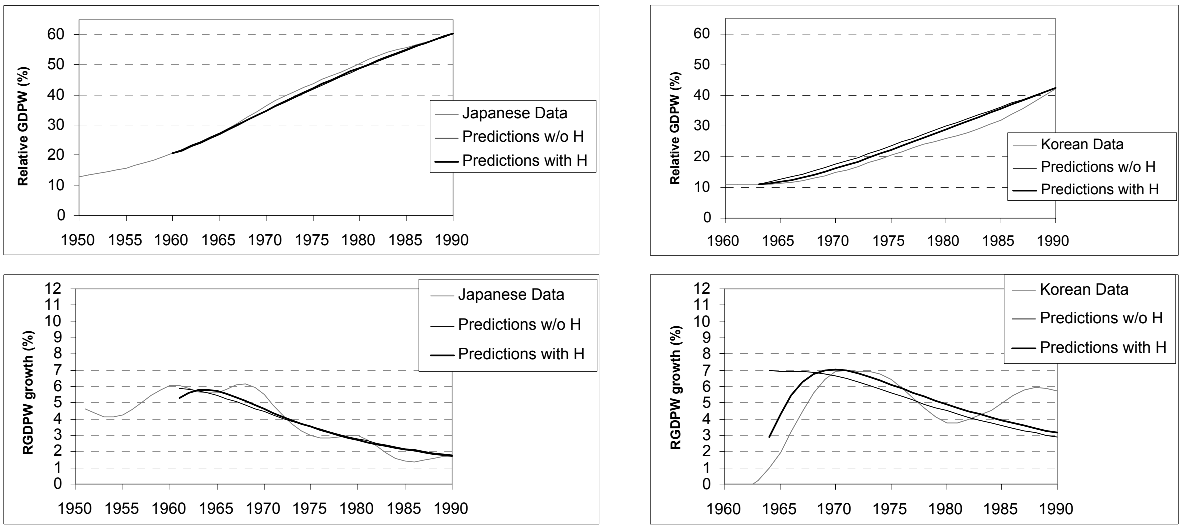
At least since the seminal work of Lucas (1993), it has been recognized that a desirable property of growth models is to be able to reproduce miraculous experiences. In terms of transitional dynamics analysis, this amounts at least to being able to reproduce the average speed of convergence, and country-specific changes in the output growth trend. In this section, we focus on the S. Korean and the Japanese output paths because they represent two very distinct growth experiences. Table 3 presents data for S. Korea and Japan on relative GDP *per worker*, relative physical capital *per worker*, and average educational attainment.[19] Between 1960 and 1990, Japan's relative output per worker increased from 20.6 to 60.3 percent. GDP per worker in S. Korea started its fast growing path around 1963; during the period 1963-1990, its relative level increased from 11.0 to 42.2 percent. During these periods, Japan and S. Korea exhibited a 5.2 and 6.5 percent average annual growth rates, respectively.

Japan had lost a substantial portion of its physical capital during WWII, but its educational attainment in 1960 of 10.2 years compared well with those of the most developed nations – e.g., the U.S. educational attainment at that time was a little over 10.7.[20] What is even more interesting is that during the period 1960-1987, average years of schooling per worker increased only by 0.8

[19] All relative measures in the paper are with respect to U.S. levels. Additionally, we follow Parente and Prescott (1994) and smooth all data series using the Hodrick-Prescott filter with the smoothing parameter equal to 25.

[20] Human capital levels in Japan were high before WWII. After the Meiji Restoration of 1868, one of the policy priorities of the Meiji government was to introduce a nationwide education system under which all children from 6 through 13 years of age were required to attend school (see Ozawa 1985).

Figure 2: Adjustment paths for Japan and S. Korea



years to reach 11.0 years. The main engine of growth in Japan seems to have been physical capital accumulation induced in part by a very important technological catch-up process.[21] In 1960, the Japanese physical capital stock per worker was only 16.9 percent; in 1990 reached a stunning 104.6 percent, which implies an average annual convergence rate of 6.3 percent.

The S. Korean growth experience is distinctly different from the Japanese experience. Even though output convergence was faster in S. Korea, physical capital accumulation was lower than in Japan, growing from 11.6 to 50.2 percent – an average annual convergence rate of 5.6 percent. As shown in table 3, human capital accumulation played a much larger role in the development process of S. Korea. In particular, the average educational attainment per worker more than doubled in the period 1963-1987, increasing from 3.2 to 7.7 years.

In this section, we examine the ability of the proposed model to replicate these two nations' convergence episodes by simulating its transitional dynamics. Because there is no analytical solution to our system of Euler and motion equations, we once again resort to numerical approximation techniques. More specifically, we follow Judd (1992) to solve the dynamic equation system, approx-

[21]For discussion on the effects of technology adoption on East Asia see Amsden (1991) and Baark (1991). For an excellent presentation of technology adoption in Japan see Minami (1994). The author explores three categories of borrowed technology which are illustrated by examples from Japanese history. He discusses in detail the introduction of the English railway technology, the machine filature technology and the silk weaving technology. According to Minami, Japan's industrialization was revolutionary in the sense that it was accomplished by the adoption of existing foreign technology.

imating the policy functions employing high-degree polynomials in the state variables.[22]

Taking the model to the data requires assigning a value to ψ. Here, we follow Parente and Prescott (1994), and calibrate the parameter ψ to each country's output data. In particular, Parente and Prescott assume that countries may differ in their degrees of technology adoption barriers and, as a consequence, they may show different values of ψ. Because we focus on two nations, Japan and South Korea, the value on which the parameter ψ takes will be the one that makes transitional dynamics be able to reproduce the output per worker evolution between 1960 and 1990 in Japan, and between 1963 and 1990 in S. Korea – i.e., their average speed of convergence.[23, 24]

Since we are interested in comparing the implications of our three-sector non-scale growth model with human capital (and imitation) to those of the two-sector non-scale growth model without human capital (but with imitation), we generate results for both frameworks. The model with human capital requires $\psi = 0.131$ to induce Japan's average speed of convergence, and $\psi = 0.162$ to produce the S. Korean output numbers. The model without human capital requires $\psi = 0.10$ for the Japanese development experience, and $\psi = 0.074$ for the S. Korean development experience. The initial values of the stock variables and output data used to calibrate ψ are presented in table 3; accuracy measures are presented in table 4.

The adjustment paths predicted by both models for the level and growth rates of relative

[22] In particular, the parameters of the approximated decision rules are chosen to (approximately) satisfy the Euler equations over a number of points in the state space, using a nonlinear equation solver. A Chebyshev polynomial basis is used to construct the policy functions, and the zeros of the basis form the points at which the system is solved; that is, we use the method of orthogonal collocation to choose these points. Finally, tensor products of the state variables are employed in the polynomial representations. This method has proven to be highly efficient in similar contexts. For example, in the one-sector growth model, Judd (1992) finds that the approximated values of the control variables disagree with the values delivered by the true policy functions by no more than one part in 10,000. All programs were written in GAUSS and are available by the authors upon request.

[23] Parente and Prescott's (1994) technology adoption equation is slightly different from ours. They do not include human capital, and equal the parameter ϕ to zero because they employ a neoclassical growth model. In addition, these authors consider that a parameter equivalent to $1/\mu$ in equation (2) is the one that is country-specific and captures the degree of technology adoption barriers. The value of ψ is, on the other hand, common to all countries. The parameters $1/\mu$ and ψ are calibrated using each country's average convergence speed. There is, therefore, a degree of freedom that forces the authors to choose ψ in an *ad hoc* manner. This formulation allows Parente and Prescott to generate very different steady-state output levels depending on the degree of the barriers. This is important for them because they propose a theory of cross-country income differences. We are, on the other hand, interested on assessing the capacity of our model to reproduce key features of the miraculous economies' convergence path. For this reason, we choose T_{ss} equal to 1 that forces μ to be common to all economies, and make ψ the country-specific parameter. We believe that renouncing in this way to the degree of freedom imposes more discipline into our analysis. The assumption that T_{ss} equals 1 implies that the U.S., Japan, and S. Korea possess the same steady state. Parente and Prescott provide convincing arguments that this is not the case. However, we think that our benchmark approach is no worse than trying to estimate each country's steady state, given that we have to make *ad hoc* assumptions over at least one of the parameters.

[24] Japan's rapid convergence toward U.S. income levels actually started right after WWII. Unfortunately, the Japanese Education Department does not possess estimates of the average educational attainment before 1960. We are grateful to Tomoya Sakagami who has attempted to obtain this data for us.

Table 4: Accuracy measures in different models

Country	Model**	ψ	Average Error (%)* C	u_H	u_A	Max. Error (%)* C	u_H	u_A
Japan	*model w/ H*	0.131	0.01	0.02	0.01	0.04	0.07	0.04
Japan	*model w/ h*	0.132	0.01	0.02	0.01	0.04	0.07	0.04
Japan	*model w/o H*	0.10	0.00	−.−	0.00	0.01	−.−	0.02
S. Korea	*model w/ H*	0.162	0.06	0.17	0.06	0.27	0.78	0.24
S. Korea	*model w/ h*	0.14	0.06	0.17	0.06	0.27	0.73	0.23
S. Korea	*model w/o H*	0.074	0.01	−.−	0.01	0.02	−.−	0.05

* We assess the Euler equation residuals over 10,000 state-space points using the approximated rules. For each variable, the measure gives the current value decision error that agents using the approximated rules make, assuming that the (true) optimal decisions were made in the previous period. Santos (2000) shows that the residuals are of the same order of magnitude as the policy function approximation error.

** *model w/ H* refers to the three-sector non-scale growth model with schooling sector.
model w/o H refers to the two-sector non-scale growth model without schooling sector.
model w/ h refers to the three-sector growth non-scale model assuming that per worker variables are obtained by dividing by L.

GDP per worker (RGDPW) are depicted in figure 2. The predicted paths replicate fairly well the Japanese and the S. Korean output paths. Our model with human capital, however, does a much better job because it predicts that output per worker growth rates do not pick at the beginning of the adjustment path but later on. This is an important feature that can not be either reproduced by the standard one-sector neoclassical growth model (see King and Rebelo (1993)), and that characterizes the output-convergence phenomenon as Easterly and Levine (1997), among others, show.

3.4 Discussion of the transition results

What are the determining factors behind our results? We can write production function (8) in per worker terms as follows:

$$\left(\frac{Y_t}{L_{Yt}+L_{At}}\right)=A_t^{\xi}e^{f(S_t)(1-\alpha)}\left(\frac{L_{Yt}}{L_{Yt}+L_{At}}\right)^{1-\alpha}\left(\frac{K_t}{L_{Yt}+L_{At}}\right)^{\alpha}. \tag{37}$$

Using a continuous time approximation, equation (37) can be rewritten in its output per worker growth (g_Y^w) form

$$g_{Yt}^{w}=\xi g_{At}+(1-\alpha)\,\frac{d\,f(S_t)}{d\,t}+\alpha g_{(K/L),t}+\Big[(1-\alpha)\,g_{u_Y,t}-g_{(1-u_H),t}\Big]. \tag{38}$$

Equation (38) presents a decomposition of output growth in its four components: (a) growth of total factor productivity (TFP), (b) change in per capita educational attainment, (c) growth of per capita physical capital, (d) net impact of labor movements across sectors (term in squared brackets). Given that the population size in our model evolves exogenously, this decomposition captures the impact of the different aggregates that enter the production function including the labor force size.

Figures 3 and 4 present the contributions of the four different components to the S. Korean and the Japanese output per worker growth, according to equation (38). We present the growth components for three different R&D models with imitation. A thin-black line represents predictions of the two-sector non-scale growth model without human capital accumulation (denoted as the *model w/o H*). Recall that in this model variables presented in their per capita or their per worker intensive form are identical as there is no schooling sector which would attract some of the labor force. As a result, the terms $(1-\alpha)\,\frac{d\,f(S_t)}{d\,t}$ and $g_{(1-u_H),t}$ in equation (38) equal zero. A thick-black line, represents predictions for our model with human capital (denoted as the *model w/ H*); the intensive form of all relevant variables are in per worker terms (dividing by $L_Y + L_A$). A dashed line represents predictions of a non-scale growth model with human capital but with the additional assumption that per worker variables come from dividing by L (the population size), instead of by $L_Y + L_A$ (the labor force) (denoted as the *model w/ h*). As a result, the second summand within the squared brackets in equation (38) vanishes. One of the determining features of this last formulation of our proposed model is that it does not consider movements in and out the labor force from and to the schooling sector. This model is examined in the hope that it will reveal which effect of the complementarity between human capital and technology dominates; the one on TFP that occurs though the R&D equation, or the one that takes place through labor movement among sectors. Finally, a grey line depicts the data for each country.

We start our analysis with a few general points. Notice that if RGDPW growth rates obtain large values early on, they must fall rapidly later on, and *vice versa*. This feature of the transitional path of output growth is due to the fact that all of the models considered are calibrated to reproduce the average convergence speed of RGDPW. Having this in mind, we can focus on model differences that occur during the early periods of the adjustment path. Another feature common to all three models is the initial values of the capital stocks from which the transition dynamics start. This implies that initial incentives to invest in physical and human capital formation (when the model includes a schooling sector) are very similar in the three cases, because by construction, so are the initial capital-output ratios and average educational attainments. As a consequence, the main forces behind the initial differences in RGDPW growth rates across models are the growth rate of relative TFP and the net contribution of labor (see figures 3 and 4; panels B and E).

Let us now compare the dashed line (denoting the *model w/ h*) and the thin-black line (denoting the *model w/o H*) in a attempt to understand the contribution of introducing human capital into the model and abstracting from the effect of movements into and out the labor force. The introduction of the new sector amplifies the effect of diminishing returns, increasing greatly initial growth rates. The new schooling sector adds a new growth engine whose contribution to the growth rate at impact lies around 4 percent for S. Korea and 0.33 percent for Japan (see panel D), and thereafter follows the standard neoclassical declining-growth-rate pattern caused by diminishing returns.

Another important effect of introducing schooling is that the final-output labor share starts further away from its steady state level and subsequently grows faster, thus making much larger its initial contribution (see panel E). The reason is that schooling is the only activity that enhances the productivity of the other two sectors, and consequently it is optimal for the economy to invest heavily in human capital at the beginning of the adjustment path, borrowing resources mainly from the consumption-good sector. Due to the same reason, physical capital suffers a slightly larger initial fall in the model with schooling, and accumulates at a faster rate during the first few periods following the evolution of output (see panel C). The big initial differences between the growth rates of output and capital in both models are due to consumption smoothing, which pulls down the investment share as output declines, causing physical capital to grow at a much lower rate than output during the first few periods.

Continuing our comparison between the dashed line (*model w/ h*) to the thin-black line (*model w/o H*) in figures 3 and 4 (panel B), we see that the relative TFP contribution is smaller in the *model w/ h*. This occurs because the shocks to physical capital and output are the same for both models, but in the *model w/ h* schooling formation also enhances the output catch-up process. As a result, the initial technology gap required by this model becomes smaller thus decreasing the productivity of the R&D sector and the contribution of TFP due to the existence of diminishing imitation opportunities. Note that R&D productivity declines so much that the technology-gap parameter ψ must rise to allow the model to reproduce the Japanese and S. Korean average convergence speeds.

Panel B in figure 3 clearly illustrates that human capital speeds up technology adoption. The contribution of TFP to output growth is presented by a hump-shaped pattern. This pattern, which turns out to also describe the evolution of the R&D labor share, is the consequence of two opposing effects. On the one hand, the technology imitation productivity declines with technology gap. On the other hand, R&D becomes more productive as the average educational attainment grows. The latter effect dominates the former during the first few periods, whereas the reverse is true later on.[25]

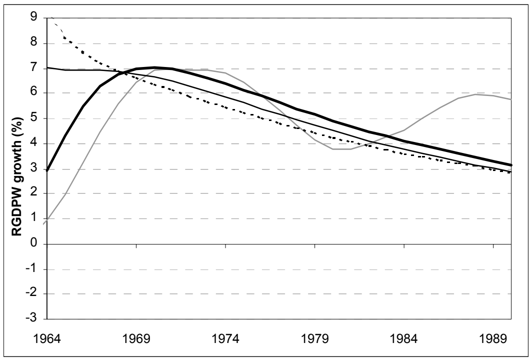
Our investigation of the *model w/o H* and the *model w/ h* reveals that neither model can replicate the hump-shaped output growth path evident in the data. We next compare the *model w/ h* (per worker variables are obtained by dividing by the population size, L) with the *model w/ H* (per worker variables are obtained by dividing by the labor force size, $L_Y + L_A$). The former model is once again illustrated by the dashed lines whereas the latter model is illustrated by the thick solid line.

Notice that the contribution of human capital is almost identical in both cases (see panel D). The hump-shaped physical capital contribution illustrated by the two lines is also the same, and complies well with the S. Korean data (see panel C). A strong and declining consumption smoothing effect causes the initial increase; but after a few periods diminishing-returns dominate and physical capital growth rates start to decrease, and continue doing so as they approach their steady-state level. There is a distinct difference between the two models though: the *model w/ H* (depicted by the thick-black line) shows a larger decrease in physical capital investment during the first two periods, and a faster physical capital growth thereafter. This is the consequence of matching the same initial data values to per worker variables, instead of per capita. Notice that at impact physical capital and output must be further away from their balanced-growth path in the former case, because the initial labor force is also below its steady-state value. The lower level of physical capital at impact produces larger returns, and raises its subsequent growth rates. The lower initial level of output along with the preference for consumption smoothing create the larger decrease in physical capital investment during the first two periods. The contribution of relative TFP in the *model w/ H* is stronger along the whole adjustment path (see panel B) because the slightly-larger initial technical gap required in the per-worker-term case raises R&D productivity. The difference is larger for S. Korea because the value of the parameter ψ required is also larger.

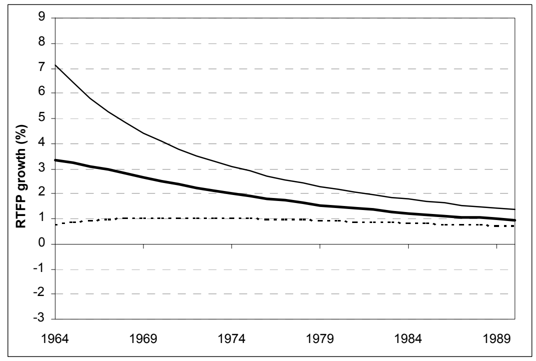
The main difference between the two models is due to the net labor contribution illustrated in panel E. Recall that net labor contribution is given by the term in brackets in equation (38), and reflects the effect of population movements across sectors. More specifically, this term takes into account that output growth rises with the amount of labor devoted to final-good production, but also that additional labor deflates output per worker. As a consequence, net labor contribution

[25] Lau and Wan (1933) suggest that the ability of human capital to enhance technology adoption may explain the miraculous experiences that achieve their maximum growth rates after trend acceleration. Our work shows that, at least in our structural *model w/ h*, human capital and TFP can not explain the output growth inverted-U path in Japan and S. Korea.

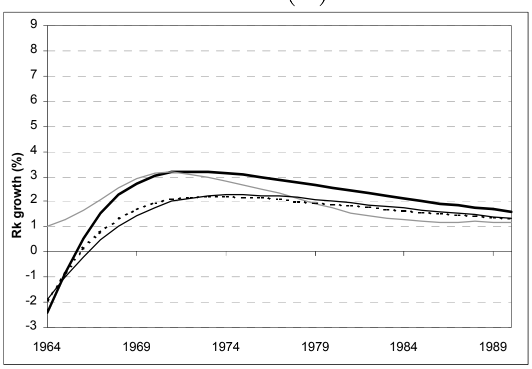
Figure 3: Contribution of different components to relative output growth, S. Korea



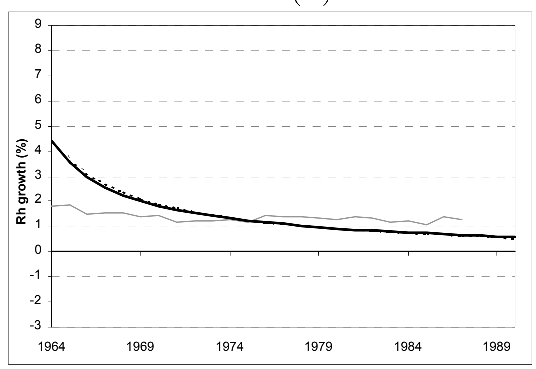
Panel (A)



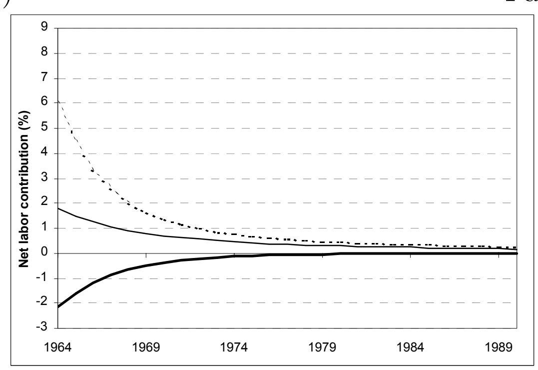
Panel (B)



Panel (C)



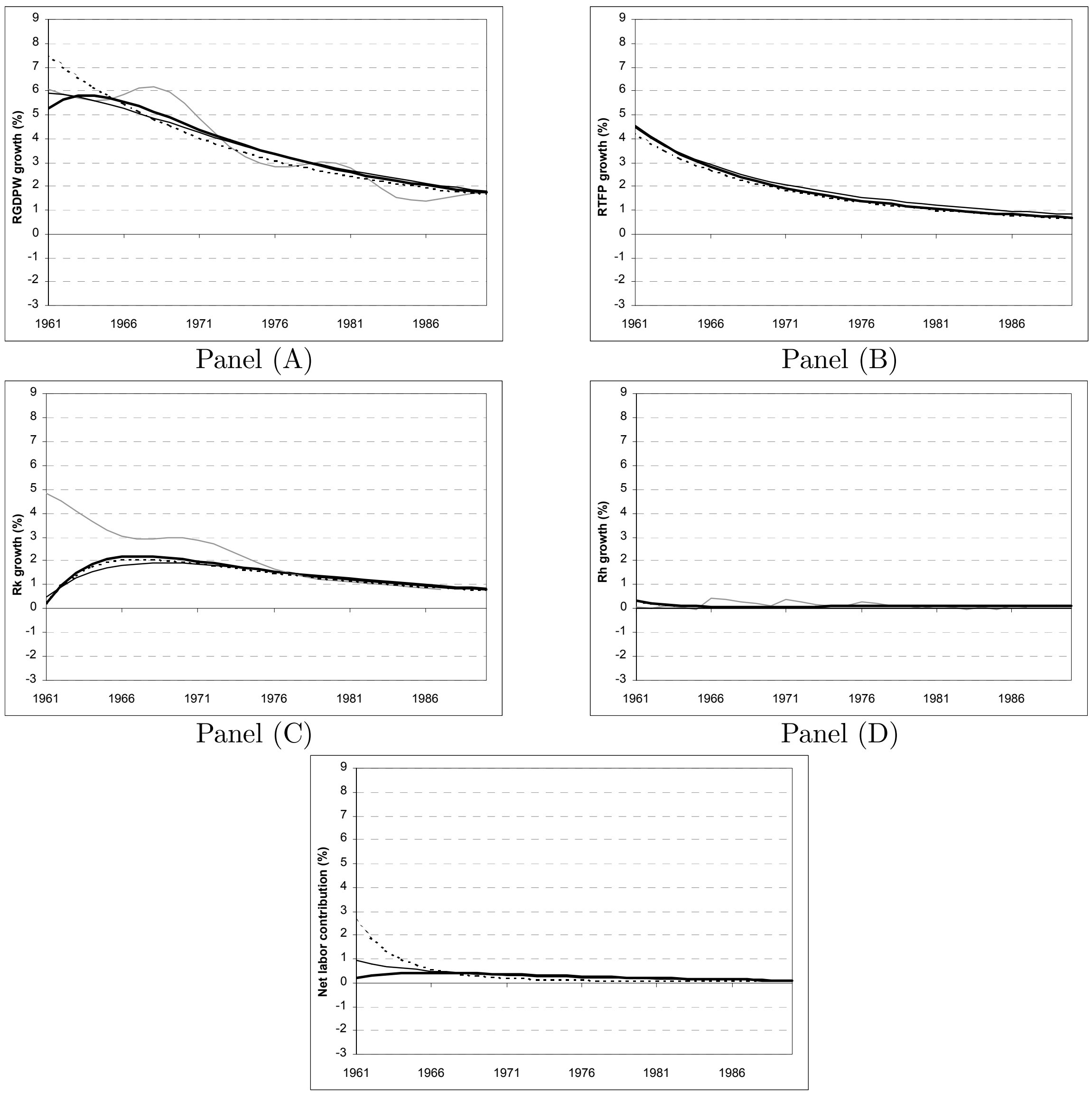
Panel (D)



Panel (E)

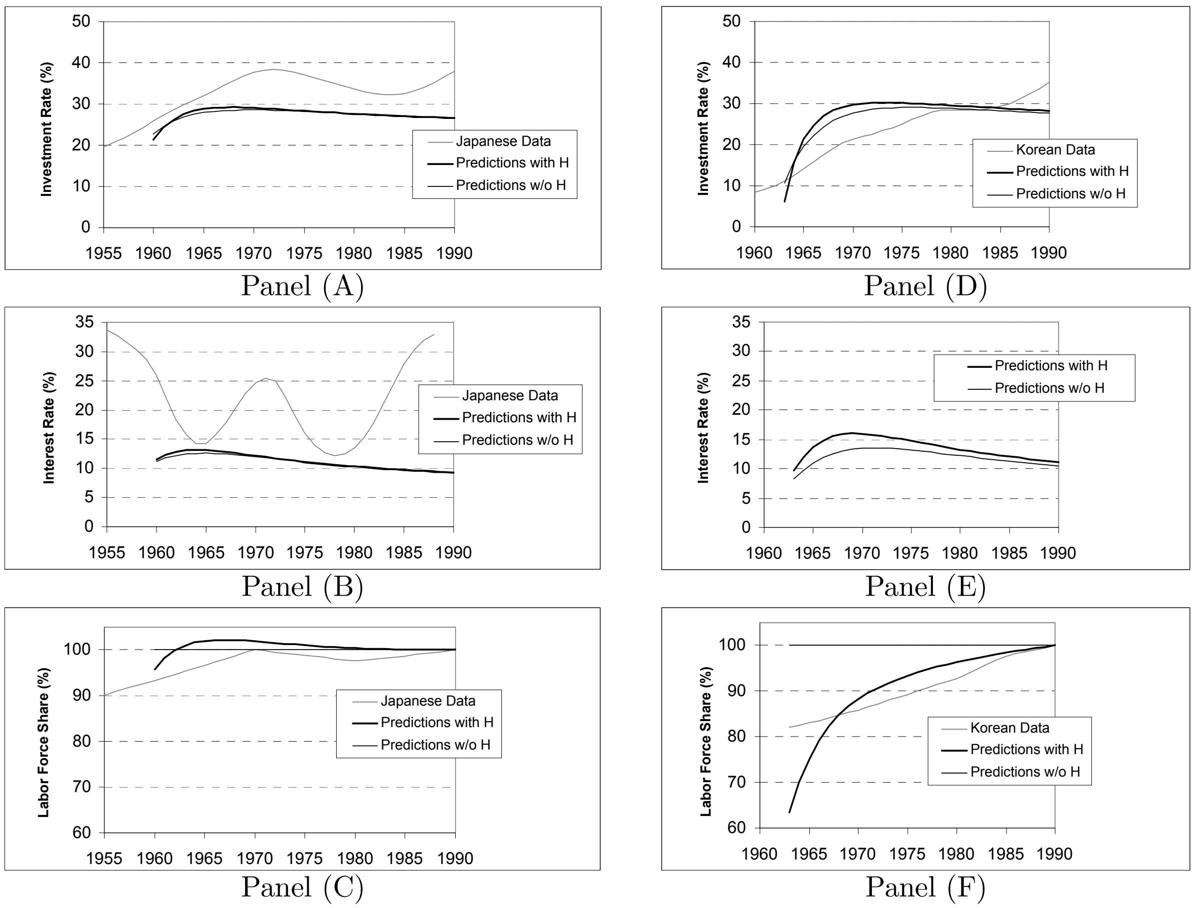
Variables: RGDPW is relative GDP per worker; Rk is relative physical capital per capita; Rh is relative human capital per capita; Net labor contribution represents the effect of the terms in brackets in equation (38).

Figure 4: Contribution of different components to relative output growth, Japan



Variables: RGDPW is relative GDP per worker; Rk is relative physical capital per capita; Rh is relative human capital per capita; Net labor contribution represents the effect of the terms in brackets in equation (38).

Figure 5: Investment, interest rates, and the labor force in S. Korea and Japan



decreases with the number of students that leave school because the amount of workers in the productive sectors rise, and increases as R&D effort declines because part of the R&D labor is reallocated to the final output sector. Along the *model w/ H* transitional dynamics, the effect of students entering the labor force is larger at the beginning, and rapidly decreases as the economy approaches the steady state, thus generating a fast declining pattern of labor force growth. This effect combined with a decreasing R&D labor share induces the initially rising net contribution of population reallocation illustrated by the thick-black line in panel E.

Our key finding here is that the main force that generates the hump-shape output path is the relatively large allocation of agents in education and R&D activities at the beginning of the convergence process, which produces large movements of agents in and out of the labor force.

3.5 Interest rates, investment, and labor force shares

We now turn attention to important variables formerly studied in the literature. King and Rebelo (1993) note that the transitional dynamics of the neoclassical one-sector growth model of physical capital accumulation needs either implausibly high interest rates or extraordinary high investment shares in order to generate the type of rapid convergence observed in East Asia. The model's adjustment path also has troubles in generating increasing investment shares. These problems can be eliminated by substantially modifying the baseline model: Christiano (1989), on the one hand, introduces a subsistence level of consumption into the utility function to correct it; Gilchrist and Williams (2001), on the other, consider a putty-clay production technology. We show that our framework is also able to avoid these counterfactual implications of the standard neoclassical growth model.

Figure 5 provides data and predictions on investment and interest rates. We see that both non-scale growth models, the one with schooling and the one without it, generate plausible investment rates that start well below their steady-state value as the evidence suggests. When we have more than one-sector, the economy deviates resources toward the activities that are relatively more productive. This is the case for the R&D and schooling sectors during the early stages of development. As the economy closes its technical gap and accumulates human capital, the relative level of investment in physical capital grows thus raising investment rates. Regarding the interest rate, we have data on inflation-adjusted returns in the Japanese stock market, obtained from Christiano (1989). These numbers show a slightly decaying trend, as predictions do.[26] Predictions are not contained within the observed values because of the calibration procedure followed that forces the steady-state interest rate to equal 7.42 percent for both Japan and S. Korea. This evidence agrees with the one supplied by King and Rebelo (1993) that suggests that interest rates do not show big variations across centuries. The difference now with the one-sector growth model is that lower levels of technology and human capital decrease the marginal productivity of capital, which mitigates the increase that interest rates suffer when physical capital declines.[27]

In addition, figure 5 provides data on the labor force share. Here, the non-scale model with human capital (*model w/ H*) clearly represents an improvement, given that by construction the labor force in the two-sector non-scale growth framework is equal to the population at any point

[26]The linear regression of the observed returns on a time trend gives a slope coefficient equal to -0.109.

[27]Perez-Sebastian (2000) makes the same point. He, however, finds a much larger variation in the interest rate than we do, and than the one suggested by the data.

in time.[28] We see that predictions replicate fairly well the main patterns. In S. Korea the labor force share starts far below its steady state value and grows monotonically, reflecting the return of student to the labor force. In Japan the labor force share at impact is below the balanced growth path and then overshoots. The overshooting is the result of the relatively high Japanese average educational attainment in 1960 which after a few periods leads the economy to borrow labor from the schooling sector and invest heavily on the final output and R&D activities in order to accumulate capital and close the big technical gap at a faster rate.

Finally, we can appreciate in all figures that the main differences between the models occur far from the steady state. This is why in the Japanese case both models offer more similar predictions along the adjustment path – Japan in 1960 was much closer to the steady state than S. Korea in 1963. This finding suggests that focusing on the asymptotic speed of convergence implied by different models may not be very informative about its overall performance to explain convergence episodes. We have shown that even though all of the models considered deliver similar asymptotic speeds of convergence that are consistent with empirical estimates, only the non-scale growth model with schooling successfully replicates important patterns of the Japanese and S. Korean experiences.

4 Conclusion

In this paper, we have constructed a non-scale growth model of endogenous technological change, physical capital accumulation, and human capital formation. The goal has been to study the model's implications once the complementarity between technology and human capital commonly found by the empirical literature is taken into account. In order to compare the model predictions to the data, we have introduced human capital following the Mincerian approach suggested in recent papers. Furthermore, we have developed a law of motion for the average educational attainment that allows for endogenous human capital formation, and preserves the non-scale nature of the model.

We have shown that the asymptotic speed of convergence of per-worker output predicted by the model is consistent with the evidence. Interestingly, we have found that the introduction of human capital makes the asymptotic speed of convergence much less sensitive to external shocks such

[28]Observed labor participation rates depend on the interval of age during which people can legally provide labor services. In our model, however, people can work all along their lives. The magnitudes shown by the data and by the predictions are therefore quite different. In order to facilitate visual comparison, we measure labor shares relative to their 1990 value. Another problem is that the actual evolution of the labor force share reflects other things than just movements between the production and schooling sectors, such as the increasing relative participation of women, etc. Unfortunately, solving this problem is no easy task.

as policy actions, which is consistent with Barro and Sala-i-Martin's (1995) result that estimated convergence speeds do not vary much across different region groups that belong to developed nations. But unlike the interpretation that the literature has assigned to Barro and Sala-i-Martin's finding, we can not conclude that policy actions have a small effect on the convergence speed, because non-scale growth frameworks deliver speeds of convergence that can vary over time. More importantly, we have shown that a model that delivers an asymptotic speed of convergence that complies better with empirical estimates does not necessarily provide a better description of the convergence process; a careful study of different adjustment paths starting far away from the balanced growth path is required to determine if this is the case.

Regarding this last point, we have shown that unlike the standard one-sector neoclassical growth model and the two-sector non-scale growth model, the framework presented in this paper is fairly successful in replicating the growth experiences of Japan and S. Korea, including important changes in the output growth-rate trend. Moreover, this is achieved by generating adjustment paths for interest rates, investment rates, and labor force shares that are in general agreement with observation.

Finally, we have shown that the hypothesis proposed in previous literature that the enhancing effect of human capital on technology-adoption is sufficient to reproduce the growth patterns shown by East Asian miracle countries does not necessarily hold in a more structural model. Our results imply that taking into account labor reallocations across sectors is crucial to replicating the Japanese and S. Korean experiences.

Our paper is not without limitations. The model predicts enrollment rates that are larger than their empirical counterparts. This suggests that the model predictions could be improved if the accumulation of human capital would not necessarily imply the transfer of resources from the final-output sector. Future research could introduce leisure in the utility function, or allow for home-production. Alternatively, we could permit human capital formation though learning-by-doing or on-the-job training. Another extension could consist of introducing different human capital technologies for final output and R&D labor, although further research is clearly necessary in determining the appropriate weights to be assigned to the effectiveness of human capital in different sectors.

In a general sense, we interpret our results as suggesting that a successful model of economic growth and development should include *both* technological progress and human capital accumulation as necessary engines, and the endogenous outcome of the economic system. It is shown that the value added from pursuing such model greatly exceeds the added complexity. In a more specific sense, our results suggest that the technology-human capital complementarity and the subsequent labor reallocation are crucial components in the making of miracles.

A Data Appendix

The data and programs used in this paper are available by the authors upon request.

- *Income (GDP), and investment rates* [Source: PWT 5.6]

Cross-country real GDP per worker (chain index), real GDP per capita (chain index), and real investment shares are taken from the Penn World Tables, Version 5.6 (PWT 5.6) as described in Summer and Heston (1991). All of the series are expressed in 1985 international prices. This data set is available on-line at: http://datacentre.chass.utoronto.ca/pwt/index.html.

- *Labor force* [Source: PWT 5.6]

The cross-country data set on the labor force is calculated from the GDP per capita and GDP per worker series. Worker for this variable is usually a census definition based on economically active population.

- *Physical capital stocks* [Source: STARS (World Bank), and PWT 5.6]

Physical capital comes from PWT 5.6. However, this data set reports physical capital starting in 1965. To obtain stocks from 1963 for S. Korea, and from 1960 for Japan, we used the growth rates implied by the STARS physical capital data to deflate the 1965 PWT 5.6 numbers.

- *Education* [Source: STARS (World Bank)]

Annual data on educational attainment are the sum of the average number of years of primary, secondary and tertiary education in labor force. These series were constructed from enrollment data using the perpetual inventory method, and they were adjusted for mortality, drop-out rates and grade repetition. For a detailed discussion on the sources and methodology used to build this data set see Nehru, Swanson, and Dubey (1995).

- *Interest rates* [Source: Christiano (1989)]

Real rates of return on physical capital for Japan are approximated using inflation-adjusted returns in the Japanese stock market. More specifically, Christiano (1989) adjusts nominal returns using the price deflator for personal consumption expenditure from the last quarter of the previous year to the last quarter of the current year, from data contained in *Annual Report on National Accounts*, and *Report on National Accounts from 1955 to 1969*. Both data sets were published in 1989 by the Economic Planning Agency in Japan.

References

Amsden A., "Diffusion and Development: The Late–Industrializing Model and Greater East Asia," *AEA Papers and Proceedings*, 81:282-286, 1991.

Baark, E., "The Accumulation of Technology: Capital Goods Production in Developing Countries Revisited," *World Development*, 19:903-914, 1991.

Barro, R.J. and Lee, J., "International Data on Educational Attainment: Updates and Implications," *working paper no. 42*, Center for International Development at Harvard University, April 2000.

Barro, R.J. and Sala-i-Martin, X., *Economic Growth*, Macmillan, New York, NY, 1995.

Bils, M. and Klenow, P., "Does Schooling Cause Growth?" *American Economic Review*, 90:1160-1183, 2000.

Caballero, R.J., and Jaffe, A.B., "How High are the Giants' Shoulders?," in Olivier Blanchard and Stanley Fischer,eds., *NBER Macroeconomics Annual*, Cambridge, MA: MIT Press, 15-74, 1993.

Caselli, F., and W. J. Coleman II, "Cross-Country Technology Diffusion: The Case of Computers," *American Economic Review: Papers and Proceedings*, 91:328-335, 2001.

Coe, D., and Helpman, E., "International R&D Spillovers," *European Economic Review*, 39:859-887, 1995.

Coe, D., Helpman, E. and Hoffmaister, A., "North-South R&D Spillovers," *Economic Journal*, 107:134-149, 1997.

Christiano, L., "Understanding Japan's Saving Rate: The Reconstruction Hypothesis," *Federal Reserve of Minneapolis Quarterly Review*, 2:10-29, 1989.

Dinopoulos, E. and Thompson P., "Schumpeterian Growth Without Scale Effects," *Journal of Economic Growth*, 3:313-337, 1998.

Easterly, W., and R. Levine, "Africa's Growth Tragedy: Policies and Ethnic Divisions," *Quarterly Journal of Economics*, 112:1202-1250, 1997.

Eicher, T., "Interaction Between Endogenous Human Capital and Technological Change," *Review of Economic Studies*, 63:127-144, 1996.

Eicher, T., and Turnovsky, S., "A Generalized Model of Economic Growth," *Economic Journal*, 109:394-415, 1999a.

Eicher, T., and Turnovsky, S., "Convergence in a Two-Sector Non-Scale Growth Model," *Journal of Economic Growth*, 4:413-429, 1999b.

Eicher, T., and Turnovsky, S., "Transition Dynamics in Non-Scale Models," *Journal of Economic Dynamics and Control*, 25:85-113, 2001.

Funke, M. and Strulik, H., "On Endogenous Growth with Physical Capital, Human Capital and Product Variety," *European Economic Review*, 44:491-515, 2000.

Gilchrist, S., and J.C. Williams, "Transitional Dynamics in Vintage Capital Models: Explaining the Postwar Catch-Up of Germany and Japan," *mimeo*, Boston University, 2001.

Griliches, Z., "Productivity Puzzles and R&D: Another Nonexplanation," *Journal of Economic Perspectives*, 2:9-21, 1988.

Hall, R. and Jones, C., "Why Do Some Countries Produce So Much More Output Per Worker Than Others?," *Quarterly Journal of Economics*, 114:83-116, 1999.

Howitt, P., "Steady-State Output Growth with Population and R&D Inputs Growing," *Journal of Political Economy*, 107:715-730, 1999.

Jones, C.I., "R&D-Based Models of Economic Growth," *Journal of Political Economy*, 103:759-784, 1995.

Jones, C.I., "Convergence Revisited," *Journal of Economic Growth*, 2:131-153, 1997.

Jones, C.I., "Sources of U.S. Economic Growth in a World of Ideas," *American Economic Review*, forthcoming.

Jones, C.I, and J. Williams, "Too Much of a Good Thing: The Economics of Investment in R&D," *Journal of Economic Growth*, 65-85, 2000.

Jovanovic, J. and Rob, R., "Solow vs. Solow," working paper, NYU, 1999.

Judd, K, "Projections Methods for Solving Aggregate Growth Models," *Journal of Economic Theory*, 58:410-452, 1992.

Keller, Wolfgang, "Absorptive capacity: On the creation and acquisition of technology in development," *Journal of Development Economics*, 49:199-227, 1996.

King, R. and Rebelo, S., "Transitional Dynamics and Economic Growth in the Neoclassical Model," *American Economic Review*, 83:908-931, 1993.

Klenow, P. and Rodríguez-Clare, A., "Economic Growth: A Review Essay," *Journal of Monetary Economics*, 40:597-617, 1997.

Kortum, S., "Equilibrium R&D and the Decline in the Patent-R&D Ratio; U.S. Evidence," *American Economic Review: Papers and Proceedings* 83, 450-457, 1993.

Krueger, A. and Lindahl, M., "Education for Growth: Why and for Whom?," *Journal of Economic Literature*, 39:1101-1136, 2001.

Lau, M-L, and Wan, H., "On the Mechanism of Catching Up," *European Economic Review* 38:952-963, 1994.

Lloyd-Ellis, H., and Roberts, J., "Twin Engines of Growth," *mimeo*, University of Toronto, 2000.

Lucas, R., "Making a Miracle," *Econometrica*, 61:251-272, 1993.

Minami, R., *The Economic Development of Japan*, St. Martin's Press, New York, 1994.

Mincer, J., *Schooling, Experience and Earnings*, Columbia University Press, 1974.

Mulligan C.B., and Sala-i-Martin, X., "Measuring Aggregate Human Capital," *Journal of Economic Growth*, 5:215-252, 2000.

Nelson, R. and Pack, H., "The Asian Miracle and Modern Growth Theory," *Economic Journal*, 109:416-436, 1999.

Nelson, R. and Phelps, E., "Investment in Humans, Technological Diffusion, and Economic Growth," *American Economic Review*, 56:69-83, 1966.

Nehru, V., Swanson, E. and Dubey, A., "A new database on human capital stocks in developing and industrial countries: sources, methodology and results," *Journal of Development Economics*, 46:379-401, 1995.

Ortigueira, S., and M. S. Santos, "On the Speed of Convergence in Endogenous Growth Models," *American Economic Review*, 87:383-399, 1997.

Ozawa, "Macroeconomic Factors Affecting Japan's Technology Inflows and Outflows: The Postwar Experience," in *International Technology Transfer: Concepts, Measures, and Comparisons*, edited by N. Rosenberg and C. Frischtak, Praeger Publishers, 1985.

Parente, S.L. and Prescott, E.C., "Barriers to Technology Adoption and Development." *Journal of Political Economy*, 102:298-321, 1994.

Perez-Sebastian, F., "Transitional Dynamics in an R&D-Based Growth Model with Imitation: Comparing its Predictions to the Data," *Journal of Monetary Economics*, 42:437-461, 2000.

Psacharopoulos, G., "Returns to Investment in Education: A Global Update," *World Development*, 22:1325-1343, 1994.

Restuccia, D., "Barriers to Capital Accumulation in a Model of Technology Adoption and Schooling," *working paper*, University of Toronto, 2001.

Romer, P., "Endogenous Technological Change," *Journal of Political Economy*, 98:S71-103, 1990.

Santos, M., "Accuracy of Numerical Solutions Using the Euler Equation Residuals," *Econometrica*, forthcoming.

Segerstrom, P. S., "Endogenous Growth Without Scale Effects," *American Economic Review*, 88:1290-1310, 1998.

Stokey, N., "Learning by Doing and the Introduction of New Goods," *Journal of Political Economy*, 99:145-176, 1988.

Summer, R. and Heston, A., "The Penn World Table (Mark 5): An Expanded Set of International Comparisons: 1950-1988," *Quarterly Journal of Economics*, 106:327-368, 1991.

Temple, J.R.W., "Robustness Test of the Augmented Solow Model," *Journal of Applied Econometrics*, 13:361-375, 1998.

Young, A. "Growth without Scale Effects," *Journal of Political Economy*, 106:41-63, 1998.