Structural Error Correction Models: Instrumental Variables Methods and an Application to an Exchange Rate Model

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Abstract

Error correction models are widely used to estimate dynamic cointegrated systems. In most applications, estimated error correction models are reduced form models. As a result, nonstructural speed of adjustment coefficients are estimated in these applications. A single equation instrumental variable method can be used to estimate a structural speed of adjustment coefficient. This paper develops a system instrumental variable method to estimate the structural speed of adjustment coefficient in an error correction model. This method utilizes Hansen and Sargent's (1982) instrumental variable estimator for linear rational expectations models, and is applied to an exchange rate model with sticky prices.

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1. Introduction

As discussed by Cooley and LeRoy (1985), Vector Autoregression (VAR) models for stationary random variables are reduced form models. Structural interpretations of VAR models require restrictions on structural form models. When some of the random variables in the system are unit root nonstationary and are not cointegrated, Blanchard and Quah's (1989) structural VAR method can be used as in Ahmed, Ickes, Wang, and Yoo (1993) among others.[1] When some of the random variables in the system are unit root nonstationary and are cointegrated, then Davidson, Hendry, Srba, and Yeo's Error Correction Model (ECM) is widely used.[2] As the Granger Representation Theorem shows (see Engle and Granger (1988)), an ECM representation exists when the variables are cointegrated and vice versa.[3] The standard ECMs are reduced form models just as VAR models are as pointed out by Urbain (1992) and Boswijk (1994,1995).

In a structural ECM, at least one linear combination of variables slowly adjusts to the long-run equilibrium level with a constant speed of adjustment. In general, the speed of adjustment coefficient in a structural ECM is different from the speed of adjustment coefficient in its reduced form ECM. As an example, we will show that they are different in an exchange rate model with sticky prices. Because the reduced form speed of adjustment coefficient mixes the structural speed of adjustment coefficient

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[1] Shapiro and Watson's (1988) method can also be used.

[2] An alternative method is levels VAR without imposing unit roots. Estimators which are based on a levels VAR are more robust but are usually less efficient than those based on an ECM if the restrictions regarding nonstationarity and cointegration imposed by the ECM are true.

[3] The theorem should be used with caution because there exist economic models in which the regularity conditions of the theorem do not apply as shown in Ogaki (1998). However, the model in this paper is subject to this criticism.

with other parameters in the system, it is not easy to interpret it. In the exchange rate model, the structural speed of adjustment coefficient in the ECM is equal to one minus the first order autoregressive coefficient for the log real exchange rate[4]. Hence the structural speed of adjustment coefficient can be used to compute the half life of the real exchange rate. However, the reduced form speed of adjustment is a nonlinear function of the structural speed of adjustment and the interest elasticity of money demand. Hence the reduced form speed of adjustment coefficient in the ECM cannot be directly compared with the half life estimates of real exchange rates in the literature (see, e.g., Rogoff (1996) for a survey, and Kilian and Zha (2001) and Murray and Papell (2001) for more recent works).

Standard estimation methods for ECMs such as Engle and Granger's two step method and Johansen's (1988) Maximum Likelihood method estimate the reduced form speed of adjustment coefficient rather than the structural speed of adjustment coefficient. A single equation instrumental variable (IV) method can be directly applied to a slow adjustment equation. The main purpose of this paper is to develop a system method that combines the single equation method with Hansen and Sargent's (1982) IV method for linear rational expectations models.

In the single equation method, an IV method is applied to a slow adjustment equation that describes how a variable slowly adjusts to the long-run equilibrium level in the structural ECM. The system method combines the single equation method with Hansen and Sargent's (1982) method

[4]As explained later, the coefficient cannot be estimated by Ordinary Least Squares with measurement errors. However, the structural coefficient is equal to one minus the first order autoregressive coefficient of the true value of the log real exchange rate even with measurement errors.

which applies Hansen's (1982) Generalized Method of Moments (GMM) to linear rational expectations models. The system method is more efficient than the single-equation method when the restrictions implied by linear rational expectations models are true. On the other hand, the single equation method is robust to misspecification in the other equations of the structural ECM. Therefore, we can form a test statistic of the restrictions by comparing the results from the two methods.

These methods are applied to an exchange rate model with sticky prices. The model is a one-good version of Mussa's (1982) model, which may be viewed as a stochastic discrete time version of Dornbush's (1976) model. This model includes a slow adjustment equation, in which the domestic price adjusts to the long run equilibrium level determined by Purchasing Power Parity (PPP) with rational expectations. We refer the speed of adjustment coefficient for this equation as the structural speed of adjustment coefficient. Because the basic idea of the ECM is that variables adjust to their long run levels, it is of interest to examine whether or not the standard estimation methods of the ECM can be used to estimate the structural speed of adjustment coefficient. We will show that, in the exchange rate model, the standard ECM estimation methods do not recover the structural speed of adjustment coefficient.

Data are for the exchange rates of currencies of Canada, France, Germany, Italy, Japan, Netherlands, Switzerland, United Kingdom, against the U.S. dollar. Using the single equation method, we obtain positive estimates for the structural speed of adjustment coefficient in most cases.

We then apply the system method to the same data set. In this case the speed of adjustment coefficient can be estimated from the slow adjustment equation for the domestic price and the rational expectations equation for the exchange rate. We form a specification test by comparing the estimates for the speed of adjustment coefficient from these two equations.

Structural ECMs have been considered by several authors. As in VAR models, identification of structural shocks is an important issue for structural ECMs. King, Plosser, Stock, and Watson (1991), Jang (2000), and Jang and Ogaki (2001) develop methods to identify structural shocks with short-run and long-run restrictions. This paper focues on estimation of the structural speed of adjustment coefficients. This issue does not arise in VAR models with stationary variables. Urbain (1992) investigates sufficient conditions for weak exogeneity for structural ECMs that are similar to ours. Boswijk (1994,1995) and Hsiao (1995) discuss the relationship between the ECM and structural simultaneous equations models. However, unlike Urbain and Hsiao, we do not assume that exogenous variables are observed by the econometrician. In our empirical application, it is not natural to assume exogeneity of any variable in the cointegrated system. Pappell (1995) derives a reduced form ECM from an exchange rate model that is similar to ours. However, the real exchangre rate is nonstationary in his model unlike ours. He applies Phillips' (1991) ML estimator to the reduced form ECM. Dolado, Galbraith and Banerjee (1991) and Gregory, Pagan, and Smith (1993) derive structural ECMs from linear quadratic models. They discuss the difficulties associated with the application of standard estimation methods such as Engle and Granger's (1987) two-step method and Johansen's (1988, 1991) Maximum Likelihood (ML) method to the ECM. They do not combine their method with Hansen and Sargent's (1982) IV method for linear rational expectations models.

The rest of this paper is organized as follows. Section 2 presents an exchange rate model in which the domestic price slowly adjusts toward the Purchasing Power Parity (PPP) level. In Section 3, a structural ECM is presented and its relationship to a reduced form ECM is discussed. Section 4 discusses the single equation and system methods for the structural ECM. In Section 5, the model of Section 2 is augmented to include measurement errors. Section 6 presents our empirical results for the single equation and system methods. Section 7 contains concluding remarks.

2. *An Exchange Rate Model with Sticky Prices*

In this section, we present a simple exchange rate model in which the domestic price adjusts slowly toward the long-run equilibrium level implied by Purchasing Power Parity (PPP). This model is used to motivate a particular form of a structural ECM in the next section. The model's two main components are a slow adjustment equation and a rational expectations equation for the exchange rate. The single equation method in Section 4 is based only on the slow adjustment equation. The system method utilizes both the slow adjustment and rational expectations equations.

2.A. The Slow Adjustment Equation

Let $p(t)$ be the log domestic price level, $p^*(t)$ be the log foreign price level, and $e(t)$ be the log nominal exchange rate (the price of one unit of the foreign currency in terms of the domestic currency). We assume that these variables are first difference stationary. We also assume that PPP holds in the long run, so that the real exchange rate, $p(t) - p^*(t) - e(t)$, is stationary, or $y(t) = (p(t), e(t), p^*(t))'$ is cointegrated with a cointegrating vector $(1,-1,-1)$. Let $\mu=E(p(t) - p^*(t) - e(t)]$, then μ can be nonzero when different units are used to measure prices in the two countries.

Using a one-good version of Mussa's (1982) model, the domestic price level is assumed to adjust slowly to the PPP level

(1) $\Delta p(t+1) = b\,[\mu + p^*(t) + e(t) - p(t)] + E_t[p^*(t+1) + e(t+1)] - [p^*(t) + e(t)]$

where $\Delta x(t+1) = x(t+1) - x(t)$ for any variable $x(t)$, $E(\cdot|I_t)$ is the expectation operator conditional on I_t, the information available to the economic agents at time t, and a positive constant $b<1$ is the adjustment coefficient. The idea behind Equation (1) is that the domestic price level slowly adjusts toward its PPP level of $p^*(t)+e(t)$, while it adjusts instantaneously to the expected change in its PPP level. The adjustment speed is slow (fast) when b is close to zero (one).

From Equation (1), we obtain

(2) $\Delta p(t+1) = d + b[\,p^*(t) + e(t) - p(t)] + \Delta p^*(t+1) + \Delta e(t+1) + \varepsilon(t+1)$

where $d=b\mu$, $\varepsilon(t+1) = E_t[p^*(t+1)+e(t+1)] - [p^*(t+1)+e(t+1)]$. Hence $\varepsilon(t+1)$ is a one-period ahead forecasting error, and $E[\varepsilon(t+1)|I_t] = 0$. Because $p^*(t) + e(t) - p(t)$ is the log real exchange rate, b coincides with one minus the first order autoregressive coefficient of the log real exchange rate. Without measurement errors, the coefficient b can be estimated by Ordinary Least Squares directly from (2). In the presence of measurment error, instrumental variables are necessary. We will consider cases with and without measurement error.

2.B. The Exchange Rate under Rational Expectations

We close the model by adding the money demand equation and the

Uncovered Interest Parity condition. Let

(3) $$m(t) = \theta_m + p(t) - hi(t)$$

(4) $$i(t) = i^*(t) + E[e(t+1)|I_t] - e(t)$$

where $m(t)$ is the log nominal money supply minus the log real national income, $i(t)$ is the nominal interest rate in the domestic country, and $i^*(t)$ is the nominal interest rate in the foreign country. In (3), we are assuming that the income elasticity of money is one. From (3) and (4), we obtain

(5) $$E[e(t+1)|I_t]-e(t) = (1/h)\ \{\theta_m + p(t) - \omega(t) - h\ E[p^*(t+1)-p^*(t)]|I_t]\}$$

where

(6) $$\omega(t) = m(t) + hr^*(t)$$

and $r^*(t)$ is the foreign real interest rate:

(7) $$r^*(t) = i^*(t) - E[p^*(t+1)|I_t] + p^*(t).$$

Following Mussa, solving (1) and (5) as a system of stochastic difference equation for $E[p(t+j)|I_t]$ and $E[e(t+j)|I_t]$ for fixed t results in

(8) $$p(t) = E[F(t)|I_{t-1}] - \sum_{j=0}^{\infty} (1-b)^j\ \{E[F(t-j)|I_{t-j}] - E[F(t-j)|I_{t-j-1}]\}$$

(9) $$e(t) = \frac{bh+1}{bh}E[F(t)|I_t] - p^*(t) - \frac{1}{bh}p(t)$$

where

(10) $$F(t) = (1 - [\delta)\ \sum_{j=0}^{\infty} \delta^j\omega(t+j)$$

and $\delta=h/(1+h)$.

We assume that $\omega(t)$ is first difference stationary. Since δ is a positive constant that is smaller than one, this implies that $F(t)$ is also first difference stationary. From (8) and (9),

$$(11) \qquad e(t)+p^{*}(t)-p(t) = \sum_{j=0}^{\infty} (1-b)^{j}\{E[F(t-j)|I_{t-j}] - E[F(t-j)|I_{t-j-1}]\}.$$

Since the right hand side of (11) is stationary,[5] $e(t)+p^{*}(t)-p(t)$ is stationary. Hence Equation (11) implies that $(p(t),\ e(t),\ p^{*}(t))$ is cointegrated with a cointegrating vector $(1,-1,-1)$.

2.C. Hansen and Sargent's Formula

In order to obtain a structural ECM representation from the exchange rate model, we use Hansen and Sargent's (1980, 1982) formula for linear rational expectations models. From (9), we obtain

$$(12) \qquad \Delta e(t+1) = \frac{bh+1}{bh}(1-\delta)\, E[\sum_{j=0}^{\infty} \delta^{j}\Delta\omega(t+j+1)|I_t] - \frac{1}{bh}\Delta p(t+1) - \Delta p^{*}(t+1) + \varepsilon_e(t+1)$$

where $\varepsilon_e(t+1) = \frac{bh+1}{bh}\{E[F_{t+1}|I_{t+1}] - E[F_{t+1}|I_t]\}$, so that the law of iterated expectations implies $E[\varepsilon_e(t+1)|I_t]=0$. The system method in Section 5 is applicable because this equation involves a discounted sum of expected future values of $\Delta\omega(t)$,

Hansen and Sargent (1982) proposes to project the conditional expectation of the discounted sum, $E[\sum \delta^{j}\Delta y_4(t+j+1)|I_t]$, onto an information set H_t, which is a subset of I_t, the economic agents' information set. Let $\hat{E}(\cdot|H_t)$ be the linear projection operator conditional on an information set H_t which is a subset of I_t.

[5]This assumes that $E_t[F(t)]-E_{t-1}[F(t)]$ is stationary, which is true for a large class of first difference stationary variable $F(t)$ and information sets.

We take the econometrician's information set at t, H_t, to be the one generated by the linear functions of the current and past values of $\Delta p^*(t)$. Then replacing the economic agents' best forecast, $E[\sum \delta^j \Delta\omega(t+j+1) | I_t]$, by the econometrician's linear forecast based on $H(t)$ in Equation (12), we obtain

$$(13) \qquad \Delta e(t+1) = \frac{bh+1}{bh}(1-\delta)\hat{E}[\sum_{j=0}^{\infty} \delta^j \Delta\omega(t+j+1) | H(t)] - \frac{1}{bh}\Delta p(t+1) - \Delta p^*(t+1) + u_2(t+1)$$

where

$$(14) \qquad u_2(t+1) = \varepsilon_e(t+1) + \frac{bh+1}{bh}(1-\delta)\, E_t\{[\sum \delta^j \Delta\omega(t+j+1)] - \hat{E}[\sum \delta^j \Delta\omega(t+j+1) | H(t)]\}.$$

Because H_t is a subset of I_t, we obtain $\hat{E}[u_2(t+1) | H_t] = 0$.

Since $\hat{E}[\cdot | H_t]$ is the linear projection operator onto H_t, there exist possibly infinite order lag polynomials $\beta(L)$, $\gamma(L)$, and $\xi(L)$, such that

$$(15) \qquad \hat{E}[\Delta p^*(t+1) | H(t)] = \beta(L)\Delta p^*(t)$$

$$(16) \qquad \hat{E}[\Delta\omega(t+1) | H(t)] = \gamma(L)\Delta p^*(t)$$

$$(17) \qquad \hat{E}[\sum_{j=0}^{\infty} \delta^j \Delta\omega(t+j+1) | H(t)] = \xi(L)\Delta p^*(t)$$

Then following Hansen and Sargent (1980, Appendix A), we obtain the restrictions imposed by (13) on $\xi(L)$:

$$(18) \qquad \xi(L) = \frac{\gamma(L)-\delta L^{-1}\gamma(\delta)\{1-\delta\beta(\delta)\}^{-1}\{1-L\beta(L)\}}{1-\delta L^{-1}}$$

Assume that linear projections of $\Delta p^*(t+1)$ and $\Delta\omega(t+1)$ onto $H(t)$ have only finite number of $\Delta p^*(t)$ terms:

$$(19)\quad \hat{E}[\Delta p^*(t+1)|H(t)] = \beta_1\Delta p^*(t) + \beta_2\Delta p^*(t-1) + \ldots + \beta_p\Delta p^*(t-p+1)$$

$$(20)\quad \hat{E}[\Delta\omega(t+1)|H(t)] = \gamma_1\Delta p^*(t) + \gamma_2\Delta p^*(t-1) + \ldots + \gamma_{p-1}\Delta p^*(t-p+2)$$

Here we assume $\beta(L)$ is of order p and $\gamma(L)$ is of order $p-1$ in order to simplify the exposition, but we do not lose generality because any of β_i and γ_i can be zero. Then as in Hansen and Sargent (1982), (18) implies that $\xi(L)=\xi_0+\xi_1 L+\ldots+\xi_p L^p$, where

$$(21)\qquad \xi_0 = \gamma(\delta)\{1-\delta\beta(\delta)\}^{-1}$$

$$\xi_j = \delta\gamma(\delta)\{1-\delta\beta(\delta)\}^{-1}(\beta_{j+1} + \delta\beta_{j+2} + \ldots + \delta^{p-j}\beta_p) + (\gamma_j + \delta\gamma_j + \ldots + \delta^{p-j}\gamma_p) \qquad \text{for } j=1,\ldots,p.$$

Thus

$$(22)\quad \hat{E}[\sum_{j=0}^{\infty}\delta^j\Delta\omega(t+j+1)|H(t)] = \xi_1\Delta p^*(t) + \xi_2\Delta p^*(t-1) + \ldots + \xi_p\Delta p^*(t-p+1).$$

Using (2), (13), (15), (16), and (22), we obtain a system of four equations:

$$(23)\quad \Delta p(t+1) = d + \Delta p^*(t+1) + \Delta e(t+1) - b[p(t) - p^*(t) - e(t)] + u_1(t+1)$$

$$(24)\quad \Delta e(t+1) = -\frac{1}{bh}\Delta p(t+1) - \Delta p^*(t+1) + \alpha\xi_1\Delta p^*(t) + \alpha\xi_2\Delta p^*(t-1) + \ldots + \alpha\xi_p\Delta p^*(t-p+1) + u_2(t+1)$$

$$(25)\quad \Delta p^*(t+1) = \beta_1\Delta p^*(t) + \beta_2\Delta p^*(t-1) + \ldots + \beta_p\Delta p^*(t-p+1) + u_3(t+1)$$

$$(26)\quad \Delta\omega(t+1) = \gamma_1\Delta p^*(t) + \gamma_2\Delta p^*(t-1) + \ldots + \gamma_{p-1}\Delta p^*(t-p+2) + u_4(t+1)$$

where $\alpha = \frac{bh+1}{bh}(1-\delta)$ and $u_1(t+1) = \varepsilon(t+1)$.

Given the data for $[\Delta p(t+1), \Delta e(t+1), \Delta p^*(t+1), \Delta\omega(t+1)]'$, GMM can be applied to these four equations as will be discussed in Section 4.B. There exist additional complications for obtaining data for $\Delta\omega(t+1)$ as we discuss in Section 4.C.

3. Structural Models and Error Correction Models

In this section, we discuss the relationship between structural models and ECMs. Let $\mathbf{y}(t)$ be an n-dimensional vector of first difference stationary random variables. We assume that there exist ρ linearly independent cointegrating vectors, so that $\mathbf{A}'\mathbf{y}(t)$ is stationary, where $\mathbf{A}'$ is a $(\rho \times n)$ matrix of real numbers whose rows are linearly independent cointegrating vectors. Consider a standard ECM

$$(27) \qquad \Delta\mathbf{y}(t+1) = \mathbf{k} + \mathbf{G}\mathbf{A}'\mathbf{y}(t) + \mathbf{F}_1\Delta\mathbf{y}(t) + \mathbf{F}_2\Delta\mathbf{y}(t-1) + \ldots + \mathbf{F}_p\mathbf{y}(t-p+1) + \mathbf{v}(t+1),$$

where $\mathbf{k}$ is a $(n \times 1)$ vector, $\mathbf{G}$ is a $(n \times \rho)$ matrix of real numbers, $\mathbf{v}(t)$ is a stationary n-dimensional vector of random variables with $\hat{E}(\mathbf{v}(t+1)|H_{t-\tau})=0$. In many applications $\tau=0$, but we will give examples of applications in which $\tau>0$.[6] There exist many ways to estimate (27). For example, Engle and Granger's two step method or Johansen's Maximum Likelihood methods can be used.

Many applications of standard ECMs give elements in $\mathbf{G}$ structural interpretations as parameters of the speed of adjustment toward the long-run equilibrium represented by $\mathbf{A}'\mathbf{y}(t)$. It is of interest to study conditions under which the elements in $\mathbf{G}$ can be given such a structural interpretation. In the model of the previous section, the domestic price level gradually

[6]We will treat more general cases in which the expectation of $\mathbf{v}(t+1)$ conditional on the economic agents' information is not zero, but the linear projection of $\mathbf{v}(t+1)$ onto an econometrician's information set (which is smaller than the economic agents' information set) is zero.

adjusts to its PPP level with a speed of adjustment parameter b. We will investigate conditions under which b can be estimated as an element in **G** from (27).

In most applications, (27) is a reduced form model. A class of structural models can be written in the following form of a structural ECM:

$$\text{(28)} \qquad \mathbf{C}_0 \Delta \mathbf{y}(t+1) = \mathbf{d} + \mathbf{B}\mathbf{A}'\mathbf{y}(t) + \mathbf{C}_1 \Delta \mathbf{y}(t) + \mathbf{C}_2 \Delta \mathbf{y}(t-1) + \ldots + \mathbf{C}_p \Delta \mathbf{y}(t-p+1) + \mathbf{u}(t+1)$$

where $\mathbf{C}_i$ is a $(n \times n)$ matrix, $\mathbf{d}$ is an $(n \times 1)$ vector, and $\mathbf{B}$ is an $(n \times \rho)$ matrix of real numbers.[7] Here $\mathbf{C}_0$ is a nonsingular matrix of real numbers with ones along its principal diagonal, $\mathbf{u}(t)$ is a stationary n-dimensional vector of random variables with $E[\mathbf{u}(t+1)|\hat{H}_{t-\tau}] = 0$. Even though cointegrating vectors are not unique, we assume that there is a normalization that uniquely determines **A**, so that parameters in **B** have structural meanings.

The exchange rate model can be written in the SECM form (28) as in the system of equations (23)-(26): we have $\mathbf{y}(t) = [\Delta p(t+1),\ \Delta e(t+1),\ \Delta p^*(t+1),\ \Delta\omega(t+1)]'$, $\mathbf{B}=[-b,0,0,0]'$, $\mathbf{A}=[1,\ -1,\ -1,\ 0]'$,

$$\text{(29)} \qquad \mathbf{C}_0 = \begin{bmatrix} 1 & -1 & -1 & 0 \\ 1/bh & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

and

[7] If the deterministic cointegration restriction (see Ogaki and Park, 1998, for this terminology) is not satisfied, then a linear trend term needs to be added to Equation (28).

$$(30) \qquad \mathbf{C}_j = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha\xi_j & 0 \\ 0 & 0 & \beta_j & 0 \\ 0 & 0 & \gamma_j & 0 \end{bmatrix}$$

for $j=1,\dots,p$. For any nonzero constant ψ, $\psi(1, -1, -1)'$ is also a cointegrating vector. However, the first row of $\mathbf{B}$ is b only when ψ is normalized to one.

In order to see the relationship between the standard ECM and the structural ECM, we premultiply both sides of (4) by $\mathbf{C}_0^{-1}$ to obtain the standard ECM (3), where $\mathbf{k} = \mathbf{C}_0^{-1}\mathbf{d}$, $\mathbf{G}=\mathbf{C}_0^{-1}\mathbf{B}$, $\mathbf{F}_i = \mathbf{C}_0^{-1}\mathbf{C}_i$, and $\mathbf{v}(t) = \mathbf{C}_0^{-1}\mathbf{u}(t)$. Thus the standard ECM estimated by Engle and Granger's two step method or Johansen's Maximum Likelihood methods is a reduced form model. Hence it cannot be used to recover structural parameters in $\mathbf{B}$, nor can the impulse-response functions based on $\mathbf{v}(t)$ be interpreted in a structural way unless some restrictions are imposed on $\mathbf{C}_0$.

As in a VAR, various restrictions are possible for $\mathbf{C}_0$. One example is to assume that $\mathbf{C}_0$ is lower triangular. If $\mathbf{C}_0$ is lower triangular, then the first row of $\mathbf{G}$ is equal to the first row of $\mathbf{B}$, and structural parameters in the first row of $\mathbf{B}$ are estimated by the standard methods used to estimate an ECM.

In the exchange rate model in the previous section, b is a structural parameter of interest. For the purpose of estimating b in the model, the restriction that $\mathbf{C}_0$ is lower triangular is not attractive. However, as is clear from Equation (29), the structural ECM from the one-good version of the exchange rate model does not satisfy the restriction that $\mathbf{C}_0$ is lower triangular for any ordering of the variables. Even though some structural models may be written in lower triangular form, this example suggests that many structural models cannot be written in that particular form.

It is instructive to observe the relationship between the structural ECM and the reduced form ECM in the exchange rate model. Because

$$(31) \qquad \mathbf{C}_0^{-1} = \begin{bmatrix} bh/(bh+1) & bh/(bh+1) & 0 & 0 \\ -1/(bh+1) & bh/(bh+1) & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$\mathbf{G} = \mathbf{C}_0^{-1}\mathbf{B} = [-b^2h/(bh+1),\ b/(bh+1),\ 0,\ 0]'$. Comparing **G** and **B** shows contemporaneous interactions between the domestic price and the exchange rate affect the speed of adjustment coefficients. The speed of adjustment coefficient for the domestic price is b in the structural model, while it is $b^2h/(bh+1)$ in the reduced form model. The error correction term does not appear in the second equation for the exchange rate in the structural ECM, while it appears with the speed of adjustment coefficient of $b/(bh+1)$ in the reduced form model.

4. The Instrumental Variables Methods

Because standard methods of estimating reduced form ECMs may not recover the structural parameters of interest in **B**, we consider two instrumental variables methods. The single equation method simply applies an IV estimator to a slow adjustment equation. The system method combines the single equation method with Hansen-Sargent IV estimator. These methods do not require restrictions on $\mathbf{C}_0$.

4.A. The Single Equation Method

First, we consider a single equation method, which applies an IV method to a slow adjustment equation. Imagine that we are interested in estimating the first row of Equation (28). In some applications, the cointegrating vectors are known, and thus the values of **A** are known. It should be noted that ordinary least squares may be applicable in this case of known cointegrating vectors.[8] In other applications, the values of **A** are unknown. In the case of the unknown cointegrating vectors, a two step method that is similar to Engle and Granger's (1987) and Cooley and Ogaki's (1996) methods can be used. In this two-step method, the cointegrating vectors are estimated in the first step.

In the first step, we estimate **A**, using a method to consistently estimate cointegrating vectors. There exist many methods to estimate cointegrating vectors. Johansen's Maximum Likelihood (ML) Estimators for Equation (27) can be used for this purpose. If ρ is equal to one, estimators based on regressions that are as efficient as Johansen's ML estimators such as Phillips and Hansen's Fully Modified Estimation Method (1990), Park's (1992) Canonical Cointegrating Regression, and Stock and Watson's (1993) estimators can be used. Ordinary Least Squares estimators are also consistent when ρ is equal to one, but not as efficient as these estimators. We assume that $\mathbf{A}_T$ is the first step estimator, where T is the sample size, and $\mathbf{A}_T$ converges to A at a faster rate than $T^{1/2}$.[9]

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[8]In our exchange rate model without measurement errors, ordinary least squres can be applied to an autoregressive regression for the real exchange rate to estimate the structural speed of adjustment coefficient.

[9]Usually, $\mathbf{A}_T$ converges at the rate of T, but there are cases where $\mathbf{A}_T$ converges at the rate of $T^{3/2}$ (see West, 1988)

In the single equation method, an IV method is applied to

$$(32)\qquad \Delta y_1(t+1) = d_1 - c^1_{02}\Delta y_2(t+1) - \dots - c^1_{0n}\Delta y_n(t+1) + \mathbf{b}_1\mathbf{A}'\mathbf{y}(t) + \mathbf{c}^1_1\Delta\mathbf{y}(t) + \mathbf{c}^1_2\Delta\mathbf{y}(t-1) + \dots + \mathbf{c}^1_p\Delta\mathbf{y}(t-p+1) + u_1(t+1),$$

where $y_i(t)$ is the i-th element of $y(t)$, d_1 is the first element in $\mathbf{d}$, c^1_{0i} is the i-th element of the first row of $\mathbf{C}_0$, $\mathbf{b}_1$ is the first row of $\mathbf{B}$, $\mathbf{c}^1_i$ is the first row of $\mathbf{C}_i$, and $u_1(t)$ is the first element of $\mathbf{u}(t)$. When $\hat{E}[u_1(t+1)|H_{t-\tau}]=0$, any stationary variable in the information set available at time t-τ that is correlated with variables in the right hand side of Equation (32) can be used as an instrumental variable. In the case of the known cointegrating vectors, the known values of $\mathbf{A}$ are used in (32). In the case of the unknown cointegrating vectors, $\mathbf{A}_T$ obtained in the first step replaces $\mathbf{A}$ in Equation (32). Because $\mathbf{A}_T$ converges to $\mathbf{A}$ at a faster rate than $T^{1/2}$, the first step estimation does not affect the asymptotic distributions of the second step estimator under regularity conditions.[10]

4.B. The System Method

In this section, we propose an econometric method that combines our single equation method with Hansen and Sargent's (1982) procedure to impose nonlinear restrictions implied by rational expectations models.

Let $\mathbf{y}(t)=(y_1(t),y_2(t),y_3(t),y_4(t))'$ be a 4×1 vector of random variables with a structural ECM representation (4). Assume that there exists only one linearly independent cointegrating vector $\mathbf{A}$ such that $\mathbf{A}'\mathbf{y}(t)$ is stationary.

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[10]As suggested by the results in de Jong (2001), the first step estimation can affect the asymptotic distributions of the second step estimator because of the nonlinear restrictions. However, because the equations are linear in this application, the reqularity conditions are likely to hold.

In the following, $\mathbf{y}(t)$ is partitioned into four subvectors, and each subvector is given a different role. For expositional simplicity, we assume that each subvector is one dimensional so that $\mathbf{y}(t)$ is a 4×1 vector, and that $\mathbf{y}(t)$ has only one cointegrating vector.

The first element of $\mathbf{y}(t)$ represents a slow adjustment as in Equation (32), with nonzero $\mathbf{b}_1$ where $\hat{E}[u_1(t+1)|H_{t-\tau}]=0$. We assume that the second element of $\mathbf{y}(t)$ is related to a discounted sum of expected future values of the fourth element in the following form:

$$(33) \qquad \Delta y_2(t+1) = d_2 - c_{01}^2\Delta y_1(t+1) - c_{03}^2\Delta y_3(t+1) - c_{04}^2\Delta y_4(t+1) + \alpha E[\sum_{j=0}^{\infty}\delta^j\Delta y_4(t+j+1)|I_t] + \varepsilon_e(t+1)$$

where δ is a positive constant that is smaller than one, and α is a constant. As pointed out by Hansen and Sargent, many linear rational expectations models imply that one variable is a geometrically declining weighted sum of expected future values of other variables.

Hansen and Sargent's (1982) methodology is to project the conditional expectation of the discounted sum, $E[\sum\delta^j\Delta y_4(t+j+1)|I_t]$, onto an information set H_t, which is a subset of I_t, the economic agents' information set. Let $\hat{E}(\cdot|H_t)$ be the linear projection operator conditional on an information set H_t which is a subset of I_t. Replacing the conditional expectation by the linear projection gives

$$(34) \qquad \Delta y_2(t+1) = d_2 - c_{01}^2\Delta y_1(t+1) - c_{03}^2\Delta y_3(t+1) - c_{04}^2\Delta y_4(t+1) + \alpha\hat{E}[\sum_{j=0}^{\infty}\delta^j\Delta y_4(t+j+1)|H_t] + u_2(t+1),$$

where

(35) $u_2(t+1) = \varepsilon_e(t+1) + E[\sum_{j=0}^{\infty} \delta^j \Delta y_4(t+j+1) | I_t] - \hat{E}[\sum_{j=0}^{\infty} \delta^j \Delta y_4(t+j+1) | H_t].$

Because H_t is a subset of I_t, we obtain $\hat{E}[u_2(t+1)|H_t] = 0.$

The current and past values of the first difference of the third element of $\mathbf{y}(t)$ are used to form the econometrician's information set H_t. Since $\hat{E}[\cdot|H_t]$ is the linear projection operator onto H_t, there exist possibly infinite order lag polynomials $\beta(L)$, $\gamma(L)$, and $\xi(L)$, such that

(36) $\hat{E}[\Delta y_3(t+1)|H(t)] = \beta(L)\Delta y_3(t)$

(37) $\hat{E}[\Delta y_4(t+1)|H(t)] = \gamma(L)\Delta y_3(t)$

(38) $\hat{E}[\sum_{j=0}^{\infty} \delta^j \Delta y_4(t+j+1)|H(t)] = \xi(L)\Delta y_3(t)$

Then following Hansen and Sargent (1980, Appendix A), we obtain the restrictions imposed on $\xi(L)$:

(39) $$\xi(L) = \frac{\gamma(L)-\delta L^{-1}\gamma(\delta)\{1-\delta\beta(\delta)\}^{-1}\{1-L\beta(L)\}}{1-\delta L^{-1}}$$

Substituting (38) into (34) gives the equation

(40) $$\Delta y_2(t+1) = d_2 - c_{01}^2\Delta y_1(t+1) - c_{03}^2\Delta y_3(t+1) - c_{04}^2\Delta y_4(t+1) + \alpha\, \xi(L)\, \Delta y_3(t) + u_2(t+1),$$

where $\xi(L)$ is given by (39). We now make an additional assumption that the lag polynomials $\beta(L)$ and $\gamma(L)$ are finite order polynomials, so that

(41) $$\Delta y_3(t+1) = \beta_1\Delta y_3(t) + \beta_2\Delta y_3(t-1) + \ldots + \beta_p\Delta y_3(t-p+1) + u_3(t+1)$$

(42) $$\Delta y_4(t+1) = \gamma_1\Delta y_3(t) + \gamma_2\Delta y_3(t-1) + \ldots + \gamma_{p-1}y_3(t-p+2) + u_4(t+1)$$

where $\hat{E}[u_i(t+1)|H_t]=0$ for $i=3,4$. Here we assume $\beta(L)$ is of order p and $\gamma(L)$

is of order $p-1$ in order to simplify the exposition, but we do not lose generality because any of β_i and γ_i can be zero. Then as in Hansen and Sargent (1982), (39) implies

$$(43)\qquad \begin{aligned} \xi_0 &= \gamma(\delta)\{1-\delta\beta(\delta)\}^{-1} \\ \xi_j &= \delta\gamma(\delta)\{1-\delta\beta(\delta)\}^{-1}(\beta_{j+1} + \delta\beta_{j+2} + \ldots + \delta^{p-j}\beta_p) \\ &\quad + (\gamma_j + \delta\gamma_j + \ldots + \delta^{p-j}\gamma_p) \qquad \text{for } j=1,\ldots,p. \end{aligned}$$

In the SECM form (28), we have $\mathbf{B}=[-b,0,0,0]'$, $\mathbf{A}=[1, -1, -1, 0]'$,

$$(44)\qquad \mathbf{C}_0 = \begin{bmatrix} 1 & c^1_{02} & c^1_{03} & c^1_{04} \\ c^2_{01} & 1 & c^2_{03} & c^2_{04} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

and

$$(45)\qquad \mathbf{C}_j = \begin{bmatrix} c^1_{j1} & c^1_{j2} & c^1_{j3} & c^1_{j4} \\ 0 & 0 & \alpha\xi_j & 0 \\ 0 & 0 & \beta_j & 0 \\ 0 & 0 & \gamma_j & 0 \end{bmatrix}$$

for $j=1,\ldots,p$, where $\gamma_p=0$.

We have now obtained a system of four equations that consist of (32), (40), (41), and (42). Because $E(u_1(t)|I_{t-\tau})=0$ and $\hat{E}(u_i(t)|H_t)=0$, we can obtain a vector of instrumental variables $\mathbf{z}_1(t)$ in $I_{t-\tau}$ for $u_1(t)$ and $\mathbf{z}_i(t)$ in H_t for $u_i(t)$ $(i=2,3,4)$.

Because the speed of adjustment b for $y_1(t)$ affects the dynamics of other variables,[11] there will be cross-equation restrictions involving b in

[11] Note that only $y_1(t)$ adjusts slowly, but b affects dynamics of other variables because of interactions between $y_1(t)$ and those variables.

many applications in addition to the restrictions in (43). Using the moment conditions $E[\mathbf{z}_i(t)u_i(t)]=0$ for $i=1,\ldots,4$, we form a GMM estimator, imposing the restrictions in (43) and the other cross-equation restrictions implied by the model.

Given estimates of cointegrating vectors from the first step, this system method provides more efficient estimators than the single equation two step method proposed in Section 3 as long as the restrictions implied by the model are true. On the other hand, the single equation two step method estimators are more robust because misspecification in other equations does not affect their consistency. The cross-equation restrictions can be tested by Wald, Likelihood Ratio type, and Lagrange Multiplier tests in the GMM framework (see, e.g., Ogaki 1993a). When restrictions are nonlinear, Likelihood Ratio type and Lagrange Multiplier tests are known to be more reliable than Wald tests.

4.C. Applying the System Method to the Exchange Rate Model

In order to apply the system method to Equations (23)-(26) of the exchange rate model in Section 2, we need data for $\Delta\omega(t)$, which requires the knowledge of h. Even though h is unknown, a cointegrating regression can be applied to money demand if money demand is stable in the long-run as in Stock and Watson (1993). For this purpose, we augment the model as follows:

(46) $$m(t) = \theta_m + p(t) - hi(t) + \zeta_m(t)$$

where $\zeta_m(t)$ is the money demand shock, which is assumed to be stationary, so that money demand is stable.

By redefining $m(t)$ in the previous section as $m(t)-\zeta_m(t)$, the same equations as those in Section 2 are obtained. For the measurement of $\Delta\omega(t)$

used in the system method, we note that the *ex ante* foreign real interest rate can be replaced by the *ex post* real foreign real interest rate because of the Law of Iterated Expectations. Using the money market clearing condition (46), we obtain

$$(47) \quad \Delta\omega(t+1) = \Delta p(t+1) - h\Delta i(t+1) + h\Delta i^*(t+1) - h\,[\Delta p^*(t+2) - \Delta p^*(t+1)].$$

With this expression, $\Delta\omega(t)$ can be measured from price and interest rate data without data for monetary aggregate and national income once h is obtained. This is useful because the latter data are available not at the monthly frequency for many countries.

5. A Measurement Model

We apply the single equation and system methods to the exchange rate model in Section 2, using monthly exchange rate and aggregate price data for Canada, France, Germany, Italy, Japan, the Netherlands, Switzerland, the United Kingdom, and the United States from January 1974 to June 1995. In the model, we assume that $\mathbf{y}(t) = (p(t), e(t), p^*(t))$ is cointegrated with a known cointegrating vector $(1,-1,-1)$. This assumption may cause a problem in applications of the model to data in the post-Bretton Woods period because many researchers have failed to reject the null hypothesis of no cointegration using similar data sets. Because more favorable evidence for the assumption is often found when a longer sample period is used, the failure to reject no cointegration may be due to low power of the no cointegration tests in small samples (see, e.g., Rogoff, 1995 for a survey). Because the evidence is mixed, a sensitivity analysis with respect to this assumption is in order. For the purpose of a sensitivity analysis, we employ Cheung and Lai (1993) and Fisher and Park's (1991) model with measurement errors to allow the cointegrating vector to be different from $(1,-1,-1)$.

5.A. Measurement Errors and the Single Equation Method

Let $p^{m}(t)$ and $p^{*m}(t)$ be the log measured domestic and foreign prices, which are related to the true prices by

(48) $$p^{m}(t) = \theta + \phi\, p(t) + v(t)$$

(49) $$p^{*m}(t) = \theta^{*} + \phi^{*}\, p^{*}(t) + v^{*}(t)$$

where $E_{t-1}[v(t)]=0$ and $E_{t-1}[v^{*}(t)]=0$. We assume that true prices follow the model of Section 2 and satisfy PPP in the long-run. Then

(50) $$p^{m}(t) - \phi e(t) - (\phi/\phi^{*})p^{*m}(t) = (\theta-\theta^{*}\phi/\phi^{*}) + \phi\, [p(t)-e(t)-p^{*}(t)] + [v(t)-(\phi/\phi^{*})v^{*}(t)]$$

is stationary. Hence, $y(t)=(p^{m}(t),\ e(t),\ p^{*m}(t))'$ is cointegrated with a cointegrating vector $(1,-\phi,-\phi/\phi^{*})$. In the first step, we run a cointegrating regression of the form

(51) $$p^{m}(t) = \psi_0 + \psi_1 e(t) + \psi_2 p^{*m}(t) + \zeta(t),$$

where $\psi_1=\phi$, $\phi_2=\phi/\phi^{*}$, and $\zeta(t)$ is stationary with mean zero.

In order to obtain the second step estimator, we use Equation (2) and $\Delta p^{m}(t+1) = \phi\ \Delta p(t+1) + \Delta v(t+1)$ to obtain

(52) $$\Delta p^{m}(t+1) = d - b\ [p^{m}(t) - \phi e(t) - (\phi/\phi^{*})p^{*m}(t)] + (\phi/\phi^{*})\Delta p^{*m}(t+1) + \phi\ \Delta e(t+1) + w(t+1)$$

where $d = b(\mu+\theta-\theta^{*}\phi/\phi^{*})$, and

(53) $w(t+1) = \phi\varepsilon(t+1) + v(t+1) - (1-b)v(t)-(b\phi/\phi^*)v^*(t+1)$

$$+ (1-b)(\phi/\phi^*)v^*(t).$$

Because $E_{t-1}[w(t+1)]=0$, we can apply the two step procedure from the last section as long as the instrumental variables are chosen from the information set available at $t-1$. In this case, the second step is to apply an IV estimation method to Equation (52), where ϕ and ϕ^* are obtained in the first step estimation. Because $E_{t-1}[w(t+1)]=0$ and $w(t+1)$ is in the information set available at $t+1$, $w(t+1)$ has a moving average (MA) representation of order one, and this serial correlation structure needs to be taken into account (see, e.g., Ogaki 1993a for an explanation of methods which treat this type of serial correlation in GMM).

5.B. Measurement Errors and the System Method

We use the measurement model for the purpose of a sensitivity analysis with respect to PPP as in the case for the single equation method. Again it is assumed that the model is true for the true price levels, but that only measured prices that follow (48) and (49) are observed. Since $p^m(t)$ and $p^{*m}(t)$ are observed instead of $p(t)$ and $p^*(t)$, (48) and (49) are substituted into Equations (23)-(26) in order to express these equations in terms of measured prices. It is also assumed that H_t is the information set generated by the current and past values of $\Delta p^{*m}(t)$ instead of $\Delta p^*(t)$.

As for the adjustment to the PPP level, (23) is replaced by (52). For $\Delta\omega(t)$, we use

(54) $\Delta\omega^m(t+1) = \frac{1}{\phi}\Delta p^m(t+1) - h\Delta i(t+1) + h\Delta i^*(t+1) - \frac{h}{\phi^*}[\Delta p^{*m}(t+2) - \Delta p^{*m}(t+1)],$

so that

(55) $$\Delta e(t+1) = d_2 + \frac{bh+1}{bh}(1-\delta)\hat{E}[\sum_{j=0}^{\infty}\delta^j\Delta\omega^m(t+j+1)|H(t)] - \frac{1}{bh\phi}\Delta p^m(t+1) - \frac{1}{\phi^*}\Delta p^{*m}(t+1) + u_2^m(t+1)$$

where

(56) $$u_2^m(t+1) = u_2(t+1) - \frac{bh+1}{bh}(1-\delta)\hat{E}[\frac{1}{\phi}v(t) - \frac{h}{\phi^*}v^*(t)|H(t)] + \frac{1}{bh\phi}\Delta v(t+1) - \frac{1}{\phi^*}\Delta v^*(t+1),$$

and $\hat{E}[u_2^m(t+1)|H_{t-1}] = 0$.

Because the price level is assumed to be measured with errors as in (48),

(57) $$m(t) = \theta_2 + (1/\phi)p^m(t) - hi(t) + \zeta_2(t)$$

where $\theta_2 = \theta_m - \theta/\phi$ and $\zeta_2(t) = \zeta_m(t) - v(t)/\phi$. Because $\zeta_2(t)$ is stationary, a cointegrating regression is applied to (57), assuming $m(t)$ and $i(t)$ are first difference stationary.

Thus we run two cointegrating regressions, (51) and (57), in the first step. In the second step, GMM is applied to the system of four equations that consist of (52),

(58) $$\Delta e(t+1) = - \frac{1}{bh\phi}\Delta p^m(t+1) - \frac{1}{\phi^*}\Delta p^{*m}(t+1)$$
$$\alpha\xi_1\Delta p^{*m}(t) + \alpha\xi_2\Delta p^{*m}(t-1) + \ldots + \alpha\xi_p\Delta p^{*m}(t-p+1) + u_2^m(t+1)$$

(59) $$\Delta p^{*m}(t+1) = = \beta_1\Delta p^{*m}(t) + \beta_2\Delta p^{*m}(t-1) + \ldots + \beta_p\Delta p^{*m}(t-p+1) + u_3(t+1)$$

(60) $$\Delta\omega^m(t+1) = \gamma_1\Delta p^{*m}(t) + \gamma_2\Delta p^{*m}(t-1) + \ldots + \gamma_{p-1}\Delta p^{*m}(t-p+2) + u_4(t+1)$$

where h is replaced by its estimate from (57) and ϕ and ϕ^* are replaced by their estimates from (51). As before, because the first step estimators are super consistent, the first step estimation does not affect asymptotic distributions of the second step GMM estimators under some regularity conditions.

6. *Empirical Results*

In this section, we present empirical results for the single equation procedure. Monthly end-of-period foreign exchange rates from the International Financial Statistics (IFS) are used. The foreign exchange rates are stated as the domestic price of one unit of foreign currency. In each regression, the United States is regarded as the foreign country, while other countries are the domestic countries. Monthly CPI is used to measure prices in the model. The sample period is from January 1974 to June 1995.

For each country, we report results for two cases. The first case is when prices are measured without error, which leads to the case of the known cointegrating vector. The second case is that of the measurement error model of Section 5.A, in which the cointegrating vector for domestic prices, exchange rates and foreign prices is not restricted to be (1,-1,-1). For the latter case, the two-step method is used. In the first step, we use CCR to obtain long-run coefficients in PPP relations. In the second step, we apply GMM to estimate the short-run coefficient.

For the measurement error model, we need estimates of the coefficients in the cointegrating relationship (51), which is based on PPP. Table 1 presents the results cointegrating regressions. We report the third stage estimates of CCR for the coefficients and the fourth stage test results.

The deterministic cointegrating restrictions are rejected for Canada, France, Germany and Italy at the five percent significance level. The restriction is rejected for France at the one percent significance level. The restriction is not rejected for Japan, the Netherlands, Switzerland and UK at the five percent level.

Stochastic cointegration is not rejected for Germany, Switzerland and UK at the one percent significance level for any H(1,q) test in the table. French and Japanese data reject the null of stochastic cointegration for any H(1,q) test.

Even though the main focus of this paper is the system method results, Table 2 reports the single equation method results for the purpose of comparisons with the system method results. The instrumental variables are $\Delta p^{*m}(t-3)$ and $\Delta p^{*m}(t-4)$, which are US prices. For each country, the first row in table 2 reports the GMM results for the case of the known cointegrating vector (1, -1, -1), and the second row shows the GMM results for the case of the unknown cointegrating vector. To obtain the half life of each estimate for b, we rearrange the ECM equation as an AR(1) process for the real exchange rates.

In most cases, we obtain positive point estimates for the structural speed of adjustment coefficient *b*. In cases where the point estimates are negative, they are not significantly different from zero at the 5 percent level.

For the system method, our estimation procedure has two steps. First, we estimate the monetary equilibrium equation to obtain interest elasticity of money demand. For the measurement error model, we also obtain the measurement error coefficients, exploiting the long-run relationship between domestic prices, foreign prices and exchange rates. In the second step, the speed of price adjustment is estimated from the adjustment equation as well as the Hansen and Sargent equations.

To estimate the interest elasticity of money demand, we use the sum of M1 and Quasi Money as the measure of the money stock, called M2, as the IFS suggests. The data for interest rates are the three month T-bill rates, but three month deposit rates are employed for Japan because T-bill rates are not available. We use nominal and real gross domestic product data in the IFS dataset for all countries except the UK, for which we use the DRI data. All data series are seasonally unadjusted. Sample periods vary across countries, according to data availability.[12]

The semi-elasticity of interest rates in the money demand equation is estimated using quarterly data. However, since we use monthly data to estimate the system, we transform the quarterly estimated elasticities into monthly elasticities. We multiply the elasticities from the quarterly data by 3, because the quarterly interest rates are divided by 3 to obtain monthly interest rates.

Table 3 shows the CCR results for the money demand equations. We assume that the income elasticity of money demand is one. For each country, the first row reports the results when the coefficient of the log price is restricted to be one, and the second row reports the results when the coefficient is allowed to differ from one. When we employ the measurement error model, we use the results reported in the second row.

The null of stochastic cointegration is rejected only for Germany, regardless of the assumption of measurement errors at the 5 percent level.

[12] Sample periods used to estimate money demand equations are 1975:1 - 1995:1 for Canada, 1986:3 - 1995:1 for France, 1978:3 - 1995:1 for Germany, 1977:1 - 1993:3 for Italy, 1974:1 - 1993:4 for Japan, 1978:2 - 1990:2 for Netherlands, 1980:1 - 1995:2 for Switzerland, and 1974:1 - 1994:1 for U.K.

The deterministic cointegrating restriction is rejected for Germany, Italy and Japan at the 5 percent level, when we allow for measurement errors. With the prespecified cointegrating vector (1, -1, -1), France and Switzerland reject the deterministic cointegrating restriction at the 5 percent level, but do not reject it at 1 percent.

In all cases, the signs of the estimates for the interest elasticity of money demand are negative, as expected from the economic model. For Canada, France and Switzerland, the specification of measurement errors does not affect the estimates for the interest elasticities. However, for Germany, Italy, Japan, the Netherlands and the UK, the estimates from the measurement error models have smaller values than those from the models without measurement error. Interestingly, these range from one fourth to one fifth of the estimates from the no measurement error models for each country.

When we restrict the cointegrating vector to (1, -1, -1), the measurement error coefficients are no longer free parameters. In this case, we have no problem when we separately run two cointegrating regressions which include a common coefficient. But, if we allow for measurement errors in price indices, then we have two estimates for the measurement error coefficient on the domestic prices. One set of estimates is obtained from the PPP regression and the other set from the money demand equation. There is no guarantee in practice for the two estimates to be the same. If the estimates from the two equations are significantly different, it might imply misspecification of the simple exchange rate model. Although this is the case, we use the estimates from the PPP equation in Table 1 because we are more interested in PPP than in the money demand equation. Park and Ogaki (1991) suggest the seemingly unrelated canonical cointegrating regressions

(SUCCR) method to deal with cross equation restrictions, when there are cointegrating vectors in the equations. However, since the small sample properties of their estimator are not better than CCR, we use the estimates from PPP.

Table 4 reports the results of GMM estimation using the system method. The instrumental variables are $\Delta p^{*m}(t\text{-}3)$ and $\Delta p^{*m}(t\text{-}4)$, which are US prices. For each country, we report results for the known cointegrating vector case and the unknown cointegrating vector case. In the system method, the structural speed of adjustment coefficient b, appears in two equations: the slow adjustment equation, (3.29) or (2.15), or the Hansen-Sargent equation, (3.30) or (3.38). The model imposes the restriction that the coefficient b in the slow adjustment equation is the same as the coefficient b in the Hansen-Sargent equation. We report results with and without this restriction imposed for the system method of estimation. In the case of unrestricted estimation, b_{hs} is the estimate of b from the Hansen-Sargent equation, and b_{sa} is the estimate of b from the slow adjustment equation. The restricted estimate is denoted by b_r. The likelihood ratio type test statistic (see, e.g., Ogaki (1993a) for an explanation of this test), denoted by C, is used to test the restriction. In most cases, this restriction is not rejected at the five percent level. The exceptions are France with the unknown cointegrating vector case, Italy for both cases, Japan for the known cointegrating vector case.

All restricted estimates for the structural speed of adjustment coefficient have the theoretically correct positive sign. Most of them are significant at the five percent level. In almost all cases, the asymptotic standard error for the speed of adjustment coefficient is smaller when the system method is used than when the single equation method is used.

The half life is calculated from the restricted estimate of the structural speed of adjustment coefficient in each case. The half life estimate is based on the first order autoregressive process of the domestic price implied by Equation (1). The half life estimates range from 0.23 to 7.10 years. Rogoff (1996) describes the consensus of 3-5 year half lives when long-horizon data are used. Only four point estimates for Italy and U.K. fall in this range. For France, Germany, the Netherlands, and Switzerland, the point estimates of half lives are shorter. For Canada and Japan, the half life estimates are very different depending on whether or not the cointegrating vector is assumed to be known.

7. Concluding Remarks

This paper compares reduced form ECMs with structural form ECMs. The speed of adjustment coefficients in reduced form ECMs are different from those in structural form ECMs in general, and in our example of an exchange rate model with sticky prices. We discussed a single equation IV method and a system IV method to estimate structural speed of adjustment coefficients. These IV methods do not require exogeneity assumptions, and can be applied to a broad range of structural ECMs.

When the single equation method is applied to the exchange rate model, the point estimate for the speed of adjustment coefficient are positive for most countries. When the system method is applied to the exchange rate model, the speed of adjustment coefficient is estimated from both the slow adjustment equation for the domestic price and the rational expectations equation for the exchange rate. Half life estimates from the system method tend to be shorter than the consensus of 3-5 years explained by Rogoff (1996). In most cases, this restriction is not rejected at the five percent level. In some cases, the restricted estimate is significantly negative, indicating that the restricted model is misspecified. For the purpose of estimating the structural speed of adjustment coefficient, the results for Canada and France in the case of the known cointegrating vector and those for Canada, Japan, and the U.K. are encouraging. In each of these cases, the restriction is not rejected and the restricted estimate is positive and significant at the five percent level.

Kim (2000) applies the system method developed in this paper to traded and nontraded good prices. He finds that traded good prices adjust faster than nontraded good prices.

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TABLE 1. Purchasing Power Parity

Country (1)	ψ_0 (2)	ϕ (3)	ϕ/ϕ^* (4)	$H(0,1)$ (5)	$H(1,2)$ (6)	$H(1,3)$ (7)
Canada	0.005	0.029	1.014	5.377	2.911	8.443
	(0.169)	(0.122)	(0.024)	(0.020)	(0.088)	(0.015)
France	-0.899	0.221	1.106	104.0	30.17	41.67
	(0.047)	(0.017)	(0.014)	(0.000)	(0.000)	(0.000)
Germany	2.796	0.073	0.376	5.402	5.571	6.068
	(0.220)	(0.022)	(0.054)	(0.020)	(0.018)	(0.048)
Italy	-4.283	0.192	1.621	4.134	1.355	3.264
	(0.064)	(0.020)	(0.028)	(0.042)	(0.244)	(0.196)
Japan	0.509	0.156	0.728	0.204	6.963	22.45
	(0.370)	(0.039)	(0.042)	(0.652)	(0.008)	(0.000)
Netherlands	1.548	0.141	0.651	2.871	0.208	19.81
	(0.107)	(0.036)	(0.021)	(0.090)	(0.648)	(0.000)
Switzerland	2.929	0.027	0.346	1.763	3.460	3.473
	(0.221)	(0.031)	(0.052)	(0.184)	(0.063)	(0.176)
U.K.	-1.902	0.369	1.458	0.003	2.686	5.078
	(0.144)	(0.071)	(0.030)	(0.958)	(0.101)	(0.079)

Note: Results for $p^m(t) = \psi_0 + \phi e(t) + (\phi/\phi^*)p^{*m}(t) + \zeta(t)$
Column (1) : domestic countries
Column (2)-(4) : Standard errors are in parentheses.
Column (5)-(7) : P-values are in parentheses.

TABLE 2. The Single Equation Method Results

Country (1)	ϕ	ϕ/ϕ^*	d (2)	b (3)	χ^2 (4)
Canada	1	1	-0.0036 (0.0024)	0.0126 (0.0104)	1.712 (0.634)
	0.029	1.014	0.0005 (0.0002)	0.0166 (0.0050)	4.688 (0.196)
France	1	1	-0.0268 (0.0224)	0.0151 (0.0125)	2.113 (0.549)
	0.221	1.106	-0.0300 (0.0131)	0.0335 (0.0146)	2.324 (0.508)
Germany	1	1	-0.0042 (0.0063)	0.0110 (0.0110)	5.793 (0.122)
	0.073	0.376	-0.0039 (0.0093)	-0.0018 (0.0033)	6.531 (0.088)
Italy	1	1	0.0701 (0.1880)	-0.0094 (0.0256)	1.612 (0.657)
	0.192	1.621	0.0058 (0.0423)	-0.0014 (0.0099)	9.025 (0.029)

TABLE 2 - *continued*

Country (1)	ϕ	ϕ/ϕ^*	d (2)	b (3)	χ^2 (4)
Japan	1	1	-0.0143 (0.0488)	0.0037 (0.0096)	5.552 (0.136)
	0.156	0.728	0.0209 (0.0083)	0.0398 (0.0164)	6.374 (0.095)
Netherlands	1	1	-0.0110 (0.0071)	0.0182 (0.0106)	0.074 (0.995)
	0.141	0.651	0.0694 (0.0197)	0.0445 (0.0127)	2.788 (0.426)
Switzerland	1	1	-0.0098 (0.0081)	0.0248 (0.0162)	1.471 (0.689)
	0.027	0.346	-0.0108 (0.0071)	-0.0041 (0.0024)	11.70 (0.008)
U.K.	1	1	0.0105 (0.0085)	0.0206 (0.0197)	2.980 (0.395)
	0.369	1.458	-0.0208 (0.0217)	0.0106 (0.0115)	4.806 (0.187)

Note: Results for $\Delta p^m(t+1) = d - b\,[p^m(t) - \phi e(t) - (\phi/\phi^*)p^{*m}(t)] + (\phi/\phi^*)\Delta p^{*m}(t+1) + \phi\,\Delta e(t+1) + w(t+1)$

Column (1) : domestic countries
Column (2)-(3) : Standard errors are in parentheses.
Column (4) : P-values are in parentheses.

TABLE 3. Money Demand Equation

Country (1)	θ_2 (2)	$1/\phi$ (3)	h (4)	$H(0,1)$ (5)	$H(1,2)$ (6)	$H(1,3)$ (7)
Canada	-0.046	1	30.03	1.019	0.445	1.098
	(0.247)		(9.875)	(0.313)	(0.505)	(0.578)
	0.464	1.899	40.79	0.523	4.681	4.792
	(0.456)	(0.240)	(16.20)	(0.469)	(0.030)	(0.091)
France	-0.341	1	5.661	4.823	0.963	1.183
	(0.071)		(3.182)	(0.028)	(0.326)	(0.554)
	-0.337	0.253	5.560	2.036	0.097	0.321
	(0.014)	(0.038)	(0.636)	(0.153)	(0.755)	(0.851)
Germany	-0.272	1	17.88	1.537	1.600	4.575
	(0.165)		(9.856)	(0.215)	(0.206)	(0.102)
	-0.454	1.597	3.003	11.50	18.61	18.65
	(0.022)	(0.050)	(1.307)	(0.001)	(0.000)	(0.000)
Italy	-0.205	1	7.847	3.755	0.753	2.424
	(0.172)		(4.807)	(0.053)	(0.386)	(0.298)
	-0.508	0.789	1.994	9.247	1.560	5.979
	(0.041)	(0.013)	(1.073)	(0.002)	(0.212)	(0.050)

TABLE 3 - *continued*

Country (1)	θ_2 (2)	$1/\phi$ (3)	h (4)	$H(0,1)$ (5)	$H(1,2)$ (6)	$H(1,3)$ (7)
Japan	-11.31 (0.059)	1	39.66 (5.827)	3.274 (0.070)	3.229 (0.072)	5.353 (0.069)
	-13.95 (0.784)	1.520 (0.171)	8.089 (5.228)	13.02 (0.000)	0.035 (0.852)	0.674 (0.714)
Netherlands	-0.178 (0.075)	1	6.658 (4.124)	10.16 (0.001)	0.292 (0.864)	0.257 (0.879)
	-1.451 (0.051)	1.778 (0.150)	1.731 (3.006)	0.565 (0.452)	3.729 (0.053)	4.826 (0.090)
Switzerland	0.282 (0.036)	1	9.858 (2.468)	4.029 (0.045)	0.876 (0.349)	1.751 (0.417)
	0.287 (0.020)	1.155 (0.056)	8.987 (1.276)	0.085 (0.771)	2.296 (0.129)	3.859 (0.145)
U.K.	10.66 (3.410)	1	111.3 (128.3)	1.853 (0.173)	0.088 (0.766)	0.597 (0.742)
	8.982 (0.273)	2.265 (0.116)	27.56 (9.306)	2.942 (0.086)	0.129 (0.720)	5.562 (0.062)

Note: Results for $m(t) = \theta_2 + (1/\phi)p^m(t) - hi(t) + \zeta_2(t)$
Column (1) : domestic countries
Column (2)-(4) : Standard errors are in parentheses.
Column (5)-(7) : P-values are in parentheses.

TABLE 4. The System Method Results

			unrestricted			*restricted*			
Country	ϕ	ϕ/ϕ^*	$b_{u,hs}$	$b_{u,sa}$	J_u	b_r	Half Life	J_r	C
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Canada	1	1	-0.0001 (0.0001)	0.0184 (0.0374)	5.229 (0.156)	0.2256 (0.0183)	0.23	5.276 (0.260)	0.047 (0.828)
	0.029	1.014	0.0081 (0.0001)	0.0081 (0.0119)	4.124 (0.248)	0.0081 (0.0002)	7.10	4.143 (0.387)	0.019 (0.891)
France	1	1	0.0589 (0.0001)	0.0162 (0.0129)	1.198 (0.754)	0.0590 (0.0002)	2.29	2.291 (0.682)	1.093 (0.296)
	0.221	1.106	0.0186 (0.0001)	-0.0843 (0.0546)	5.068 (0.167)	0.0186 (0.0001)	0.24	8.293 (0.081)	7.297 (0.007)
Germany	1	1	0.0186 (0.0001)	-0.0843 (0.0546)	5.068 (0.167)	0.0186 (0.0001)	1.23	7.519 (0.111)	2.451 (0.117)
	0.073	0.376	0.1334 (0.1550)	-0.0058 (0.0060)	4.986 (0.173)	0.1134 (0.0017)	0.94	6.037 (0.196)	1.051 (0.305)
Italy	1	1	0.0042 (0.0112)	0.0425 (0.0008)	4.020 (0.259)	0.0424 (0.0006)	3.95	7.992 (0.092)	3.971 (0.046)
	0.192	1.621	0.1755 (0.0054)	-0.0196 (0.0089)	4.612 (0.203)	0.0194 (0.0101)	4.74	12.363 (0.015)	7.751 (0.005)

TABLE 4 - *Continued*

			unrestricted			*restricted*			
Country	ϕ	ϕ/ϕ^*	$b_{u,hs}$	$b_{u,sa}$	J_u	b_r	Half Life	J_r	C
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Japan	1	1	0.0084	-0.0612	6.191	0.0084	6.85	11.561	5.369
			(0.0005)	(0.0322)	(0.103)	(0.0002)		(0.021)	(0.020)
	0.156	0.728	0.0414	0.0790	6.774	0.0414	1.37	6.823	0.049
			(0.0002)	(0.0606)	(0.079)	(0.0001)		(0.146)	(0.825)
Netherlands	1	1	0.0503	0.0247	7.573	0.0249	2.29	8.584	1.012
			(0.0010)	(0.0300)	(0.056)	(0.0145)		(0.072)	(0.315)
	0.141	0.651	0.1762	0.2963	3.732	0.2157	0.24	4.574	0.842
			(0.0085)	(0.0466)	(0.292)	(0.0131)		(0.334)	(0.359)
Switzerland	1	1	0.0018	0.0463	7.052	0.0457	1.23	7.079	0.028
			(0.0011)	(0.0362)	(0.070)	(0.0351)		(0.132)	(0.868)
	0.027	0.346	0.0373	0.0209	7.008	0.0596	0.94	7.407	0.399
			(0.0003)	(0.0121)	(0.072)	(0.0181)		(0.354)	(0.528)
U.K.	1	1	0.0030	-0.0411	6.497	0.0145	3.95	8.097	1.600
			(0.0044)	(0.0613)	(0.090)	(0.0601)		(0.088)	(0.206)
	0.369	1.458	0.0121	0.0047	5.377	0.0121	4.74	5.500	0.123
			(0.0009)	(0.0204)	(0.146)	(0.0001)		(0.240)	(0.726)

Note: For the unrestricted estimaion, $b_{u,hs}$ is the estimate for the speed of adjustment coefficient obtained from Hansen and Sargent equations, and $b_{u,sa}$ is the estimate for the coefficient obtained from the price adjustment equation.

Column (1) : domestic countries

Column (2) and (3) are from Table 3.

Column (4),(5) & (7) : Standard errors are in parentheses.

Column (6),(9) & (10) : P-values are in parentheses.

Column (8): Half life in years.