

The Pakistan Development Review
45 : 3 (Autumn 2006) pp. 369–381

Predictability in Stock Returns in an Emerging Market: Evidence from KSE 100 Stock Price Index

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We investigate the persistence in monthly KSE100 excess stock returns over the Treasury bills rates using non-Gaussian state space or unobservable component model with stable distributions and volatility persistence.

Results from our non-Gaussian state space model, which is an improvement over Conard and Kaul (1988), show that the conditional distribution has a stable α of 1.748 and normality is rejected even after accounting for GARCH. There exists a statistically significant predictable component in the KSE 100 excess stock returns. The optimal predictor in the unconditional expectation of the series is estimated to be 0.18 percent per annum. An evidence of highly non-constant scales in different periods of time exhibits a tendency towards stock market crashes which invites remedial policy action.

JEL classification: C22, C53, G14

Keywords: Stock Return Predictability; Unobserved Components; Fat Tails; Stable Distributions

1. INTRODUCTION

There has been a growing tendency on forecasting stock return predictability over time because stock return predictability, if it exists, can help attain large economic gains with suitable trading strategies. In his survey article, Fama (1991) demonstrates that predictability in stock returns has been explored extensively in the literature.

While there are a number of studies that include Summers (1986), Fama and French (1988), Lo and MacKinlay (1998), Poterba and Summers (1988) and Bailey, *et al.* (1990) that show evidence against random walk hypothesis in emerging markets, there are others that include studies by Divecha, Drach and Stefek (1992), and Wilcox (1992) that demonstrate that high volatility exists in stock returns.

A number of other parametric and non-parametric approaches have been employed to forecast predictability in stock returns but most of these approaches do not consider fat tails in return series that are widely documented in the literature. For example McQueen and Thorely (1991) used Markov chains to test stock returns predictability. However, in Markov chains the outcome from current period experiment is assumed to affect the outcome of the next period with some probability and so on. Similarly, in non-parametric approaches, for example, artificial neural networks seems to be a candidate approach to predict stock returns although neural networks are under

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heavy criticism for not having an explicit functional form (being black boxes) and overfitting¹ issues associated with them. However, in this study we are not merely working with stock return predictability, therefore, we feel that state space models encompassing a signal extraction approach might do a better job to extract predictable signals (if any) in return series while taking into account non-normality and time varying volatility that is well-documented in the literature.

While a number of techniques were used in the empirical literature to forecast stock return predictability, researchers focused on two aspects of empirical distribution of stock returns, which, they think, are important for accurate predictability. For example, Akgiray and Booth (1988), Jensen (1991), De Vries (1991), Buckel (1995), Mantegna and Stanley (1995), and McCulloch (1997) show existence of non-normality in stock returns. On the other hand, Nelson (1991) Danielsson (1994), Pagan and Schwert (1990), Diebold and Lopez (1995), and Goose and Kroner (1995) show evidence of volatility persistence in stock returns.

Conard and Kaul (1988) employed state space or unobservable component model for stock return predictability considering that shocks in both the observation and state equations are i.i.d. normal. Similarly, Harvey (1989) and Watson (1986) also used state space models with assumptions that the underlying errors are i.i.d. normal. However, McCulloch (1996a) and Bidarkota and McCulloch (2004) considered modeling stock returns to be non-Gaussian with fat tails because the empirical literature recommends using non-normality and volatility persistence in models that are employed for accurate forecasts of stock returns. One might employ either t-distributions or stable distribution to account for fat tails while modeling stock returns, however, following Bidarkota and McCulloch (2004) we employ stable distributions for technical reasons that will be elaborated in the later sections.

In this study, we employ state space or unobservable component model to investigate whether persistent predictable signal is present in Karachi Stock Exchange (KSE) 100 index monthly excess returns over the risk free rates (Treasury bill rates). In order to account for non-Gaussian data, we model returns within the framework of Parisian stable distributions that were also employed by Mantegna and Stanley (1995), Buckel (1995), and McCulloch (1997). Therefore, as in Oh (1994) and Bidarkota and McCulloch (1998) we relax the normality assumption in favour of stable distributions because the Kalman filter is not operable efficiently with stable distributions. Similarly, to explicitly account for volatility persistence in the return series we employ GARCH-like model.

The remaining paper is organised as follows. Section 2 outlines the most general model used in this paper and some estimation issues. In Section 3, we present data sources and empirical results and hypotheses tests. Finally Section 4 concludes the study.

¹Even with their tendency to overfit, artificial neural networks can be applied efficiently with adequate selection of neural network architecture [Kiani (2005)].

2. STATE SPACE MODEL FOR STOCK RETURNS

In this research we use six types of models. Model 1 is the most general model that encompasses unobservable component in stock returns including non-normal errors and GARCH-like effects. The most general unobserved component or state space model is shown in the following three Equations:

$$r_t = x_t + \varepsilon_t \quad \varepsilon_t \sim c_t z_{1t}, \quad z_{1t} \sim iid S_\alpha(0,1) \quad \dots \quad \dots \quad \dots \quad (1a)$$

$$(x_t - \mu) = \phi(x_{t-1} - \mu) + \eta_t \quad \eta_t \sim c_\eta c_t z_{2t} \\ z_{2t} \sim iid S_\alpha(0,1) \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (1b)$$

$$c_t^\alpha = \omega + \beta c_{t-1}^\alpha + \delta |r_{t-1} - E(r_{t-1} | r_1, \dots, r_{t-1})|^\alpha \\ + \gamma d_{t-1} |r_{t-1} - E(r_{t-1} | r_1, \dots, r_{t-1})|^\alpha \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (1c)$$

Where,

$$d_{t-1} = \begin{cases} 1 & \text{if } r_{t-1} - E(r_{t-1} | r_1, r_2, \dots, r_{t-2}) < 0 \\ 0 & \text{otherwise} \end{cases}$$

Here r_t is the observed one-period excess return, x_t is an unobserved persistence components in the series, and Z_1 , and Z_2 are independent white noise processes.

Model 2 is obtained restricting $\alpha=2$ in model 1 which is shown in Equations 2a and 2b.

$$r_t = x_t + \varepsilon_t, \quad \varepsilon_t \sim \sqrt{2}c_t z_{1t}, \quad z_{1t} \sim iid N(0,1) \quad \dots \quad \dots \quad \dots \quad \dots \quad (2a)$$

$$(x_t - \mu_t) = \phi(x_{t-1} - \mu) + \eta_t, \quad \eta_t \sim \sqrt{2}c_\eta c_t z_{2t}, \quad z_{2t} \sim iid N(0,1) \quad \dots \quad \dots \quad (2b)$$

$$c_t^2 = \omega + \beta c_{t-1}^2 + \delta |r_{t-1} - E(r_{t-1} | r_1, r_2, \dots, r_{t-2})|^2 + \gamma d_{t-1} \\ - E(r_{t-1} | r_1, r_2, \dots, r_{t-2})|^2 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (2c)$$

Setting $\beta = \delta = \gamma = 0$ in model 1, gives model 3 which is given in Equations 3a and 3b.

$$r_t = x_t + \varepsilon_t, \quad \varepsilon_t \sim S_\alpha(0,c) \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (3a)$$

$$(x_t - \mu) = \phi(x_{t-1} - \mu) + \eta_t, \quad \eta_t \sim S_\alpha(0, c_\eta, c) \quad \dots \quad \dots \quad \dots \quad \dots \quad (3b)$$

We restrict $\phi = 0$ in model 1 to obtain model 4. In this case the shocks ε_t and η_t are not separately identified so $c_\eta = 0$ resulting in model 4 that is shown in Equations 4a and 4b.

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t \sim c_t z_t, \quad z_t \sim iid S_\alpha(0,1) \quad \dots \quad \dots \quad \dots \quad \dots \quad (4a)$$

$$c_t^\alpha = \omega + \beta c_{t-1}^\alpha + \delta |r_{t-1} - \mu|^\alpha + \gamma d_{t-1} |r_{t-1} - \mu|^\alpha \quad \dots \quad \dots \quad \dots \quad \dots \quad (4b)$$

where,

$$d_{t-1} = \begin{cases} 1 & \text{if } r_{t-1} - \mu < 0 \\ 0 & \text{otherwise} \end{cases}$$

Model 5 is obtained restricting $\alpha = 2$ in model 5 which is presented in Equations 5a and 5b.

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t \sim \sqrt{2}c_t z_t, \quad z_t \sim iid N(0,1) \quad \dots \quad \dots \quad \dots \quad (5a)$$

$$c_t^2 = \omega + \beta c_{t-1}^2 + \delta |r_{t-1} - \mu|^2 + \gamma d_{t-1} |r_{t-1} - \mu|^2 \quad \dots \quad \dots \quad \dots \quad (5b)$$

Restricting $\beta = \delta = \gamma = 0$ in model 4 results in model 6 results that is shown in Equation 6.

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t \sim S_\alpha(0, c) \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (6)$$

A random variable x will have stable distribution $S_\alpha(0, c)$ when its log characteristic function can be represented as:

$$\ln E \exp(ixt) = i\delta t - |c_t|^\alpha \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (7)$$

The parameter $c > 0$ measures scale whereas the parameter $\delta(-\infty, \infty)$ measures location and $\alpha \in (0, 2)$ is the characteristic exponent that governs the tail behaviour. A small value of α indicates thicker tail and normal distribution pertaining to a symmetric stable family but when $\alpha = 2$ errors come from the normal family whose variance is equal to $2c^2$.

In the process contained in Equation 1c, we restrict $\omega > 0$, $\beta \geq 0$, $\delta \geq 0$, and $\gamma \geq 0$. The theoretical term involving dummy variable d_{t-1} captures leveraged effects that is transmitted from negative shock to increase in future volatility more than a positive shock of equal magnitude [Nelson (1991) and Hamilton and Susmel (1994)]. Abstracting from the threshold term, when the errors are normal, the model of volatility persistence reduces to GARCH-normal process.

Any predictable variation in excess return is because of persistent component x_t , which are assumed to follow a simple AR (1) process. When the predictable component in Equation 1 becomes significant, then $E(r_t | r_1, \dots, r_{t-1})$ provides a useful forecast of returns. However, when c_η and ϕ or one of these is negligible, the returns are purely random, so these may display spurious predictions.

2.1. Estimation Issues

Non-Gaussianity of the state space model shown in Equations 1a – 1c creates complication in estimation even without the presence of conditional heteroskedasticity. This happens because the Kalman filter is no longer optimal due to the non-Gaussian nature of shocks. In such situations, the general recursive-filtering algorithm due to Sorenson and Alspach (1971) works better and provides optimal filtering and predictive densities under any distribution for the errors and the formula for computing the log likelihood function. These formulas are shown in Appendix-A.

The recursive equation that is employed to compute filtering and predicting densities are given in the form of integrals whose close form analytical expressions are generally intractable, especially in very special cases. Therefore, in this study, these integrals are numerically evaluated.

Stable distribution and density may be evaluated using Zolotarev’s (1986) proper integral representation or by taking the inverse Fourier transformation of the characteristic function. However, McCulloch (1996a) developed a fast numerical approximation to stable distribution and density that has an expected relative density of the precision of 10^{-6} for $\alpha \in [0.84, 2]$. Therefore, we restrict α in this range for computational convenience.

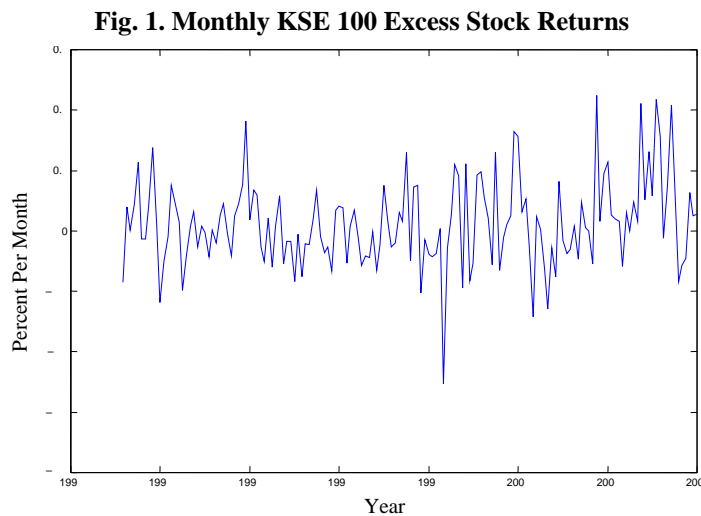
Lumsdaine (1996) shows that the effect of initial values in GARCH volatility process on the properties of the parameter estimates in GARCH (1,1) is asymptotically negligible. Diebold and Lopez (1995) suggests to set the initial conditional variance ($2c_0^2$, where it exists) equal to sample variance at the first iteration and the subsequent iterations to sample variance from simulated realisations with estimated parameters from the previous iterations. Engle and Bollerslev (1986) suggests initialising the GARCH process using unconditional estimates of c_0 obtained from the volatility process contained in Equation 1c.

3. EMPIRICAL RESULTS

3.1. Data Sources

We obtained KSE 100 index stock price data for Karachi Stock Exchange from DataStream and Treasury bill rates for Pakistan from September 2004 version of International Financial Statistic CD-ROM. The treasury bill rates are used as risk free rates that are used to calculate excess returns that are expressed as percent per month throughout the study. The data span for excess returns used in the study ranges from March 1991 to February 2004.

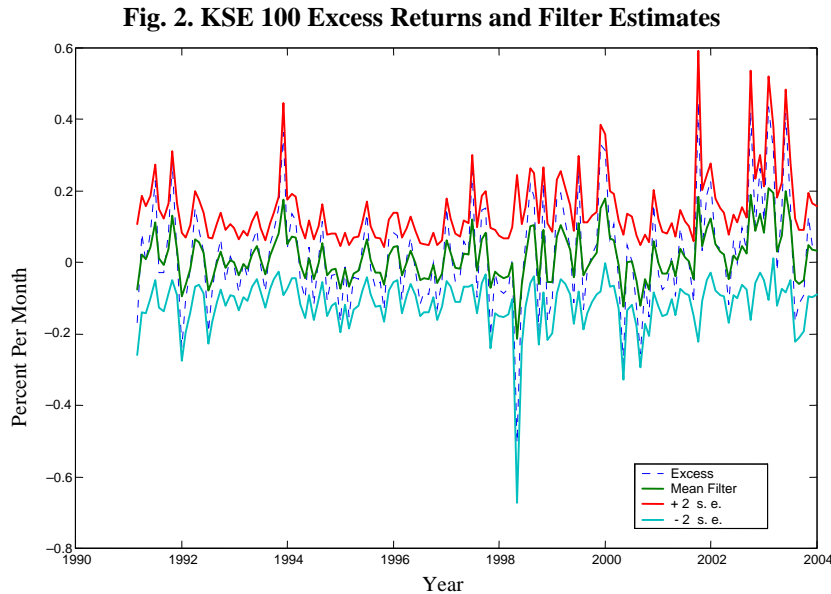
Figure 1 plots excess return series for KSE 100 index. These plots encourage us to employ state space or unobserved component model that is presented in Equations 1a – 1c for detecting possible persistence of predictable component in mean returns.



3.2. Estimation Results

Table 1 show estimation results for models 1 through 6 estimated for this study. Parameter estimates for characteristic exponent α , volatility persistence parameter β , the ARCH parameter δ , leverage parameter γ , signal to noise ratio c_η , and AR coefficient of persistent component for returns ϕ , respectively are 1.610, 0.000, 0.002, 18.170, 0.000, and 0.191.

Figure 2 illustrate mean of the filter density $E(x_t \mid r_1, r_2, r_3, \dots, r_r)$ which demonstrates that due to constant predictable component, any variation in its parameter estimates may be of a little importance in forecasting excess returns.



3.3. Hypotheses Test

In the following sub-sections, we elaborate the test for normality, volatility persistence, and persistence in mean returns. All the tests are based on likelihood ratio test.

3.3.1. Test for Normality

This test is based on the null of normality ($\alpha = 2$) in model 1. The LR test statistics for this test has non-standard distribution because the null hypothesis lies on the boundary of the admissible values for α , therefore, standard regularity conditions are not satisfied. Therefore, inferences are drawn from test statistics based on critical values due to McCulloch (1997).

Based on LR test statistics, the null hypothesis can easily be rejected at a significance level of better than 0.005 using critical values from McCulloch (1997). Consequently, even after accounting for GARCH-like behaviour, the excess returns are significantly non-normal.

Table 1

Estimates for non-Gaussian Space State Models and its Restricted Versions

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
α	1.610 (0.110)	2 (restricted)	1.749 (0.131)	1.748 (0.130)	2 (restricted)	1.748 (0.177)
μ	0.013 (0.013)	0.024 (0.016)	0.012 (0.002)	0.015 (0.011)	0.023 (0.011)	0.088 (0.008)
ω	0.000 (0.000)	0.007 (0.007)		0.014 (0.006)	0.010 (0.009)	
β	0.000 (0.000)	0.014 (1.079)		0.031 (0.468)	0.004 (0.946)	
δ	0.002 (0.005)	6.45e-13 (8.46e-10)		2.85e-13 (1.06e-10)	8.58e-13 (2.36e-9)	
γ	18.170 (5.916)	0.541 (0.442)		1.66e-9 (6.34e-7)	3.038e-10 (3.20e-7)	
c_η	6.25e-9 (6.77e-7)	1.25e-10 (1.65e-7)	0.086 (0.007)			
C			0.001 (0.003)			0.016 (0.012)
ϕ	0.191 (0.070)	0.613 (0.313)	0.156 (0.001)			
Log L	88.494	83.738	85.411	82.529	79.792	82.528
LR ($\alpha = 2$)	9.512			5.474		
LR ($\beta = \delta = \gamma = 0$)	6.166			0.002		
LR ($\phi = c_\eta = 0$)	11.930	2.418				

Notes: on Table 1.

1. The following unobserved component or state space model with non-normality (stable model) is employed to estimate the results shown in Table 1.

$$r_t = x_t + \varepsilon_t \quad \varepsilon_t \sim c_t z_{1t}, \quad z_{1t} \sim iid S_\alpha(0,1) \quad \dots \quad \dots \quad \dots \quad (1a)$$

$$(x_t - \mu_t) = \phi(x_{t-1} - \mu) + \eta_t, \quad \eta_t \sim c_\eta c z_{2t}, \quad z_{2t} \sim iid S_\alpha(0,1) \quad \dots \quad \dots \quad \dots \quad (1b)$$

- All estimates are rounded off to the third decimal place.
- Hessian-based standard errors for the parameter estimates are reported in parentheses. LR ($\phi = c_\eta = 0$) gives the value of the likelihood ratio test statistic. It is a test for no predictable components in excess returns. Under this null, the distribution of the LR test statistic is non-standard (see section 3.2 in the text for an elaboration).
- P-values generated by estimating Gaussian versions of Models 1 and 2 with data simulated from the estimated Gaussian Model 2 are reported in parentheses.
- LR ($\alpha = 2$) gives the value of the likelihood ratio test statistic for the null hypothesis of normality.
- The small-sample critical value at the 0.01 significance level for a sample size of 300 is reported to be 4.764 from simulations in McCulloch (1997).
- LR ($\beta = \delta = \gamma = 0$) is the test for no volatility persistence. The χ^2_3 p-values equal 2.5e-26.

3.3.2. Test for Volatility Persistence

The test for no volatility persistence (homoskedasticity) can be constructed restricting $\beta = \delta = \gamma = 0$ in the most general state space model shown in Equations 1a – 1c. Statistical inferences for this test are based on χ_3^2 distributions.

The LR for the null of no GARCH which is to test the restriction of $\beta = \delta = \gamma = 0$ that is reported in Table 1 showing that homoskedasticity is strongly rejected with χ_3^2 critical values.

3.3.3. Test for Persistence in Mean Returns

The null hypothesis for this test assumes that return series are random. The null is obtained by setting $\phi = 0$ in the most general state space model shown in Equations 1a – 1c. In this case the shocks ε_t and η_t are not identified separately, so $c_\eta = 0$.

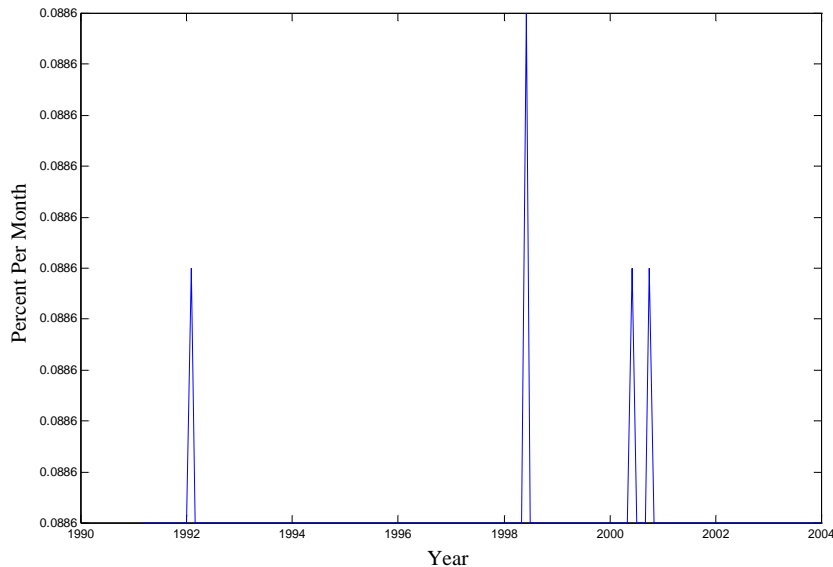
The standard likelihood ratio test statistics for this test are not applicable because the two shocks ε_t and η_t are not separately identified, so the scale ratio c_η is also not identified. Similarly the bound for the asymptotic distribution of a standardised likelihood ratio test statistics due to Hansen (1992) which is applicable in such cases may result in under-rejection of the null or a subsequent power loss as noticed by Hansen himself. In addition, the test statistics is computationally very intense especially in our case, so we abstain using it. Therefore, the inferences are drawn based on the critical values obtained from both χ_1^2 , and χ_2^2 distributions.

Based on the LR test statistics for null hypothesis for no predictable component ($\phi = c_\eta = 0$) is not rejected. Accordingly, after accounting for normality and volatility persistence, there exists a statistically significant evidence of persistence component in monthly KSE 100 Index excess returns.

3.4. Results on Additional Tests on Normality and Volatility Persistence

The test for non-normality and volatility persistence is repeated considering model 4 as alternative model. Needless to mention that model 4 is a version of the most general state space model that restricts predictable components ($\phi = c_\eta = 0$) in the general model. Using model 4 as an alternate model, we use model 5 as null model for testing non-normality and model 6 as null model for testing homoskedasticity. The rationale for the additional tests is to discover the effects of excluding predictable components from our most general model on the significance level of the tests for non-normality and volatility persistence.

LR test statistics for normality and volatility persistence are reported in the last two rows of column 5 in Table 1. Once again the hypothesis of normality and no volatility persistence are rejected. Figure 3 plots scales from model 4 which show the evidence of highly non-constant scales. Moreover, scales in monthly KSE100 index shows spikes in different time periods exhibiting a tendency towards stock market crashes. This should be an indication for the policy-makers to step ahead and take necessary policy measure for the stability of the major stock market of the country.

Fig. 3. Stock Returns Volatility-stable GARCH(1, 1) Model 4

3.5. Test for Leverage Effect

The leverage effect imply that negative shock do not necessarily lead to negative increase in future volatility more than the positive shocks of the same magnitude. This hypothesis can be tested setting $\gamma = 0$ for the null and $\gamma > 0$ for the alternate hypothesis showing that the leverage effect exists. The results (not reported for brevity) strongly reject the null hypothesis in favour of leverage effects.

3.6. Discussions on Results

Our results on hypothesis tests reveal that the monthly KSE100 index excess returns from March 1991 through February 2004 do posses significant non-normality that is predictable even after accounting for conditional heteroskedasticity. Similarly, volatility persistence is also statistically significant. The leverage effect in volatility is insignificant, however, there is an evidence of statistically significant predictable component in this market.

An evidence of highly non-constant scales in the monthly KSE100 index shows spikes in different time periods exhibiting a tendency towards stock market crashes. Our analysis reveals that the KSE 100 index was affected significantly is due to sudden external shocks. For example, in the year 1992, due to the Gulf war, in 1998 due to the economic sanctions on India and Pakistan, and in 2001-02 due to the incidence of September 11, 2001 and recession in the US economy. These shocks did not affect that drastically to S&P CNX 500 Indian stock price index.

The value of the characteristic exponent α for KSE 100 stock market excess returns equal 1.748 that is well below the value that would show normal behaviour in a market. The value of characteristic exponent for KSE 100 index is in line with

developed, transition, and emerging stock markets in the world. For example the value of characteristic exponent α for USA due to Bidarkota and McCulloch (2004) is 1.890. Similarly, the values of α due to Kiani and Bidarkota (2004) for Canada, France, Germany, Japan, UK, and USA are 1.645, 1.867, 1.748, 2.00, 1.999, 1.879, and 1.866 respectively.

Similarly, we can also compare the values of KSE 100 characteristic exponent α with that of the emerging markets of the world. The study results due to Kiani (2006) show that the value of characteristic exponent is 1.476 for Argentina, 1.694 for Brazil, 1.668 for Chile, 1.485 for Greece, 1.999 for India, 1.623 for Indonesia, 1.645 for Jordan, 1.820 for Malaysia, 1.806 for Mexico, 1.803 for Nigeria, 1.831 for Philippines, 1.494 for Portugal, 1.872 for Thailand, 1.759 for Turkey, and 1.612 for Venezuela. Moreover, the value of characteristic exponent α for KSE 100 index can also be compared with those transition economies. For example the value of the characteristic exponent is 1.649 for Croatia, 1.647 for Hungary, 1.749 for Latvia, 1.999 for Russia, 1.722 for Slovakia, and 1.659 for Ukraine. The results on characteristic exponent α in stock markets in transition economies are due to Kiani (2005).

The values of characteristic exponent from developed economies, transition economies, and emerging economies of the world show that most stock markets encompass non-normality. Our results from the KSE 100 stock price excess returns are in line with the results from most developed, transition, and emerging stock markets of the world. However, the results on characteristic exponent α for Germany, Italy, India, and Russia stock markets appear to be in sharp contrast showing normal behaviour in these markets.

4. CONCLUSION

In this study, we employ non-Gaussian state space or unobservable component model to find possible predictability in KSE100 index excess returns. Our state space models account for non-normality and volatility persistence that might be present in the series. The KSE100 index excess stock returns demonstrate significant leptokurtosis. The estimated value of characteristic exponent α is well away from value pertaining to normal behaviour. Similarly, excess stock returns exhibit persistence in stock return volatility that can be characterised by a GARCH-like process. In addition, there is an insignificant leverage effect in the stock return volatility indicating that the negative shocks lead to greater increases in future volatility than the positive shocks of equal magnitude.

Our results on predictability of monthly stock returns are statistically significant. Moreover, the stock returns encompass statistically significant predictable components. The efficiently estimated excess returns are 0.015 percent per month (0.18 percent per annum).

An evidence of highly non-constant scales in monthly KSE100 index shows spikes in different time periods exhibiting a tendency towards stock market crashes. Our analyses show that the KSE 100 index was affected significantly is due to sudden external shocks in the years 1992, 1998, and 2001-2. However, S&P CNX 500 Indian stock price index was not affected that severely with these sudden external shocks. This is an issue that policy-makers should seriously consider so that even non-constant scales should show a pattern avoiding chances of stock market crashes.

APPENDIX A

Sorenson-Alspach Filtering Equations

Let $y_t, t = 1, \dots, T$, be an observed time series and x_t an unobserved state variable, stochastically determining y_t . Denote $Y_t = \{y_1, \dots, y_t\}$. The recursive formulae for obtaining one-step ahead prediction and filtering densities, due to Sorenson and Alspach (1971), are as follows:

$$p(x_t | Y_{t-1}) = \int_{-\infty}^{\infty} p(x_t | x_{t-1}) p(x_{t-1} | Y_{t-1}) dx_{t-1}, \quad \dots \quad \dots \quad \dots \quad \dots \quad (A1)$$

$$p(x_t | Y_t) = p(y_t | x_t) p(x_t | Y_{t-1}) / p(y_t | Y_{t-1}), \quad \dots \quad \dots \quad \dots \quad \dots \quad (A2)$$

$$p(y_t | Y_{t-1}) = \int_{-\infty}^{\infty} p(y_t | x_t) p(x_t | Y_{t-1}) dx_t. \quad \dots \quad \dots \quad \dots \quad \dots \quad (A3)$$

Finally, the log-likelihood function is given by:

$$\log p(y_1, \dots, y_T) = \sum_{t=1}^T \log p(y_t | Y_{t-1}). \quad \dots \quad \dots \quad \dots \quad \dots \quad (A4)$$

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