WZB

WISSENSCHAFTSZENTRUM BERLIN
FÜR SOZIALFORSCHUNG

SOCIAL SCIENCE RESEARCH
CENTER BERLIN

Andreas Blume *
Paul Heidhues **

Private Monitoring in Auctions

* University of Pittsburgh
** WZB - Wissenschaftszenturm Berlin

SP II 2003 – 14

September 2003

ISSN Nr. 0722 – 6748

Research Area
Markets and Political Economy

Research Unit
Competitiveness and Industrial Change

Forschungsschwerpunkt
Markt und politische Ökonomie

Abteilung
Wettbewerbsfähigkeit und industrieller Wandel

Zitierweise/Citation:

Andreas Blume and Paul Heidhues, **Private Monitoring in Auctions**, Discussion Paper SP II 2003 – 14, Wissenschaftszentrum Berlin, 2003.

Wissenschaftszentrum Berlin für Sozialforschung gGmbH,
Reichpietschufer 50, 10785 Berlin, Germany, Tel. (030) 2 54 91 – 0
Internet: www.wz-berlin.de

ABSTRACT

Private Monitoring in Auctions*

by Andreas Blume and Paul Heidhues

We study collusion in repeated first-price auctions under the condition of minimal information release by the auctioneer. In each auction a bidder only learns whether or not he won the object. Bidders do not observe other bidders' bids, who participates or who wins in case they are not the winner. We show that for large enough discount factors collusion can nevertheless be supported in the infinitely repeated game. While there is a unique Nash equilibrium in public strategies, in which bidders bid competitively in every period, there are simple Nash equilibria in private strategies that support bid rotation. Equilibria that either improve on bid rotation or satisfy the requirement of Bayesian perfection, but not both, are only slightly more complex. Our main result is the construction of perfect Bayesian equilibria that improve on bid rotation. These equilibria require complicated inferences off the equilibrium path. A deviator may not know who has observed his deviation and consequently may have an incentive to use strategic experimentation to learn about the bidding behavior of his rivals.

Keywords: tacit collusion, repeated auctions, supergames, contagion, bid-rotation, trigger strategies.

JEL Classification: C73, D44

* We are grateful for comments made by seminar audiences at the Harvard/MIT Theory Workshop, the Universität Bielefeld, the University of Pittsburgh, the Midwest Theory Meetings and the SAET conference, and by Galit Ashkenazi-Golan, Glenn Ellison, Drew Fudenberg, Sergei Izmalkov, Dipjyoti Majumdar, Stephen Morris, and Michael Schwarz. Andreas Blume thanks the WZB and Paul Heidhues thanks the University of Pittsburgh for its hospitality.

ZUSAMMENFASSUNG

Privates Monitoring in Auktionen

Der Beitrag untersucht, inwieweit Bieter Kollusion, bzw. stillschweigende Abkommen, in wiederholten Erstpreisauktionen aufrecht erhalten können, in welchen der Auktionator alle Informationen zurückhält. Nach jeder Auktion lernt ein Bieter nur, ob er das Objekt gewonnen hat oder nicht. Ein Bieter kann weder die Gebote der anderen Bieter beobachten, noch kann er beobachten, welche Bieter an der Auktion teilgenommen haben und wer gewonnen hat - solange er nicht selbst das Objekt erhält. Wir zeigen, dass in dem unendlich wiederholten Spiel für hinreichend geduldige Bieter Kollusion möglich ist. Es existiert zwar ein eindeutiges Gleichgewicht in öffentlichen Strategien, in welchem die Bieter in jeder Periode kompetitiv bieten, aber es gibt einfache Nash-Gleichgewichte in privaten Strategien, die Bieterrotation durchsetzen. Wir zeigen auch, dass Bieterrotation das Ergebnis eines perfekt bayesianischen Gleichgewichtes sein kann. Nash-Gleichgewichte, die höhere erwartete Gewinne als Bieterrotation erzielen, sind nur ein wenig komplexer. Das Hauptergebnis ist die Konstruktion von (essentiell) perfekt bayesianischen Gleichgewichten, welche höhere Gewinne als Bieterrotation erzielen. Nach Abweichungen vom Gleichgewichtspfad, müssen die Bieter in diesen Gleichgewichten komplizierte Rückschlüsse auf das Verhalten Ihrer Wettbewerber ziehen. So weiß ein Bieter nach bestimmten Abweichungen nicht, ob diese von seinen Mitspielern beobachtet wurden, und hat ein Interesse daran, durch strategisches experimentieren das Bietverhalten seiner Rivalen kennenzulernen.

1 Introduction

Many resources are allocated through auctions and collusion in auctions is widespread.[1] Experimental evidence as well as theoretical arguments support the intuitive belief that collusion becomes harder if the auctioneer releases less information about bidders' behavior in the auction.[2] Thus - unless there are other benefits from information release - the auctioneer appears to have an incentive to suppress as much information as he can to fight collusion. We say that the auctioneer *withholds all information* if after each auction each bidder has only the information that cannot be concealed from him. In the case of the first-price auction of an indivisible object, which we consider in this paper, the information that cannot be concealed includes a bidder's own value, his bid, and whether or not he received the object. Perhaps somewhat surprisingly, we will show that even by withholding all information, the auctioneer cannot prevent collusion.

Collusion in first-price auctions with and without transfers has been analyzed by McAfee and McMillan [1992] using a mechanism design framework. They assume that both the identity and the bid of the winning bidder are publicly available information. To ensure adherence to the rules of the mechanism, they appeal to repeated game effects, without explicitly specifying a repeated game. Marshall and Marx [2002] study mechanisms that do not use information provided by the auctioneer.

We study the effect of limiting information release on collusion in a repeated-game environment. The same set of bidders repeatedly participates in a first-price auction for a single object. Valuations are drawn independently across bidders and time. The literature on repeated auctions, e.g. Aoyagi [2002a,2002b], Skrzypacz and Hopenhayn [2002], and Blume and Heidhues [2002] thus far has focused on studying perfect public equilibria under variety of assumptions on communication among and information available to bidders. It follows

[1]For example, Froeb [1988] points out that 81% of all Sherman Act cases filed by the Department of Justice between 1979 and 1988 involved auctions. Aoyagi [2002b] mentions that in 2001 alone, the Japan Fair Trade Commission (JFTC) issued warnings to 928 firms in thirty three collusion cases regarding government procurement auctions. In all cases the warning was based on allegation that firms "collaborated to predetermine a winning bidder." Porter and Zona [1993] discuss bid rigging in procurement auctions. Baldwin, Marshall and Richard [1997], Cassady [1967], Hendricks and Porter [1988], and Pesendorfer [1996] report evidence on the occurrence of collusion in auctions for timber, antiques, fish, wool, oil drainage leases, and school milk.

[2]See Dufwenberg and Gneezy [2002] and Marshall and Marx [2002].

from standard results on repeated games (see Fudenberg, Levine and Maskin [1994]) that in similar environments approximately efficient collusion, where bidders pay an approximately zero price and the good is allocated efficiently in every period, can be supported as a perfect public equilibrium if bidders are sufficiently patient and the auctioneer reveals all information that is available to him - i.e. all bids and the identities of the bidders who made them. In contrast, we have shown elsewhere (Blume and Heidhues [2002]) that the payoffs from perfect public equilibria are bounded away from the efficient frontier for repeated second-price auctions in which only the identity of each period's winner is observed. In the present paper, we permit bidders to condition behavior on information that is only privately observed.

In the extreme case, where the auctioneer withholds all information, it is easy to see that given a unique equilibrium in the stage game, no collusion is supported by perfect public equilibria. This raises the question of whether any collusion can be supported by *private equilibria,* i.e. perfect Bayesian equilibrium strategies that condition on more than only publicly available information.

The dynamic programming techniques used to analyze games with imperfect public monitoring such as in Abreu, Pearce and Stacchetti [1990] and Fudenberg, Levine and Maskin [1994] are only of limited use in games with imperfect private monitoring, such as ours. Indeed, sequential equilibria in repeated games with imperfect private monitoring are intricate and have only recently been studied more carefully. One cannot exploit the recursive structure of perfect public equilibria because off the equilibrium path continuation strategies may not be Nash or even correlated equilibria of the continuation game.

Recent contributions on sustaining cooperation through private monitoring typically focus on a repeated prisoners' dilemma in which each player observes a private signal of the action chosen by his rival. Within such an environment, Bhaskar and Obara [2002] and Ely and Välimäki [2002] derive folk theorems. The differences between our paper and the above literature are manifold: We consider an incomplete information stage game. The action space in our underlying stage game is infinite. Auction rules provide a natural link between the action profile and the monitoring technology in our environment. For example, a bidder who does not bid receives no signal of his rivals' behavior. The signal a player receives depends only on his bid and the highest rival bid. This implies that a bidder cannot use the signaling technology to distinguish between rivals. The monitoring technology hence distinguishes our paper from the above literature in which all distinct action profiles have distinct signal distributions.

Our paper is perhaps most closely related to the literature on private monitoring games with partial observability, e.g. Kandori [1992], Ellison [1994] and Ben-Porath and Kahneman [1996]. A leading example, studied in Kandori's seminal paper on social norms, is a random matching game in which each player only observes what his opponents have done to him. The main difference between our setting and the above literature is that partial observability is not a feature of the exogenous environment (matching technology) in our model. Rather, we choose strategies such that partial observability of payoff relevant deviations is ensured. That is in equilibrium any deviation that yields a short term benefit is detected by some rival with probability one. Once observed, the deviation initiates contagion as in Kandori [1992] and Ellison [1994].

In the environment we study there is a stark contrast between the set of public equilibria and the set of private equilibria. On the one hand, we show that the repeated game has a unique public Nash equilibrium regardless of the discount factor. On the other hand, with sufficiently patient bidders bid rotation can be supported as a private Nash equilibrium in simple trigger strategies that induce contagion. Along similar lines one can construct Nash equilibria that improve on bid rotation by permitting some players to choose to "trade places" with other players in periods in which their valuation is relatively high. If it is the case that the payoff from bid rotation exceeds the payoff from competitive bidding, one can construct perfect Bayesian equilibria that support bid rotation. The construction relies on a similar contagion argument as above, except that after observing a deviation bidders revert to competitive bidding assuming that they are in an environment where they are the last ones to have observed a deviation. Our central result is that improvements on bid rotation can be supported by perfect Bayesian equilibria as well. Here the construction is more subtle because after some histories bidders may find themselves in a situation where they are uncertain about whom they bid against. Then they face a dynamic programming problem similar to a monopolist who experiments with prices in order to learn about an uncertain demand function, as for example in Rothschild [1974], McLennan [1984], Aghion, Bolton, Harris and Jullien [1991], and Mirman, Samuelson and Urbano [1993].

Our observation that in a repeated auction with limited observability, private equilibria can lead to efficiency improvements over the set of public equilibria complements a similar observation made by Kandori and Obara [2000] for repeated partnership games. Radner, Myerson and Maskin [1986] had shown that for an example of such a game the folk theorem fails. Among other things, Kandori and Obara show that for the same example private strategies may attain efficiency.

2 Auctions Without Public Information Release

We consider the following repeated auction environment with $n \geq 3$ bidders. Denote the set of bidders by N. Bidders participate in an infinite sequence of auctions. Auctions are for separate single objects that are indivisible. While many of our results extend to other auction environments, we focus on first-price sealed-bid auctions. Before an auction in period t each bidder i's valuation v_{it} is drawn from a distribution F_i with expected value v_i^e. All these distributions have common support $[0, v^h]$, with continuously differentiable densities f_i that are bounded away from zero on $[0, v^h]$. Valuations are drawn independently across bidders and time. Under these assumptions, there exists a unique equilibrium in the stage game. In this equilibrium bidder i's bid function is a strictly monotonically increasing and differentiable function of v_{it}. All bid functions have the same range with the lowest bid being equal to the lowest valuation (see Athey [2001], Bajari [1997, 2001], Maskin and Riley [1996], Lebrun [1999]). For the repeated auction game, we assume that all bidders are risk neutral and have common discount factor δ. Hence bidder i's (average) expected payoff in the repeated auction equals

$$(1-\delta)E\left[\sum_{t=1}^{\infty} \delta^{t-1}(v_{it} - b_{it})1_{\{b_{it} \text{ is the winning bid in period } t\}}\right].$$

After each round bidders only observe whether they won that round or not. This structure and the auction rules are common knowledge.

A private history $h_{it} = (h_0, (v_{i,1}, b_{i,1}, w_{i,1}), \ldots, (v_{i,t-1}, b_{i,t-1}, w_{i,t-1}))$ begins with the null history ("nothing has happened yet") and further records for each of the first $t-1$ periods bidder i's valuation $v_{i,\tau}$, his bid $b_{i,\tau}$ and whether he won ($w_{i,\tau} = 1$) or lost ($w_{i,\tau} = 0$) in period τ. Denote by H_{it} the set of all such histories and define $H_i := \cup_{t=1}^{\infty} H_{it}$. A strategy $\sigma_i : H_i \to \{\text{no bid}\} \cup [0, \infty)$ for player i maps private histories into bids.

We start by observing that no collusion is possible if bidders rely on public strategies only.

Proposition 1 *There exists a unique public Nash equilibrium. In this equilibrium bidders bid competitively following every history.*

Proof: A bidder's behavior in any given period does not affect the public information in future periods. Hence, a public Nash equilibrium strategy must maximize current payoffs after every history. Therefore, a public Nash equilibrium strategy induces a stage-game equilibrium after every history. The result follows because in our environment there is a unique stage-game equilibrium. $\square$

The intuition for the above simple, yet strong, antifolk theorem is obvious: If there exists no public information then bidders cannot collude on the basis of public information alone.[3] It extends to other auction formats that have a unique stage game equilibrium. In an auction format in which there are multiple stage game equilibria, bidders relying on public strategies must play a stage game equilibrium in every period.

Next, we show that the bid-rotation outcome can always be sustained as a Nash equilibrium outcome if bidders use private strategies. The key insight is that along the equilibrium path of a bid-rotation equilibrium, the only temptation a bidder faces is to submit a bid when it is not his turn. If successful, however, this deviation will be observed with probability one by the bidder whose turn it is to obtain the good. Following this deviation, the betrayed bidder simply bids the highest possible value in all future periods and thereby eliminates all future benefits from participating in the auction. Hence, for high enough discount factors, no bidder wants to bid when it is not his turn. Observe that the partial observability of payoff relevant deviations is a property of the collusive scheme the bidders adopt rather than of the economic environment alone, which is the case in Kandori [1992].

[3]Note however that the time period is public information. In the second-price sealed-bid auction in which there exists a continuum of equilibria (see Blume and Heidhues [2002]) bidders could use the time period to coordinate on which stage game equilibrium to play. In particular, bidders could enforce bid rotation as a perfect public equilibrium by designating a winner for each time period who bids above the highest possible valuation while all other bidders refrain from bidding in that time-period.

Proposition 2 *There exists a Nash equilibrium that supports bid rotation for sufficiently high discount factors.*

Proof: Number bidders from 1 to n. Along the equlibrium path bidder j bids 0 in periods $\tau \equiv j \mod n$ and refrains from submitting a bid in all other periods. Any bidder who detects a deviation, i.e. does not win when it is his turn, bids v^h forever after. $\square$

While the above private strategies are simple, they rely on out-of-equilibrium threats that are not sequentially optimal. We will address this issue below when we show that bid rotation can be sustained as a perfect Bayesian equilibrium outcome.

Before doing so, however, observe that the above construction can be generalized to allow for a commonly known reserve price r as long as $v_i^e - r > 0$ for all bidders. This condition ensures that a bidder who does not know his realized value is willing to pay r for the right to receive the object. Along the equilibrium path, bidder i bids $r + \epsilon$ in periods $\tau \equiv i \mod n$ for some sufficiently small ϵ. In periods $\tau \equiv i + 1 \mod n$ he randomizes uniformly over the interval $[r, r+\epsilon]$, and he refrains from bidding otherwise. Any bidder who detects a deviation, i.e. does not win when it is his turn to win or wins when it is his turn to randomize over the interval $[r, r + \epsilon]$, bids v^h forever after.[4] Note that this construction also handles the case in which bidders are forced to submit a bid. Furthermore, it can be adopted to allow for randomly drawn reserve prices that are announced at the beginning of each period, as long as the expected future reserve price is low enough. Similarly, a scheme like this works if the reserve price is randomly drawn in each period and only announced after the auction.

We prove next that there exists a collusive scheme that ensures that bidders receive a higher payoff than the payoff from bid rotation. This result is perhaps somewhat surprising as McAfee and McMillan [1992] have shown that bid rotation is the optimal static collusive scheme in the absence of side-payments.[5] Bid rotation is an extreme collusive scheme in that it reduces bidders' payments to zero at the cost of any allocative efficiency from assigning the good to the bidder with the highest value. In a dynamic environment in which the winners'

[4]The randomization over the interval $[r, r+\epsilon]$, sometimes referred to as aggressive mixing, is the continuous action space analog to undercutting ones rival by the smallest bid increment. Its been used, for example, in Hirschleifer and Riley (1992).

[5]Indeed, McAfee and McMillan conjectured that this would also be true in a dynamic environment.

identity is announced, Skrzypacz and Hopenhayn [2002] have constructed a perfect public equilibrium in which bidders use continuation values as implicit side-payments in order to obtain a higher payoff than the payoff from bid-rotation.[6] The side-payment is established by ensuring that, in equilibrium, a bidder who obtains the good in the current period is less likely to win the object in future periods. In general, one may expect this to be considerably harder if bidders have to rely on private monitoring.

We solve this problem by starting with the above bid-rotation scheme and introducing a simple (bilateral) trading scheme that is mutually beneficial for the participants and retains the property that payoff relevant deviations are partially observable. The trading scheme works as follows: Select two bidders i and $i+1$. All bidders $j \neq i, i+1$ behave exactly as in the bid-rotation scheme. Furthermore, in all periods other than i and $i+1$, bidders i and $i+1$ behave as in the bid rotation scheme. In contrast to the bid-rotation scheme, however, bidder i gets the right to choose whether he wants the good in period i or in period $i+1$. In particular, if bidder i has a low value in period i he chooses to wait and obtain the good in period $i+1$ instead, which makes him better of. Bidder $i+1$ has the chance of getting the good in an earlier period, which makes him better of as he discounts the future.

To implement this trading scheme, bidder $i+1$ bids 0 in period i and bidder i bids slightly above zero if he chooses to take the good in period i rather than $i+1$. Obviously, a technical issue arises as there is no lowest bid above 0. To circumvent this problem, we permit a bidder to submit a bid $b = 0^+$, which is identical to a bid $b = 0$ except that it wins the object if the highest competing bid is zero. We view this as innocuous, because in reality every currency is discrete and we use continuity of possible bids as a modeling device to facilitate the analysis. As the fact that no lowest higher bid exists makes the analysis more complicated and in our view misleading, we extend the model to allow for bids to take the value 0^+.[7] If bidder i decided to trade places by not bidding in period i, then he bids 0

[6] Athey and Bagwell [2001] point out the usefulness of future market shares as implicit side-payments in an infinitely repeated Bertrand game. Their setup resembles a procurement auction with private observations of costs.

[7] Alternatively, one could view the tie-breaking rule as part of the equilibrium concept. This has been proposed by Jackson, Simon, Swinkels, and Zame [2002] for games with discontinuous payoffs such as ours. They argue that an endogenous tie-breaking rule allows one to interpret the idealization of a continuous action space as the limit of finer and finer discrete action spaces through ensuring the existence of best replies and, hence, equilibrium. In our case, a tie-breaking rule which specifies that bidder i gets the good if multiple bidders bid zero in period i would also allow one to use the proposed trading scheme in equilibrium.

in period $i+1$. Otherwise, bidder $i+1$ did not obtain the good in period i, and in this case bidder $i+1$ claims the good in period $i+1$ by bidding 0.

In the above construction, the only temptation a bidder faces is to bid for the good when it is not his turn. But any such deviation is detected within two periods with probability 1. (Consider for example a bidder $j \neq i, i+1$ who submits a bid in period i and wins. In case bidder i chose to trade and not bid in period i, this deviation is not detected immediately. In the next period, however, both bidder i and $i+1$ will bid 0 and expect to win. At least one of these two bidders loses and thereby detects that a deviation occurred.) Once a bidder detects a deviation, he simply bids v^h forever after and thereby eliminates all future benefits for all bidders. Thus any sufficiently patient bidder does not want to deviate.

The collusive scheme is just one simple example on how bidders can beat bid rotation. It is easy to improve on this scheme by repeatedly allowing for the above type of bilateral trade. Furthermore, it is easy to think of multilateral trades that may further increase efficiency.

Proposition 3 *There exists a Nash equilibrium in which each bidder's expected payoff exceeds the payoff from bid rotation for sufficiently high discount factors.*

Proof: Number bidders from 1 to n. Randomly select a bidder i. In case $i = n$, denote bidder 1 as bidder $i+1$. We formulate strategies with respect to a path of play. On the path of play, bidders behave as follows. Bidders $j \neq i, i+1$ bid zero in periods $\tau \equiv j \bmod n$ and refrain from submitting a bid in periods $\tau \not\equiv j \bmod n$. Bidder $i+1$ bids zero in period i, does not submit a bid in period $i+1$ if he won the object in period i, bids zero in period $i+1$ if he lost in period i, bids zero in periods $\tau \equiv i+1 \bmod n$, $\tau \neq i+1$ and does not submit a bid in all other periods. Bidder i bids 0^+ in period i if $v_{i,i} \geq \delta v_i^e$, refrains from bidding in period i if $v_{i,i} < \delta v_i^e$, bids zero in period $i+1$ if he did not submit a bid or lost in period i with a zero bid, does not submit a bid in period $i+1$ if he won in period i, bids zero in periods $\tau \equiv i \bmod n$, $\tau \neq i$ and does not bid in all other periods. Each bidder's strategy specifies to bid consistent with this path of play unless he observed a deviation from this path of play in some prior period. A bidder who observed a deviation bids v^h.

First note that conditional on the outcome of the randomization, the expected payoff of bidder i and $i+1$ induced by the above strategy profile are higher than their payoff under bid-rotation for the following reasons: Because values are independent across players and time, bidder $i+1$'s expected contemporary value is the same in period i and $i+1$, and he thus benefits because with positive probability he obtains the good earlier. If bidder i always submitted a bid in period i his payoff would be identical to the payoff from bid-rotation. However, he chooses to wait in period $i+1$ whenever this increases his expected payoff and therefore he is better off. Furthermore, it easy to check that no bidder can gain by deviating for sufficiently high discount factors. $\square$

We now argue that bid rotation can be sustained as a perfect Bayesian equilibrium outcome,[8] if for all bidders the payoff from bid rotation, having a $\frac{1}{n}$ chance of getting the object for free, exceeds the payoff from competitive bidding, i.e. each bidder using the bid function of the unique equilibrium of the stage game.[9]

The equilibrium is a slight modification of the collusive scheme introduced in Proposition 2. Along the equilibrium path behavior is identical to our earlier scheme and therefore induces partial observability of payoff relevant deviations. A bidder who submits a bid and wins the object when it it is not his turn initiates competitive bidding in a contagious fashion. After the first deviation at least two bidders (the deviator and the bidder who lost when it was his turn to win) are aware of the fact that a deviation occurred. The correctly anticipate each other to submit a positive bid in all following periods. Hence no bidder can unilaterally deviate and stop or slow down the contagious process as all following bidders who submit a zero bid when it is their turn to win will lose. Within n periods, all bidders bid competitively.

More specifically, bidders who observe a deviation, i.e. do not win when it is their turn, have pessimistic beliefs: They believe that a deviation occurred as early as possible. That is either in the first period or in the period following their last win (a bidder who wins when submitting a zero bid knows that no deviation has occurred so far). Given these pessimistic beliefs and ignoring the first n periods, they bid competitively as soon as they observe a deviation thinking that all other bidders are already engaged in competitive bidding. In the first $n-1$ periods bidders believe that bidder n deviated and submitted a bid in the first period. From this belief they can deduce which rivals they are facing and they use the

[8] A pair (σ, μ) is a perfect Bayesian equilibrium if the strategy profile σ induces a best response after every history given a player's belief and beliefs μ are updated according to Bayes' rule whenever possible.

[9] McAfee and McMillan [1992] show that if all bidders draw their valuations from the same distribution F this assumption is satisfied provided that $1 - F(v)$ is log-concave.

static equilibrium bid function of the corresponding game. We show that these beliefs are consistent after the proposition.[10]

A bidder who deviates knows that he initiates competitive bidding. In equilibrium, he knows how many rivals he faces in every period and which bid functions his rivals use. Furthermore, once he deviated, he knows that he cannot affect the speed with which the contagion spreads. He thus simply plays a myopic best response to his rivals bidding behavior. The following lemma makes the technical observation that this best response always exists. Denote the set of bidders by N. For any subset $M \subset N$ refer to the one-shot auction in which M is the set of participants as the M-auction.

Lemma 1 *Let $M \subset N \setminus \{i\}$. Suppose all bidders $j \in M$ use the equilibrium bid-function of the M-auction. Then there exists a best response for bidder i.*

Proof: Note that bidder i's probability of winning the object is a continuous function of his bid if all bidders $j \in M$ use the equilibrium bid-function of the one-shot M-bidder auction. Hence, his expected payoff is a continuous function of his bid. Clearly, it is never a best response to bid above the highest possible valuation. Hence, bidder i maximizes a continuous function over a compact set and thus a maximum exists by Weierstrass' theorem. □

To characterize bidder i's equilibrium strategy it is useful to partition the set of his private histories into three subsets: (i) cooperative histories, denoted H_{it}^0, after which the bidder has no evidence that a deviation occurred (Following these histories the bidder plays according to the bid-rotation scheme.); (ii) histories, denoted H_{it}^1, in which he observed someone else taking the good when it was his turn to win (Following these histories, the bidder bids competitively.); and, (iii) histories, denoted H_{it}^2, in which he himself has deviated and obtained the good when it was not his turn (After these histories he plays a myopic best-response to the rivals he is facing.). We want to emphasize that while bidders have to rely on private strategies to sustain bid-rotation, the strategies are appealingly simple. In essence, bidders use trigger strategies.

[10] More precisely, while the formal definition of sequential equilibrium by Kreps and Wilson [1982] does not apply to games with uncountable action spaces such as ours, we argue below that these beliefs satisfy the spirit of their consistency requirement.

Number bidders from 1 to n. Let $n(t) := \min\{t, n\}$. We say that bidder j uses the $n(t)$-competitive bid function if he uses his equilibrium bid function for the one-shot auction with bidders in the set $\{1, \ldots, n(t)-1\} \cup \{n\}$. For any $\underline{s} < \overline{s}$, $\underline{s}, \overline{s} \in \mathbb{N}$, let $S(\underline{s}, \overline{s}) := \{j \in N | t \equiv j \bmod n \text{ for some } t \in \{\underline{s}, \ldots, \overline{s}-1\}\}$.

Proposition 4 *Suppose that for each bidder the expected payoff from bid rotation exceeds the expected payoff from competitive bidding. Then, bid rotation can be sustained as the outcome of a perfect Bayesian equilibrium for sufficiently high discount factors.*

Proof:

Partition the set of private histories h_{it} of bidder i that precede period t as follows. Let H_{it}^0 be the set of all h_{it} such that for all periods $\tau < t$ in which $\tau \equiv i \bmod n$ bidder i either won or did not bid, and for all periods $\tau < t$ in which $\tau \not\equiv i \bmod n$ he either did not bid or lost. Let $H_{it}^{1\theta}$ be the set of all h_{it} such that there exists $\theta < t$ with $\theta \equiv i \bmod n$, $h_{i\theta} \in H_{i\theta}^0$, bidder i submitted a bid in period θ and lost in period θ. Let $H_{it}^{2\theta}$ be the set of all h_{it} such that there exists $\theta < t$ with $\theta \not\equiv i \bmod n$, $h_{i\theta} \in H_{i\theta}^0$, bidder i submitted a bid in period θ and won.

Bidder i uses the following strategy:

1. In any period t with $h_{it} \in H_{it}^0$ and $t \equiv i \bmod n$, bid zero.

2. In any period t with $h_{it} \in H_{it}^0$ and $t \not\equiv i \bmod n$, do not submit a bid.

3. In any period t with $h_{it} \in H_{it}^{1\theta}$, use the $n(t)$-competitive bid function.

4. In any period t with $h_{it} \in H_{it}^{2\theta}$, use a myopic best response against all bidders $j \in S(\theta, t) \setminus \{i\}$ using the $n(t)$-competitive bid function. This best response exists by Lemma 1.

Whenever possible bidder i's beliefs are given by Bayes' rule. This is the case for all $h_{it} \in H_{it}^0$ in which bidder i has no evidence that someone else has deviated from the postulated strategy. The only histories $h_{it} \in H_{it}^0$ in which he does have such evidence are those in which in some period $\tau < t$ with $\tau \not\equiv i \bmod n$ he submitted a bid $b > 0$ and lost. In that case we assume that he believes that the bidder j for whom $\tau \equiv j \bmod n$ bid above b and won and any bidder $k \neq j, i$ did not submit a bid.

Following any history $h_{it} \in H_{it}^{1\theta}$ with $\theta \geq n$, bidder i believes that bidder j with $\theta + 1 \equiv j \bmod n$ lost in period $\theta + 1 - n$ to some bidder $k \neq i, j$. Following any history $h_{it} \in H_{it}^{1\theta}$ with $\theta < n$, he believes that bidder 1 lost in the first period because bidder n submitted a bid.

Following any history $h_{it} \in H_{it}^{2\theta}$ bidder i's beliefs can be derived from Bayes' rule for any bid b by bidder i in period $t-1$ that is in the range of the $n(t-1)$-competitive bid functions. Provided bidder i won, Bayes' rule also applies if b is above the range of the $n(t-1)$-competitive bid functions. Otherwise assume that bidder i believes that one of the bidders in the set $S(\theta, t-1) \setminus \{i\}$ bid above him.

It remains to show that bidder i's strategy is optimal after every history given his beliefs. After any history $h_{it} \in H_{it}^{0}$ given his beliefs, bidder i expects that every other bidder j continues to bid 0 in periods τ with $\tau \equiv j \bmod n$ and abstains from bidding in other periods, unless i submits a bid in a period τ with $\tau \not\equiv i \bmod n$ and wins. Thus, given any $\epsilon > 0$, by following his prescribed strategy, for sufficiently high δ bidder i obtains a payoff ϵ-close to the bid-rotation payoff whereas by deviating he induces competitive bidding in no more than n periods. For large δ the contribution to average payoffs from the first n periods following a deviation goes to zero. Thus deviations are not profitable following histories $h_{it} \in H_{it}^{0}$.

After any history $h_{it} \in H_{it}^{1\theta}$ bidder i expects that the other bidders use the $n(\tau)$-competitive bid function for $\tau \geq t$ regardless of his own bidding behavior in period t. Hence it is optimal for him to use the $n(t)$-competitive bid function in period t, since it is a myopic best reply.

After any history $h_{it} \in H_{it}^{2\theta}$, given his beliefs, for $\tau \geq t$ bidder i expects that bidders in the set $S(\theta, \tau) \setminus \{i\}$ use the $n(\tau)$-competitive bid function regardless of his own bidding behavior in period t. Hence, it is optimal for him to use a myopic best response against bidders $j \in S(\theta, t) \setminus \{i\}$ using the $n(t)$-competitive bid function. $\square$

While the definition of sequential equilibria by Kreps and Wilson [1982] does not apply to our game, we capture the idea that consistent beliefs are derived as the limit of totally mixed strategy profiles as follows: Consider any distribution function $\tilde{F}$ that puts positive probability on every subinterval $[0, \infty)$ and has a finite mean. For each bidder i, let $\sigma_i(\epsilon)$ be constructed as follows: After any history $h_{it} \in H^0_{it}$, bidder i draws a bid from $\tilde{F}$ with probability ϵ^t in periods $t \equiv i \bmod n$ and with probability ϵ^{n+t} in periods $t \not\equiv i \bmod n$ and $t > 1$. In period 1, let bidder n draw a bid from $\tilde{F}$ with ϵ^{n+1} and let bidders $i \neq 1, n$ draw a bid from $\tilde{F}$ with probability ϵ^{n+2}. After any history $h_{it} \notin H^0_{it}$, let bidder i draw a bid from $\tilde{F}$ with probability ϵ. Otherwise, let every bidder bid as prescribed by the equilibrium strategy σ. For the profile $\sigma(\epsilon)$ beliefs $\mu(\epsilon)$ can be derived from Bayes' rule after every private history. Taking the limit of those beliefs as $\epsilon \to 0$, gives our belief system.

Next, we augment the construction of Proposition 4 to allow two bidders to engage in the same trade that we used in Proposition 3. Incentives to cooperate are again given by the threat of reversion to the static Nash equilibrium. An additional complication arises, however, because there exist unprofitable deviations in the trading periods that will only be detected with probability less than 1. Following such a deviation, a bidder is uncertain as to whether his rivals reverted back to competitive bidding or still bid according to the bid-rotation scheme. After one such deviation a bidder has an incentive to mimic cooperative behavior for high enough discount factors in the hope that his deviation has not been detected. If this bidder, however, continues to deviate, the problem of assigning optimal continuation behavior becomes exceedingly difficult and we cannot prove the existence of an optimal continuation strategy following every possible private history. Nevertheless, as it is easy to show that such deviations are unprofitable in the first place - independent of the behavior the bidder adopts following these histories - we view this as a technical difficulty that arises due to the fact that we have a continuous action space. A practical problem to specify a perfect Bayesian equilibrium also arises: As the optimal behavior in any given period may depend on the entire private history of a player, it is an enormous and tedious task to keep track of all possible histories. We circumvent this problem by relaxing the solution concept and allowing bidders to use suboptimal continuation strategies following irrelevant histories. We formalize this idea, which might prove useful more generally in analyzing games with private monitoring, next.

Let $\mathcal{H}$ be a subset of the set of private histories such that if $h \notin \mathcal{H}$ then $h' \notin \mathcal{H}$ for any continuation history h' of h. For any strategy profile σ define $\tilde{\Sigma}(\sigma, \mathcal{H})$ as the set of strategy profiles that coincide with σ on $\mathcal{H}$. Denote by $\overline{\mathcal{H}}$ the complement of $\mathcal{H}$.

Definition 1 *A triple* $(\sigma, \mu, \mathcal{H})$ *is an* essentially perfect Bayesian equilibrium ($EPBE$) *if:*

1. *$\mathcal{H}$ contains all histories that are reached on the path of play of σ.*

2. *σ induces a best response against any $\tilde{\sigma} \in \tilde{\Sigma}(\sigma, \mathcal{H})$ after every history in $\mathcal{H}$ given the system of beliefs μ.*

3. *Beliefs μ are derived from Bayes' rule whenever possible.*

When convenient, we will simply refer to the profile σ as an $EPBE$ without explicitly mentioning the system beliefs μ and the set of histories $\mathcal{H}$.

For finite games $EPBE$ and PBE outcomes coincide. Trivially, any PBE is an $EPBE$ in which $\mathcal{H}$ contains all histories. Now consider any $EPBE$ strategy profile σ of the finite game Γ. To show that there exists a PBE that yields the same outcome, we first construct an induced game Γ' as follows: Let Γ' be identical to Γ, except that for all histories $h \in \mathcal{H}$, the probability distribution over the possible actions according to σ is implemented by nature. Because the induced game Γ' is finite, it has a perfect Bayesian equilibrium. Now consider the strategy profile σ' that for all histories $h \in \mathcal{H}$ coincides with σ and for all others prescribes the same behavior as the PBE of the induced game. This strategy profile together with an appropriate set of beliefs μ constitutes a PBE because all choices following histories $h \in \mathcal{H}$ are optimal independent of the choices taken following all other histories and the choices following all other histories are optimal for the beliefs of the PBE of the induced game.

Much of the literature on repeated games with private monitoring exploits a fundamental insight developed in Sekiguchi [1997]. He observed that in repeated games with finite action and signaling spaces in which the distribution of private signals has full support independent of the action profile chosen, the set of Nash equilibrium outcomes and the set of sequential equilibrium outcomes coincide. His insight can be interpreted in our framework by observing that all histories following a deviation by a given player are irrelevant, i.e. can be placed into $\overline{\mathcal{H}}$. More precisely, any Nash equilibrium σ in this environment is an EPBE in which $\mathcal{H}$ is the set of histories reached on the path of play. The reason is that player i cannot reach histories in $\overline{\mathcal{H}}_j,\ j \neq i$. Hence, the specification of behavior after those histories is irrelevant for his decisions. The full-support condition ensures that for players $j \neq i$, the histories that player i *can* reach by way of a deviation remain in $\mathcal{H}_j$.

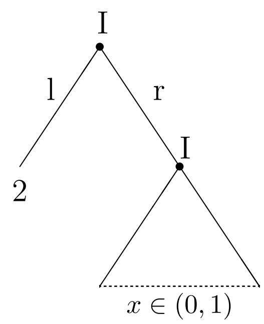

FIGURE 1

In games with infinite action spaces, $EPBE$ and PBE outcomes do not have to coincide. Consider the decision problem illustrated in FIGURE 1. At the first decision node player I can decide whether to get a payoff of 2 by choosing l or whether to advance to the second period. In the second period, the player can choose any $x \in (0, 1)$. His payoff is identical to his choice of x. Observe that this decision problem has no PBE outcome as there is no optimal action in the second period. The choice of l in the first period, however, can be sustained as an $EPBE$ outcome. The reason is that it is better than any action the player may take in the second period. Clearly in this example, $EPBE$ captures the essence of perfection and predicts the appropriate outcome. The careful reader will observe that we use $EPBE$ to eliminate potential openness problems of this kind in the equilibrium construction below.[11]

In the remainder of the paper, we prove that there exists an $EPBE$-outcome with higher expected payoffs than the payoffs from bid-rotation (as long as bid-rotation yields higher payoffs than competitive bidding). The central idea is straightforward: Following any history $h_{it} \in H^0_{it}$ in which a bidder has no evidence that a deviation occurred, he bids as in the

[11]Observe that the Nash equilibrium profile constructed in the proof of Proposition 2 (and 3) is not an *EPBE.* The reason is as follows: Consider any private history h^θ_{it} in which bidder i has observed a deviation in period θ and is assigned to bid v^h in period $t > \theta$. Given that no one ever bids above v^h, bidding v^h is not a best response in the continuation game following any h_{it} because bidder i risks obtaining the object with positive probability at a price above his value. Therefore $h^\theta_{it} \notin \mathcal{H}$, $\forall\ t > \theta$. Now consider the strategy profile $\tilde{\sigma}$, which differs from σ only in that bidder i refrains from bidding after any history h^θ_{it}. Given this strategy profile $\tilde{\sigma}$, it is profitable to deviate and submit a bid in θ. Therefore σ is not a *EPBE* for any set $\mathcal{H}$.

collusive scheme with a bilateral trade that we introduced in Proposition 3.[12] Following histories $h_{it} \in H_{it}^1$ in which a bidder observed a deviation and lost when he was supposed to win, bidders revert to bidding competitively. Loosely speaking, bidders have the same type of pessimistic beliefs as introduced in Proposition 4. As before, the threat of Nash reversion makes deviations unprofitable.

Off the equilibrium path, however, a bidder who deviated himself may face complicated decision problems that require strategic experimentation. Consider, for example, a bidder who is not involved in the bilateral trade and deviates in the second "trading period." The deviator knows that his deviation has been detected and initiates contagion, but he does not know which of the bidders involved in the trade detected his deviation. n periods after his deviation, all bidders will have detected his deviation and bid competitively thereafter. In the meantime, however, it is in general useful for the deviator to know which rivals he faces as they may use different bid functions. When submitting a zero bid (or a bid above the range of the competitive bid function), the deviator loses (wins) with probability one independent of which rivals he faces. For all other bids, however, his probability of losing may depend on which bidder detected his deviation. He therefore updates his beliefs as to which rivals he faces after every round. When bidding, the deviator thus has to trade off the effect of his bids on his contemporaneous profits with the expected informational value his bid generates. In particular, this implies that the optimal behavior within the first n periods following his initial deviation depends not only on the first time the bidder deviated but also on the exact bids he submitted in the meantime, because these bids can affect his beliefs about which rival he is facing. This already indicates the necessity of keeping track of histories in a far more detailed fashion.

As another example, consider a bidder $i+1$ who is meant to bid 0 in the first trading period and obtain the good only if his trading partner prefers to get the good in the second period. In the out-of-equilibrium event in which he submits a positive bid in the first trading period, he is uncertain about whether his deviation has been detected. For high enough discount factors, it is optimal for him to refrain from bidding in subsequent periods in which the good is assigned to his rivals in the hope that his deviation has not been detected and

[12] For notational simplicity we consider trade between bidders 1 and 2. The argument generalizes straightforwardly to any randomly selected pair of bidder.

that he will continue to enjoy the benefits of the bid-rotation scheme. When it is his turn to bid according to the bid-rotation scheme, he can submit a zero bid; in this case he knows for certain that his deviation has been detected if he loses and that he has not been caught if he wins. Nevertheless, if he has a high value, he may prefer to submit a positive bid because this increase his chance of winning in case his rivals bid competitively. Bidder $i+1$ needs to balance this contemporaneous profit effect with the informational value his bid generates. Note that bidder $i+1$ faces a similar issue every n periods when it is his turn to win until he either loses or wins with a zero bid. He thus has to solve an infinite horizon strategic experimentation problem. The problem becomes even more delicate if bidder $i+1$ continued to deviate, repeatedly submitted zero bids when it was not his turn to win, and lost. After every bid he has to update his beliefs as to whether his deviation has been detected and if he repeatedly loses with zero bids, he may think that it is extremely likely that he has been detected earlier (his beliefs also depend on the exact bids he submitted when it was his turn to win). Rather than trying to solve all the different types of strategic experimentation problems that can arise following repeated deviations by bidder $i+1$, we group these histories into the set $\overline{\mathcal{H}}$ of irrelevant histories because - no matter how he behaves following these histories - it is easy to show that it is unprofitable for him to deviate in the first place. Similar issues arise if bidder $i+1$ deviates in period i by not submitting a bid and then deviates again and submits a winning bid in period $i+1$, because in this case he initiated contagion if and only if bidder i decided to obtain the good in period $i+1$ rather than in period i. Due to the same uncertainty as to whether or not he initiated contagion, he must solve similar strategic experimentation problems. In the proof below we fully specify sequentially rational and consistent strategies for all bidders $j \neq i+1$ and restrict the use of $\overline{\mathcal{H}}$ to the type of histories discussed above. Observe also that bidder $i+1$'s rivals never become aware of the fact that bidder i "deviated himself" into the set of irrelevant histories.

Proposition 5 *Suppose that for each bidder the expected payoff from bid rotation exceeds the expected payoff from competitive bidding. Then, there exists an essentially perfect Bayesian equilibrium outcome with higher payoffs than from bid rotation for sufficiently high discount factors.*

3 Conclusion

We constructed a simple collusive scheme that allows bidders to support bid-rotation in repeated first-price auctions even if the auctioneer withholds all possible information. Our result highlights the contrast between studying collusion through the repeated game approach employed in the current paper and the static mechanism design framework used in McAfee and McMillan [1992]. Following this approach, Marshall and Marx [2002] have shown that bid-rotation cannot be sustained by mechanisms that do not rely on information provided by the auctioneer. The contrast in results can be explained by the fact that in our repeated game environment equilibrium strategies can ensure partial observability of potentially profitable deviations even if the auctioneer withholds all possible information. One lesson of this paper is thus that the information available to colluding players in a repeated game framework depends itself on the equilibrium strategies employed. In addition, we illustrated how a simple bilateral trade can be used in equilibrium to improve on the bid-rotation outcome. This, again, is in contrast to the static mechanism design framework in which McAfee and McMillan [1992] have shown that bid-rotation is the optimal form of collusion in the absence of explicit sidepayments under weak distributional assumptions.

Our analysis also highlights the important difference between studying collusion through perfect public equilibria and through perfect Bayesian equilibria. Bidders relying on public strategies cannot collude at all in our framework. Often restricting attention to public strategies is defended as focusing on relatively simple strategies. Simple collusive schemes that employ private strategies, however, allow the bidders to improve on competitive bidding in our setting. When constructing a perfect Bayesian equilibrium that improves upon bid-rotation, one needs to investigate complicated dynamic programming problems of the equilibrium path. Nevertheless, we think of the collusive scheme as extraordinarily simple from the participants perspective because the required behavior along the equilibrium path is simple and the collusive scheme is supported through the straightforward threat of Nash reversion. Furthermore, it is easy to foresee for every bidder that a deviation from the equilibrium behavior is unprofitable - independent of the behavior he adopts following such an initial deviation. In addition, his behavior following certain deviations does not affect how his rivals play in the continuation game and it is thus unnecessary to assign optimal behavior after such deviations in order to predict the outcome of the game. From a methodological point of view, we introduced the concept of essentially perfect Bayesian equilibria, which allows a researcher to capture the spirit of the perfection refinement without the cost of having to specify behavior following such irrelevant histories.

Appendix

Proof of Proposition 5

Proof: We first partition the set of histories, then describe strategies in terms of this partition, then describe bidders' beliefs and finally show that the strategies are optimal given the specified beliefs. The reader may want to consult Figures 2-8 while reading the proof. Those figures graphically represent the partitions of private histories that we employ in the proof. To save space, the figures only deal with the smallest interesting number of bidders, which is three but are easily adapted to the general case of n bidders that is addressed in the proof.

Histories

For bidders $i = 3, \ldots, n$, partition their sets of private histories h_{it} that precede period t as follows. Let H_{it}^{0} be the set of all h_{it} such that for all periods $\tau < t$ in which $\tau \equiv i \bmod n$ bidder i either won or did not bid, and for all periods $\tau < t$ in which $\tau \not\equiv i \bmod n$ he either did not bid or lost. Let $H_{it}^{1\theta}$ be the set of all h_{it} such that there exists $\theta < t$ with $\theta \equiv i \bmod n$, $h_{i\theta} \in H_{i\theta}^{0}$, bidder i submitted a bid in period θ and lost in period θ. Let $H_{it}^{2\theta}$ be the set of all h_{it} such that there exists $\theta < t$ with $\theta \not\equiv i \bmod n$, $h_{i\theta} \in H_{i\theta}^{0}$, bidder i submitted a bid in period θ and won.

For bidder 1, partition his set of private histories h_{1t} that precede period t as follows. For $t \leq n+1$, let H_{1t}^{0} be the set of all h_{1t} such that in period 1 bidder 1 either won or did not bid, and for all periods $\tau < t$ other than period 1 he either did not bid or lost. For $t = n+2$, let $H_{1t}^{0,\text{grab}}$ be the set of all h_{1t} such that $h_{1,t-1} \in H_{1,t-1}^{0}$ and bidder 1 submitted a winning bid in period $n+1$. For $t = n+3$, let $H_{1t}^{0,\text{grab}}$ be the set of all h_{1t} such that $h_{1,t-1} \in H_{1,t-1}^{0,\text{grab}}$ and bidder 1 either abstained from bidding in period $n+2$ or lost. For $t > n+3$, let $H_{1t}^{0,\text{grab}}$ be the set of all h_{1t} such that $h_{1,t-1} \in H_{1,t-1}^{0,\text{grab}}$ and if $t-1 \equiv 1 \bmod n$ bidder 1 either won or did not bid in period $t-1$ and if $t-1 \not\equiv 1 \bmod n$ he either did not bid or lost in period

$t-1$.

For $t = n+2$, let $H_{1t}^{0,\text{leave}}$ be the set of all h_{1t} such that $h_{1,t-1} \in H_{1,t-1}^{0}$ and bidder 1 refrained from bidding in period $n+1$ or bid $b=0$ in period $n+1$ and lost. For $t = n+3$, let $H_{1t}^{0,\text{leave}}$ be the set of all h_{1t} such that $h_{1,t-1} \in H_{1,t-1}^{0,\text{leave}}$ and bidder 1 either won in period $n+2$ or abstained from bidding. For $t > n+3$, let $H_{1t}^{0,\text{leave}}$ be the set of all h_{1t} such that $h_{1,t-1} \in H_{1,t-1}^{0,\text{leave}}$ and if $t-1 \equiv 1 \bmod n$ bidder 1 either won or did not bid in period $t-1$ and if $t-1 \not\equiv 1 \bmod n$ he either did not bid or lost in period $t-1$. For $t \geq n+2$, let $H_{1t}^{0} = H_{1t}^{0,\text{grab}} \cup H_{1t}^{0,\text{leave}}$.

Let $H_{1t}^{1\theta}$ be the set of all h_{1t} such that either (i) there exists a $\theta < t$ with $\theta \neq n+1$, $\theta \equiv 1 \bmod n$, $h_{1\theta} \in H_{1\theta}^{0}$, and bidder 1 submitted a losing bid in period θ or (ii) bidder 1 submitted a bid $b \neq 0$ in period $\theta = n+1$ and lost or (iii) bidder 1 submitted a bid in period $\theta = n+2$ for a history $h_{1,n+2} \in H_{1,n+2}^{0,\text{leave}}$ and lost.

Let $H_{1t}^{2\theta}$ be the set of all h_{1t} such that there exists $\theta < t$ for which $h_{1\theta} \in H_{1\theta}^{0}$, and either (i) $\theta \neq n+2$, $\theta \not\equiv 1 \bmod n$, and bidder 1 submitted a winning bid in period θ or (ii) $\theta = n+2$, and bidder 1 submitted a winning bid in period θ following $h_{1\theta} \in H_{1\theta}^{0,\text{grab}}$.

For bidder 2, partition his set of private histories h_{2t} that precede period t as follows. For $t \leq n+1$, let H_{2t}^{0} be the set of all h_{2t} such that in period 2 bidder 2 either won or did not bid, and for all periods $\tau < t$ other than period 2 he either did not bid or lost. For $t = n+2$, let $H_{2t}^{0,\text{grab}}$ be the set of all h_{2t} such that $h_{2,t-1} \in H_{2,t-1}^{0}$ and bidder 2 submitted a losing bid $b \geq 0$ in period $n+1$. For $t = n+3$, let $H_{2t}^{0,\text{grab}}$ be the set of all h_{2t} such that $h_{2,t-1} \in H_{2,t-1}^{0,\text{grab}}$ and bidder 2 submitted a bid and won or refrained from bidding. For $t > n+3$, let $H_{2t}^{0,\text{grab}}$ be the set of all h_{2t} such that $h_{2,t-1} \in H_{2,t-1}^{0,\text{grab}}$ and if $t-1 \equiv 2 \bmod n$ bidder 2 either won or did not bid in period $t-1$ and if $t-1 \not\equiv 2 \bmod n$ he either did not bid or lost in period $t-1$.

For $t = n+2$, let $H_{2t}^{0,\text{leave}}$ be the set of all h_{2t} such that $h_{2,t-1} \in H_{2,t-1}^{0}$ and bidder 2 submitted a winning bid $b=0$ in period $n+1$. For $t = n+3$, let $H_{2t}^{0,\text{leave}}$ be the set of all h_{2t} such that $h_{2,t-1} \in H_{2,t-1}^{0,\text{leave}}$ and bidder 2 abstained from bidding in period $n+2$ or submitted a losing bid. For $t > n+3$, let $H_{2t}^{0,\text{leave}}$ be the set of all h_{2t} such that $h_{2,t-1} \in H_{2,t-1}^{0,\text{leave}}$ and if $t-1 \equiv 2 \bmod n$ bidder 2 either won or did not bid in period $t-1$ and if $t-1 \not\equiv 2 \bmod n$ he either did not bid or lost in period $t-1$.

For $t = n + 2$, let $H_{2t}^{0,\emptyset}$ be the set of all h_{2t} such that $h_{2,t-1} \in H_{2,t-1}^{0}$ and bidder 2 refrained from bidding in period $n+1$. For $t = n+3$, let $H_{2t}^{0,\emptyset}$ be the set of all h_{2t} such that $h_{2,t-1} \in H_{2,t-1}^{0,\emptyset}$ and bidder 2 abstained from bidding in period $n+2$ or submitted a losing bid. For $t > n+3$, let $H_{2t}^{0,\emptyset}$ be the set of all h_{2t} such that $h_{2,t-1} \in H_{2,t-1}^{0,\emptyset}$ and if $t-1 \equiv 2 \bmod n$ bidder 2 either won or did not bid in period $t-1$ and if $t-1 \not\equiv 2 \bmod n$ he either did not bid or lost in period $t-1$.

For $t = n + 3$, let $H_{2t}^{0,\emptyset,2}$ be the set of all h_{2t} such that $h_{2,t-1} \in H_{2,t-1}^{0,\emptyset}$ and bidder 2 submitted a winning bid in period $n+2$. For $t > n+3$, let $H_{2t}^{0,\emptyset,2}$ be the set of all h_{2t} such that $h_{2,t-1} \in H_{2,t-1}^{0,\emptyset,2}$ and if $t-1 \equiv 2 \bmod n$ bidder 2 either won with a bid $b > 0$ or did not bid in period $t-1$ and if $t-1 \not\equiv 2 \bmod n$ he did not bid in period $t-1$. Let $H_{2t}^{0,\emptyset,0}$ be the set of all h_{2t} such that (i) there exists a θ with $h_{2,\theta} \in H_{2,\theta}^{0,\emptyset,2}$, $\theta \equiv 2 \bmod n$, and bidder 2 won with a bid $b = 0$ and (ii) for all $\tau = \theta, \dots, t-1$ if $\tau \equiv 2 \bmod n$ bidder 2 either won or did not bid in period τ and if $\tau \not\equiv 2 \bmod n$ he either did not bid or lost in period τ. Any history h_{2t} for which there exists a θ such that $h_{2,\theta} \in H_{2,\theta}^{0,\emptyset,2}$, $\theta \not\equiv 2 \bmod n$, and bidder 2 submitted a losing bid $b = 0$ belongs to the complement of $\mathcal{H}$ denoted by $\overline{\mathcal{H}}$.

For $t = n+2$, let $H_{2t}^{0,2}$ be the set of all h_{2t} such that $h_{2,t-1} \in H_{2,t-1}^{0}$ and bidder 2 submitted a winning bid $b \neq 0$ in period $n+1$. For $t = n+3$, let $H_{2t}^{0,2}$ be the set of all h_{2t} such that $h_{2,t-1} \in H_{2,t-1}^{0,2}$ and bidder 2 abstained from bidding in period $n+2$. For $t > n+3$, let $H_{2t}^{0,2}$ be the set of all h_{2t} such that $h_{2,t-1} \in H_{2,t-1}^{0,2}$ and if $t-1 \equiv 2 \bmod n$ bidder 2 either won with a bid $b > 0$ or did not bid in period $t-1$ and if $t-1 \not\equiv 2 \bmod n$ he did not bid in period $t-1$. Let $H_{2t}^{0,2,0}$ be the set of all h_{2t} such that (i) there exists a θ with $h_{2,\theta} \in H_{2,\theta}^{0,2}$, $\theta \equiv 2 \bmod n$ and $\theta \neq n+2$, and bidder 2 won with a bid $b = 0$ and (ii) for all $\tau = \theta, \dots, t-1$ if $\tau \equiv 2 \bmod n$ bidder 2 either won or did not bid in period τ and if $\tau \not\equiv 2 \bmod n$ he either did not bid or lost in period τ. Any history h_{2t} for which there exists a θ such that $h_{2,\theta} \in H_{2,\theta}^{0,2}$, $\theta \not\equiv 2 \bmod n$ or $\theta = n+2$ and bidder 2 submitted a losing bid $b = 0$ belongs to the set $\overline{\mathcal{H}}$.

For $t \geq n+2$, let $H_{2t}^{0} = H_{2t}^{0,\text{grab}} \cup H_{2t}^{0,\text{leave}} \cup H_{2t}^{0,\emptyset,0} \cup H_{2t}^{0,2,0}$.

Let $H_{2t}^{1\theta}$ be the set of all h_{2t} such that there exists a θ with $\theta < t$ and $\theta \equiv 2 \bmod n$, $h_{2\theta} \in H_{2\theta}^{0} \cup H_{2\theta}^{0,\emptyset}$, bidder 2 submitted a bid in period θ and lost in period θ.

Let $H_{2t}^{0,\emptyset,2,1}$ be the set of all h_{2t} such that there exists a θ with $\theta < t$, $h_{2\theta} \in H_{2t}^{0,\emptyset,2}$, and either (i) $\theta \equiv 2 \bmod n$ and bidder 2 submitted a losing bid or (ii) $\theta \not\equiv 2 \bmod n$ and bidder 2 submitted a losing bid $b > 0$.

Let $H_{2t}^{0,2,1}$ be the set of all h_{2t} such that there exists a θ with $\theta < t$, $h_{2\theta} \in H_{2t}^{0,2}$, and either (i) $\theta \equiv 2 \bmod n$, $\theta \neq n+2$, and bidder 2 submitted a losing bid or (ii) bidder 2 submitted a losing bid $b > 0$ in period θ (where $\theta \not\equiv 2 \bmod n$ or $\theta = n+1$).

Let $H_{2t}^{2\theta}$ be the set of all h_{2t} such that there exists $\theta < t$ for which $h_{2\theta} \in H_{2\theta}^{0}$, and either (i) $\theta \neq n+1$, $\theta \not\equiv 2 \bmod n$, and bidder 2 submitted a winning bid in period θ or (ii) $\theta = n+2$, $h_{2\theta} \in H_{2\theta}^{0,\text{leave}}$, and bidder 2 submitted a winning bid in period θ.

Let $H_{2t}^{0,\emptyset,\theta}$ be the set of all h_{2t} such that there exists $\theta < t$ for which $h_{2\theta} \in H_{2\theta}^{0,\emptyset}$, $\theta \not\equiv 2 \bmod n$, and bidder 2 submitted a winning bid in period θ.

Let $H_{2t}^{0,\emptyset,2,\theta}$ be the set of all h_{2t} such that there exists $\theta < t$ for which $h_{2\theta} \in H_{2\theta}^{0,\emptyset,2}$, $\theta \not\equiv 2 \bmod n$, and bidder 2 submitted a winning bid in period θ.

Let $H_{2t}^{0,2,2,\theta}$ be the set of all h_{2t} such that there exists $\theta < t$ for which $h_{2\theta} \in H_{2\theta}^{0,2}$, and either (i) $\theta \neq n+1$, $\theta \not\equiv 2 \bmod n$, and bidder 2 submitted a winning bid in period θ or (ii) $\theta = n+2$ and bidder 2 submitted a winning bid in period θ.

Stategies

A bidder $i \in \{3, \ldots, n\}$ uses the following strategy:

1. In any period t with $h_{it} \in H_{it}^{0}$ and $t \equiv i \bmod n$, bid zero.

2. In any period t with $h_{it} \in H_{it}^{0}$ and $t \not\equiv i \bmod n$, do not submit a bid.

3. In any period t with $h_{it} \in H_{it}^{1\theta}$, use the $n(t)$-competitive bid function.

4. In any period t with $h_{it} \in H_{it}^{2\theta}$ and $t > \theta > n+2$ use a myopic best response against all bidders $j \in S(\theta, t) \setminus \{i\}$ using the n-competitive bid function, which exists by Lemma 1.

5. In any period $t > 2n+2$ for $h_{it} \in H_{it}^{2,n+2}$, use the n-competitive bid function.

Call the sequence of sets of bidders $\{1\}, \{1,3\} \setminus \{i\}, \{1,3,4\} \setminus \{i\}, \ldots, N \setminus \{i,2,n\}, N \setminus \{i,2\}, N \setminus \{i,2\}, N \setminus \{i\}, N \setminus \{i\}, \ldots$ the "1-sequence" and the sequence of sets of bidders $\{2\}, \{2,3\} \setminus \{i\}, \{2,3,4\} \setminus \{i\}, \ldots, N \setminus \{i,1,n\}, N \setminus \{i,1\}, N \setminus \{i\}, N \setminus \{i\}, \ldots$ the "2-sequence". Denote the kth element of the 1-sequence by s_{1k}, and similarly for the 2-sequence. Let

$\zeta = F_1(\delta v_1^e)$ denote the probability that bidder 1 refrained from bidding in period $n+1$. Therefore, ζ is the probability that following a winning bid by bidder i in period $n+1$ bidder i bids against the 1-sequence and $1-\zeta$ is the probability that he bids against the 2-sequence. Let $\xi(h_{it}, \zeta)$ denote the corresponding posterior probability following i's private history h_{it}.

6. In period $t \in \{n+3, \ldots, 2n+2\}$ for $h_{it} \in H_{it}^{2,n+2}$, use the bid function that corresponds to first period-behavior under the continuation strategy that is a best reply against beliefs given by $\xi(h_{i,t}, \zeta)$.

This best reply can be found recursively as follows: Bidder i's expected continuation payoff given his belief $\xi(h_{i,t}, \zeta)$ and the realization of his value v_{it} in period t if he bids b in period is

$$V_i\Big(\xi(h_{i,t}, \zeta)\,, v_{it}\Big) = \max_b \Big(v_{it} - b\Big) P(b, \xi(h_{i,t}, \zeta))$$
$$+\delta \int_0^{v^h} \Big\{ V_i\Big(\xi(h_{i,t} \circ (b, w), \zeta), v\Big) P(b, \xi(h_{i,t}, \zeta))$$
$$+V_i\Big(\xi(h_{i,t} \circ (b, l), \zeta), v\Big) [1 - P(b, \xi(h_{i,t}, \zeta))] \Big\} \; f_i(v)\; dv,$$

where

$$P(b, \xi(h_{i,t}, \zeta)) := \left(\xi(h_{i,t}, \zeta) \prod_{j \in s_{1,t-(n+2)}} F_j(\beta_j^{-1}(b)) + (1 - \xi(h_{i,t}, \zeta)) \prod_{j \in s_{2,t-(n+2)}} F_j(\beta_j^{-1}(b)) \right),$$

$h_{i,t} \circ (b, w)$ denotes the history $h_{i,t}$ followed by a winning bid b, $h_{i,t} \circ (b, l)$ denotes the history $h_{i,t}$ followed by a losing bid b, and $V_i(\xi(h_{i,2n+3}, \zeta), \cdot)$ is the present discounted value of infinitely repeated n-bidder competitive bidding for all $h_{i,2n+3}$.

Note that it is never optimal to bid above $\frac{v^h}{1-\delta}$ as this gives a negative continuation payoff. Furthermore, observe that in period $2n+2$, the bid b does not affect the continuation value, and hence for any $\xi(h_{i,2n+2}, \zeta)$ and any $v_{i,2n+2}$, $V_i\Big(\xi(h_{i,2n+2}, \zeta)\,, v_{i,2n+2}\Big)$ is well defined by Weierstrass's theorem since we can restrict attention to bids $b \in [0, \frac{v^h}{1-\delta}]$. Furthermore, by Berge's maximum theorem, $V_i\Big(\xi(h_{i,2n+2}, \zeta)\,, v_{i,2n+2}\Big)$ is continuous in $\xi(h_{i,2n+2}, \zeta)$. Therefore, the objective function in period $2n+1$ is continuous and recursively applying the above argument shows that a best response exists in every period $t \in \{n+3, \ldots, 2n+2\}$ for all $\xi(h_{it}, \zeta)$.

7. In period $n+2$ for $h_{it} \in H_{it}^{2,n+1}$, use a best reply against facing bidder 1 with probability ζ or bidder 2 with probability $1-\zeta$.

8. In any period $t > n+2$ with $h_{it} \in H_{it}^{2,n+1}$, use a myopic best response against all bidders $j \in S(n+1,t) \setminus \{i\}$ using the n-competitive bid function.

10. In any period $t > \theta$ and $t \neq n+2$ for $h_{it} \in H_{it}^{2,\theta}$, for $\theta \in \{3,\ldots,n\}$, use a myopic best response against all bidders $j \in S(\theta,t) \setminus \{i\}$ using the $n(t)$-competitive bid function.

11. In period $t = n+2$ for $h_{it} \in H_{it}^{2,\theta}$, for $\theta \in 3,\ldots,n$, use a myopic best response against facing bidder 1 with probability ζ or bidder 2 with probability $1-\zeta$, and all bidders $j \in S(\theta,t) \setminus \{i,1\}$ for certain, all of whom use the n-competitive bid function.

12. In any period $t > 1$ for any history $h_{it} \in H_{it}^{2,1}$, use the myopic best response against all bidders $j \in S(1,t) \setminus \{i\}$ using the $n(t)$-competitive bid function.

13. In any period $t > 2$ and $t \neq n+2$ for any history $h_{it} \in H_{it}^{2,2}$, use the myopic best response against all bidders $j \in S(2,t) \setminus \{i\}$ using the $n(t)$-competitive bid function.

14. In period $t = n+2$ for any history $h_{i,n+2} \in H_{i,n+2}^{2,2}$, use a myopic best response against facing bidder 1 with probability ζ and all bidders $j \in S(2,n+2) \setminus \{i,1\}$ for certain, all of whom use the n-competitive bid function.

Bidder 1 uses the following strategy:

1. In any period $t \neq n+1$ with $h_{1t} \in H_{1t}^{0}$ and $t \equiv 1 \mod n$, bid zero.

2. In any period $t \neq n+2$ with $h_{1t} \in H_{1t}^{0}$ and $t \not\equiv 1 \mod n$, do not submit a bid.

3. In period $t = n+1$ with $h_{1,n+1} \in H_{1,n+1}^{0}$ bid 0^{+} if $v_{1,n+1} \geq \delta v_1^e$ and do not bid otherwise.

4. In period $t = n+2$ with $h_{1,n+2} \in H_{1,n+2}^{0,\text{grab}}$ do not bid.

5. In period $t = n+2$ with $h_{1,n+2} \in H_{1,n+2}^{0,\text{leave}}$ bid 0.

6. In any period t with $h_{1t} \in H_{1t}^{1\theta}$, use the $n(t)$-competitive bid function.

7. In any period $t \neq n+2$ with $h_{1t} \in H_{1t}^{2\theta}$, use a myopic best response against all bidders $j \in S(\theta, t) \setminus \{1\}$ using the $n(t)$-competitive bid function, which exists by Lemma 1.
8. In period $t = n+2$ with $h_{1,n+2} \in H_{1,n+2}^{2\theta}$, use a myopic best response against all bidders $j \in S(\theta, t) \setminus \{1, 2\}$ using the n-competitive bid function, which exists by Lemma 1.

Bidder 2 uses the following strategy:

1. In any period $t \neq n+2$ with $h_{2t} \in H_{2t}^{0} \cup H_{2t}^{0,\emptyset}$ and $t \equiv 2 \bmod n$, bid zero.
2. In any period $t \neq n+1$ with $h_{2t} \in H_{2t}^{0} \cup H_{2t}^{0,\emptyset}$ and $t \not\equiv 2 \bmod n$, do not submit a bid.
3. In period $t = n+1$ with $h_{2,n+1} \in H_{2,n+1}^{0}$ bid 0.
4. In period $t = n+2$ with $h_{2,n+2} \in H_{2,n+2}^{0,\text{grab}}$ bid 0.
5. In period $t = n+2$ with $h_{2,n+2} \in H_{2,n+2}^{0,\text{leave}} \cup H_{2,n+2}^{0,\emptyset}$ do not bid.
6. In any period t with $h_{2t} \in H_{2t}^{1\theta}$, use the $n(t)$-competitive bid function.
7. In any period $t \neq n+2$ with $h_{2t} \in H_{2t}^{2\theta}$ and $\theta \neq n+2$, use a myopic best response against all bidders $j \in S(\theta, t) \setminus \{2\}$ using the $n(t)$-competitive bid function, which exists by Lemma 1.
8. In any period t with $h_{2t} \in H_{2t}^{2,n+2}$ use a myopic best response against all bidders $j \in \{S(\theta, t) \cup \{1\}\} \setminus \{2\}$ using the n-competitive bid function.
9. In period $t = n+2$ with $h_{2,n+2} \in H_{2,n+2}^{2\theta}$ for $\theta \neq 1$, use a myopic best response against facing bidder 1 with probability ζ and all bidders $j \in S(\theta, t) \setminus \{1, 2\}$ for certain, all of whom use the n-competitive bid function. Observe that the myopic best response exists by Lemma 1.
10. In period $t = n+2$ with $h_{2,n+2} \in H_{2,n+2}^{2,1}$ use the n-competitive bid function.
11. In periods $t \not\equiv 2 \bmod n$ with $h_{2,t} \in H_{2,t}^{0,2}$ refrain from bidding.
12. In periods $t \equiv 2 \bmod n$ with $h_{2,t} \in H_{2,t}^{0,2}$ bid as described in the dynamic programming problem in the final subsection of the appendix, with initial beliefs being equal to $\zeta = F_1(\delta v_1^e)$.

13. In period t with $h_{2,t} \in H_{2,t}^{0,2,1}$ use a myopic best response against bidders in the set $S(n+1,t) \setminus \{2\}$ using the n-competitive bid function.

14. In period t with $h_{2,t} \in H_{2,t}^{0,2,2,n+2}$ use a myopic best response against bidders in the set $S(n+1,t) \setminus \{2\}$ using the n-competitive bid function.

    Let $\xi(h_{2,t}, \zeta)$ be bidder 2's posterior belief following private history $h_{2,t}$ that bidder 1 refrained from bidding in period $n+1$.

15. In period t with $h_{2,t} \in H_{2,t}^{0,2,2,\theta}$, $\theta > n+2$, $\theta \not\equiv 2 \bmod n$ use the bid function that corresponds to first-period behavior under the continuation strategy that is a best reply against facing the set of bidders $S(\theta,t) \setminus \{2\}$ with probability $\xi(h_{2,t}, \mu_0)$ and the set of bidders $S(n+1,t) \setminus \{2\}$ with probability $1 - \xi(h_{2,t}, \mu_0)$.

    This best reply is found in a similar manner as described above for a bidder i following history $h_{it} \in H_{it}^{2,n+2}$. Note that the posterior probability will now depend on the bids made in periods $t \equiv 2 \bmod n$.

16. In period t with $h_{2,t} \in H_{2,t}^{0,\emptyset,\theta}$, use a myopic best response against bidders in the set $S(\theta,t) \setminus \{2\}$ using the n-competitive bid function.

17. In periods $t \not\equiv 2 \bmod n$ with $h_{2,t} \in H_{2,t}^{0,\emptyset,2}$ refrain from bidding.

18. In periods $t \equiv 2 \bmod n$ with $h_{2,t} \in H_{2,t}^{0,\emptyset,2}$ bid as described in the dynamic programming problem in the final subsection of the appendix, with initial beliefs being equal to $\zeta = F_1(\delta v_1^e)$.

19. In period t with $h_{2,t} \in H_{2,t}^{0,\emptyset,2,1}$ use a myopic best response against bidders in the set $S(n+1,t) \setminus \{2\}$ using the n-competitive bid function.

20. In period t with $h_{2,t} \in H_{2,t}^{0,\emptyset,2,\theta}$, $\theta > n+2$, $\theta \not\equiv 2 \bmod n$ use the bid function that corresponds to first-period behavior under the continuation strategy that is a best reply against facing the set of bidders $S(\theta,t) \setminus \{2\}$ with probability $\xi(h_{2,t}, \zeta)$ and the set of bidders $S(n+1,t) \setminus \{2\}$ with probability $1 - \xi(h_{2,t}, \zeta)$.

21. Bid competitively after all histories $h_{2t} \in \overline{\mathcal{H}}$.

Beliefs

For all histories h_{it}, $i \neq 1, 2$ in which bidder i has no evidence of another bidder's deviation, his beliefs are derived from Bayes' rule. The histories $h_{it} \in H_{it}^0$ in which he does have such evidence are those in which in some period $\tau < t$ with $\tau \not\equiv i \bmod n$ he submitted a bid $b > 0$ and lost. In these cases we assume that bidder i believes that the bidder j for whom $\tau \equiv j \bmod n$ bid above b and won and any bidder $k \neq j, i$ did not submit a bid.

For bidder 1, we recall that $H_{1t}^0 = H_{1t}^{0,\text{grab}} \cup H_{1t}^{0,\text{leave}}$ and make the same assumption. Note that a losing bid following history $h_{1,n+2} \in H_{1,n+2}^{0,\text{leave}}$ takes bidder 1 out of the sets of histories $H_{1t}^{0,\text{leave}}$ into the sets $H_{1t}^{1,n+2}$.

For bidder 2 we make the same assumption for histories $h_{2t} \in H_{2,t}^0 \cup H_{2,t}^{0,\emptyset}$, except that in addition after a history in which bidder 2 following $h_{2,n+2}^{0,\text{leave}}$ submitted a bid in period $n+2$ and lost, we assume that he believes bidder 1 bid above b and won and any bidder $k \neq 1, 2$ did not submit a bid.

For any bidder $i \neq 1, 2$, following any history $h_{it} \in H_{it}^{1\theta}$ with $\theta \geq n$ and $\theta \neq 2n$ bidder i believes that bidder j with $\theta + 1 \equiv j \bmod n$ lost in period $\theta + 1 - n$ to some bidder $k \neq i, j$. Following any history $h_{nt} \in H_{nt}^{1,2n}$ bidder n believes that bidder 1 submitted a positive bid b in period $n + 2$ following $h_{1t}^{0,\text{grab}}$. Following any history $h_{it} \in H_{it}^{1\theta}$ with $\theta < n$, he believes that bidder 1 lost in the first period because bidder n submitted a bid.

For any bidder $i = 1, \ldots, n$, following any history $h_{it} \in H_{it}^{2\theta}$ bidder i's beliefs can be derived from Bayes' rule for any bid b by bidder i in period $t - 1$ that is in the range of the $n(t-1)$-competitive bid functions. Provided bidder i won, Bayes' rule also applies if b is above the range of the $n(t-1)$-competitive bid functions. Otherwise assume that bidder i believes that one of the bidders who detected his deviation bid above him.

For bidder 1 following any history $h_{1t} \in H_{1t}^{1\theta}$ with $\theta \geq n$ and $\theta \neq 2n + 1$ bidder 1 believes that bidder j with $\theta + 1 \equiv j \bmod n$ lost in period $\theta + 1 - n$ to some bidder $k \neq 1, j$. Similarly, following any history $h_{1t} \in H_{1t}^{1,2n+1}$ in which bidder 1 submitted a winning bid in period $n + 1$, bidder 1 believes that bidder 2 lost in period $n + 2$ to some bidder $k \neq 1, 2$. Following any history $h_{1t} \in H_{1t}^{1,2n+1}$ in which bidder 1 submitted a winning bid in period $n+2$, bidder 1 believes that bidder 2 submitted a winning bid in period $n+3$. Following any history $h_{1t} \in H_{1t}^{1,1}$, he believes that bidder n submitted the winning bid in the first period.

Following any history $h_{2t} \in H_{2t}^{0,\emptyset,2}$ bidder 2's beliefs can be derived from Bayes' rule with initial beliefs given by $\zeta = F_1(\delta v_1^e)$.

Following any history $h_{2t} \in H_{2t}^{0,\emptyset,\theta} \cup H_{2t}^{0,\emptyset,2,\theta} \cup H_{2t}^{0,2,2,\theta}$ with $\theta \geq n+2$, bidder 2's beliefs can be derived from Bayes' rule for any bid b by bidder i in period $t-1$ that is in the range of the n-competitive bid functions. Provided bidder 2 won, Bayes' rule also applies if b is above the range of the n-competitive bid functions. Otherwise assume that bidder 2 believes that one of the bidders who detected his deviation bid above him.

Following any history $h_{2t} \in H_{2t}^{0,\emptyset,2,1}$ bidder 2 believes, consistent with Bayes' rule, to be bidding against bidders in the set $S(n+1,t) \setminus \{2\}$, all of whom use the n-competitive bid function

Following any history $h_{2t} \in H_{2t}^{0,2}$ bidder 2's beliefs can be derived from Bayes' rule with initial beliefs given by $\zeta = F_1(\delta v_1^e)$.

Following any history $h_{2t} \in H_{2t}^{0,2,1}$ bidder 2 believes, consistent with Bayes' rule, to be bidding against bidders in the set $S(n+1,t) \setminus \{2\}$, all of whom use the n-competitive bid function

For bidder 2 following any history $h_{2t} \in H_{2t}^{1\theta}$ with $\theta \geq n$ bidder 2 believes that bidder j with $\theta + 1 \equiv j \bmod n$ lost in period $\theta + 1 - n$ to some bidder $k \neq 2, j$. Following any history $h_{2t} \in H_{2t}^{1,2}$ he believes that bidder 1 lost in the first period because bidder n submitted a winning bid in the first period.

Optimality

It remains to show that bidders' strategies are optimal given their beliefs after every history except those in $\overline{\mathcal{H}}$.

After any history $h_{it} \in H_{it}^0$ for any period $t \neq n+1$ bidder i faces the choice of bidding when he is meant to abstain or bidding higher than necessary when he is meant to win. Submitting a bid in case a bidder is meant to lose only matters if he wins in which case it induces all rivals to revert to the n-competitive bid function within n periods. As the payoff from competitive bidding is lower than the payoff from bid rotation, this is unprofitable for high δ. By the same reasoning no bidder $i \neq 1, 2$ can gain by submitting a bid in period $n+1$.

Bidder 2 cannot gain from bidding above 0 in period $n+1$ because with positive probability he bids above bidder 1 and induces competitive bidding by all rivals within n periods. It is optimal for bidder 1 to submit a bid 0^+ in period $n+1$ if and only if $v \geq \delta v_1^e$ because the difference in his discounted continuation value between losing and winning is δv_1^e, i.e. the discounted expected value of obtaining the good in period $n+2$ for free.

After any history $h_{it} \in H_{it}^{1\theta}$ bidder i expects that the other bidders use the $n(\tau)$-competitive bid function for periods $\tau \geq t$ regardless of his own bidding behavior in period t. Hence it is optimal for him to use the $n(t)$-competitive bid function in period t, since it is a myopic best reply.

After any history $h_{it} \in H_{it}^{2\theta} \cup H_{2t}^{0,\emptyset,\theta} \cup H_{2t}^{0,\emptyset,2,1} \cup H_{2t}^{0,\emptyset,2} \cup H_{2t}^{0,\emptyset,2,\theta} \cup H_{2t}^{0,2} \cup H_{2t}^{0,2,1} \cup H_{2t}^{0,2,2,\theta}$, it is straightforward to check that the strategies are best replies against the beliefs derived from Bayes' rule.

□

Dynamic Programming off the Equilibrium Path

Consider bidder 2 who is uncertain whether either all of his rivals bid competitively or all of his rivals engage in bid rotation and who assumes that any discovery of a deviation from bid rotation will result in everyone bidding competitively in no more than n periods. For any prior belief $\mu_0 < 1$ of facing competitive bidding, there is a discount factor $\underline{\delta}$ such that for any $\delta > \underline{\delta}$ bidder 2 will refrain from bidding in periods in which another bidder is designated to win the object. However in periods, in which he himself is designated to win the object, he faces a trade-off. He can submit a zero bid in which case he learns with probability one whether or not everyone else conforms with competitive bidding or put in a positive bid, which may increase his current payoff at the expense of learning about his rivals. He faces this decision every n periods and we refer to those periods as *decision periods.*

Let μ_k denote bidder 2's posterior probability at the end of period $k-1$ of facing competitive bidding and let $G(b_{2,k})$ denote the probability that $b_{2,k}$ exceeds all other bids when all of bidder 2's rivals bid competitively. If bidder 2 places a bid $b_{2,k}$ in the kth decision period then with probability $\mu_k(1-G(b_{2,k}))$ he loses in that period, therefore becomes convinced his rivals bid competitively, and continues with his unique best reply of himself bidding competitively ever after from the next period on, which earns him a payoff of $\frac{\delta v_c}{1-\delta.}$ With probability $1-\mu_k(1-G(b_{2,k}))$ he wins the object in that period, earns a payoff of $(v_{2,k}-b_2)$ in that period, and enters the next decision period with a posterior $\mu_{k+1}=\frac{\mu_k G(b)}{\mu_k G(b)+1-\mu_k}$.

Let $U := [0, v^h]$. Denote the bidding rule in the decision period k of a bidder 2 who won in all previous decision periods by $b_{2,k}(\cdot) : U^k \to \mathbb{R}$ and define $\beta_2 = \{b_{2,k}(\cdot)\}_{j=1}^{\infty}$. Let $\mu(\beta_2, \mu_0, v_2^{k-1})$ be his posterior at the beginning of decision period k conditional on winning in all prior decision periods, his bidding rule, his prior, and the past realizations of his valuations. Define

$$\pi(v, \mu, b) := [1-\mu(1-G(b))](v-b) + \delta\mu(1-G(b))\frac{v^c}{1-\delta},$$

and

$$\rho(v_2^k) := \mu(\beta_2, \mu_0, v_2^{k-1})[1-G(b_{2,k}(v_2^k))].$$

Then bidder 2's K-period interim expected payoff from following the bidding rule β_2 equals

$$\Upsilon(\beta_2, v_{2,1}, \mu_0, K) = \pi(v_{2,1}, \mu_0, b_{2,1}(v_2^1))$$
$$+\sum_{k=2}^{K}\int_{U^k}\delta^{(k-1)\times n}\left[\prod_{r=1}^{k-1}(1-\rho(v_2^r))\right]\pi\left(v_{2,k}, \mu(\beta_2, \mu_0, v_2^{k-1}), b_{2,k}(v_2^k)\right)dF_2(v_{2,2})\ldots dF_(v_{2,k}).$$

Bidder 2's interim expected payoff from following the bidding rule β_2 equals

$$\Upsilon(\beta_2, v_{2,1}, \mu_0) = \lim_{K\to\infty}\Upsilon(\beta_2, v_{2,1}, \mu_0, K).$$

Since $G(\cdot)$ is the probability that all bidders other than 2 who use the competitive bid function submit a bid below $b_{2,k}$ and each bidder's competitive bid function is continuous and monotone, $G(\cdot)$ is continuous and monotone.

Lemma 2 *Let $G(\cdot) : [0, v^h] \to [0, 1]$ continuous and monotone. Then the Bellman equation*

$$V(v,\mu) = \max_b\left\{[1-\mu(1-G(b))](v-b) + \delta\mu(1-G(b))\frac{v^c}{1-\delta}\right.$$
$$\left. + \delta^n\left\{[1-\mu(1-G(b))]_2\int_0^{v^h}V\left(\lambda, \frac{\mu G(b)}{\mu G(b)+1-\mu}\right)dF(\lambda)\right\}\right\}$$

has a solution in the space of bounded continuous functions on $[0, v^h] \times [0, 1]$.

Proof: Consider the operator T defined by

$$(TV)(v,\mu) = \sup_b \left\{ [1-\mu(1-G(b))](v-b) + \delta\mu(1-G(b))\frac{v^c}{1-\delta} \right.$$

$$\left. + \delta^n \left\{ [1-\mu(1-G(b))] \int_0^{v^h} V\left(\lambda, \frac{\mu G(b)}{\mu G(b)+1-\mu}\right) dF_2(\lambda) \right\} \right\}$$

Since the factor multiplying the integral is nonnegative, it is clear that for all bounded functions V', V'' on $[0,v^h]\times[0,1]$ with $V'(v,\mu) \le V''(v,\mu) \ \ \forall (v,\mu) \in [0,v^h]\times[0,1]$, it is the case that $(TV')(v,\mu) \le (TV'')(v,\mu)$ for all $(v,\mu) \in [0,v^h]\times[0,1]$. Hence the operator T is monotonic. Furthermore, one easily checks that $(T(V+a))(v,\mu) \le (TV)(v,\mu) + \delta a$ for all bounded functions $V(\cdot,\cdot)$ on $[0,v^h]\times[0,1]$, $a \ge 0$ and $(v,\mu) \in [0,v^h]\times[0,1]$. Therefore Blackwell's sufficient condition for a contraction implies that the operator T is contraction with modulus δ. Simple inspection shows that the operator T maps the space of bounded continuous functions on $[0,v^h]\times[0,1]$ into itself. The space of bounded continuous functions on $[0,v^h]\times[0,1]$ with the sup-norm is a complete metric space. Therefore the contraction mapping theorem implies that the operator T has a fixed point in this space, i.e. the functional equation has a unique solution in the class of bounded continuous functions on $[0,v^h]\times[0,1]$.

Since the solution of the functional equation is a continuous function, the supremum is attained by Weierstrass's theorem. $\square$

Let $B(v,\mu)$ be set of $\tilde{b}$ such that

$$\tilde{b} \in \arg\max_b \left\{ [1-\mu(1-G(b))](v-b) + \delta\mu(1-G(b))\frac{v^c}{1-\delta} \right.$$

$$\left. + \delta^n \left\{ [1-\mu(1-G(b))] \int_0^{v^h} V\left(\lambda, \frac{\mu G(b)}{\mu G(b)+1-\mu}\right) dF(\lambda) \right\} \right\}.$$

Let $b(v,\mu)$ be a selection from $B(v,\mu)$ and note that any such selection is monotonically increasing and therefore measurable. Let μ_0 be defined as above and let $\mu_{k+1}(v^k) = \frac{\mu_k G(b(v_k,\mu_k))}{\mu_k G(b(v_k,\mu_k))+1-\mu_k}$. Let $b^*_{2,k}(v_2^K) := b(v_{2,k},\mu_k)$.

Lemma 3 *The bidding rule* $\beta_2^* = \{b_{2,k}^*(\cdot)\}_{k=1}^{\infty}$ *maximizes* $\Upsilon(\beta_2, \mu_0)$.

Proof: Consider any alternative bidding rule β_2. Then

$$\begin{aligned}
V(v_{2,1}, \mu_0) &\geq \pi(v_{2,1}, \mu_0, b_{2,1}(v_2^1)) + \delta^n(1 - \rho(v_2^1)) \int_0^{v^h} V\left(v_{2,2}, \mu(\beta_2, \mu_0, v_2^1)\right) dF(v_{2,2}) \\
&\geq \pi(v_{2,1}, \mu_0, b_{2,1}(v_2^1)) + \delta^n(1 - \rho(v_2^1)) \times \\
&\quad \int_0^{v^h} \Bigg\{ \pi(v_{2,2}, \mu(\beta_2, \mu_0, v_2^1), b_{2,2}(v_2^2)) + \\
&\qquad\qquad \delta^n(1 - \rho(v_2^2)) \int_0^{v^h} V\left(v_{2,3}, \mu(\beta_2, \mu_0, v_2^2)\right) dF(v_{2,3}) \Bigg\} dF(v_{2,2}) \\
&= \pi(v_{2,1}, \mu_0, b_{2,1}(v_2^1)) + \delta^n(1 - \rho(v_2^1)) \int_0^{v^h} \pi(v_{2,2}, \mu(\beta_2, \mu_0, v_2^1), b_{2,2}(v_2^2))\ dF(v_{2,2}) \\
&+ \delta^{2n}(1 - \rho(v_2^1))(1 - \rho(v_2^2)) \int_0^{v^h} V\left(v_{2,3}, \mu(\beta_2, \mu_0, v_2^2)\right) dF(v_{2,3}) dF(v_{2,2}) \\
&\geq \Upsilon(\beta_2, v_{2,1}, \mu_0, K) \\
&+ \delta^{Kn} \left[\prod_{r=1}^{K} (1 - \rho(v_2^r))\right] \int_0^{v^h} V\left(v_{2,K+1}, \mu(\beta_2, \mu_0, v_2^K)\right) dF(v_{2,K+1}) \dots dF(v_{2,2}).
\end{aligned}$$

Taking the limit as $K \to \infty$, it follows that

$$V(v_{2,1}, \mu_0) \geq \Upsilon(\beta_2, v_{2,1}, \mu_0).$$

Replacing β_2 above with β_2^*, all inequalities become equalities. Hence

$$\Upsilon(\beta_2^*, v_{2,1}, \mu_0) = V(v_{2,1}, \mu_0) \geq \Upsilon(\beta_2, v_{2,1}, \mu_0) \quad \forall \beta_2.$$

□

References

Abreu, D., D. Pearce, and E. Stacchetti [1990], "Toward a Theory of Discounted Repeated Games with Imperfect Monitoring," *Econometrica,* **58**, 1041-1063.

Aghion, P., P. Bolton, C. Harris and B. Jullien [1991] "Optimal Learning by Experimentation," *Review of Economic Studies,* **58**, 621-654.

Aoyagi, M. [2002a], "Bid Rotation and Collusion in Repeated Auctions," *Journal of Economic Theory,* forthcoming.

Aoyagi, M. [2002b], "Efficient Collusion in Repeated Auctions with Communication," ISER-University of Osaka Working Paper.

Athey, S. [2001], "Single Crossing Properties and the Existence of Pure Strategy Equilibria in Games of Incomplete Information," *Econometrica,* **69**, 861-889.

Athey, S. and K. Bagwell [2001], "Optimal Collusion with Private Information," *Rand Journal of Economics,* **32**, 428-465.

Athey, S., K. Bagwell and C. Sanchirico [2000], "Collusion and Price Rigidity," MIT, Columbia University, and University of Virginia Law School Working Paper.

Baldwin, L. H. R. C. Marshall and J.-F. Richard [1997], "Bidder Collusion at Forest Service Timber Sales," *Journal of Political Economy* **105**, 657-699.

Bajari, P. [1997], "The First-Price Auction with Asymmetric Bidders: Theory and Applications," University of Minnesota Ph.D. Thesis.

Bajari, P. [2001], "Comparing Competition and Collusion in Procurement Auctions: A Numerical Approach," *Economic Theory,* **18**, 187-205.

Ben-Porath, E. and M. Kahneman [1996], "Communication in Repeated Games with Private Monitoring," *Journal of Economic Theory,* **70**, 281-297.

Bhaskar, V. and I. Obara [2002], "Belief-Based Equilibria in the Repeated Prisoners' Dilemma with Private Monitoring," *Journal of Economic Theory,* **102**, 40-69.

Blume, A. and P. Heidhues [2002], "All Equilibria of the Vickrey Auction," *Journal of Economic Theory,* forthcoming.

Blume, A. and P. Heidhues [2002], "Modeling Tacit Collusion in Auctions," University of Pittsburgh and Social Science Research Center Berlin (WZB) Working Paper.

Cassady, Jr., R. [1967], *Auctions and Auctioneering*, University of California Press.

Dufwenberg, M. and U. Gneezy [2002], "Information Disclosure in Auctions: An Experiment," *Journal of Economic Behavior & Organization,* **48**, 431-444.

Froeb, L. [1988], "Auctions and Antitrust," Department of Justice Working Paper.

Ellison, G. [1994], "Cooperation in the Prisoner's Dilemma with Anonymous Random Matching," *Review of Economic Studies,* **61**, 567-588.

Ely, J.C. and J. Välimäki [2002], "A Robust Folk-Theorem for the Prisoner's Dilemma" *Journal of Economic Theory,* **102**, 84-105.

Fudenberg, D., D. Levine and E. Maskin [1994], "The Folk Theorem with Imperfect Public Information," *Econometrica,* **62**, 997-1039.

Hendricks, K. and R. H. Porter [1993], "An Empirical Study of an Auction with Assymetric Information," *American Economic Review* **78(5)**, 865-883.

Hirshleifer, J. and J. Riley [1992], *The Analysis of Uncertainty and Information*, Cambridge: Cambridge University Press.

Jackson, M. O., L. K. Simon, J. M. Swinkels and W. R. Zame [2002], "Communication and Equilibrium in Discontinuous Games of Incomplete Information," *Econometrica,* **70**, 1711-1740.

Kandori, M. [1992], "Social Norms and Community Enforcement," *Review of Economic Studies,* **59**, 63-80.

Kandori, M. and I. Obara [2000], "Efficiency in Repeated Games Revisited: The Role of Private Strategies," University of Tokyo and University of Pennsylvania Working Paper.

Kreps, D. and R. Wilson [1982], "Sequential Equilibrium," *Econometrica,* **50**, 863-894.

Lebrun, B. [1999], "First-Price Auctions in the Asymmetric N Bidder Case," *International Economic Review,* **40**, 125-142.

Marshall, R.C. and L.M. Marx [2002], "Bidder Collusion," Penn State University and Duke University Working Paper.

Maskin, E. and J. Riley [1996], "Uniqueness in Sealed High Bid Auctions," Harvard University and University of California at Los Angles Working Paper .

McAfee, R.P. and J. McMillan [1992], "Bidding Rings," *American Economic Review,* **82**, 579-599.

McLennan, A. [1984], "Price Dispersion and Incomplete Learning in the Long Run," *Journal of Economic Dynamics and Control,* **7**, 331-347.

Mirman, L. J., L. Samuelson and A. Urbano [1993], "Monopoly Experimentation," *International Economic Review* **34**, 549-563.

Pesendorfer, M. [2000], "A Study of Collusion in First-price Auctions," *Review of Economic Studies* **67**, 381-411.

Porter, R. and J. D. Zona [1993], "Detection of Bid Rigging in Procurement Auctions," *Journal of Political Economy* **101**, 518-538.

Radner, R., R. Myerson, and E. Maskin [1986], "An Example of a Repeated Partnership Game with Discounting and with Uniformly Inefficient Equilibria" *Review of Economic Studies,* **53**, 59-69.

Rothschild, M. [1974], "A Two-Armed Bandit Theory of Market Pricing," *Journal of Economic Theory,* **9**, 185-202.

Sekiguchi, T. [1997], "Efficiency in Repeated Prisoner's Dilemma with Private Monitoring," *Journal of Economic Theory,* **76**, 345-361.

Skrzypacz, A. and H. Hopenhayn [2002], "Tacit Collusion in Repeated Auctions," *Journal of Economic Theory*, forthcoming.

Bidder $i \neq 1, 2$

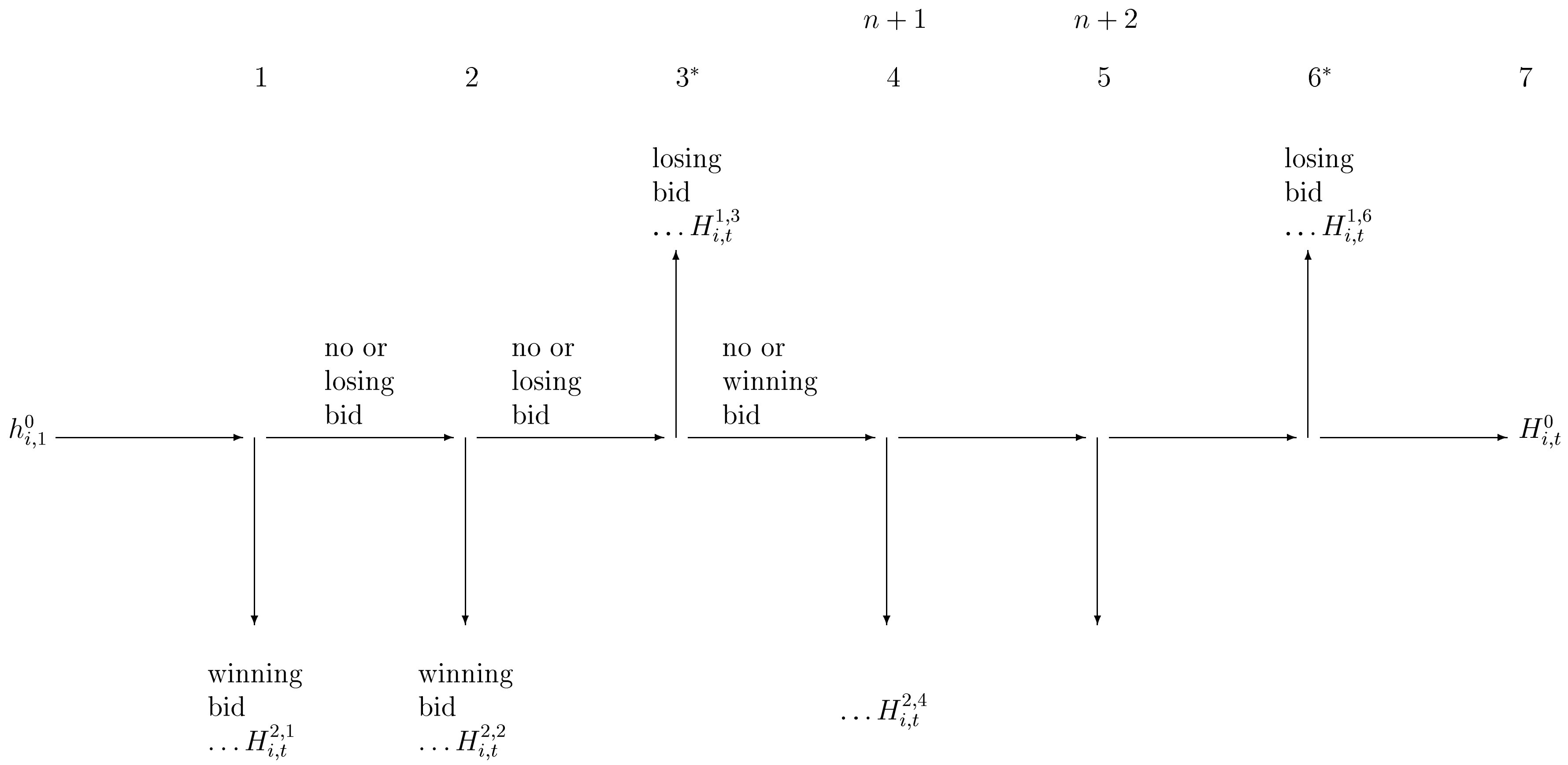

FIGURE 2

Bidder 1

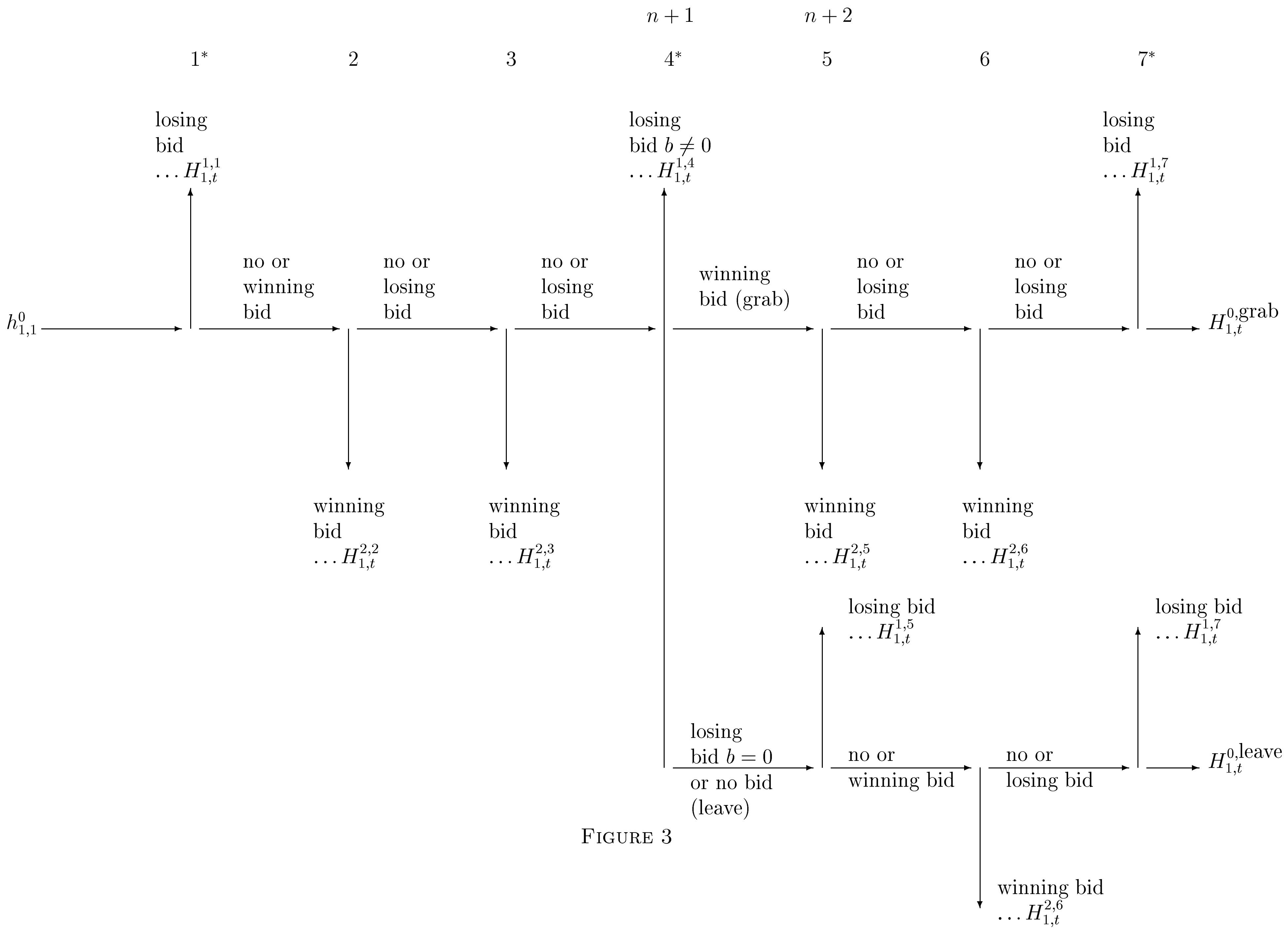

Figure 3

Bidder 2

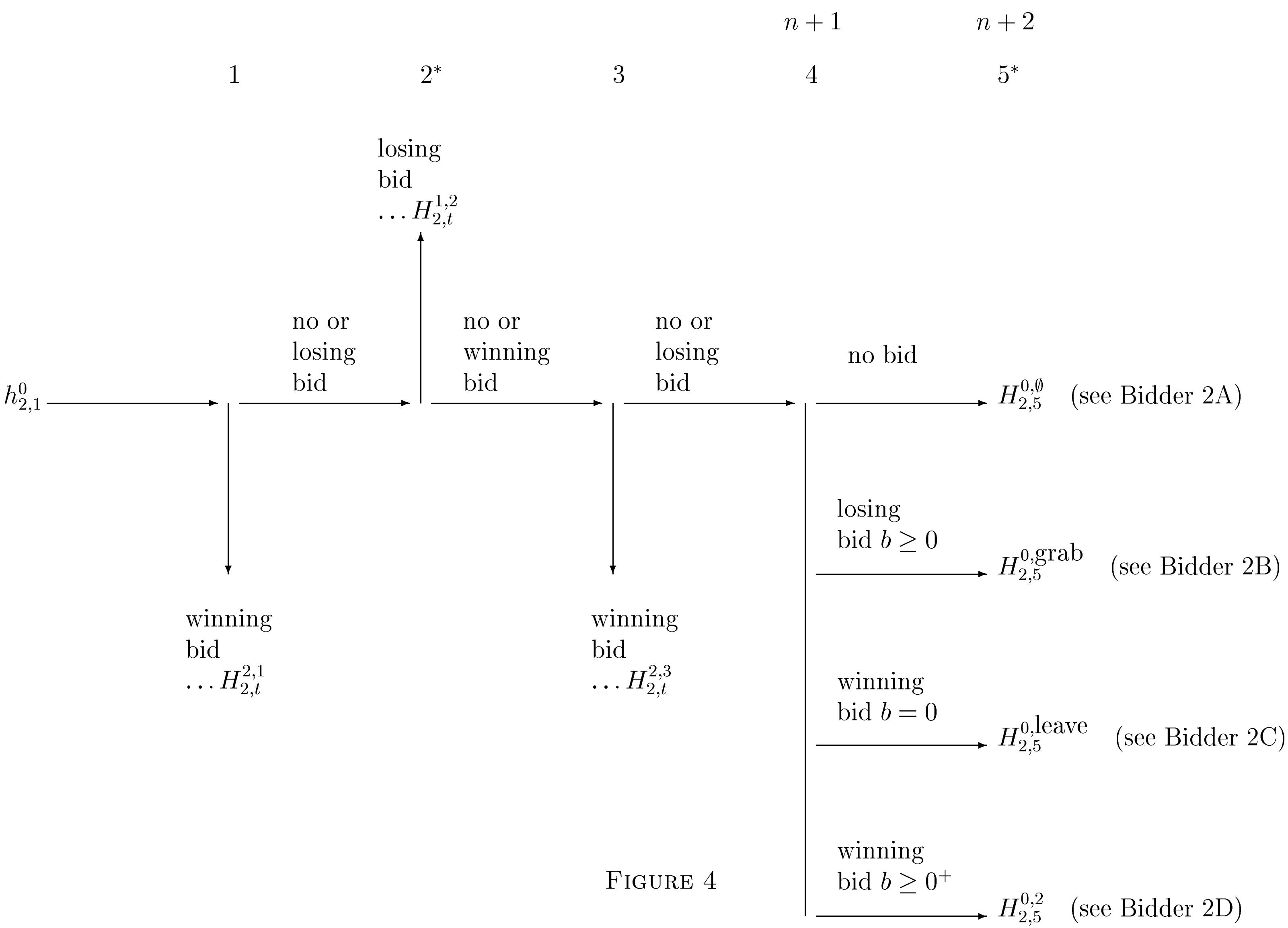

FIGURE 4

Bidder 2A

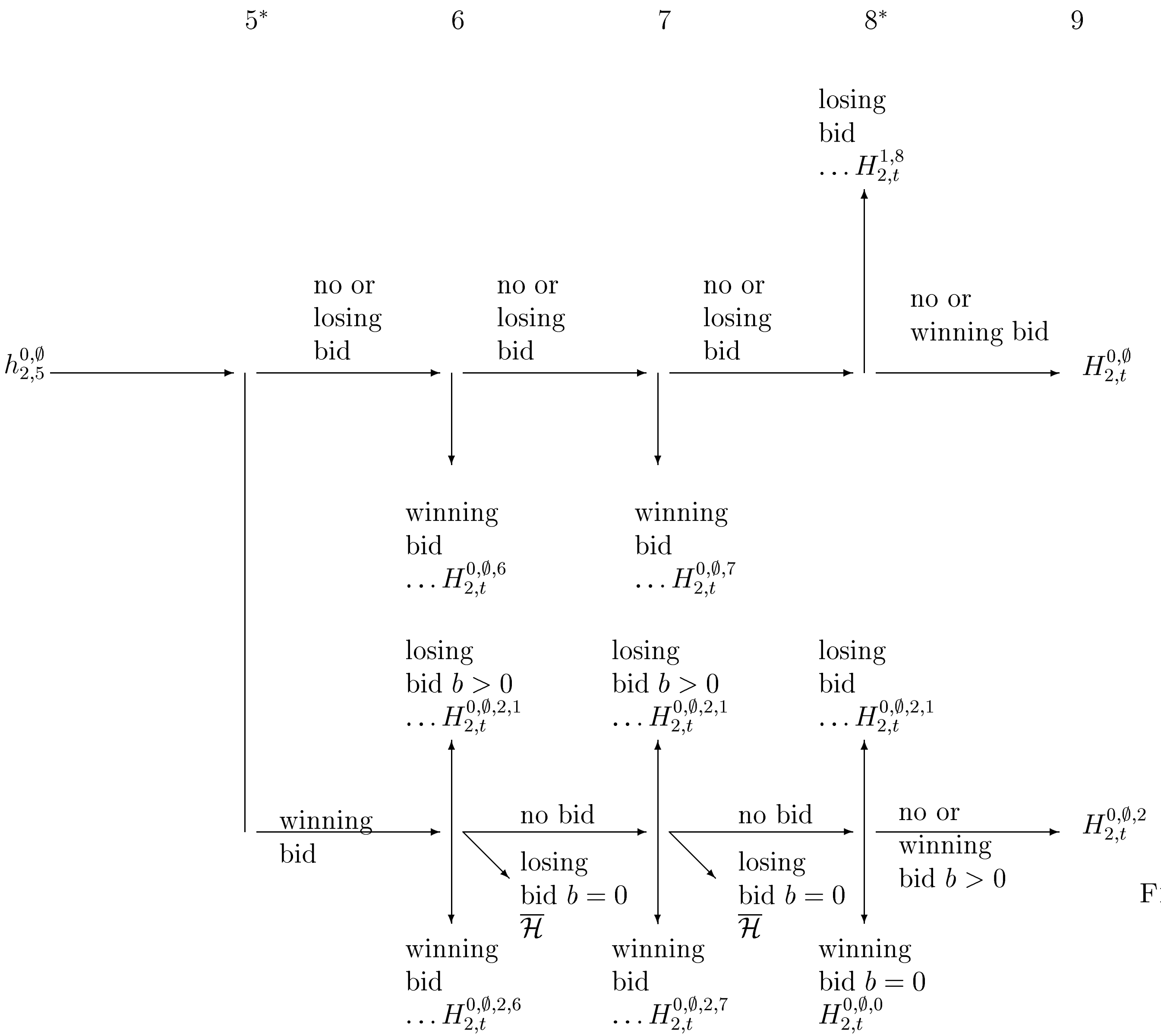

FIGURE 5

Bidder 2B

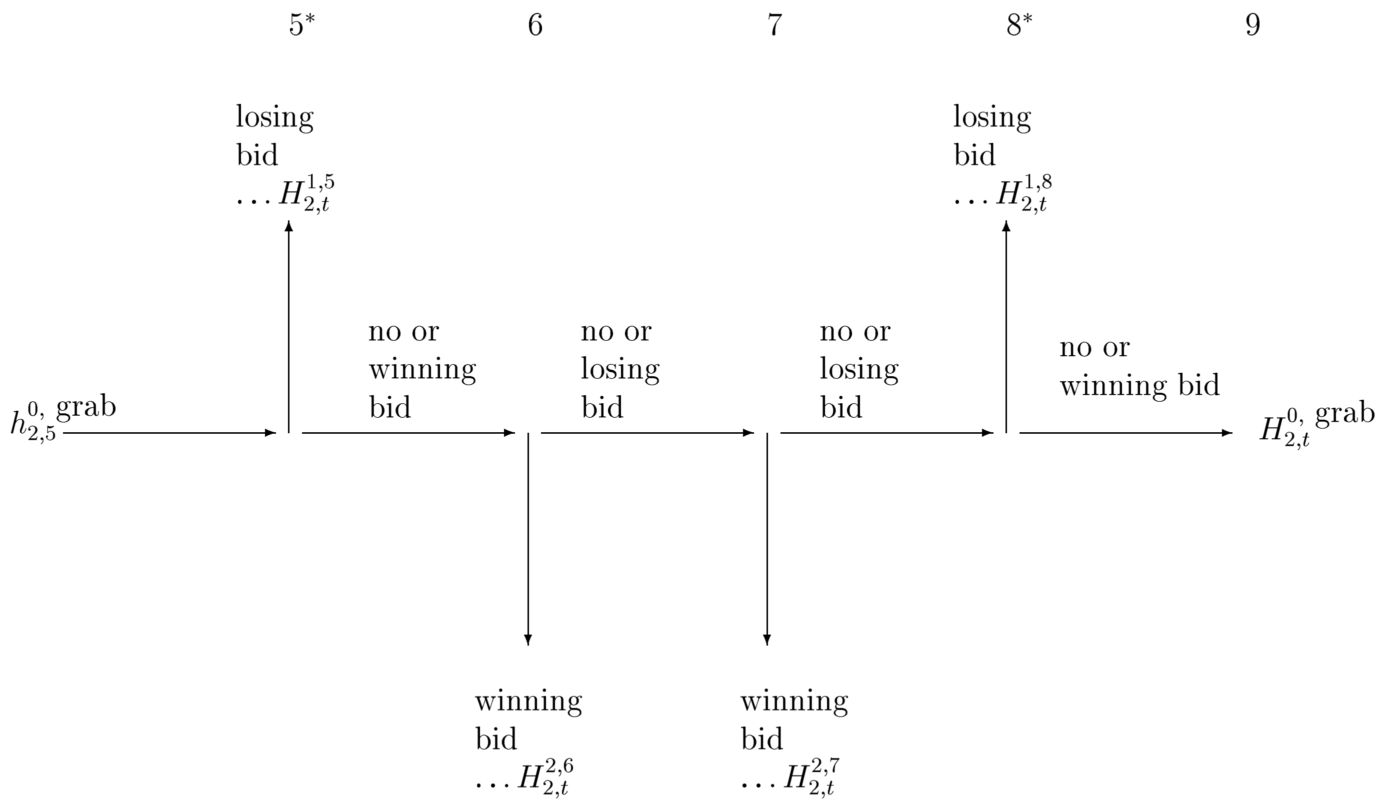

FIGURE 6

Bidder 2C

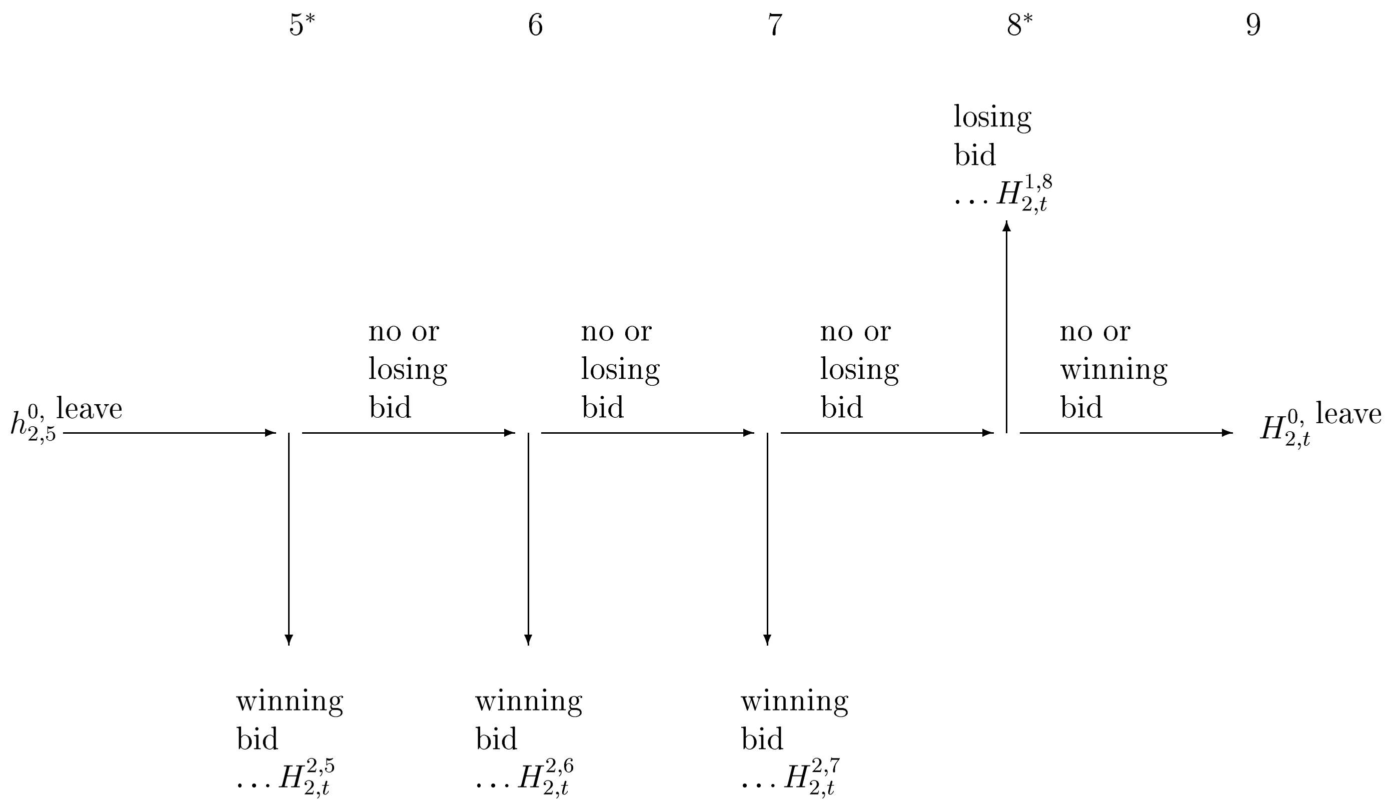

FIGURE 7

Bidder 2D

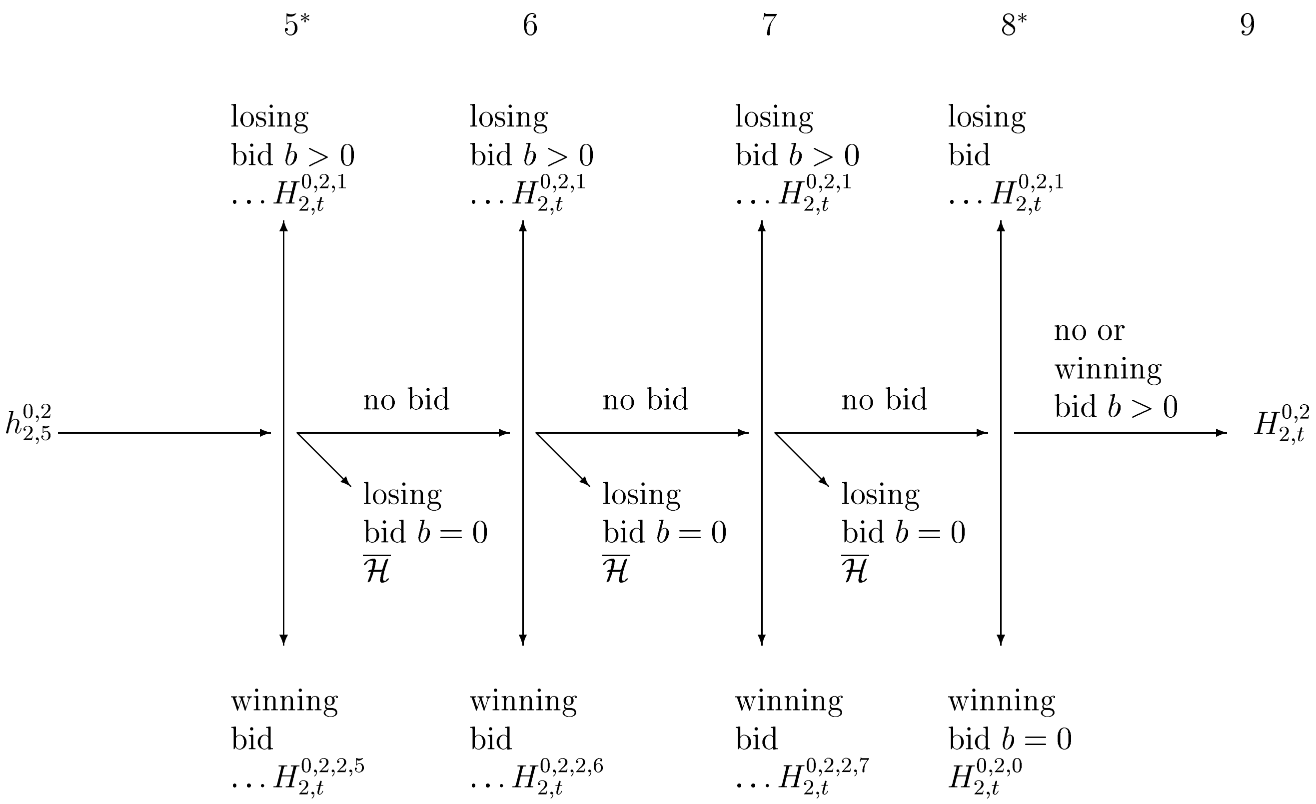

FIGURE 8

Bücher des Forschungsschwerpunkts Markt und politische Ökonomie
Books of the Research Area Markets and Political Economy

Andreas Stephan
Essays on the Contribution of Public Infrastructure to Private: Production and its Political Economy
2002, dissertation.de

Hans Mewis
Essays on Herd Behavior and Strategic Delegation
2001, Shaker Verlag

Andreas Moerke
Organisationslernen über Netzwerke – Die personellen Verflechtungen von Führungsgremien japanischer Aktiengesellschaften
2001, Deutscher Universitäts-Verlag

Silke Neubauer
Multimarket Contact and Organizational Design
2001, Deutscher Universitäts-Verlag

Lars-Hendrik Röller, Christian Wey (Eds.)
Die Soziale Marktwirtschaft in der neuen Weltwirtschaft, WZB Jahrbuch 2001
2001, edition sigma

Michael Tröge
Competition in Credit Markets: A Theoretic Analysis
2001, Deutscher Universitäts-Verlag

Tobias Miarka
Financial Intermediation and Deregulation: A Critical Analysis of Japanese Bank-Firm-Relationships
2000, Physica-Verlag

Rita Zobel
Beschäftigungsveränderungen und organisationales Lernen in japanischen Industriengesellschaften
2000, Humboldt-Universität zu Berlin
http://dochost.rz.hu-berlin.de/dissertationen/zobel-rita-2000-06-19

Jos Jansen
Essays on Incentives in Regulation and Innovation
2000, Tilburg University

Ralph Siebert
Innovation, Research Joint Ventures, and Multiproduct Competition
2000, Humboldt-Universität zu Berlin
http://dochost.rz.hu-berlin.de/dissertationen/siebert-ralph-2000-03-23/

Damien J. Neven, Lars-Hendrik Röller (Eds.)
The Political Economy of Industrial Policy in Europe and the Member States
2000, edition sigma

Jianping Yang
Bankbeziehungen deutscher Unternehmen: Investitionsverhalten und Risikoanalyse
2000, Deutscher Universitäts-Verlag

Christoph Schenk
Cooperation between Competitors – Subcontracting and the Influence of Information, Production and Capacity on Market Structure and Competition
1999, Humboldt-Universität zu Berlin
http://dochost.rz.hu-berlin.de/dissertationen/schenk-christoph-1999-11-16

Horst Albach, Ulrike Görtzen, Rita Zobel (Eds.)
Information Processing as a Competitive Advantage of Japanese Firms
1999, edition sigma

Dieter Köster
Wettbewerb in Netzproduktmärkten
1999, Deutscher Universitäts-Verlag

Christian Wey
Marktorganisation durch Standardisierung: Ein Beitrag zur Neuen Institutionenökonomik des Marktes
1999, edition sigma

Horst Albach, Meinolf Dierkes, Ariane Berthoin Antal, Kristina Vaillant (Hg.)
Organisationslernen – institutionelle und kulturelle Dimensionen
WZB-Jahrbuch 1998
1998, edition sigma

Lars Bergman, Chris Doyle, Jordi Gual, Lars Hultkrantz, Damien Neven, Lars-Hendrik Röller, Leonard Waverman
Europe's Network Industries: Conflicting Priorities - Telecommunications
Monitoring European Deregulation 1
1998, Centre for Economic Policy Research

Manfred Fleischer
The Inefficiency Trap
Strategy Failure in the
German Machine Tool Industry
1997, edition sigma

Christian Göseke
Information Gathering and Dissemination
The Contribution of JETRO to
Japanese Competitiveness
1997, Deutscher Universitäts-Verlag

DISCUSSION PAPERS 2002

Fredrik Andersson Kai A. Konrad	**Human Capital Investment and Globalization in Extortionary States**	FS IV 02 – 01
Lars-Hendrik Röller Christian Wey	**Merger Control in the New Economy**	FS IV 02 – 02
Talat Mahmood Klaus Schömann	**Die Determinanten der Mirgrationsentscheidung von IT-Hochschulabsolventen aus Pakistan – Empirische Befunde zur Ausgestaltung der deutschen „Green Card“**	FS IV 02 – 03
Talat Mahmood Klaus Schömann	**The Determinants of the Migration Decision of IT-graduates from Pakistan: Empirical Evidence for the Design of a German "Green Card"**	FS IV 02 – 03a
Jos Jansen	**The Effects of Disclosure Regulation on Innovative Firms: Common Values**	FS IV 02 – 04
Jos Jansen	**The Effects of Disclosure Regulation on Innovative Firms: Private Values**	FS IV 02 – 05
Günter Franke Harris Schlesinger Richard C. Stapleton	**Multiplicative Background Risk**	FS IV 02 – 06
Tomaso Duso	**On the Politics of the Regulatory Reform: Econometric Evidence from the OECD Countries**	FS IV 02 – 07
Johan Lagerlöf Paul Heidhues	**On the Desirability of an Efficiency Defense in Merger Control**	FS IV 02 – 08
Olivier Cadot Lars-Hendrik Röller Andreas Stephan	**Contribution to Productivity or Pork Barrel? The Two Faces of Infrastructure Investment**	FS IV 02 – 09
Justus Haucap Christian Wey	**Unionization Structures and Firms’ Incentives for Productivity Enhancing Investments**	FS IV 02 – 10
Heidrun C. Hoppe Emre Ozdenoren	**Intermediation in Innovation**	FS IV 02 – 11
Rainer Nitsche	**On the Effectiveness of Anti-Predation Rules**	FS IV 02 – 12
Daniel Krähmer	**Entry and Experimentation in Oligopolistic Markets for Experience Goods**	FS IV 02 – 13
J. Peter Murmann	**The Coevolution of Industries and National Institutions: Theory and Evidence**	FS IV 02 – 14
Kai A. Konrad	**Terrorism and the State**	FS IV 02 – 15
Robert Nuscheler	**Physician Reimbursement, Time-Consistency and the Quality of Care**	FS IV 02 – 16
Fredrik Andersson Kai A. Konrad	**Taxation and Education Investment in the Tertiary Sector**	FS IV 02 – 17

Jan Boone	**'Be nice, unless it pays to fight': A New Theory of Price Determination with Implications for Competition Policy**	FS IV 02 – 18
Kai A. Konrad	**Altruism and Envy in Contests: An Evolutionarily Stable Symbiosis**	FS IV 02 – 19
Helmut Bester Kai A. Konrad	**Delay in Contests**	FS IV 02 – 20
Kjell Erik Lommerud Bjørn Sandvik Odd Rune Straume	**Good Jobs, Bad Jobs and Redistribution**	FS IV 02 – 21
Steffen Huck Vicki Knoblauch Wieland Müller	**On the Profitability of Collusion in Location Games**	FS IV 02 – 22
Ralph Siebert	**Learning by Doing and Multiproduction Effects over the Life Cycle: Evidence from the Semiconductor Industry**	FS IV 02 – 23
Jürgen Bracht Saul Lach Eyal Winter	**Modeling Oligopolistic Price Adjustment in Micro Level Panel Data**	FS IV 02 – 24
Steffen Huck Kai A. Konrad	**Strategic Trade Policy and the Home Bias in Firm Ownership Structure**	FS IV 02 – 25
Daniel Krähmer	**Delegation versus Authority**	FS IV 02 – 26
Thomas Knaus Robert Nuscheler	**Incomplete Risk Adjustment and Adverse Selection in the German Public Health Insurance System**	FS IV 02 – 27
Kurt R. Brekke Robert Nuscheler Odd Rune Straume	**Quality and Location Choices under Price Regulation**	FS IV 02 – 28
Kai A. Konrad	**Inverse Campaigning**	FS IV 02 – 29
Sebastian Kessing	**A Note on the Determinants of Labour Share Movements**	FS IV 02 – 30
Sebastian Kessing	**Employment Protection and Product Market Competition**	FS IV 02 – 31
Michal Grajek	**Identification of Network Externalities in Markets for Non-Durables**	FS IV 02 – 32
Robert M. Adams Lars-Hendrik Röller Robin C. Sickles	**Market Power in Outputs and Inputs: An Empirical Application to Banking**	FS IV 02 – 33
Tomaso Duso Damien J. Neven Lars-Hendrik Röller	**The Political Economy of European Merger Control: Evidence using Stock Market Data**	FS IV 02 – 34
Tomaso Duso Astrid Jung	**Market Conduct and Endogenous Lobbying: Evidence from the U.S. Mobile Telecommunications Industry**	FS IV 02 – 35

DISCUSSION PAPERS 2003

Bei Ihren Bestellungen von WZB-Papers schicken Sie bitte unbedingt einen an Sie adressierten Aufkleber mit sowie je paper eine Briefmarke im Wert von 0,51 Euro oder einen "Coupon Reponse International " (für Besteller aus dem Ausland)

Please send a self addressed label and postage stamps in the amount of 0.51 Euro or a "Coupon-Reponse International" (if you are ordering from outside Germany) for each WZB-paper requested

Bestellschein *Order Form*

Absender / Return Address:

Wissenschaftszentrum Berlin
für Sozialforschung
Presse- und informationsreferat
Reichpietschufer 50

D-10785 Berlin-Tiergarten

Hiermit bestelle ich folgende(s) Discussion paper(s):

Please send me the following Discussion paper(s):

Bestell-Nr. / Order no.	**Autor/in, Kurztitel /Author(s) / Title(s) in brief**