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Evidence on Allocative Efficiency and Elasticities of Substitution in the Manufacturing Sector of Pakistan

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I. INTRODUCTION

Lack of effective competition in factor markets often produces allocative or price inefficiencies in the manufacturing sector of developing countries like Pakistan. Such inefficiencies are common due to distortion in factor markets leading to the use of inappropriate factor proportions Lau and Yotopoulos (1971, 1972), Yotopoulos and Lau (1973), Burki, *et al.* (1997), Khan (1998), Ahmed (1999), Zafar (2000). Pakistan is also one of the country where labour is abundant but capital and raw material are scarce. Our finding undermine estimates of elasticities of demand and substitution based on classical assumption that factor markets are perfectly competitive i.e. Kazi, *et al*. (1976), Kemal (1981), Battese and Malik (1987, 1988, 1993), Malik, *et al.* (1989), Mahmood (1989, 1992), Zahid, *et al*. (1992) and Khan and Rafiq (1993). In order to discuss the cost structure of the manufacturing sector we will estimate well behaved translog cost function.

II. THE MODEL

To estimate underlying technology one can use either production or associated cost function. The choice between them is a matter of statistical convenience. As a firm may not be able to have optimal combination of inputs due to imperfection in decision-making and imposition of distortionary government regulations. The role of these potential sources of misspecification in a firms behaviour slither introducing concept of shadow prices Lau and Yotopoulos (1971), Atkinson and Halvorsen (1980, 1984), Burki, *et al*. (1997), Burki and Mahmood (2004). The concept of shadow prices here after represented by $p_i^* = f\ (\tau_i, p_i)$. Where τ_i is inefficiency parameter. The firms dual total shadow cost function is defined as $c^s = c^s\ (\tau p, y)$.

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Where τp is vector of input specific shadow price. We can derive actual input demand function with the help of shadow cost function by applying Shepherd's lemma $\partial c^a/\partial \tau_i p_i = x$. The firm's total actual cost is given by $c^a = \sum_{i=1}^{3} p_i x_i$. *If* $\tau_i = \tau_i \forall_{ij}$ firm's total actual cost function reduces to total shadow cost function. Hicks own and cross price elasticities of demand for input *i* with respect to its market price turns out

$$\sigma_{ij}^{h} = k_i(k_i - 1) + \beta_{ii} / k_i \qquad \forall i \neq j$$

$$\sigma_{ij}^{h} = (k_i k_j + \beta_{ij}) k_i \qquad \forall i \neq j$$

The Allen partial elasticities of substitution turns out

$$\sigma_{ij}^{a} = (1 + \beta_{ij} / k_i \ k_j) \qquad \forall i \neq j$$

III. DATA

The Census of Manufacturing Industries (CMI) is the only major source of data on different aspects of manufacturing industries in Pakistan. All data in CMI are on aggregate level and on groups of industries. Most of the data are taken from its sixteen most recent publication (1969-71, 1970-71, 1975-76 through 1987-88 and 1990-91). Some supplementary information is collected from *Monthly Statistical Bulletin* and *Economic Survey of Pakistan*.

Value of production consist of value of finished products and by products receipts for repairs and maintenance, value of electricity sold, receipt for work done for others, value of the sale goods purchased for resale, wastes and used goods, the net increase in the value of working capital, and value of processed and fixed assets produced by establishment for its own use. Valuation is made at ex-factory prices, which include indirect taxes and exclude transport cost outside the factory gate.

To estimate unit labour cost we divide employment cost with average daily persons engaged. The data on employment cost and average daily persons engaged are given in Census of Manufacturing and Industries. The most appropriate price of capital for our purpose is user cost of capital calculated as follows $P_k = P_{k_{ind}}(r + \delta - \pi_{k_{ind}})$. Where p_k is user cost of capital and $p_{k_{ind}}$ is price index of capital goods *r* is real rate of interest δ is capital depreciation rate and $\pi_{k_{ind}}$ is defined as $\pi_{k_{ind}}(p_{k_{ind}t} - p_{k_{ind}t-1}) / p_{k_{ind}t-1}$. It is rate of growth in price index of capital. Thus user cost of capital increases with an increase in price of capital real rate of interest and capital depreciation. On the other hand user cost decreases with appreciation in value of capital due to increase in rate of growth in capital price. The price index of machinery is taken from Monthly Statistical Bulletin and is used as proxy for price index of capital both for Punjab and Sindh. The rate of interest is average schedule banks rate on long term advances for manufacturing sector. These information are available in Monthly Statistical Bulletin Depreciation rate is calculated by dividing total depreciation amount with value of fixed assets at the beginning of the year. Total depreciation amount and value of fixed assets at beginning of the year are available in CMI.

The quantity of capital is calculated by dividing value of capital by price index of machinery. Finally multiplying quantity of capital by user cost marks out total user cost of capital. Total cost is obtained by summing up value of capital stock total employment cost and value of raw material whereas input shares are obtained by dividing cost of each input with total costs.

We estimate parameters of translog cost function along with share equations in a system of equations. We use Iterative Zullner Efficient (IZEF) method for seemingly unrelated regression equations. Since shares satisfy adding up restriction it means all shares sum equal to one. To solve the problem of singularity one of these equations is dropped Christensen and Jorgenson (1969), Berndt and Christensen (1973), Barten (1969) showed maximum likelihood estimates are independent of the equation omitted. We drop share equation of raw material and recover its parameters with the help of adding up restrictions. Since IZEF estimation converages to Maximum Likelihood Estimate which are unique it follows that IZEF estimates are invariant to the choice of equation dropped.

IV. EMPIRICAL FINDINGS

Our results show that estimated cost shares at each point of data set are positive confirming monotonicity, while curvature condition also holds i.e. the mean shares of capital labour and raw material are 0.06, 0.09 and 0.85. We have tested homotheticity condition for translog cost model using the X^2 test. The calculated value of statistic is 74.99 which exceeds critical value at 5 percent level of significance. Thus homotheticity does not hold in the estimated model. It follows that underlying production function is not homogenous either. Parameters of translog cost are reported in Table 1. It can be seen that most of the parameter estimate are statistically different from zero at conventional level of significance. The coefficient of price output interaction variable β_{iy} interpret change in input intensity as level of output increase. It measures change in cost share of input *i* with respect to increase in output with price of inputs held constant. Value of β_{iy} would be (negative) positive if intensity of input *i* (decrease) increase with the level of output.

The estimated value of β_{ly} is –0.04 which is statistically different from zero, implying that intensity of labour is lower at higher levels of output. On the other hand β_{ky} is 0.01 and β_{my} is 0.03 showing that intensities of capital and raw material are higher at higher level of output however rate of increase in capital, intensity with

Table 1

Results of Translog Cost Function

	Shadow Cost		Actual Cost	
	Estimate	*t*-statistic	Estimate	t-statistic
α_o	–0.04287*	–1.99084	–0.17871**	–1.68223
α_k	0.054163*	9.03673	0.069541**	1.68227
α_l	0.113966*	34.775	0.254508*	3.65572
α_m	0.83187*	107.352	0.675951*	7.7462
β_{kk}	0.036264*	5.93955	0.038917	0.598841
β_{ll}	0.020921*	2.30363	0.063863*	2.22854
β_{mm}	0.038497*	2.60915	0.06632**	1.69725
β_{kl}	–9.34E-03*	–2.81302	–0.01823	–0.71517
β_{km}	–0.02692*	–3.2527	–0.02069	–0.49398
β_{lm}	–0.01158	–1.17085	–0.04563	–1.38047
β_y	0.798442*	14.3678	0.77331*	11.3929
β_{ky}	9.03E-03	1.26604	0.016801	0.662313
β_{ly}	–0.03866*	–5.30534	–0.10473*	–3.95765
β_{my}	0.029626*	2.59179	0.087926*	2.50531
β_{yy}	0.284274*	4.158	0.293844*	3.81874
τ_k			0.326372	0.547926
τ_m			0.248006*	2.27675
τ_l			1	

*Significant at 5 percent level of significance.
**Significant at 10 percent level of significance.

level of output is statistically insignificant. In any case this pattern of factor intensities confirms our result that underlying production structure is non-homothetic and there exist biases in use of factor inputs.

Allen and Hicks price elasticities of demand, based on estimates of translog cost share equations are calculated for each input pair and are shown in Table 3. The Allen elasticity of substitution for combination of capital and labour is negative, but the estimated value is statistically insignificant where as cross price elasticities $\sigma^h<0$ between labour and raw material and between capital and raw material.

The results of Hicksian cross price elasticities of factor demand are in agreement with the results of Allen partial elasticities of substitution. As one should expect the magnitudes of Hicksian elasticities are smaller than the corresponding Allen elasticities. This is a natural result as Hicksian cross-price elasticity of factor demand is obtained by Allen (1938) multiplying Allen partial elasticity of substitution with average input share that is a positive fraction. Thus effect of change in price of one input on demand for another input is high if cost share of first input is large. This explains for example, why elasticity of capital with respect to price of raw material is larger than elasticities of raw material with respect to capital.

Hicks own price elasticities are of correct sign $\sigma^h<0$ showing with an increase in price of an input the utilisation decreases. The results show that raw material and labour are substitutes. This means that when wage relative to price of raw material increase firms will increase raw material intensity relative to labour. This result is quite consistent with observed factor prices and factor intensities within our sample. Over the years unit labour cost has risen faster than users cost of raw material as a result raw material utilisation has increased substantially. Measure of Hicks elasticity of capital with respect to labour is –0.07 showing a complemetary relationship but the value is statistically insignificant and is in no harmony with factor prices and factor intensities within the sample.

The translog cost function along with share equations allows allocative inefficiency, estimated with iterative Zellner efficient technique. The function satisfies the monotonicity condition where as the curvature condition does not hold. The test of homotheticity is rejected implying that homotheticity does not hold either. The results of parameters along with their *t*-statistic are reported in Table 1. The results show that about half of the estimates are statistically different from zero at 5 percent level of significance. The estimated value of β_{ly} is –0.11 hence intensity of labour is lower at the higher levels of output. On the other hand β_{ky} is 0.02 and β_{my} is 0.09 showing intensities of capital and energy are higher at higher output levels. These results are qualitatively similar to ones obtained without allowing allocative inefficiency.

We attain relative price efficiency equalise $\tau_k = \tau_l = \tau_m$. The actual cost and cost shares are homogeneous of degree zero for all τ therefore we cannot estimate the values of τ for each input. The values shown in Table 2 exhibit that $\tau_k \neq \tau_m$ while all other inputs are relatively equally price inefficient.

Relative to labour raw material is inefficiently utilised. In particular raw material is over used in relation to labour. We also find that capital is inefficiently utilised as compared to labour and raw material but the *t*-statistic indicate that the extent of inefficiency is insignificant. This result is consistent with the findings of

Table 2

Relative Efficiency Test

Hypothesis	*t*-statistic	Ratios	Estimates
$\tau_l = \tau_k$	1.13	τ_l / τ_k	3.07
$\tau_l = \tau_m$	2.28	τ_l / τ_m	4.03*
$\tau_m = \tau_k$	0.13	τ_m / τ_k	0.08

*Significant at 5 percent level of significance.

Burk, *et al.* (1997) that capital and raw material are over utilised relative to labour. The effects of relative price inefficiency on cost of production can be evaluated by comparing actual total cost with the cost when relative price efficiency has been attained. The efficient level of cost is estimated by imposing restriction $\tau_k = \tau_m = 1$. A comparison with fitted total cost indicates that at the mean values of data over the period of our analysis relative price inefficiency increases total cost by 0.62 percent. It implies that allocative inefficiency increase cost of production or reduces profitability of production units beneath full potential.

The results of own and cross-price elasticities of demand at average values of variables along with *t*-statistics are shown in Table 3 we can observe that qualitative nature of our results are same as in shadow cost in particular signs of all elasticities

Table 3

Results of Elasticities of Substitution

	Shadow Cost		Actual Cost	
	Allen Elasticities			
	Estimate	*t*-statistic	Estimate	*t*-statistic
σ^a_{kl}	–0.74101**	–1.17445	–2.3764	–0.38798
σ^a_{km}	0.442952**	1.76429	0.57503	0.600408
σ^a_{lm}	0.85531*	5.58513	0.428584	0.960245
	Hicks Elasticities of Substitution			
σ^h_{kk}	0.30613*	–3.88458	–0.94263*	–55.2265
σ^h_{ll}	–0.6838*	–28.5823	–0.22733*	–1.96296
σ^h_{mm}	–0.10585*	–4.35944	–0.15149*	–4.79984
σ^h_{kl}	0.06985	–1.2833	–0.22366	–0.40998
σ^h_{lk}	–0.04219	–1.15216	–0.13799	–0.43703
σ^h_{km}	0.375977**	1.70158	0.487921	0.610938
σ^h_{mk}	0.025219	1.43918	0.032989	0.517381
σ^h_{lm}	0.725987*	4.83761	0.36366	0.956618
σ^h_{ml}	0.080626*	2.76234	0.040337	0.888283

*Significant at 5 percent level of significance.
**Significant at 10 percent level of significance.

are same as before. However in quantitative terms there is a significant change in the magnitudes of elasticities compared with results without controlling inefficiency. The absolute magnitude of the Allen elasticity has increased for capital and labour but the degree of complementaries $\sigma^a < 0$ has remained statistically insignificant. Like wise Hicks own elasticities of capital and labour have increased.

The cross-price elasticity also indicates an increase in magnitude for labour with respect to capital while that of raw materiel with respect to labour has decreased.

V. CONCLUSION

This study has been an attempt to investigate nature of allocative inefficiencies in Pakistan's large scale manufacturing sector using pooled provincial level time series data for Punjab and Sindh. The nature of allocative inefficiencies here after focus distortion effect in standard translog cost function. We estimated two models that are generated by appropriate adjustments in globally known translog cost function to explain substitutability of different inputs. In first model we simply take translog cost function without introducing allocative efficiency and then we include distortion parameters to represent allocative inefficiency in the cost structure.

The relative price efficiency between each pair of inputs provide evidence that raw material is over utilised as compared to labour while other inputs are equally efficiently utilised. It turns out capital and labour are complement in use while both of these inputs are substitutable with raw material. However complementarities or substitutability relationship are weak. Capital and labour are found to be complement in analysis where manufacturing sector is treated as a whole.

Hicks own price elasticities are of correct sign depict with increase in price of an input utilisation decrease further raw material and labour are substitutes. This means that when wage relative to price of raw material increase firm will increase raw material intensity. This result is quite consistent with observed factor prices and factor intensities within our sample. Further comparative analysis is useful to observe how estimate of substitution elasticities affect due to conventional assumption that firms at disaggregates are able to minimise cost in the light of observed input prices.

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