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ABSTRACT

Policy-Motivated Candidates, Noisy Platforms, and Non-Robustness

by Johan Lagerlöf*

A model of a two-candidate election is developed in which the candidates are mainly office-motivated but also to some arbitrarily small extent policy-motivated, and their chosen platforms are to some arbitrarily small extent noisy. The platforms' being noisy means that if a candidate has chosen a particular platform, the voters' perception is that she has, with positive probability, actually chosen some other platform. It is shown that (i) an equilibrium in which the candidates play pure exists whether or not there is a Condorcet winner among the policy alternatives, and (ii) in this equilibrium the candidates choose their ideal points, which means that the platforms do not converge.

JEL classification: D64; D82
Keywords: Electoral competition; Policy motivation; Noisy commitment; Convergence; Robustness

ZUSAMMENFASSUNG

Politikmotivierte Kandidaten, Plattformen mit Rauschen und Nichtrobustheit

Es wird ein Wahlkampf mit zwei Kandidaten modelliert, in dem die Kandidaten im wesentlichen ein Interesse am Amt haben, aber auch in geringem Umfang eine bestimmte Politik bevorzugen. Die politischen Plattformen, für die sie sich entschieden haben, werden in geringem Umfang mit einem „Rauschen" an den Wähler weitergegeben. Das Rauschen der Plattformen ist wie folgt zu definieren. Wenn ein Kandidat sich für eine bestimmte Plattform entschieden hat, dann führt das Rauschen dazu, daß der Kandidat in der Wahrnehmung des Wählers mit einer positiven Wahrscheinlichkeit sich für eine andere als die gewählte Plattform entschieden hat. Es wird gezeigt, daß: (i) ein Gleichgewicht existiert, in dem die Kandidaten reine Strategien spielen, unabhängig davon, ob es einen Condorcet-Gewinner zwischen den bestehenden politischen Alternativen gibt; und (ii) in diesem Gleichgewicht entscheiden sich die Kandidaten für ihre eigenen Ideal-Positionen, d. h. die Plattformen konvergieren nicht.

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"The strong convergence results in one dimension have proven rather robust; equally robust have been the instability results in multiple policy dimensions." Gary M. Miller (1997, p. 1185)

1 Introduction

Since the classic contributions of Hotelling (1929) and Downs (1957), a large body of literature on two-candidate elections has developed. Two main themes in this literature have been the questions: (i) Where are the equilibrium platforms of the candidates located? and (ii) Under what restrictions do equilibria (in pure strategies) exist in settings that are more general than the Hotelling-Downs model? Concerning the first question, the famous answer is the so-called full-convergence result: in equilibrium, the candidates choose the same platform, namely the favorite policy of the median voter. As for the second question, a common conclusion in the literature is that, if the policy space has more than one dimension, an equilibrium in which the candidates do not randomize in their platform choices exists only under conditions which are very stringent and which fail to hold in many natural settings.[1]

This paper sheds light on the question whether these results are *robust*, that is, whether the full-convergence result and the result that pure-strategy equilibria typically do not exist in settings with more than one dimension (or when there is no Condorcet winner among the policy alternatives) still hold true *if one alters the model only slightly.* There are two things that I simultaneously add to the standard setting, although both are needed only in an arbitrarily small amount for my results go through. First, I assume that the candidates do not only care about winning the election; they also care about policy per se. Second, I assume that the candidates' platforms are noisy in the eyes of the voters; that is, if a candidate has chosen a particular platform, the voters' perception is that she has, with positive probability, actually chosen some other platform. One may think of this assumption as representing (exogenous) errors or misunderstandings in the transmission

[1] See for example Plott (1967) and McKelvey (1979) or the survey in Austen-Smith and Banks (1999).

of information about the true platforms from candidates via mass media to voters.

In the literature, the most commonly used approach for obtaining non-convergence of policy platforms is to assume that (i) the candidates have policy-preferences and (ii) they are uncertain about the outcome of the election. Papers that make these assumptions include Calvert (1985), Roemer (1994), and Wittman (1983). The degree to which the platforms diverge in these papers, however, varies continuously with the amount of uncertainty and with the relative weight the candidates put on policy. In particular, as either the amount of uncertainty or the weight on policy tends to zero, the equilibrium platforms approach each other. Hence, with regard to these two assumptions, the full convergence result is robust. As Calvert (1985: 70) argues, this is an important result:

> Such robustness against departures from the basic assumptions is vital to the development of any positive theory. As a model of general process, the basic multidimensional voting model serves mainly as a tractable guide to our thinking about electoral competition (and to more detailed modeling), and not necessarily as a direct generator of hypotheses about real-world elections. It must assume away many features of the real world. If the model's conclusions are robust against complications of that abstract picture, then it has captured the essence of electoral competition; it is a useful abstraction.

In the present paper, however, I show that such a robustness result fails to hold if we allow the candidates' platforms to be noisy in the sense described above. More precisely, if the candidates are to some arbitrarily small extent policy motivated *and* if the platforms are to some arbitrarily small extent noisy, then there is an equilibrium in which both candidates choose their favorite policies. This means that the platforms do *not* converge, and this non-convergence result is obtained by altering the original Hotelling-Downs model only slightly. Although the literature has identified many alternative ways of obtaining non-convergence, there is, to the best of my knowledge, no other result of non-robustness.

The result that the candidates choose their favorite policies is close in spirit to that of Alesina (1988). He points out that if the candidates care about policy to some small extent and if they are *not* able to precommit to policy platforms, then the candidates will, once they have been elected, choose their ideal policy. In the present paper, however, it is assumed that the candidates *can* commit. Still, with the additional assumption that the platforms are to some small extent noisy, it turns out that the candidates' ideal policies have a very strong drawing power; as a result, the equilibrium outcome is the same as in Alesina's model.

The second main contribution of this paper is to show that the typical non-existence of pure-strategy equilibria in a setting where there is no Condorcet winner among the policy alternatives can be remedied by, again, assuming that the candidates are to some arbitrarily small extent policy motivated and the platforms are to some arbitrarily small extent noisy. It is shown that, if we slightly alter the standard model in this fashion, there is an equilibrium in which the candidates play pure; in particular, in this equilibrium, the candidates choose their own favorite policies.

In order to facilitate an explanation of the results and the intuition behind them, the paper starts out in Sections 2 and 3 by considering two simple examples. The example in Section 2 concerns the full convergence result whereas the example in Section 3 concerns the non-existence of pure-strategy equilibria in a setting where there is no Condorcet winner among the policy alternatives. In Section 4 a more general model is analyzed, and it is shown that the main results from the two examples still hold true. The results and the insights in this paper draw heavily on an important and thought-provoking paper by Bagwell (1995) on the robustness of the first-mover-advantage result in the industrial organization literature. Section 5 reviews this paper and some others that extend and criticize Bagwell's analysis. That section also provides a concluding discussion. An appendix contains proofs of those results that are not proven in the main body of the paper.

2 Non-convergence of equilibrium platforms

Let us consider the following very stylized electoral-competition game played between two candidates and one voter. The candidates first, simultaneously and independently, choose one electoral platform each; candidate i's (for $i \in \{1,2\}$) chosen platform is denoted by x_i. The voter then votes for one of the candidates. The candidate who wins the election (i.e., the one who is voted for by the single voter) must implement her previously chosen platform.[2] There are three policy alternatives, Left (L), Center (C), and Right (R). In order to make the example as simple as possible, however, the candidates choice sets are restricted so that candidate 1 is constrained to choose her platform x_1 from the set $\{L, C\}$, and candidate 2 must choose her platform x_2 from the set $\{C, R\}$.

I will carry out the analysis under two different assumptions about the informational structure of the game. The first assumption says that the voter, prior to making his voting decision, can observe the candidates' chosen platforms perfectly. The second assumption says that the platforms are *noisy*; that is, the voter can observe only an (exogenous) signal about the candidates' platforms. The signal is denoted by $s = (s_1, s_2)$, where $s_1 \in \{L, C\}$ and $s_2 \in \{C, R\}$. The signal technology works as follows. When a candidate chooses a particular platform, the voter will observe a signal specifying that same platform with probability $1 - \varepsilon$; if the signal does not specify the right platform, then the other possible platform is specified (see Table 1). The realizations of s_1 and s_2 are independent. The noise parameter, ε, is strictly positive, but it should be thought of as being small; in particular it is assumed that $\varepsilon \in \left(0, \frac{1}{3}\right]$.

$\Pr(s_1 = \widetilde{s} \mid x_1 = \widetilde{x})$	$\widetilde{s} = L$	$\widetilde{s} = C$	$\Pr(s_2 = \widetilde{s} \mid x_2 = \widetilde{x})$	$\widetilde{s} = C$	$\widetilde{s} = R$
$\widetilde{x} = L$	$1 - \varepsilon$	ε	$\widetilde{x} = C$	$1 - \varepsilon$	ε
$\widetilde{x} = C$	ε	$1 - \varepsilon$	$\widetilde{x} = R$	ε	$1 - \varepsilon$

Table 1: *The signal technology.* The probability that each "sub-signal" s_i is correct equals $(1 - \varepsilon)$, and the two sub-signals s_1 and s_2 are independent.

[2] The assumption that electoral promises are binding is discussed in Section 4.

The three players have preferences over the policy alternatives L, C, and R, although the two candidates primarily care about being in office (we say that a candidate "is in office" if she has won the election). In particular, each candidate gets the incremental payoff 1 if being in office and the incremental payoff 0 otherwise. In addition, each candidate gets the incremental payoff $a \in \left(0, \frac{1}{4}\right)$ if her favorite policy is implemented; candidate 1's favorite policy is L and candidate 2's favorite policy is R. The voter gets the payoff 1 if his favorite policy C is implemented and the payoff 0 if either policy L or policy R is implemented. In sum, the players' preferences are described by Table 2.

	candidate 1	candidate 2	voter
$1L$	$1+a$	0	0
$1C$	1	0	1
$2C$	0	1	1
$2R$	0	$1+a$	0

Table 2: *The players' payoffs.* The abbreviation $1C$ (respectively, $2C$) means that candidate 1 (respectively, 2) is in office and chooses policy C, and similarly with the abbreviations $1L$ and $2R$. It is assumed that $a \in \left(0, \frac{1}{4}\right)$.

Perfect observability To start with, let us analyze the model where the platforms are perfectly observable. The solution concept that will be employed is that of subgame perfect equilibrium. Let us begin by considering the voter's decision for whom to vote, given that he has observed some platforms x_1 and x_2. Let r_{jk} be the probability with which the voter votes for candidate 1 given that $x_1 = j$ (for $j \in \{L, C\}$) and $x_2 = k$ (for $k \in \{C, R\}$). Clearly, if candidate 1 has chosen C and candidate 2 has chosen R, then the voter will vote for candidate 1 ($r_{CR} = 1$); and if candidate 1 has chosen L and candidate 2 has chosen C, then the voter will vote for candidate 2 ($r_{LB} = 0$). For the two remaining possible configurations of platforms, the voter is indifferent between the candidates; this means that the probabilities r_{CC} and r_{LR} can take on any value between zero and one.

We can substitute this optimal behavior on the part of the voter into the expressions for the candidates' expected payoffs. Doing this yields the following 2×2 game matrix:

	$x_2 = C$	$x_2 = R$
$x_1 = L$	$0, 1$	$r_{LR}(1+a), (1-r_{LR})(1+a)$
$x_1 = C$	$r_{CC}, 1 - r_{CC}$	$1, 0$

(1)

Candidate 1 is here the row player and candidate 2 is the column player. The first expression in each cell of the matrix is candidate 1's payoff and the second one is player 2's payoff. Let us first consider the special case where the candidates get elected with equal probability whenever the voter is indifferent between them: $r_{CC} = r_{LR} = 1/2$. The game matrix above then simplifies to the following.

	$x_2 = C$	$x_2 = R$
$x_1 = L$	$0, 1$	$\frac{1+a}{2}, \frac{1+a}{2}$
$x_1 = C$	$\frac{1}{2}, \frac{1}{2}$	$1, 0$

(2)

Clearly, since $a < 1$, this game between the two candidates has a unique Nash equilibrium where both of them, with probability one, choose platform C.

For the general case where the probabilities r_{CC} and r_{LR} can take on any value between zero and one, the analysis is a bit more involved, and it is therefore relegated to the Appendix. The result, however, is in line with the above: in any equilibrium, the candidate who wins the election chooses platform C with probability one. Indeed, in those equilibria in which $r_{CC} \in (0,1)$, *both* candidates choose platform C with probability one. Letting p be the probability with which candidate 1 chooses her favorite policy L, and letting q be the probability with which candidate 2 chooses her favorite policy R, we have the following.

Proposition 1. *Consider the game where the platforms are perfectly observable. This game has a continuum of subgame perfect equilibria. In any one of these equilibria, either:*

a) $(p, q, r_{LB}, r_{LR}, r_{CC}, r_{CR}) = (0, 0, 0, r_{LR}, r_{CC}, 1)$, *for any* $r_{CC} \in (0,1)$ *and* $r_{LR} \in [0,1]$*; or*

b) $(p, q, r_{LB}, r_{LR}, r_{CC}, r_{CR}) = (p, 0, 0, r_{LR}, 0, 1)$, *for any* p *and* r_{LR} *such that* $p(1 - r_{LR})(1+a) \leq 1$*; or*

c) $(p, q, r_{LB}, r_{LR}, r_{CC}, r_{CR}) = (0, q, 0, r_{LR}, 1, 1)$, *for any* q *and* r_{LR} *such that* $qr_{LR}(1 + a) \leq 1$.

This is an example of the full-convergence result. The candidates primarily care about winning the election, and the voter will vote for a candidate who is choosing the voter's favorite policy C (if anyone of the candidates is doing this). Hence, there is a pressure on the candidates to choose the voter's favorite policy instead of their own one. Although there always exist equilibria in which one of the candidates is choosing her own favorite policy platform with positive probability (and sometimes even with probability one), such a candidate must be losing the election for sure.

Noisy platforms Let us now consider the version of the model where the candidates' platforms are noisy, that is, where the voter only can observe the signal s. Notice that this game has no subgames (except for the subgame that consists of the whole game), since for any configuration of platforms the voter will observe all possible signals with positive probability. This means that subgame perfection does not have any bite, and I will therefore focus attention on the set of all Nash equilibria.

Let us start with looking for an equilibrium in which both candidates choose platform C, i.e., where $(x_1, x_2) = (C, C)$. Surprisingly, it turns out that such an equilibrium does not exist. To see why, recall that the informational structure of the game is such that the voter cannot directly observe the candidates' actions, only the noisy signal. By definition of a Nash equilibrium, however, the voter's beliefs about the candidates' actions are correct. This means that in an equilibrium in which $(x_1, x_2) = (C, C)$, the signal about the candidates' actions is completely uninformative: The voter correctly believes that both candidates have chosen platform C with probability one, so any signal not specifying $s_1 = C$ and $s_2 = C$ must be an incorrect signal.

To see in a simple way how the logic of the remaining part of the argument works, let us momentarily assume that each candidate gets elected with the same probability whenever the voter is indifferent between them; that is, if the voter is indifferent between the candidates after having observed, say, the signals $s = (C, C)$ and $s = (L, C)$, candidate 1 gets elected with the same probability whether $s_1 = C$ or $s_1 = L$. Now suppose that we are in an equilibrium in which $(x_1, x_2) = (C, C)$, and imagine that one of the candidates, say candidate 1, deviates from her action $x_1 = C$ and instead plays $x_1 = L$. Then the voter will indeed observe the sub-signal $s_1 = L$ with a much higher probability. Yet, the voter will not be able to infer that candidate 1 has deviated; he will believe that $x_1 = C$ regardless of whether $s_1 = C$ or $s_1 = L$. Hence, the voter will be indifferent between the candidates both when he has observed the signal $s = (C, C)$ and when he has observed the signal $s = (L, C)$. Accordingly, candidate 1 will be winning with the same probability if she deviates to $x_1 = L$ as if she did not and, if she indeed wins, she will win with her favorite policy. It follows that there cannot be an equilibrium where $(x_1, x_2) = (C, C)$, since then the candidates would have an incentive to deviate to their favorite policies.

The general case — where we allow for the possibility that the voter, when he is indifferent between the candidates, can randomize with *any* probability and where he possibly makes this probability contingent on the signal that he observes — is a bit more involved. In the proof of Proposition 2 below, however, it is shown that the result still goes through: there cannot be an equilibrium where both candidates choose platform C. Indeed, it turns out that in any Nash equilibrium of the game between the three players, both candidates choose their favorite policies with probability one.

Before stating Proposition 2, we need to introduce some more notation. For the voter's voting decision I use the same notation as in the model with perfect observability, although the voting decision is now made contingent on the signal s and not the actual platforms. Hence, let r_{jk} be the probability with which the voter votes for candidate 1 given that $s_1 = j$ and $s_2 = k$ (for $j \in \{L, C\}$ and $k \in \{C, R\}$). Moreover, let r be a vector having these four probabilities as its components; that is, $r = (r_{LC}, r_{LR}, r_{CC}, r_{CR})$. Finally, let Ω be the set of all $r \in [0, 1]^4$ such that inequalities (3) and (4) below are met:

$$\frac{r_{LC}\varepsilon + r_{LR}(1-\varepsilon)}{1-\varepsilon-\varepsilon(1+a)} \geq \frac{r_{CC}\varepsilon + (1-\varepsilon)r_{CR}}{(1-\varepsilon)(1+a)-\varepsilon}, \tag{3}$$

$$\frac{1-r_{LR}+(r_{LR}-r_{CR})\,\varepsilon}{1-\varepsilon-\varepsilon\,(1+a)} \geq \frac{1-r_{LC}+(r_{LC}-r_{CC})\,\varepsilon}{(1-\varepsilon)\,(1+a)-\varepsilon}. \tag{4}$$

One can easily verify that, for example, the vectors $\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$, $(1,1,1,1)$, and $(0,0,0,0)$ all belong to Ω; that is, if the voter always randomizes fifty-fifty or always votes for one of the candidates, then r belongs to Ω.

The following result is proven in the Appendix.

Proposition 2. *Consider the game with noisy platforms. This game has a continuum of Nash equilibria. In any Nash equilibrium, however, candidate 1 chooses L with probability 1 and candidate 2 chooses R with probability 1; the voter may vote according to any r such that $r \in \Omega$.*

Hence, there are many Nash equilibria of the game with noisy platforms, because the voter may randomize in many different ways. Yet, *all of them* involve both candidates' choosing their favorite platform. This result is in sharp contrast to Proposition 1 and the full-convergence result in general. For the present result to hold, the candidates must to some extent care about policy ($a > 0$) and the platforms must to some extent be noisy ($\varepsilon > 0$). However, the result holds for *any* small a and ε greater than zero.

3 No Condorcet winner

Now consider the following simple electoral-competition game played between two candidates and three voters. The candidates are indexed by $i \in \{1,2\}$, and the voters are indexed by $j \in \{1,2,3\}$. There are again three policy alternatives, Left (L), Center (C), and Right (R), although now both candidates are free to choose any policy. The sequence of events is also the same as in the previous section. First the two candidates simultaneously and independently choose platforms, $x_i \in \{L,C,R\}$. Then the three voters simultaneously and independently vote for one of the candidates. The candidate who receives a majority of votes wins office and must implement his chosen platform.

In a first version of the game all three voters can, prior to their voting, observe the candidates' chosen platforms perfectly. In a second version of the game, voter 2 can only observe a noisy signal $s \in \{L, C, R\}$ about candidate 2's platform; voter 2 can observe candidate 1's platform perfectly, and voters 1 and 3 can observe both candidates' platforms perfectly.[3] The signal technology is similar to the one in the previous section. More specifically, when candidate 2 chooses a particular platform, then voter 2 will observe a signal specifying that same platform with probability $1-\varepsilon$; if the signal does not specify the right platform, then the two other platforms have an equal chance of being specified (see Table 3). The noise parameter, ε, is strictly positive, but it should be thought of as being small; in particular it is assumed that $\varepsilon \in \left(0, \frac{1}{2}\right)$.

$\Pr\left(s = \widetilde{s} \mid x_2 = \widetilde{x}\right)$	$\widetilde{s} = L$	$\widetilde{s} = C$	$\widetilde{s} = R$
$\widetilde{x} = L$	$1-\varepsilon$	$\varepsilon/2$	$\varepsilon/2$
$\widetilde{x} = C$	$\varepsilon/2$	$1-\varepsilon$	$\varepsilon/2$
$\widetilde{x} = R$	$\varepsilon/2$	$\varepsilon/2$	$1-\varepsilon$

Table 3: *The signal technology.* The probability that the signal s is correct equals $(1-\varepsilon)$, where $\varepsilon \in \left(0, \frac{1}{2}\right)$.

The five players' preferences are specified in Table 4. The voters' preferences are such that voter 1 prefers L to R and R to C; voter 2 prefers C to L and L to R; and voter 3 prefers R to C and C to L. These preferences are chosen to make sure that there is no policy alternative that is a Condorcet winner. That is, if the voters vote directly on policy alternatives (and if they vote for their favorite policy), then there is no alternative that can beat both the other ones in a pairwise comparison: $L \prec C \prec R \prec L$ (where "$\prec$" means "is beaten in a pairwise comparison by"). It is also assumed that if the candidates have chosen the same platform, then all voters prefer candidate 1 (this is captured by the term $\lambda \in (0,1)$).[4] The candidates have preferences

[3]That is, it is only the platform of *one* of the candidates that is noisy, and this platform is noisy only in the eyes of *one* of the three voters. This modeling choice serves two purposes. First, it shows that only a very small amount of noise is needed for the results to through (but it is important both *which* candidate's platform is noisy and in the eyes of *which* voter). Second (and perhaps more importantly), it simplifies parts of the analysis. The more general model in Section 4 will assume that *both* candidates' platforms are noisy in the eyes of *all* voters.

[4]The assumption that the voters are biased in favor of candidate 1 is made for the sake of tractability: it simplifies the analysis considerably since it breaks ties.

similar to the ones in the model of the previous section: they mainly care about winning the election, but they also care to some extent about policy (this is captured by the term $a \in (0,1)$); candidate 1's favorite policy is L, and candidate 2's favorite policy is R.

	candidate 1	candidate 2	voter 1	voter 2	voter 3
$1L$	$1+a$	0	$1+\lambda$	λ	$-1+\lambda$
$2L$	a	1	1	0	-1
$1C$	1	0	$-1+\lambda$	$1+\lambda$	λ
$2C$	0	1	-1	1	0
$1R$	1	a	λ	$-1+\lambda$	$1+\lambda$
$2R$	0	$1+a$	0	-1	1

Table 4: *The players' preferences.* It is assumed that $a, \lambda \in (0,1)$

Perfect observability To start with, let us consider the benchmark case where the platforms are perfectly observable. We are looking for the subgame perfect equilibria of this game. I also impose the requirement that the voters do not use a weakly dominated strategy. That is, even if a voter's vote does not change the outcome of the election, he votes for a candidate if he prefers that candidate to the other candidate. From Table 4 it follows that candidate 2 will win the election in three cases, namely if the platforms are $(x_1, x_2) = (R, L)$, $(x_1, x_2) = (L, C)$, or $(x_1, x_2) = (C, R)$. For any other platform configuration candidate 1 will win the election.

Substituting this optimal behavior on the part of the voters into the expressions for the candidates' expected payoffs yields the following 3×3 game matrix:

	L	C	R
L	$1+a, 0$	$0, 1$	$1+a, 0$
C	$1, 0$	$1, 0$	$0, 1+a$
R	$a, 1$	$1, a$	$1, a$

(5)

As in the model in the previous section, candidate 1 is the row player and candidate 2 is the column player, and the first expression in each cell of the matrix is candidate 1's payoff and the second one is player 2's payoff. It can be easily verified that the strategic-form game depicted in (5) does not have any pure-strategy equilibrium. However, there is of course a mixed-strategy equilibrium of this game. In fact, there is a unique equilibrium of the game in which both candidates are randomizing over all three policy alternatives. Let $\sigma_i(x_k)$ be the probability with which candidate i chooses platform x_k. The proof of the following proposition is fairly straightforward and is therefore omitted.

Proposition 3. *Consider the model with perfect observability. This game has only one Nash equilibrium which is both subgame perfect and in which the voters do not use weakly dominated strategies. In this equilibrium the candidates behave as follows:*

$$\sigma_1(x_L) = \frac{1-a^2}{3-a^2}; \qquad \sigma_1(x_C) = \frac{1-a}{3-a^2}; \qquad \sigma_1(x_R) = \frac{1+a}{3-a^2};$$

$$\sigma_2(x_L) = \frac{1}{3-a^2}; \qquad \sigma_2(x_C) = \frac{1+a-a^2}{3-a^2}; \qquad \sigma_2(x_R) = \frac{1-a}{3-a^2}.$$

We see that if a, the parameter that captures the candidates' policy preferences, is small, both candidates choose all three platforms with approximately the same probability (one third).

Noisy platforms Let us now consider the version of the model where candidate 2's platform is noisy in the eyes of voter 2. As in the previous section, subgame perfection does not have any bite when the platforms are noisy, and the natural solution concept is therefore Nash equilibrium. As in the case with perfect observability, however, I also impose the requirement that the voters do not use a weakly dominated strategy. In the rest of this section and in the proof of Proposition 4, I will refer to such a Nash equilibrium simply as an "equilibrium." It turns out that in this model there is indeed an equilibrium where none of the candidates is randomizing in their choice of platform. In particular, there is an equilibrium in which both candidates choose their favorite policies.

To see that such an equilibrium exists, suppose that candidate 1 indeed chooses L and candidate 2 indeed chooses R. Voter 2 cannot observe candidate 2's choice directly, only the noisy signal; he correctly believes, however, that candidate 2 chooses R. This means that voter 2, given the candidates' equilibrium behavior, will vote for candidate 1 no matter which signal s he has observed (remember that voter 2 prefers L to R; see Table 4). Moreover, given the candidates' equilibrium behavior, voter 1 will vote for candidate 1 and voter 3 will vote for candidate 2. Hence, the winner is candidate 1. It remains to check that none of the candidates has an incentive to deviate. Candidate 1 certainly does not have such an incentive, since she is winning with her favorite policy. Nor does candidate 2 have a (strict) incentive to deviate since this would not make her win. The reason for this is that voter 2 cannot observe her chosen platform directly and ignores the signal. Hence, if candidate 2 deviated to C, for example, voter 2 would not change his voting behavior. Voters 1 and 3 can indeed observe candidate 2's deviation; their preferences, however, are such that they still would not change their voting behavior (see Table 4). If deviating to L, candidate 2 certainly would not win.

There also exists another equilibrium where both candidates play pure, namely where both candidates choose L with probability one. Also in this equilibrium candidate 2 loses the election with probability one. The following is proven in the Appendix.

Proposition 4. *Consider the model with noisy platforms. In this model there is an equilibrium in which candidate 1 chooses L with probability one and candidate 2 chooses R with probability one. There is also an equilibrium in which both candidates choose L with probability one. There is no other equilibrium in which none of the candidates is randomizing in her choice of platform.*

4 A general model

Consider an electoral competition with two candidates, labeled $j \in \{1, 2\}$, and $N \geq 1$ voters, labeled $i \in \{1, \ldots, N\}$. The sequence of events is as follows. First the candidates simultaneously commit to platforms $x_j \in X \subset \Re^d$ (for some positive integer d). Second, the voters observe the chosen platforms with some, possibly very small, noise. Formally, each voter i observes a signal $s_i = (s_{i,1}, s_{i,2}) \in S = X^2$, where each $s_{i,j}$ is independently drawn from a distribution $F(\cdot \mid x_j)$ which has positive support on the whole of X. Finally, each voter votes for one of the candidates. The candidate who receives the largest number of votes wins office and gets his chosen policy implemented. In case of a tie, each candidate wins with equal probability.

On the part of the candidates, only pure strategies will be considered. Hence, a strategy for candidate j is a d-dimensional vector x_j. The voters, however, are allowed to randomize in their voting decisions. A strategy for voter i is a mapping r_i from the set of possible signals, S, to the one-dimensional unit simplex, $[0, 1]$. We make the interpretation that $r_i(s)$ is the probability that voter i votes for candidate 1 conditional on having observed a signal s. Denote a vector of voting decisions by $r = (r_1, \ldots, r_N)$.

Each voter has preferences over the policy space X and the set of candidates $\{1, 2\}$ that can be represented by the utility function $U^i(x, j)$, where x is the winning candidate's policy. The candidates care about being in office and about policy, although the latter may possibly be of only slight importance. Candidate j's utility if policy x is implemented and she is (respectively, is not) in office is $V^j(x, 1)$ (respectively, $V^j(x, 0)$). It is assumed that there is an $\widehat{x}_j \in X$ such that, for $j \in \{1, 2\}$,

$$V^j(\widehat{x}_j, 1) > V^j(x, 1) \qquad \text{all } x \neq \widehat{x} \tag{6}$$

and that

$$V^j(\widehat{x}_j, 1) > V^j(\widehat{x}_j, 0)\,. \tag{7}$$

Hence, both candidates have an "ideal policy" $\widehat{x}_j$; and, at least if her ideal policy is implemented, a candidate prefers to be in office to not be in office.

From what is said above it follows that, at the stage where a candidate is to choose her platform, she does not know what signals the voters will observe. Nor does she know what the realizations of any randomized strategies on the part of the voters will be. Hence, the outcome of the election will be uncertain for her.[5] Given a vector of voting decisions r and a pair of platforms (x_1, x_2), let $P^j(x_1, x_2, r)$ be the probability that candidate j wins the

[5] Yet another source of uncertainty is the above-mentioned fact that, in case of a tie, a lottery will decide which candidate wins.

election. The candidates and the voters are assumed to be expected utility maximizers. Candidate j's expected utility is denoted by EV^j, where

$$EV^j(x_1, x_2, r) = P^j(x_1, x_2, r)\, V^j(x_j, 1) + \left[1 - P^j(x_1, x_2, r)\right] V^j(x_k, 0) \quad (8)$$

for $j, k \in \{1, 2\}$ and $k \neq j$. Voter i's expected utility at the stage where he makes his voting decision is denoted by EU^i. Since the candidates are constrained to use pure strategies and, in an equilibrium, the voters correctly anticipate the candidates' actions, the signals that the voters observe will not be informative. Hence, we can write voter i's expected utility EU^i as

$$EU^i(x_1, x_2, r) = P^1(x_1, x_2, r)\, U^i(x_1, 1) + P^2(x_1, x_2, r)\, U^i(x_2, 2). \quad (9)$$

The equilibrium concept employed is that of Nash equilibrium (conditions E1 and E3 below). In addition it is required that none of the candidates is in equilibrium using a strategy that is weakly dominated (conditions E2 and E4). Formally, an *equilibrium* of the model described is a pair of candidate platforms (x_1^*, x_2^*) and a list of voting decisions $(r_1^*, \ldots, r_N^*)$ such that: (E1) for all $j, k \in \{1, 2\}$ and $k \neq j$,

$$EV^j(x_1^*, x_2^*, r^*) \geq EV^j(x_j, x_k^*, r^*), \qquad \forall x_j \in X; \quad (10)$$

(E2) there is no $x_j^{'} \in X$ such that

$$EV^j\left(x_j^{'}, x_k, r\right) \geq EV^j(x_j^*, x_k, r), \qquad \forall (x_k, r) \in X \times \{1, 2\}^N, \quad (11)$$

for all $j, k \in \{1, 2\}$ and $k \neq j$, with the inequality holding strictly for some (x_k, r); (E3) for all $i \in \{1, \ldots, N\}$,[6]

$$EU^i(x_1^*, x_2^*, r^*) \geq EU^i\left(x_1^*, x_2^*, r_i, r_{-i}^*\right), \qquad \forall r_i \in \{1, 2\}; \quad (12)$$

and (E4) there is no $r_i^{'} \in \{1, 2\}$ such that

$$EU^i\left(x_1, x_2, r_i^{'}, r_{-i}\right) \geq EU^i(x_1, x_2, r_i^*, r_{-i}), \qquad \forall (x_1, x_2, r_{-i}) \in X^2 \times \{1, 2\}^{N-1}; \quad (13)$$

with the inequality holding strictly for some (x_1, x_2, r_{-i}).

[6] I here use the standard notation $r_{-i}^* = \left(r_1^*, \ldots, r_{i-1}^*, r_{i+1}^*, \ldots, r_N^*\right)$. A similar notation is also used below.

Proposition 5. *An equilibrium exists. Moreover, in any equilibrium, both candidates choose their own ideal policies.*

PROOF: Recall from the model description that the candidates are constrained to play pure. That is, candidate j must choose $x_j = x_j^{'}$ with probability one for some $x_j^{'} \in X$. We need to show that (i) there exists such an equilibrium and (ii) in any such equilibrium $x_j^{'} = \widehat{x}_j$ for all $j \in \{1, 2\}$. Let us first prove (i). Suppose both candidates choose their own ideal policies with probability one, i.e., $x_j = \widehat{x}_j$. Moreover, suppose that none of the voters makes his voting choice contingent on the signal but votes for candidate j if $U^i(\widehat{x}_j) > U^i(\widehat{x}_k)$ for $k \neq j$; and if $U^i(\widehat{x}_1) = U^i(\widehat{x}_2)$, then voter i randomizes in some fashion that is not contingent on the signal. Let the corresponding vector of voting decisions be denoted by $\widehat{r}$; the probability that candidate j wins the election is thus $P^j(\widehat{x}_1, \widehat{x}_2, \widehat{r})$. We must show that none of the candidates and none of the voters has an incentive to deviate unilaterally from this behavior, and that their strategies are not weakly dominated (i.e., that conditions (E1)-(E4) are met). First, consider candidate 1. For this candidate not to have an incentive to deviate unilaterally, condition (E1) must be met. This requires that

$$EV^1(\widehat{x}_1, \widehat{x}_2, \widehat{r}) \geq EV^1(x_1, \widehat{x}_2, \widehat{r}) \Leftrightarrow P^1(\widehat{x}_1, \widehat{x}_2, \widehat{r}) \left[V^1(\widehat{x}_1, 1) - V^1(x_1, 1)\right] \geq 0 \tag{14}$$

If $P^1(\widehat{x}_1, \widehat{x}_2, \widehat{r}) = 0$, then this inequality holds with equality. If $P^1(\widehat{x}_1, \widehat{x}_2, \widehat{r}) > 0$, then (14) simplifies to $V^1(\widehat{x}_1, 1) \geq V^1(x_1, 1)$, which is true by definition of $\widehat{x}_1$. For candidate 1's strategy not to be weakly dominated, condition (E2) must hold. For condition (E2) *not* to hold, there must exist an $x_j^{'}$ such that

$$EV^1\left(x_1^{'}, \widehat{x}_2, r\right) \geq EV^1(\widehat{x}_1, \widehat{x}_2, r) \Leftrightarrow P^1(\widehat{x}_1, \widehat{x}_2, r)\left[V^1\left(x_1^{'}, 1\right) - V^1(\widehat{x}_1, 1)\right] \geq 0 \tag{15}$$

for all r. However, this inequality does not hold for any r such that $P^1(\widehat{x}_1, \widehat{x}_2, r) > 0$. Hence, condition (E2) must hold. For candidate 2 the arguments above are analogous. Now consider a voter. For a voter not to have a unilateral incentive to deviate, condition (E3) must be met. If voter i's vote cannot change the outcome of the election, then this condition clearly holds (with equality). Suppose that voter i's vote *can* change the outcome of the election. Then condition (E3) requires that he votes for his favorite candidate. This is consistent with what we postulated above about the voters' behavior. Finally, for a voter's strategy not to be weakly dominated, condition (E4) must hold. This condition is met if voter i always votes for his favorite candidate whenever $U^i(\widehat{x}_1) \neq U^i(\widehat{x}_2)$. This is also consistent with the above postulated behavior.

Let us now prove (ii). Suppose per contra that there is an equilibrium in which $x_1^{'} \neq \widehat{x}_1$ (the case $x_2^{'} \neq \widehat{x}_2$ follows the same logic). Now candidate 1's strategy, however, is weakly dominated; i.e., condition (E2) is violated. To see this one only needs to note that whenever r is such that $P^1(x_1, x_2, r) > 0$, candidate 1 can gain by deviating to $\widehat{x}$; and whenever r is such that $P^1(x_1, x_2, r) = 0$, candidate 1's choice of platform does not affect her payoff. $\square$

5 Concluding discussion

As already mentioned in the Introduction, the results and the insights of this paper draw heavily on Bagwell's (1995) analysis of the robustness of the first-mover advantage result. He investigates a noisy-leader game in which one player moves first and then a second player, before making his own move, observes a (possibly almost perfect) signal of the first player's action. Bagwell shows that, under a certain regularity condition,[7] "the set of pure-strategy Nash equilibrium outcomes for the noisy-leader game coincides exactly with the set of pure-strategy Nash equilibrium outcomes for the associated simultaneous-move game" (Bagwell, 1995: 272). Hence, he concludes, the first-mover advantage is eliminated. This conclusion is criticized by van Damme and Hurkens (1997), who show that in Bagwell's game and under his regularity condition, there always exists a mixed equilibrium that induces an outcome that is close to the equilibrium outcome of Bagwell's game without any noise. They also suggest an equilibrium selection theory that selects this

[7] Namely that the player moving last has, given any action of the first player, a unique best-reply action.

mixed equilibrium.[8]

The present paper is also closely related to a recent paper by Güth, Kirchsteiger, and Ritzberger (1998). They study an extension of Bagwell's noisy-leader game in which there are a set of leaders who all move simultaneously, whereupon another set of players, the followers, observe noisy signals about the leaders' actions and then choose their actions simultaneously. The main result of Güth et al. (1998) is that, for almost all games that they consider, there exists a subgame perfect equilibrium outcome of the game with no noise that is approximated by (possibly mixed) equilibrium outcomes of games with small noise. This result, however, relies on generic payoffs, so it does not necessarily hold true in knife-edge cases.

The contribution of the present paper has been to show what the implications of Bagwell's result are for models of electoral competition. We have seen that if the candidates' platforms are just a little bit noisy and if the candidates are to some small extent policy motivated, then: (i) there is an equilibrium in which both candidates choose their own ideal policies, which means that the equilibrium platforms do not converge; and (ii) even if there is no Condorcet winner among the policy alternatives, there is an equilibrium in which both candidates play pure.

In light of the literature review above, it is natural to ask the question how sensitive result (i) is to the focus on pure-strategy equilibria. We know from the more general analysis in Section 4 that the result that an equilibrium in which both candidates choose their favorite platforms exists is robust. It could be, however, that there also exists a mixed equilibrium in which the candidates' platforms approach each other as the noise gets very small. In the simple model considered in Section 2, mixed strategies were indeed allowed for, and it turned out that the candidates never randomize in equilibrium. Hence, in that model it does not exist an equilibrium that approximates

[8] There are also a couple of other papers that, using different approaches, investigate the problem of equilibrium selection in Bagwell's model. Oechssler and Schlag (1997) carry out an evolutionary-game-theory analysis of a simple version of Bagwell's model. They find that the pure-strategy equilibrium is selected by most evolutionary dynamics. Huck and Müller (2000) report on an experimental test of Bagwell's prediction. They do not find empirical support for his result. For other work related to Bagwell's result, see Adolph (1996), Levine and Martinelli (1998), and Maggi (1999).

the full-convergence result of the model in which platforms are perfectly observable. The results of Güth et al. (1998), however, suggest that if we generalized the model of Section 2 in some way, such a mixed equilibrium may appear. In particular, I conjecture that if the voter in that model also cared about the identity of the elected candidate and not only about policy, and if he thus strictly preferred, say, candidate 1 whenever the candidates' policies are identical, then a mixed equilibrium that approximates the full convergence result would indeed exist.[9] This kind of asymmetry, however, makes for a much more complex model, and I can unfortunately not report any results affirming or disaffirming the conjecture.

An important assumption that was made throughout the paper is that the candidates can commit to electoral platforms (even though these commitments are not perfectly observable). The usual justification for this assumption is that, although in principle feasible, deviating from an announced platform is prohibitively costly because of reputational concerns. Such reputational concerns were formally modeled by Alesina (1988), within the framework of a repeated game with an infinite horizon; see also Dixit, Grossman, and Gul (2000) for a generalization of Alesina's work. In Alesina's as well as in Dixit et al.'s framework, a political outcome "in the middle" is sustained through punishment strategies on the part of the two candidates; the punishments are carried out by each candidate against her competitor in case the competitor deviates. Alternatively, one could construct a similar model where the punishments were carried out by the voters. In either case, it is important that any deviation can be observed by the punisher.

This suggests that, also in a setting in which candidates cannot com-

[9] This conjecture of mine may very well be false. One reason for this is that what Güth et al. (1998) can show is that, except for knife-edge cases, *some* subgame perfect equilibrium of the game without noise can be approximated by an equilibrium of a game with small noise. In an electoral competion game with many voters who vote strategically, however, there are typically many subgame perfect equilibria (some of them unreasonable). The reason for this is that in each subgame (i.e., given a pair of platforms) there are always many Nash equilibria; for example, all voters voting for one of the candidates is an equilibrium, because given this voting behavior no single voter can gain by a unilateral deviation. This unreasonable behavior in the subgames can support equilibrium behavior on the part of the candidates that also is unreasonable. Hence, it might be, in principle, that only one of the unreasonable subgame perfect equilibrium outcomes can be approximated by (possibly mixed) equilibrium outcomes of games with small noise.

mit, the model feature that the candidates' choices (of actual policies or of announced policy platforms) can only be imperfectly observed may have important implications for the location of equilibrium policies. This should be particularly true if the signals that the players observe about the candidates actions are private to the receiver of the signal: What is the credibility of, say, candidate 2's (or the voters') threat to punish if candidate 1 cannot observe candidate 2's (noisy) observation of her action? An interesting topic for future research would be to explore these questions in a formal model; for a related analysis within a seller-buyer framework, see Bhaskar and van Damme (2000). It is my hope that the insights from the analysis of the present paper, with its single-period setting and its commitment assumption, will constitute a first step in understanding those more far-reaching questions.

Let me conclude by relating to a recent discussion in Persson and Tabellini (2000) on the role of the commitment assumption. They refer to models making this assumption as models of *preelection politics*. In such models, "[t]he essential political action ... takes place in the electoral campaign, and the role of the election is to select a particular policy" (p. 11). In models of *postelection politics* (i.e., where electoral promises are *not* binding), "the role of elections is very different .... Rather than directly selecting policies, voters select politicians on the basis of their ideology, competence, or honesty, or more generally, their behavior as incumbents" (pp. 12-13). Persson and Tabellini are of the opinion that existing research has not produced a clear consensus concerning the question whether electoral promises are binding or unimportant, and they call for more work on this issue: " ... progress in the field depends on our finding a way of resolving some of these tensions, building a bridge between pre- and postelection politics ..." (p. 14).

The results of the present paper can hopefully constitute a small contribution to that research agenda by showing that, even if electoral promises are assumed to be binding, a small amount of noise in the transmission of these promises to the electorate can make the model look like one of post- rather than preelection politics: The candidates' own policy preferences have an even stronger drawing power than has been acknowledged in the previous literature.

6 Appendix

Proof of Proposition 1 Let us first look for Nash equilibria of the reduced-form game in (1) where $r_{CC} \in (0,1)$. It is immediate from the game matrix that $(p,q) = (0,0)$ is a Nash equilibrium of this game. One may also easily verify that neither $(p,q) = (0,1)$ nor $(p,q) = (1,0)$ is a Nash equilibrium. It turns out that under the assumption that $a < 1$, nor can $(p,q) = (1,1)$ be a Nash equilibrium. To see this, suppose that this is indeed a Nash equilibrium. Then for candidate 1 not to have an incentive to deviate, we must have $r_{LR}(1+a) \geq 1$, and for candidate 2 not to have an incentive to deviate, we must have $(1-r_{LR})(1+a) \geq 1$. These two conditions together imply that $a \geq 1$ — a contradiction. Hence, when $r_{CC} \in (0,1)$, the only pure-strategy Nash equilibrium of the game is the one where both candidates play C, i.e., where $(p,q) = (0,0)$. Since there is only one pure-strategy Nash equilibrium, there can be no equilibrium in mixed strategies. This gives us the result stated under a). Next, let us look for Nash equilibria of the reduced-form game in matrix (1) where $r_{CC} = 0$. First, in any Nash equilibrium of this game, $q = 0$. To see this, suppose that $q > 0$. Then we must have $p(1-r_{LR})(1+a) \geq 1$; otherwise candidate 2 will have an incentive to play C with probability 1. This condition implies $(1-r_{LR})(1+a) \geq 1$ and $p > 0$. The condition $p > 0$ in turn implies that $qr_{LR}(1+a) \geq q$; otherwise candidate 1 will have an incentive to play C with probability 1. This condition in turn implies that $r_{LR}(1+a) \geq 1$, which together with the previously derived condition $(1-r_{LR})(1+a) \geq 1$ imply that $a \geq 1$ — a contradiction. Given that $q = 0$, candidate 1 is indifferent between playing L and C. In order to have a Nash equilibrium where candidate 2 plays C with probability one and where candidate 1 randomizes with some probability p, we must have $p(1-r_{LR})(1+a) \leq 1$. This gives us the result stated under b). It only remains to consider the possibility that $r_{CC} = 1$. This case, however, is symmetric to the previous one; the result is stated under c). $\square$

Proof of Proposition 2 Let p be the probability with which candidate 1 chooses L and let q be the probability with which candidate 2 chooses R. I first show that in any Nash equilibrium, $p = q$. Suppose, per contra, that $p \neq q$. Because of symmetry, we can without any further loss of generality assume that $p < q$. This implies $r_{CC} = r_{LR} = r_{CR} = 1$. To see this, let us calculate the voter's expected utility if voting for candidate 1 respectively candidate 2 upon observing the signal $s = (s_1, s_2) \in \{L, C\} \times \{C, R\}$. The voter uses Bayes' rule to update his beliefs about candidate i's platform upon observing the signal s_i:

$$\Pr(x_i = \widetilde{x}_i \mid s_i = \widetilde{s}_i) = \frac{\Pr(s_i = \widetilde{s}_i \mid x_i = \widetilde{x}_i)\Pr(x_i = \widetilde{x}_i)}{\sum_{\widetilde{x}_i \in \{L,C,R\}} \Pr(s_i = \widetilde{s}_i \mid x_i = \widetilde{x}_i)\Pr(x_i = \widetilde{x}_i)}. \tag{16}$$

By using the formula in (16), we get

$\Pr(x_1 = \widetilde{x}_1 \mid s_1 = \widetilde{s}_1)$	$\widetilde{s}_1 = L$	$\widetilde{s}_1 = C$
$\widetilde{x}_1 = L$	$\frac{(1-\varepsilon)p}{(1-\varepsilon)p+\varepsilon(1-p)}$	$\frac{\varepsilon p}{\varepsilon p+(1-\varepsilon)(1-p)}$
$\widetilde{x}_1 = C$	$\frac{\varepsilon(1-p)}{(1-\varepsilon)p+\varepsilon(1-p)}$	$\frac{(1-\varepsilon)(1-p)}{\varepsilon p+(1-\varepsilon)(1-p)}$

(17)

and

$\Pr(x_2 = \widetilde{x}_2 \mid s_2 = \widetilde{s}_2)$	$\widetilde{s}_2 = C$	$\widetilde{s}_2 = R$
$\widetilde{x}_2 = C$	$\frac{(1-\varepsilon)(1-q)}{(1-\varepsilon)(1-q)+\varepsilon q}$	$\frac{\varepsilon(1-q)}{\varepsilon(1-q)+(1-\varepsilon)q}$
$\widetilde{x}_2 = R$	$\frac{\varepsilon q}{(1-\varepsilon)(1-q)+\varepsilon q}$	$\frac{(1-\varepsilon)q}{\varepsilon(1-q)+(1-\varepsilon)q}$

(18)

Now, using the above tables and the payoff matrix in Table 2 in the main body of the paper, we get the following expressions for the voter's expected utility:

	$\widetilde{s}_i = L$	$\widetilde{s}_i = C$	$\widetilde{s}_i = R$
$EU(\text{vote for } 1 \mid s_1 = \widetilde{s}_1)$	$\frac{\varepsilon(1-p)}{(1-\varepsilon)p+\varepsilon(1-p)}$	$\frac{(1-\varepsilon)(1-p)}{\varepsilon p+(1-\varepsilon)(1-p)}$	$---$
$EU(\text{vote for } 2 \mid s_2 = \widetilde{s}_2)$	$---$	$\frac{(1-\varepsilon)(1-q)}{(1-\varepsilon)(1-q)+\varepsilon q}$	$\frac{\varepsilon(1-q)}{\varepsilon(1-q)+(1-\varepsilon)q}$.

(19)

Using the expressions in the table in (19) it is easily verified that $p < q$ implies $EU(\text{vote for } 2 \mid s_2 = C) < EU(\text{vote for } 1 \mid s_1 = C)$, which in turn implies $r_{CC} = 1$. In a similar fashion one can check that $p < q$ also implies $r_{LR} = r_{CR} = 1$.

Next, note that $p < q$ and $r_{CC} = r_{LR} = r_{CR} = 1$ together imply $r_{LC} < 1$. This is because if $r_{LC} = 1$, candidate 1 would win with certainty no matter which platform she chose, which means that she should choose her favorite platform $x_1 = L$; but this in turn implies $p = 1 \geq q$.

Finally, let us show that the above implications in turn imply $q = 0$, which contradicts the assumption that $p < q$. The equality $q = 0$ holds if candidate 2's expected utility if choosing C is strictly greater that her expected utility if choosing R:

$$[p(1-\varepsilon) + (1-p)\varepsilon](1-r_{LC})(1-\varepsilon) > [p(1-\varepsilon) + (1-p)\varepsilon](1-r_{LC})(1+a)\varepsilon \tag{20}$$

which always holds since we know from above that $r_{LC} < 1$ and by assumption $\varepsilon \in \left(0, \frac{1}{3}\right)$ and $a \in \left(0, \frac{1}{4}\right)$.

We have thus established that in any Nash equilibrium, $p = q$. Next step is to show that in any Nash equilibrium, $p = q = 1$. It follows from the arguments in the paragraphs preceding Proposition 2 that we cannot have $p = q = 0$. It thus remains to show that $p = q \in (0,1)$ cannot be part of a Nash equilibrium. Suppose, per contra, that there exists a Nash equilibrium where $p = q \in (0,1)$. This implies that $r_{LC} = 0$ and $r_{LC} = 1$; this follows from the table in (19) above. It is also implied that both candidates must be indifferent between their two available actions. For candidate 1 we get the following condition:

$$\begin{aligned} & \begin{pmatrix} 1-\varepsilon & \varepsilon \end{pmatrix} \begin{pmatrix} 0 & r_{LR}(1+a) \\ r_{CC}(1+a) & 1+a \end{pmatrix} \begin{pmatrix} (1-q)(1-\varepsilon) + q\varepsilon \\ q(1-\varepsilon) + (1-q)\varepsilon \end{pmatrix} \\ = & \begin{pmatrix} \varepsilon & 1-\varepsilon \end{pmatrix} \begin{pmatrix} 0 & r_{LR} \\ r_{CC} & 1 \end{pmatrix} \begin{pmatrix} (1-q)(1-\varepsilon) + q\varepsilon \\ q(1-\varepsilon) + (1-q)\varepsilon \end{pmatrix}, \end{aligned} \tag{21}$$

which (using $q = p$) simplifies to

$$\begin{aligned} & r_{LR}[p(1-\varepsilon) + (1-p)\varepsilon][(1-\varepsilon)(1+a) - \varepsilon] \\ = & \{r_{CC}[(1-p)(1-\varepsilon) + p\varepsilon] + p(1-\varepsilon) + (1-p)\varepsilon\}[(1-\varepsilon) - \varepsilon(1+a)] \end{aligned} \tag{22}$$

or $r_{LR} = r_{CC}K_1 + K_2$, where

$$K_1 = \frac{[(1-p)(1-\varepsilon) + p\varepsilon][(1-\varepsilon) - \varepsilon(1+a)]}{[p(1-\varepsilon) + (1-p)\varepsilon][(1-\varepsilon)(1+a) - \varepsilon]}$$

and

$$K_2 = \frac{(1-\varepsilon) - \varepsilon(1+a)}{(1-\varepsilon)(1+a) - \varepsilon}.$$

For candidate 2 we get the following condition:

$$\begin{aligned}
& \begin{pmatrix} p(1-\varepsilon) + (1-p)\varepsilon & (1-p)(1-\varepsilon) + p\varepsilon \end{pmatrix} \\
& \times \begin{pmatrix} 1+a & (1-r_{LR})(1+a) \\ (1-r_{CC})(1+a) & 0 \end{pmatrix} \begin{pmatrix} \varepsilon \\ 1-\varepsilon \end{pmatrix} \\
= & \begin{pmatrix} p(1-\varepsilon) + (1-p)\varepsilon & (1-p)(1-\varepsilon) + p\varepsilon \end{pmatrix} \\
& \times \begin{pmatrix} 1 & 1-r_{LR} \\ 1-r_{CC} & 0 \end{pmatrix} \begin{pmatrix} 1-\varepsilon \\ \varepsilon \end{pmatrix}, \qquad (23)
\end{aligned}$$

which simplifies to

$$\begin{aligned}
& \{(1-r_{CC})[(1-p)(1-\varepsilon) + p\varepsilon] + p(1-\varepsilon) + (1-p)\varepsilon\}[(1-\varepsilon) - \varepsilon(1+a)] \\
= & (1-r_{LR})[p(1-\varepsilon) + (1-p)\varepsilon][(1-\varepsilon)(1+a) - \varepsilon] \qquad (24)
\end{aligned}$$

or $r_{LR} = r_{CC}K_1 + 1 - K_1 - K_2$. Notice that this equation and $r_{LR} = r_{CC}K_1 + K_2$ are parallel, and they coincide if and only if $K_1 = 1 - 2K_2$. Under the conditions that $p \in (0,1)$, $\varepsilon \in \left(0, \frac{1}{3}\right)$, and $a \in \left(0, \frac{1}{4}\right)$, however, $K_1 > 0$ whereas $(1 - 2K_2) < 0$. Hence, we cannot have an equilibrium in which $p = q \in (0,1)$.

Finally, let us investigate under what conditions $p = q = 1$ is part of a Nash equilibrium. This is the case if and only if candidate 1 has a weak incentive to choose L and candidate 2 has a weak incentive to choose R, given the other candidate's behavior and given some $r \in [0,1]^4$. For candidate 1 we get:

$$\begin{aligned}
& (1+a)\begin{pmatrix} 1-\varepsilon & \varepsilon \end{pmatrix} \begin{pmatrix} r_{LC} & r_{LR} \\ r_{CC} & r_{CR} \end{pmatrix} \begin{pmatrix} \varepsilon \\ 1-\varepsilon \end{pmatrix} \\
\geq & \begin{pmatrix} \varepsilon & 1-\varepsilon \end{pmatrix} \begin{pmatrix} r_{LC} & r_{LR} \\ r_{CC} & r_{CR} \end{pmatrix} \begin{pmatrix} \varepsilon \\ 1-\varepsilon \end{pmatrix}, \qquad (25)
\end{aligned}$$

which simplifies to (3). The corresponding condition for candidate 2 must, because of symmetry, be identical to (3) but with $r_{LC} = (1 - r_{CR})$, $r_{CC} = 1 - r_{CC}$, and $r_{LR} = 1 - r_{LR}$. This gives us (4). □

Proof of Proposition 4 The existence of an equilibrium in which both candidates choose their favorite policies is already proven in the paragraphs preceding Proposition 4. Let us verify that there indeed exists an equilibrium in which both candidates choose L with probability 1. Clearly, for this platform configuration, candidate 1 will win the election (all three voters will vote for her). Hence, candidate 1 will not have an incentive to deviate. Nor does candidate 2 have a (strict) incentive to deviate to any one of the other platforms. If candidate 2 deviated to either platform R or platform C, then she would get voter 3's vote but not voter 1's, and voter 2 would not change his voting behavior since he cannot observe the deviation. Hence, candidate 2 cannot change the outcome of the election by deviating.

Let us finally check that there is no other equilibrium in which none of the candidates is randomizing in her choice of platform. There are seven remaining platform configurations to consider: (i) $(x_1, x_2) = (R, R)$, (ii) $(x_1, x_2) = (C, L)$, (iii) $(x_1, x_2) = (C, C)$, (iv) $(x_1, x_2) = (R, C)$, (v) $(x_1, x_2) = (C, R)$, (vi) $(x_1, x_2) = (R, L)$, and (vii) $(x_1, x_2) = (L, C)$. It is a straightforward exercise to verify the following. In case (i)-(iv) candidate 1 wins. In case (i) and (ii), however, candidate 1 has an incentive to deviate to L; in case (iii) and (iv), candidate 2 has an incentive to deviate to R respectively to L. In case (v)-(vii) candidate 2 wins. In case (v) and (vi), candidate 1 has an incentive to deviate to L; in case (vii), candidate 1 has an incentive to deviate to C or R. $\square$

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