TIME ALLOCATION AND SELLING MECHANISMS IN OUTCRY AUCTIONS

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Abstract

This paper examines the effect of binding time constraints on the auctioneer's strategy in an outcry auction, using data from an original survey of auctioneers. A time-allocation model is introduced, in which the length of time allocated to the sale of an item is traded off against its realized price, and is applied to (1) the relationship between the time allocation and the ex ante value of an item; (2) the effect of the time-dependence of the number of bidders present, on both the time allocation and the order in which items of differing value are presented for sale; and (3) the auctioneer's choice of selling technique, such as batching and selling by choice, which can save time despite the fact that they reduce selling prices. The results corroborate and provide theoretical underpinning for observed auctioneer behavior.

TIME ALLOCATION AND SELLING MECHANISMS

IN OUTCRY AUCTIONS[1]

1. Introduction

In this paper we use data from an original survey to study the effect of binding time constraints on auctioneer strategies in outcry, or English, auctions.[2] With few exceptions, the typical outcry auction is a race against the clock, as many lots need to be sold within a given period. Hence the auctioneer is pressed to keep the auction and bidding moving along. In preliminary field investigations for this research, auctioneers reported that *time* was one of the most important considerations in the conduct of an auction. The time factor imposes significant transactions costs on the auctioneer, including not only the cost of holding over merchandise that does not get presented for sale during the designated period of the auction, but also the more important implicit cost that arises from declining audience size at lengthy auctions. Here we examine auctioneer behavior in a time allocation model based on the inherent time-revenue tradeoff of the outcry auction.

For this work, we designed and tested a survey to elicit information on aspects of the time element in auctions. A sample of auctioneers was sampled randomly from the membership of the American Association of Auctioneers. One-hundred and forty-eight professional auctioneers representing a broad range of auction experience responded to the mail survey. The average respondent had been in the auction business approximately 22 years and conducted an average of 65 auctions per year.[3] The

[1] We wish to thank the National Auctioneers Association and its officers and members for much useful information, data, and insight used in this paper. We are especially grateful to Joseph Keefhaver, Executive Vice President of the National Auctioneers Association, and Dennis Eberhart, President of the Toledo Auction School.

[2] The NAA estimates that the approximately 11,000 auction firms in the U.S. sell about $162 billion of property annually through outcry auction. It is a common mechanism for selling commercial and personal real estate, automobiles, livestock, heavy machinery, wine, art, antiques, and a wide variety of other agricultural and nonagricultural commodities.

[3] Further detail on the survey and sample are presented in Appendix B.

results are presented below along with a formal representation of how auctioneers report that time affects their techniques for conducting a sale.

Since the data for this research represents auction conduct from the *auctioneer's* point of view, here we will focus primarily on the auctioneering decision. To realistically introduce the complexities of bidder behavior in a bidder utility maximization model would require data beyond the scope of the present survey and will be the subject for future research. However, the current paper will explore and model critical factors in the auctioneer's maximization process.

Institutional features of auctions have been investigated by Ashenfelter (1989), Vanderporten (1992), Gordy (1996), and Beggs and Graddy (1997), among others.[4] In the English, or outcry, auction, the auctioneer establishes an opening bid, and the bidding proceeds upward in increments called out by the auctioneer until the gavel falls at the last bid the auctioneer is able or willing to elicit. It is assumed in the literature that the gavel price is equal to the second-highest evaluation in the audience. One contribution of the current paper is to demonstrate that there are circumstances under which this will not be true. Note also that auction houses typically operate on a predetermined commission basis. Consequently, an auctioneer can be thought of as a "total take", or total revenue, maximizer.[5]

In the first part of the paper, we show how the optimal time allocation comes from weighing the gain from continuing the bidding on a lot against the benefit from allocating time to other lots, holding bidder attendance constant. In our survey of auctioneers, 92 percent report that they have closed bidding even though they realized a higher bid was possible, in order to move on to other lots in a timely fashion. In our model, the auctioneer is faced with the problem of maximizing revenue subject to a binding *time constraint*. The time interval between bids plays a crucial role here. Two-thirds of the auctioneers in our

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[4] For a review of the auction literature, see McAfee and McMillan (1987) and Milgrom (1989).

[5] State laws governing auctions vary in the U. S., but this variation does not affect the auctioneering principles described here. We also ignore illegal practices such as calling a nonexistent bid and the use of in-house agents bidding in the audience.

sample reported that the time interval between bids lengthens as the bid approaches the gavel price.[6] This pattern is incorporated in our model: time devoted to selling an item increases the sale price but at a decreasing rate, and it follows that the auctioneer may stop the bidding before the highest possible bid is reached. We show that because of the time-revenue tradeoff, the winning bid may not be equal to the second-highest bidder valuation. We also show that the auctioneer will spend more time on items with higher *ex ante* value.

Next, we extend our model to examine formally the effects of varying bidder attendance on the auctioneer's time allocation problem. We argue that the implicit value of time rises with attendance, and we show that the auctioneer spends less time on the sale of any one lot if attendance is expected to decline with time. (Of the sample, 72 percent reported that attendance increases to a certain maximum size after an auction has begun and then decreases steadily thereafter.) Approximately 90 percent of auctioneers reported that they strategically set their order of sale to take account of attendance. In our model, we show formally how the auctioneer does this, and we derive conditions under which higher value lots are offered during higher attendance periods.

Finally, we examine the choice of auctioneer's selling technique in light of the time-revenue tradeoff. Selling technique refers to how items are presented for bid and how bids are executed by the auctioneer. The existing literature mainly focuses on what we shall refer to as the "default" auction technique where items are offered one at a time, with the high bidder taking permanent possession at the fall of the gavel. In practice, there are four commonly used alternatives to the default auction: *batching*, *choice*, *by the piece*, and *together and separately*. We find that in some cases selling techniques which result in time savings are chosen even though they have been shown to directly reduce sale price. Our results corroborate and provide theoretical underpinning for auctioneer behavior which has been observed and has been reported in our survey.

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[6] Some auctioneers reported variations on this pattern that depended on the price range of the item being auctioned.

2. The Time Factor

Time plays an important role in an auction because of the transactions costs it can impose. Transactions costs include first the direct costs associated with holding unsold merchandise. These exist because auctions are typically conducted under contract to present all items and/or property under consideration for sale. Although the auction literature has commonly taken price discovery as the primary *raison d'être* for the auction mechanism, a rationale frequently cited by auctioneers is the need for sure sale by a certain deadline.[7] If items do not get presented for sale during the specified time of the auction, they will have to be disposed of at a future auction, requiring additional costs. Second, and more importantly, attendance varies over time: the buying crowd will thin out as the day (or evening) wears on.[8] Since few bidders are interested in every item presented for sale, much of the time at any auction is a 'waste' for the typical bidder. This time spent waiting must be counted as a cost of acquiring the item for the bidder and may cause some bidders to leave, thus resulting in a lower sale price than would have occurred with a full house. A slow pace of sale may also have a negative "reputation effect" for an auctioneer and may result in loss of business. For these reasons, auctioneers often speed up the pace at peak attendance times. As noted, approximately 90 percent of our survey sample reported that they try to place the most valuable merchandise strategically in the *order of the sale* at the time of maximum attendance.

The realized price of a lot will depend on the length of time for which the auctioneer continues the bidding on that lot. In deciding to stop, the auctioneer must weigh the gain in price from continuing the bidding against the benefit from allocating time to other lots. The pattern of time intervals between bids reported as the norm by two-thirds of the auctioneers in our survey, and which will be modeled here, is as follows: it is short early in the bidding, and lengthens as the bid approaches the final price. The slowdown in bidding has two effects on the auction process: (1) it increases the marginal time cost to the

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Incorporation of all variants is again beyond the scope of the present paper, and will be examined in future work.

[7] For example, estates being sold to meet probate deadlines and court ordered bankruptcy sales choose the auction mechanism for this reason in a large fraction of cases.

auctioneer of eliciting a higher bid for the current lot; and (2) it signals to the bidders that the bid is close to the high-value bidder's valuation.

The slow-down is an observed and empirically reported effect, and the time allocation model developed in this paper does not depend on a particular interpretation of the slow-down. It is interesting to note, however, that the commonly cited interpretation of auctioneers is that it arises from bidder uncertainty and the need to process information. Under this interpretation, as the bid approaches the bidder's *ex ante* private valuation, which equals the sum of a noisy signal of common resale value and a bidder specific component, the bidder updates his valuation in order to improve the precision of his estimate. Hence, according to this interpretation, without bidder uncertainty there should be little variation in the time intervals, and the bid would rise linearly with time. However, with bidder uncertainty, the closer the bid gets to his *ex ante* valuation, the more important it is for the bidder to improve his estimate, because winning at a bid closer to the *ex ante* value carries a higher probability of regrets or losses from overpaying. The bidder may improve his estimate by simply reprocessing his own information, by consulting a colleague or price guide, or by using information from others' bids to update his prior beliefs – all requiring time.[9] Initially, the effect of information processing may not affect the time interval if the bidders with the higher valuations (and hence less need to process information in the early stages) take the lead in bidding. As the bid approaches the second highest *ex ante* valuation, however, even the higher-valuation bidders must re-evaluate, and this will slow their bids. Hence the time interval between bids increases as the difference between the current bid and the second highest *ex ante* value decreases, which in turn decreases the auctioneer's gain per unit of time allocated to the sale of this

[8] Here we assume no absentee bids, which are not universally accepted in the industry.
[9] At high-stakes auctions, dealers bidding on behalf of clients may be connected to the client via phone or computer and need to convey the situation and elicit instructions for further bidding.

lot. There can, of course, be other interpretations for the slow-down in bidding, but this will not affect our results.[10]

3. The Time Allocation Model[11]

In this section, we present a model of the revenue-time tradeoff inherent in an outcry auction and characterize the auctioneer's optimal time allocation. The model is first introduced in a simplified case of what we have called the default auction, where attendance is constant and the order of sale is fixed. We then consider the effect of varying attendance, which leads to the problem of optimizing the order of sale (i.e., determining the time at which an item of given value should be offered for sale).

Basic Model

The auctioneer sells item *I* using the outcry mechanism with uncertainty over the price he will attain. We suppose that τ units of time spent on the sale of item *i* bring a bid price of $b_i(\tau)$, where $b_i(\tau)$ is modeled as a nondecreasing stochastic process with mean $p_i(\tau)$, i.e., $b_i(\tau) = p_i(\tau) + u_i(\tau)$ with $\mathrm{E}[u_i(\tau)] = 0$. We assume that each item is individually negligible relative to the total worth of the auction. This will imply that the shadow value of time can be considered invariant over the potential set of sales techniques which the auctioneer considers, for any item or set of items.

At the start of bidding, the auctioneer's expectation of the sale price, given a time allocation of τ is $\mathrm{E}[b_i(\tau)] = p_i(\tau)$. For simplicity, we represent this as

$$p_i(\tau) = v_i\, H(\tau)$$

[10] The slowdown in bidding could also be interpreted as the result of the drop-out of bidders as the bid level increases. However, this does not account for the fact the slowdown is observed even when only two bidders are active.

[11] In what follows, for simplicity we ignore the mechanisms by which the bidding is begun. This does not affect the conduct of the selling techniques which are the primary focus of this paper.

where v_i is a fixed item-specific parameter, interpreted as the expected sale price for item *i* if auctioned with no time constraint, and taken in the existing literature to be the second-highest evaluation in the audience. For convenience, we refer to v_i as the "valuation" of the item. $H(\tau)$ is a function independent of the item, with $0 \leq H(\tau) \leq 1$, and with $H(\tau) \rightarrow 1$ as $\tau \rightarrow \infty$. Denoting $h(\tau) = dH(\tau)/d\tau$, we assume that $h(\tau) > 0$ and $h'(\tau) < 0$ for $\tau > 0$, in accordance with the observed slow-down in the rate of bidding as τ increases.[12]

As the bidding proceeds, the auctioneer updates his expected sale price as a function of τ, in order to weigh the expected gain from continuing the bid against the implicit value of allocating time on other items. We assume here that $u_i(\tau)$, the unanticipated component of $b_i(\tau)$, can be represented as a martingale process, i.e., $\mathrm{E}[u_i(\tau+t) \mid \tau] = u_i(\tau)$ for $t > 0$.[13] It then follows that the auctioneer's expectation at elapsed time τ of the bid after t more units of time is

$$\mathrm{E}[b_i(\tau + t) \mid \tau] = \mathrm{v}_i\,[H(\tau + t) - H(\tau)] + b_i(\tau)$$

The decision whether to continue the bidding depends on expected incremental revenue, i.e. the function $\mathrm{E}[b_i(\tau + t) \mid \tau] - b_i(\tau)$ for $t > 0$, which is equal to $v_i\,[H(\tau + t) - H(\tau)]$. Since this does not depend on the current bid $b_i(\tau)$, the auctioneer's stopping rule is identical to what it would be if the bid followed the fixed path $\mathrm{v}_i H(\tau)$. This allows us to replace $b_i(\tau)$ by the function $v_i H(\tau)$ in formulating the auctioneer's objective function.

The auctioneer maximizes total receipts from sale subject to a time constraint. The simplest case is when the auction is restricted to a fixed time period of length *T* (after which, for example, the bidding audience may dissipate or the auctioneer may have to vacate the premises). As we have seen, the

[12] The function *H* incorporates an optimal choice of bid increment by the auctioneer.
[13] This implies that only the current level of the bid is relevant to the auctioneer in updating his beliefs.

optimization problem is equivalent to one in which the bid $b_i(\tau)$ follows the nonstochastic path $v_i H(\tau)$. The objective function is therefore given by

$$\max_{\{\tau_i\}} \sum_i v_i\, H(\tau_i) + \lambda[T - \sum_i \tau_i] \tag{1}$$

where τ_i is the amount of time allocated to the sale of item *i*. The resulting optimal allocation is

$$\tau_i = \tau^*(v_i) = \begin{cases} h_i\left(\dfrac{\lambda}{v_i}\right) & \text{if } \lambda / v_i < h(0) \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where λ, the shadow value of time, is determined by

$$\sum_i h^{-1}\left(\frac{\lambda}{v_i}\right) 1\{v_i > \lambda / h(0)\} = T \tag{3}$$

For a large number of items, we may approximate the distribution of valuations v_i by a continuous distribution with density *f,* and formulate equation (3) as

$$\int h^{-1}\left(\frac{\lambda}{v}\right) f(v)\, dv = T \tag{4}$$

If this distribution *f* is representative of the general class of items sold in markets of this type, this determines λ without the need to enumerate the values of all individual items in the sale.

This basic model has several useful features. Most importantly it highlights the revenue-time tradeoff. On the one hand, the more time devoted to selling a particular item or items, the greater the receipts from their sale; on the other hand, this results in less time available for other items and hence lower receipts from their sale. The optimal time allocation gives the result that the auctioneer spends more time on the more valuable items, i.e., $d\tau^*(v)/dv > 0$. The model implies that the auctioneer will not waste time on items at or below a certain threshold valuation. Specifically, an item is not offered for auction unless its value exceeds $v_{\min} = \lambda / h(0)$, and $\tau^*(v_i) > 0$ for all $v_i > v_{\min}$. In practice, items below this minimum are batched (see Section 4.1).

Under the assumption of a large number of items, the auctioneer can consider an item or group of items in isolation, as the value of each item is small relative to the whole. This implies that the auctioneer does not have to compute all possible scenarios for all items. Rather the auctioneer need only have a sense of the worth of time in order to decide on the optimal time allocation for a particular item.

It is important to note that in a more realistic auction setting, the bidder will undoubtedly develop his own strategy for responding to the auctioneer's time allocation decisions. However, a full development of the bidder's response is beyond the scope of the current paper and will be dealt with in future research. We therefore proceed under the simplifying assumption that bidder strategy is passive. This assumption has the advantage of allowing us to focus on the behavior of the auctioneer.

Attendance Factor

Until now, we have assumed that attendance is constant over the period of the auction. However, bidders are faced with heterogeneous time constraints, which means that attendance should be variable over the length of the auction. In our survey of auctioneers, over 90 percent reported that the size of the bidding audience changes significantly during an auction. The reported patterns of change are given in Table 1 below.

Table 1: Percentage Reporting Patterns of Change in Bidding Audience

Pattern of Change	Percentage of Sample
Increases steadily	4.9 %
Increases to certain maximum size, then decreases	71.5
Decreases steadily	5.6
Variable pattern	8.3
No significant changes	9.7

We see that the total percentage reporting *decreases* in the bidding audience, either monotonically or after some point as an auction proceeds, is 77 percent. As attendance drops, so does the expected sale price. Changes in attendance over the duration of the auction require the auctioneer not only to take attendance into account in determining the optimal time allocation for each item, but also to optimize the order in which items are presented for sale. In this subsection, we assume a fixed, exogenously-given order of sale. In the next section, we allow the auctioneer to strategically organize the order of sale as well.

The expected sale price is now a function of the expected sale price with unlimited time, the time devoted to the sale, and the level of attendance at the time of the sale. We rewrite the sale price as

$$p_i = \nu_i \, G(A(t_i)) \, H(\tau_i)$$

where v_i, τ_i and $H(\cdot)$ are as previously defined, and $G(\cdot)$ is a function which is increasing in attendance A. Attendance is a function of the clock time t_i, which ranges from 0 to T. The attendance function $A(\cdot)$ is allowed to take a general shape. We require, in our exposition, that the auctioneer know the functions $A(\cdot)$ and $G(\cdot)$.[14] For notational simplicity, we define $B(t_i) = G(A(t_i))$.

[14] Our model extends naturally to the case of attendance uncertainty and the main results do not change.

Let the item number (i = 1, 2, …, N) denote the order of sale, which is taken as fixed for the moment. Using this convention, we can write the clock time as $t_i = \sum_{j<i} \tau_j$. Time spent on the current item i has three effects: as τ_i increases, (1) the expected sale price p_i increases, (2) the time available for selling other items decreases, and (3) the clock time t_j at which each subsequent item j is presented for sale is postponed, thus shifting it to a different attendance level.

The auctioneer maximizes

$$\max_{\tau_i} \sum_i v_i \, B(\sum_{j<i} \tau_j) \, H(\tau_i) + \lambda [T - \sum_i \tau_i] \tag{5}$$

The optimal time allocation for item i is

$$\tau_i^* = h^{-1}\left(\frac{\lambda - \sum_{j>i} v_j \, B'(t_j) \, H(\tau_j^*)}{v_i \, B(t_i)} \right) \tag{6}$$

where $B(t)$ is assumed to be differentiable with derivative $B'(t) = \partial B(t) / \partial t$, and the function $h^{-1}(\cdot)$ is understood to be zero if its argument is greater than $h(0)$. This allocation is determined by both current and future attendance. On the one hand, higher current attendance increases the term $\nu_i \, B(t_i)$, which increases the marginal value of time spent on items currently being sold, and so provides an incentive to increase τ_i. On the other hand, if attendance is decreasing, time spent on the current item will shift subsequent items to a period of lower attendance, reflected by the fact that the term $\sum_{j>i} v_j \, B'(t_j) H(\tau_j^*)$ is negative. This shift increases the marginal cost of time and provides an incentive to rush through items to take advantage of periods of peak attendance, i.e., to decrease τ_i. The converse holds when attendance is increasing.

Strategic Ordering of Sales

With variable attendance, the auctioneer will strategically order the items for sale.[15] In this section we study the accepted view that items with higher valuation ν should be sold during periods of higher attendance. Indeed, 90 percent of our survey sample reported that they ordered their sales to this end. However, this ordering does not follow from the results so far. For discrete items, a general solution to the order-of-sale problem is not tractable. Consequently, we turn to the limiting case of a continuum model (i.e. a model in which each item sold is infinitesimally small in both time and value), which should provide a good approximation when the number of items is large, and determine the conditions under which the optimal order of sale has $v_i \geq v_j$ if and only if $A(t_i) \geq A(t_j)$.

First, define a new time variable $s = s(t)$ such that the function $\widetilde{A}(s)$ defined by $\widetilde{A}(s(t)) = A(t)$ is non-increasing in s and is differentiable (except possibly at $s = s(0)$ and $s = s(T)$). In particular, if $A(t)$ is not a constant in any time interval, we can define

$$s(t) = \int dt' \, 1\{A(t') > A(t)\} \tag{7}$$

i.e., $s(t)$ is the total amount of time during which attendance A is greater than $A(t)$. In general, there may be more than one clock time t corresponding to a given value of s. We also define $\widetilde{B}(s)$ by $\widetilde{B}(s(t)) = B(t)$, and $\widetilde{v}(s)$ as the expectation of the unconstrained sale price for an item offered for sale when $s(t) = s$.[16]

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[15] In this treatment, we consider the auctioneer's strategy with attendance given exogenously. The auctioneer can also attempt to influence attendance through the order of sale; we would expect this to have less effect because, while the auctioneer can freely adjust the order of sale, buyers may incur substantial costs in changing their time of attendance.

[16] In practice, attendance may not be the only determinant of the value of items sold at a given time. For example, the order of sale may include groups of items of similar type but differing individual values, which may be considered as inseparable in optimizing the order of sale. In that case, we interpret $\widetilde{v}(s)$ as the average value of v in a small interval.

To formulate the continuum model, τ and H are rescaled. Let $\tilde{\tau}(s)/N$ be the time taken to sell an item when $s(t) = s$, where N is the number of items to be sold, so that the normalized *rate of sale* $1/\tilde{\tau}(s)$ is the quantity (as a fraction of the total number of items) sold per unit time. Define also $\tilde{H}(\tilde{\tau}) = H(\tilde{\tau}/N)$. Then, taking the limit as $N \to \infty$ with T and $\tilde{H}(\cdot)$ fixed, the function $\tilde{H}$ has the same properties as were previously assumed for H, i.e., $0 \le \tilde{H}(\tilde{\tau}) \le 1$, $\tilde{H}(\tilde{\tau}) \to 1$ as $\tau \to \infty$, $\tilde{h}(\tilde{\tau}) > 0$, and $\tilde{h}'(\tilde{\tau}) < 0$.

The objective function for the order-of-sale problem is then

$$\int_0^T ds\,[\tilde{\tau}(s)]^{-1}\,\tilde{v}(s)\,\tilde{B}(s)\,\tilde{H}(\tilde{\tau}(s)) - \int dv\,\mu(v)\left(\int_0^T ds\,[\tilde{\tau}(s)]^{-1}\,1\{\tilde{v}(s) \le v\} - F_v(v)\right) \tag{8}$$

The first term is the average revenue per item sold, and the second term, with Lagrange multiplier μ(·), restricts the value distribution of items sold to the given distribution function $F_\nu(\cdot)$, i.e., for any ν', the number of items with $\nu \le \nu'$ is held fixed at $N\,F_\nu(\nu')$. The first-order conditions for stationarity with respect to the functions $\tilde{v}(\cdot)$, $\tilde{\tau}(\cdot)$, and $\mu(\cdot)$ are

$$\begin{gathered}
\tilde{B}(s)\,\tilde{H}(\tilde{\tau}(s)) + \mu(\tilde{v}(s)) = 0 \\
\tilde{v}(s)\,\tilde{B}(s)\left[-\tilde{H}(\tilde{\tau}(s)) + \tilde{\tau}(s)\,\tilde{h}(\tilde{\tau}(s))\right] + \int_{\tilde{v}(s)} dv\,\mu(v) = 0 \\
\int_0^T ds\,[\tilde{\tau}(s)]^{-1}\,1\{\tilde{v}(s) \le v\} = F_v(v)
\end{gathered} \tag{9}$$

The first expression represents the effect of offering at this time items with a higher valuation ν; it equates the marginal gain in revenue to μ, the gradient with respect to v of the shadow value of unsold items with valuation $\tilde{v}$. The second expression represents the effect of slowing the rate of sale (i.e. allocating more time to current items with valuation $\tilde{v}$); it equates the marginal loss of revenue to the shadow value of unsold items with valuation $\tilde{v}$. The third expression simply reflects the constraint associated with the Lagrange multiplier.

To see what this characterization implies about the value of time, we eliminate the Lagrange function μ from the first two equations of (9) and write the result, after some cancellations, as

$$\frac{d\Lambda(s)}{ds} = [\,\tau(s)]^{-1}\,\tilde{v}(s)\,\tilde{B}'(s)\,\tilde{H}(\tilde{\tau}(s)) \tag{10}$$

where

$$\Lambda(s) \equiv \tilde{v}(s)\,\tilde{B}(s)\,\tilde{h}(\tilde{\tau}(s)) \tag{11}$$

is the (time-varying) local value of time. We note that this is just the continuous-model version of equation (6), which can be rewritten as

$$v_i\;B(s_i)\,h(\tau_i) = \lambda - \sum_{j>i} v_j\;B'(s_j)\,H(\tau_j)$$

To see that $\Lambda(s)$ is indeed the value of time, consider a small quantity of items of measure dn being sold at time s, bringing in revenue $\tilde{v}(s)\,\tilde{B}(s)\,\tilde{H}(\tilde{\tau}(s))\,dn$ in a time interval of length $\tilde{\tau}(s)\,dn$. A small change $d\,\tilde{\tau}$ in $\tilde{\tau}(s)$ increases revenue by $\tilde{v}(s)\,\tilde{B}(s)\,\tilde{h}(\tilde{\tau}(s)\,)\,dn\,d\tilde{\tau}$. At the optimum this must equal the cost of lengthening the time interval by $dn\;\;d\tilde{\tau}$, and the resulting expression for the value of time is (11).

Proposition 1, proven in Appendix A, gives conditions under which items with higher valuation v (i.e. higher expected unconstrained sale prices) will be offered during periods of higher attendance. This condition involves the ratio of average rate of increase in sale price, $\tilde{H}(\tilde{\tau})\,/\,\tilde{\tau}$, to the marginal rate of increase, $\tilde{h}(\tilde{\tau})$.

Proposition 1. If the function $\widetilde{H}$ satisfies the condition

$$\frac{d}{d\tilde{\tau}}\left(\frac{\widetilde{H}(\tilde{\tau})}{\tilde{\tau}\,\tilde{h}(\tilde{\tau})}\right) > 0 \tag{12}$$

and if the order of sale $\tilde{v}(s)$ is an optimal solution of (9), then the value v is monotonically related to attendance, i.e., $\tilde{v}(s_1) \geq \tilde{v}(s_2)$ if and only if $A(s_1) \geq A(s_2)$.

4. Choice of Selling Techniques

The auctioneer selects the selling technique for lots presented at auction taking into account the effect on the time-revenue tradeoff. The auctioneer may use the default or standard outcry auction technique described above, or one of several other techniques. Auctioneers responding to our survey reported the most commonly used alternatives to the default auction were as indicated in Table 2 below.

Table 2. Incidence of Use of Auctioneering Techniques

Auctioneering Technique	Percentage of Sample
Batching	91.9%
Choice	92.6
By the Piece	93.2
Separately & Together	70.9
Other	16.9

In the discussion that follows, we consider the sale of two or more lots with maximum expected sale prices v_i, expected sale prices $p_i = v_i\, H(\tau_i)$, and selling times $\tau_i = \tau^*(v_i)$ in the standard auction. We compare the net revenue $\sum_i (p_i - \lambda\,\tau_i)$ in the default auction with the corresponding net revenue under the alternative technique, holding constant the selling techniques for all other lots. This determines when the auctioneer prefers an alternative to the default auction. In particular, the auctioneer sometimes prefers a technique that raises less gross revenue for the current lot in order to save time for other lots.

For simplicity, we assume that the value of each lot is small relative to the value of the whole auction and that attendance is constant.

4.1 Batching[17]

Batching is selling together as a single lot multiple items which do not naturally form a set as defined below. If the auctioneer chooses to batch items, he announces that the items are to be sold together and proceeds to elicits bids for a single lot consisting of the group of items. This technique is common in a variety of auction markets. For example, batching was heavily used in the recent large-scale auctions of properties for the Resolution Trust Corporation.

Auctioneers report many diverse circumstances in which batching is preferred, with the common explanation being that batching allows more items to be sold in a given period of time. Frequently batching will involve a less desirable item being offered for sale in consort with a more desirable item, since this will insure a more speedy and certain sale of the less desirable item. Auctioneers will, at times, batch spontaneously if a less desirable item fails to bring an initial opening bid.[18]

Batching often forces the bidder to purchase unwanted items in order to obtain desired items, and it also requires a larger financial commitment. Consequently the technique can be objectionable, particularly to small bidders. Auctioneers report that batching is less objectionable to *dealers*, who can more easily dispose of unwanted objects.

We consider the sale of two items, as described above. Let the expected maximum price for the batch be

$$\nu = \gamma(\nu_1 + \nu_2)$$

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[17] This phenomenon also goes by the terms "bundling" and "box-lotting".

[18] A drawback of batching is that it provides extra incentive for bidders to form *pools,* or bidders' conspiracies, where two or more bidders agree not to bid against each other and then later divvy up the goods. Pooling is illegal in most states.

If $\gamma > 1$, the items form a *natural set*; that is, they are worth more together than apart, and will be offered for sale as a single lot. If $\gamma \leq 1$, the items do not form a natural set and, absent time constraints, the auctioneer should prefer the default auction to batching.[19] Comparison of the net revenues for the two techniques shows that batching is preferred to the default auction if

$$v\,H(\tau^*(v)) - \lambda\,\tau^*(v) > v_1\,H(\tau^*(v_1)) + v_2\,H(\tau^*(v_2)) - \lambda[\tau^*(v_1) + \tau^*(v_2)]$$

or, equivalently,

$$p - p_1 - p_2 > \lambda[\tau - \tau_1 - \tau_2]$$

where p and τ are the expected sale price and the selling time of the batch. With a time constraint, the auctioneer may prefer to batch even though $\gamma < 1$.

Items that are not worth selling separately are usually batched with other items rather than not offered for sale. Recall that an item is not worth selling separately, $\tau^*(v_i) = 0$, if its maximum expected sale price is $0 < v_i \leq \lambda / h(0)$. In most cases, this item still has positive marginal value when combined with another lot with maximum expected price v_j, i.e., $v > v_j$. Thus, even if the auctioneer did not increase the time $\tau_j^*(v)$ allocated to the sale of the batched lot, more revenue would be raised than by not offering item i for sale separately.

4.2 Choice

Under this mechanism, n items are placed on the block simultaneously, and the winning bidder is allowed to take any number from one to n of those items, each at the price of the top bid. If the bidder takes fewer than n of the items, then the bidding restarts in a similar fashion for the remainder. Choice

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[19] The case $\gamma < 1$ assumes that there is sufficient competition; if there are only two bidders for the batched items, then necessarily $\gamma \geq 1$ (Palfrey, 1983).

alters bidder strategy in two ways. On the one hand, it creates more bidder competition by getting bidders to bid against each other on the same call for *different* items. On the other hand, it causes bidders to bid less aggressively by lowering their bid due to the possibility that rival bidders are actually bidding for different items. Vanderporten (1992) shows, in a two bidder/two item choice auction, that the second effect dominates the first. That is, absent other considerations, the choice mechanism lowers expected revenues from sale, weakly in some cases but strictly in others. In practice, however, auctioneers report that the following two time-related considerations prompt their use of this mechanism in outcry auctions.

(a) Auctioneer Uncertainty. An auctioneer may prefer the choice mechanism, despite its effect on bidder strategy, due to the combination of time considerations and auctioneer uncertainty. Suppose, for example, that the auctioneer has ten items to sell, all of a similar type but different enough to have a range of values. Suppose also that the optimal strategy - because of time constraints - is to sell the higher value items individually and then to batch the rest. Auctioneers report that if they are uncertain about which of the items will have the higher value, they will sell "choice" one or more rounds until the price falls to a level at which it is optimal to batch the remaining items.[20]

(b) Multiple Takes Per Round. Even without auctioneer uncertainty, the choice mechanism may still give rise to a time saving which will justify its use, since the top bidder will often take more than one item at the price of the winning bid. Many auctioneers implicitly encourage multiple takes by occasionally offering the choice option on the remaining items (after the winning bidder has chosen) to the second-highest bidder at the price of the winning bid. This practice introduces uncertainty over which items will remain in the next round and thus provides an incentive for the winning bidder to take all items in which he or she may have an interest at the first round, rather than waiting to bid again in hopes of obtaining a lower price.

[20] Many auctioneers report that it takes at least two rounds to get adequate information in these cases, unless they discover on the first round that the highest-value item in the set falls at or below the price level at which it is optimal to batch.

4.3 By the Piece

Under this mechanism, a group of n items is sold as a lot, but the bid called by the auctioneer is the price per item. The winning bidder is required to take all n items for a total outlay of n times the top bid. Items sold in this manner are almost always related, forming either a set or a collection. With regard to the issue of time, this mechanism is equivalent to batching. However, it is believed by many in the auctioneering industry to have added psychological value by focusing attention on the intrinsic *individual* value of the items for sale in an attempt to eliminate any quantity discount that might occur.

4.4 Separately and Together

This is the least-used of the auction mechanisms considered in this paper. It is appropriate when a number of related items (a set or a collection) are to be sold, but the auctioneer is *uncertain* whether a higher total selling price could be realized by selling the items individually or batched as a set. Typically this will occur when there are two distinct types of bidders: those interested only in individual items or subsets of the items, and those interested in acquiring all the items as a set. While a bidder interested in the complete set may, of course, bid on individual items, he may be unwilling to do so (or may lower his bid) because of the risk of being outbid on a later item and thus being left with an incomplete set.

In these circumstances, an auctioneer may employ the technique of "Separately and Together" (S&T). Under the S&T technique, the items are first sold individually and then "resold" together as a set. If the auctioneer is able to elicit a "together" price which is greater than the sum of the "separate" prices, the high bidder in the "together" round takes all the items. Otherwise, the original separate bids stand.

Usually this technique will be used only if the auction house considers the value of the items to be large by its standards. The technique requires *additional* time to execute (for example, three rounds for two items) and it has the potential to alienate customers who lose in the final "together" round after a successful "separate" round bid. Thus a faulty decision to use this technique may have a high cost. A classic example of the use of the S&T technique appeared in an auction announcement in the September

11, 1995 issue of *Antique Week*, a leading trade publication for the antique auction business. The announcement, for a turn-of-the-century carousel from a New York amusement park, ran as follows: "This magnificent Carousel features 41 wooden handcrafted horses and a fabulous lion.... Will be sold *individually and offered complete*."[21]

5. Summary and Conclusions

We have examined the phenomena of time constraints as a key factor in selling mechanisms. We have introduced these constraints in the framework of outcry auctions and have studied their effects on auctioneers' strategies. Our work is based upon information obtained from a random mail survey of the membership of the American Association of Auctioneers, as well as field investigations and interviews with auctioneers who report that at time is perhaps the most important consideration in their conduct of an auction. Time introduces transactions costs, and hence the auctioneer needs to maintain a swift selling pace in order to minimize these costs.

Our model begins with the survey finding that the time interval between bids usually lengthens as the bid approaches its final level due to factors which include the increasing importance of information processing by bidders. We then use this observed phenomenon to model the time-revenue tradeoff inherent in an outcry auction. In a time allocation model, we show first that the auctioneer has a time incentive to close bidding *before* the last possible bid is made. We also show that time constraints magnify the importance of varying attendance levels, which gives the auctioneer an incentive to strategically order items in the sale. We derive conditions to support the intuition that more valuable items should be offered during periods of high attendance.

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[21] An auction house may occasionally state a minimum premium for a successful together bid, or the house may set such a premium by the interval of the bids that are called. An example of the former appeared in the February 5, 1995 issue of *Antique Week* in the following auction announcement for five large, carved architectural features from a building in St. Louis: "To be sold by the unit, then altogether – bidder on all five must raise combined bids by 5 percent."

We have considered the role of time constraints in the choice of selling techniques. We document the most common selling techniques reported in our survey and describe how they relate to the auctioneer's time allocation decision. We argue that the auctioneer's choice of selling technique results from the time-revenue tradeoff. Certain techniques, such as "choice", are used in practice because they result in time savings even though they have been shown previously to lower selling price.

Appendix A

Strategic Ordering of Sales

Proof of Proposition 1: The first step is to show that the function $\tilde{v}(s)$ can be inverted when $\tilde{B}$ is decreasing. From the first two equations of (9), we find that $\tilde{\tau}\,\tilde{h}(\tilde{\tau})/\tilde{H}(\tilde{\tau})$ is a function of $\tilde{v}$. From the monotonicity implied by (12), it follows that $\tilde{\tau}$ itself is a function of $\tilde{v}$. Going back to the first equation of (9), we now find that $\tilde{B}$ is a function of $\tilde{v}$. In other words, the value v of an item determines a unique attendance level at which it should be offered for sale.

Since the order of sale does not matter in an interval in which $\tilde{B}(s)$ is constant, consider an interval in which $\tilde{B}(s)$ is strictly decreasing, and therefore s is also a function of $\tilde{v}$. The third equation of (9) then implies that

$$[\tilde{\tau}(s)]^{-1} = f_v(\tilde{v}(s))\ |\tilde{v}'(s)| \tag{A1}$$

where $f_v(v) = dF_v(v)/dv$ and $\tilde{v}'(s) = d\tilde{v}(s)/ds$.

The last step is to determine the sign of $\tilde{v}'(s)$. Consider an interval in $\tilde{v}$ of width $d\,\tilde{v}$ in which $\tilde{v}'(s) \geq 0$, and where equations (10) and (A1) hold. We need to find the change in net revenue (i.e., actual revenue less the cost of time) when the sign of $\tilde{v}'(s)$ is reversed and $\tilde{\tau}(s)$ within the interval is re-optimized. Net revenue is given by

$$\int dv\, f(v)\,\{v\,\tilde{B}(s(v))\ \ \tilde{H}(\tilde{\tau}(s(v))) - \tilde{\tau}(s(v))\,\Lambda(s(v))\} \tag{A2}$$

integrated over the relevant interval, with $\Lambda(s)$ held fixed at its initial value. After some calculations, the change in net revenue is found to be, to lowest order,[22]

$$\tfrac{1}{6}[\,f(v)\,\tilde{\tau}\,\tilde{h}(\tilde{\tau})]^2\,\frac{\tilde{B}'(s)}{\tilde{h}'(\tilde{\tau})}\,\frac{d}{d\tilde{\tau}}\left(\frac{\tilde{H}(\tilde{\tau})}{\tilde{\tau}\,\tilde{h}(\tilde{\tau})}\right)(d\tilde{v})^3 \tag{A3}$$

evaluated at $s = s(v)$ and $\tilde{\tau} = \tilde{\tau}(s(v))$. Under condition (12), this is positive, i.e., the solution with $\tilde{v}\,'(s) \geq 0$ is better. Since this argument can be applied to any small interval, the optimal order of sale has $\tilde{v}$ decreasing as attendance drops.[23]

Appendix B

The Survey

The survey from which the data for this study were taken was implemented by the Economic Indicators Division of the Ohio State Center for Survey Research. A random sample of auctioneers was drawn from the membership rolls of the American Association of Auctioneers, which is the major auction trade association in the United States. The survey was conducted through a mailed questionnaire in the

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[22] Details are available from the authors on request.

[23] It is possible to construct functions $\tilde{H}$, satisfying our previous assumptions, for which condition (12) does not hold over some range of $\tilde{\tau}$, although it is unclear whether such functions have practical significance. If the optimal values of $\tilde{\tau}$ fell in this range for some items, then standard order of sale would be reversed for those items.

third quarter of 1997. One-hundred and forty-eight responding auctioneers form the sample whose data are reported here. The descriptive statistics for this sample are reported in the tables below.

Table B1: Descriptive Statistics for Survey Sample

Variable	Mean (*Standard Deviation)*
Years in Business	21.8 (*13.6*)
Number of Employees	8.7 (*8.1)*
Number of Auctions per Year	64.9 (*58.1*)

n = 148

The following table shows the percentage of the sample who regularly conduct various types of auctions.

Table B2: Types of Auctions Conducted by the Sample

Type of Auction	Percentage of the Sample
General Merchandise	80.1 %
Automobiles	42.6
Machinery and Equipment	66.2
Antiques/Art	73.0
Real Estate	66.9
Livestock or Crops	20.9
Other	22.3

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