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ABSTRACT

Capacity Choice and Duopoly Incentives for Information Sharing

by William Novshek and Lynda Thoman

We examine a three-stage game in which duopolists face a random demand intercept. The firms first choose capacities, then decide whether to commit to share the private information they will receive about the intercept. After the private information is observed, firms choose output levels. Comparing the results to an alternative model without capacity choice or capacity constraints, we show the existence of a capacity choice stage may reverse the incentives to share information, and lead to equilibria in which information sharing occurs. We use binary uncertainty since the common linear-normal model cannot handle capacity constraints.

ZUSAMMENFASSUNG

Kapazitätsentscheidungen und Duopolanreize bei Informationsaustausch

In dem Beitrag wird ein dreistufiges Spiel vorgestellt, in dem sich Duopolisten einer Nachfrage gegenübersehen, die eine Zufallsgröße ist. Die Unternehmen entscheiden auf der ersten Stufe über Kapazitäten, dann können sie sich freiwillig verpflichten, ihre privaten Information auszutauschen, die sie über das Marktvolumen erhalten werden. Nach der Beobachtung der privaten Information entscheiden die Unternehmen über die Angebotsmenge. Ein Vergleich der Ergebnisse dieses Spiels mit einem alternativen Modell ohne Kapazitätsentscheidung oder Kapazitätsbeschränkungen zeigt, daß die Existenz einer Kapazitätsentscheidungsstufe die Anreize zum Austausch von Informationen umkehren und zu Gleichgewichten führen kann, in denen Informationsaustausch stattfindet. In dem Spiel wird Unsicherheit binär modelliert, da das übliche linear-normalverteilte Modell keine Kapazitätsbeschränkungen erfassen kann.

Capacity Choice and Duopoly Incentives for Information Sharing

1. Introduction

We are interested in the role of capacity and capacity choice in the incentives of firms to share information about a random demand intercept. Standard information sharing models contain no capacity restrictions. In essence, capacity and output choices are made simultaneously. Our point is simple but powerful: Separation of the capacity and output choices into different stages sometimes reverses previous results on firms' incentives to share information.

To understand why the existence of a capacity choice stage may reverse the incentives to share information, it is instructive to consider first a case with capacity constraints which are exogenously set. When set at certain levels, the capacity constraints can reduce one of the profit-reducing effects of information sharing. When information is shared and both firms receive high signals, they are fairly confident that the demand intercept is large. In their resulting output choices, the capacity constraints are binding, and prevent the firms from being "too competitive" (i.e., they keep the firms closer to the collusive output). When the firms do not share information, they are less sure that the demand intercept is large when they receive a high signal, so the capacity constraint does not bind as often or with as much impact. The result is that, for certain exogenous capacity levels, information sharing becomes relatively more attractive to the firms than it would have been without the capacity constraints.

The remaining question is whether such capacities, at which the information sharing incentives are reversed, would be endogenously chosen by firms in an equilibrium. We show that the answer is sometimes, but not always, yes. We consider a multistage game: in the First Stage, firms choose capacities; in the Second Stage, firms decide whether to share information; in the Third Stage, firms play a Cournot game, choosing outputs given their capacity and information-sharing decisions. We focus on two types of parameter combinations in which, without capacity constraints, there is a unique equilibrium, and that equilibrium does not involve information sharing. The first combination involves equally well informed firms while the second combination involves firms with very different levels of information. In both, we show that the existence of a capacity choice stage sometimes leads to equilibria in which information sharing occurs. We conclude that the endogenously chosen information sharing is due to the capacity choice stage.

The most commonly used model for examining information sharing about a random demand parameter includes linear marginal cost for each firm and linear demand with a random, normally-distributed intercept. The firms receive signals which are also normally distributed. To simplify the analysis, the nonnegativity constraints on quantities and prices are ignored. This is justified by the argument that, by appropriate restrictions on the parameters, at the “equilibrium,” the probability with which the nonnegativity constraints are violated may be made smaller than any pre-specified, strictly positive target.

The linear-normal model cannot be applied successfully when capacities are endogenously chosen. Even if one is completely happy with the approximation argument when there are no capacity constraints, the probability of violating the capacity constraints cannot be made arbitrarily small without pre-specifying the capacities, as if they were exogenous parameters rather than endogenously determined choice variables. For some endogenously chosen capacity levels, firms will be capacity constrained with high probability, and the standard linear-normal formulas will be seriously wrong. In equilibrium, the capacity constraints will be binding with significant probability. Roughly speaking, if that were not true, the firm could decrease capacity in the First Stage, for a first-order reduction in cost, with only a minor reduction in expected revenue.

By taking strict account of the capacity and nonnegativity constraints, we introduce substantial complications into the analysis. For example, given the information structure (including the sharing or nonsharing choice), the Third Stage equilibrium in outputs will take one of several forms, depending on the capacity levels chosen in the First Stage. The capacity choices will determine the number of states in which capacity is binding, and this will lead to complicated weighted sums for the corresponding expected payoffs to use in the Second Stage. The level of complication has motivated us to use the simplest possible structure for the uncertainty, the binary model, as well as to focus on special parameter combinations for our specific results. While the examples are fairly specific, the results are quite robust. It is a straightforward, though tedious, exercise to use continuity arguments to show our results apply to a much wider range of parameter values. (Actual solution of the problem for general parameter values is extremely tedious, and does not change our main point: the incentives to share information are sometimes reversed when there is a capacity choice stage.)

In the binary model, even without capacity constraints, for certain parameter values firms have an incentive to share information, and information sharing is a Nash equilibrium. To ensure we do not confuse the effects of capacity constraints with the effects of these special parameter values, we examine a Comparison Model which matches our model except for its lack of capacity constraints. Capacity choice can be viewed as being made simultaneously with quantity choice in the final stage of the Comparison Model. For the two types of parameter combinations we consider, the Comparison model has a unique equilibrium, and that equilibrium does not involve information sharing. For both parameter combinations, we show that the existence of a capacity choice stage sometimes leads to equilibria in which information sharing occurs.

In our time line, capacity choices are made before information sharing decisions. Capacity choice occurs first because we view it as a longer-run decision than the decision whether to commit to information sharing. If one were to consider capacity choice as a shorter-run decision than information sharing, the first and second stages of our model would be reversed. Though the results are not reported in this paper, an analysis of this alternative time line leads to the same overall conclusion: the incentives to share information are sometimes reversed when there is a capacity choice stage. In fact, the analysis becomes somewhat simpler because, in the capacity choice stage, as capacities are changed, the information sharing decision has already been made at an earlier stage, and cannot vary with capacities.

Incentives for information sharing about a random demand intercept have been examined by Novshek and Sonnenschein (1982), Clarke (1983), Vives (1984), Gal-Or (1985), Li (1985) and many others. Jin (1992) and Raith (1996) provide more general analyses. Several papers have shown how the incentives reverse when certain parameters of the problem change: Kirby (1988) varies the slope of the (linear) marginal cost curve; Malueg and Tsutsui (1996), in a model with random demand slope rather than intercept, change the amount of variation in the slope. In our analysis, the incentives also change as parameters vary, but that is not our focus. Instead, our main point is that the introduction of a capacity choice stage sometimes, but not always, reverses the incentives to share information.

Section 2 contains a description of the General Model and solution procedure. The results for the Comparison Model, which is just a modification of the General Model, are contained in Section 3. Section 4 contains the results for the two special examples, one with equally-well-informed firms, and the other with firms with very different levels of information. It also includes a brief discussion of unilateral information disclosure in the context of the second example. The details of all the results are contained in an appendix.

2. General Model

In this Section we set out the General Model. The actual circumstances we will analyze as well as the Comparison Model are special cases or modifications of this General Model. Here we introduce the notation, general assumptions, and time line common to the cases of interest, and outline the solution procedure.

Two risk-neutral, expected-profit-maximizing firms, $i = 1,2$, produce a homogeneous product, to be sold in a single market. The market inverse demand is $A - BQ$ where Q is the aggregate quantity produced by the two firms, B is a strictly positive constant, and A is a random variable which takes the value H with probability t and L with probability 1-t, where $H > L > 0$. Firms know B and the distribution of A, but they do not know the realization of A when they make their initial decisions.

With no additional information about the realization of A, firms first simultaneously and independently choose capacity levels. Both firms have the same cost of capacity, c per unit. No additional capacity may be purchased in subsequent stages, so these capacity choices become absolute upper bounds on production.

Next, and again without any additional information about the realization of A, firms simultaneously and independently decide whether to join a group of firms who will share information among themselves. When additional information about A becomes available later, those who join the group will have access to all the information received by members of the group, while those who do not join will have access to their own information only. With just two firms, this means firms have access to only their own information unless both agree to join the group and share information. [In a special case in which only one firm receives additional information, we will also briefly discuss information disclosure. By this we mean the informed firm, before receiving the additional information, unilaterally decides whether the other firm will also see the additional information.]

After the firms have decided whether to share information, additional information about A is received. Firm i, receives a signal, s_i, which also takes on the values H and L; the signal is correct with probability p_i (i.e., prob(s_i=S|A=S) = p_i for S=H,L and i = 1,2). Conditional on the realization of A, the signals s_1 and s_2 are independent. If both firms have decided to share information, then they both receive both signals. Otherwise each firm receives its own signal only. Each firm uses its additional information to update its belief about the realization of the demand intercept. Note when $p_i = 0.5$, the signal is totally uninformative while when $p_i = 1$, the signal is perfectly informative. Without loss of generality each p_i satisfies $0.5 \leq p_i \leq 1$. (If not, let $p_i^* = 1-p_i$ and use s_i=L as a signal that A=H.)

Given their updated beliefs about the demand intercept, the firms simultaneously and independently choose output levels. Both firms have the same constant cost d per unit for production levels up to their capacities, with d < L. It is impossible for either firm to produce more than its capacity.

In summary, the game proceeds through the following time line of three strategic stages and two non-strategic steps.

Strategic Stage 1: Firms simultaneously and independently choose capacity levels, K_i, i = 1,2, at cost c per unit of capacity.

Strategic Stage 2: Firms simultaneously and independently decide whether to share information. Information sharing occurs if and only if both firms decide to share.

Non-strategic information reception step: The true intercept of the inverse demand function is realized and firms receive their signals. If the Stage 2 result is information sharing, then both firms receive both signals. Otherwise each firm receives only its own signal.

Strategic Stage 3: Given their information, firms simultaneously and independently choose output levels, $q_i \leq K_i$, i = 1,2, at cost d per unit of output (up to capacity K_i).

Non-strategic payoff step: Given the realized value of the intercept of the inverse demand function, A, and the firms' choices of capacity, K_1 and K_2, and outputs, q_1 and q_2, the payoff for firm i is $(A - Bq_1 - Bq_2)q_i - dq_i - cK_i$ (or $-dq_i - cK_i$ when $q_1 + q_2 > A/B$).

We will now outline the solution procedure for the game. Firms are risk-neutral, expected-profit maximizers. Our equilibrium concept is subgame-perfect Nash equilibrium. We pay careful attention to the solution at each stage so that only economically meaningful prices and quantities obtain. That is, we ensure the realized quantities and market prices are always nonnegative and capacity constraints are never exceeded.

Solving the game backwards, in the final strategic stage we need to find an equilibrium in quantities given the capacities K_i , i = 1,2, and the information structure. If firms are not sharing information, then each firm chooses two output levels, one for each potential value of the signal it receives, q_i^H and q_i^L, i = 1,2. Given the value of the signal received by firm i, say s_i = H, the firm knows the probability distribution over the possible realizations of the pair (true inverse demand intercept, signal received by the other firm) given s_i = H. Given anticipated outputs q_j^H and q_j^L for the other firm, j, this leads to an expected value for the realization of $A - Bq_j$, call it $E^H(q_j^H, q_j^L)$. As long as $A - Bq_j$ is nonnegative for every realization of the pair (true inverse demand intercept, signal received by firm j), the expected profit for firm i is $(E^H - Bq_i)q_i - dq_i - cK_i$ whenever q_i is less than E^H/B. Taking account of the capacity constraint and the nonnegativity constraint, the corresponding best response function for firm i having seen signal s_i = H is

$$q_i^H(q_j^H, q_j^L) = \text{maximum}\{0, \text{minimum}[K_i, (E^H(q_j^H, q_j^L) - d)/2B]\}.$$

Similarly, we find the best response functions for firm i having seen signal s_i = L and for firm j having seen either of the two possible signal values. The four equations are then solved simultaneously to find the Third Stage equilibrium outputs. The weighted average of the expected profits for firm i conditional on the signal received by firm i, where the weights are the probabilities of receiving the different signals, is the overall expected profit for firm i. Thus we have derived the payoff functions $\pi_i^{NS}(K_1, K_2)$, i = 1,2 for the Third Stage equilibrium without information sharing and given the capacities.

If firms are sharing information, then in the final strategic stage each firm chooses four output levels, one for each potential value of the pair of signals it receives, q_i^{HH}, q_i^{LH}, q_i^{HL} and q_i^{LL}, i = 1,2. Given the value of the pair of signals received by both firms, say HH, each firm knows the other firm received the same pair of signals, and knows the probability distribution over the possible realizations of the true inverse demand intercept given the pair of signals HH. Given anticipated output q_j^{HH} for the other firm, j, this leads to an expected value for the realization of A - Bq_j, call it $E^{HH}(q_j^{HH})$. As long as A - Bq_j is nonnegative for every realization of the true inverse demand intercept, the expected profit for firm i is $(E^{HH} - Bq_i)q_i - dq_i - cK_i$ whenever q_i is less than E^{HH}/B. Taking account of the capacity constraint and the nonnegativity constraint, the corresponding best response function for firm i having seen the pair of signals HH is

$$q_i^{HH}(q_j^{HH}) = \text{maximum}\{0, \text{minimum}[K_i, (E^{HH}(q_j^{HH}) - d)/2B]\}.$$

Similarly, we find the best response functions for firm j having seen the same pair of signals. The two equations are then solved simultaneously to find the equilibrium outputs and corresponding expected profits for the sharing case when the pair of signals is HH. We solve a similar problem for each possible pair of signals. The weighted average of the expected profits for firm i conditional on the pair of signals received by firm i, where the weights are the probabilities of receiving the different pairs of signals, is the overall expected profit for firm i. Thus we have derived the payoff functions $\pi_i^{S}(K_1, K_2)$, i = 1,2 for the Third Stage equilibrium with information sharing and given the capacities.

By appropriate restrictions on the parameters, we ensure that the procedure used to solve the Third Stage for the sharing and the nonsharing cases is correct, and yields the correct profit formulas. In particular, we restrict the parameters so that e.g., for the nonsharing case, $A - Bq_1^{s1} - Bq_2^{s2}$ is nonnegative for every possible realization of the triple (A, s_1, s_2). That such conditions do not automatically hold is easily seen by considering a monopolist with p = 0.5 (so no updating of beliefs occurs), t near 1, K large, B = 1, and H very large relative to L and d. Such a monopolist would find it optimal to "ignore" the state in which the intercept is L, producing q* = (H - d)/2B. When the intercept is L, L - Bq* < 0, and a very large reduction in output would be necessary to obtain a strictly positive price.

Now that we have obtained Third Stage equilibrium payoff functions with and without information sharing, we can analyze the second strategic stage of the game. Given (K_1, K_2), firm i prefers to share information if $\pi_i^S(K_1, K_2) > \pi_i^{NS}(K_1, K_2)$ i = 1,2. Information sharing occurs if and only if both firms agree to share information. Thus there are three regions of (K_1, K_2) values: $M^{NS} = \{(K_1, K_2) \mid \pi_i^S(K_1, K_2) < \pi_i^{NS}(K_1, K_2)$ for i = 1 or i = 2$\}$ is the set of capacity pairs at which the Second Stage equilibrium involves nonsharing of information; $M^S = \{(K_1, K_2) \mid \pi_i^S(K_1, K_2) > \pi_i^{NS}(K_1, K_2)$ for i = 1 and i = 2$\}$ is the set of capacity pairs at which the Second Stage equilibrium involves sharing of information; and $M^E = \{(K_1, K_2) \mid \pi_i^S(K_1, K_2) \geq \pi_i^{NS}(K_1, K_2)$ for i = 1 and i = 2 with equality for at least one firm$\}$ is the set of capacity pairs at which there are two Second Stage equilibria, one with sharing of information and the other with nonsharing of information. Note that for (K_1, K_2) in M^E, the two equilibria at (K_1, K_2) have the same payoff for the firm that is indifferent, but not for the other firm. In general, as we move through (K_1, K_2)-space, when we cross the boundary, M^E, between M^S and M^{NS}, one of the Second Stage equilibrium payoff functions (actually correspondences), $\pi_i(K_1, K_2)$, i = 1,2, is discontinuous, with a jump down in profit as we move from M^S to M^{NS}. At the boundary, the other firm vetoes information sharing, leading to the drop in profit.

With the Second Stage equilibrium payoff functions, $\pi_i(K_1, K_2)$, i = 1,2, we can analyze the first stage capacity choice. To solve this stage we find Nash equilibria in capacities with the given payoff functions. If one were interested in finding all equilibria for the general model, it would be necessary to check for a type of "boundary" equilibria in which the equilibrium capacities lie in M^E and information sharing occurs. The equilibria are at a "boundary" in the sense that the firm whose payoff function is discontinuous at the equilibrium capacities would prefer to change capacity if information sharing could be maintained, but the (conditionally) desired change in its capacity would lead the other firm to veto information sharing. This difficulty will not arise in the equilibria we find in our special cases.

We will consider two special cases. In the first, the two firms are equally well informed and the high and low demand states are equally likely: $0.5 < p_1 = p_2 = p < 1$ and $t = 0.5$. In the second, the first firm's signal is perfectly informative while the second firm's signal is totally uninformative: $p_1 = 1$ and $p_2 = 0.5$, with $0 < t < 1$.

3. Comparison model

Since our point is to show how the existence of a capacity choice stage changes the incentives to share information, and leads to information sharing in equilibrium, we want to start from a situation in which a corresponding model without the capacity choice stage would have no information sharing in equilibrium. This leads us to define a Comparison Model and to analyze it for cases in which the parameters match those of our special cases of interest.

Consider a model without capacity choice or capacity constraints obtained from the General Model by (1) eliminating the first stage (capacity choice), (2) removing the capacity constraint in the third stage (output choice), and (3) using production cost of c+d per unit in the third stage (output choice). In this Comparison Model, "capacity" and output decisions are made simultaneously in the final strategic stage. There is never any unused capacity in any state of the world, and there is never a capacity constraint. The production cost is set at c+d per unit so the cost matches that in the General Model when a firm always uses all of its capacity.

The solution for this Comparison Model follows easily from the solution for the General Model. For the Third Stage of the General Model, use $d^* = c + d$ as the per unit cost of production, $c^* = 0$ as the cost of capacity, and $K_i^* = H/B$ as capacity (since it is larger than any output the firm would ever find optimal). The resulting Third Stage equilibria with and without information sharing lead to expected profits π_i^S and π_i^{NS}, i = 1,2 respectively. The Second Stage equilibrium involves a comparison of these profits just as in the General Model, and there is no First Stage to solve. If $\pi_i^S > \pi_i^{NS}$ for i = 1,2, then there is a unique subgame perfect equilibrium and it involves information sharing. If $\pi_i^S < \pi_i^{NS}$ for i = 1 or i = 2, then there is a unique subgame perfect equilibrium and it does not involve information sharing. If $\pi_i^S \geq \pi_i^{NS}$ for i = 1,2, with equality for at least one firm, then there are two subgame perfect equilibria, one with information sharing and the other without information sharing.

For the Comparison Model, when $p_1 = p_2 = p$, the Third Stage equilibrium is symmetric, with equal expected profits (written as functions of the parameters p and t), $\pi_1^S(p, t) = \pi_2^S(p, t) = \pi^S(p, t)$ and $\pi_1^{NS}(p, t) = \pi_2^{NS}(p, t) = \pi^{NS}(p, t)$. The Second Stage equilibrium involves information sharing if $\pi^S(p, t) > \pi^{NS}(p, t)$, and involves nonsharing if $\pi^S(p, t) < \pi^{NS}(p, t)$. When $\pi^S(p, t) = \pi^{NS}(p, t)$, there are two Second Stage equilibria. The (p, t) combinations at which the equilibrium involves information sharing are shown as the shaded area in Figure 1 (see the Appendix for derivation of this figure). They are combinations in which p is "large" and t is either "near 0" or "near 1." In the first of our special cases, with equally well informed firms ($0.5 < p_1 = p_2 = p < 1$), we have chosen t = 0.5 since for each p value it is the t value which both maximizes the profit advantage of the nonsharing payoff over the sharing payoff, $\pi^{NS}(p, t) - \pi^{S}(p, t)$ and is furthest from the region in which information sharing occurs in the equilibrium of the Comparison Model.

Figure 1 Here

In the second of our special cases, with $p_1 = 1$ and $p_2 = 0.5$, for the Comparison Model the uninformed firm would like to share information but the firm that will receive the completely informative signal prefers not to share. In equilibrium, sharing does not occur. (See the Appendix for a derivation of this result.)

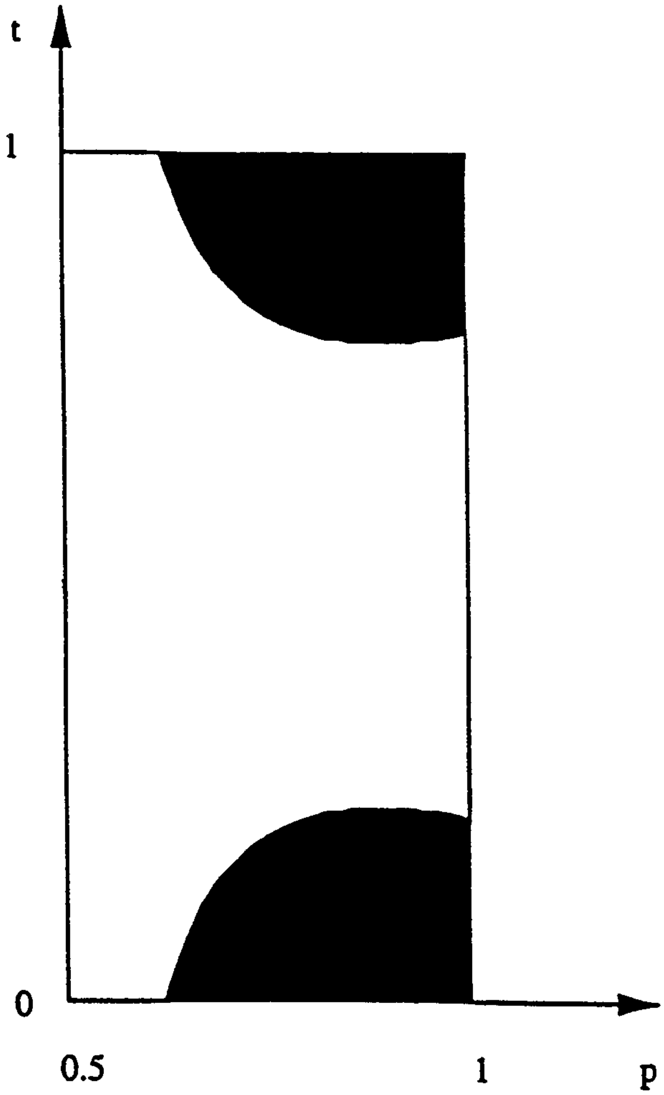
4. Results

With capacity constraints, the analysis becomes more complicated than that in the Comparison Model. The capacity constraint might never, always, or sometimes be binding in the Third stage equilibrium.

If each firm's capacity is sufficiently large, then in the Third Stage, the capacity constraints will never be binding, and the result of the Third Stage is as in the Comparison Model. However, as long as the cost of capacity is strictly positive, this will never occur in a subgame perfect equilibrium of the overall game. The savings in cost of capacity for a slight reduction in capacity is a first-order effect while the reduction in revenue (net of production cost) due to the sometimes (slightly) binding capacity constraint in the Third Stage is a second-order effect.

If each firm's capacity is sufficiently small, then in the Third Stage, the capacity constraints will always be binding, and the result of the Third Stage is production at capacity by both firms, independent of the information structure (sharing or nonsharing) and the signals received. This case will occur in a subgame perfect equilibrium of the overall game whenever

Figure 1: Parameter Values with Sharing Equilibrium in Eqal Information Comparison Model



the cost of capacity is sufficiently high. In such a case, there are always two equilibria of the overall game, one with information sharing and one without, with identical quantity choices (always capacity) and expected profit. The information sharing issue is totally uninteresting here.

The effect of capacity choice on the information sharing decision is potentially interesting only when the cost of capacity is strictly positive, but not so large that firms are always capacity constrained in the Third Stage. In this case, in the Third Stage equilibrium firms are sometimes capacity constrained and sometimes not, depending on the information structure (sharing or nonsharing) and the signals received. The analysis becomes complicated because each different combination of the information structure, signals received, and whether the capacity constraint is binding leads to a different version of the formulas for Second Stage profits, $\pi_i^S(K_1, K_2)$ and $\pi_i^{NS}(K_1, K_2)$, i = 1,2. In (K_1, K_2)-space, we must find the boundaries dividing the regions in which these different formulas apply. Given (K_1, K_2), once the appropriate formulas for $\pi_i^S(K_1, K_2)$ and $\pi_i^{NS}(K_1, K_2)$, i = 1,2 have been determined, the Second Stage equilibrium can be determined. To determine whether (K_1^*, K_2^*) is a First Stage equilibrium, we need to check whether $\pi_1(K_1^*, K_2^*) \geq \pi_1(K_1, K_2^*)$ for all K_1, which typically involves comparison between several different regions (and thus formulas), and similarly, whether $\pi_2(K_1^*, K_2^*) \geq \pi_2(K_1, K_2^*)$ for all K_2. This process is particularly tedious for the first example, in which firms are equally well informed.

4A. First Example

Assume the firms are equally well informed, with $0.5 < p_1 = p_2 < 1$ and t = 0.5. Recall that in the Comparison Model, for these parameter values the subgame perfect equilibrium never involved information sharing. In the following statement of results, we restrict our attention to values of the cost of capacity, c, at which chosen capacities are not always binding in the Third Stage. (We also impose parameter restrictions to insure that the Third Stage equilibrium quantities and prices are always strictly positive. See the Appendix.)

Result 1: With equal information and t = 0.5, unless the cost of capacity is extremely low, there always is a subgame perfect equilibrium in which the second stage choice is information sharing. (See the Appendix for details of this result.)

When the cost of capacity is extremely low, there is no such equilibrium. In these cases, in equilibrium the firms are virtually unconstrained by capacity in the Third Stage, so the result matches that in the Comparison Model.

4B. Second Example

Assume firm 1 has perfect information ($p_1 = 1$) while firm 2 has no information ($p_2 = 0.5$). Recall that in the Comparison Model, for these parameter values the subgame perfect equilibrium never involved information sharing. In the following statement of results, we restrict our attention to values of the cost of capacity, c, at which chosen capacities are not always binding in the Third Stage.

Here the existence of an equilibrium involving information sharing is not so pervasive as in the first example, but is still robust. To indicate what we mean by robust, consider the following special case. The condition (H-d)/(L-d) = (1+t)/t is added to allow us to graph the result, and is not essential for the qualitative nature of the result. (This is the largest (H-d)/(L-d) ratio at which, independent of the cost of capacity, for any realization of the intercept and signals, the Third Stage equilibrium quantities always leave A - Bq_1 - Bq_2 nonnegative.) The other added restrictions (B=1 and L=1) are merely normalizations which play no substantive role.

Result 2: Let p_1=1, p_2=0.5, B=1, L=1 and (H-d)/(L-d) = (1+t)/t. For this special case, the shaded region in Figure 2 indicates the combinations of the parameters, t and c/(1 - d), at which there is a subgame perfect equilibrium in which the second stage choice is information sharing. (See the Appendix for details of this result.)

Figure 2 Here

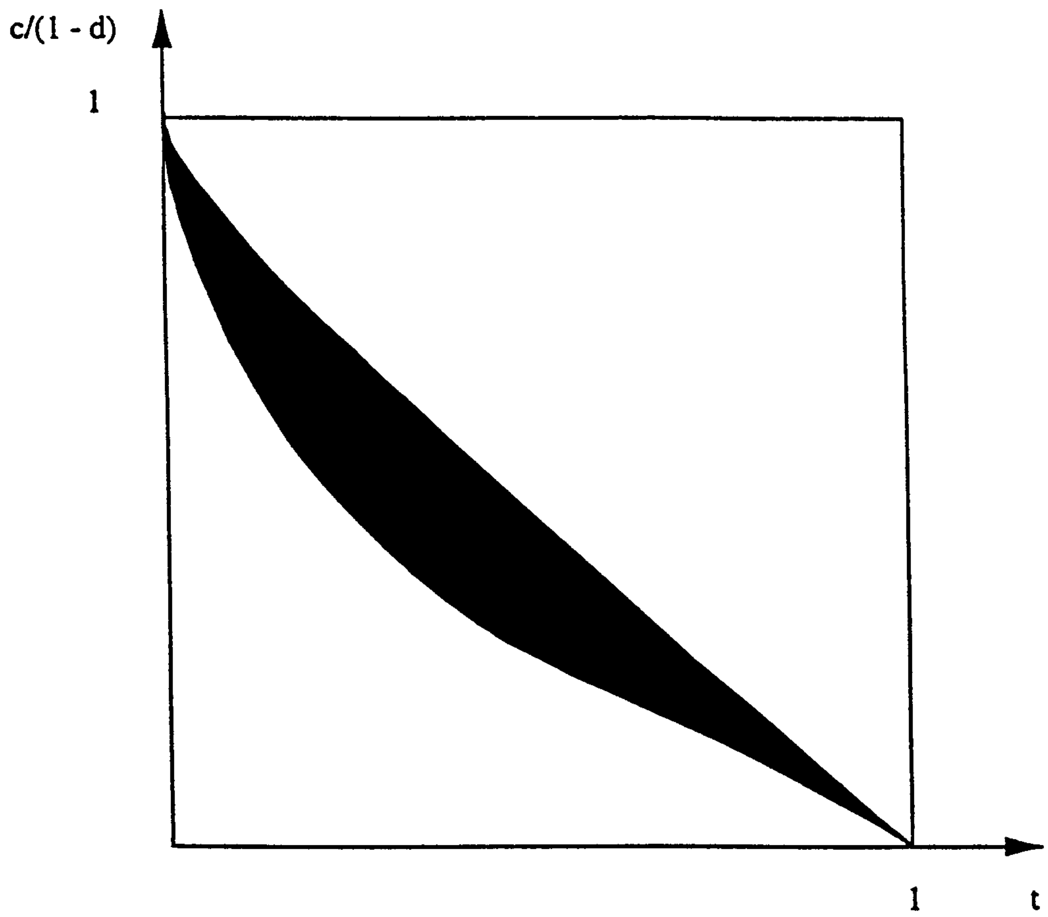
For every t value satisfying $0 < t < 1$, there are $c/(1 - d)$ values such that an equilibrium involves information sharing, with mid-level t values leading to the largest range of corresponding $c/(1 - d)$ values. (With the given parameter values, $c/(1 - d) \geq 1$ corresponds to the uninteresting case in which the firms are always capacity constrained.)

4C. Disclosure

In the context of the Second Example, it is easy to examine the impact of a capacity choice stage on the incentive to disclose information. By information disclosure we mean that in the Second Stage, each firm unilaterally decides whether the other firm will receive both signals. By deciding to disclose its information, the firm commits to reveal its information without any *quid pro quo* from the other firm. Also, the firm receiving the disclosure cannot commit not to use the revealed information. (A forward induction argument might be used to justify this assumption. By having disclosed its information, the disclosing firm is indicating that it will play its part in a Third Stage equilibrium in which the other firm has received both signals.) With $p_2 = 0.5$, firm two has no information to disclose, so its disclosure decision is irrelevant. Firm one will receive perfect information, so its disclosure decision is relevant.

How do the incentives to share information compare to the incentives to disclose information? With $p_1 = 1$ and $p_2 = 0.5$, from firm one's perspective, sharing and disclosure are identical: firm one wants to disclose if and only if it would want to share information. From firm two's perspective, sharing information is the same as having firm one disclose its information. In the Comparison Model, firm two always wanted to share information, but that

Figure 2: Parameter Values with Sharing Equilibrium for Second Example



is no longer the case when there is a capacity choice stage. In the Second Example, information sharing sometimes does not occur because it is vetoed by firm two. But firm two cannot veto disclosure. To prevent disclosure it must have chosen a capacity in the First Stage such that firm one does not want to disclose. Thus the range of parameter values with information sharing in Result 2 understates the range of parameter values for which disclosure would occur.

Result 3: Let p_1=1, p_2=0.5, B=1, L=1 and (H-d)/(L-d) = (1+t)/t. For this special case, there is a subgame perfect equilibrium in which the second stage choice is disclosure for any (t, c/(1 - d)) with 0 < t < 1 and 0 < c/(1 - d) < 1 that lies above the lower boundary of the shaded region in Figure 2. (See the Appendix for details of this result.)

5. Conclusion

By comparing the results from a three-stage game of capacity choices, information-sharing decisions, and output choices, with those from a corresponding two-stage game of information-sharing decisions and output choices (without capacity choices or capacity constraints), we have shown that the existence of a capacity-choice stage may reverse the incentives to share information. Though our results are presented in the form of two special examples, they are much more robust.

The intuition is most easily understood when capacities are exogenously fixed rather than choice variables: When set at certain levels, the capacity constraints can reduce one of the profit-reducing effects of information sharing. With information sharing, when both firms receive high signals they are confident that the demand intercept is large. In their resulting output choices, the capacity constraints are binding, and prevent the firms from being "too competitive." Without information sharing, when firms receive a high signal they are less sure that the demand intercept is large. In their resulting output choices, the capacity constraint does not bind as often or with as much impact. Thus, for certain exogenous capacity levels, information sharing becomes relatively more attractive to the firms than it would have been without the capacity constraints. We have shown that such capacity levels are sometimes, but not always, endogenously chosen in the equilibria of the three-stage game.

Appendix

This appendix contains the proofs for the Second Example, Disclosure, the First Example, and the Comparison Model, in that order. The level of detail diminishes in later parts of the appendix, as the method of analysis becomes familiar.

Second Example

Most of the analysis will use $p_1 = 1$, $p_2 = 0.5$, $(H-d)/(L-d) \leq (1+t)/t$ but general values for the other parameters. Near the end of the proof, the specific values $B = 1$, $L = 1$, and $(H-d)/(L-d) = (1+t)/t$ will be added.

Third Stage with information sharing: This section solves for the Third Stage equilibrium in quantities given K_1, K_2, and the assumption that information will be shared. Both firms have perfect information so the states can be solved separately. The Third Stage equilibrium quantities depend on the known, true intercept and whether either firm is capacity constrained. If I is the known realization of the intercept (I=H or I=L), then the quantity equilibrium given that realization takes one of three forms:

$q_1^I = q_2^I = (I - d)/3B$ if $K_i \geq (I - d)/3B$ for i = 1,2

$q_1^I = K_1$, $q_2^I = K_2$ if $K_i \leq (I - d - BK_j)/2B$ for i,j = 1,2, i≠j

$q_i^I = K_i$, $q_j^I = (I - d - BK_i)/2B$ if $K_i \leq (I - d - BK_j)/2B$ and $K_j \geq (I - d - BK_i)/2B$

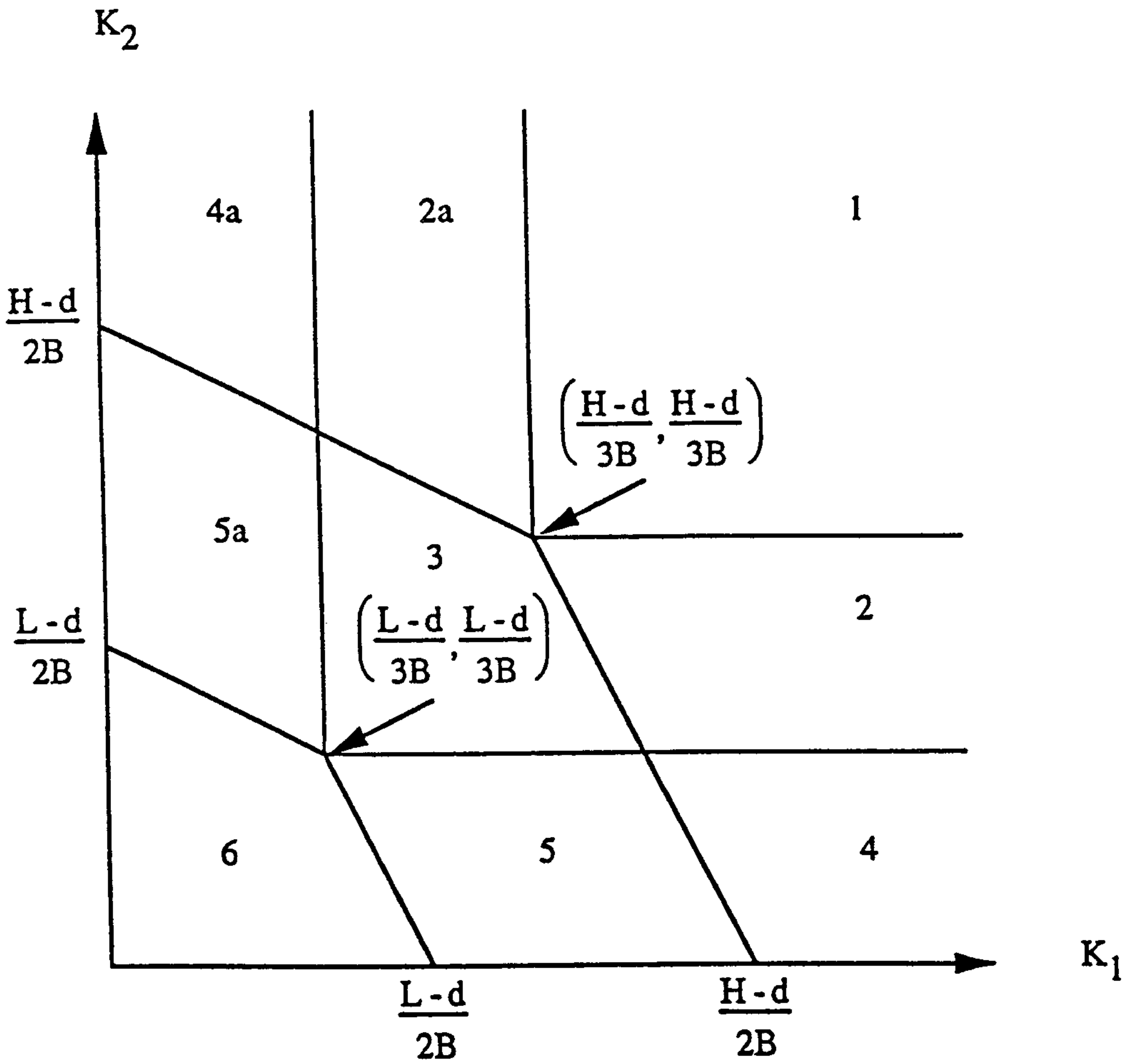
for i,j = 1,2, i≠j

Since each firm may be capacity constrained in neither state, in the high state, or in both states, there are nine cases, as indicated in Figure 3.

Figure 3 Here

The expected profits in each region are as follows.

Figure 3: Third Stage Sharing Equilibrium in Second Example



1. Neither is capacity constrained.

$\pi_i^S(K_1, K_2) = t(H-d)^2/9B + (1-t)(L-d)^2/9B - cK_i$ for i = 1,2

2. Firm 2 is capacity constrained in the high state.

$\pi_1^S(K_1, K_2) = t(H-d-BK_2)^2/4B + (1-t)(L-d)^2/9B - cK_1$

$\pi_2^S(K_1, K_2) = tK_2(H-d-BK_2)/2 + (1-t)(L-d)^2/9B - cK_2$

3. Both firms are capacity constrained in the high state.

$\pi_1^S(K_1, K_2) = tK_1(H-d-BK_1-BK_2) + (1-t)(L-d)^2/9B - cK_1$

$\pi_2^S(K_1, K_2) = tK_2(H-d-BK_1-BK_2) + (1-t)(L-d)^2/9B - cK_2$

4. Firm 2 is capacity constrained in both states.

$\pi_1^S(K_1, K_2) = t(H-d-BK_2)^2/4B + (1-t)(L-d-BK_2)^2/4B - cK_1$

$\pi_2^S(K_1, K_2) = K_2(tH + (1-t)L-d-BK_2)/2 - cK_2$

5. Firm 2 is capacity constrained in both states while firm 1 is capacity constrained in the high state.

$\pi_1^S(K_1, K_2) = tK_1(H-d-BK_1-BK_2) + (1-t)(L-d-BK_2)^2/4B - cK_1$

$\pi_2^S(K_1, K_2) = K_2(2tH+(1-t)L-d-2tBK_1-(1+t)BK_2)/2 - cK_2$

6. Both firms are capacity constrained in both states.

$\pi_i^S(K_1, K_2) = K_i(tH+(1-t)L-d-BK_1-BK_2) - cK_i$ for i = 1,2.

For regions 2a, 4a, and 5a, just switch the roles of the two firms in regions 2, 4, and 5 respectively.

Third Stage without information sharing: This section solves for the Third Stage equilibrium in quantities given K_1, K_2, and the assumption that information will not be shared. Firm 1 has perfect information, and chooses q_1^H and q_1^L while firm 2 gets no information from its signal, and chooses q_2 (technically, firm 2 chooses q_2^H and q_2^L, but since the signals H and L provide no information, the optimal choices will always have $q_2^H = q_2^L$). Since firm 2 is either capacity constrained or not, there are only six regions, as indicated in Figure 4.

Figure 4 Here

The Third Stage equilibrium quantities and expected profits in each region are as follows.

1. Neither firm is capacity constrained.

$q_1^H = ((3-t)H-(1-t)L-2d)/6B$

$q_1^L = ((2+t)L-tH-2d)/6B$

$q_2 = (tH+(1-t)L-d)/3B$

$\pi_1^{NS}(K_1, K_2) = t((3-t)H-(1-t)L-2d)^2/36B+(1-t)((2+t)L-tH-2d)^2/36B - cK_1$

$\pi_2^{NS}(K_1, K_2) = (tH+(1-t)L-d)^2/9B - cK_2$

2. Firm 1 is capacity constrained in the high state.

$q_1^H = K_1$

$q_1^L = ((1+t)L-tH-d+BtK_1)/(3+t)B$

$q_2 = (2tH+(1-t)L-(1+t)d-2BtK_1)/(3+t)B$

$\pi_1^{NS}(K_1, K_2) = tK_1((3-t)H-(1-t)L-2d-BK_1(3-t))/(3+t)+(1-t)((1+t)L-tH-d+BtK_1)^2/B(3+t)^2 - cK_1$

$\pi_2^{NS}(K_1, K_2) = (2tH+(1-t)L-(1+t)d-2BtK_1)^2/B(3+t)^2 - cK_2$

3. Firm 1 is capacity constrained in both states.

$q_1^H = q_1^L = K_1$

$q_2 = (tH+(1-t)L-d-BK_1)/2B$

$\pi_1^{NS}(K_1, K_2) = K_1(tH+(1-t)L-d-BK_1)/2 - cK_1$

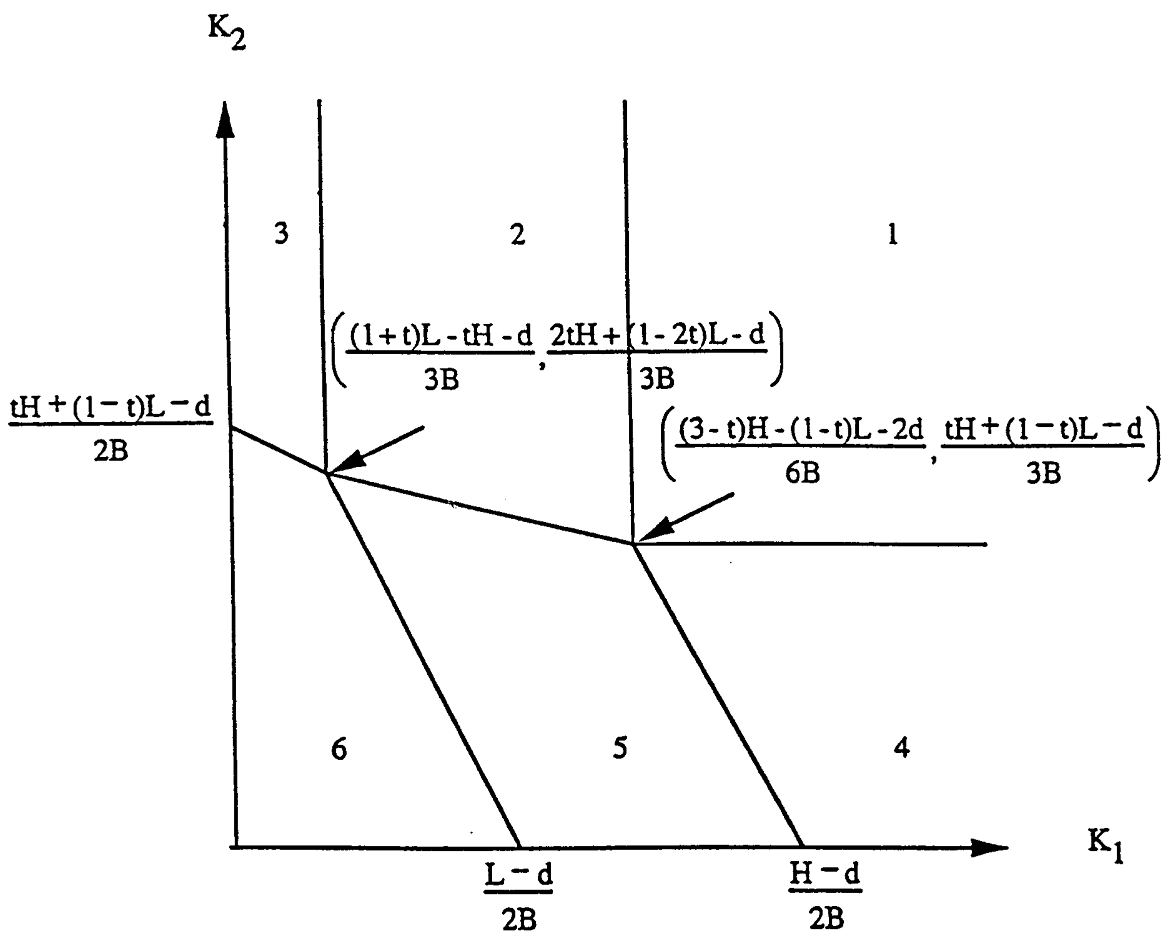
$\pi_2^{NS}(K_1, K_2) = (tH+(1-t)L-d-BK_1)^2/4B - cK_2$

4. Firm 2 is capacity constrained.

$q_1^H = (H-d-BK_2)/2B$

$q_1^L = (L-d-BK_2)/2B$

Figure 4: Third Stage Nonsharing Equilibrium in Second Example



$q_2 = K_2$

$\pi_1^{NS}(K_1, K_2) = t(H\text{-}d\text{-}BK_2)^2/4B + (1\text{-}t)(L\text{-}d\text{-}BK_2)^2/4B - cK_1$

$\pi_2^{NS}(K_1, K_2) = K_2(tH+(1\text{-}t)L\text{-}d\text{-}BK_2)/2 - cK_2$

5. Firm 2 is capacity constrained and firm 1 is capacity constrained in the high state.

$q_1^H = K_1$

$q_1^L = (L\text{-}d\text{-}BK_2)/2B$

$q_2 = K_2$

$\pi_1^{NS}(K_1, K_2) = tK_1(H\text{-}d\text{-}BK_1\text{-}BK_2)+(1\text{-}t)(L\text{-}d\text{-}BK_2)^2/4B - cK_1$

$\pi_2^{NS}(K_1, K_2) = K_2(2tH+(1\text{-}t)L\text{-}(1+t)d\text{-}2tBK_1\text{-}(1+t)BK_2)/2 - cK_2$

6. Both firms are always capacity constrained.

$q_1^H = q_1^L = K_1$

$q_2 = K_2$

$\pi_1^{NS}(K_1, K_2) = K_1(tH+(1\text{-}t)L\text{-}d\text{-}BK_1\text{-}BK_2) - cK_1$

$\pi_2^{NS}(K_1, K_2) = K_2(tH+(1\text{-}t)L\text{-}d\text{-}BK_1\text{-}BK_2) - cK_2$

Note with $(H\text{-}d)/(L\text{-}d) \leq (1+t)/t$, as we have assumed, the formulas lead to nonnegative quantities in each region. For $(H\text{-}d)/(L\text{-}d) > (1+t)/t$ some of the formulas would need to change to reflect the nonnegativity constraints.

Equilibrium conditional on sharing: Since we are interested in overall equilibria in which information sharing occurs, in this section we will skip the Second Stage and solve the First Stage assuming the firms decide to share information in the Second Stage. After finding these "sharing equilibria," in the next section we will determine parameter values for which these "equilibria" are true equilibria of the actual three stage game.

Assume the firms will share information in the Second Stage independent of the capacity levels. Now consider the First Stage. Since the problem is symmetric, we will determine the optimal K_2 given K_1 only. Within each of the nine regions, for each value of K_1, π_2^S is concave in K_2. Ignoring the boundaries of the regions, the list below indicates the solutions to the first-order-conditions for the problem of maximizing the profit formula for π_2^S within each region with respect to K_2 (or, if the profit formula is everywhere declining in K_2, 0 is listed as the solution).

1. 0

2. (tH-td-2c)/2Bt

3. $(tH-td-c-BtK_1)/2Bt$

4. (tH+(1-t)L-d-2c)/2B

5. $(2tH+(1-t)L-(1+t)d-2c-2BtK_1)/2B(1+t)$

6. $(tH+(1-t)L-d-c-BK_1)/2B$

2a. 0

4a. 0

5a. $(tH-td-c-BtK_1)/2Bt$

The actual maximizer within each region is given by the solution above if that capacity level lies within the region. If not, the maximizer is the closest boundary point of the region with the same value of K_1. Comparing the “regional maximizers” yields the best response for firm 2, $K_2(K_1)$.

Next combine the best responses, $K_2(K_1)$ and $K_1(K_2)$, to find a Nash equilibrium in capacities conditional on sharing in the Second Stage. For each value of c, there is a unique "sharing equilibrium," and it is symmetric. There are three cases.

1. For $c \geq tH+(1-t)L-d$, $K_1^* = K_2^* = 0$, and profit is zero.

2. For $tH+(1-t)L-d>c\geq t(H-L)$, $K_1^* = K_2^* = (tH+(1-t)L-d-c)/3B$ and each firm's profit is $(tH+(1-t)L-d-c)^2/9B$. Both firms are always capacity constrained.

3. For $t(H-L)>c>0$, $K_1^* = K_2^* = (tH-td-c)/3Bt$ and each firm's profit is $(tH-td-c)^2/9Bt + (1-t)(L-d)^2/9B$. Both firms are capacity constrained in the high but not the low state.

Checking whether "sharing equilibria" are true equilibria: For $c \geq t(H-L)$, the "equilibria" assuming nonsharing in the Second Stage are identical to those assuming sharing. The firms are always capacity constrained, and there is no interesting information-sharing question. We now turn to the case $t(H-L)>c>0$, and introduce the special parameter assumptions B=1, L=1, and $(H-d)/(L-d) = (1+t)/t$ (so $H=1+(1-d)/t$). With these parameter values, the case becomes $1-d>c>0$. Given the "sharing equilibrium," which for these parameter value is $K_1^* = K_2^* = ((1-d)(1+t)-c)/3t$ with each firm's profit $((1-d)(1+t)-c)^2/9t + (1-t)(1-d)^2/9$, all that remains is to show that, in the First Stage, neither firm wants to change capacity to a level at which nonsharing would be the Second Stage outcome. In Figure 3, as c varies, these "equilibria" trace out the diagonal of region 3. In Figure 4, these points fall in regions 2 and 5.

First consider firm 2. Given K_1 at one of these "equilibria," varying K_2 leads to points in regions 2, 5, and possibly 6 (if c is sufficiently large, so that the K_1 value is sufficiently small) of Figure 4. At these points, $\pi_2^{NS}(K_1, K_2)$ is decreasing in K_2 within region 2 and increasing in K_2 within region 6, and the K_2 value that maximizes π_2^{NS} given K_1 lies in the interior or along the top boundary of region 5. These maximizing values for K_2 (written as a function of c rather than K_1) take two forms: if $c>(1-t)(7+t)(1-d)/8(2+t)$, $K_2 = ((1-d)(7+t)-4c)/6(1+t)$ with $\pi_2^{NS} = ((1-d)(7+t)-4c)^2/72(1+t)$; if c **(1-t)(7+t)(1-d)/8(2+t), K** $_2 = ((1-d)(7+t)+2c)/3(3+t)$ with $\pi_2^{NS} = ((1-d)(7+t)+2c)((1-d)(7+t)-(7+3t)c)/9(3+t)^2$. Comparing this best nonsharing profit to the "sharing equilibrium" profit, firm 2 does not want to deviate from the "sharing equilibrium" if and only if $c/(1-d)\leq 1-t/2-\sqrt{t(1+t)/8}$.

Now consider possible deviations by firm 1. For $(1-t)/(1+t) \le c/(1-d) < 1$, the "sharing equilibrium" is in nonsharing region 5, and given K_2^* and assuming nonsharing in the Second Stage, the best K_1 from regions 4, 5, and 6 is the sharing equilibrium K_1^*. At (K_1^*, K_2^*), $\pi_1^S - \pi_1^{NS} = (1-t)(1-d)^2(4-(3-(1+t)/t+c/t(1-d))^2)/36$ which is positive for $c/(1-d)<1$, which holds here.

We must still check whether firm 1 would prefer to deviate to nonsharing regions 1, 2, or 3. When $c/(1-d)>(1-t)$, K_2^* is too low for any point in those regions to be feasible for firm 1, so firm 1 will not deviate from the "sharing equilibrium." Now consider $c/(1-d) \le (1-t)$, so parts of regions 1, 2, and 3 are available. For $c/(1-d)>4t/3(3+t)$, the best K_1 from these regions is in the interior of 2, with $K_1^{**} = (3+t)^2(1-c)(1-d)/2t(9-t)$. The corresponding profit difference is $\pi_1^S(K_1^*, K_2^*) - \pi_1^{NS}(K_1^{**}, K_2^*) = ((1-d)^2/36t(9-t))((c/(1-d))^2(-45-58t-9t^2)+(8(1-d))(90+44t+26t^2)-(45-50t+21t^2))$ which is positive (within this region) when $c/(1-d)>(45+22t+13t^2-2\sqrt{t(9-t)(45+94t+5t^2)})/(45+58t+9t^2)$.

For $c/(1-d) \le 4t/3(3+t)$, the best K_1 from regions 1, 2, and 3 is at the boundary between regions 1 and 2, at $K_1^{**} = (1-d)(3+t)/6t$. The corresponding profit difference is $\pi_1^S(K_1^*, K_2^*) - \pi_1^{NS}(K_1^{**}, K_2^*) = ((1-d)^2/36t)(4(c/(1-d))^2 + 2(c/(1-d))(5-t)+5(1-t))$ which is positive (within this region) when $c/(1-d)>(-5+t+\sqrt{45-30t+t^2})/4$.

Combining all the observations, we obtain Figure 2 in the text. The upper boundary is given by the condition for firm 2: $c/(1-d) \le 1-t/2-\sqrt{t(1+t)/8}$. The lower boundary is given by the condition for firm 1 deviating to a point in regions 1, 2, and 3: $c/(1-d) \ge (45+22t+13t^2-2\sqrt{t(9-t)(45+94t+5t^2)})/(45+58t+9t^2)$ when $c/(1-d)>4t/3(3+t)$ [this is approximately when $t<0.58$] and $c/(1-d) \ge (-5+t+\sqrt{45-30t+t^2})/4$ when $c/(1-d) \le 4t/3(3+t)$.

Disclosure

The disclosure result follows from the Second Example with these additional observations. Firm 1 wants to disclose if and only if it prefers to share information, i.e., if and only if (t, c/(1-d)) is above the bottom boundary in Figure 2. Since disclosure is unilateral, to overturn the "equilibrium" firm 2 must choose a K_2 in the First Stage such that firm 1 does not want to disclose at (K_1^*, K_2) [since K_2^* is best for firm 2 conditional on K_1^* and sharing/disclosure occurring in the Second Stage] and the profit for firm 2 without disclosure at (K_1^*, K_2) is better than its profit with disclosure at (K_1^*, K_2^*). Among the K_2 that move (K_1^*, K_2) to regions 5 or 6 in Figure 3, the one that maximizes π_2^{NS} is $K_2 = (1-d)/3$, at which $\pi_2^{NS} < \pi_2^S(K_1^*, K_2^*)$. At all K_2 that move (K_1^*, K_2) to region 2 in Figure 4, $\pi_2^{NS} < \pi_2^S(K_1^*, K_2^*)$. At all remaining K_2, firm 1 wants to disclose. Thus disclosure occurs in equilibrium at every (t, c/(1-d)) above the bottom boundary in Figure 2.

First Example

The initial analysis uses $p_1 = p_2 = p$ but allows a general t. This allows us to use the results for the Comparison Model as well. We will use the following notation: $\mu = tH+(1-t)L-d$, $D = H-L$, and $R = t(1-t)(2p-1)$. Let P_S be the probability that a firm has information S. For the nonsharing case S = H or S = L, and $P_H = 1-p-t+2tp$, $P_L = p+t-2tp$. For the sharing case, S = HH, HL, LH, or LL, and $P_{HH} = tp^2 + (1-t)(1-p)^2$, $P_{HL} = P_{LH} = p(1-p)$, $P_{LL} = t(1-p)^2 + (1-t)p^2$.

Recall the corresponding E^S for firm i is the expected value for the realization of $A-Bq_j$ given information S. We impose sufficient assumptions so that, in the relevant region, both firms will always have non-zero outputs in the Third Stage equilibrium in outputs. (The conditions are noted at the end of each discussion of the Third Stage.) Thus, in the relevant region, the best response for firm i given information S is $q_i^S = \text{minimum}\{K_i, (E^S - d)/2B\}$with expected payoff, conditional on S, of $(E^S - d - c - BK_i)K_i$ if the firm is capacity constrained and

$(E^S - d)^2/4B - cK_i$ if it is not capacity constrained. The overall expected profit is the weighted sum of these conditional expected profits, with weights P_S for information S.

For the nonsharing case, for firm $i \neq j$,

$E^H - d = \mu + (DR/P_H) - B((P_{HH}/P_H)q_j^H + (P_{HL}/P_H)q_j^L)$

$E^L - d = \mu - (DR/P_L) - B((P_{LH}/P_L)q_j^H + (P_{LL}/P_L)q_j^L)$.

For the sharing case, for firm $i \neq j$,

$E^{HH} - d = \mu + (DR/P_{HH}) - Bq_j^{HH}$

$E^{HL} - d = \mu - Bq_j^{HL}$

$E^{LH} - d = \mu - Bq_j^{LH}$

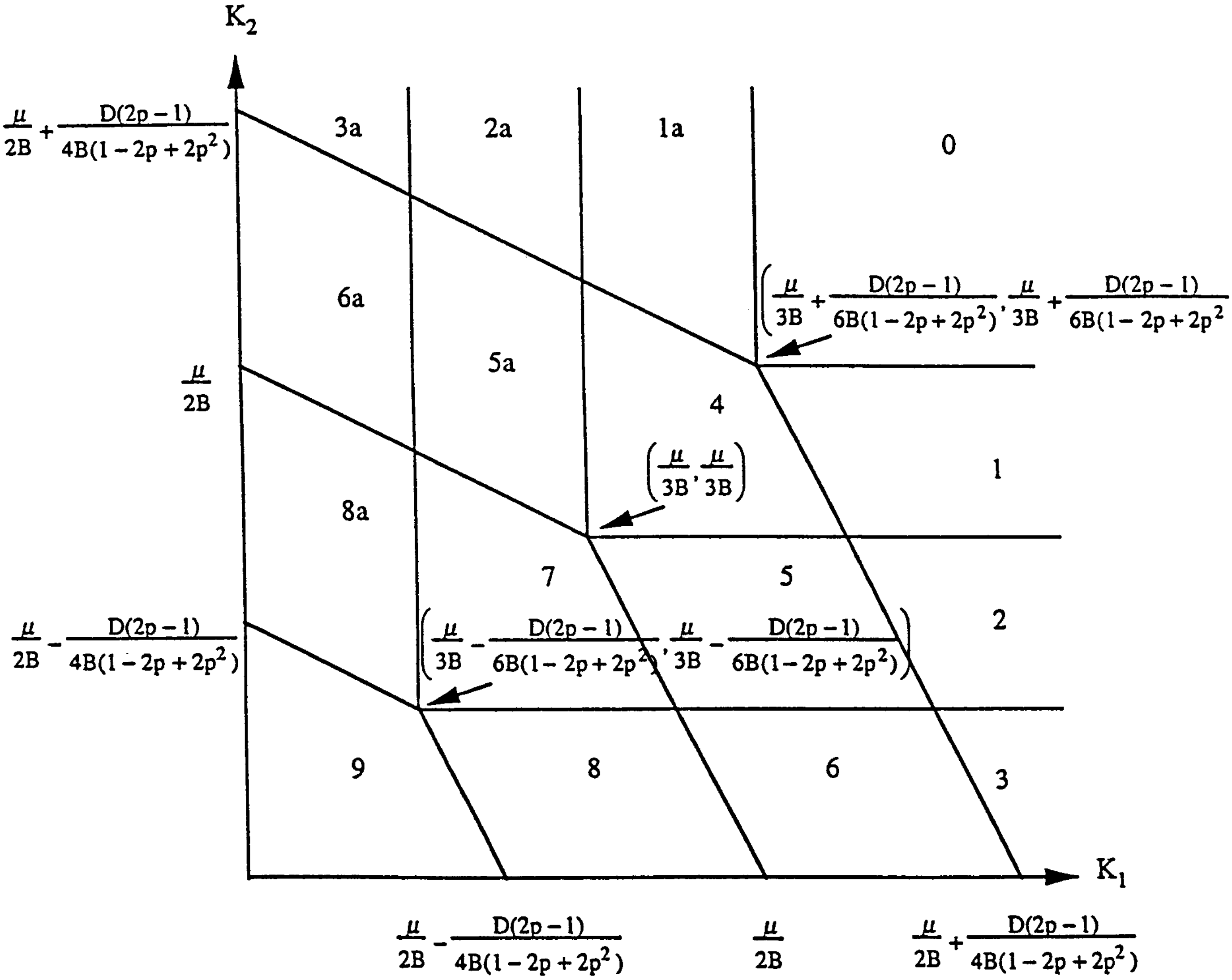
$E^{LL} - d = \mu - (DR/P_{LL}) - Bq_j^{LL}$

Third Stage with information sharing: We now impose the condition t = 0.5. Figure 5 indicates the regions in which the capacity constraint is binding in the Third Stage equilibrium with information sharing. For firm 1: in regions 0, 1, 2, and 3 capacity is not binding; in regions 1a, 4, 5, and 6 capacity is binding for S = HH; in regions 2a, 5a, 7, and 8 capacity is binding for S = HL, S = LH, and S = HH; in regions 3a, 6a, 8a, and 9 capacity is always binding. The situation is symmetric for firm 2 (i.e., switch the roles of regions 1a and 1 etc.).

Figure 5 Here

The Third Stage equilibrium quantities with sharing are as follows. In each region, for each S, there are three cases. If both firms are capacity constrained given S, then $q_i^S = K_i$ i = 1,2. If one firm, say firm 2, is capacity constrained but the other isn't, given S, then $q_2^2 = K_2$ and $q_1^S = (E^{S*} - d)/2B$ where E^{S*} is E^S evaluated at $q_2^S = K_2$. If neither firm is capacity constrained given S, then $q_1^S = q_2^S = X/3B$ where $x = \mu$, if S = HL or S = LH, $x = \mu +$

Figure 5: Third Stage Sharing Equilibrium in First Example



(DR/P_{HH}) if S = HH, and x = μ - (DR/P_{LL}) if S = LL. (We assume the parameters are such that $Bq_1^{HH} + Bq_2^{HH} < L$, so all formulas are correct.)

Third Stage without information sharing: Again with t = 0.5, Figure 6 indicates the regions in which the capacity constraint is binding in the Third Stage equilibrium without information sharing. For firm 1: in regions 0, 1, and 2 capacity is not binding; in regions 1a, 3, and 4 capacity is binding for S = H; in regions 2a, 4a, and 5 capacity is always binding. The situation is symmetric for firm 2 (i.e., switch the roles of regions 1a and 1 etc.).

Figure 6 Here

In regions 2, 2a, 4, 4a, and 5 the Third Stage equilibrium quantities with nonsharing can be found as in the sharing case since at least one firm is always capacity constrained. In the other regions, $q_i^H \neq q_i^L$ for i = 1,2 so both outputs enter into the determination of all non-capacity-constrained outputs of the other firm. The non-capacity quantities in the remaining regions are as follows.

In region 0,

$$q_1^H = q_2^H = \frac{m}{3B} + \frac{D(2p-1)}{2B(3-4p+4p^2)}$$

$$q_1^L = q_2^L = \frac{m}{3B} - \frac{D(2p-1)}{2B(3-4p+4p^2)}$$

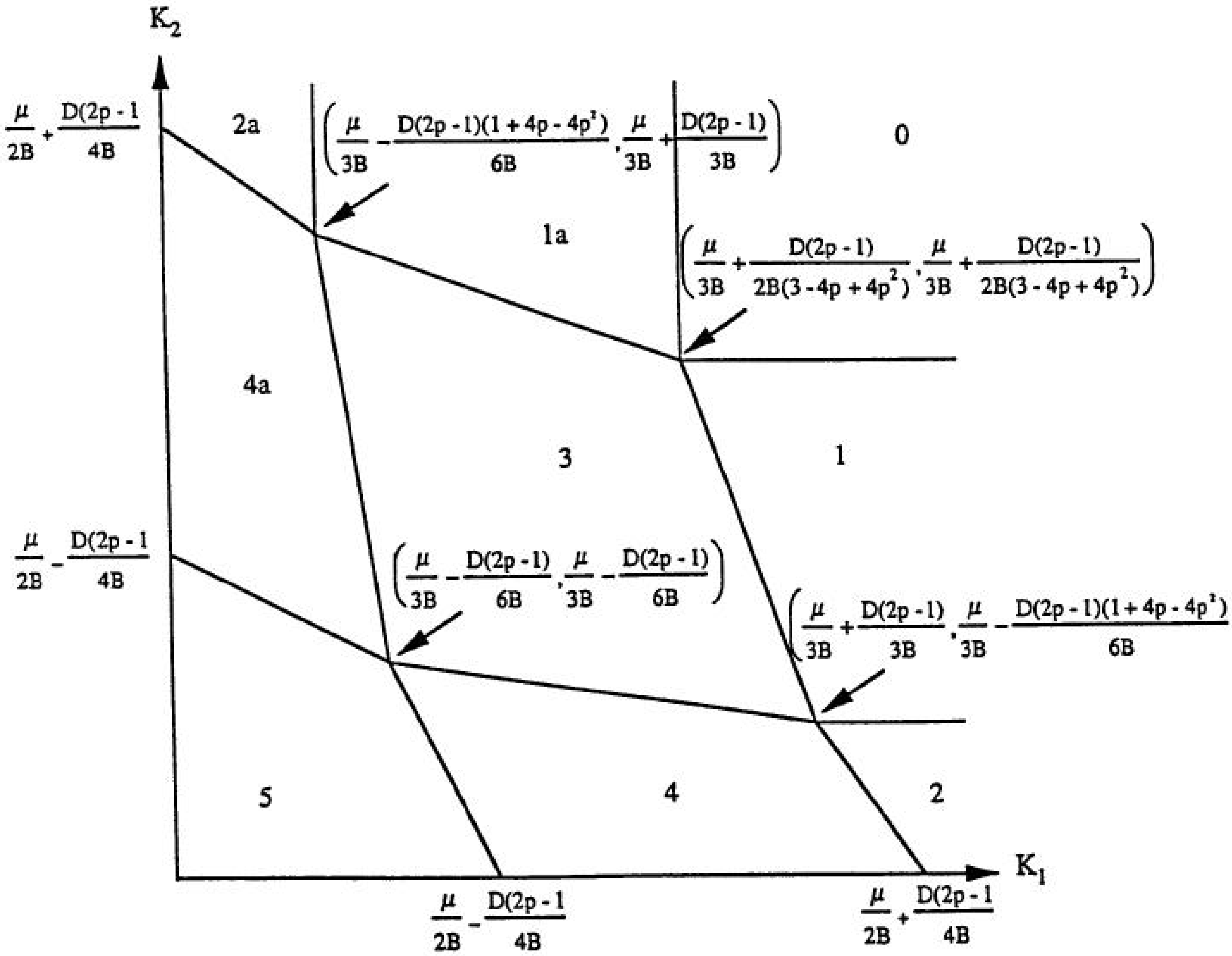
In region 1,

$$q_1^H = \frac{2m(3+2p-10p^2+16p^3-8p^4) - 2BK_2(3-2p-6p^2+16p^3-8p^4) - 3D(2p-1)(1+2p-2p^2)}{4B(3+4p-12p^2+16p^3-8p^4)}$$

$$q_1^L = \frac{2m(1+3p-7p^2+8p^3-4p^4) - 2BK_2 p(5-9p+8p^2-4p^3) - D(2p-1)(1+p-p^2)}{2B(3+4p-12p^2+16p^3-8p^4)}$$

$$q_2^L = \frac{2m + 8BK_2 p(1-p)(1-2p+2p^2) - D(2p-1)(1+4p-4p^2)}{2B(3+4p-12p^2+16p^3-8p^4)}$$

Figure 6: Third Stage Nonsharing Equilibrium in First Example



For region 1a, switch the roles of 1 and 2 from region 1.

In region 3,

$$q_1^L = \frac{2m - D(2p-1)}{2B(3-2p+2p^2)} - \frac{2(1-p)p(-K_1 + 2K_2 + 2K_1 p - 2K_1 p^2)}{3+4p-8p^2+8p^3-4p^4}$$

$$q_2^L = \frac{2m - D(2p-1)}{2B(3-2p+2p^2)} - \frac{2(1-p)p(2K_1 - K_2 + 2K_2 p - 2K_2 p^2)}{3+4p-8p^2+8p^3-4p^4}$$

(We assume the parameters are such that $Bq_1^H + Bq_2^H < L$, so all formulas are correct.)

Equilibrium conditional on sharing: Continuing with t = 0.5, as in the details of the Second Example, we now solve the First Stage assuming the firms decide to share information in the Second Stage. For every c there is a symmetric "sharing equilibrium", which takes one of four forms.

1. For $c \geq \mu$, $K_1^* = K_2^* = 0$.

2. For $\mu > c \geq DR/P_{LL}$, $K_1^* = K_2^* = (\mu\text{-}c)/3B$

and both firms are always capacity constrained. This is in region 9 of Figure 5.

3. For $DR/P_{LL} > c \geq D(2p\text{-}1)/4$, $K_1^* = K_2^* = (\mu/3B) - (2c/3B(1+2p-2p^2)) + (D(2p-1)/6B(1+2p-2p^2))$ and both firms are capacity constrained except when S = LL. This is in region 7 of Figure 5.

4. For $D(2p\text{-}1)/4 > c \geq 0$, $K_1^* = K_2^* = (\mu/3B) - (2c/3B(1-2p+2p^2)) + (D(2p-1)/6B(1-2p+2p^2))$ and both firms are capacity constrained only when S = HH. This is in region 4 of Figure 5. (At c = 0, the capacity constraint stops being binding even when S = HH.)

Checking whether "sharing equilibria" are true equilibria: Comparing Figures 5 and 6, the upper-right corner of regions 3 (5) in the nonsharing case lies in the interior of region 4 (7) of the sharing case. For $c \geq DR/P_{LL}$ the firms are always capacity constrained and there is no difference between the sharing and nonsharing cases. Consider $c < DR/P_{LL}$ with corresponding "sharing equilibrium" (K_1^*, K_2^*). Since the problem is symmetric, we need only check whether firm 1 would prefer to deviate. Since the initial (K_1^*, K_2^*) is a "sharing equilibrium," we need only check whether $\pi_1^{NS}(K_1, K_2^*) \leq \pi_1^S(K_1^*, K_2^*)$ for all $K_1 \geq 0$. For $Dp(2p-1)(1-p)/2(3-4p+4p^2) < c < DR/P_{LL}$, if firm 1 deviates to nonsharing, the other firm's capacity constraint is binding for at least one S, and no deviation improves the payoff of firm 1.

For $c < Dp(2p-1)(1-p)/2(3-4p+4p^2)$, at (K_1^*, K_2^*) both firms always have excess capacity for the Third Stage in the nonsharing case. By deviating to K_1 with no excess capacity, firm 1 obtains expected profit $\pi^{NS}(p, 0.5) - cK_1$ (where π^{NS} is the nonsharing profit in the Comparison Model). As c converges to 0, the profit in the "sharing equilibrium" converges to $\pi^S(p, 0.5)$ (again from the Comparison Model), which is less than $\pi^{NS}(p, 0.5)$ by the result for the Comparison Model. Thus the "sharing equilibrium" cannot be a true equilibrium if c is sufficiently near 0. In fact there is a strictly positive c* in this range such that the "sharing equilibrium" is a true equilibrium if and only if $c \geq c^*$.

Comparison Model

Let $p_1 = p_2 = p$. For both sharing and nonsharing, the quantity-setting equilibrium matches that in region 0 of the Third Stage in the First Example, and the equilibrium expected profits correspond to c = 0 in region 0 of the First Example. Then $\pi^S(p, t) - \pi^{NS}(p, t)$ has the same sign as $(-3/8) + (7p/4) - (15p^2/4) + 4p^3 - 2p^4 + (6 - 27p + 39p^2 - 24p^3 + 12p^4)z + (-18 + 144p - 432p^2 + 576p^3 - 288p^4)z^2$ where $z = (t - 0.5)^2$. For $0.5 < p < 1$ and $0 < t < 1$, this is positive if and only if (p, t) lies in the shaded region of Figure 1, which is symmetric about t = 0.5.

Let $p_1 = 1$, $p_2 = 0.5$. For both sharing and nonsharing the quantity-setting equilibrium matches that in region 1 of the Third Stage in the Second Example, and the equilibrium expected profits correspond to c = 0 in region 1 of the Second Example. Equilibrium expected profit is convex in output and the equilibrium mean output is the same for each firm whether sharing or not. Thus the preferred case is that in which the firm's output differs most between S = H and S = L. Since $q_2^H = q_2^L$ in the nonsharing case, firm 2 always prefers sharing. Since firm 2 has constant output in the nonsharing case, firm 1 is more responsive to the signal when nonsharing, and thus firm 1 always prefers nonsharing.

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References

Clarke, R., 1983, Collusion and the incentives for information sharing, *Bell Journal of Economics* 14, 383-394.

Gal-Or, E., 1985, Information sharing in oligopoly, *Econometrica* 53, 329-343.

Jin, J., 1992, Information sharing in oligopoly: A general model, mimeo Wissenschaftszentrum Berlin.

Kirby, A., 1988, Trade associations as information exchange mechanisms, *Rand Journal of Economics* 19, 138-146.

Li, L., 1985, Cournot oligopoly with information sharing, *Rand Journal of Economics* 16, 521-536.

Malueg, D. and S. Tsutsui, 1996, Duopoly information exchange: The case of unknown slope, *International Journal of Industrial Organization* 14, 119-136.

Novshek, W. and H. Sonnenschein, 1982, Fulfilled expectations Cournot duopoly with information acquisition and release, *Bell Journal of Economics* 13, 214-218.

Raith, M., 1996, A general model of information sharing in oligopoly, *Journal of Economic Theory* 71, 260-288.

Vives, X., 1984, Duopoly information equilibrium: Cournot and Bertrand, *Journal of Economic Theory* 34, 71-94.