Computational Procedures for Input-Output Analysis: A Comment

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One of the more serious problems facing the input-output analyst is that of minimising the computational burden involved in the inversion of Leontief matrices. The article under review[1] describes the result of an exercise undertaken with a view to evolving procedures for determining total output levels, corresponding to a given "bill of goods", without inverting the detailed Leontief matrix.

Most of the short-cut procedures of this sort are based on certain simplifying assumptions about the character of technical co-efficients matrix, which in turn rest on the observed properties of the flow matrix. The simplifying assumptions commonly aim at transforming the Leontief model of general inter-dependence between industries into a model with a considerably reduced inter-dependence between groups (or clusters) of industries. Within a particular group of industries, however, the general inter-dependence is assumed to be maintained.

Using Dr. Ghosh's own notation, if there are two blocks of r and s industries in the economy the technical co-efficients matrix can be written in the following partitioned form.

$$A = \left(\begin{array}{c|c} A_{rr} & A_{rs} \\ \hline A_{sr} & A_{ss} \end{array} \right)$$

If A_{rs} and A_{sr} can both be equated to zero, then the matrix A becomes.

$$\left(\begin{array}{c|c} A_{rr} & O \\ \hline O & A_{ss} \end{array} \right)$$

In other words, the matrix has now become separable. It is no longer necessary to invert the whole matrix (I-A), but only to invert the two smaller matrices $(I-A_{rr})$ and $(I-A_{ss})$.

Another simplification arises when we assume that the real cost structures of all industries in a particular group are identical. If the last r industries such a group this implies that all the columns in the sub-matrix (A_{rs}) are identical. If this is the case, the block s is aggregable in the ordinary sense, and (A_{rs}) can be replaced by a single column of that sub-matrix and $(A_{sr} \mid A_{ss})$ by a row vector which is the sum of all the s row vectors. This simplification reduces the computational task from inverting a matrix of r + s order to that of inverting a matrix of r + 1 order only.

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[1]A Ghosh, "Input-Output Analysis with Substantially Independent Groups of Industries", *Econometrica*, 1960, Vol. 28, No. 1. pp. 88-96.

However, neither of the two simplified situations obtains in reality. Dr. Ghosh considers approximations to both the cases, *viz.*, the "separable" case and the "identical cost structure" case, by making certain simplified assumptions. He calls the approximation to Case I (*i.e.*, the separable case) the "partial equilibrium approach" and the approximation to Case II (*i.e.*, the identical cost structure case) the "aggregate approach". Dr. Ghosh's criterion for identifying the situations in which his two approaches may be applied is that the magnitude of the elements in, at least, one of the off-diagonal blocks—*e.g.*, A_{rs} —should be small. When this condition is satisfied, Dr. Ghosh terms the first r industries a "self-contained block" and the last s industries as forming the "non-selfcontained" block.

For the partial-equilibrium approach, the specific simplifying assumption made by Dr. Ghosh is that the total absorption of the products of an industry belonging to a self-contained block by all the industries outside such a block is proportional to the total output of the producing industry, rather than that of the consuming industry in the non-selfcontained block. Under this assumption, Dr. Ghosh is able to transform the original A matrix into the separable form.

In the aggregate approach, which involves the aggregation of all industries outside a self-contained block, Dr. Ghosh makes the specific assumption that the average cost structure represents the cost structure of all the industries in the non-self-contained block.[2]

The empirical part of Dr. Ghosh's paper consists of an experiment with a 50 order base year input-output table of the United Kingdom for 1948. First of all, Dr. Ghosh picks out clusters or groups of industries which can reasonably be called substantially independent or self-contained. The actual criterion used by Dr. Ghosh for identifying the independent or self-contained blocks is that between 80 and 90 per cent of the output of all sectors in a self-contained block is consumed within the block.

Ordinarily, all aggregation procedures result in certain error of estimation. The magnitude of this error is tested by comparing the inverses of the aggregated Leontief matrices with those of the original matrix. For example, if we denote the three co-efficients matrices used by Dr. Ghosh as follows,

A	A_s	A_a
(The 'ideal' detailed matrix)	(The 'partial equilibrium' matrix)	(The 'aggregate' matrix)

then A is the 'correct' technical co-efficients matrix, while A_s and A_a are matrices involving certain errors. The ordinary procedure of investigating the seriousness of the errors involved would consist of applying an ideal 'bill of goods', say Y, to the ideal Leontief inverse and the Leontief inverses of the error matrices and to compare the resulting vectors of total outputs[3]. In other words to compare X (the ideal total output vector) with X_s and X_a (the 'error' total output vectors),

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[2]. If $A_1, A_2 \ldots\ldots A_s$ are the s columns of $\begin{bmatrix} A_{rs} \\ \hline A_{ss} \end{bmatrix}$ then the column $\frac{I}{s}$ $(A_1+A_2+\ldots A_s)$ represents the average cost structure of the non-self contained block of s industries.

[3]. Since A_s and A_a do not have the same dimension as A, the ideal bill of goods Y should be adjusted to become 'conformable' with A_s and A_a.

where X, X_s and X_a are given by:

$$(1)\ X = (I-A)^{-1}Y$$
$$(2)\ X_s = (I-A_s)^{-1}Y$$
$$(3)\ X_a = (I-A_a)^{-1}Y$$

Perhaps to avoid the inversion of the Leontief matrices which the above procedure necessarily involves,[4] Dr. Ghosh reverses the procedure by applying the three Leontief matrices on a supposedly ideal total output vector, X, to arrive at three different 'bill of goods', say, Y, Y_s and Y_a, *i.e.*,

$$(4)\ Y = (I-A)X$$
$$(5)\ Y_s = (I-A_s)X$$
$$(6)\ Y_a = (I-A_a)X$$

where Y is the 'ideal' bill of goods and Y_s, Y_a are the bills of goods containing certain errors.

The rationale of the reverse procedure, although not explicitly mentioned by Dr. Ghosh, seems to be the following. Assuming the availability of the two smaller matrices, $(I-A_s)^{-1}$ and $(I-A_a)^{-1}$, equations (5) and (6) imply that in order to obtain the total output vector X, the "erroneous" bills of goods, Y_s and Y_a, have to be applied on the "erroneous" Leontief inverses, $(I-A_s)^{-1}$ and $(I-A_a)^{-1}$, respectively. In other words, if we start with an ideal bill of goods Y, it should be adjusted to become Y_s and Y_a before it can be applied to $(I-A_s)$, $(I-A_a)$ to obtain the correct total outputs X. Dr. Ghosh finds that, in fact, the three bills of goods Y, Y_s and Y_a, for the years 1949-55 for the United Kingdom are very nearly the same. The adjustment of Y is therefore unnecessary; X_s and X_a would nearly be the same as X, in (1) to (3) above.

The result of Dr. Ghosh's exercise turn out to be quite satisfactory and the three methods give close agreement. He also devises two co-efficients of variation of the estimates obtained by the "aggregate" and "partial" approaches. For the self-contained sectors, they turn out to be about 6 per cent, although for the non-self contained sectors these are much higher.

Dr. Ghosh has made a valuable contribution in a field where the analyst is always looking for more efficient computational procedures. However, as he himself points out, the methods are valid only so long as the basic clustering property of the table is maintained. If changes in technology are so rapid as to render a self-contained block non-self-contained, the two approaches can no longer be applied. Another limitation of such an approach is also obvious. It cannot be applied to tables of a highly aggregative nature. To that extent, the methods outlined in this paper are not very useful in an under-developed country, where the prime need is to improve the quality of the basic table, rather than to apply efficient computational procedures for the inversion of matrices.

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[4]. The reason actually given by Dr. Ghosh is that for the period of his forecasts, 1949-55, "we have firm estimates of total outputs, but not for final demands." However, this does not seem to be a legitimate reason, since the comparison was between the total, partial and aggregate approaches, and not with the actual outputs in the given period. Any reasonable estimate of the "bill of goods" could have served the purpose of the exercise.