# Predicting Money Multiplier in Pakistan 

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#### Abstract

The paper has developed time-series models for the monthly money multiplier and its components, viz., currency-deposit ratio, reserve-deposit ratio, etc. A comparison is made between the predictive performance of the aggregate multiplier and the component models. It is found that the projected values of the multiplier on the basis of the aggregate model are closer to actual values as compared to those worked out on the basis of the component models. Thus, for the purposes of projecting the money multiplier, it may be preferable to focus on the aggregate money multiplier model. Stability tests, applied to the identified models for each component and the overall multiplier, suggest that all the models are stable.


## 1. INTRODUCTION

The recent liberalisation and restructuring of the financial system in Pakistan has necessitated designing a new set of instruments to conduct monetary policy. Of these instruments the most important is the Open Market Operations (OMO). Through Open Market Operations, the State Bank brings about changes in the monetary base in accordance with the money supply target and the expected money multiplier. The success of the OMO in keeping the money supply within targets depends to a great extent on the accuracy of the estimated money multiplier. The objective of this paper is to develop a forecasting model of money multiplier for Pakistan following the Box-Jenkins (1976) time-series methodology. Money multiplier can be predicted by using two approaches, viz., (a) modelling the overall money multiplier, and (b) first modelling components of the multiplier and then estimating the overall money multiplier on the basis of predicted components. In the paper, both approaches have been adopted and compared with respect to their forecast abilities. It is found that the model for the overall multiplier performs better than the components models.

The paper has been organised as follows. Section 2 contains a brief review of the existing literature on the subject. Section 3 defines the money multiplier and

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Author's Note: The views expressed in this paper are the author's personal opinion of the usual disclaimer applies.
its components in the context of Pakistan, and applies tests of stationarity to the series. Section 4 explores appropriate ARMA models for the monthly money multiplier and its components. In Section 5, diagnostic checking of the models has been undertaken. Stability and forecast abilities of the aggregate and component models have been examined in Section 6. The last section contains the concluding remarks.

## 2. REVIEW OF LITERATURE

One of the first studies using time-series models to analyse the money multiplier was undertaken by Bomhoff (1977). He used the time-series technique for the United States and the Netherlands. Büttler et al. (1979) and Fratianni and Nabli (1979) also used this technique to forecast the money multiplier in Switzerland and in seven EEC countries respectively. Johannes and Rasche (1979) further extended the time-series approach to the components of money multiplier. They claimed that the predictive performance of this dis-aggregated model was superior to the aggregate model. When Hafer and Hein (1984) tested this claim, they found that the gain in terms of forecast accuracy from the component procedure was not significant. In the context of Pakistan, Hamdani (1976) used the money multiplier as a determinant of money supply. Mangla and Ladenson (1978) and Siddique and Ahmad (1994) developed different models for the projection of the money multiplier. However, both these studies used M1-multiplier that was little relevant for the State Bank of Pakistan from the policy point of view because monetary policy in Pakistan focused on broad money (M2). The present paper models the money multiplier (both aggregate- and component-wise) based on $M 2$ for forecasting purposes by using the latest available data.

## 3. DEFINING MONEY MULTIPLIER

Money multiplier is defined as a ratio of monetary aggregates (M2 in Pakistan) to reserve money, i.e.,

$$
\begin{equation*}
m_{t}=M 2_{t} / R M_{t} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \tag{1}
\end{equation*}
$$

where ' $m_{t}$ ' is the money multiplier, $M 2_{\mathrm{t}}$ is monetary aggregate, and $R M_{t}$ is the reserve money or the monetary base. Money multiplier can also be defined in terms of components of $M 2$ and $R M^{1}$ —all expressed as ratios to deposits as given below;

$$
\begin{equation*}
m_{t}=\frac{1+c_{t}+o_{t}}{c_{t}+k_{t}+r_{t}+o_{t}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \tag{2}
\end{equation*}
$$

${ }^{1}$ In Pakistan, M2 includes currency in circulation, deposits (demand, time, and foreign currency) with the scheduled banks, and other deposits with the SBP. Reserve Money is the sum of currency in circulation, cash in the tills, reserves of the scheduled banks with the SBP, and other deposits with the SBP.
where
$c_{t}=$ Currency/ Deposit. ${ }^{2}$
$o_{t}=$ Other deposits with the State Bank/Deposit.
$k_{t}=$ Cash in the tills of scheduled banks/Deposit.
$r_{t}=$ Reserves of the scheduled banks with the State Bank/Deposit.

Monthly series of money multiplier and components ratios from July 1989 to June 1999 are presented in Figure 1. Data for all these variables have been extracted from various issues of the State Bank's Annual Report. The overall money multiplier shows volatility, with some upward jumps over the sample period. The first upward jump by the money multiplier may be noted in 1992, soon after the introduction of residents’ foreign currency deposits. Residents’ foreign currency deposits not only discouraged currency holdings on the part of the public but also accelerated the process of deposit creation on the part of commercial banks, as banks often used these deposits as collateral to give advances to their depositors. This two-fold impact of RFCDs reduced the currency-to-deposit ratio significantly and thus increased the money multiplier. The other contributory factor for a jump in the money multiplier was a reduction in the required reserve ratio back to 5 percent from 13 percent by the State Bank in January 1992. The next significant jump in the money multiplier and a coinciding reduction in the currency/deposit ratio may be observed in 1996. In fact, mid-1990s is the period when Pak rupee was devalued frequently, thereby increasing the opportunity cost of currency holding. Particularly, during 1996, devaluation of more than 11 percent was undertaken within a month or so (September 10-October 22). Associated with devaluation, inflation during this period was about 12 percent, which fuelled dollarisation of the economy. As a result, the currency/deposit ratio declined and the money multiplier increased. Again the reduction of the reserve requirement on Ist July, 1996 also contributed to the upward jump in the money multiplier. During 1990s, the reserve requirement was changed quite frequently, which also caused fluctuation in the money multiplier. Since 1968, the reserve requirement had been 5 percent of the demand and time liabilities of commercial banks. In October 1991, it was raised from 5 to 13 percent ( 5 percent without interest, and 7 percent remunerable at the rate of 10 percent). As mentioned above, in January 1992, it was again fixed at 5 percent. During 1995, it was changed three times, in the range of 5 to 8.5 percent. Effective June 1998, it was fixed at 3.75 percent for rupee liabilities and 5 percent for FCDs of the banks. The reserve ratio as depicted in Figure 1 gives a combined impact of required reserves and excess reserves of banks. If the impact of required reserves is filtred out, the excess reserves-to-deposit ratio has a declining trend over time (not shown
${ }^{2}$ Deposits include demand deposits, time deposits, and residents' foreign currency deposits with the scheduled banks. These deposits are the base of the Statutory Reserve Requirement.


Fig. 1. Trend in Money Multiplier and Its Components.
separately). The ratio of cash in the tills remained fairly stable, except a few outliers during early 1995. The Other deposits with the SBP includes the State Bank employees’ G.P. Fund, Staff Welfare Fund, sundry deposits account, etc. Therefore, any behavioural attributes cannot be associated with this variable.

A quantitative evaluation of the properties of these series may be made by using the Augmented Dicky-Fuller (ADF) test. In fact, the first step in modelling time-series is to test the stationarity of the series by applying the ADF test and to find some stationary transformations if the original series are non-stationary.

We have applied the following Augmented Dicky-Fuller test to each series.

$$
\begin{equation*}
\Delta z_{t}=\alpha+\beta z_{t-1}+\gamma_{i} \sum_{1}^{n} \Delta z_{t-i}+\varepsilon_{t} \quad \ldots \quad \ldots \quad \ldots \tag{3}
\end{equation*}
$$

where $\Delta$ indicates the first difference, $z_{t}$ is any series of $m_{t}$ or its components. A trend variable may also be included in the above equation to test trend-stationarity of a series. The null hypothesis for the ADF test is the following;

$$
H_{0}: \beta=0
$$

The series $z_{t}$ is non-stationary if $H_{0}$ is accepted. We have used 5 percent Mckinnon critical values for testing the hypothesis. If a series is found non-stationary in its level form, the same test is applied at its first or higher-order difference to examine its difference-stationarity. The results of the ADF tests are given in Table 1, which shows that the money multiplier and currency ratios are non-stationary in their level forms. They become stationary in the first differences. Thus the ARMA models for their first differences have been developed. ${ }^{3}$ The models of Other Deposit Ratio, Cash in the Tills Ratio, and Reserves Ratio are constructed on their level forms because they are level stationary series.

Table 1
Results of the ADF Test on Level and First Difference of the Series
(Sample Size: 1989:07 1999:06)

| Series Name | Level Stationary | Trend Stationary | Difference Stationary |
| :--- | :---: | :---: | :---: |
| $m_{t}$ | No | No | Yes |
| $c_{t}$ | No | Yes | Yes |
| $o_{t}$ | Yes | No | Yes |
| $k_{t}$ | Yes | Yes | Yes |
| $r_{t}$ | Yes | Yes | Yes |

Notes: (i) In case of $k_{t}$ six outliers (from 1994:12 to 1995:05) have been excluded from the sample.
(ii) Computer software 'Econometric Views 3.1 has been used for the application of tests.

[^0]
## 4. IDENTIFICATION OF ARMA MODELS

To determine the order of Autoregressive (AR) and/or Moving average (MA), use is made of autocorrelation function (AC) and partial autocorrelation function (PAC) of the series. Both these functions for the first differences of $m_{t}$, and $c_{t}$, and level forms of $o_{t}, k_{t}$ and $r_{t}$ are given in the Annexure. The results show that for all variables, both AC and PAC are significant (on the basis of QStatistic) at almost all lags included in the functions. An appropriate ARMA model should have to include all ACs/PACs that are significant. We have applied various-order mix of AR and MA terms on each series and selected relatively better ARMA model in terms of Akaike Information Criterion (AIC). In order to ensure the principle of parsimony we have considered only those parameters which are significantly different from zero at least 95 percent confidence level. However, in some models, insignificant intercepts have been allowed. We have found that the following ARMA models are good approximations of true datagenerating processes;
(i) $\left(1-\alpha_{1} L-\alpha_{2} L^{8}-\alpha_{3} L^{36}\right)(1-L) m_{t}=\alpha_{0}+\left(1+\beta_{1} L^{31}\right) \varepsilon_{t}$
(ii) $\left(1-\delta_{1} L^{12}\right)(1-L) c_{t}=\delta_{0}+\left(1+\eta_{1} L+\eta_{2} L^{5}+\eta_{3} L^{9}+\eta_{4} L^{12}\right) \varepsilon_{t}$
(iii) $\left(1-\phi_{1} L\right) \mathrm{o}_{t}=\phi_{0}+\left(1+\pi_{1} L\right) \varepsilon_{t}$
(iv) $\left(1-\theta_{1} L\right) k_{t}=\theta_{0}+\left(1+\omega_{1} L^{1}+\omega_{2} L^{3}\right) \varepsilon_{t}$
(v) $\left(1-\rho_{1} L-\rho_{2} L^{2}-\rho_{3} L^{3}\right) r_{t}=\rho_{0}+\left(1+\sigma_{1} L+\sigma_{2} L^{2}\right) \varepsilon_{t} \quad \ldots \quad \ldots$
where $L$ is lag operator, such that $L^{i} z_{t}=z_{t-1}$.
The estimates of parameters are given in Table 2.
Other characteristics of the equations are given below:
(i) $\mathrm{R}^{2}=0.59, \mathrm{DW}=1.9$, Akaike Information Criterion $=-2.40$, F -statistic $=27.5$
(ii) $\mathrm{R}^{2}=0.63, \mathrm{DW}=2.0$, Akaike Information Criterion $=-5.19$, F -statistic $=39.4$
(iii) $\mathrm{R}^{2}=0.74$, $\mathrm{DW}=1.9$, Akaike Information Criterion $=-9.27$, F-statistic $=169.5$
(iv) $\mathrm{R}^{2}=0.65$, $\mathrm{DW}=1.9$, Akaike Information Criterion $=-9.60$, F -statistic $=68.5$
(v) $\mathrm{R}^{2}=0.55, \mathrm{DW}=2.0$, Akaike Information Criterion $=-5.56$, F -statistic $=27.8$

It may be noted that the selection of models has been made on the basis of the Akaike Information Criterion by keeping it as small as possible. Any attempt to include more lags as regressors in order to improve R $^{2}$ will increase the AIC, making the model less attractive. In fact, in the case of the ARMA models, the most appropriate measure of the overall "goodness of fit" is the Akaike Information Criterion or the Schwarz Bayesian Criterion instead of $\mathrm{R}^{2}$.

Table 2
Estimates of Parameters
(Sample Size: 1989:07 1999:06)

| Parameter | Estimate | $t$-value |
| :--- | ---: | ---: |
| Equation - i: |  |  |
| $\alpha_{0}$ | 0.013 | 3.234 |
| $\alpha_{1}$ | -0.188 | -2.514 |
| $\alpha_{2}$ | -0.232 | -3.161 |
| $\alpha_{3}$ | 0.252 | 3.755 |
| $\beta_{1}$ | -0.731 | -2974 |
| Equation - ii: | -0.001 |  |
| $\delta_{0}$ | 0.876 | -0.155 |
| $\delta_{1}$ | -0.200 | 22.167 |
| $\eta_{1}$ | -0.017 | -2.534 |
| $\eta_{2}$ | -0.143 | -35.870 |
| $\eta_{3}$ | -0.552 | -2.102 |
| $\eta_{4}$ |  | -6.765 |
| Equation - iii: | 0.002 |  |
| $\phi_{0}$ | 0.799 | 7.766 |
| $\phi_{1}$ | 0.193 | 12.991 |
| $\pi_{1}$ |  | 1.881 |
| Equation - iv: | 0.003 |  |
| $\theta_{0}$ | 0.873 | 2.214 |
| $\theta_{1}$ | -0.522 | 14.657 |
| $\omega_{1}$ | 0.373 | -5.909 |
| $\omega_{2}$ |  | 4.542 |
| Equation - v: | 0.055 |  |
| $\rho_{0}$ | -0.448 | 9.229 |
| $\rho_{1}$ | 0.458 | -1.953 |
| $\rho_{2}$ | 0.454 | 3.378 |
| $\rho_{3}$ | 0.951 | 3.204 |
| $\sigma_{1}$ | 0.424 | 4.050 |
| $\sigma_{2}$ |  | 2.730 |

Notes: Inverted roots of AR and MA terms in all models are less than one.

## 5. DIAGNOSTIC CHECKING

Diagnostic checking is also an important ingredient of the Box-Jenkins methodology. In fact, identification and diagnostic checking run parallel in the process of selecting an appropriate ARMA model. The standard practice is to see if the residuals estimated from a particular model are white noise; if they are, then the model is acceptable; if not, it may be re-specified and re-estimated. In the present case, residuals from all the models estimated above are white noise as determined by the Autocorrelation and Partial Autocorrelation functions. As Table 3 shows, none of the autocorrelations and partial autocorrelations is individually significantly different from zero on the basis of Q -statistic. The probability of Q -statistic of all the ACs/PACs (not shown) remained below 95 percent confidence level.

Table 3
Autocorrelation and Partial Autocorrelation Functions of Residuals

| Lags | Equation-i |  | Equation-ii |  | Equation-iii |  | Equation-iv |  | Equation-v |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | AC | PAC | AC | PAC | AC | PAC | AC | PAC | AC | PAC |
| 1 | -0.06 | -0.06 | 0.00 | 0.00 | -0.03 | -0.03 | 0.00 | 0.00 | -0.01 | -0.01 |
| 2 | -0.03 | -0.04 | -0.06 | -0.06 | 0.11 | 0.11 | 0.05 | 0.05 | 0.01 | 0.01 |
| 3 | -0.04 | 0.04 | 0.07 | 0.07 | 0.00 | 0.01 | 0.04 | 0.04 | -0.01 | -0.01 |
| 4 | -0.05 | -0.06 | -0.07 | -0.08 | 0.04 | 0.03 | 0.04 | 0.04 | -0.01 | -0.11 |
| 5 | -0.05 | 0.06 | -0.01 | 0.00 | 0.03 | 0.03 | -0.04 | -0.04 | -0.08 | -0.08 |
| 6 | -0.05 | -0.06 | 0.06 | 0.04 | -0.01 | -0.01 | 0.05 | 0.05 | 0.10 | 0.10 |
| 7 | 0.01 | -0.01 | -0.01 | -0.01 | -0.03 | 0.04 | -0.09 | -0.19 | 0.01 | 0.02 |
| 8 | -0.06 | -0.08 | -0.03 | -0.03 | 0.08 | 0.08 | 0.01 | 0.01 | 0.01 | 0.00 |
| 9 | 0.06 | 0.04 | -0.02 | -0.02 | -0.02 | -0.01 | -0.04 | -0.02 | 0.06 | 0.03 |
| 10 | -0.05 | -0.06 | -0.05 | -0.05 | 0.04 | 0.03 | -0.03 | -0.02 | -0.04 | -0.02 |
| 11 | -0.09 | -0.10 | 0.03 | 0.03 | -0.05 | -0.03 | -0.01 | 0.01 | -0.04 | 0.02 |
| 12 | 0.01 | -0.02 | 0.09 | 0.08 | 0.04 | 0.01 | 0.00 | -0.01 | -0.07 | -0.05 |
| 13 | 0.04 | 0.02 | 0.03 | 0.04 | 0.06 | 0.07 | 0.10 | 0.13 | 0.12 | 0.09 |
| 14 | -0.02 | -0.03 | 0.00 | -0.10 | -0.09 | -0.12 | 0.02 | -0.03 | 0.05 | 0.01 |
| 15 | 0.02 | 0.01 | -0.10 | -0.10 | 0.04 | 0.03 | -0.07 | -0.07 | -0.04 | -0.07 |
| 16 | 0.02 | 0.00 | -0.02 | -0.02 | -0.02 | -0.01 | 0.07 | 0.06 | 0.09 | 0.05 |
| 17 | -0.09 | -0.10 | 0.03 | 0.04 | 0.02 | 0.01 | 0.02 | 0.01 | -0.11 | -0.08 |
| 18 | -0.07 | -0.10 | -0.01 | -0.02 | 0.08 | 0.07 | 0.02 | 0.02 | -0.06 | -0.03 |
| 19 | 0.07 | 0.05 | -0.05 | -0.06 | -0.05 | -0.03 | 0.04 | 0.02 | 0.05 | 0.04 |
| 20 | 0.07 | 0.07 | 0.05 | 0.06 | 0.10 | 0.06 | 0.03 | 0.07 | 0.00 | -0.05 |
| 21 | 0.03 | 0.03 | -0.03 | -0.12 | -0.08 | -0.08 | 0.01 | 0.01 | 0.03 | 0.04 |
| 22 | -0.10 | -0.02 | -0.01 | 0.01 | 0.08 | 0.03 | 0.02 | -0.01 | -0.03 | -0.02 |
| 23 | 0.05 | 0.04 | 0.10 | 0.06 | -0.01 | 0.00 | -0.03 | -0.01 | 0.07 | 0.07 |
| 24 | -0.04 | -0.03 | -0.09 | -0.10 | -0.08 | -0.01 | 0.01 | 0.00 | 0.02 | 0.09 |
| 25 | 0.06 | 0.05 | -0.11 | -0.13 | 0.07 | 0.05 | 0.03 | 0.03 | 0.01 | 0.02 |
| 26 | -0.10 | -0.10 | 0.08 | -0.09 | -0.04 | -0.10 | -0.05 | -0.06 | -0.12 | -0.16 |
| 27 | 0.02 | 0.01 | 0.07 | 0.13 | -0.04 | -0.04 | 0.00 | 0.02 | 0.18 | 0.19 |
| 28 | 0.02 | 0.02 | 0.01 | 0.00 | 0.04 | 0.05 | -0.04 | -0.13 | -0.11 | -0.07 |
| 29 | -0.08 | -0.10 | -0.01 | -0.06 | 0.07 | 0.11 | -0.07 | -0.18 | -0.07 | -0.10 |
| 30 | -0.02 | -0.04 | 0.09 | 0.06 | 0.00 | -0.07 | -0.02 | -0.02 | 0.08 | 0.02 |
| 31 | -0.08 | -0.07 | -0.10 | -0.10 | 0.04 | 0.07 | -0.07 | -0.05 | -0.08 | -0.10 |
| 32 | 0.09 | 0.05 | -0.03 | -0.01 | -0.13 | -0.19 | -0.01 | 0.03 | 0.09 | 0.09 |
| 33 | 0.03 | 0.02 | 0.05 | 0.03 | 0.01 | -0.03 | -0.04 | -0.07 | 0.06 | 0.08 |
| 34 | 0.01 | -0.03 | -0.01 | 0.00 | -0.10 | 0.01 | -0.11 | -0.10 | 0.02 | -0.06 |
| 35 | -0.02 | -0.04 | 0.16 | 0.10 | 0.03 | 0.08 | 0.07 | 0.06 | -0.13 | -0.12 |
| 36 | -0.07 | -0.10 | -0.05 | -0.11 | -0.02 | -0.11 | 0.04 | -0.02 | 0.03 | 0.05 |

## 6. STABILITY AND FORECAST ABILITY

To evaluate the forecasting ability, the above models were re-estimated with a truncated sample size by dropping the last 12 observations. The re-estimated parameters are given in Table 4.

Table 4
Re-estimated Parameters
(Sample Size: 1989:07 1998:06)

| Parameter | Estimate | $t$-value |
| :---: | :---: | :---: |
| Equation - i: |  |  |
| $\alpha_{0}$ | 0.013 | 2.483 |
| $\alpha_{1}$ | -0.178 | -2.215 |
| $\alpha_{2}$ | -0.189 | -2.410 |
| $\alpha_{3}$ | 0.295 | 3.834 |
| $\beta_{1}$ | -0.731 | -2884 |
| Equation -ii: |  |  |
| $\delta_{0}$ | -0.001 | -8.200 |
| $\delta_{1}$ | 0.857 | 20.309 |
| $\eta_{1}$ | -0.212 | -2.496 |
| $\eta_{2}$ | -0.020 | -0.225 |
| $\eta_{3}$ | -0.159 | -279.6 |
| $\eta_{4}$ | -0.521 | -5.512 |
| Equation - iii: |  |  |
| $\phi_{0}$ | 0.002 | 7.521 |
| $\phi_{1}$ | 0.790 | 11.838 |
| $\pi_{1}$ | 0.199 | 1.838 |
| Equation -iv: |  |  |
| $\theta_{0}$ | 0.005 | 4.756 |
| $\theta_{1}$ | 0.801 | 9.678 |
| $\omega_{1}$ | -0.470 | -4.601 |
| $\omega_{2}$ | 0.426 | 4.774 |
| Equation - v: |  |  |
| $\rho_{0}$ | 0.058 | 9.316 |
| $\rho_{1}$ | -0.441 | -1.912 |
| $\rho_{2}$ | 0.426 | 2.971 |
| $\rho_{3}$ | 0.451 | 3.096 |
| $\sigma_{1}$ | 0.950 | 4.067 |
| $\sigma_{2}$ | 0.456 | 2.875 |
| $\boldsymbol{y}$ : |  |  |

Note: Econometric Views 3.1 is used for estimation.
Other characteristics of the equations are given below:
(i) $\mathrm{R}^{2}=0.58$, $\mathrm{DW}=1.9$, Akaike Information Criterion $=-2.30$, F -statistic $=22.5$
(ii) $\mathrm{R}^{2}=0.63$, $\mathrm{DW}=2.0$, Akaike Information Criterion $=-5.10$, F -statistic $=35.1$
(iii) $\mathrm{R}^{2}=0.74$, $\mathrm{DW}=1.9$, Akaike Information Criterion $=-9.16$, F -statistic $=146.0$
(iv) $\mathrm{R}^{2}=0.57$, $\mathrm{DW}=1.9$, Akaike Information Criterion $=-9.57$, F -statistic $=42.3$
(v) $\mathrm{R}^{2}=0.54$, $\mathrm{DW}=2.0$, Akaike Information Criterion $=-5.48$, F -statistic $=23.5$

A comparison of the re-estimated parameters presented in Table 4 with those in Table 2 indicates the stability of the models given in (4). All parameters have the same signs and almost the same values in both the tables. Stability of the reestimated co-efficients has further been checked by using the following Chow forecast test.

$$
\begin{equation*}
F=\frac{\sum_{1}^{T} \varepsilon_{t}^{2}-\sum_{1}^{T_{1}} \bar{\varepsilon}_{t}^{2}}{\sum_{1}^{T_{1}} \bar{\varepsilon}_{t}^{2}} * \frac{T-k}{T_{1}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \tag{5}
\end{equation*}
$$

Where $\sum_{1}^{T} \varepsilon_{t}^{2}$ is the sum of squared residuals of the original models; $\sum_{1}^{T_{1}} \bar{\varepsilon}_{t}^{2}$ is the sum of squared residuals of the re-estimated model with truncated sample; $T$ is the total number of observations in the original models; $T_{1}$ is the observations in reduced models; and ' $k$ ' is the number of parameters estimated. F-statistic has an exact finite sample F-distribution, given the errors are independent, and identically, normally distributed. The Null hypothesis of the test is that the model is not unstable. The results of the test are given below. These show that all the models are stable (Table 5).

## Table 5

## Chow Forecast Test

| Equation | F-Statistic | Probability | Remarks |
| :---: | :---: | :---: | :--- |
| (i) | 0.680 | 0.76 | $\mathrm{H}_{0}$ accepted, the model is stable |
| (ii) | 0.265 | 0.99 | $\mathrm{H}_{0}$ accepted, the model is stable |
| (iii) | 0.110 | 0.99 | $\mathrm{H}_{0}$ accepted, the model is stable |
| (iv) | 0.780 | 0.67 | $\mathrm{H}_{0}$ accepted, the model is stable |
| (v) | 0.307 | 0.99 | $\mathrm{H}_{0}$ accepted, the model is stable |

A comparison has also been made between the projected values from the reestimated models and actual observations for the period of 1998:07-1999:06 on the basis of (i) the root mean squared error, (ii) mean absolute error, (iii) mean absolute percentage error, and (iv) Theil inequality coefficient (Table 6). As a rule, the smaller the values of these statistics, the better would be the forecast. The statistics of Table 6 show that the aggregate model of money multiplier performs quite well in projections as compared to models of component ratios in terms of the mean absolute
percentage error and the Theil inequality coefficient. ${ }^{4}$ Projected $m_{t}$ on the basis of the aggregate model is also closer to the actual values as compared to that obtained indirectly ( $m_{t}^{*}$ ) through projected components. Thus, contrary to the findings of Johannes and Rasche (1979), the model of the overall money multiplier gives a better forecast in the case of Pakistan as compared to the component models. The results are intuitive as in the case of the component approach. The forecast errors of all component models accumulate while working out the money multiplier, whereas in the case of the aggregate approach, the forecast errors are already minimum.

Table 6
Forecast Evaluation

|  | Root Mean <br> Squared <br> Error | Mean <br> Absolute <br> Error | Mean Absolute <br> Percentage <br> Error | Theil <br> Inequality <br> Coefficient |
| :--- | :---: | :---: | :---: | :---: |
| Aggregate Model: |  |  |  |  |
| $m_{t}$ | 0.0643 | 0.0492 | 1.5903 | 0.0103 |
| Component Models: |  |  |  |  |
| $c_{t}$ | 0.0112 | 0.0086 | 2.6609 | 0.0179 |
| $o_{t}$ | 0.0008 | 0.0006 | 8.6351 | 0.0520 |
| $k_{t}$ | 0.0019 | 0.0017 | 10.4292 | 0.0527 |
| $r_{t}$ |  |  |  |  |
| $m_{t}^{*}$ | 0.0665 | 0.0544 | 1.7267 | 0.0154 |

$\overline{m_{t}^{*}}=$ Money multiplier estimated on the basis of projected components.

## 7. CONCLUSION

The paper has developed time-series models for the monthly money multiplier and its component ratios. A comparison is made between the predictive performance of the aggregate multiplier and the component models. It is found that the projected values of the multiplier on the basis of the aggregate model are closer to actual values as compared to those worked out on the basis of the component models. Thus, for the purposes of projecting money multiplier, it may be preferable to focus on the performance and modelling of the aggregate money multiplier.
${ }^{4}$ The root mean squared error and the mean absolute error are very low in the case of component ratios as compared to those for the multiplier. But these statistics are not comparable because the component ratios are in fractions while the multiplier has values more than one. Thus, for the purposes of comparison in this case, the mean absolute percentage error and the Theil inequality coefficients are more relevant.

Annexure
Autocorrelation and Partial Autocorrelation Functions

|  | $\Delta m_{t}$ |  |  |  | $\Delta c_{t}$ |  |  |  | $o_{t}$ |  |  |  | $k_{t}$ |  |  |  | $r_{t}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lags | AC | PAC | Q-Stat | Prob | AC | PAC | Q-Stat | Prob | AC | PAC | Q-Stat | Prob | AC | PAC | Q-Stat | Prob | AC | PAC | Q-Stat | Prob |
| 1 | -0.25 | -0.25 | 7.79 | 0.01 | -0.15 | -0.15 | 2.81 | 0.09 | 0.88 | 0.88 | 94.26 | 0.00 | -0.34 | -0.34 | 14.31 | 0.00 | 0.67 | 0.67 | 55.86 | 0.00 |
| 2 | 0.10 | 0.04 | 8.97 | 0.01 | 0.01 | -0.01 | 2.83 | 0.24 | 0.75 | -0.09 | 163.14 | 0.00 | -0.25 | -0.41 | 21.77 | 0.00 | 0.65 | 0.35 | 107.40 | 0.00 |
| 3 | 0.00 | 0.03 | 8.97 | 0.03 | -0.07 | -0.08 | 3.51 | 0.32 | 0.63 | -0.03 | 211.96 | 0.00 | 0.34 | 0.11 | 35.78 | 0.00 | 0.46 | -0.11 | 134.29 | 0.00 |
| 4 | -0.05 | -0.05 | 9.30 | 0.05 | 0.03 | 0.01 | 3.61 | 0.46 | 0.53 | 0.04 | 247.44 | 0.00 | -0.09 | 0.01 | 36.72 | 0.00 | 0.39 | -0.04 | 152.98 | 0.00 |
| 5 | -0.15 | -0.19 | 12.30 | 0.03 | -0.27 | -0.28 | 13.10 | 0.02 | 0.45 | 0.00 | 273.41 | 0.00 | -0.14 | -0.03 | 39.03 | 0.00 | 0.37 | 0.18 | 170.67 | 0.00 |
| 6 | -0.01 | -0.10 | 12.32 | 0.06 | 0.13 | 0.04 | 15.13 | 0.02 | 0.39 | 0.01 | 292.78 | 0.00 | 0.16 | 0.04 | 42.43 | 0.00 | 0.36 | 0.11 | 187.11 | 0.00 |
| 7 | -0.08 | -0.09 | 13.11 | 0.07 | -0.29 | -0.30 | 25.73 | 0.00 | 0.35 | 0.07 | 308.77 | 0.00 | -0.20 | -0.22 | 47.53 | 0.00 | 0.38 | 0.08 | 206.21 | 0.00 |
| 8 | -0.15 | -0.20 | 15.92 | 0.04 | 0.05 | -0.08 | 26.05 | 0.00 | 0.34 | 0.09 | 323.90 | 0.00 | -0.14 | -0.30 | 49.91 | 0.00 | 0.28 | -0.18 | 216.41 | 0.00 |
| 9 | 0.10 | -0.01 | 17.29 | 0.04 | -0.07 | -0.13 | 26.78 | 0.00 | 0.33 | -0.02 | 337.96 | 0.00 | 0.37 | 0.13 | 67.49 | 0.00 | 0.25 | -0.04 | 224.73 | 0.00 |
| 10 | 0.01 | 0.02 | 17.30 | 0.07 | 0.05 | -0.13 | 27.05 | 0.00 | 0.33 | 0.08 | 352.16 | 0.00 | -0.19 | 0.01 | 72.30 | 0.00 | 0.15 | -0.03 | 227.69 | 0.00 |
| 11 | -0.07 | -0.12 | 18.01 | 0.08 | -0.10 | -0.14 | 28.29 | 0.00 | 0.31 | -0.07 | 364.64 | 0.00 | -0.17 | -0.06 | 76.07 | 0.00 | 0.12 | -0.01 | 229.45 | 0.00 |
| 12 | 0.33 | 0.25 | 32.44 | 0.00 | 0.67 | 0.57 | 88.44 | 0.00 | 0.29 | 0.03 | 375.84 | 0.00 | 0.34 | 0.11 | 91.42 | 0.00 | 0.08 | -0.04 | 230.22 | 0.00 |
| 13 | -0.08 | 0.03 | 33.27 | 0.00 | -0.14 | 0.03 | 90.96 | 0.00 | 0.24 | -0.14 | 383.40 | 0.00 | -0.07 | 0.05 | 92.18 | 0.00 | 0.07 | 0.01 | 230.95 | 0.00 |
| 14 | 0.09 | 0.05 | 34.35 | 0.00 | -0.02 | -0.07 | 91.00 | 0.00 | 0.17 | -0.06 | 387.42 | 0.00 | -0.16 | -0.05 | 95.73 | 0.00 | 0.05 | -0.04 | 231.25 | 0.00 |
| 15 | -0.15 | -0.17 | 37.60 | 0.00 | -0.09 | -0.07 | 92.18 | 0.00 | 0.14 | 0.11 | 390.12 | 0.00 | 0.28 | 0.12 | 106.33 | 0.00 | -0.03 | -0.14 | 231.41 | 0.00 |
| 16 | 0.11 | 0.04 | 39.27 | 0.00 | 0.04 | 0.00 | 92.40 | 0.00 | 0.13 | 0.02 | 392.31 | 0.00 | -0.24 | -0.17 | 114.72 | 0.00 | -0.01 | 0.10 | 231.42 | 0.00 |
| 17 | -0.19 | -0.05 | 44.54 | 0.00 | -0.24 | -0.03 | 100.58 | 0.00 | 0.13 | 0.05 | 394.62 | 0.00 | -0.06 | -0.08 | 115.30 | 0.00 | -0.07 | 0.02 | 232.11 | 0.00 |
| 18 | -0.02 | -0.09 | 44.60 | 0.00 | 0.10 | -0.06 | 101.86 | 0.00 | 0.13 | -0.02 | 396.96 | 0.00 | 0.27 | 0.01 | 125.46 | 0.00 | -0.05 | -0.01 | 232.44 | 0.00 |
| 19 | -0.12 | -0.15 | 46.78 | 0.00 | -0.25 | -0.04 | 110.99 | 0.00 | 0.12 | -0.04 | 398.99 | 0.00 | -0.20 | -0.01 | 131.29 | 0.00 | -0.03 | 0.06 | 232.57 | 0.00 |
| 20 | 0.02 | 0.03 | 46.84 | 0.00 | 0.05 | -0.06 | 111.29 | 0.00 | 0.12 | 0.02 | 401.08 | 0.00 | -0.07 | 0.08 | 131.93 | 0.00 | -0.02 | 0.03 | 232.60 | 0.00 |
| 21 | 0.04 | 0.03 | 47.11 | 0.00 | -0.11 | -0.14 | 113.08 | 0.00 | 0.07 | -0.19 | 401.90 | 0.00 | 0.23 | 0.05 | 139.86 | 0.00 | -0.01 | 0.00 | 232.62 | 0.00 |
| 22 | -0.02 | -0.07 | 47.15 | 0.00 | 0.09 | -0.01 | 114.37 | 0.00 | 0.03 | 0.01 | 402.05 | 0.00 | -0.14 | -0.06 | 142.71 | 0.00 | 0.00 | 0.06 | 232.62 | 0.00 |
| 23 | 0.07 | -0.01 | 47.78 | 0.00 | -0.04 | 0.02 | 114.60 | 0.00 | -0.03 | -0.09 | 402.16 | 0.00 | -0.06 | -0.02 | 143.18 | 0.00 | 0.00 | 0.00 | 232.62 | 0.00 |
| 24 | 0.18 | 0.15 | 52.90 | 0.00 | 0.49 | 0.05 | 151.11 | 0.00 | -0.08 | -0.02 | 403.13 | 0.00 | 0.21 | -0.04 | 149.71 | 0.00 | -0.02 | -0.05 | 232.70 | 0.00 |
| 25 | -0.01 | 0.03 | 52.93 | 0.00 | -0.16 | -0.11 | 155.18 | 0.00 | -0.08 | 0.17 | 404.22 | 0.00 | -0.09 | 0.10 | 151.00 | 0.00 | -0.02 | -0.03 | 232.73 | 0.00 |
| 26 | -0.04 | -0.16 | 53.17 | 0.00 | 0.00 | -0.03 | 155.18 | 0.00 | -0.10 | -0.10 | 405.80 | 0.00 | -0.14 | -0.06 | 154.22 | 0.00 | -0.09 | -0.18 | 234.07 | 0.00 |
| 27 | 0.13 | 0.23 | 55.99 | 0.00 | -0.03 | 0.09 | 155.36 | 0.00 | -0.09 | 0.08 | 407.01 | 0.00 | 0.28 | 0.08 | 166.14 | 0.00 | -0.02 | 0.15 | 234.12 | 0.00 |
| 28 | -0.09 | 0.00 | 57.32 | 0.00 | 0.02 | -0.05 | 155.44 | 0.00 | -0.06 | 0.01 | 407.60 | 0.00 | -0.16 | 0.01 | 170.05 | 0.00 | -0.11 | -0.12 | 236.00 | 0.00 |
| 29 | -0.17 | -0.17 | 61.69 | 0.00 | -0.24 | -0.10 | 164.61 | 0.00 | -0.06 | -0.12 | 408.12 | 0.00 | -0.20 | -0.17 | 176.53 | 0.00 | -0.10 | -0.14 | 237.45 | 0.00 |
| 30 | 0.05 | -0.03 | 62.12 | 0.00 | 0.11 | 0.02 | 166.69 | 0.00 | -0.08 | -0.12 | 409.11 | 0.00 | 0.25 | -0.09 | 186.97 | 0.00 | -0.08 | 0.11 | 238.41 | 0.00 |
| 31 | -0.31 | -0.20 | 77.91 | 0.00 | -0.24 | -0.06 | 176.30 | 0.00 | -0.11 | -0.01 | 411.20 | 0.00 | -0.16 | -0.11 | 190.95 | 0.00 | -0.15 | -0.10 | 241.90 | 0.00 |
| 32 | 0.06 | -0.08 | 78.60 | 0.00 | 0.02 | -0.12 | 176.36 | 0.00 | -0.16 | -0.13 | 415.65 | 0.00 | -0.05 | -0.01 | 191.28 | 0.00 | -0.06 | 0.11 | 242.54 | 0.00 |
| 33 | 0.00 | -0.10 | 78.61 | 0.00 | -0.04 | 0.07 | 176.57 | 0.00 | -0.18 | 0.17 | 421.21 | 0.00 | 0.14 | -0.06 | 194.33 | 0.00 | -0.11 | 0.03 | 244.46 | 0.00 |
| 34 | 0.07 | 0.03 | 79.37 | 0.00 | 0.11 | 0.01 | 178.45 | 0.00 | -0.22 | -0.08 | 429.17 | 0.00 | -0.13 | -0.08 | 197.21 | 0.00 | -0.10 | -0.15 | 246.32 | 0.00 |
| 35 | -0.06 | -0.04 | 79.90 | 0.00 | -0.04 | -0.09 | 178.67 | 0.00 | -0.24 | -0.02 | 438.70 | 0.00 | 0.00 | -0.01 | 197.21 | 0.00 | -0.16 | -0.04 | 250.75 | 0.00 |
| 36 | 0.29 | 0.06 | 94.80 | 0.00 | 0.38 | -0.06 | 203.23 | 0.00 | -0.27 | -0.08 | 451.33 | 0.00 | 0.18 | -0.05 | 202.76 | 0.00 | -0.14 | 0.11 | 254.00 | 0.00 |

Note: Econometric views 3.1 is used to estimate correlograms.
$\mathrm{AC}=$ Autocorrelation function, PAC = Partial Autocorrelation Function.

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[^0]:    ${ }^{3}$ Though currency ratio is also trend-stationary, its first-difference transformation is used because that is more useful for short-run forecasting. However, if the objective is long-run forecast, then the series may be used in level-form, including trend, or it may be detrended by a suitable polynomial trend.

