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Mining and Incentive Concession Contracts

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Abstract

This paper studies the design of a mining concession contract as a multi-period autoselection problem where production is the depletion of a non renewable resource. As compared to symmetric information, we show that overproduction (resp. underproduction) is optimal in the initial phase (resp. terminal phase) of the resource extraction program. Also, asymmetric information lengthens the contract duration but reduces the scarcity rent. Finally, when there are several agents competing for contract bid, we show that optimal auctioning could be used to award the concession, assigning the lowest cost agent to carry out the extraction.

Keywords: Adverse selection, Exhaustibility, Overproduction

JEL Codes: D 82, Q 30.

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1 Introduction

The theory of mechanism design has made rapid advances over the last twenty years. Standard results show that production is lower with asymmetric information than with symmetric information in order to reduce the agent's informational rent. This conclusion can be generalized on several periods if the principal can commit itself at the beginning of the relationship and the conditions (technology or preferences) of the principal-agent relationship are time invariant.

In this paper, we analyze the particular case where the multi-period production concerns a non renewable resource. More precisely, we study the impacts of informational asymmetric context related to autoselection, when the time invariance assumption is relaxed. Because of a dynamic exhaustibility constraint, this problem can be referred to as the design of mining concession contracts: autoselection is fitted in the exhaustible resource management over time

In the informational framework above, quite a few earlier analyses exist in the field of natural resources economics. Noticeable are the study of Dynamic Taxation with asymmetric information about reserves (Osmuden, 1998) and of Resource Royalty Contract with asymmetric information about the extraction cost (Gaudet G, P Lasserre and NV Long, 1995). In short, these analyses have shown that asymmetric information - more precisely, the adverse selection problem affecting cost efficiency in Gaudet *et al.* and reserves in Osmundsen - involves distortions of both the extent and the pace of resource depletion.

Among other results of interest, Gaudet *et.al* (op.cit) have shown that the optimal royalty scheme will generate a distorted extraction which favors the earlier phase of production less than under symmetric information, provided resource complete exhaustion over the time horizon. The basic assumption adopted is that extraction costs - the variable of adverse selection - are uncorrelated over time so that one can use the revelation principle which is crucial in the analysis.

In this paper, we want to precise the direction of these distortions using a fairly general model of dynamic adverse selection. Our work is much akin to Gaudet *et.al,* except that we walk away from their basic assumption but aim at obtaining the same above result in a generalized context. First, as compared to the outcome under symmetric information, we show that overproduction is optimal at the end of the contract because the gains from exhaustibility dominate the rent loss under asymmetric information. We also show that asymmetric information lengthens the endogenous contract duration but reduces scarcity rent. Theses results are in some sense non standard comparatively to the theory of mechanism design, but we argue that they could be interpreted as a simple "cost raising effect" in the resource economics. Finally, we see that optimal auctioning could be an easy way to award the concession contract, when there are several agents in bidding competition. We show that selling by auction the concession leads to a efficient separation procedure among agents so that the lowest cost agent is assigned to carry out the concession contract.

The paper is organized as follows. In the next, we present the model and the assumptions. In section 3, we derive the optimal concession contract with complete and incomplete information. In section 4, we make a comparison of information context and examine the issue of overproduction. In section 5, we discuss an auctioning procedure to award the concession contract among potential agents (or operators). Sections 6 concludes and 7 is devoted to the proofs of some Propositions.

2 The model

A principal is the owner of the non renewable resource stock $\bar{S}$. It delegates the extraction to a risk neutral agent on T periods. The agent's cost function is:

$$\theta q_t$$

with θ its constant efficiency and q_t the extraction at time $t, t = 0, 1, ..., T$. The principal's surplus is a strictly concave function $V(q)$ with $V(0) = 0$. The marginal surplus $V'(q) = v(q) > 0$ is decreasing so that $\omega \equiv v^{-1}$ exists and $\omega'(.) < 0$. The owner is not able to observe θ but has a prior belief summarized by the regular density $f(.) > 0$ on $\left[\underline{\theta}, \bar{\theta}\right]$. Regularity implies that $F(.)/f(.)$ is non decreasing in θ, where $F(\theta) = \int_{\underline{\theta}}^{\theta} f(\varepsilon)d\varepsilon$. Note $\beta, \beta < 1$, is the discounting factor.

We assume that the principal can commit itself on the T periods.[1] Thus, it can construct a credible mechanism. Such mechanism is function of the horizon of the contract, that could be exogenous or endogenous. In the sequel, we focus on the more general framework of an endogenous horizon of the contract. In this case, the horizon of the contract must depend on the private information. The mechanism offered by the principal is $\langle Y_t(\hat{\theta}), q_t(\hat{\theta}), T(\hat{\theta})\rangle$ specifying for a report $\hat{\theta}$, the monetary transfer $Y_t(\hat{\theta})$ and the extraction $q_t(\hat{\theta})$ at time $t, t = 0, 1, ..., T(\hat{\theta})$, where $T(\hat{\theta})$ represents the time at which the contract ends if the agent reports $\hat{\theta}$. The utility of the agent becomes:

$$U(\hat{\theta}, \theta) = \sum_{t=0}^{t=T(\hat{\theta})} \beta^t \left[Y_t(\hat{\theta}) - \theta q_t(\hat{\theta})\right]$$

when its type is θ and it announces $\hat{\theta}$. The individual rationality implies the agent is not forced to contract, thus we must have:

$$U(\theta) \equiv U(\theta, \theta) \geq 0, \forall \theta \in \left[\underline{\theta}, \bar{\theta}\right] \tag{1}$$

where zero is the (normalized) agent's utility of reservation. Moreover, the principal must satisfy the incentive compatibility constraint which ensures truth-telling by the agent. So:

$$U(\theta, \theta) \geq U(\theta, \hat{\theta}), \forall \theta, \hat{\theta} \in \left[\underline{\theta}, \bar{\theta}\right] \tag{2}$$

Finally, the agent can not extract more than the initial stock. It follows the exhaustibility constraint (hereafter EC):

$$\sum_{t=0}^{t=T(\theta)} q_t(\theta) \leq \bar{S}, \forall \theta \in \left[\underline{\theta}, \bar{\theta}\right] \tag{3}$$

The principal's objective being to maximize the expected discounted sum of net surplus, the problem is:

$$\max_{q_t(.), Y_t(.), T(.)} \int_{\underline{\theta}}^{\bar{\theta}} \sum_{t=0}^{t=T(\theta)} \beta^t \left[V(q_t(\theta)) - Y_t(\theta)\right] f(\theta) d\theta \tag{4}$$

[1] If there is no commitment, pooling equilibrium arises (see Laffont and Tirole, 1988). Further discussion about this assumption is postponed until section 6, neverthless note that we abandon the assumption of temporally uncorrelated costs made in Gaudet *et.al* (op.cit).

subject to (1), (2) and (3). Determining feasible mechanisms typically (see appendix A), relations (1) and (2) can be rewritten as:

$$U(\bar{\theta}) = 0 \tag{1'}$$

$$U'(\theta) = -Q(\theta) \tag{2'}$$

$$Q'(\theta) = \beta^{T(\theta)} q_{T(\theta)}(\theta)\, T'(\theta) + \sum_{t=0}^{t=T(\theta)} \beta^t q_t'(\theta) \leq 0 \tag{2''}$$

With $Q(\theta) = \sum_{t=0}^{t=T(\theta)} \beta^t q_t(\theta)$. Note that $y_t'(\theta)$ is the derivative of y with respect to θ at the t period. After standard manipulations[2], the problem becomes:

$$\max_{q_t(.)} \int_{\underline{\theta}}^{\bar{\theta}} \sum_{t=0}^{t=T(\theta)} \beta^t \left[V(q_t(\theta)) - K(\theta) q_t(\theta)\right] f(\theta) d\theta \tag{4'}$$

(4') subject to: (2') and (3), with $K(\theta) = \theta + \frac{F(\theta)}{f(\theta)}$is non decreasing. In the adverse selection literature $K(\theta)$ is known as the adjusted (unit and marginal) cost, which includes the informational cost to the principal. More precisely, $\frac{f(\theta)}{F(\theta)}$ is the conditional probability that efficiency is no more increasing (*i.e.* θ is falling) given that there have already been an increase. Then the hazard rate ($\frac{F(\theta)}{f(\theta)}$) can be interpreted as the cost of screening the information about agent's efficiency. Moreover, because of the regularity assumption, this cost is increasing with inefficiency of the agent.

3 Optimal concession contract

If we ignore (in a first step) the incentive compatibility constraint (2"), the problem can be viewed as an optimal control one in discrete time, introducing $S_t(\theta)$ as the state variable representing the current reserve of agent θ. As a consequence, relation (3) can be also written as a motion equation:

$$\forall \theta \in \left[\underline{\theta}, \bar{\theta}\right], \forall t \in \{0, \ldots, T(\theta)\}, \; S_{t+1}(\theta) - S_t(\theta) = -q_t(\theta) \text{ with } S_0(\theta) = \bar{S} \tag{3'}$$

[2]Using (1') and (2'), $U(\theta) = U(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} \sum_{t=0}^{t=T(\theta)} \beta^t q_t(\varepsilon) d\varepsilon$. In addition $\sum_{t=0}^{t=T} \beta^t Y_t(\theta) = U(\theta) + \theta \sum_{t=0}^{t=T(\theta)} \beta^t q_t(\theta)$. Substituting in (4) and integrating by parts yields (4').

Moreover at the terminal period the reserve can be exhausted or not, i.e. $S_{T(\theta)}(\theta) \geq 0$. Optimizing point-wise over θ, the parameterized Hamiltonian of this problem is:

$$H_t(\theta) = \beta^t \left[V(q_t(\theta)) - K(\theta)q_t(\theta)\right] - \lambda_t(\theta)q_t(\theta)$$

where $\lambda_t(\theta)$ is the costate variable associated to $S_t(\theta)$. Applying Pontryagin principle in a discrete time version, we have necessary (and sufficient[3]) conditions for $q_t(\theta) \geq 0$[4], $\forall t \in \{0, \ldots, T(\theta)\}$:

$$\begin{cases} \frac{\partial H_t(\theta)}{\partial q_t(\theta)} = 0 \\ -\frac{\partial H_t(\theta)}{\partial S_t(\theta)} = \lambda_{t+1}(\theta) - \lambda_t(\theta) \\ \lambda_{T(\theta)}(\theta) S_{T(\theta)}(\theta) = 0 \\ \lambda_{T(\theta)}(\theta) \geq 0 \end{cases} \Leftrightarrow \begin{cases} v\left(q_t(\theta)\right) - K(\theta) = \beta^{-t}\lambda_t(\theta) \\ \lambda_{t+1}(\theta) - \lambda_t(\theta) = 0 \Rightarrow \lambda_t(\theta) = \lambda(\theta) \geq 0 \\ \lambda(\theta) S_{T(\theta)}(\theta) = 0 \end{cases} \quad (5)$$

The first relation in (5) is the usual Hotelling rule modified to account for informational constraints. Hence production is chosen so that the net marginal surplus corrected for the informational cost, must grow at the discount rate. The time-invariant variable $\lambda(\theta)$ is called scarcity rent that is opportunity cost of one unit of "ore" in ground.

Furthermore and because of an endogenous termination of the contract, this supplementary Arrow's type terminal condition must hold:

$$H_{T(\theta)}(\theta) = 0 \Leftrightarrow \beta^{T(\theta)} \left[V(q_{T(\theta)}(\theta)) - K(\theta)q_{T(\theta)}(\theta)\right] - \lambda(\theta)q_{T(\theta)}(\theta) = 0$$

This terminal condition, which first has been derived by Levari and Liviathan (1977), implies that either the terminal extraction level is nil, $q_{T(\theta)}(\theta) = 0$, or if not ($q_{T(\theta)}(\theta) > 0$), the discounted average surplus equals the scarcity rent at the end of the contract, so should satisfy:

$$\beta^{T(\theta)} \frac{V(q_{T(\theta)}(\theta)) - K(\theta)q_{T(\theta)}(\theta)}{q_{T(\theta)}} = \lambda(\theta)$$

[3] Mangasarian sufficient conditions holds here because the discrete time Hamiltonian $H_t(\theta)$ is concave (but not strictly) in (q, S) for all t. Indeed the hessian of $H_t(\theta)$ with respect to (q, S) is negative semidefinite because : $\partial^2 H_t(\theta)/\partial q_t(\theta)^2 = \beta^t v'(q_t(\theta)) < 0$ and the determinant of that hessian is zero.

[4] Indeed positive extraction is only possible if $v(0) > K\left(\overline{\theta}\right)$.

However this latter case cannot arise. For, using (5), the above equation can be rewritten as:

$$\beta^{T(\theta)}\left[V(q_{T(\theta)}(\theta))-\left(v(q_{T(\theta)}(\theta))-\beta^{-T(\theta)}\lambda(\theta)\right)q_{T(\theta)}(\theta)\right]-\lambda(\theta)q_{T(\theta)}=0$$
$$\Leftrightarrow V(q_{T(\theta)}(\theta))-v(q_{T(\theta)}(\theta))q_{T(\theta)}(\theta)=0$$

By assumption put on $V(x), V(x)-v(x)x$ is a non decreasing function of x and equals 0 only iff $x=0$.Thus $q_{T(\theta)}(\theta)=0$, and therefore $T(\theta)$ is such that :

$$\frac{\lambda(\theta)}{v(0)-K(\theta)}=\beta^{T(\theta)}\Leftrightarrow T(\theta)=\frac{1}{\ln(\beta)}\ln\left(\frac{\lambda(\theta)}{v(0)-K(\theta)}\right) \tag{6}$$

This optimal stopping condition implies that the incentive compatibility constraint (2") can be rewritten now as:

$$Q'(\theta)=\sum_{t=0}^{t=T(\theta)}\beta^t q_t'(\theta)\leq 0 \tag{2"'}$$

Finally, we prove now that exhaustion time and contract duration are concomitant. Assume the contrary for all θ, i.e $S_{T(\theta)}>0$. Then $\lambda(\theta)=0, \forall\theta$, and $q_t(\theta)=\omega(K(\theta))\neq 0$, for all t, a contradiction because the horizon cannot be not optimal with $q_{T(\theta)}(\theta)\neq 0$. It follows that $S_{T(\theta)}=0$. Consequently, the exhaustibility constraint (3) binds, that is $\sum_{t=0}^{t=T(\theta)}q_t(\theta)=\bar{S}$.
We can sum up these results in the following proposition.

Proposition 1 *With incomplete information, the optimal production is such that the modified Hotelling rule applies and resource exhaustion takes place at the end of the contract.*

The incentive compatibility constraint (2"') is verified in appendix B.

The complete information context can be derived as a particular case of proposition 1.

Corollary 2 (Complete information) *With complete information, the optimal production is such that the standard Hotelling rule applies and resource exhaustion takes place at the end of the contract.*

Proof. Indeed, if we put $K(\theta) \equiv \theta, \forall \theta$, that is collecting information about the agent's type is not costly (by assumption), we have from (5): $v'\left(q_t(\theta)\right) = \theta + \beta^{-t}\lambda(\theta)$, that is the standard Hotelling rule: net marginal surplus grows at the discount rate. ■

4 Comparison of information context

4.1 Non exhaustible benchmark case

We begin by recalling the standard results in the following lemma.

Lemma 3 (Baron and Besanko, 1984) *In the non exhaustible case, at each period, the optimal production with incomplete information is lesser (except at $\theta = \underline{\theta}$) than with complete information.*

A "good" approximation of this benchmark case can be found using the conditions (5) and (6) and setting down $\lambda(\theta) = 0, \forall \theta$, because physical resource scarcity is irrelevant now. When there is commitment, the optimal static scheme with inexhaustible resource is reiterated at each period (so horizon is infinite a priori). So, with asymmetric information, the principal diminishes the first-best quantity in order to reduce the informational rent. If we note $\Delta q_t^*(\theta) = q_t^{c*}(\theta) - q_t^{i*}(\theta)$, where superscripts c^* and i^* refer respectively to complete and incomplete information in the inexhaustible case, we must have:

$$\Delta q_t^*(\theta) \geq 0, t = 0, 1, \ldots, T, \text{ (holding with equality at } \theta = \underline{\theta})$$

. Indeed, from (5) and with $\lambda(\theta) = 0, \forall \theta$, $q_t^{c*}(\theta) = \omega(\theta) \geq q_t^{i*}(\theta) = \omega(K(\theta))$ because $\forall \theta$, $K(\theta) > \theta$.

4.2 Exhaustible resources

We return to the exhaustible case and we use c and i superscripts to refer respectively to complete and incomplete information.

Proposition 4 i) *In the exhaustible case and for all* $\theta \in \left]\underline{\theta}, \bar{\theta}\right]$*:*

$$\lambda^c(\theta) \geq \lambda^i(\theta)$$

with equality holding only at $\theta = \underline{\theta}$*.*

ii) *Moreover, let* $\Delta q_t(\theta) = q_t^c(\theta) - q_t^i(\theta)$ *denote the extraction path differential with respect to information context. Then it does exist a date* $\tau(\theta)$ *such that:*

$$\Delta q_t(\theta) \geq 0, \forall t \in [0, \tau(\theta)] \text{ and } \Delta q_t(\theta) < 0, \forall t \in \left]\tau(\theta), \min\left\{T^i(\theta), T^c(\theta)\right\}\right]$$

with equality holding at $\theta = \underline{\theta}$*.*

iii) *And finally,*

$$T^c(\theta) \leq T^i(\theta)$$

with equality holding only at $\theta = \underline{\theta}$*.*

Proof. i) If the contrary is true, $\lambda^c(\theta) < \lambda^i(\theta)$, we have:

$$K(\theta) + \beta^{-t}\lambda^i(\theta) \geq \theta + \beta^{-t}\lambda^c(\theta)$$

and thus $q_t^c(\theta) > q_t^i(\theta)$ for $t = 0, 1, \ldots, \min\{T^i(\theta), T^c(\theta)\}$ because $\omega(.)$ is decreasing. This leads to the contradiction of the (binding) exhaustibility constraint:

$$\bar{S} = \sum_{t=0}^{t=T^c(\theta)} q_t^c(\theta) > \sum_{t=0}^{t=T^i(\theta)} q_t^i(\theta) = \bar{S}$$

ii) Let us define a date $\tau(\theta)$ such that $\Delta q_{\tau(\theta)}(\theta) = 0$:

$$\tau(\theta) = \frac{1}{\ln(\beta)} \ln\left(\frac{\lambda^c(\theta) - \lambda^i(\theta)}{K(\theta) - \theta}\right)$$

For all $\theta \geq \underline{\theta}$, this date is unique and lying between 0 and $\min\{T^i(\theta), T^c(\theta)\}$. Indeed, if $\tau(\theta) > T^i(\theta)$ then $\Delta q_t(\theta) \geq 0, \forall t$, contradicting EC[5]. Similarly, if $\tau(\theta) = 0$ then $\Delta q_t(\theta) < 0, \forall t$, and a same conclusion occurs.

iii) For the last part, if the contrary is true, $T^c(\theta) > T^i(\theta)$ then $\Delta q_{T^i(\theta)}(\theta) = q_t^c(\theta) - 0 > 0$, contradicting the result of part ii). So $T^c(\theta) \leq T^i(\theta)$.[6] ■

[5] Actually, for all dates t such that $t = \tau(\theta) - \varepsilon, \varepsilon > 0$:

$$\begin{aligned} v\left(q_t^c(\theta)\right) - v\left(q_t^i(\theta)\right) &\Leftrightarrow \theta + \beta^{-t}\lambda^c(\theta) - \left[K(\theta) + \beta^{-t}\lambda^i(\theta)\right] \\ &\Leftrightarrow (\beta^\varepsilon - 1)(K(\theta) - \theta) < 0 \end{aligned}$$

and the reverse for $t = \tau(\theta) + \varepsilon, \varepsilon > 0$. Hence since $v' < 0, q_t^c(\theta) \gtrless q_t^i(\theta)$ if $t \lessgtr \tau(\theta)$

[6] Notice that this date is also anterior to $T^c(\theta)$. Hence $\min\left\{T^i(\theta), T^c(\theta)\right\} = T^c(\theta)$. Indeed if it was not the case (i.e. $\tau(\theta) > T^c(\theta)$), we would have the contradiction: $\Delta q_{T^c(\theta)}(\theta) = 0 - q_t^c(\theta) < 0$

Scarcity rent or mining rent appears to be reduced because of asymmetric information. Accordingly, the principal is compelled to forgo some rents in order to induce incentives in the contract, so its marginal evaluation of a unit of resource is relatively lower than if it could pays zero rents. This is precisely what we propose as a "raising cost" argument to interpret this mining rent reduction.

The contract duration is increased by adverse selection. In fact, if it is not the case, some agents are not concerned by the terms of the contract given in Proposition 4[7] and the mechanism is then suboptimal. Indeed, prolonging the bilateral relation is a "good" way to postpone rent disbursement. Intuitively, this is a logical and direct consequence of the next result.

The main result is that for almost all agents overproduction arises at the end of the contract when information is asymmetric. When the mine owner designs a non renewable resource mining contract, he or she retains a double targets in the case of asymmetric information. First, as the principal, the owner wants to reduce the informational rent. But, second, he or she would also like to fully use the scarce mining resource. The optimal policy puts these two effects in play in turn. In a first period (*i.e.* $t \in [0, \tau]$), the quantity extracted is lower, and so is the informational rent. But in a second (*i.e.* $t \in [\tau, T]$), the principal prefers to exhaust its stock rather than continue to reduce the rent. In this event, the quantity becomes greater with asymmetric information, allowing the mine owner to recapture some of the forgone earnings made in the initial periods. Actually from the τ period on, the gains from exhaustibility dominate the rent loss.

This phenomenon can be reinterpreted as a simple result of extraction unit cost increase. In non renewable resource economics, it is well known[8] that an increase in the marginal extraction cost reduces the discounted scarcity rent, increases the exhaustion date and twists the extraction path. More precisely, the Hotelling's arbitrage principle tells us that the discounted marginal net surplus must be equal at each period at the discounted scarcity rent. If marginal costs are higher, the

[7] For these agents the scarcity rent would be zero and production would be based only on $K(\theta)$, so Hotelling rule would not apply.

[8] See J. Hartwick (1989), p. 37-38, for statics comparative results.

discounted marginal net surplus is lower at each period, and so is the mining rent. As a consequence, extraction is reduced in the initial periods but is increased in the terminal ones, because of the optimality of exhausting completely the reserve. All in all, the adverse selection problem could be reduced to a simple cost increase to the principal through prospecting costly information.

5 Auctioning mining contract

In the previous section, it has been assumed that the mine owner (the principal) is facing only one operator (the agent) who carries out the contract. But in most of the real cases[9], it can be observed some *ex ante* competition between potential agents. In order to enlarge the scope of our model, we introduce auctioning to select the "best" agent from the principal point of view. We show that by auctioning the concession leads to a efficient separation procedure among agents so that the lowest cost agent is assigned to carry out the mining contract.

Assume there are n risk-neutral agents, indexed by j, $j = 1, 2, ..., n$, who can carry out the resource extraction. Each of them incurs an unit cost θ^j, a private information issued independently from the random variable with density $f(.) > 0$ defined on $\left[\underline{\theta}, \bar{\theta}\right]$. The mechanism becomes $\left\langle Y_t^j(\hat{\theta}), q_t^j(\hat{\theta}), T^j(\hat{\theta}), \rho^j(\hat{\theta})\right\rangle$ specifying for a vector of agents' report $\hat{\theta} = (\hat{\theta}^1, ..., \hat{\theta}^j, ..., \hat{\theta}^n)$ the monetary transfer $Y_t^j(\hat{\theta})$, the extraction rate $q_t^j(\hat{\theta})$, at time $t, t = 0, 1, ..., T^j(.)$, the terminal date $T^j(\hat{\theta})$ and the probability to wins the auction $\rho^j(\hat{\theta})$ for agent j, $j = 1, 2, ..., n$. The incentive compatibility, the individual rationality and the resource exhaustibility constraints become respectively, for each j, $j = 1, 2, ..., n$ with type θ^j and report $\hat{\theta}^j$, $\forall \theta^j, \hat{\theta}^j \in \left[\underline{\theta}, \bar{\theta}\right]$, $\forall \hat{\theta}$:

$$U^j(\theta^j, \theta^j) \geq U^j(\hat{\theta}^j, \theta^j) = E_{\theta^{-j}} \sum_{t=0}^{t=T^j(\hat{\theta})} \beta^t \left[Y_t^j(\hat{\theta}^j, \theta^{-j}) - \rho^j(\hat{\theta}^j, \theta^{-j})\theta^j q_t^j(\hat{\theta}^j, \theta^{-j})\right] \quad (7)$$

$$U^i(\theta^j) = U^j(\theta^j, \theta^j) \geq 0 \quad (8)$$

[9]For example in hydrocarbon industrial sector, attribution of exploration-production contracts are often submitted to auctions.

$$\sum_{t=0}^{t=T^i(\hat{\theta})} q_t^j(\hat{\theta}) \leq \bar{S} \tag{9}$$

where $E_{\theta^{-j}}$ represents agent's j expectations over $\theta^{-j} = (\theta^1, ..., \theta^{j-1}, \theta^{j+1}, ..., \theta^n)$. Moreover, we must satisfy the feasibility constraints (FC), $\forall \hat{\theta}$:

$$\sum_{j=1}^{j=n} \rho^j(\hat{\theta}) \leq 1 \quad \text{and} \quad 0 \leq \rho^j(\hat{\theta}) \leq 1, \forall j = 1, 2, ..., n \tag{10}$$

The principal's objective being to maximize the expected discounted sum of net surplus, the problem is:

$$\max_{\rho^j(.), q_t^j(.), Y_t^j(.), T^j(.)} E_\theta \sum_{i=1}^{j=n} \sum_{t=0}^{t=T^j(\theta)} \beta^t \left[\rho^j(\theta) V(q_t^j(\theta)) - Y_t^j(\theta)\right] \tag{11}$$

where E_θ are the principal's expectations over the n agents and $\theta = (\theta^1, ..., \theta^j, ..., \theta^n)$ subject to (7), (8), (9) and (10). Following similar arguments as before, we can show that IC and IR are satisfied if the following is true[10]:

$$q_t^j(.) \text{ and } T^j(.) \text{ are only functions of } \theta^j$$

$$U^{j\prime}(\theta^j) = -E_{\theta^{-j}} \rho^j(\theta^j, \theta^{-j}) Q^j(\theta^j) \text{ with } Q^j(\theta^j) = \sum_{t=0}^{t=T^j(\theta^j)} \beta^t q_t^j(\theta^j)$$

$$U^j(\bar{\theta}) = 0$$

$$\rho^j(\theta^j, \theta^{-j}) \text{ and } Q^j(\theta^j) \text{ are decreasing in } \theta^j$$

Ignoring in a first time the last relation, the problem can be rewritten:

$$\max_{\rho^j(.), q_t^j(.)} E_\theta \sum_{j=1}^{j=n} \rho^j(\theta) \sum_{t=0}^{t=T^j(\theta^j)} \beta^t \left[V(q_t^j(\theta^j)) - K(\theta^j) q_t^j(\theta^j)\right]$$

with $K(\theta^j) = \theta^j + \frac{F(\theta^j)}{f(\theta^j)}$.

It is clear that the problem can be decomposed in two steps. First is the maximization with respect to $q_t^j(.)$, second is the maximization with respect to $\rho^j(\theta)$.

Now, let $W^j(\theta^j)$ the discounted sum of net "adjusted" surpluses if the agent j is selected:

$$W^j(\theta^j) = \sum_{t=0}^{t=T^{j*}(\theta^j)} \beta^t \left[V(q_t^{j*}(\theta^j)) - K(\theta^j) q_t^{j*}(\theta^j)\right]$$

[10] See Laffont and Tirole (1987) for the first relation.

Then, the second step of the problem reduces to:

$$(\mathcal{P})\begin{cases} \max\limits_{\rho^j(.)} E_\theta \sum\limits_{j=1}^{j=n} \rho^j(\theta) W^j(\theta^j) \\ \text{subject to:} \\ \sum\limits_{j=1}^{j=n} \rho^j(\theta) \leq 1 \text{ and } 0 \leq \rho^j(\theta) \leq 1 \end{cases}$$

We now establish :

Proposition 5 *i)The optimal contract is such that exhaustion occurs for all* θ^j *and:*

$$q_t^{j*}(\theta^j) = \omega\left(K(\theta^j) + \beta^{-t}\lambda^j(\theta^j)\right)$$
$$T^{j*}(\theta^j) = \frac{1}{\ln\beta}\ln\left(\frac{\lambda^j(\theta^j)}{v(0)-K(\theta^j)}\right)$$

ii) The discounted sum of net "adjusted" surpluses if the agent j *is selected, is decreasing for all* θ^j *:*

$$W^{j\prime}(\theta^j) < 0.$$

iii) And the award of the mining contract is such that:

$$\rho^{j*}(\theta) = 1 \quad \textit{if } \theta^j = \min_l \theta^l$$
$$\rho^{j*}(\theta) = 0 \quad \textit{otherwise.}$$

Proof. i) See proofs of the previous section. Using appendix B, it also can be shown that $Q^j(\theta^j)$ is decreasing in θ^j.

ii) This part helps solving the problem $(\mathcal{P})$, see appendix C.

iii) We solve the problem $(\mathcal{P})$. The Langrangean for agent j,$j = 1, 2, ..., n$, is:

$$\mathcal{L}^j(\theta) = E_\theta\left\{\sum_{j=1}^{j=n} \rho^j(\theta)W^j(\theta^j) + \mu(\theta)(1 - \sum_{j=1}^{j=n} \rho^j(\theta)) + \overline{\delta}^j(\theta)(1-\rho^j(\theta)) + \underline{\delta}^j(\theta)\rho^j(\theta)\right\}$$

where $\overline{\delta}^j(\theta)$,$\underline{\delta}^j(\theta)$ and $\mu(\theta)$ are Kuhn and Tucker multipliers. Necessary conditions are:

$$W^j(\theta^j) - \mu(\theta) - \overline{\delta}^j(\theta) + \underline{\delta}^j(\theta) = 0$$

If there exists an agent j such that $\rho^j(\theta^j) = 1$, then $\underline{\delta}^j(\theta) = 0$ and:

$$W^j(\theta^j) \geq \mu(\theta) \geq 0$$

Since from lemma 2, $W^j(\theta^j)$ is decreasing in θ^j, the proposition must hold and $\rho^j(\theta^j, \theta^{-j})$ is effectively (non continuously) decreasing in θ^j. ■

A glance at the Proposition 5 tells us that auctioning does not affect the design of the concession contract. Indeed, Proposition 4 is still valid whatever efficient the agent is, the contract structure remains identical. It is clear that auctioning yields to a separation procedure among agents, in fact the separation property of the auction theory applies here. In few words (see Laffont, Tirole (1993) p. 328 for details), the separation property tells us "*that the winner faces the same incentives as if there had been no bidding competition*", *ibid.* The winner's revelation strategy is then unaffected by auctioning.[11]

Auctioning[12] is an easy way to award the concession contract, when there is several agents in competition. Selling by auction the concession leads to a efficient separation procedure among agents so that the lowest cost agent is assigned to carry out the concession contract. This result can be viewed as an application of Laffont and Tirole (1987) analysis to exhaustible resource management problem when it is solved using incentive contracts. The bilateral *ex ante* situation analyzed in sections 3 and 4 is then robust when agents compete to win the concession.

6 Extensions and conclusion

At least two central assumptions could be relaxed in our framework.

First it could be relevant to make a specific assumption concerning the agent's cost function related to non renewable stylized facts. Second, we assume so far that the principal can commit itself on all periods, in that case *ex post* inefficiency of the contract can arise.

[11] In return its award is expected to be lesser, because alternative bidders simply reduce the transfert given to the winner.

[12] Laffont, Tirole (1993) show that the implementation of these auctions by a dominant strategy auction is possible. We could use the argument in our mining context.

6.1 Stock effects and cost of depletion

In the exhaustible resources literature, it is well known (Livernois and Uhler, 1987) that extraction costs are greatly influenced by stock effects. As long as the resource is depleted, its stock *in situ* diminishes and extraction cost rises owing to the fact that it becomes more and more difficult to dig. As pointed out in Osmundsen (1998), this stock effect introduces the possibility for the agent (firms or operators) to learn the level of *in situ* reserves and keeps it as private informations. Beside, with a very general but drastic cost structure, Osmundsen shows that the overproduction result is no longer valid. In fact, with his asymptotic assumption states that the cumulative extraction costs approach infinity when the remaining resource base approaches zero, it follows that exhaustibility constraint are non binding, henceforth the Baron and Besanko result (see our lemma 1) applies for each period.

Using a less general stock effect model (*e. g.* linear), it could be possible[13] to reintroduce some overproduction in the last periods. Let $\theta(S_t)$ be a non decreasing function which is bounded above by $\theta(0)$. Resource exhaustibility could also be warranted by assuming $\theta(0) < v(0)$, that is, in resource economics, the choked-off price exceeds the highest unit cost of extraction (of the most efficient agent). In this case, the results so far obtained remain valid. Nevertheless, the impacts of stock effects on the exhaustion date are more ambiguous.

6.2 Commitment and renegotiation

The central question of the principal commitment has been addressed in the literature[14] we cite before. One problem in studying the dynamics of non renewable resource management is to obtain consistent paths in the principal-agent game (where the principal is Stackelberg leader) so as to ensure the credibility of equilibrium strategies. Gaudet *et al.* (1995) analyze a closed-loop royalties mechanism which is renegotiation- proof, but they suppose that the efficiency parameters (*i.e.* the types) are temporally independent so they brush aside the ratchet effect problem.

[13] See Poudou and Thomas (2000) for this point.

[14] Osmundsen (1998) assumes also commitment of the principal.

In our work, we allow to consider this effect, but adopt the assumption of principal long term commitment over the whole time horizon. Even though this assumption points out an alternative way to reach the overproduction in later stage of resource extraction under asymmetric information, the weakness resides in that ex-post inefficiency results and motivates the principal to renegotiate the terms of the mining contract. The next step of this study should overcome this difficulty.

As a conclusion, this paper points out a non standard overproduction result in incentive theory. Exhaustibility of the production resource reserve is the stepping stone: when the resource base is very scarce (the exhaustion happens in finite time...), overproduction is optimal at the end of the contract because the gains from exhaustibility dominate the informational rent loss, moreover these phenomena lengthen the horizon of the contract but reduce mining rent.

7 Appendices

7.1 Appendix A: Feasible mechanisms

In order to simplify the developments, we suppose that $T'(\theta) = \lim_{d\theta \to 0} \frac{T(\theta+d\theta)-T(\theta)}{d\theta}$ exists as a continuous application $T' : \left[\underline{\theta}, \bar{\theta}\right] \to \mathsf{Z}$, so $\forall\theta \in \left[\underline{\theta}, \bar{\theta}\right]$, $T'(\theta) \in \{-1, 0, 1\}$. Incentive constraint (2) can be written as:

$$\theta = \arg \max_{\hat{\theta} \in \left[\underline{\theta}, \bar{\theta}\right]} U(\hat{\theta}, \theta)$$

which is equivalent to

1.
$$\frac{\partial U(\hat{\theta}, \theta)}{\partial \hat{\theta}}_{|\hat{\theta}=\theta} = \sum_{t=0}^{t=T(\theta)} \beta^t \left[Y_t'(\theta) - \theta q_t'(\theta)\right] + T'(\theta) \left(\beta^{\mathcal{T}(\theta)} \left[Y_{\mathcal{T}(\theta)}(\theta) - \theta q_{\mathcal{T}(\theta)}(\theta)\right]\right) = 0$$
where $\mathcal{T}(\theta) = T(\theta) + T'(\theta)$ is also an optimal horizon due to the variation of the type θ

2.
$$\frac{\partial^2 U(\hat{\theta}, \theta)}{\partial \hat{\theta}^2}_{|\hat{\theta}=\theta} \leq 0$$

Applying envelop theorem to the agent's utility, we have:

$$\forall\theta \in \left[\underline{\theta}, \bar{\theta}\right], \; U'(\theta) = -\sum_{t=0}^{t=T(\theta)} \beta^t q_t(\theta) \equiv -Q(\theta) < 0 \qquad \text{(A.1)}$$

Moreover, totally differentiating $\partial U(\hat{\theta}, \theta)/\partial\hat{\theta}_{|\hat{\theta}=\theta} = 0$ leads to:

$$\frac{\partial^2 U(\hat{\theta}, \theta)}{\partial\hat{\theta}^2}_{|\hat{\theta}=\theta} = -\frac{\partial^2 U(\hat{\theta}, \theta)}{\partial\hat{\theta}\partial\theta}_{|\hat{\theta}=\theta} \leq 0$$

$$\Leftrightarrow \forall\theta \in \left[\underline{\theta}, \bar{\theta}\right], Q'(\theta) \equiv \sum_{t=0}^{t=T(\theta)} \beta^t q_t'(\theta) + T'(\theta)\beta^{T(\theta)} q_{T(\theta)}(\theta) \leq 0 \qquad \text{(A.2)}$$

Relations (A.1) and (A.2.) are incentive constraints (2') and (2") in the text. These necessary conditions are also sufficient because U verifies the Spence-Mirlees condition that is $\partial^2 U / \partial Q \partial\theta = -1 < 0$.

7.2 Appendix B: IC2 checking

Because of the optimal horizon condition (recall: $q_{T(\theta)} = 0$), checking (2"') or (A.2) just above, comes to verify:

$$Q'(\theta) \leq 0 \Leftrightarrow \sum_{t=0}^{t=T(\theta)} \beta^t q_t'(\theta) \leq 0 \qquad \text{(B.1)}$$

From (5), computing $q_t'(\theta)$ gives for all periods:

$$q_t'(\theta) = \omega'\left(K(\theta) + \beta^{-t}\lambda(\theta)\right)\left[K'(\theta) + \beta^{-t}\lambda'(\theta)\right] \qquad \text{(B.2)}$$

A glance at (B.2) show us that it does a date $t_0(\theta)$ such that

$$\forall t \lesseqqgtr t_0(\theta) \Leftrightarrow q_t'(\theta) \lesseqqgtr 0$$

Indeed and because $\omega'(\cdot) < 0$,

$$\begin{aligned} q_{t_0(\theta)}'(\theta) &= \omega'\left(v\left(q_{t_0(\theta)}(\theta)\right)\right)\left[K'(\theta) + \beta^{-t_0(\theta)}\lambda'(\theta)\right] = 0 \\ \Leftrightarrow \; & t_0(\theta) = \frac{1}{\ln(\beta)} \ln\left(-\frac{\lambda'(\theta)}{K'(\theta)}\right) \end{aligned}$$

For the moment we just assume that $0 < t_0(\theta) < T(\theta)$, for all θ.

>From the binding exhaustibility constraint (and $q_{T(\theta)} = 0$) we can derive that:

$$\sum_{t=0}^{t=T(\theta)} q_t'(\theta) = 0 \Leftrightarrow -\sum_{t=0}^{t=t_0(\theta)} q_t'(\theta) = \sum_{t=t_0(\theta)}^{t=T(\theta)} q_t'(\theta) > 0 \qquad \text{(B.3)}$$

Then using (B.3), the mean theorem can be apply to (B.1) for each relevant interval, that is:

$$\sum_{t=0}^{t=T(\theta)} \beta^t q_t'(\theta) = \beta^{t_1} \sum_{t=0}^{t=t_0(\theta)} q_t'(\theta) + \beta^{t_2} \sum_{t=t_0(\theta)}^{t=T(\theta)} q_t'(\theta)$$

where $t_1 \in]0, t_0(\theta)[$ and $t_2 \in]t_0(\theta), T(\theta)[$, so $t_1 < t_2$. So if $0 < t_0(\theta) < T(\theta)$, this finishes the IC2 checking:

$$\sum_{t=0}^{t=T(\theta)} \beta^t q_t'(\theta) = \left(\beta^{t_2} - \beta^{t_1}\right) \sum_{t=t_0(\theta)}^{t=T(\theta)} q_t'(\theta) < 0 \tag{B.4}$$

because $t_2 > t_1$ so $\beta^{t_2} - \beta^{t_1} < 0$.

Finally let us prove that $0 < t_0(\theta) < T(\theta)$, for all θ. From (B.3) and using again the mean theorem, we can derive that:

$$\begin{aligned} \sum_{t=0}^{t=T(\theta)} \omega'(v(q_t(\theta)))\left[K'(\theta) + \beta^{-t}\lambda'(\theta)\right] &= 0 \Rightarrow \lambda'(\theta) = -K'(\theta)\frac{\sum_{t=0}^{t=T(\theta)} \omega'(v(q_t(\theta)))}{\sum_{t=0}^{t=T(\theta)} \beta^{-t}\omega'(v(q_t(\theta)))} < 0 \\ &\Leftrightarrow \lambda'(\theta) = -K'(\theta)\beta^{t_3} < 0 \end{aligned} \tag{B.5}$$

with $t_3 \in]0, T(\theta)[$. This last relation shows us that $-\frac{\lambda'(\theta)}{K'(\theta)} = \beta^{t_3} < 1$ so $t_0(\theta) > 0$ for all θ. More from (B.2) and (B.5):

$$\begin{aligned} q_{T(\theta)}'(\theta) &= \omega'(v(0))\left[K'(\theta) + \beta^{-T(\theta)}\lambda'(\theta)\right] \\ &= \omega'(v(0))\, K'(\theta)\left[1 - \beta^{t_3 - T(\theta)}\right] > 0 \end{aligned} \tag{B.6}$$

So if $t_0(\theta) \geq T(\theta)$, then $q_{T(\theta)}'(\theta) < 0$ which contradicts (B.6) then $t_0(\theta) < T(\theta)$, so IC2 is checked.

As an extension of this proof, we can now see that $T'(\theta) > 0$. >From (6) in the text and (B.5):

$$\begin{aligned} T'(\theta) &= \frac{1}{\ln(\beta)}\left[\frac{1}{v(0) - K(\theta)}\right]\left[\lambda'(\theta)(v(0) - K(\theta)) + \lambda(\theta) K'(\theta)\right] \\ &= \beta^{t_3} K'(\theta)\left[\ln(\beta)(v(0) - K(\theta))\right]^{-1}\left[\beta^{-t_3}\lambda(\theta) - (v(0) - K(\theta))\right] \end{aligned}$$

Using (6) again implies:

$$T'(\theta) = \lambda(\theta) K'(\theta)\left[\ln(\beta)(v(0) - K(\theta))\right]^{-1}\left[1 - \beta^{t_3 - T(\theta)}\right] > 0 \tag{B.7}$$

7.3 Appendix C: Proof of Proposition 5

First, let us proof that $W^j(\theta^j)$ is decreasing in θ^j.

$$W^{j\prime}(\theta^j) = \sum_{t=0}^{t=T^{j*}(\theta^j)} \beta^t \left[v\left(\left(q_t^{j*}(\theta^j)\right) - K(\theta^j)\right] q_t^{j*\prime}(\theta^j) - K'(\theta^j) \sum_{t=0}^{t=T^{j*}(\theta^j)} \beta^t q_t^{j*}(\theta^j) \quad \text{(C-1)}$$
$$+T^{j*\prime}(\theta^j)\left[V\left(q^{j*}_{\mathcal{T}^{j*}(\theta^j)}(\theta^j)\right) - K(\theta^j) q^{j*}_{\mathcal{T}^{j*}(\theta^j)}(\theta^j)\right]$$

where $\mathcal{T}^{j*}(\theta^j) = T^{j*}(\theta^j) + T^{j*\prime}(\theta^j)$ is also an optimal horizon due to the variation of the type θ^j. From previous results, we know that $q^{j*}_{\mathcal{T}^{j*}(\theta^j)}(\theta^j) = 0$ so $V\left(q^{j*}_{\mathcal{T}^{j*}(\theta^j)}(\theta^j)\right) - K(\theta^j) q^{j*}_{\mathcal{T}^{j*}(\theta^j)}(\theta^j) = 0$. Moreover from the proposition 4, we know that exhaustion occurs for all j, so $\sum_{t=0}^{t=T^{j*}(\theta^j)} q_t^{j*}(\theta^j) = \overline{S}$ and

$$\frac{d}{d\theta}\left[\sum_{t=0}^{t=T^{j*}(\theta^j)} q_t^{j*}(\theta^j)\right] = \frac{d\overline{S}}{d\theta}$$
$$\Leftrightarrow \sum_{t=0}^{t=T^{j*}(\theta^j)} q_t^{j*\prime}(\theta^j) + T^{j*\prime}(\theta^j) q^{j*}_{\mathcal{T}^{j*}(\theta^j)}(\theta^j) = 0$$
$$\Leftrightarrow \sum_{t=0}^{t=T^{j*}(\theta^j)} q_t^{j*\prime}(\theta^j) = 0$$

Hence (12) becomes:

$$W^{j\prime}(\theta^j) = \lambda^j(\theta^j) \sum_{t=0}^{t=T^{j*}(\theta^j)} q_t^{j*\prime}(\theta^j) - K'(\theta^j) \sum_{t=0}^{t=T^{j*}(\theta^j)} \beta^t q_t^{j*}(\theta^j)$$
$$= -K'(\theta^j)\beta^{\widetilde{t}^{j*}(\theta^j)}\overline{S} < 0$$

where $\widetilde{t}^{j*}(\theta^j) \in \left]0, T^{j*}(\theta^j)\right[$.

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