JOB MATCHING: A MULTI-PRINCIPAL, MULTI-AGENT MODEL*

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Abstract

A version of Spence's "job market" model is constructed and studied: There are two groups of individuals, the job applicants (the informed), and the incumbents (the uninformed). The applicants have private information about their types, and their actions (choice of education levels) serve as messages. The incumbents (employers) have only one type, and are endowed with differentiated information structures on actions of the informed. A contract is a pair of an education level and a wage level, and a wage schedule specifies a contract for each education level. The incumbent set is finite, and is fixed throughout the analysis (so free entry/exit is excluded). The paper studies endogenous determination of the wage schedules offered by the incumbents. The applicants behave noncooperatively. Two equilibrium concepts are proposed: a noncooperative equilibrium, a version of the Nash equilibrium which postulates noncooperative and passive behavior of the incumbents, and a cooperative equilibrium, a version of the strong equilibrium which postulates cooperative and passive behavior of the incumbents. It is shown that a cooperative equilibrium does not exist. By studying noncooperative equilibria, which do exist in many cases, it is concluded that it is not the informational advantage (defined as the abundance of measurable sets), but rather possession of the right information (in the sense that it best serves the needs of applicants) that enables an incumbent to win.

1 INTRODUCTION

The role of asymmetric information in allocation of resources, together with the associated information-revelation process, has long been a central focus of economic research. While the bulk of the literature addresses these issues within the framework of principal-agent relationship, which essentially reduces the problem to the sole principal's (the sole Stackelberg leader's) optimization problem subject to the agents' (the Stackelberg followers') responses, there are recent attempts to extend analysis to other economic setups characterized by different relationships among decision-makers.

A notable strand of such attempts is the core analysis of incomplete information. Here, there is no Stackelberg-type relationship, and more importantly the players can talk to each other for coordinated choice of strategies. See, e.g., Wilson (1978) for a pioneering work; Yannelis (1991) for formulation of feasibility of a strategy as its measurability; Ichiishi and Idzik (1996) for introduction of Bayesian incentive-compatibility to this strand; Ichiishi, Idzik and Zhao (1994) for information revelation (that is, endogenous determination of updated information structures); Ichiishi and Radner (1997) and Ichiishi and Sertel (1998) for studies of a specific model of Chandler's firm in multidivisional form for sharper results; and Vohra (1999) for a recent work. It is a common postulate in these works that every player takes part in design of a mechanism and also in execution of the signed contract.

The present paper provides an analysis of the role of asymmetric information, given yet another player relationship: We retain the principal-agent relationship, but allow for *several* principals in addition to several agents. Interaction of the principals is a focus of the paper. It is true that the traditional principal-agent literature frequently postulates existence of many principals, in fact infinitely many potential principals as required by the pure competition assumption (specifically, by the free entry and exit assumption), but this assumption in a nutshell reduces the model to the one-principal case in which the principal's only economically feasible strategy is the competitive strategy; this point was emphasized in Ichiishi (1997, Sections 7.4 and 7.6).

Given a multi-principal, multi-agent setup, we intend to study the roles of incomplete information about exogenous data and of incomplete information about endogenous variables. A general theory is yet to be developed, and our work reported in this paper is modest: As the first step towards a healthy general theory, we construct and study a very specific model, a variant of

Spence's (1974) education model. There are two groups of individuals, the *job applicants* (the informed), and the *incumbents* (the uninformed). The applicants have private information about their types (productivity levels), and their actions (choice of education levels) serve as messages to the incumbents. The incumbents (employers) have only one type, have no information about applicants' types, only partially observe applicants' actions, and their strategies are to determine wage schedules. We are following Spence in modelling incomplete information about exogenous data, namely about types of applicants. Our modelling of incomplete information about endogenous variables, namely about applicants' actions, on the other hand, is quite different from the way the traditional literature on moral hazard has modelled unobservability, but is suited to the nature of the present setup.

In our model, the principals are the incumbents, and the agents are the job applicants. The game is played in the following sequence: (1) Each incumbent first designs a wage schedule as his strategy. (2) Each applicant then chooses an education level, and (3) finally chooses the best contract for him. Anticipating optimal reactions of the applicants in (2) and (3), the incumbents play a game in the above stage (1) (called the *first-stage game*).[3] We analyze the first-stage game; analysis of the subsequent stages is trivial.

We consider two situations: one in which the incumbents behave noncooperatively and passively, and the other in which the incumbents behave cooperatively and passively. Associated with each situation, we propose an equilibrium concept: a *noncooperative equilibrium*, a version of the Nash equilibrium for the noncooperative behavioral principle, and a *cooperative equilibrium*, a version of the strong equilibrium for the cooperative behavioral principle. Our first observation is negative: a cooperative equilibrium does not exist. On the other hand, we obtain positive results on noncooperative equilibria; they do exist in many cases. By studying typical noncooperative equilibria, we conclude that it is not the informational advantage (defined as the abundance of measurable sets), but rather possession of the right information (in the sense that it best serves the needs of applicants) that enables an incumbent to win.

The negative result on a cooperative equilibrium is analogous to the

[3] Our theory in the present paper is in line with the mechanism theory, which postulates that the uninformed move ¯rst and the informed move second, rather than the signalling game, which postulates that the informed move ¯rst and the uninformed move second.

nonexistence of a strong equilibrium in the prisoner's dilemma game. As Ichiishi and Idzik (1996) stressed, this is due to the very structure of the model (which is simplistic). For a cooperative equilibrium to exist, there have to be merits of coordination of strategies. Let F^S be the set of all feasible strategies available to coalition S, coordinated and uncoordinated. There would be merits if F^S strictly contains $\prod_{j \in S} F^j$. Roughly stated, however, our present model postulates that F^S is identical to $\prod_{j \in S} F^j$ (apart from the informational aspect), as in the prisoner's dilemma game, hence the nonexistence result. There are countless situations in the real economy in which the above strict inclusion holds true (including the basic situations, like the pure exchange economy). We expect that future research will establish positive results on a cooperative equilibrium, given such situations.

Our conclusion that possession of the right information enables an incumbent to win appears to be robust. We expect that this can be taken as one of the general principles that prevail in most models.

2 MODEL

The player set consists of the *applicants*, who first go through education and then look for a job, and the *incumbents*, who offer jobs to applicants.

There are two types of applicants, type L (low quality) and type H (high quality). An applicant's type is his private information. An applicant of type t, when employed by an incumbent, brings in to the employer the marginal revenue r_t, $t = L, H$. Denote by M the set of possible education levels; for simplicity, we assume that $M = [0, \overline{m}]$, a nondegenerate interval. A pair $(m, w) \in M \times \mathbf{R}_+$ then signifies the education level and the wage level of an applicant; the pair is called a *contract*. The preference relation of an applicant of type t is defined on the contract space $M \times \mathbf{R}_+$, and is represented by a continuous utility function $u(\cdot | t) : M \times \mathbf{R}_+ \to \mathbf{R}$, which is decreasing in $m \in M$ and is increasing in $w \in \mathbf{R}_+$. We postulate that each applicant has a reservation wage level, $\underline{w}$, that is, he will leave this "job market"[4] if no incumbent offers a job with a wage greater than or equal to $\underline{w}$; for simplicity we assume that this level is the same regardless of a type and also regardless of an education level. Since any contract $(m, \underline{w})$ gives the worst utility level,

[4] Although we adopt the conventional terminology of "job market" for convenience, the game played by the applicants and the incumbents is far from the neoclassical market.

for any contract (m, w) such that $w > \underline{w}$, there exists $w' > \underline{w}$ such that $u(0, w' \,|t) = u(m, w \,|t)$. We postulate

$$0 \leq \underline{w} < r_L < r_H.$$

We also postulate that the high-quality applicant can endure education more than the low-quality applicant. This is precisely formulated by the assumption that at any contract (m, w) for which $w > \underline{w}$, the slope of the indifference curve of the type-H applicants is lower than that of the type-L applicants. In the case $u(\cdot|t)$ is differentiable, it means that the high-quality applicant's marginal rate of substitution of wage for education is lower than that of the low-quality applicant:

$$\left| \frac{\frac{\partial u(m,w|H)}{\partial m}}{\frac{\partial u(m,w|H)}{\partial w}} \right| < \left| \frac{\frac{\partial u(m,w|L)}{\partial m}}{\frac{\partial u(m,w|L)}{\partial w}} \right|.$$

There are n_t applicants of type t. Set $n := n_L + n_H$.

Let I be the finite set of incumbents. An applicant's education level may serve as a signal of his type, hence set M is considered a message space. However, each incumbent can observe messages only imperfectly. While he may discern a college graduate from a high school graduate, he may not be able to discern different intensities of the education that two college graduates have gone through. On the other hand, he may be able to discern different intensities, perhaps due to the personal contact he has with the faculty of a college. Thus two incumbents may have different abilities to discern education levels. Incumbent i's discerning ability is formulated as an information structure, formally defined as a finite algebra $\mathcal{M}^i$ on M; incumbent i can discern education levels m and m', iff there exists $A \in \mathcal{M}^i$ for which $m \in A$ and $m' \notin A$. For simplicity we assume that each minimal member of $\mathcal{M}^i$ is of the form, $[m, m')$, a half-closed and half-open interval in M, in case $m' \neq \overline{m}$, or of the form $[m'', \overline{m}]$.

3 ENDOGENOUS DETERMINATION OF WAGE SCHEDULES

We analyze the "job market" in which (1) each incumbent i first decides either to stay in the "market" or to quit, and if he stays, he announces a wage schedule, $g^i : M \to \mathbf{R}_+$, which offers a job with wage level $g^i(m)$ to the applicants of every possible education level m, (2) each applicant j then chooses his education level $\tilde{m} \in M$, and (3) applicant j accepts a job from among those offered to the applicants of his education level, thereby choosing his wage level from $\{g^i(\tilde{m})\}_{i\in I_+}$, where I_+ is the set of all incumbents who stay in the "job market." A wage schedule is considered a mechanism.

The applicants behave noncooperatively in the above stages (2) and (3) as the Stackelberg followers. The incumbents are the Stackelberg leaders: Anticipating optimal reactions of the applicants, the incumbents play a game (with the player set I) in the above stage (1); this game will henceforth be called the *first-stage game*. We will analyze two situations: one in which the incumbents also behave noncooperatively, and the other in which the incumbents behave cooperatively, that is, they may merge into a larger firm and jointly design their mechanism. The overall game is, therefore, a specific instance of a multi-principal, multi-agent problem.

Our main focus here is analysis of the first-stage game. The subsequent subgame played by the applicants, (2) and (3), is trivial. Indeed, if each remaining incumbent i ($\in I_+$) chooses a wage schedule g^i, then the applicants of type t choose education level m_t and sign the employment contract with any of the incumbents i_t, hence receive wage $g^{i_t}(m_t)$, so that m_t is a solution to

$$\begin{array}{ll} \text{Maximize} & u\left(m, \bigvee_{i\in I_+} g^i(m) \,\middle|\, t\right) \\ \text{subject to} & m \in M, \end{array}$$

where

$$\bigvee_{i\in I_+} g^i(m) := \max_{i\in I_+} g^i(m),$$

and i_t satisfies

$$g^{i_t}(m_t) = \bigvee_{i\in I_+} g^i(m_t).$$

A *strategy* of incumbent i in the first-stage game is a wage-schedule $g^i : M \to \mathbf{R}_+$. It is feasible if it takes the same value for any two undiscernible messages, that is, if it is $\mathcal{M}^i$-measurable. It keeps the applicants in the "job market" if the offered wages are no lower than the reservation wage, that is, if $g^i(m) \geq \underline{w}$ for all $m \in M$. An outcome of a strategy bundle $\{g^i\}_{i\in I_+}$

is the applicants' strategy-choice in the subgame (2) and (3) in accordance with the offered wage schedules $\{g^i\}_{i\in I_+}$; it is the education level $\tilde{m}_t$ and the number n^i_t of applicants of type t who accept a contract with incumbent i, $t \in \{L, H\}$, $i \in I_+$. The $2\#I_+$ nonnegative integers $\{n^i_L, n^i_H\}_{i\in I_+}$ are called *assignment*. The *gain* of incumbent i is then defined as the profit,

$$n^i_L \left(r_L - g^i(\tilde{m}_L) \right) + n^i_H \left(r_H - g^i(\tilde{m}_H) \right) .$$

If i anticipates in the first-stage game that his gain will be negative, he will change his strategy, or else quit from the "job market." Prospect for a gain thus endogenously determines the set I_+ of incumbents in the "market." The prospect in turn is determined by strategies currently chosen by the other incumbents. We postulate that when behaving noncooperatively, each incumbent is passive *vis-à-vis* the other incumbents' strategy-choice. Given a strategy bundle $\{g^i\}_{i\in I}$ with the associated assignment $\{n^i_L, n^i_H\}_{i\in I}$, incumbent i is called *active* if $n^i_L > 0$ or $n^i_H > 0$. A *noncooperative equilibrium* of the first-stage game is an $\#I$-tuple of mechanisms $\{g^{*i}\}_{i\in I}$ together with the associated assignment $\{n^{*i}_L, n^{*i}_H\}_{i\in I}$ such that

- each mechanism g^{*i} is feasible, keeps the applicants in the "job market," and receives a nonnegative gain; and

- it is not true that there is an incumbent who can improve upon the outcome of $\{g^{*i}\}_{i\in I}$, that is, it is not true that there are incumbent $j \in I$ and his feasible strategy $g^j : M \to \mathbf{R}_+$ such that j remains active and receives a nonnegative gain given strategy bundle $\left(g^j, \{g^{*i}\}_{i\in I\setminus\{j\}} \right)$, and such that denoting by I_+ the set of the incumbents remaining in the "market," j's gain from the remaining strategy bundle $\{g^j, \{g^{*i}\}_{i\in I_+\setminus\{j\}}\}$ is greater than his gain from the outcome of the strategy bundle $\{g^{*i}\}_{i\in I}$.

As an alternative to the passive noncooperative behavior of the incumbents, we also model a passive cooperative behavior. Denote by $\mathcal{I}$ the family of nonempty coalitions of incumbents, $2^I \setminus \{\emptyset\}$. A coalition structure is a partition of I. The members of a coalition T jointly design a mechanism $g^T : M \to \mathbf{R}_+$; in so doing, they can pool their private information, so g^T is feasible if it is $\bigvee_{i\in T} \mathcal{M}^i$-measurable. Suppose coalition structure $\mathcal{P}$ is realized and each coalition $T \in \mathcal{P}$ chooses a wage schedule g^T. The applicants then play the subsequent subgame, in accordance with the most advantageous schedule, $\bigvee_{T\in\mathcal{P}} g^T$. The outcome in turn determines a gain of each coalition in $\mathcal{P}$. As a part of the first-stage game, the members of a coalition in $\mathcal{P}$ agree in the first stage on distribution of the anticipated coalitional gain among themselves. A *cooperative equilibrium* of the first-stage game is a triple of a coalition structure $\mathcal{P}^*$, a $\#\mathcal{P}^*$-tuple of mechanisms $\{g^{*T}\}_{T\in\mathcal{P}^*}$, and a gain distribution among the incumbents $\{\pi^{*i}\}_{i\in I}$, such that

- for each realized coalition $T \in \mathcal{P}^*$, its mechanism g^{*T} is feasible and keeps the applicants in the "job market," and the nonnegative gain distribution is feasible, that is, $\pi^{*i} \geq 0$, and $\sum_{i\in T} \pi^{*i}$ is less than or equal to T's coalitional gain; and

- it is not true that there is a coalition of incumbents which can improve upon the gain distribution $\{\pi^{*i}\}_{i\in I}$, that is, it is not true that there are coalition $S \in \mathcal{I}$, its feasible strategy $g^S : M \to \mathbf{R}_+$ and its feasible gain distribution $\{\pi^i\}_{i\in S}$, such that $\pi^i > \pi^{*i}$ for every $i \in S$.

Here, the second equilibrium condition (the coalitional stability condition) is ambiguous, and there are actually many precise versions. The ambiguity arises, because in analyzing the effects of a deviating coalition, we need to specify actions of the non-deviating incumbents: We postulate that when behaving cooperatively, the members of each coalition is passive *vis-à-vis* the other coalitions' strategy-choice. The members of a deviating coalition S perceive, therefore, that those coalitions T in $\mathcal{P}^*$ that do not lose their members to S (those $T \in \mathcal{P}^*$ for which $T \bigcap S = \emptyset$) keep the same strategies g^{*T}. We need to specify, however, strategies chosen by the incumbents who lose some colleagues to S (for the coalitions $T \setminus S$ for $T \in \mathcal{P}^*$ for which $\emptyset \neq T \setminus S \neq T$, we need to specify their strategies perceived by the members of S). There are many specifications, hence many versions of the coalitional stability condition. One scenario for the deviating coalition S's perception is that for each $T \in \mathcal{P}^*$, the members of T who are left behind at the time of formation of S stay together afterwards, that is, the coalition structure $\{S\} \bigcup \{T \setminus S \mid T \in \mathcal{P}^*, T \setminus S \neq \emptyset\}$ is realized as a result of formation of S, and that, for each $T \in \mathcal{P}^*$ for which $T \setminus S \neq \emptyset$, the coalition of the remaining players $T \setminus S$ keep choosing g^{*T} as its feasible strategy, since each member in $T \setminus S$ has learnt the information structure $\bigvee_{i\in T} \mathcal{M}^i$ through the earlier cooperation of the members of T. The gain of each coalition that co-exists after formation of S is then determined by the subgame given the wage schedule:

$$g^S \vee \left(\bigvee_{T\in\mathcal{P}^*:T\cap S\neq\emptyset} g^{*T} \right).$$

There are other scenarios; in particular, we may allow some incumbents to leave the "job market," as we did in formulating the noncooperative equilibrium. It will turn out that the results on the cooperative equilibrium in this paper are obtained for a wide class of scenarios. The only postulate we make is:

- Suppose that each coalition T in the prevailing coalition structure $\mathcal{P}^*$ is choosing strategy g^{*T}, and that coalition S is formed against $\mathcal{P}^*$ and chooses strategy g^S. Then, coalition S can attract all the applicants who have education level m, only if $g^S(m) > \bigvee_{T\in\mathcal{P}^*:T\cap S\neq\emptyset} g^{*T}(m)$.

In the following analysis, we will concentrate on the nontrivial case of multi-principals, $\#I \geq 2$. In the trivial case of $\#I = 1$, say $I = \{i\}$, the wage schedule $g^{*i} : m \mapsto \underline{w}$ is an equilibrium.

We first state a basic negative result on the cooperative equilibrium:

PROPOSITION 3.1 *Assume $\#I \geq 2$. If the grand coalition I and the singleton coalitions can form, then there is no cooperative equilibrium.*

The rest of this section is devoted to study of the noncooperative equilibrium. We establish existence results for several cases by constructing specific noncooperative equilibria. The specific formula of equilibria provides an insight into the role that information structures $\mathcal{M}^i$, $i \in I$, play in the "job market."

Define $w_0 \in \mathbf{R}_+$ by

$$n_L(r_L - w_0) + n_H(r_H - w_0) = 0,$$

and let U_{H0} be the indifference curve of the type-H applicants that passes through the contract $(0, w_0)$. See figure 1.

Insert figure 1 here.

Let U_t be an indifference curve of the type-t applicants $t = H, L$. By abuse of notation, U_t and r_t also denote the functions from M to $\mathbf{R}_+$ whose graphs are U_t and the horizontal line of height r_t, respectively. Thus, $(m, w) \in U_t$ iff $w = U_t(m)$. The function $U_L \wedge r_H : M \to \mathbf{R}_+$ is then defined by $(U_L \wedge r_H)(m) := \min\{U_L(m), r_H(m)\}$.

For $h = 1, 2, \ldots$, define $m_h \in M$ so that $[0,\ m_h)$ is the hth smallest non-degenerate interval that can be distinguished by some incumbents. Clearly,

$$0 < m_1 < m_2 < \ldots .$$

For each h, choose $w_{L,h}$ and $w_{H,h}$ so that

$$\begin{aligned} n_L(r_L - w_{L,h}) + n_H(r_H - w_{H,h}) &= 0 \\ u(0, w_{L,h} \mid L) &= u(m_h, w_{H,h} \mid L). \end{aligned}$$

The pair $(w_{L,h}, w_{H,h})$ is uniquely determined. Denote by $U_{L,h}$ the indifference curve of the type-L applicants that passes through $(0, w_{L,h})$ and $(m_h, w_{H,h})$. Denote also by $U_{H,h}$ the indifference curve of the type-H applicants that passes through $(m_h, w_{H,h})$. See figure 2. Let I_h be the set of all incumbents who can distinguish the interval $[0,\ m_h)$; $I_h := \{i \in I \mid [0,\ m_h) \in M^i\}$.

Insert figure 2 here.

Let k be the positive integer for which $u(m_k, w_{H,k} \mid H)$ is the highest, that is,

$$u(m_k, w_{H,k} \mid H) \geq u(m_h, w_{H,h} \mid H), \quad \text{for all } h.$$

If there is a tie, choose k so that $w_{L,k}$ is the highest among such maximizers of $u(m_h, w_{H,h} \mid H)$ (or equivalently, m_k is the smallest among such maximizers). For the required characterizations, we need to consider several mutually exclusive and exhaustive cases:

Case (1): $w_{H,k} \leq U_{H0}(m_k)$,

Case (2): $r_H \geq w_{H,k} > U_{H0}(m_k)$,

Case (3): $r_H < w_{H,k}$, and $w_{H,k} > U_{H0}(m_k)$.

Notice that $w_{L,k} \geq r_L$ ($w_{L,k} < r_L$, resp.) in case (2) (in case (3), resp.).

THEOREM 3.2 *Assume $\#I \geq 2$, and consider case (1). Strategy bundle $\{g^{*i}\}_{i\in I}$ is a noncooperative equilibrium, if*

$$g^{*i}(m) = w_0, \quad \text{for all } i \in I \text{ and all } m \in M.$$

Any assignment $\{n_L^i, n_H^i\}_{i\in I}$ may prevail with this equilibrium, provided that the gain of each incumbent is zero,

$$n_L^i(r_L - w_0) + n_H^i(r_H - w_0) = 0, \text{ for all } i \in I.$$

Case (2) is divided into three subcases:

> *Subcase (2.1)*: $\#I_k \geq 2$,
>
> *Subcase (2.2)*: $\#I_k = 1$, say $I_k = \{i_k\}$, there is a tie in obtaining $\max_h u(m_h, w_{H,h} \mid H)$, that is, there is k' ($\neq k$) such that $u(m_{k'}, w_{H,k'} \mid H) = u(m_k, w_{H,k} \mid H)$, and for at least one such k', $I_{k'} \setminus \{i_k\} \neq \emptyset$;
>
> *Subcase (2.3)*: $\#I_k = 1$, say $I_k = \{i_k\}$, and for any $i \in I \setminus \{i_k\}$ and any h for which $I_h \ni i$, $u(m_h, w_{H,h} \mid H) < u(m_k, w_{H,k} \mid H)$.

See figure 3.

Insert figure 3 here.

THEOREM 3.3 *Consider subcase (2.1). Then, feasible strategy bundle $\{g^{*i}\}_{i\in I}$ is a noncooperative equilibrium, if*

$$g^{*i}(\cdot) \leq \left(U_{L,k} \wedge U_{H,k}\right)(\cdot), \quad \text{for all } i \in I,$$

and for at least two distinct members i_1 and i_2 in I_k,

$$g^{*i_1}(m) = g^{*i_2}(m) = \begin{cases} w_{L,k} & \text{if} \quad m = 0 \\ w_{H,k} & \text{if} \quad m = m_k. \end{cases}$$

Given this $\{g^{*i}\}_{i\in I}$, the applicants of type L sign the contract $(0, w_{L,k})$, and the applicants of type H sign the contract $(m_k, w_{H,k})$. Any assignment (n_L^i, n_H^i) may prevail to those incumbents $i \in I_k$ for whom $g^{*i}(0) = w_{L,k}$ and $g^{*i}(m_k) = w_{H,k}$, provided that i's gain is zero, i.e., $n_L^i/n_H^i = n_L/n_H$.

THEOREM 3.4 *Consider subcase (2.2). Feasible strategy bundle* $\{g^{*i}\}_{i\in I}$ *is a noncooperative equilibrium, if*

$$g^{*i}(\cdot) \le \left(U_{L,k} \wedge U_{H,k}\right)(\cdot), \quad \textit{for all} \ \ i \in I,$$

$$g^{*i_k}(m) = \begin{cases} w_{L,k} & \textit{if} \quad m = 0 \\ w_{H,k} & \textit{if} \quad m = m_k, \end{cases}$$

and for some k' $(\neq k)$ *for which* $u(m_{k'}, w_{H,k'} \mid H) = u(m_k, w_{H,k} \mid H)$,

$$\exists\, i' \in I_{k'} \setminus \{i_k\} : \ g^{*i'}(m) = \begin{cases} w_{L,k} & \textit{if} \quad m = 0 \\ w_{H,k'} & \textit{if} \quad m = m_{k'}. \end{cases}$$

Given this $\{g^{*i}\}_{i\in I}$, *the applicants of type* L *sign* i_k*'s contract* $(0, w_{L,k})$, *and the applicants of type* H *sign* i_k*'s contract* $(m_k, w_{H,k})$.

For subcase (2.3), let l be any positive integer such that $u(m_l, w_{H,l} \mid H)$ is the highest level achievable by the incumbents other than i_k, i.e.,

$$u(m_l, w_{H,l} \mid H) = \max \ \{u(m_h, w_{H,h} \mid H) \mid I_h \setminus \{i_k\} \neq \emptyset\},$$

and denote by $U_{H,l}$ the indifference curve of the type-H applicants that passes through $(m_l, w_{H,l})$. For each h, let $U'_{L,h}$ be the indifference curve of the type-L applicants that passes through $(m_h, U_{H,l}(m_h))$, and set $w'_{L,h} := U'_{L,h}(0)$. Let $U''_{L,h}$ be the indifference curve of the type-L applicants that passes through $(0, \max\ \{r_L, w'_{L,h}\})$. See figure 4. Let J_{i_k} be the set of all integers h such that the interval $[0, m_h)$ is discernable to incumbent i_k, $\{h \mid I_h \ni i_k\}$. We are going to compare mechanisms parameterized by $h \in J_{i_k}$, so that the mechanism for h is intended to offer contract $(0, \max\ \{r_L, w'_{L,h}\})$ to type-L applicants and contract $(m_h, U_{H,l}(m_h))$ to type-H applicants. Let $k^* \in J_{i_k}$ be the parameter that maximizes i_k's gain, of all such parameterized mechanisms: the parameter k^* solves

$$\begin{array}{ll} \text{Maximize} & n_L \left(r_L - \max\ \{r_L, w'_{L,h}\}\right) + n_H \left(r_H - U_{H,l}(m_h)\right), \\ \text{subject to} & h \in J_{i_k}. \end{array}$$

Set $w'^*_L := w'_{L,k^*}$, $U'^*_L := U'_{L,k^*}$, and $U''^*_L := U''_{L,k^*}$.

Insert figure 4 here.

THEOREM 3.5 *Assume $\#I \geq 3$, and consider subcase (2.3). Assume that there exists an integer p for which $m_{k^*} \leq m_p$, and $I_p \setminus \{i_k\} \neq \emptyset$. Feasible strategy bundle $\{g^{*i}\}_{i \in I}$ is a noncooperative equilibrium, if*

$$g^{*i}(\cdot) \leq \left(U''^*_L \wedge U_{H,l}\right)(\cdot), \quad \textit{for all} \quad i \in I,$$

$$g^{*i_k}(m) = \begin{cases} \max \{r_L, w'^*_L\} & \textit{if} \quad m = 0 \\ U_{H,l}(m_{k^*}) & \textit{if} \quad m = m_{k^*}, \end{cases}$$

$$\exists\, i_p \in I_p \setminus \{i_k\} : \; g^{*i_p}(m) = \begin{cases} \max \{r_L, w'^*_L\} & \textit{if} \quad m = 0 \\ U_{H,l}(m_p) & \textit{if} \quad m = m_p, \end{cases}$$

and

$$\exists\, i' \in I \setminus \{i_k, i_p\} : \; g^{*i'}(m) = r_L \quad \textit{for all } m \in M.$$

Given this $\{g^{*i}\}_{i \in I}$, the applicants of type L sign i_k's contract $(0, \max \{r_L, w'^*_L\})$, and the applicants of type H sign i_k's contract $(m_{k^*}, H_{H,l}(m_{k^*}))$.

REMARK 3.6 A typical equilibrium for subcase (2.1) is given by: for all $i \in I_k$,

$$g^{*i}(m) = \begin{cases} w_{L,k} & \text{if} \quad 0 \leq m < m_k \\ w_{H,k} & \text{if} \quad m_k \leq m. \end{cases}$$

In subcase (2.2), incumbent i_k seemingly has a strict informational advantage since $w_{L,k} > w_{L,k'}$, but by adopting the strategy g^{i_k} defined by

$$g^{i_k}(m) = \begin{cases} w_{L,k'} & \text{if} \quad 0 \leq m < m_k \\ w_{H,k} & \text{if} \quad m_k \leq m, \end{cases}$$

he could not separate applicants of different types (all applicants would sign the contract $(m_k, w_{H,k})$). A typical equilibrium for subcase (2.3) is given by

$$g^{*i_k}(m) = \begin{cases} \max \{r_L, w'_{L,k}\} & \text{if} \quad 0 \leq m < m_k \\ U_{H,l}(m_k) & \text{if} \quad m_k \leq m. \end{cases}$$

The equilibrium of Theorem 3.2 is a *pooling* equilibrium. The equilibria of Theorems 3.3-3.5 are *separating* equilibria.

In order to analyze case (3), define for each $h = 1, 2, \ldots$,

$$\begin{aligned} \underline{w}_{H,h} &:= \min \{r_H, w_{H,h}\}, \\ \overline{w}_{L,h} &:= \max \{r_L, w_{L,h}\}, \end{aligned}$$

and denote by $\underline{U}_{H,h}$ ($\overline{U}_{L,h}$, resp.) the indifference curve of the type-H applicants (of the type-L applicants, resp.) that passes through $(m_h, \underline{w}_{H,h})$ (through $(0, \overline{w}_{L,h})$, resp.) Notice that $\underline{w}_{H,h} = w_{H,h}$ iff $\overline{w}_{L,h} = w_{L,h}$, and that $u(0, \overline{w}_{L,h} \mid L) \geq u(m_h, \underline{w}_{H,h} \mid L)$. Re-define k as the positive integer for which $u(m_k, \underline{w}_{H,k} \mid H)$ is the highest, that is,

$$u(m_k, \underline{w}_{H,k} \mid H) \geq u(m_h, \underline{w}_{H,h} \mid H), \quad \text{for all } h.$$

If there is a tie, choose k so that $\overline{w}_{L,k}$ is the highest among such maximizers of $u(m_h, \underline{w}_{H,h} \mid H)$. We consider mutually exclusive and exhaustive subcases:

Subcase (3.1): $\underline{w}_{H,k} \leq U_{H0}(m_k)$,

Subcase (3.2): $\underline{w}_{H,k} > U_{H0}(m_k)$.

By definition, $r_H \geq \underline{w}_{H,k}$. Subcase (3.2) is divided into three subsubcases:

Subcase (3.2.1): $\#I_k \geq 2$,

Subcase (3.2.2): $\#I_k = 1$, say $I_k = \{i_k\}$, and there is k' ($\neq k$) such that $u(m_{k'}, \underline{w}_{H,k'} \mid H) = u(m_k, \underline{w}_{H,k} \mid H)$ and $I_{k'} \setminus \{i_k\} \neq \emptyset$.

Subcase (3.2.3): $\#I_k = 1$, say $I_k = \{i_k\}$, and for any $i \in I \setminus \{i_k\}$ and any h for which $I_h \ni i$, $u(m_h, \underline{w}_{H,h} \mid H) < u(m_k, \underline{w}_{H,k} \mid H)$.

For subcase (3.2.3), define $\underline{U}_{H,l}$, k^*, $\underline{w}'^*_L$, $\underline{U}'^*_L$ and $\underline{U}''^*_L$ as in subcase (2.3).[5] The next theorem says that in case (3), essentially the same conclusions as in cases (1)-(2) hold true by substituting $\underline{w}_{H,k}$, $\overline{w}_{L,k}$, $\underline{U}_{H,k}$, $\overline{U}_{L,k}$, $\underline{U}_{H,l}$, $\underline{w}'^*_L$, $\underline{U}'^*_L$ and $\underline{U}''^*_L$ for $w_{H,k}$, $w_{L,k}$, $U_{H,k}$, $U_{L,k}$, $U_{H,l}$, w'^*_L, U'^*_L and U''^*_L, respectively.

[5] The number l is defined as any positive integer such that $u(m_l, \underline{w}_{H,l} \mid H)$ is the highest level achievable by the incumbents other than i_k, $\underline{U}_{H,l}$ is the indifference curve of the type-H applicants that passes through $(m_l, \underline{w}_{H,l})$. For each h, $\underline{U}'_{L,h}$ is the indifference curve of the type-L applicants that passes through $(m_h, \underline{U}_{H,l}(m_h))$, $\underline{w}'_{L,h}$ is defined as $\underline{U}'_{L,h}(0)$, and $\underline{U}''_{L,h}$ is the indifference curve of the type-L applicants that passes through $(0, \max\{r_L, \underline{w}'_{L,h}\})$. The integer k^* is the specific $h \in J_{i_k}$ at which $n_L \left(r_L - \max\{r_L, \underline{w}'_{L,h}\}\right) + n_H \left(r_H - \underline{U}_{H,l}(m_h)\right)$ is maximized. Then, $\underline{w}'^*_L := \underline{w}'_{L,k^*}$, $\underline{U}'^*_L := \underline{U}'_{L,k^*}$, and $\underline{U}''^*_L := \underline{U}''_{L,k^*}$.

THEOREM 3.7 *Consider case (3), and let* $\{g^{*i}\}_{i\in I}$ *be a feasible strategy bundle.*

(i) *In subcase (3.1), suppose* $\#I \geq 2$, *and*

$$g^{*i}(m) = w_0, \quad \text{for all } i \in I \quad \text{and all } m \in M.$$

(ii) *In subcases (3.2.1) and (3.2.2), suppose*

$$g^{*i}(\cdot) \leq \left(\overline{U}_{L,k} \wedge \underline{U}_{H,k}\right)(\cdot), \quad \text{for all } i \in I.$$

(iii) *In subcase (3.2.1), suppose for at least two distinct members* i_1 *and* i_2 *in* I_k,

$$g^{*i_1}(m) = g^{*i_2}(m) = \begin{cases} \overline{w}_{L,k} & \text{if} \quad m = 0 \\ \underline{w}_{H,k} & \text{if} \quad m = m_k. \end{cases}$$

(iv) *In subcase (3.2.2), suppose*

$$g^{*i_k}(m) = \begin{cases} \overline{w}_{L,k} & \text{if} \quad m = 0 \\ \underline{w}_{H,k} & \text{if} \quad m = m_k, \end{cases}$$

and for some k' $(\neq k)$ *for which* $u(m_{k'}, \underline{w}_{H,k'} \mid H) = u(m_k, \underline{w}_{H,k} \mid H)$,

$$\exists\, i_{k'} \in I_{k'} \setminus \{i_k\}: \; g^{*i_{k'}}(m) = \begin{cases} \overline{w}_{L,k} & \text{if} \quad m = 0 \\ \underline{w}_{H,k'} & \text{if} \quad m = m_{k'}, \end{cases}$$

$$\exists\, i' \in I \setminus \{i_k, i_{k'}\}: \; g^{*i'}(m) = r_L \quad \text{for all } m \in M.$$

(v) *In subcase (3.2.3), suppose that there exists an integer* p *for which* $m_{k^*} \leq m_p$, *and* $I_p \setminus \{i_k\} \neq \emptyset$, *and that*

$$g^{*i}(\cdot) \leq \left(\underline{U}_L^{\prime\prime *} \wedge \underline{U}_{H,l}\right)(\cdot), \quad \text{for all } i \in I,$$

$$g^{*i_k}(m) = \begin{cases} \max\{r_L, \underline{w}_L^{\prime *}\} & \text{if} \quad m = 0 \\ \underline{U}_{H,l}(m_{k^*}) & \text{if} \quad m = m_{k^*}, \end{cases}$$

$$\exists\, i_p \in I_p \setminus \{i_k\}: \; g^{*i_p}(m) = \begin{cases} \max\{r_L, \underline{w}_L^{\prime *}\} & \text{if} \quad m = 0 \\ \underline{U}_{H,l}(m_p) & \text{if} \quad m = m_p, \end{cases}$$

$$\exists\, i' \in I \setminus \{i_k, i_p\}: \; g^{*i'}(m) = r_L \quad \text{for all } m \in M.$$

Then, $\{g^{*i}\}_{i\in I}$ *is a noncooperative equilibrium.*

The final proposition in this paper is intended to be the first step towards characterizing the noncooperative equilibria. Let k be the positive integer for which $u(m_k, w_{H,k} \mid H)$ is the highest (this definition is the same as before for cases (1) and (2), but is different from the earlier definition for case (3)).

PROPOSITION 3.8 *Let $\{g^i\}_{i \in I}$ be a noncooperative equilibrium. Then,*

$$g^i(\cdot) \leq \left(U_{H0} - U_{H,k} \right)(\cdot), \quad \textit{for all} \;\; i \in I.$$

4 PROOFS

Proof of Proposition 3.1 Choose any coalition structure $\bar{\mathcal{P}}$ and any feasible strategy $\bar{g}^T$ for each $T \in \bar{\mathcal{P}}$ which keeps the applicants on the "job market," and define $\bar{g} := \bigvee_{T \in \bar{\mathcal{P}}} \bar{g}^T$. Let $\bar{m}_t$ be the message that the applicants of type t send, given $\bar{g}$, and let $\bar{\pi}_i$ be a gain distributed to incumbent i. We need to show that some coalition improves upon $(\bar{\mathcal{P}}, \{\bar{g}^T\}_{T \in \mathcal{P}}, \{\bar{\pi}_i\}_{i \in I})$.

If $\bar{g}(\bar{m}_t) > \underline{w}$ for some type $t \in \{L, H\}$, then

$$\begin{aligned} \sum_{i \in I} \bar{\pi}_i &\leq n_L \left(r_L - \bar{g}(\bar{m}_L)\right) + n_H \left(r_H - \bar{g}(\bar{m}_H)\right) \\ &< n_L \left(r_L - \underline{w}\right) + n_H \left(r_H - \underline{w}\right). \end{aligned}$$

So the grand coalition I can improve by adopting $g^I : m \mapsto \underline{w}$.

If on the other hand $\bar{g}(\bar{m}_t) = \underline{w}$ for each type $t \in \{L, H\}$,

$$\sum_{i \in I} \bar{\pi}_i \leq n_L \left(r_L - \underline{w}\right) + n_H \left(r_H - \underline{w}\right).$$

If strict inequality holds true here, then the grand coalition can improve by adopting $\bar{g}$ and a more efficient gain distribution. So assume that equality holds true. Then, there exists $i_0 \in I$ for whom $\bar{\pi}_{i_0} > 0$, and consequently for each $i \in I \setminus \{i_0\}$ $(\neq \emptyset)$,

$$\bar{\pi}_i \leq n_L \left(r_L - \underline{w}\right) + n_H \left(r_H - \underline{w}\right) - \bar{\pi}_{i_0}.$$

Therefore the singleton $\{i\}$ forms, adopts strategy $g^i : m \mapsto \underline{w} + \varepsilon$, where ε is a positive real number, attracts all the applicants, and obtains the entire gain

$$n_L \left(r_L - \underline{w} - \varepsilon\right) + n_H \left(r_H - \underline{w} - \varepsilon\right).$$

For ε small enough, this gain is greater than $\bar{\pi}_i$.

In order to prove Theorems 3.2-3.5 and 3.7, we first establish lemmas:

LEMMA 4.1 *Assume* $\#I \geq 2$. *Let* $\{g^{*i}\}_{i \in I}$ *be a noncooperative equilibrium, and let* $\{n_L^{*i}, n_H^{*i}\}_{i \in I}$ *be the associated assignment. For* $i \in I$ *and* $t \in \{L, H\}$ *for which* $n_t^{*i} > 0$, *let* $(m_t^{*i}, g^{*i}(m_t^{*i}))$ *be* i*'s offered contract that is signed by type-*t *applicants.*
(i) *If* $n_L^{*i} > 0$, *then* $g^{*i}(m_L^{*i}) \geq r_L$.
(ii) *If* $n_L^{*i} > 0$, *then* $m_L^{*i} = 0$.
(iii) *If* $n_L^{*i} > 0$ *and* $n_L^{*j} > 0$, *then* $g^{*i}(0) = g^{*j}(0)$.
(iv) *If* $n_H^{*i} > 0$, *then* $g^{*i}(m_H^{*i}) \leq r_H$.

Proof (i) Suppose the contrary, i.e., suppose

$$\exists\, i_0 : \ n_L^{*i_0} > 0, \quad \text{and} \quad g^{*i_0}(m_L^{*i_0}) < r_L.$$

Denote by U_L^* the indifference curve of the type-L applicants that passes through the contract $(m_L^{*i_0}, g^{*i_0}(m_L^{*i_0}))$. Notice that $g^{*i_0}(m_L^{*i_0}) \geq U_L^*(0)$, and that the equality holds true iff $m_L^{*i_0} = 0$. Choose any $\varepsilon > 0$, and define $w_\varepsilon := U_L^*(0) + \varepsilon$. Choose any $i \in I \setminus \{i_0\}$, and consider i's strategy g_ε^i defined by

$$g_\varepsilon^i(m) := \max \ \{g^{*i}(m), w_\varepsilon\}.$$

It suffices to show that i can take away applicants from i_0 and improve upon the outcome of $\{g^{*j}\}_{j \in I}$. Since g^{*i} is $\mathcal{M}^i$-measurable, so is g_ε^i. Since $U_L^*(m) \geq g^{*i}(m)$ for all m, it follows that

$$g_\varepsilon^i(0) = w_\varepsilon.$$

If i changes his strategy from g^{*i} to g_ε^i while the other incumbents h keep their strategies g^{*h}, all type-L applicants will choose i's new contract $(0, w_\varepsilon)$, since

$$\forall\, m : \ u\,(0, w_\varepsilon \mid L) > u\left(m_L^{*i_0}, g^{*i_0}(m_L^{*i_0}) \,\Big|\, L\right) \geq u\left(m, g^{*h}(m) \,\Big|\, L\right).$$

So i increases his gain from the type-L applicants at least by

$$\begin{cases} (n_L - n_L^{*i})\,(r_L - w_\varepsilon) + n_L^{*i}(-\varepsilon), & \text{if} \quad n_L^{*i} > 0, \\ n_L\,(r_L - w_\varepsilon)\,, & \text{if} \quad n_L^{*i} = 0. \end{cases}$$

If, on the one hand, all type-H applicants also choose i's contract $(0, w_\varepsilon)$, i increases his gain from the type-H applicants at least by $n_H(-\varepsilon)$, since $g^j(m_H^{*j}) \geq U_L^*(0)$ for all j for which $n_H^j > 0$. If, on the other hand, no applicant of type H switches his contract, then i's gain from the type-H applicants remains constant. Therefore, by changing a strategy from g^{*i} to g_ε^i, i increases his gain at least by

$$\begin{cases} (n_L - n_L^{*i})(r_L - w_\varepsilon) + (n_L^{*i} + n_H)(-\varepsilon), & \text{if} \quad n_L^{*i} > 0, \\ n_L(r_L - w_\varepsilon) + n_H(-\varepsilon), & \text{if} \quad n_L^{*i} = 0. \end{cases}$$

Since $n_L - n_L^{*i} \geq n_L^{*i_0} > 0$, i strictly increases his gain for all ε sufficiently close to 0, contradicting the definition of $\{g^{*h}\}_{h \in I}$ as a noncooperative equilibrium.

(ii) Suppose the contrary, i.e., suppose

$$\exists\, i_0 : \ n_L^{*i_0} > 0, \quad \text{and} \quad m_L^{*i_0} > 0.$$

Let U_L^* be the indifference curve of the type-L applicants that passes through the contract $(m_L^{*i_0}, g^{*i_0}(m_L^{*i_0}))$. For each type $t \in \{L, H\}$, let I_t^* be the set of incumbents whose contract is signed by type-t applicants, $\{i \in I \mid n_t^{*i} > 0\}$.

Since all type-L applicants sign contracts on U_L^*,

$$\begin{aligned} g^{*i}(m_L^{*i}) &\geq U_L^*(0) \ \text{ for all } i \in I_L^*, \\ g^{*i_0}(m_L^{*i_0}) &> U_L^*(0). \end{aligned}$$

Let $k \in I_H^*$ be the incumbent whose contract signed by type-H applicants requires the least amount of education:

$$\forall\, i \in I_H^* : \ m_H^{*k} \leq m_H^{*i}.$$

Then,

$$\forall\, i \in I_H^* : \ g^{*k}\left(m_H^{*k}\right) \leq g^{*i}\left(m_H^{*i}\right).$$

Moreover, in view of the facts, $u(m_L^{*i_0}, g^{*i_0}(m_L^{*i_0}) \mid L) \geq u(m_H^{*k}, g^{*k}(m_H^{*k}) \mid L)$ and $u(m_L^{*i_0}, g^{*i_0}(m_L^{*i_0}) \mid H) \leq u(m_H^{*k}, g^{*k}(m_H^{*k}) \mid H)$, the assumption on the slopes of the two types of indifference curves implies

$$m_L^{*i_0} \leq m_H^{*k}.$$

Since each applicant tries to minimize his education level given a wage level, it follows that $[m_t^{*i}, \overline{m}] \in \mathcal{M}^i$. For two positive real numbers $\varepsilon := (\varepsilon_L, \varepsilon_H) \gg 0$, define incumbent k's strategy g_ε^k by

$$g_\varepsilon^k(m) := \begin{cases} U_L^*(0) + \varepsilon_L & \text{if} \quad 0 \leq m < m_H^{*k}, \\ g^{*k}(m_H^{*k}) + \varepsilon_H & \text{if} \quad m_H^{*k} \leq m. \end{cases}$$

Strategy g_ε^k is $\mathcal{M}^k$-measurable. For any $\delta > 0$ sufficiently small, we may choose $\varepsilon \ll (\delta, \delta)$ so that

$$\begin{aligned} u(0, g_\varepsilon^k(0) \mid L) &> u(m_H^{*k}, g_\varepsilon^k(m_H^{*k}) \mid L) \\ u(0, g_\varepsilon^k(0) \mid H) &< u(m_H^{*k}, g_\varepsilon^k(m_H^{*k}) \mid H). \end{aligned}$$

Strategy g_ε^k is intended to offer contract $(0, g_\varepsilon^k(0))$ to type-L applicants, and contract $(m_H^{*k}, g_\varepsilon^k(m_H^{*k}))$ to type-H applicants. The preceding two inequalities say that g_ε^k is indeed incentive-compatible. In the following, we choose such ε.

Now, given $\{g^{*i}\}_{i \in I}$, incumbent i's gain is

$$\pi^{*i} := n_L^{*i} \left(r_L - g^{*i}(m_L^{*i}) \right) + n_H^{*i} \left(r_H - g^{*i}(m_H^{*i}) \right) \geq 0,$$

where $n_t^{*i} := 0$ and m_t^{*i} is arbitrary if $i \notin I_t^*$. When k changes his strategy from g^{*k} to g_ε^k, while the others keep their strategies, all the type-L applicants come to k to sign contract $(0, g_\varepsilon^k(0))$ and all the type-H applicants also come to k to sign contract $(m_H^{*k}, g_\varepsilon^k(m_H^{*k}))$, so k's gain becomes

$$\begin{aligned} \pi_\varepsilon^k \;&:=\; n_L (r_L - U_L^*(0) - \varepsilon_L) + n_H \left(r_H - g^{*k}(m_H^{*k}) - \varepsilon_H \right) \\ &= \left(n_L - n_L^{*i_0} \right) (r_L - U_L^*(0)) + n_L^{*i_0} \left(r_L - g^{*i_0}(m_L^{*i_0}) \right) \\ &\quad + n_H \left(r_H - g^{*k}(m_H^{*k}) \right) \\ &\quad + A_\varepsilon \end{aligned}$$

where

$$A_\varepsilon := n_L^{*i_0} \left(g^{*i_0}(m_L^{*i_0}) - U_L^*(0) \right) - n_L \varepsilon_L - n_H \varepsilon_H.$$

But

$$\left(n_L - n_L^{*i_0} \right) (r_L - U_L^*(0)) + n_L^{*i_0} \left(r_L - g^{*i_0}(m_L^{*i_0}) \right)$$

$$
\begin{aligned}
&+n_H\left(r_H - g^{*k}(m_H^{*k})\right)\\
&\ge \sum_{i\in I_L^*} n_L^{*i}\left(r_L - g^{*i}(m_L^{*i})\right) + \sum_{i\in I_H^*} n_H^{*i}\left(r_H - g^{*i}(m_H^{*i})\right)\\
&= \sum_{i\in I}\pi^{*i}\\
&\ge \pi^{*k}.
\end{aligned}
$$

Thus,

$$\pi_\varepsilon^k \ge \pi^{*k} + A_\varepsilon.$$

For ε sufficiently small, $A_\varepsilon > 0$, so k's gain increases as he changes his strategy from g^{*k} to g_ε^k, contradicting the definition of $\{g^{*i}\}_{i\in I}$ as an equilibrium.

(iii) If $n_L^{*i} > 0$ and $n_L^{*j} > 0$, then in view of (ii),

$$u\left(0, g^{*i}(0)\,\Big|\,L\right) = u\left(0, g^{*j}(0)\,\Big|\,L\right),$$

so $g^{*i}(0) = g^{*j}(0)$

(iv) If there exists i for whom $n_H^{*i} > 0$ and $g^{*i}(m_H^{*i}) > r_H$, then for this i to survive,

$$n_L^{*i} > 0 \quad\text{and}\quad g^{*i}(m_L^{*i}) < r_L,$$

which contradicts (i).

LEMMA 4.2 *Assume $\#I \ge 2$, and let $\{g^{*i}\}_{i\in I}$ be a feasible strategy bundle. In case (1), assume*

$$
\begin{aligned}
&\forall i \in I:\ g^{*i}(\cdot) \le U_{H0}(\cdot),\\
&\exists i_k \in I:\ g^{*i_k}(0) = w_0.
\end{aligned}
$$

In subcases (2.1) and (2.2), assume

$$
\begin{aligned}
&\forall i \in I:\ g^{*i}(\cdot) \le \left(U_{L,k} \wedge U_{Hk}\right)(\cdot),\\
&\exists i_k \in I_k:\ g^{*i_k}(m) = \begin{cases} w_{L,k}, & \text{if } m = 0\\ w_{H,k}, & \text{if } m = m_k. \end{cases}
\end{aligned}
$$

In subcase (2.3), assume

$$
\begin{aligned}
&\forall i \in I:\ g^{*i}(\cdot) \le \left(U_L''^{*} \wedge U_{Hl}\right)(\cdot),\\
&\exists i_k \in I_k:\ g^{*i_k}(m) = \begin{cases} \max\{r_L, w_L'^{*}\}, & \text{if } m = 0\\ U_{H,l}(m_{k^*}), & \text{if } m = m_{k^*}. \end{cases}
\end{aligned}
$$

Then, in cases (1) and (2), no incumbent $i \neq i_k$ can adopt a feasible strategy g which takes applicants away from i_k, and then eventually receive a nonnegative gain.

Proof Let $\{g^{*i}\}_{i\in I}$ be the strategy bundle and let i_k be the incumbent given in the statement of the lemma. Suppose there exist $i \in I \setminus \{i_k\}$ and i's feasible strategy g such that i takes applicants away from i_k, and then eventually receives a nonnegative gain.

We first claim that g cannot attract only type-L applicants. Indeed, denote by $(m^i_L, g(m^i_L))$ the incumbent i's offered contract that type-L applicants would sign. In case (1) we have

$$u\left(m^i_L, g(m^i_L) \mid L\right) > u\left(0, w_0 \mid L\right),$$

so $g(m^i_L) > w_0 > r_L$. In subcases (2.1) and (2.2) we have

$$u\left(m^i_L, g(m^i_L) \mid L\right) > u\left(0, w_{L,k} \mid L\right),$$

so $g(m^i_L) > w_{L,k} \geq r_L$. In subcase (2.3) we have

$$u\left(m^i_L, g(m^i_L) \mid L\right) > u\left(0, \max\{r_L, w'^*_L\} \mid L\right),$$

so $g(m^i_L) > r_L$. In both cases (1) and (2), therefore, $g(m^i_L) > r_L$, consequently $(m^i_L, g(m^i_L))$ yields a negative gain, and the claim was proved.

Therefore, g attracts some type-H applicants; let $(m^i_H, g(m^i_H))$ be i's offered contract which is signed by type-H applicants. Then,

$$u\left(m^i_H, g(m^i_H) \mid H\right) > \begin{cases} u(0, w_0 \mid H), & \text{in case (1);} \\ u(m_k, w_{H,k} \mid H), & \text{in subcases (2.1) and (2.2);} \\ u(m_{k^*}, U_{H,l}(m_{k^*}) \mid H), & \text{in subcase (2.3).} \end{cases}$$

The contract $(m^i_H, g(m^i_H))$ then attracts *all* the type-H applicants in both cases (1) and (2). Let U_L be the indifference curve of the type-L applicants that passes through $(m^i_H, g(m^i_H))$.

Denote by n^i_L the number of type-L applicants who eventually sign a contract with i after instituting $\left\{g, \{g^{*j}\}_{j\in I\setminus\{i\}}\right\}$. Then, $0 \leq n^i_L \leq n_L$. We claim that $n_L^i < n_L$. If $n_L^i = n_L$, then denoting by $(m_L^i, g(m_L^i))$ the contract eventually signed by type-L applicants,

$$\begin{aligned} u\left(m_L^i, g(m_L^i) \,\middle|\, L\right) &\geq u\left(m_H^i, g(m_H^i) \,\middle|\, L\right) \\ &= u\left(0, U_L(0) \mid L\right), \end{aligned}$$

so that $g(m_L^i) \geq U_L(0)$, and consequently

$$\begin{aligned} & n_L\left(r_L - g(m_L^i)\right) + n_H\left(r_H - g(m_H^i)\right) \\ \leq\ & n_L\left(r_L - U_L(0)\right) + n_H\left(r_H - g(m_H^i)\right) \\ <\ & 0. \end{aligned}$$

Here, the last inequality is: a consequence of $g(m_H^i) > U_{H0}(m_H^i)$ in case (1); a consequence of $g(m_H^i) > U_{H,k}(m_H^i)$ in subcases (2.1) and (2.2); and a consequence of $g(m_H^i) > U_{H,l}(m_H^i)$ and $i \in I \setminus \{i_k\}$ in subcase (2.3). Thus g would eventually receive a negative gain, and the claim was proved.

Due to the claim, there exists $j \neq i$ who eventually receives some type-L applicants only; let (m_L^j, w_L^j) be j's offered contract signed by type-L applicants. For j to survive, $r_L \geq w_L^j$. Then,

$$\begin{aligned} u\left(0, r_L \mid L\right) &\geq u\left(m_L^j, w_L^j \,\middle|\, L\right) \\ &\geq u\left(m_H^i, g(m_H^i) \,\middle|\, L\right) \\ &= u\left(0, U_L(0) \mid L\right), \end{aligned}$$

so that $r_L \geq U_L(0)$. Then, the three inequalities,

$$\begin{aligned} n_L\left(r_L - U_L(0)\right) + n_H\left(r_H - g(m_H^i)\right) &< 0 \\ n_L^i\left(r_L - U_L(0)\right) + n_H\left(r_H - g(m_H^i)\right) &\geq 0 \\ n_L &\geq n_L^i \end{aligned}$$

are inconsistent. Thus, no incumbent i ($\neq i_k$) can choose a strategy which takes applicants away from i_k and then eventually receive a nonnegative gain.

Proof of Theorem 3.2 Let $\{g^{*i}\}_{i \in I}$ be the strategy bundle given in the statement of the theorem. Choose $i_1 \in I$. We need to show that i_1 cannot improve upon $\{g^{*i}\}_{i \in I}$. Suppose that i_1 changes his strategy from g^{*i_1} to g^{i_1}.

In general, if an incumbent i designs a mechanism g to induce type-L applicants to sign contract $(m_L, g(m_L))$ given $\{g, \{g^{*j}\}_{j\neq i}\}$, and if $m_L > 0$, then i can do better by another mechanism g' such that the type-L applicants would choose contract $(0, g'(0))$ and $g'(0) < g(m_L)$. Indeed, let U_L be the indifference curve of the type-L applicants which passes through $(m_L, g(m_L))$. Since each applicant wants to minimize his education level given a wage level, we may assume $[m_L, \overline{m}] \in M^i$. In view of the assumption on the slopes of the two types of indifference curves, if type-H applicants also sign i's offered contract $(m_H, g(m_H))$, then $(m_L, g(m_L)) \leq (m_H, g(m_H))$. The required mechanism g' is given as

$$g'(m) := \begin{cases} U_L(0) & \text{if} \quad 0 \leq m < m_L \\ g(m) & \text{if} \quad m_L \leq m. \end{cases}$$

If, on the other hand, i is to induce only type-H applicants with his contract $(m_H, g(m_H))$, then again without loss of generality, $u(0, g(0) \mid L) \geq u(m_H, g(m_H) \mid L)$.

Thus, we may assume without loss of generality that

$$\forall m \in M: \; u(0, g^{i_1}(0) \mid L) \geq u(m, g^{i_1}(m) \mid L).$$

We can re-define U_L as the indifference curve of the type-L applicants that passes through $(0, g^{i_1}(0))$. The above inequality means $U_L(\cdot) \geq g^{i_1}(\cdot)$.

Now, if $g^{i_1}(0) > w_0$, then i_1 attracts all the applicants of both types and the gain becomes negative, so i_1 cannot improve upon $\{g^{*i}\}_{i\in I}$.

If $g^{i_1}(0) < w_0$, then in view of $U_L(\cdot) \geq g^{i_1}(\cdot)$, i_1 loses all type-L applicants. If i_1 also loses type-H applicants, he becomes inactive, so he cannot improve upon $\{g^{*i}\}_{i\in I}$. Therefore, suppose there exists m_H for which $g^{i_1}(m_H) > U_{H0}(m_H)$. Then i_1 attracts all type-H applicants, the other incumbents suffer a loss from type-L applicants so drop out of the "market," and i_1 eventually gets all applicants. But then he cannot make a positive gain in case (1).

Suppose $g^{i_1}(0) = w_0$. If i_1 is to have a positive eventual gain, he has to attract all the type-H applicants given $\{g^{i_1}, \{g^{*i}\}_{i\neq i_1}\}$. this means:

$$\exists\, m_H \in M: \; g^{i_1}(m_H) > U_{H0}(m_H).$$

But then i_1 attracts all type-H applicants, the other incumbents, as long as they keep type-L applicants, suffer from a loss and drop out, so i_1 eventually gets all the applicants of both types. In case (1), i_1's eventual gain becomes negative.

Proof of Theorem 3.3 Let $\{g^{*i}\}_{i\in I}$ be the strategy bundle given in the statement of the theorem. Choose $i_1 \in I_k$. In view of Lemma 4.2, it suffices to show that i_1 cannot improve upon $\{g^{*i}\}_{i\in I}$. Suppose that i_1 changes his strategy from g^{*i_1} to g^{i_1}. Let U_L be the indifference curve of the type-L applicants that passes through $(0, g^{i_1}(0))$. By the same argument as in the second paragraph of the proof of Theorem 3.2, we may assume without loss of generality that $U_L(\cdot) \geq g^{i_1}(\cdot)$.

If $g^{i_1}(0) > w_{L,k}$, then i_1 attracts all the applicants of type L, and the gain becomes negative (regardless whether i_1 attracts type-H applicants or not) because of the definition of k, so i_1 cannot improve upon $\{g^{*i}\}_{i\in I}$.

If $g^{i_1}(0) < w_{L,k}$, then i_1 loses all type-L applicants. So in order for him to stay active, g^{i_1} has to attract all type-H applicants. If $g^{i_1}(0) = w_{L,k}$, in order to make a change for increase in his gain, i_1 has to attract all the type-H applicants. Thus, if $g^{i_1}(0) \leq w_{L,k}$, which we assume in the rest of the proof, i_1 has to attract all type-H applicants given $\{g, \{g^{*i}\}_{i\neq i_1}\}$. Somebody other than i_1, say i_2, is taking strategy g^{*i_2}, which guarantees utility level $u(m_k, w_{H,k} \mid H)$ to the type-H applicants, and utility level $u(0, w_{L,k} \mid L)$ to the type-L applicants. Since i_1 has to supercede i_2's guarantee to the type-H applicants,

$$\exists h: \ I_h \ni i_1, \quad \text{and} \quad g^{i_1}(m_h) > U_{H,k}(m_h).$$

Incumbent i_2, and possibly some members $i \in I \setminus \{i_1, i_2\}$, receive only type-$L$ applicants (all type-L applicants, in case $g^{i_1}(0) < w_{L,k}$). We consider two cases separately: (A) $r_L < w_{L,k}$, and (B) $r_L = w_{L,k}$.

Suppose (A). Then those incumbents who received type-L applicants suffer from a loss and drop out of the "market." As long as $g^{i_1}(0) > r_L$, incumbent i_1 eventually gets all applicants. But then i_1's eventual gain becomes negative, in view of the definition of k. If $g^{i_1}(0) \leq r_L$, then i_1 may or may not get all type-L applicants eventually, since somebody other than i_1 and i_2 may be able to keep type-L applicants. In case i_1 eventually gets all type-L applicants, his eventual gain is negative, in view of the definition of k. In case i_1 does not get all type-L applicants eventually, his eventual gain is even lower than in the situation in which he gets all type-L applicants, since each type-L applicant brings in nonnegative gain. Thus, i_1 cannot improve upon $\{g^{*i}\}_{i\in I}$ in case (A).

Suppose (B). Then $w_{H,k} = r_H$, and the conditions,

$$U_L(\cdot) \geq g^{i_1}(\cdot) \text{ and } g^{i_1}(m_h) > U_{H,k}(m_h),$$

mean that i_1 receives a negative gain given $\{g, \{g^{*i}\}_{i \neq i_1}\}$, so he cannot survive.

Proof of Theorem 3.4 Let $\{g^{*i}\}_{i \in I}$ be the strategy bundle given in the statement of the theorem. It suffices to show that i_k cannot improve upon $\{g^{*i}\}_{i \in I}$. Literally the same proof as the proof of theorem 3.3 (except that i_k replaces i_1) applies.

Proof of Theorem 3.5 Let $\{g^{*i}\}_{i \in I}$ be the strategy bundle given in the statement of the theorem. It suffices to show that i_k cannot improve upon $\{g^{*i}\}_{i \in I}$. Suppose i_k changes his strategy from g^{*i_k} to g^{i_k}. Let U_L be the indifference curve of the type-L applicants that passes through $(0, g^{i_k}(0))$. Without loss of generality, $U_L(\cdot) \geq g^{i_k}(\cdot)$.

If $g^{i_k}(0) \geq g^{*i_k}(0) \geq r_L$, then all type-$L$ applicants stay with i_k, contributing a nonpositive gain, so i_k has to keep all type-H applicants. Then i_k's eventual gain is no higher than that before his change of a strategy, in view of the definition of k^*. So i_k cannot improve upon $\{g^{*i}\}_{i \in I}$.

If $g^{i_k}(0) < g^{*i_k}(0)$, then i_k loses all type-L applicants, so in order to stay active, the mechanism g^{i_k} is designed so that it keeps all type-H applicants given $\{g^{i_k}, \{g^{*i}\}_{i \neq i_k}\}$. We consider two cases separately: (A) $r_L < w_L'^*$ (so that $g^{*i_k}(0) = w_L'^* > r_L$), and (B) $r_L \geq w_L'^*$ (so that $g^{*i_k}(0) = r_L$).

Suppose (A). Then incumbent i_p, and possibly some members $i \in I \setminus \{i_k, i_p\}$, get all the type-$L$ applicants, who bring in only a loss, so those incumbents who receive type-L applicants eventually drop.

If $g^{i_k}(0) > r_L$, incumbent i_k eventually takes back all the type-L applicants. But then i_k's eventual gain becomes no greater than his original gain given $\{g^{*i}\}_{i \in I}$, in view of the definition of k^*.

If $g^{i_k}(0) < r_L$, then incumbent i' eventually gets all the type-L applicants. Incumbent i_k eventually ends up only with the type-H applicants with a contract $(m_h, g^{i_k}(m_h))$ for some $h \in J_{i_k}$. But then i_k's eventual gain becomes:

$$\begin{aligned} & n_H \left(r_H - g^{i_k}(m_h) \right) \\ & \leq n_H (r_H - U_{H,l}(m_h)) \end{aligned}$$

$$
\begin{aligned}
&= n_L \left(r_L - \max \{r_L, g^{i_k}(0)\} \right) + n_H \left(r_H - U_{H,l}(m_h)\right) \\
&\leq \left(r_L - \max \{r_L, w'^{*}_L\}\right) + n_H \left(r_{HL} - U_{H,l}(m_{k^*})\right),
\end{aligned}
$$

so i_k cannot improve upon $\{g^{*i}\}_{i\in I}$.

If $g^{i_k}(0) = r_L$, incumbent i_k may get back some of the type-L applicants, but they bring in only zero gain, so the above inequalities apply here as well; i_k's eventual gain becomes no greater than his original gain given $\{g^{*i}\}_{i\in I}$.

Suppose (B). Then, i_k, if he survives, eventually ends up only with the type-H applicants. But i_k's gain given $\{g^{*i}\}_{i\in I}$ comes only from the type-H applicants, so as in the preceding two paragraphs, i_k cannot improve upon $\{g^{*i}\}_{i\in I}$.

Sketch of Proof of Theorem 3.7 Let $\{g^{*i}\}_{i\in I}$ be the strategy bundle given in the theorem. For subcase (3.1), choose any $i_k \in I$; for subcase (3.2.1), set $i_k := i_1$; for the other subcases, i_k is already defined.

We first establish that no incumbent $i \neq i_k$ can adopt a feasible strategy g which takes applicants away from i_k, and eventually receive a nonnegative gain. The proof follows closely the proof of Lemma 4.2. Here is an outline: Suppose there exist an incumbent $i \neq i_k$ and his feasible strategy g such that i takes applicants away from i_k, and eventually receives a nonnegative gain. As in the proof of Lemma 4.2, we claim that i cannot attract only type-L applicants, so g attracts all type-H applicants, that is, there exists $m^i_H \in M$ such that

$$
u(m^i_H, g(m^i_H) \mid H) > \begin{cases} u(0, w_0 \mid H), & \text{in subcase (3.1);} \\ u(m_k, \underline{w}_{H,k} \mid H), & \text{in subcases (3.2.1)-(3.2.2);} \\ u(m_{k^*}, \underline{U}_{H,l}(m_{k^*}) \mid H), & \text{in subcase (3.2.3).} \end{cases}
$$

Without loss of generality, $[m^i_H, \overline{m}] \in \mathcal{M}^i$. By definition of k and by the fact that $i \neq i_k$, either (A) $g(m^i_H) > r_H$ or else (B) $n_L \left(r_L - U_L(0)\right) + n_H \left(r_H - g(m^i_H)\right) < 0$. If (A) is the case, i receives a loss from type-H applicants, so he has to attract type-L applicants also. But the only way to attract type-L applicants is to offer a wage higher than r_L (thereby receiving a loss also from type-L applicants), in view of $g^{*i_k}(0) \geq r_L$. So, i cannot survive. If (B) is the case, the same argument as in the proof of Lemma 4.2 applies.

We only need to show that i_k cannot improve upon $\{g^{*i}\}_{i\in I}$. Suppose i_k changes his strategy from g^{*i_k} to g^{i_k}. Let U_L be the indifference curve of the type-L applicants that passes through $(0, g^{i_k}(0))$. Without loss of generality, $U_L(\cdot) \geq g^{i_k}(\cdot)$.

Consider subcase (3.1). If $g^{i_k}(0) > w_0$, then the proof of Theorem 3.2 applies. If $g^{i_k}(0) \leq w_0$, then, as in the proof of Theorem 3.2, there exists $m_H \in M$ such that $g^{i_k}(m_H) > U_{H0}(m_H)$. In subcase (3.1), this means either (A) $g^{i_k}(m_H) > r_H$, or else (B) $n_L\left(r_L - g^{i_k}(0)\right) + n_H\left(r_H - g^{i_k}(m_H)\right) < 0$. If (A) is the case, i_k receives a loss from type-H applicants. In order to survive, therefore, i_k has to attract type-L applicants with wage $g^{i_k}(0)$ lower than r_L, but this is impossible in view of the fact that $g^{*i}(0) = w_0 \geq r_L$ for all $i \neq i_k$. If (B) is the case, the proof of Theorem 3.2 applies.

The idea of the proofs of the theorem for subcases (3.2.1)-(3.2.3) are the same as above: We follow the proofs of Theorems 3.3-3.5. The only situation in which we have to modify the proof is the case $g^{i_k}(m_H) > r_H$ (in which case, it is possible that $n_L\left(r_L - g^{i_k}(0)\right) + n_H\left(r_H - g^{i_k}(m_H)\right) \geq 0$). But in this situation i_k receives a loss from type-H applicants, so in order for him to receive a nonnegative eventual gain, he has to attract type-L applicants with lower wage than r_L, which is impossible in the presence of i'.

Proof of Proposition 3.8 Suppose that there exists a noncooperative equilibrium $\{g^i\}_{i\in I}$ for which

$$\exists\, i^* \in I : \exists\, m^* \in M : g^{i^*}(m^*) > \left(U_{H0} \vee U_{H,k}\right)(m^*).$$

Let $\{(n_L^i, n_H^i)\}_{i\in I}$ be the associated assignment, and let I_t be the set of incumbents i whose contract $(m_t^i, g^i(m_t^i))$ is actually signed by some applicants of type $t \in \{L, H\}$. By Lemma 4.1, $g^i(m_H^i) \leq r_H$ for all $i \in I_H$, $m_L^i = 0$ for all $i \in I_L$, and $w_L := g^i(0) = g^j(0)$ for all $i, j \in I_L$.

For $i \in I_H$, type-H applicants sign the contract $(m_H^i, g^i(m_H^i))$, when they could sign the contract $(m^*, g^{i^*}(m^*))$, so

$$\forall\, i \in I_H : u\left(m_H^i, g^i(m_H^i) \,\middle|\, H\right) \geq u\left(m^*, g^{i^*}(m^*) \,\middle|\, H\right),$$

in short, each contract $(m_H^i, g^i(m_H^i))$ is strictly above the indifference curves $U_{H,k}$ and U_{H0}, for all $i \in I_H$. For each $i \in I_H$, $m_H^i \in \{0, m_1, m_2, \cdots\}$, since every applicant minimizes the needed education level. Define $w_L'^i \in \mathbf{R}_+$ by

$$u\left(0, w_L'^i \,\middle|\, L\right) = u\left(m_H^i, g^i(m_H^i) \,\middle|\, L\right).$$

Choose any $j \in I_L$. Since type-L applicants sign the contract $(0, w_L)$ rather than the contract $(m_H^i, g^i(m_H^i))$ for any $i \in I_H$,

$$\begin{aligned} u\left(0, w_L \mid L\right) &\geq u\left(\left(m_H^i, g^i(m_H^i)\right) \mid L\right) \\ &= u\left(\left(0, w_L'^i\right) \mid L\right), \end{aligned}$$

consequently,

$$w_L \geq w_L'^i.$$

Therefore,

$$\begin{aligned} \sum_{j \in I_L} n_L^j \left(r_L - w_L\right) &= \sum_{i \in I_H} \frac{n_H^i}{n_H} n_L \left(r_L - w_L\right) \\ &\leq \sum_{i \in I_H} \frac{n_H^i}{n_H} n_L \left(r_L - w_L'^i\right). \end{aligned}$$

In view of the present assumption, $g^i(m_H^i) > \max\{U_{H,k}(m_H^i), U_{H0}(m_H^i)\}$ for all $i \in I_H$, and the definition of k,

$$\begin{aligned} &\sum_{j \in I_L} n_L^j \left(r_L - w_L\right) + \sum_{i \in I_H} n_H^i \left(r_H - g^i(m_H^i)\right) \\ &\leq \sum_{i \in I_H} \frac{n_H^i}{n_H} \left[n_L \left(r_L - w_L'^i\right) + n_H \left(r_H - g^i(m_H^i)\right) \right] \\ &< 0. \end{aligned}$$

Thus, some active incumbents suffer from a loss, contradicting the definition of $\{g^i\}_{i \in I}$ as a noncooperative equilibrium.

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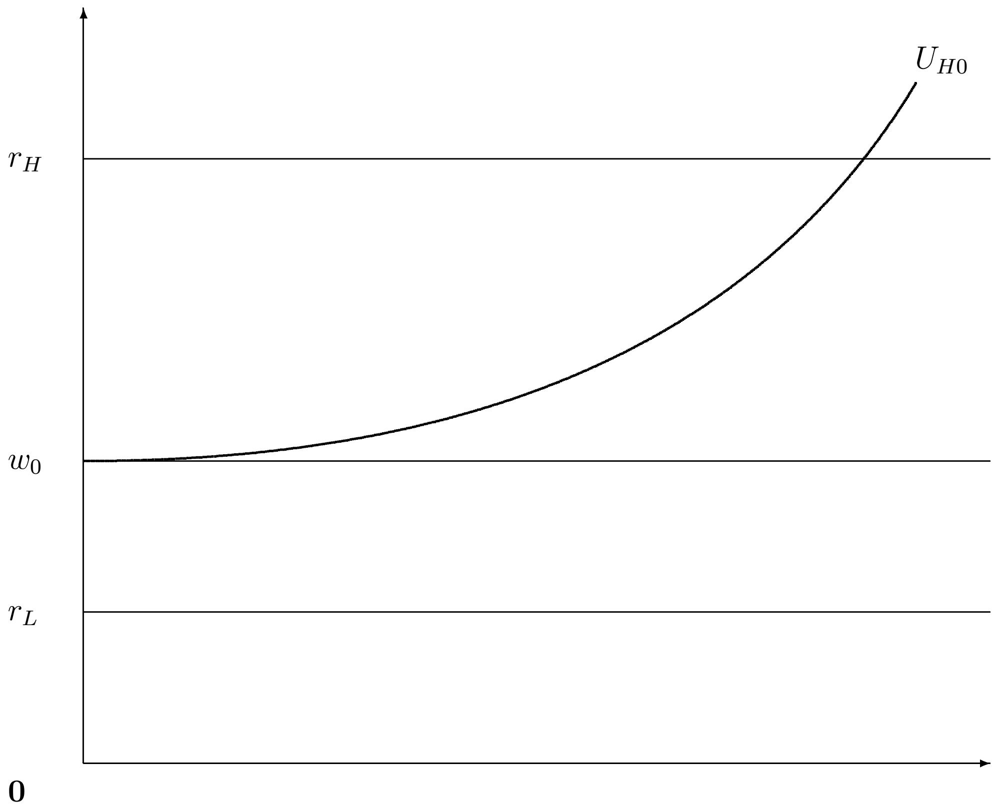

Figure 1: definitions of w_0 and U_{H0}

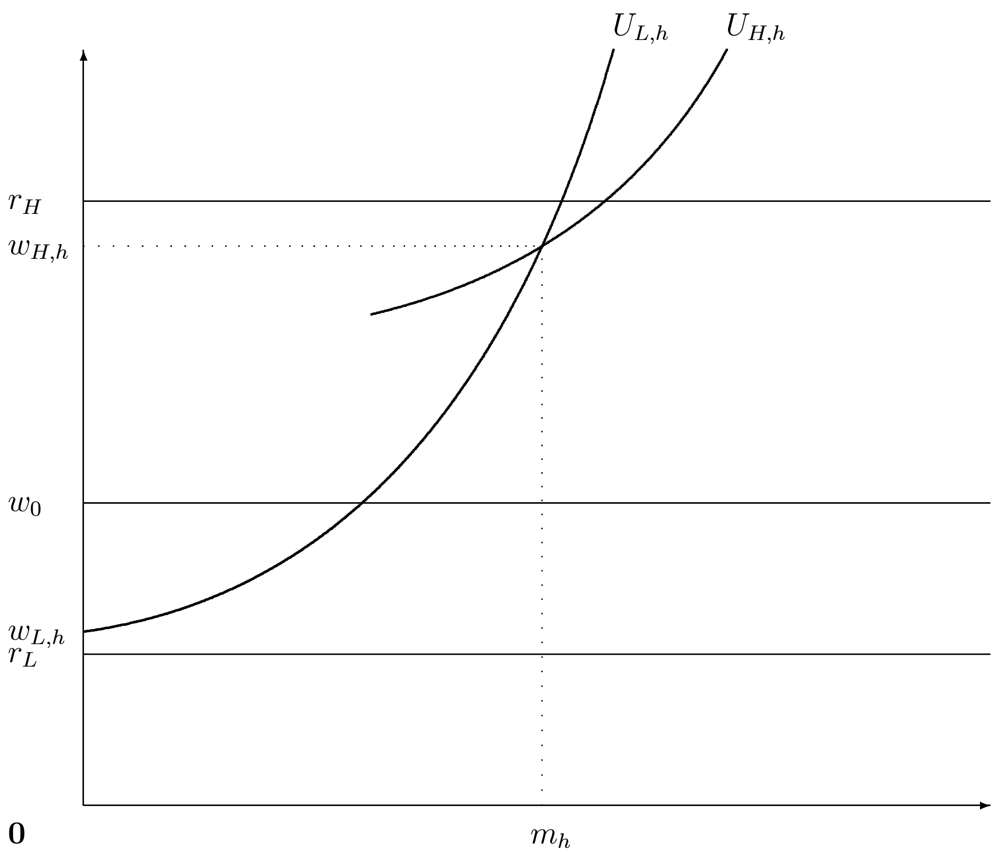

Figure 2: definitions of $w_{H,h}$, $w_{L,h}$, $U_{H,h}$ and $U_{L,h}$

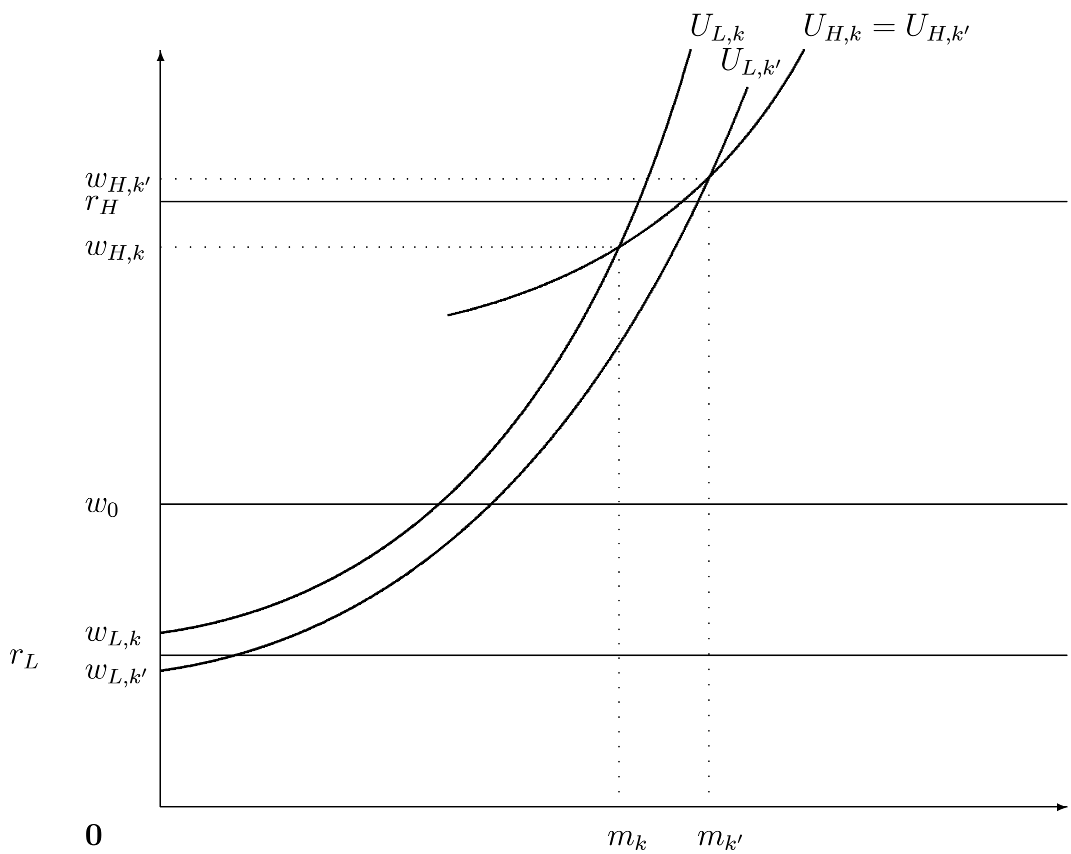

Figure 3: subcase (2.2)

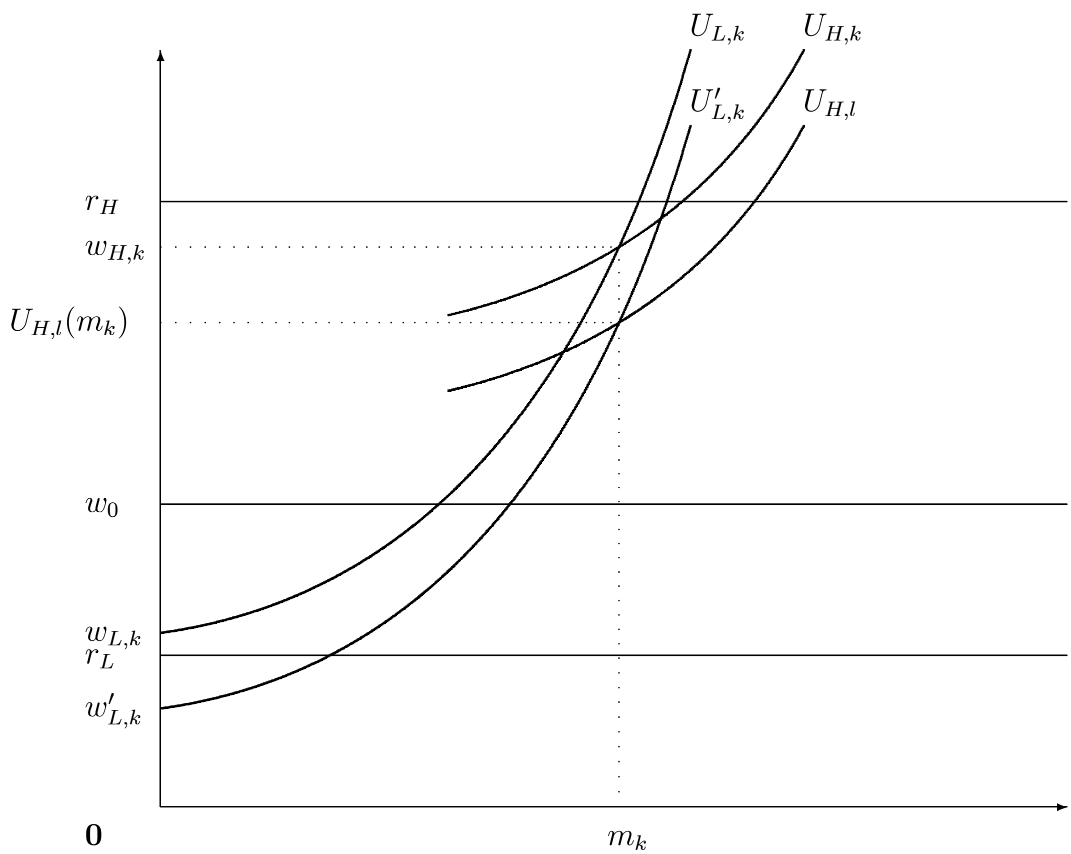

Figure 4: subcase (2.3)