Resale Price Maintenance in an Oligopoly with Uncertain Demand*

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This paper tries to clarify the effects of resale price maintenance (RPM) in a market where there is demand uncertainty and the manufacturers have to compete with one another. It shows that with RPM, competing manufacturers can earn more profits, promote wholesale demand for inventories from the retailers, enhance the social welfare, and preclude the possible coordination failure between the manufacturers. But the effect of RPM on the consumer surplus is ambiguous.

Key Words: RPM, Demand uncertainty, Oligopoly

JEL Classification Code: L1, L4, M3

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I. Introduction

This paper intends to clarify the effects of strategic interaction among the manufacturers on the advantages of resale price maintenance (RPM) in a market with demand uncertainty. Particularly, it focuses on the impact of RPM on the manufacturers' profits, consumer surplus, and social welfare. In the model, the oligopolistic manufacturers sell their products to the consumers through competitive retailers in a marketplace where the number of consumers in the market is a random variable. This paper considers the games without and with resale price maintenance and then observes the difference brought by RPM. The games defined in this paper significantly differ from those in previous works by modeling the strategic interaction between the manufacturers.

RPM has been a contentious topic in the industrial organization literature for decades. Different theories have been developed to address this behavior of manufacturing firms. One line of justifications is the free riding theories represented by Telser [1960]. The free riding theories assume that the demand for a manufacturer's product depends on some informational services provided by the retailers. RPM enables the retailers to capture the demand generated by the services and thus provides incentive for them to invest in those services. RPM then improves social efficiency by enhancing demand. However, new theory is needed to justify the use of RPM for the products that do not need extensive sale services. Deneckere, Marvel and Peck [1996, 1997] explain the RPM use from another standpoint. They find that a manufacturer facing uncertain demand has an incentive to support adequate retail inventories by preventing the emergence of discount retailers. They analyze a model with a monopoly manufacturer selling to competitive retailers in a market where the demand is uncertain. They find that with RPM, the monopoly manufacturer has higher wholesale demand and makes more profits.

The results reported by Deneckere, Marvel and Peck are impressive. However, the results are based on the analysis of a model with a monopoly manufacturer, which is restrictive. In the real life, RPM or its equivalent programs are more frequently observed in markets with inter-manufacturer competition. It is unclear whether the incentive that works in a monopoly still works in an oligopoly. Thus some issues (especially some anti-trust issues) cannot be addressed satisfactorily without a theory capable of extending those results on RPM to markets where the manufacturers have to compete with each other.

The competition among the manufacturers adds another layer of complication to the model considered by Deneckere, Marvel and Peck [1996]. In a monopoly, the retailers' behaviors are simply determined by the manufacturer's wholesale price. But in an oligopoly, the manufacturers have to anticipate the strategic interaction among the retailers before choosing their own strategies. However, if the demand is uncertain, as the price dispersion found in a monopoly (Prescott [1975], Bryant [1980], Eden [1990]), this paper finds that in an oligopoly, the entire set of retail prices of each manufacturer's product is still solely determined by that manufacturer's wholesale price, which makes the model tractable. However, the retail inventories are now determined by both manufacturers' wholesale prices. This paper also finds that RPM encourages the retailers to stock greater inventories, even when the manufacturers have to compete with one another. The manufacturers thus enhance the retailers' expected sale and make more profits. Under the assumption that the demand is either high or low, the game without RPM can result in two types of symmetric equilibria: low wholesale price equilibrium and high wholesale price equilibrium. And it is possible for the game to have the two types of equilibria simultaneously. In this case, high wholesale price equilibrium represents a high level of coordination between the manufacturers. It results in high profits for the manufacturers but low consumer surplus. On the contrary, the low wholesale price equilibrium represents a coordination failure. However, if RPM is allowed, the possibility of coordination failure is ruled out. Our model makes it easy to discuss welfare issues since the consumers have unit demand, though there is product differentiation in the model. RPM can enhance the social welfare by encouraging the retailers to stock greater inventories and thus facilitating greater expected sales to the consumers. RPM can also transfer some benefits from the consumers to the manufacturers, because the average retail prices are not lower under RPM. The consumers as a whole can be better off if significantly more consumers are served under RPM. Otherwise they are worse off with RPM. The advantages of RPM found by Deneckere, Marvel and Peck [1996] in markets with demand uncertainty are well preserved even if the strategic interaction between the manufacturers is taken into consideration.

This paper will advance as follows. Section II depicts the model and the definitions of the games. Sections III and IV analyze and solve for the subgame perfect Nash equilibria of the games. The welfare issues are addressed in section V. Section VI discusses an example about the recent use of Minimum Advertised Price (MAP) programs in prerecorded music market. Some summary remarks are given in section VII. The Appendix provides the proofs of the three propositions in Section III.

II. The Model

The market and the games

This is a symmetric model with duopoly manufacturers and competitive retailers. The two manufacturers are denoted by 0 and 1. They produce horizontally differentiated, non-storable products. The manufacturers have identical cost functions, which are assumed to be zero[1]. The feasible specifications of the goods are normalized to interval [0,1]. Assume the specification of manufacturer 0's product is 0 and that of manufacturer 1's product is 1.

The consumers have unit demand. It is common knowledge that the consumers' most preferred specifications are evenly distributed along [0, 1]. But each particular consumer's taste is private information. If a consumer's most preferred specification is x, the consumer's utility from consuming manufacturer 0's product is $1-tx$ and the utility from consuming manufacturer 1's product is $1-t(1-x)$. Parameter t represents the degree of product differentiation.

The market demand is uncertain. The measure of active consumers $\boldsymbol{q}$ is a random variable and $prob\{\boldsymbol{q}=\boldsymbol{q}_i\}=\boldsymbol{a}_{i+1}-\boldsymbol{a}_i$, for $i\in\{1,2,...,n\}$, where $0\le\boldsymbol{q}_1<\boldsymbol{q}_2<...<\boldsymbol{q}_n=1$ and

[1] The results of this paper remain valid with positive marginal costs c>0.

$0 = a_1 < a_2 < ... < a_n < a_{n+1} = 1$. We can define *demand pocket* i as the portion of demand that would show up in the market if and only if $q \geq q_i$.[2] Therefore we have n demand pockets denoted as $1, 2, ..., n$. The measure of demand pocket i is $q_i - q_{i-1}$ (let $q_0 = 0$). The demand pocket i's probability of entering the market is $1 - a_i$ because $prob\{q \geq q_i\} = 1 - a_i$. The consumers' probabilities of entering the market are independent to the consumers' preferences.

The retail market is perfectly competitive. The retailers have zero dealing costs. Unsold retail inventories cannot be returned to the manufacturers. Without loss of generality, we adopt the convention that every retailer carries only one manufacturer's product and charges a single price.[3]

From the paper by Deneckere, Marvel and Peck [1996], each manufacturer should always prefer to use RPM, given the other manufacturer's strategy. Thus this paper only defines two games depending on whether RPM is allowed: the niche competition game and the RPM game.[4]

The niche competition game: *First, the two manufacturers simultaneously announce their wholesale prices. Second, the retailers order inventories from the manufacturers and decide the retail prices. Third, the demand uncertainty resolves and the consumers come to the market. The*

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[2] If $q \geq q_i$, the consumers in the market can be represented by interval [0, q_i]. If $q < q_i$, the consumers NOT in the market can be represented by interval [q_{i-1}, 1]. Therefore the consumers in the market if and only if $q \geq q_i$ is [q_{i-1}, q_i], whose measure is $q_i - q_{i-1}$.

[3] As long as the retailers are perfectly competitive, every single unit of retail inventory with a price tag on it should yield zero expected profit for the retailer who carries that unit. Assuming a retailer can carry both manufacturers' products and charge multiple prices would not affect the price-inventory configuration in equilibrium. Therefore it would not change any result of this paper except requiring more notations.

[4] The rationing rule employed in the games is first-come-first-served.

consumers enter the market sequentially in random order. Each active consumer chooses a retailer to purchase such that his/her utility is maximized.

The RPM game: *First, the two manufacturers simultaneously announce their wholesale prices and retail prices. Second, the retailers order inventories from the manufacturers. Third, the demand uncertainty resolves and the consumers come to the market. The consumers enter the market sequentially in random order. Each active consumer chooses a retailer to purchase such that his/her utility is maximized. Since a unique price is charged for each manufacturer's product, I assume the ratios of sale to inventory are identical for all retailers selling the same product.*

A note on the model with certain demand

As a benchmark, consider the price competition game played by two vertically integrated manufacturers, where the market demand is certain. Suppose there is a continuum of consumers with measure of 1. Denote the prices as p_0 and p_1. Let $p_0 \le 1$ and $p_1 \le 1$ since the consumers' reservation prices are not greater than 1. There are three possible types of symmetric equilibria for this game, depending on the degree to which the manufacturers' products are differentiated.

1. *Typical Oligopoly*: If $t < \frac{2}{3}$, the equilibrium prices are $p_0^* = p_1^* = t$ (See Tirole [1988] for details). In this equilibrium, the manufacturers compete at the margin and all active consumers receive positive consumer surplus. A result that can apply to all games defined in this paper is

Lemma 2.1 *If the retail prices p_0 and p_1 satisfy $|p_0 - p_1| \le t$, then all consumers are served by the manufacturers if and only if $p_0 + p_1 \le 2 - t$.*

Proof: The consumer that is least likely to purchase the product is the "marginal consumer" who is indifferent between purchasing from either manufacturer. Denoting the marginal consumer's most preferred specification as x, it should satisfy $p_0 + tx = p_1 + t(1-x)$. Hence $x = \frac{p_1 - p_0}{2t} + \frac{1}{2}$. But $0 \le x \le 1$ if and only if $|p_0 - p_1| \le t$. The marginal consumer would purchase the product if and only if $(\frac{p_1 - p_0}{2t} + \frac{1}{2})t + p_0 \le 1$, which is equivalent to $p_0 + p_1 \le 2 - t$. *Q.E.D.*

2. *Restricted oligopoly*: If $\frac{2}{3} \le t \le 1$, the equilibrium prices are $p_0^* = p_1^* = 1 - \frac{t}{2}$. In this case the manufacturers still interact with each other, but the competition is limited. Given $p_0 = 1 - \frac{t}{2}$, manufacturer 1 faces a demand curve with a kink point at $1 - \frac{t}{2}$: If $p_1 < 1 - \frac{t}{2}$, the manufacturers compete at the margin since $p_0 + p_1 < 2 - t$ (Lemma 2.1). Thus the demand function would be $x_1 = \frac{p_0 - p_1}{2t} + \frac{1}{2}$. If $p_1 > 1 - \frac{t}{2}$, the manufacturers do not compete with each other since $p_0 + p_1 > 2 - t$. Thus the demand function would be $x_1 = \frac{1 - p_1}{t}$. Both manufacturers choose price $1 - \frac{t}{2}$ in equilibrium when $\frac{2}{3} < t \le 1$, and the marginal consumer receives zero surplus.

3. *Monopoly*: If $t > 1$, the equilibrium prices are $p_0^* = p_1^* = \frac{1}{2}$. The market is segmented into two monopoly markets.

III. The Niche Competition Game

In this section we will find the subgame perfect Nash equilibrium of the niche competition game. We solve the retailers' subgame first, then solve the manufacturers' problems. In order to focus on the oligopoly situations, we assume $0 \le t < \frac{2}{3}$ from now on.

The retailers' subgame

In the niche competition game, the wholesale prices, denoted as w_0 and w_1, are exogenous to the retailers. Lemma 3.1 characterizes the equilibrium of the retailers' subgame. An important characteristic of the equilibrium configuration of the niche competition game derived from Lemma 3.1 is presented in Proposition 3.2.

Lemma 3.1 *In the niche competition game, the retailers' subgame has a unique equilibrium configuration of prices and quantities. In that equilibrium configuration, there is a group of retailers catering to each demand pocket. The retailers catering to demand pocket i charge retail price of $\frac{w_0}{1-\alpha_i}$ or $\frac{w_1}{1-\alpha_i}$, and stock the pre-sale total inventories (denoted as I_0^i and I_1^i) that are exactly enough to serve demand pocket i at those retail prices.*[5]

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[5] Inventories I_0^i and I_1^i can be uniquely determined given w_0 and w_1. But explicitly specifying them is very complex for arbitrary w_0 and w_1. Essentially, we find the measure of consumers in demand pocket 1 who can to purchase from each manufacturer at prices w_0 and w_1. Next, we find the measure of consumers in demand pockets 1 and 2 who are not able to purchase at prices w_0 and w_1, but wish to purchase at prices $w_0/(1-\alpha_1)$ and $w_1/(1-\alpha_1)$. Iterating this procedure yields I_0^i and I_1^i. We omit the details.

Proof: According to the rules of the game and the price configuration depicted in the lemma, for any $i \in \{1,2,...,n\}$, the retailers charging retail price $\frac{w_0}{1-\alpha_i}$ or $\frac{w_1}{1-\alpha_i}$ can sell their inventories with probability of $1-\alpha_i$ (when $\theta \geq \theta_i$). So they earn zero profits. This price configuration is an equilibrium because any deviation by any single retailer would not be profitable: raising its price leads to a loss because of lower probability of selling; lowering its price also leads to a loss because of less revenue from sale.

Now we show that any departure from the configuration stated in the lemma cannot be an equilibrium. *First*, apparently no retailer would charge retail price lower than the wholesale price w_0 (or w_1) in equilibrium. *Second*, if the retail inventory at w_0 (or w_1) is not I_0^1 (or I_1^1), the configuration cannot be an equilibrium because: if the total retail inventory at w_0 (or w_1) is greater than I_0^1 (or I_1^1), the retailers cannot break even because its probability of selling is less than $1-\alpha_1 = 1$; if the total retail inventory at w_0 (or w_1) is less than I_0^1 (or I_1^1), the residual demand can be profitably served by a retailer charging $w_0 + \varepsilon$ (for small ε). *Third*, any retailer charging a price between $\frac{w_0}{1-\alpha_1}$ and $\frac{w_0}{1-\alpha_2}$ (or a price between $\frac{w_1}{1-\alpha_1}$ and $\frac{w_1}{1-\alpha_2}$) cannot break even, because they can only sell their inventories with probability of $1-\alpha_2$ (note that demand pocket 1 is served by retailers charging w_0 and w_1). Iterating the reasoning of the second and third steps, we can see that for any $i \in \{2,3,...,n\}$, if the retail inventory at $\frac{w_0}{1-\alpha_i}$ (or $\frac{w_1}{1-\alpha_i}$) is not I_0^i (or I_1^i), the configuration cannot be an equilibrium. And any retailer charging a price between $\frac{w_0}{1-\alpha_i}$ and $\frac{w_0}{1-\alpha_{i+1}}$ (or between $\frac{w_1}{1-\alpha_i}$ and $\frac{w_1}{1-\alpha_{i+1}}$) cannot break even. Thus the configuration stated in the lemma is the unique equilibrium configuration of the retailers' subgame. *Q.E.D.*

Proposition 3.2 *In the niche competition game, the retail prices of a manufacturer's product only depend on the wholesale price of that manufacturer.*

Lemma 3.1 says that the price that a competitive retailer charges depends on the probability of selling. For instance, if a retailer expects its inventory to be sold with probability of 0.5, then this retailer would charge retail price that is twice as much as the wholesale price in order to break even. Introducing the competition by manufacturers into the model complicates the retailers' subgame because the retailers now face inter-brand competition as well as intra-brand competition. Proposition 3.2 points out that the whole set of retail prices of a manufacturer's product is still solely determined by the manufacturer's wholesale price, but not affected by its rival's behavior. In other words, the retail prices in this oligopoly game are still determined by the intra-brand competition in the same way as that in a monopoly. However, the retailers' pre-sale inventories are now determined by both manufacturers' wholesale prices. The inter-brand competition only influences the retailers' inventory decisions, but not their pricing decisions. This proposition also makes the manufacturers' problem tractable.

The manufacturers' problem

Each manufacturer, anticipating the ensuing retailer's subgame, chooses its optimal reacting wholesale price w_0 or w_1 to maximize its profit. The manufacturers' profits are simply their wholesale revenues since the production is costless. Unfortunately, even with the relief brought by Proposition 3.2, solving the manufacturers' problem in a general model is still exceptionally difficult because too many situations could arise. We will consider a simplified model where the demand is either low or high. The measure of active consumers is 0.5 in the low demand and 1 in the high demand. Assume $prob(q = 0.5) = q$ and $prob(q = 1) = 1 - q$. Hence we have two demand pockets, denoted by 1 and 2. Each demand pocket has the size of 0.5. Demand pocket 1 enters the market with probability of 1 and pocket 2 enters with probability of $1 - q$.

We only consider the symmetric equilibria in order to compare them to the equilibrium of the RPM game, which is always symmetric. The following strategy is used to locate the equilibria of the niche competition game. First, assume the symmetric equilibrium wholesale prices are in a certain interval so that the manufacturers' profit functions can be identified. Second, solve for the corresponding candidate equilibrium prices. At last, characterize the necessary and sufficient conditions under which the manufacturers cannot profitably deviate from the candidate equilibrium prices. There are many potential types of equilibrium. In particular, for each niche, competition can be characterized as "typical oligopoly" where the marginal consumer of the demand pocket receives surplus; "restricted oligopoly" where the marginal consumer receives no surplus yet the demand pocket is fully served; "monopoly" where the demand pocket is partially served; and the case in which the demand pocket is not served at all. However, it turns out that only three types of equilibria can actually occur, as shown by the following lemma.

Lemma 3.3 *In the niche competition game, the equilibrium can only take three types:*

Type I. $w_0^*, w_1^* \in (0,(1-\frac{t}{2})(1-q))$.

Type II. $w_0^* = w_1^* = (1-\frac{t}{2})(1-q)$.

Type III. $w_0^*, w_1^* \in (1-q, 1-\frac{t}{2})$, *(when* $t < 2q$ *).*

Proof. **(**See the Appendix)

The proof of Lemma 3.3 is put in the Appendix. Types I, II and III equilibria are characterized in the following Propositions 3.4, 3.5 and 3.6 respectively. In type I equilibrium, the manufacturers compete at the margin in both demand pockets because $\frac{w_0^*}{1-q} + \frac{w_1^*}{1-q} < 2-t$ (Lemma 2.1) and all active consumers receive positive surplus. In type II equilibrium, both demand pockets are fully served but the competition between the manufacturers is limited in demand pocket 2. In type III equilibrium, demand pocket 1 is fully served, but pocket 2 is not served at all. The proofs of the following propositions are put in the Appendix.

Proposition 3.4 *In the niche competition game, the prices*

$$w_0^* = w_1^* = \frac{2-2q}{2-q}t \tag{3.1}$$

are a Nash equilibrium of the game if and only if

(a). $0 < q \le \frac{12-4\sqrt{2}}{7} (\approx 0.906)$ *and* $0 \le t \le \frac{4-2q}{6-q}$, *OR*

(b). $\frac{12-4\sqrt{2}}{7} < q < 1$ *and* $0 \le t \le \frac{(2-2q)(2-q)}{4-3q}$.

All active consumers are served in this equilibrium. The corresponding equilibrium profits of the manufacturers are

$$\pi_0^* = \pi_1^* = \frac{1-q}{2-q}t\,. \tag{3.2}$$

Proof. (See the Appendix)

Conditions (a) and (b) show that it is likely for all active consumers to be served if parameters t and q are relatively small. A small t implies low degree of product differentiation, which means that a manufacturer can easily attract more customers by lowering its price. A small q implies high likelihood for demand pocket 2 to enter the market, which also induces the manufacturers to set relatively low prices in order to make it possible for the demand pocket 2 getting served. The gray area in Figure[6] 3.1, called area 1, illustrates the set of parameters (q, t) satisfying condition (a) or (b).

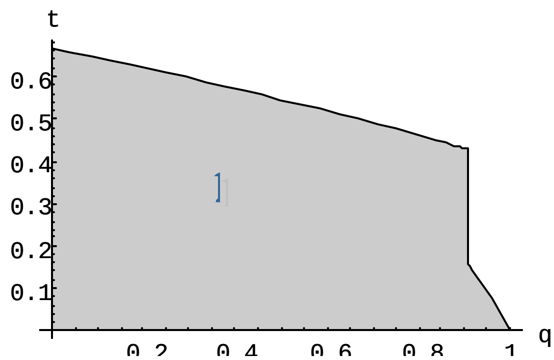

Figure 3.1

Proposition 3.5 *In the niche competition game, the prices*

$$w_0^* = w_1^* = (1 - \frac{t}{2})(1 - q) \tag{3.3}$$

are a Nash equilibrium of the game if and only if

(c). $\frac{4-2q}{6-q} \le t \le \frac{2-2q}{7-4\sqrt{2}-q}$.

All active consumers are served in this equilibrium. The corresponding equilibrium profits of the manufacturers are

$$p_0^* = p_1^* = \frac{1}{2}(1 - \frac{t}{2})(1 - q). \tag{3.4}$$

Proof. (See the Appendix)

Figure 3.2 shows the set of parameters (q, t) that satisfy condition (c), called area 2, and thus supports type II equilibrium.

[6] The figures in this paper are created in Mathematica 4, Wolfram Research [1999].

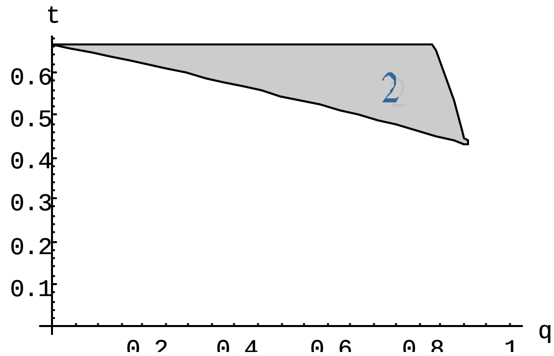

Figure 3.2

Proposition 3.6 *In the niche competition game, the prices*

$$w_0^* = w_1^* = t \tag{3.5}$$

are a Nash equilibrium of the game if and only if

(d). $0 \le t \le \frac{-1+\sqrt{3}}{2} (\approx 0.366)$ *and* $\frac{1-(3-\sqrt{3})t}{1-t} \le q < 1$, *OR*

(e). $\frac{-1+\sqrt{3}}{2} < t < \frac{2}{3}$ *and* $1 - \frac{2t^2}{1+2t} \le q < 1$.

Only demand pocket 1 is served in this equilibrium. The corresponding equilibrium profits of the manufacturers are

$$\pi_0^* = \pi_1^* = \frac{t}{4}. \tag{3.6}$$

Proof. (See the Appendix)

Type III equilibrium is likely to occur with relatively great parameter t and q, which imply that the product differentiation is significant and demand pocket 2 is unlikely to enter the market. Therefore the manufacturers are likely to give up demand pocket 2 and serve pocket 1 only. Type III equilibrium is not Pareto efficient because some consumers are not served though they have positive evaluations on the products and the production is costless. Figure 3.3 shows the set of parameters (q,t) satisfy condition (d) or (e), called area 3, and thus supports type III equilibrium.

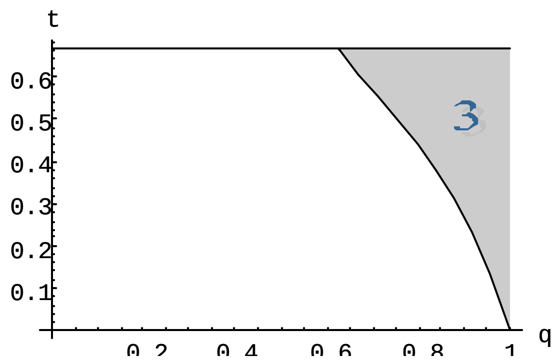

Figure 3.3

Putting the three areas above into one figure, we have the following Figure 3.4. It is notable to see that both areas 1 and 2 overlap with area 3, which implies that it is possible to have multiple symmetric equilibria for a niche competition game. For instance: the parameters $q = 0.83$ and $t = 0.4$ satisfy both conditions (a) and (e). By Proposition 3.4 and 3.6, we have two symmetric equilibria for this game: wholesale prices $w_0^* = w_1^* = 0.116$ with profits $p_0^* = p_1^* = 0.058$, and wholesale prices $w_0^* = w_1^* = 0.4$ with profits $p_0^* = p_1^* = 0.1$. In the case of multiple equilibria, the high wholesale price equilibrium represents a high level of coordination between the manufacturers and it yields high profits for them. But the high price equilibrium is Pareto inefficient since some consumers may not be served. The low wholesale price equilibrium represents a coordination failure between the manufacturers. Notice that both situations are equilibria, but not temporary collusion or price war. This is an interesting result because it shows that in a given marketplace, we could have "cutting-throat" competition among the manufacturers, or just moderate competition. Which case would occur depends on the starting point of the competition game. The possibility of multiple equilibria in an oligopoly is jointly caused by the (wholesale) pricing externality and demand uncertainty. There is also a small area in Figure 3.4 representing the set of parameters that cannot support any symmetric equilibrium. It could be the case of asymmetric equilibrium (where one manufacturer charges low wholesale price and serve both demand pockets and another manufacturer charges high price and serve demand pocket 1 only), or no equilibrium at all.

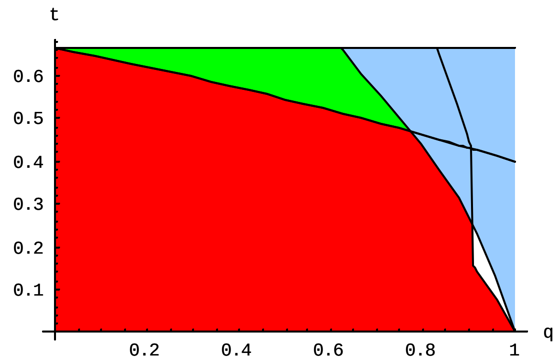

Figure 3.4

IV. The RPM Game

The retailers' subgame

In the RPM game, the manufacturers specify both the wholesale prices and retail prices. The retailers only decide how much inventory to order. The retailers can earn a margin from sale, but the competition drives the retailers' inventories to be higher than their expected sales. Therefore they still make zero profits in equilibrium. The following lemma characterizes the equilibrium of the retailers' subgame.

Lemma 4.1 *In the equilibrium of the retailers' subgame in the RPM game, the ratio of a retailer's inventory to its expected sale equals the manufacturer's retail price markup.*

Proof: Without loss of generality, consider manufacturer 0 only. Denote: w_0— wholesale price of manufacturer 0, r_0— retail price of manufacturer 0, d_0— expected sale of a retailer dealing manufacturer 0's product, i_0— inventory of the retailer, p_0^r— expected profit of the retailer. Since the retail market is competitive and the retailers are risk-neutral, the expected profit of the retailer in equilibrium is $\pi_0^r = r_0 \cdot d_0 - w_0 \cdot i_0 = 0$, which implies $\dfrac{i_0}{d_0} = \dfrac{r_0}{w_0}$. *Q.E.D.*

The manufacturers' problem

The next lemma is about the manufacturers' profits. It is easy to prove, noting that the retailers earn zero profits and there is no production or dealing cost.

Lemma 4.2 *Each manufacturer's profit equals the total expected revenue of its retailers.*

The retail prices and inventories determine the retailers' revenues, and therefore the manufacturers' profits. Thus the manufacturers can maximize their profits by optimally choosing their retail prices. Given the retail prices, the manufacturers can stimulate high enough retail inventories by choosing low enough wholesale prices (Lemma 4.1). The retailers therefore should never stock out in equilibrium. Denote the retail prices of the manufacturers as r_0 and r_1. According to Lemma 4.2, when $t < \dfrac{2}{3}$, the manufacturers' profits are

$$\pi_0 = \sum_{i=1}^{n} r_0 \left(\frac{r_1 - r_0}{2t} + \frac{1}{2}\right)(1-\alpha_i)(\theta_i - \theta_{i-1}) = r_0 \left(\frac{r_1 - r_0}{2t} + \frac{1}{2}\right)\sum_{i=1}^{n}(1-\alpha_i)(\theta_i - \theta_{i-1}) \qquad (4.1)$$

$$\pi_1 = \sum_{i=1}^{n} r_1 \left(\frac{r_0 - r_1}{2t} + \frac{1}{2}\right)(1-\alpha_i)(\theta_i - \theta_{i-1}) = r_1 \left(\frac{r_0 - r_1}{2t} + \frac{1}{2}\right)\sum_{i=1}^{n}(1-\alpha_i)(\theta_i - \theta_{i-1}) \qquad (4.2)$$

Notice that $\sum_{i=1}^{n}(1-\alpha_i)(\theta_i - \theta_{i-1})$ is the expected measure of active consumers in the market. The equilibrium of the RPM game can be characterized by the following proposition. We omit the proof because it is essentially the same as that of the vertically intergraded situation.

Proposition 4.4 *There exists a unique Nash equilibrium for the RPM game. If* $t < \frac{2}{3}$, *the equilibrium retail prices are*

$$r_0^* = r_1^* = t. \tag{4.3}$$

All active consumers are served in this equilibrium. The corresponding equilibrium profits of the manufacturers are

$$\boldsymbol{p}_0^{r*} = \boldsymbol{p}_1^{r*} = \frac{t}{2}\sum_{i=1}^{n}(1-\boldsymbol{a}_i)(\boldsymbol{q}_i - \boldsymbol{q}_{i-1}).$$

Note that the equilibrium retail prices of the RPM game are the same as that of the niche competition game when only demand pocket 1 is served. Therefore if a niche competition game had multiple equilibria, the low wholesale price equilibrium would be precluded by RPM. Compared to the high price equilibrium of the niche competition game, RPM allows the manufacturers to earn even more profits by allowing more consumers served. Apply Proposition 4.4 on the simplified model defined in Section III, we have

$$\boldsymbol{p}_0^{r*} = \boldsymbol{p}_1^{r*} = \frac{2-q}{4}t. \tag{4.4}$$

V. Welfare Effects

In this section we study the welfare effects brought by RPM in a market with oligopolistic manufacturers and competitive retailers. The analyses are based on the simplified model defined in Section III. Consider the effects of RPM on the manufacturers' profits first.

Proposition 5.1 *In the symmetric equilibria of the games, the manufacturers' profits are strictly higher in the RPM game than that in the niche competition game.*

Proof: We have explicitly found the equilibrium profits for both games. The manufacturers' profits in the equilibrium of the RPM game are $\pi^{RPM} = \frac{2-q}{4}t$ by (4.4). In the niche competition game, there are three situations. (1). In type I equilibrium, the manufacturers' profits are $\pi^{niche} = \frac{1-q}{2-q}t$ by (3.2). We have $\pi^{RPM} = \frac{2-q}{4}t > \frac{1-q}{2-q}t = \pi^{niche}$ if and only if $q > 0$. (2). In type II equilibrium, the manufacturers' profits are $\pi^{niche} = \frac{1}{2}(1-\frac{t}{2})(1-q)$ by (3.4). But $\pi^{RPM} = \frac{2-q}{4}t > \frac{1}{2}(1-\frac{t}{2})(1-q) = \pi^{niche}$ if and only if $t > \frac{2-2q}{3-2q}$, which is true if condition (c) is satisfied because $\frac{2-2q}{3-2q} < \frac{4-2q}{6-q}$. (3). In type III equilibrium, the manufacturers' profits are $\pi^{niche} = \frac{t}{4}$ by (3.6). But $\pi^{RPM} = \frac{2-q}{4}t > \frac{t}{4} = \pi^{niche}$ if and only if $q < 1$. Summing up, the manufacturers' profits in symmetric equilibrium are strictly higher in the RPM game. *Q.E.D.*

By specifying a unique retail price and supporting the margins from sales for the retailers, a manufacturer can ensure that all active consumers are charged the same optimal price that can maximize its profit, given the pricing of its rival. Proposition 5.1 shows that in equilibrium, the strategic interaction under RPM results in higher profits for both manufacturers. Therefore the competition by the manufacturers does not take away the advantage of RPM on the manufacturers' profits.

Our next proposition is on the total inventory of the retailers. Recall that in the niche competition game, we may have an equilibrium that only demand pocket 1 is served and no retail inventory is ordered for demand pocket 2. But in the RPM game, the retailers always have enough inventories to serve all active consumers. We thus immediately have

Proposition 5.2 *In the symmetric equilibria of the games, the total inventory of all retailers in the RPM game is as high as, or strictly higher than that in the niche competition game.*

Now consider the social welfare resulting from the two games. Since the production is costless, the social welfare from this market (includes the expected consumer surplus and the producer surplus) equals the expected consumer benefits from the consumption, which can be measured by the expected consumption quantity.[7] We have shown that the expected consumption quantity in the RPM game is not lower than that in the niche competition game: If only demand pocket 1 is served in the niche competition game, the expected consumption quantity is strictly higher in the RPM game. Otherwise they are the same in both games. We therefore have

Proposition 5.3 *In the symmetric equilibria of the games, the social welfare in the RPM game is as high as, or strictly higher than that in the niche competition game.*

Proposition 5.3 is logically related with Proposition 5.2, because the consumption quantity depends on the retail inventories. RPM can improve the social welfare because it encourages the retailers to stock greater inventory and thus facilitates greater expected sale to the consumers. At last we observe the consumer surplus, which is the benefit from consumption net of the consumers' payments, or equivalently, the social welfare net of the manufacturers' profits.

Proposition 5.4 *In the symmetric equilibria of the games, if both demand pockets are served in the niche competition game, the consumer surplus is higher in the niche competition game; If*

[7] For each unit of the product consumed, assuming each manufacturer supply half unit, it can be showed that the consumer benefit from the consumption is $1-\frac{t}{4}$. Also notice a demand pocket is either served in full or not served at all in the equilibrium of the niche competition game.

only demand pocket 1 is served in the niche competition game, the consumer surplus is higher in the RPM game.

Proof: Denote p as the total profit of the manufacturers, t as the consumer surplus, and w as the social welfare. Then $w = p + t$, or $t = w - p$. (1). If both demand pockets are served in the niche competition game, the social welfare w is identical in both games. But the total profit of the manufacturers p is lower in the niche competition game (Proposition 5.1). So the consumer surplus t is higher in the niche competition game. (2). If only demand pocket 1 is served in the niche competition game, the equilibrium retail prices are the same in both games, which are t. So the consumer surplus of demand pocket 1 is the same for both games. But the consumer surplus of demand pocket 2 is positive in the RPM game while is zero in the niche competition game. So the consumer surplus t is higher in the RPM game. *Q.E.D.*

Proposition 5.4 shows that RPM favors the manufacturers more than the consumers. The gain of the manufacturers comes not only from the efficiency gain brought by RPM, but also from the consumer surplus. If RPM encourages greater wholesale demand from the retailers, the efficiency gain can make not only the manufacturers but also the consumers better off. Otherwise, the manufacturers can still earn more profits, but the consumers are liable to be worse off.

VI. Application: Prerecorded Music Market in the United States

The Minimum Advertised Price (MAP) programs[8] of the major distributors of prerecorded music in the United States provide an illustration of RPM use in an oligopoly. In the United States, five distributors, Sony Music Distribution, Universal Music & Video Distribution, BMG

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[8] The FTC file No. 971 0070 "Five Consent Agreements Concerning the Market for Prerecorded Music in the United States." (http://www.ftc.gov/os/2000/05/index.htm).

Distribution, Warner-Elektra-Atlantic Corporation and EMI Music Distribution account for approximately 85% of the industry's $13.7 billion sales. The Federal Trade Commission (FTC) recently (May 2000) found that the MAP programs adopted by these five companies violated Section V of the Federal Trade Commission Act. Those companies adopted much stricter MAP programs between late 1995 and 1996. The programs prevent the retailers from advertising prices below the distributors' minimum advertised prices by denying the cooperative advertising funds to any retailer that promotes discounted prices, even in advertisements funded solely by the retailers. The advertisements are broadly defined and include even in-store displays. The penalty to violating the MAP provisions is serious: failure to stick to the provisions for any particular music title would subject the retailer to a suspension of all cooperative advertising funding offered by the distributor for about 60 to 90 days. This ensures that even the most aggressive retailer would stop advertising prices below the MAP, which makes the MAP programs essentially equivalent to RPM.

1. As the FTC found, the sale of music CDs does not require extensive sale services. Thus the "free rider" theory cannot explain the use of MAP in this market very well.

2. Each of the major distributors has substantial market power. Their copyrighted products are well differentiated from each other. Thus this is a typical oligopoly market with horizontal product differentiations.

3. Potential music retailers do not face significant entry barriers. It is reasonable to treat the retail market as perfectly competitive.

4. Though music CDs are physically storable, its market value cannot be regarded as perfectly storable because most music titles are fashion goods. Most people's valuations on those music titles are significantly lower after the enthusiasm fades.

5. The consumers can readily be modeled as having unit demand.

6. The demand is uncertain: It is difficult to predict the demand toward a certain music title.[9]

Also notice that as footnote 3 states, our model applies perfectly even if every music retailer carries all distributors' products, as long as the retail market is perfectly competitive. The FTC argues that the distributors can preclude retail price competition through the stricter MAP programs and thus eventually increase their own wholesale prices. This is intuitive not necessarily true according to this paper. Our model shows that if all demand pockets are served in the niche competition game, it is true that the wholesale prices in the RPM game are higher. But if only the demand pocket coming with high probability is served in the niche competition game, it can be shown that the wholesale price is strictly lower under the RPM game (while the retail prices are the same in both games). Thus it is not generally true that the MAP programs lead to higher wholesale prices. This is somewhat against the common intuition. But we have to refrain from that intuition because it overlooks the strategic interactions among the manufacturers and retailers in a new market mechanism.

People may be concerned about the fact that the consumers have to pay higher prices under the MAP programs. While this is likely to be the case, they may neglect the important effect of the MAP programs on the quantity of sale. The judgment on the MAP programs would be fairer if we notice that the MAP programs may facilitate greater expected sales to the consumers and thus result in higher consumer surplus. As a music dealer Joan C. Bradley from Northeast One Stop, Inc. argued,

"… They (mass merchants) also discriminate against all music except the top sellers. When less than top sellers stop being sold, future generations will be deprived of new music and older

[9] In the music CD industry, the distributors usually buy back the unsold copies from the retailers. This violates an assumption of this paper. However, our model still applies as long as the retailers have to incur some notable costs for unsold CDs, for example, costs associated with organizing, displaying and shipping.

generations will be deprived of catalog product (including classical, jazz and their favorite tunes) not carried by the mass merchants and not promoted with loss-leader prices."[10]

The "mass merchants" refer to the discount stores. They charge relatively low retail prices for the music titles and cover their costs by selling quickly. Those discount stores cannot profitably deal the catalog products at low prices because the sales of those items are slow and uncertain. The catalog items have to be sold at high prices under the niche competition, because the retailers have to confront the costs associated with unsold copies. But the high prices may suffocate the demand and destroy the business. The retailers under the MAP programs are able to carry the catalog products because the margins from sales are guaranteed.

The MAP programs are likely to improve the social welfare in the prerecorded music market because they encourage the retailers to stock greater inventories and serve more consumers. To estimate its effect on the consumer surplus, it is important to observe whether those programs can stimulate significantly higher wholesale demand for inventories from the retailers. The MAP programs are beneficial to the consumers if considerably greater wholesale demand is observed. It may not be adequate to consider the effects of the MAP programs on the prices only, which could be misleading in an oligopoly with demand uncertainty.

VII. Concluding Remarks

This paper tries to extend the results found by Deneckere, Marvel and Peck [1996] to a market where the manufacturers face competition from rivals. One might conjecture that the competition by manufacturers may take away the advantages of RPM, because the competition tends to drive the prices down and thus may discourage the manufacturers from imposing RPM. But this paper shows that may not be the case. The manufacturers still have incentive to impose

[10] Public comments to FTC file No. 971 0070 (http://www.ftc.gov/os/2000/05/index.htm).

RPM even after the competition by manufacturers is taken into consideration. RPM can also preclude the possible coordination failure among the manufacturers, which means RPM can ensure a better market condition for the manufacturers.

RPM can enhance the social welfare by encouraging the retailers to stock greater inventories and thus facilitating greater expected sales to the consumers. Though the economy as a whole can benefit from it, the consumers can be strictly worse off with RPM. Only when the efficiency gain is significant, RPM can make not only the manufacturers but also the consumers better off. It is interesting to see that RPM is able to rule out the possible coordination failure among the manufacturers, which makes the market outcome more certain and preferable from the perspective of the manufacturers who face competition and demand uncertainty.

One might also think that the competition by manufacturers should have very complicated interactions in the niche competition game, since there are many different retail prices prevailing in the market and the interaction among the retailers could be exceptionally complex. But this paper shows that it could be analyzed tractably. Part of the reason is that even with oligopolistic manufacturers (and competitive retailers), the entire set of retail prices of each manufacturer's product is still solely determined by the manufacturer's wholesale price, though the retail inventories are now determined by both manufacturers' wholesale prices. With similar methodology, we may be able to study the effect of RPM on some other types of wholesale markets, for example, a market with monopolistically competitive manufacturers.

Appendix

Following are the proofs of Lemma 3.3 and Proposition 3.4, 3.5 and 3.6 of Section III about the niche competition game.

If $t < 2q$, the following price ranges need to be considered separately in order to locate the equilibrium of the game. Note each price range is either an open interval or a point.

A. $w_0^*, w_1^* \in (0,(1-\frac{t}{2})(1-q))$. This is akin to the "typical oligopoly".

B. $w_0^* = w_1^* = (1-\frac{t}{2})(1-q)$. This is akin to the "restricted oligopoly".

C. $w_0^*, w_1^* \in ((1-\frac{t}{2})(1-q),1-q)$. Demand pocket 1 is fully served. Pocket 2 is partially served.

D. $w_0^* = w_1^* = 1-q$. This is the boundary situation of C and E.

E. $w_0^*, w_1^* \in (1-q,1-\frac{t}{2})$. Demand pocket 1 is fully served and pocket 2 is not served.

F. $w_0^* = w_1^* = 1-\frac{t}{2}$. "Restricted oligopoly" in demand pocket 1. Pocket 2 is not served.

G. $w_0^*, w_1^* \in (1-\frac{t}{2},1)$. Demand pocket 1 is partially served and pocket 2 is not served.

If $t \geq 2q$, the price ranges that need to be considered separately are

A', B'. (Same as A or B).

C'. $w_0^*, w_1^* \in ((1-\frac{t}{2})(1-q),1-\frac{t}{2})$. Demand pocket 1 is fully served. Pocket 2 is partially served.

D'. $w_0^* = w_1^* = 1-\frac{t}{2}$. "Restricted oligopoly" in demand pocket 1. Pocket 2 is partially served.

E'. $w_0^*, w_1^* \in (1-\frac{t}{2},1-q)$. Both demand pockets are partially served.

F'. $w_0^* = w_1^* = 1-q$. This is the boundary situation of E' and G'.

G'. $w_0^*, w_1^* \in (1-q,1)$. Demand pocket 1 is partially served. Pocket 2 is not served.

Fortunately, not all these situations can happen in equilibrium. Lemma 3.3 claims the situations that can occur in equilibrium are A (A'), B (B') and E. Situation A, B and E leads to type I, II and III equilibrium respectively. It is easy to see situations F, G, F'

happen: same to the "typical oligopoly" of Section II, when the manufacturers just compete in one demand pocket and $t < \frac{2}{3}$, the equilibrium wholesale price is t and the whole demand pocket 1 is served. Two claims are presented at the end of this appendix. Claim 1 precludes situations C, C', D' and E'. Claim 2 precludes situation D. Lemma 3.3 is reached automatically after all the propositions and claims are proven.

Proposition 3.4 *In the niche competition game, the prices*

$$w_0^* = w_1^* = \frac{2-2q}{2-q}t \tag{A1}$$

are a Nash equilibrium of the game if and only if

(a). $0 < q \le \frac{12-4\sqrt{2}}{7} (\approx 0.906)$ *and* $0 \le t \le \frac{4-2q}{6-q}$, *OR*

(b). $\frac{12-4\sqrt{2}}{7} < q < 1$ *and* $0 \le t \le \frac{(2-2q)(2-q)}{4-3q}$.

All active consumers are served in this equilibrium. The corresponding equilibrium profits of the manufacturers are

$$\pi_0^* = \pi_1^* = \frac{1-q}{2-q}t\,. \tag{A2}$$

Proof: Suppose the manufacturers' wholesale prices satisfy $w_0, w_1 \in (0,(1-\frac{t}{2})(1-q))$. By Lemma 2.1, demand pocket 2 is fully served since $\frac{w_0}{1-q}+\frac{w_1}{1-q} < 2-t$. So is pocket 1 of course. For manufacturer 1, the demand for its products from demand pocket 1 is $\frac{1}{2}(\frac{w_0-w_1}{2t}+\frac{1}{2})$ and the demand from pocket 2 is $\frac{1}{2}(\frac{w_0-w_1}{2t(1-q)}+\frac{1}{2})$. Thus manufacturer 1's profit function is

$$\boldsymbol{p}_1 = \frac{1}{2} w_1 (\frac{w_0 - w_1}{2t} + \frac{1}{2}) + \frac{1}{2} w_1 (\frac{w_0 - w_1}{2t(1-q)} + \frac{1}{2}) . \quad \text{(A3)}$$

The first order condition is $w_1 = \frac{1}{2} w_0 + \frac{1-q}{2-q} t$. Similarly, we have $w_0 = \frac{1}{2} w_1 + \frac{1-q}{2-q} t$ from manufacturer 0's problem. Solve for the candidate equilibrium from them $w_0^* = w_1^* = \frac{2-2q}{2-q} t$, which is (A1). The manufacturers' profits are $\boldsymbol{p}_0^* = \boldsymbol{p}_1^* = \frac{1-q}{2-q} t$. A necessary condition for (A1) to be the equilibrium can be obtained by observing $w_0^*, w_1^* \in (0, (1-\frac{t}{2})(1-q))$, which leads to

$$t \le \frac{4-2q}{6-q} \text{, or equivalently } q \le \frac{4-6t}{2-t} . \quad \text{(A4)}$$

To ensure prices (A1) are really the equilibrium of the game, we need to guarantee that neither manufacturer has incentive to deviate from (A1). Without loss of generality, we only examine whether manufacturer 1 can profitably deviate.

First, we see whether manufacturer 1 can profitably deviate to $w_1 \in [(1-\frac{t}{2})(1-q), 1-q]$. We have two cases to consider. 1. $w_0^* + w_1 > 2 - t$. The manufacturers do not compete in either demand pocket. The profit function is $\boldsymbol{p}_1 = \frac{1}{2} w_1 (\frac{1-w_1}{t}) + \frac{1}{2} w_1 (\frac{1}{t} - \frac{w_1}{t(1-q)})$. It can be showed that manufacturer 1 cannot do better than that when $w_1 = 2 - t - w_0^*$ (by checking the first order derivative). So we enter the second case. 2. $w_0^* + w_1 \le 2 - t$. If the manufacturers compete at the margin in demand pocket 2, we have showed that the optimal wholesale price is $w_1^* = \frac{2-2q}{2-q} t$. Now we suppose they compete only in demand pocket 1. Manufacturer 1's customers from demand pocket 2, denoted as x , should satisfy $\frac{w_1}{1-q} + tx = 1$, which implies $x = \frac{1}{t} - \frac{w_1}{t(1-q)}$.

So the profit function is $\pi_1 = \frac{1}{2} w_1 (\frac{w_0^* - w_1}{2t} + \frac{1}{2}) + \frac{1}{2} w_1 (\frac{1}{t} - \frac{w_1}{t(1-q)})$. It can be showed that manufacturer 1 cannot do better than that when $w_1 = (2-t)(1-q) - w_0^*$, which yields a lower profit than (A2). Thus manufacturer 1 cannot profitably deviate to $w_1 \in [(1-\frac{t}{2})(1-q), 1-q]$.

Second, we see if manufacturer 1 wishes to deviate to $w_1 > 1-q$ and serve demand pocket 1 only. Again we have two cases: 1. If $w_0^* + w_1 > 2-t$, the manufacturers do not compete even in demand pocket 1. It can be showed that manufacturer 1's profit is always lower than that when $w_1 = 2 - t - w_0^*$. So we enter the second case. 2. If $w_0^* + w_1 \le 2-t$ Manufacturer 1's profit function is $\pi_1 = \frac{1}{2} w_1 (\frac{w_0^* - w_1}{2t} + \frac{1}{2})$. The first order condition is $w_1^{**} = \frac{1}{2} w_0^* + \frac{t}{2}$. Substituting $w_0^* = \frac{2-2q}{2-q} t$ into it, we have $w_1^{**} = \frac{4-3q}{4-2q} t \le 2 - t - w_0^*$. If $\frac{4-3q}{4-2q} t \le 1-q$, or equivalently

$$t \le \frac{2(1-q)(2-q)}{4-3q}, \tag{A5}$$

manufacturer 1's optimal wholesale price cannot possibly be greater than $1-q$. If $t > \frac{2(1-q)(2-q)}{4-3q}$, manufacturer 1's profit from the deviation is

$\pi_1^{**} = \frac{1}{2} w_1^{**} (\frac{w_0^* - w_1^{**}}{2t} + \frac{1}{2}) = \frac{1}{2} \cdot \frac{4-3q}{4-2q} \cdot t \cdot (\frac{\frac{2-2q}{2-q} t - \frac{4-3q}{4-2q} t}{2t} + \frac{1}{2}) = \frac{(4-3q)^2}{16(2-q)^2} t$. This profit is not greater than its original profit if and only if $\frac{(4-3q)^2}{16(2-q)^2} t \le \frac{1-q}{2-q} t$, or equivalently

$$q \le \frac{12 - 4\sqrt{2}}{7} \approx 0.906. \tag{A6}$$

Summing up, prices (A1) are the equilibrium wholesale prices of the game if and only if (A4) and (A5), or, (A4) and (A6) are satisfied. That is equivalent to (a). $0 < q \le \frac{12-4\sqrt{2}}{7}$ and $0 \le t \le \frac{4-2q}{6-q}$, OR

(b). $\frac{12-4\sqrt{2}}{7} < q < 1$ and $0 \le t \le \frac{(2-2q)(2-q)}{4-3q}$. *Q.E.D.*

Proposition 3.5 *In the niche competition game, the prices*

$$w_0^* = w_1^* = (1-\frac{t}{2})(1-q) \tag{A7}$$

are a Nash equilibrium of the game if and only if

(c). $\frac{4-2q}{6-q} \le t \le \frac{2-2q}{7-4\sqrt{2}-q}$.

All active consumers are served in this equilibrium. The corresponding equilibrium profits of the manufacturers are

$$\boldsymbol{p}_0^* = \boldsymbol{p}_1^* = \frac{1}{2}(1-\frac{t}{2})(1-q). \tag{A8}$$

Proof: Suppose $w_0^* = w_1^* = (1-\frac{t}{2})(1-q)$. At those wholesale prices, the profits of the manufacturers are $\boldsymbol{p}_0^* = \boldsymbol{p}_1^* = \frac{1}{2}(1-\frac{t}{2})(1-q)$. Now we check what conditions are needed for $w_1^* = (1-\frac{t}{2})(1-q)$ to be the optimal price of manufacturer 1, given $w_0^* = (1-\frac{t}{2})(1-q)$.

First, consider any deviation price $w_1 < (1-\frac{t}{2})(1-q)$, which implies the manufacturers compete at the margin in demand pocket 2 (Lemma 2.1). Manufacturer 1's profit is $\boldsymbol{p}_1 = \frac{1}{2}w_1(\frac{w_0^*-w_1}{2t}+\frac{1}{2}) + \frac{1}{2}w_1(\frac{w_0^*-w_1}{2t(1-q)}+\frac{1}{2})$. The first order condition is $w_1 = \frac{1}{2}w_0^* + \frac{1-q}{2-q}t$.

Substituting $w_0^* = (1-\frac{t}{2})(1-q)$ into it, we have $w_1^{**} = \frac{1}{2}(1-\frac{t}{2})(1-q) + \frac{1-q}{2-q}t$. This is less than $(1-\frac{t}{2})(1-q)$ if and only if $t < \frac{4-2q}{6-q}$. Otherwise if

$$t \geq \frac{4-2q}{6-q}, \tag{A9}$$

it can be showed that manufacturer 1 cannot do better than that when $w_1 = (1-\frac{t}{2})(1-q)$. Thus the condition for manufacturer 1 not to deviate to $w_1 < (1-\frac{t}{2})(1-q)$ is (A9).

Second, consider deviation to $w_1 \in ((1-\frac{t}{2})(1-q), 1-q]$, which implies demand pocket 2 is served in partial. We have two cases: 1. If $w_0^* + w_1 > 2-t$, the manufacturers do not compete even in demand pocket 1. The profit function is $\pi_1 = \frac{1}{2}w_1(\frac{1-w_1}{t}) + \frac{1}{2}w_1(\frac{1}{t} - \frac{w_1}{t(1-q)})$. It can be showed that manufacturer 1 cannot do better than that when $w_1 = 2-t-w_0^*$. So we enter the second case. 2. If $w_0^* + w_1 \leq 2-t$, the manufacturers compete in and only in demand pocket 1. The profit function is $\pi_1 = \frac{1}{2}w_1(\frac{w_0^* - w_1}{2t} + \frac{1}{2}) + \frac{1}{2}w_1(\frac{1}{t} - \frac{w_1}{t(1-q)})$. It can be showed that manufacturer 1 cannot do better than that when $w_1 = (1-\frac{t}{2})(1-q)$. Hence deviating to $w_1 \in ((1-\frac{t}{2})(1-q), 1-q]$ cannot possibly be an advantage.

Third, consider deviation to the range of $w_1 > 1-q$, which implies manufacturer 1 now serves demand pocket 1 only. Consider two cases: 1. If $w_0^* + w_1 > 2-t$, the manufacturers do not compete even in demand pocket 1. It can be showed that manufacturer 1 cannot do better than that when $w_1 = 2-t-w_0^*$. So we enter the second case. 2. If $w_0^* + w_1 \leq 2-t$, the profit function is $\pi_1 = \frac{1}{2} w_1 (\frac{w_0^* - w_1}{2t} + \frac{1}{2})$. The first order condition gives $w_1^{**} = \frac{1}{2} w_0^* + \frac{t}{2}$. Substituting $w_0^* = (1 - \frac{t}{2})(1-q)$ into it, we have $w_1^{**} = \frac{1}{2} \cdot (1 - \frac{t}{2})(1-q) + \frac{t}{2} < 2 - t - w_0^*$. This is manufacturer 1's optimal price in $w_1 > 1-q$ only if $\frac{1}{2} \cdot (1 - \frac{t}{2})(1-q) + \frac{t}{2} > 1-q$, which requires $t > \frac{2-2q}{1+q}$. Substitute w_1^{**} to the profit function, we have $\pi_1^{**} = \frac{(2-2q+t+qt)^2}{64t}$.

This is not better than manufacturer 1's original profits (A8) if and only if

$\pi_1^{**} = \frac{(2-2q+t+qt)^2}{64t} \le \frac{1}{2}(1-\frac{t}{2})(1-q)$, which can be simplified to $t \le \frac{2-2q}{7-4\sqrt{2}-q}$.

Therefore manufacturer 1 cannot profitably deviate to $w_1 > 1-q$ if $t \le \frac{2-2q}{1+q}$ or $\frac{2-2q}{1+q} < t \le \frac{2-2q}{7-4\sqrt{2}-q}$, which is equivalent to

$$t \le Max\{\frac{2-2q}{1+q}, \frac{2-2q}{7-4\sqrt{2}-q}\}, \tag{A10}$$

Both $\frac{2-2q}{1+q}$ and $\frac{2-2q}{7-4\sqrt{2}-q}$ are greater than $\frac{2}{3}$ when $q < \frac{1}{2}$. Thus (A10) is not binding when $q < \frac{1}{2}$. If $q \ge \frac{1}{2}$, we have $\frac{2-2q}{1+q} < \frac{2-2q}{7-4\sqrt{2}-q}$. Thus (A10) is equivalent to

$$t \le \frac{2-2q}{7-4\sqrt{2}-q}. \tag{A11}$$

Summing up, the conditions for (A7) to be the equilibrium wholesale prices of the game are (A9) and (A11), which can be written as

(c). $\frac{4-2q}{6-q} \le t \le \frac{2-2q}{7-4\sqrt{2}-q}$. *Q.E.D.*

Proposition 3.6 *In the niche competition game, the prices*

$$w_0^* = w_1^* = t \tag{A12}$$

are a Nash equilibrium of the game if and only if

(d). $0 \le t \le \dfrac{-1+\sqrt{3}}{2} (\approx 0.366)$ *and* $\dfrac{1-(3-\sqrt{3})t}{1-t} \le q < 1$, *OR*

(e). $\dfrac{-1+\sqrt{3}}{2} < t < \dfrac{2}{3}$ *and* $1 - \dfrac{2t^2}{1+2t} \le q < 1$.

Only demand pocket 1 is served in this equilibrium. The corresponding equilibrium profits of the manufacturers are

$$p_0^* = p_1^* = \frac{t}{4}. \tag{A13}$$

Proof: Suppose $w_0, w_1 \in (1-q, 1-\frac{t}{2})$. Only demand pocket 1 is served. The manufacturers' profit functions are $p_0 = \frac{1}{2} w_0 (\frac{w_1 - w_0}{2t} + \frac{1}{2})$ and $p_1 = \frac{1}{2} w_1 (\frac{w_0 - w_1}{2t} + \frac{1}{2})$. It is easy to solve for the candidate equilibrium as $w_0^* = w_1^* = t$. The equilibrium profits are $p_0^* = p_1^* = \frac{t}{4}$. Note a necessary condition for (A12) to be a type III equilibrium is $1-q < t < 2q$.

When $t < \frac{2}{3}$ and $1-q < t < 2q$, the manufacturers cannot profitably deviate to $w_1 \ge 1 - \frac{t}{2}$, or $w_1 = 1-q$. The proofs are the same as that with vertical integrated manufacturers. Now we check if one of the manufacturers, say 1, has incentive to deviate to $w_1 < 1-q < t$ and serve both demand pockets. Given $w_0^* = t$, manufacturer 1 faces demand of $(\frac{t-w_1}{2t} + \frac{1}{2})$ from demand pocket 1, and demand from pocket 2, denoted as x, satisfying $\frac{w_1}{1-q}+tx=1$ if $x<1$, and $x=1$ otherwise, i.e., $x=\begin{cases}\frac{1}{t}-\frac{w_1}{t(1-q)} & if \quad w_1\geq(1-q)(1-t)\\ 1 & if \quad w_1<(1-q)(1-t)\end{cases}$.

If $w_1\geq(1-q)(1-t)$, the profit function is $\pi_1=\frac{1}{2}w_1(\frac{t-w_1}{2t}+\frac{1}{2})+\frac{1}{2}w_1(\frac{1}{t}-\frac{w_1}{t(1-q)})$. The first order condition is $w_1=\frac{(1-q)(t+1)}{3-q}$. Note $\frac{(1-q)(t+1)}{3-q}<1-q$ constantly holds. Denote w_1^{**} as manufacturer 1's optimal price below $1-q$. We have two possible cases:

(1). If $\frac{(1-q)(t+1)}{3-q}\geq(1-q)(1-t)$, or equivalently $q\geq\frac{2-4t}{1-t}$, the optimal wholesale price is $w_1^{**}=\frac{(1-q)(t+1)}{3-q}$. Substituting it to the profit function, we have $\pi_1^{**}=\frac{(1-q)(1+t)^2}{4t(3-q)}$. This profit is not greater than its original profit $\frac{t}{4}$ if and only if $q\geq1-\frac{2t^2}{1+2t}$. Thus (A12) gives the equilibrium of the game if $q\geq\frac{2-4t}{1-t}$ and $q\geq1-\frac{2t^2}{1+2t}$, which is equivalent to

$$q\geq\begin{cases}1-\frac{2t^2}{1+2t}, & if \qquad t\in(\frac{-1+\sqrt{3}}{2},\frac{2}{3}]\\ \frac{2-4t}{1-t}, & if \qquad t\in(0,\frac{-1+\sqrt{3}}{2}]\end{cases}. \tag{A14}$$

(2). If $\frac{(1-q)(t+1)}{3-q}<(1-q)(1-t)$, or equivalently, $q<\frac{2-4t}{1-t}$, we would have $w_1^{**}=(1-q)(1-t)$. The manufacturer's profit with the deviation would be

$$\pi_1^{**}=\frac{1}{2}w_1^{**}(\frac{t-w_1^{**}}{2t}+\frac{1}{2})+\frac{1}{2}w_1^{**}=\frac{1}{4t}(1-q)(1-t)(5t-qt+q-1).$$

This is not greater than the original profit $\frac{t}{4}$ if and only if $q \le \frac{1-(3+\sqrt{3})t}{1-t}$ or $q \ge \frac{1-(3-\sqrt{3})t}{1-t}$. But $q \le \frac{1-(3+\sqrt{3})t}{1-t}$ conflicts with the condition $t > 1-q$. Thus for (A12) to be an equilibrium, we only need $q < \frac{2-4t}{1-t}$ and $q \ge \frac{1-(3-\sqrt{3})t}{1-t}$, which imply

$$\frac{1-(3-\sqrt{3})t}{1-t} \le q \le \frac{2-4t}{1-t}, \quad \text{and} \quad t \le \frac{-1+\sqrt{3}}{2}. \tag{A15}$$

Condition (A14) OR (A15) can be summarized as

(d). $0 \le t \le \frac{-1+\sqrt{3}}{2}$ and $\frac{1-(3-\sqrt{3})t}{1-t} \le q < 1$, OR

(e). $\frac{-1+\sqrt{3}}{2} < t \le \frac{2}{3}$ and $1-\frac{2t^2}{1+2t} \le q < 1$,

One can check that condition $1-q < t < 2q$ is satisfied in (d) and (e). Thus they are the sufficient and necessary conditions for (A12) to be the equilibrium prices of the game. *Q.E.D.*

Claim 1: *In the niche competition game, the equilibrium wholesale prices cannot possibly satisfy* $w_0^*, w_1^* \in ((1-\frac{t}{2})(1-q), 1-q)$.

Proof: Suppose wholesale prices $w_0, w_1 \in ((1-\frac{t}{2})(1-q), 1-q)$. Consider three cases. 1. If $w_0 + w_1 \le 2-t$, the manufacturers compete in and only in demand pocket 1. Manufacturer 1's profit function is $\pi_1 = \frac{1}{2} w_1 (\frac{w_0 - w_1}{2t} + \frac{1}{2}) + \frac{1}{2} w_1 (\frac{1}{t} - \frac{w_1}{t(1-q)})$. It can be showed that $\frac{\partial \pi_1}{\partial w_1} < 0$ whenever $w_0 + w_1 \le 2-t$ and $w_0, w_1 \in ((1-\frac{t}{2})(1-q), 1-q)$. So this case cannot contain an equilibrium. 2. If $w_0 + w_1 > 2-t$, the manufacturers do not compete even in demand pocket 1.

Manufacturer 1's profit function is $\pi_1 = \frac{1}{2} w_1 (\frac{1-w_1}{t}) + \frac{1}{2} w_1 (\frac{1}{t} - \frac{w_1}{t(1-q)})$. The first order condition is $w_1^* = \frac{1-q}{2-q}$. Similarly, we have $w_0^* = \frac{1-q}{2-q}$. But $w_0^* + w_1^* > 2-t$ if and only if $t > \frac{2}{2-q} > 1$, which is impossible. Therefore it is impossible to have equilibrium prices $w_0^*, w_1^* \in ((1-\frac{t}{2})(1-q), 1-q)$. *Q.E.D.*

Claim 2: *The equilibrium of the niche competition game cannot be* $w_0^* = w_1^* = 1-q$.

Proof: At wholesale prices $w_0^* = w_1^* = 1-q$, only demand pocket 1 is served. The equilibrium wholesale prices would be t if only demand pocket 1 is served (Proposition 3.6). Thus $w_0^* = w_1^* = 1-q$ is not an equilibrium when $t \neq 1-q$. If $t = 1-q$, it can be showed that lowering the price from $1-q$ and thus serving demand pocket 2 enables a manufacturer to earn more profit (the opportunity to serve demand pocket 2 destroys the first order conditions at t). Hence $w_0^* = w_1^* = 1-q$ cannot be an equilibrium of the game. *Q.E.D.*

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