WISSENSCHAFTSZENTRUM BERLIN
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SOCIAL SCIENCE RESEARCH
CENTER BERLIN

discussion papers

FS IV 98 - 14

The Stability of Information Cascades: How Herd Behavior Breaks Down

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November 1998

ISSN Nr. 0722 - 6748

**Forschungsschwerpunkt
Marktprozeß und Unternehmensentwicklung**

**Research Area
Market Processes and
Corporate Development**

Zitierweise/Citation:

Hans Mewis, **The Stability of Information Cascades: How Herd Behavior Breaks Down,** Discussion Paper FS IV 98 - 14, Wissenschaftszentrum Berlin, 1998.

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ABSTRACT

The Stability of Information Cascades: How Herd Behavior Breaks Down

by Hans Mewis*

We extend the standard model of information cascades to situations where agents have to choose between switching to a new alternative or not. In particular, we add two new features to the standard model. First, agents continue to receive signals after they have chosen 'switching'; and second agents may revert their earlier decisions. The focus of the paper is to analyze how information is passed on within an information cascade, and the conditions under which breakdown occurs. We identify rules which describe the learning process and run simulations to estimate the properties of the information cascade.

ZUSAMMENFASSUNG

Zur Stabilität der Informationskaskade: Wann bricht Herdenverhalten ab?

Wir erweitern das Standardmodel der Informationskaskade. Durch die Erweiterung lassen sich Situationen modellieren, die den Individuen die Wahl zwischen einer neuen Alternative und dem bewährten lassen. Die Erweiterung besteht aus zwei Aspekten. Erstens erhalten die Individuen weiterhin Signale, nachdem sie sich für die neue Alternative entschieden haben. Zweitens können sie ihre Entscheidung später wieder zurücknehmen. Wir untersuchen wie sich Information innerhalb der Informationskaskade ausbreitet und unter welchen Bedingungen die Informationskaskade abbricht. Wir beschreiben den Lernprozeß der Individuen und führen eine Simulation des Prozesses durch.

* This paper has benefited from discussions with Ulrich Kamecke, Lars-Hendrik Röller, Martin Strobel, Zhentang Zhang and Christine Zulehner. Remaining errors are mine. The author would like to gratefully acknowledge the financial support through a grant from the Deutsche Forschungsgemeinschaft (DFG) while he was a fellow of the Graduiertenkolleg „Applied Microeconomics“ at the Humboldt-Universität Berlin.

1 Introduction

The theory of information cascades was pioneered by Banerjee (1992) and Bikhchandani et al. (1992). Information cascades integrate the phenomenon of herd behavior into the paradigm of rationality. The set up is very simple. An infinite sequence of agents have to take the same decision in an uncertain world. Decisions are taken in some predetermined order. Just before an agent has to make a choice he receives a private and imperfect signal on the state of the world. Signals are not observable by others but each agent observes the decisions of his predecessors. Agents then form assessments of the true state of the world based on a common prior beliefs on the true state, the private signal, and the observed behavior of other agents. This assessment forms the basis of the decision. An information cascade occurs if agents do not follow their own signals but rather do what their predecessors have done. The main result in the literature is that, even if the signal quality is very high, cascades occur with a probability of one. Moreover, even at a relatively high signal quality, a wrong cascade cannot be excluded. A wrong cascade describes a situation where agents make a decision which they would not have chosen if they knew the true state. This points to an inefficiency in the aggregation of information. Bikhchandani et al. (1992) demonstrate further the fragility of information cascades to external shocks.

The concept has been applied to quite a wide range of problems. Bikhchandani et al. (1992) present fashion, fads and conventions as examples. Zhang and Zhang (1995) use information cascades to investigate the asymptotic efficiency of an oligopolistic market with uncertain demand.

Caplin and Leahy (1992) look at a related problem. In their model, firms have the same three sources of information as in the standard information cascade model: a prior knowledge on some state of the world, private signals, and the observation of past actions by other firms. Unlike the information cascade concept there is a fixed number of firms which start the game at the same time. Moreover, Caplin and Leahy assume a fixed horizon at which the game stops. Firms may suspend their participation and then decide whether to continue or to exit from the game. They focus also on how private information is released and how the market reacts to the new information.

However, the standard model has some troubling features. Zhang (1997) points out that the exogenously given order of players is hard to justify. He endogenizes the order of movers by introducing individually different precision of information and the option to delay which is associated with a cost of delay. The signal quality is private information. He shows in a continuous time framework that the agent with the highest signal quality moves …rst, thereby generating an information cascade, i.e. all players follow immediately. The superior quality of the …rst mover's signal dominates the others' signals. Moreover, there is always delay before the …rst move.

In addition there are at least two more shortcomings to the standard model. First, once an information cascade has started it runs for ever unless some exogenous event, such as an information release, occurs. Second, from the point of view of an agent, once the decision has been taken the game is over. In other words, agents are assumed to be at a crossroad and directions can not be later changed. However, there are many situations where this is not plausible. Think of the adoption of a new technology or even the adoption of some new management fad. Once an agent has switched to the new alternative, the game is not over. The agent struggles to implement the new technology or gains experience applying the new management strategy. He learns whether the new alternative is of any value. This is true for set ups which let the agent chose between a new alternative and the old way. The inclusion of the option to reverse leads to the possibility of breakdown.[1] The richer modelling provides more insights into the di¤usion process of information.

Suppose the state of the world described the quality of a new technology. Managers have to decide whether they want to buy the new technology or stick with their old one. They do not have complete information on the quality of the new technology. However, they share a common prior belief about the potential of the innovation. Besides this prior belief managers may have private information about the quality of the new technology. The experience gained through working with the new technology leads the managers to update their beliefs. Initial failures might be due to bad handling or just bad luck, or indeed might be due to bad quality. In addition to his own experience, the manager also bases his decision on the behavior of other managers at other …rms. Our manager cannot observe the other managers' experience directly but only indirectly through observed behavior. From observing that a manager returns to the old technology he concludes that the manager in question must have had bad experiences with the new technology. He adds

[1] Bikhchandani et al. (1992, p. 1007) point to a case study by Apodaca (1952). "[T]he introduction of one variety of hybrid seed corn for 84 growers in a New Mexican village from 1954 to 1949 in which a trend reversed before settling on an outcome. [...]"

this conclusion to his stock of information. This implies he might now be willing to also reverse his earlier decision. Yet, imagine our manager observes no such reversion for some time. He then concludes that the others cannot have had such bad results, since otherwise they would have returned to the old technology. This leads him to a more optimistic attitude. He might then swallow more bad news from his production plant than otherwise. The modelling of this type of learning from observed behavior is the core of the paper. How does an optimistic attitude among managers evolve? How much can an optimistic atmosphere strengthen the stability of a cascade?

In this paper we adopt information cascades to situations where agents have to decide whether they want to switch to some new alternative or rather stay with the old one. We augment the standard model by two new features. First, agents continue to receive signals after they have decided whether to switch. Second, firms may change their earlier decision to go for the new. Firms are allowed to reverse only once. The focus of the paper is how information is passed on within an "enter"-cascade and the conditions under which breakdown occurs. A "non-enter" cascade is essentially the same as a standard information cascade.

This paper is organized as follows. Section 2 presents the model and establishes condition under which learning occurs. Furthermore it is shown how an optimistic attitude spreads as no reversion occurs. Section 3 deals with the simulation and section 4 concludes.

2 The Model

Suppose there is an ordered line of decision makers (call them firms). These firms have to make the same binary decision one after the other. Think of it as whether to enter or not into a new market. Each period one firm is called upon to announce its decision according to a predetermined order. Those firms which have entered are also asked to announce each period whether they will remain by their choice, i.e. to stay in the market or change their decision and exit[2]. A firm which has decided not to enter or exit cannot enter later or "reenter". It leaves the game for ever.

There are two states of the world and in each state only one decision is optimal. State space is $\Theta = \{\theta_1; \theta_2\}$. The state of the world does not

[2] We use the terms "enter", "adopt" and "switching" interchangeably, just as "exit" and "revert".

change during the game. Think of it as if nature chooses a state of the world before the first firm makes a move. Suppose both states are equally likely. Furthermore assume that the payoff π for decision "enter" in a positive state of the world is 1 with probability p and in a negative state -1 with probability p. With probability $(1 - p)$ the payoff structure is reversed. Vice versa for "not enter". The following table then summarizes the payoff structure:

state	$\theta_1 = 1$		$\theta_2 = -1$	
prob.	p	$1 - p$	p	$1 - p$
enter	1	-1	-1	1
not enter	-1	1	1	-1

The firms do not know in which of the two states of the world they are. However, they all share the same prior belief ψ about the probabilities of being in one of the two states. Assume all firms think the two states are equally likely[3]:

$$\psi = \Pr(\text{positive state}) = \frac{1}{2}$$

In addition to the prior belief each firm receives a private signal before it makes a decision. The private signal is the payoff of that firm. Therefore, the table above also represents the signal structure. Signals are binary. Signal space is $\Sigma = \{-1; 1\}$. p can be interpreted as the signal quality. If p is 0.5 than the signal does not contain any information. If $p = 1$ the signal quality is perfect. In the interval $(0.5; 1)$ the signal is imperfect but does contain some information, since the indicated state is then more likely to be the true state. Signal quality below 0.5 would mean that the signals are systematically misleading. Therefore, we assume signal quality to be in the interval $(0.5; 1)$. The following table describes signal quality:

signal/state	$\theta_1 = 1$	$\theta_2 = -1$
$\sigma_t = 1$	p	$(1 - p)$
$\sigma_t = -1$	$(1 - p)$	p

A firm then makes its decision. The decision is based on three factors. First, there is the common prior belief; second, there is the private signal; and third the firm observes with a one period delay what other firms have

[3] Alternatively, it is possible to assume that ψ is from the intervall $(0; 1)$. However, then one has to make sure that one positive private signal is sufficient for entry, since otherwise no information cascade would ever develop. If $\psi > (1 - p)$ holds, one positive private signal is sufficient for entry. The results of this paper do not depent on this assumption.

decided. From these decisions it may draw conclusions about the signals of other …rms. It then uses the revealed signals to update its belief according to Bayes' rule. In other words, the …rm can learn something about the true state by observing the history of actions: some …rms reverse their decisions and others stick to their initial choice. These three sources of information are then used to update the …rms belief according to Bayes' rule. Once a …rm gets to belief entry was a mistake and the true state is negative rather than positive it reverses its earlier decision and exits. As a tie breaking rule we assume that an indi¤erent …rm stays where it is. A potential entrant does not enter, an incumbent stays with its initial choice.

2.1 Notation

The state of information $istate_{t;e}$ of each …rm e at time t is described by two variables, the public state $pstate_t$ and the private state $xstate_{t;e}$. These two states essentially determine the action of …rm e in period t. Denote the action by $v_{t;e}$. $v_{t;e}$ can assume two values f ¡1; 1g. These represent for an incumbent …rm exit (no entry) and stay (entry), respectively. The corresponding action of an entering …rm is in brackets. $v_{6;2} =$ ¡1 represents the exit of the second …rm in period 6.

The public state $pstate_t$ summarizes the publicly known signals at time t. Therefore, it has no …rm speci…c index. Public state in period t is the sum of all public signals, that is the public state of the previous period plus the sum of all those private signals, that have been revealed through last periods actions :

$$pstate_t = pstate_{t¡1} + \sum_{e=1}^{t} s_{t¡1;e} \; ,$$

where t refers to the current period, e is the time of entry of a …rm, $s_{t¡1;e}$ is the public signal sent by …rm e in period t (through its action in t ¡ 1, see below). (Note that $pstate_1 = 0$)

The private state $xstate_{t;e}$ summarizes the private information of a …rm e at period t. It is simply the private state of the previous period plus the new private signal:

$$xstate_{t;e} = xstate_{t¡1;e} + ¾_{t;e}$$

where $¾_{t;e}$ is the private signal received in period t by …rm e.

The variable $S_{t;e}$ counts the signals revealed by firm e to other firms up to period t. (Note the $s_{t;e}$ is received in the next period. The index t refers to the time when the underlying action takes place and the signal is sent. There is a one period delay between action and receiving a signal.) This variable is important because the same signals are included in the public state and in the private state. To make sure we do not double count the revealed signals of some firm when calculating the stock of information of that firm we need $S_{t;e}$:

$$S_{t;e} = S_{t-1;e} + s_{t-1;e}$$

The variable $S_{t;e}$ also provides the value of the revealed signals. Note that $s_{t-1;e}$ is always positive as long as it does not result from the exit of firm e. Therefore, as long as firm e has not exited this variable counts and gives the value of the revealed signals at the same time. Once a firm reveals signals by its exit, these signals are of course negative and the variable is not able to count the signals any more but just provides the value of all revealed signals by that particular firm. However, now that this firm has exited we no longer need to calculate its state of information. Hence, how many signals this firm has revealed is no longer of interest.

The overall state of knowledge or information of firm e at time t is characterized by $istate_{t;e}$.

$$istate_{t;e} = pstate_t + xstate_{t;e} - S_{t;e}$$

Note that there is no difference in weights for own signals and revealed signals.

2.2 Determination of Action

We assume firms are risk-neutral and maximize expected discounted profits. The behavior of firm e at time t can be characterized as follows. As soon as a firm has knowledge of more negative signals than positive signals the firm reverses. Equally, if a firm knows of more positive than negative signals and it is asked whether it wants to enter, it enters. If a firm is indifferent, i.e. knows of the same number of positive and negative signals, it stays where it is: incumbents stay in the market and potential entrants stay out. Looking at the intertemporal payoff function and given that the discount factor δ is sufficiently small[4] it is easy to see that the expected payoff is only positive if

[4] The relaxation of this assumption leads to the question of an optimal stopping rule. Simulations show that even at a discount rate of $\delta = 1$ the optimal stopping rule is $\alpha = -1$,

$\phi > \frac{1}{2}$, it is zero if $\phi = \frac{1}{2}$ and negative if $\phi < \frac{1}{2}$.

$$\begin{aligned} E(\pi) &= \frac{1}{1-\delta}(\phi(p - (1-p)) + (1-\phi)((1-p)-p)) \\ &= \frac{1}{1-\delta}(2\phi - 1)(2p-1), \end{aligned}$$

where ϕ is the updated Bayesian assessment of the probability of being in a good state and δ is the discount rate. ϕ depends on received private signals and revealed signals:

$$\phi(n;m) = \frac{\frac{1}{2}p^n(1-p)^m}{\frac{1}{2}p^n(1-p)^m + \frac{1}{2}(1-p)^n p^m},$$

where n and m are the number of positive and negative signals, respectively. The conditional probability of a good state can also be formulated in terms of the difference between positive and negative signals $\alpha = n - m$, and $n > m$.

$$\phi(\alpha) = \frac{p^\alpha}{p^\alpha + (1-p)^\alpha}$$

Straight forward calculation yields:

$$\begin{aligned} \phi(\alpha) &> \frac{1}{2} \text{ if } \alpha > 0 \\ \phi(\alpha) &= \frac{1}{2} \text{ if } \alpha = 0 \\ \phi(\alpha) &< \frac{1}{2} \text{ if } \alpha < 0. \end{aligned}$$

The following Lemma summarizes the behavior of firms:

Lemma 1 Given a sufficiently small discount factor δ, an incumbent firm stays as long as it has knowledge of at least as many positive signals as negative signals. If this is not the case it exits:

$$\begin{aligned} v_{t;e} &= 1 \text{ if } istate_{t;e} \geq 0 \\ v_{t;e} &= -1 \text{ if } istate_{t;e} < 0. \end{aligned}$$

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where α is the difference between all negative and positive signals.In other words, if an agent has knowledge of one more negative signal than positive signals he exits.

A potential entrant enters if it knows of more positive than negative signals. Otherwise it stays out:

$$v_{t;e} = 1 \text{ if } istate_{t;e} > 0$$
$$v_{t;e} = \text{¡}1 \text{ if } istate_{t;e} \cdot 0.$$

The di¤erence between positive and negative signals ® is identical to the above de…ned istate. Remember we adopted as a tie-breaking rule that an indi¤erent …rm stays were it is. In other words, the absolute number of signals does not matter, but only the di¤erence of positive and negative signals.

2.3 An Example

To see the structure of the process and how observing the others' action improves a …rm's stock of knowledge we look at an example of signals and the resulting decisions:

EXAMPLE

t	1			2			3			4		
$pstate_t$	0-0			1-0			2-0			2-0		
Firm	¾	I	A	¾	I	A	¾	I	A	¾	I	A
1	1	1-0	In	-1	1-1	In	-1	2-2	In	1	3-2	In
2				1	2-0	In	-1	2-1	In	-1	2-2	In
3							-1	2-1	In	-1	2-2	In
4										-1	2-1	In

¾=Private Signal; I=all known signals (positive-negative); A=action ,decision)

In period 2, every …rm knows that …rm 1 has received a positive signal in period 1 since it would not have otherwise entered. Therefore, the public information in period 2 is 1:0, $pstate_2 = 1$. Firm 1 has sent one positive signal in period 1 which is received in period 2: $s_{1;1} = 1$. In period 2 the same applies to …rm 2. Therefore, the public information in period 3 is 2:0, $pstate_3 = 2$, $s_{2;1} = 0$, $s_{2;2} = 1$. In period 4 nothing can be learned from the observation that …rm 3 has entered and that no one exited in the previous period. This is because whatever the unknown private signals were, the observed action would have taken place. Therefore $s_{3;3} = 0$, and $s_{3;1} = s_{3;2} = 0$.

EXAMPLE cont'd

t	5			6			7			8		
$pstate_t$		3-0			3-0			6-0			6-0	
Firm	¾	I	A	¾	I	A	¾	I	A	¾	I	A
1	-1	3-3	In	1	4-3	In	-1	6-4	In	-1	6-5	In
2	-1	3-3	In	1	4-3	In	-1	6-4	In	-1	6-5	In
3	-1	3-3	In	1	4-3	In	-1	6-4	In	-1	6-5	In
4	-1	3-2	In	-1	3-3	In	-1	6-4	In	-1	6-5	In
5	-1	3-1	In	-1	3:2	In	-1	6-3	In	-1	6-4	In
6				-1	3:1	In	-1	6-2	In	-1	6-3	In
7							-1	6-1	In	-1	6-2	In
8										-1	6-1	In

In period 5, the …rms observe that …rm 1 has not exited. This means that …rm 1 has received at least 1 positive signal after its initial one. If all its signals from period 2 onwards had been negative, …rm 1 would have exited in period 4. Therefore, the public information in 5 is 3:0, $pstate_5 = 3$, $s_{4;1} = 1$, $S_{5;1} = 2$, that is …rm 1 has sent 2 signals so far. In period 6, nothing can be learned from the observed behavior in period 5. In period 7, …rms observe again that no one has left. However, three …rms could have left: 1, 2 and 3. This is because they had enough time to accumulate su¢cient negative signals to outweigh the publicly known positive signals. Yet, none of these …rms has left. Hence, each of these …rms must have received at least one positive signal since they have entered in addition to their already revealed signals. All …rms now know that …rm 1 has received at least 3 positive signals, …rm 2 has received at least 2 positive signals, and …rm 3 at least 1 positive signal. Therefore, $s_{6;1} = 1$, $S_{7;1} = 3$, and $s_{6;2} = 1$, $S_{7;2} = 2$, and $s_{6;3} = 1$, $S_{7;3} = 1$. Hence, public information is 6:0, $pstate_7 = 6$. In period 8, nothing can be learned from the observations.[5] Note that this example contains the maximum number of negative signals such that no exit occurs.

2.4 How much and when can …rms learn?

In this section we investigate how optimistic and pessimistic attitudes in a market evolve. In other words, how much and when can …rms learn from

[5] Note that the observation of a new entry does not contain any information, since given the publicly known information, …rms will enter no matter what their own private signals look like.

observing the action of other firms? The answer to that question determines the value of $s_{t;e}$. To this end, we introduce a new variable:

$$f_{t;e} = (t - e + 1) - S_{t-1;e}$$

$f_{t;e}$ is the number of publicly unknown private signals of firm e at period t. A firm that tries to infer from the action of some other firm the value of its unknown signals has to know as much as possible about that firm. One important piece of information that is available is the number of its unrevealed private signals.

It makes a difference whether the action of a potential entrant or incumbent firm is observed. Therefore, we look first at the entrant's case and than at the incumbent's case.

2.4.1 Entrant's Case

The following proposition shows the condition under which the action of a potential entrant is informative and the exact information it conveys.

Proposition 2 The action of an entrant conveys only information if its private signal influences its decision. This is the case if $pstate_t$ is either 1 or 0. Entry reveals one positive signal and non entry reveals one negative signal.

$$\text{If } v_{t;e} = 1 \wedge pstate_t \in \{0;1\} \text{ than } s_{t;e} = 1 \text{ and 0 otherwise.}$$
$$\text{If } v_{t;e} = -1 \wedge pstate_t \in \{0;1\} \text{ than } s_{t;e} = -1 \text{ and 0 otherwise.}$$

Proof: Note first that the action of any firm only contains information if the number of unrevealed private signals of that firm is sufficiently large to outweigh available public information. A necessary but not sufficient condition for an action to be informative is therefore:

$$|pstate_t| \leq f_{t;e} \qquad (1)$$

If the opposite were true then the unrevealed private signals had no influence on the action of the particular firm and therefore the action would not convey any information.

In the entrant's case $f_{t;e} = 1$ since the information cascade is running and entry is uninformative. Consequently, only if $pstate_t$ is either 0 or 1, the action of an entrant will carry information. Note that if $pstate_t = -1$, which satisfies (1), entry never occurs and hence non entry is uninformative. Since $f_{t;e} = 1$, the number of revealed signals is at most 1. As $v_{t;e} = 1$ and $pstate_t \in \{0;1\}$, it follows $s_{t;e} = 1$; and from $v_{t;e} = -1$ and $pstate_t \in \{0;1\}$, it follows $s_{t;e} = -1$. $\blacksquare$

2.4.2 Incumbent's Case

Define the ordered set $\Gamma_{t;e}$ of pairs $(x;y)$: $\Gamma_{t;e} = \{(x;y) \,|\, x + y = f_{t;e}\}$. The ordered set starts with the smallest x, where x equals the number of positive signals and y represents the number of negative signals that might make up the unknown signals $f_{t;e}$.

Proposition 3 : The action of an incumbent conveys only information if its private signals have an influences on its decision. This is the case whenever the number of private unknown signals is sufficiently large to outweigh the public available information. Stay (exit) then reveals the minimum number of positive (negative) signals such that the particular decision comes about.

(1) An incumbent does not exit:

(1.1) If $v_{t;e} = 1 \wedge |pstate_t| < f_{t;e} \wedge pstate_t \geq 0$ then $s_{t;e} = \underline{x}$ of the pair $(x;y)$, where $\underline{x}$ is the smallest x such that $pstate_t + x - y \geq 0$. If $|pstate_t| \geq f_{t;e}$ then nothing can be learned.

(1.2) If $v_{t;e} = 1 \wedge |pstate_t| \leq f_{t;e} \wedge pstate_t < 0$ then $s_{t;e} = \underline{x}$ of the pair $(x;y)$, where $\underline{x}$ is the smallest x such that $pstate_t + x - y \geq 0$. If $|pstate_t| > f_{t;e}$ then nothing can be learned.

(2) An incumbent exits:

(2.1) If $v_{t;e} = -1 \wedge |pstate_t| < f_{t;e} \wedge pstate_t \geq 0$ then $s_{t;e} = \underline{y}$ of the pair $(x;y)$, where $\underline{y}$ is the smallest y such that $pstate_t + x - y < 0$. If $|pstate_t| \geq f_{t;e}$ then nothing can be learned.

(2.2) If $v_{t;e} = -1 \wedge |pstate_t| \leq f_{t;e} \wedge pstate_t < 0$ then $s_{t;e} = \underline{y}$ of the pair $(x;y)$, where $\underline{y}$ is the smallest y such that $pstate_t + x - y < 0$. If $|pstate_t| > f_{t;e}$ then nothing can be learned.

Proof: The proof is carried out in two steps. First we show the condition under which an action is informative (Step A). The second step then shows what an action by some firm can tell the observer about its signals.

Step A: Look at the case (1.1). We have $pstate_t \geq 0$ and some incumbent firm e stays in the market ($v_{t;e} = 1$), that is $istate_{t;e} \geq 0$. Suppose the number of unrevealed private signals of that firm were smaller than the balance of positive and negative signals in the stock of public signals ($|pstate_t| \geq f_{t;e}$). Obviously, the private information of firm e cannot be strong enough to overcome public information. Therefore, in this case $v_{t;e} = 1$ reveals nothing. Now suppose $|pstate_t| < f_{t;e}$. There are enough unrevealed private signals in firm e's stock of information to turn the balance either way. We observe that firm e has stayed in the market, i.e. $istate_{t;e} \geq 0$. Since $istate_{t;e} < 0$ was possible firm e's action is informative. It cannot be the case that all of firm e's unrevealed private signals are negative. This completes step A for (1.1).

Step B: Look at case (1.1) again. We know that $istate_{t;e} < 0$ is not true, since $v_{t;e} = 1$. Consequently it must be the case that $istate_{t;e} \geq 0$. From that we infer that at least as many positive signals are in e's stock of unrevealed private signals such that $istate_{t;e} \geq 0$. That is we know that there are $\underline{x} = \arg\min_x (x - y + pstate_{t;e}) \geq 0$ positive signals. Therefore, $s_{t;e} = \underline{x}$. This completes the proof of (1.1). The argument for the rest of the proposition is analogous to this one. $\blacksquare$

The following table summarizes the condition under which an action is informative:

A firm observes the action of an incumbent firm:

observation		public knowledge		
	$v_{t;e}$	$pstate_t$	case	Something to learn?
(1.1)	1	positiv or zero	$\|pstate_t\| < f_{t;e}$	yes
			$\|pstate_t\| = f_{t;e}$	no
			$\|pstate_t\| > f_{t;e}$	no
(1.2)	1	negativ	$\|pstate_t\| < f_{t;e}$	yes
			$\|pstate_t\| = f_{t;e}$	yes
			$\|pstate_t\| > f_{t;e}$	no (not possible)
(2.1)	-1	positiv or zero	$\|pstate_t\| < f_{t;e}$	yes
			$\|pstate_t\| = f_{t;e}$	no (not possible)
			$\|pstate_t\| > f_{t;e}$	no (not possible)
(2.2)	-1	negativ	$\|pstate_t\| < f_{t;e}$	yes
			$\|pstate_t\| = f_{t;e}$	yes
			$\|pstate_t\| > f_{t;e}$	no

The reader might wonder why firms do not try to estimate unknown signals. However, this is a useless exercise. On what basis is such an estimation possible? Firms have to use their assessment Á of the probability of a good state. Thereby the estimation just reinforces their assessment. There is no more information to be extracted.

The action of a …rm is informative if it has had a choice, i.e. if the value of its unrevealed signals makes a di¤erence. In particular, the number of unrevealed signals must be large enough so that the overall state of information of the …rm can lead to either "exit" or "continue". Otherwise, the action depends only on the already known public state and it is therefore uninformative. The value of the revealed signals is the minimum number of positive or negative signals in the private stock of signals such that the …rm's action comes about.

In this section we have identi…ed rules according to which an action of a …rm reveals information. Furthermore, we have shown that the amount of information depends on the public state $pstate_t$ and the number of unrevealed private signals $f_{t;e}$ of a particular …rm. The larger in absolute terms is the public state and the more unrevealed signals are left, the more information is conveyed.

2.5 Stability of an "entry"-cascade

Before we move on to the simulation we will take a look at a special case of how an information cascade might evolve. In particular we investigate the conditions under which an "entry"-cascade continues. We call this case the "business as usual" scenario (BUS), where …rms continue to enter but none leaves. The BUS is de…ned by the minimum number of positive signals to keep an "enter"-cascade going. In other words, we look at the case where no reversions occur and the spread of negative information is suppressed. Note that the example above was the BUS. This exercise will lead to the conclusion that all information cascades, which do not experience any reversion gain in stability.

As we have seen …rms learn from observing that no one has reversed. Firms learn whenever a reversion is possible. Call any period at which such a reversion might occur reversion date (RD).

De…nition 4 A period at which a …rm might want to reverse is called reversion date.

The …rst RD is at period 4 (See Example). Firm 1 could have left leave. In the following we state …ve lemmas to establish the properties of information cascades under the BUS. The analysis of this case sheds some light on the conditions under which the cascade keeps going and on how an optimistic attitude gradually builds up. The cascade becomes more stable the longer it lasts.

We partition the population of firms in to those which might exit at a RD and those which do not reverse in any case. As we will see later the former group of firms is always older than the latter. Therefore, call the former group mature firms and the latter young firms.

Definition 5 A firm that might reverse at a RD, i.e. $pstate_{t,e} < f_{t,e}$, is called mature. Other firms are labeled young firms.

The following lemma shows how the behavior of mature firms adds to the stock of public information.

Lemma 6 "No reversion" by a mature firm at a RD reveals exactly one positive signal given the BUS.

Proof. As we have seen in the previous section no reversion reveals the minimum number of positive signals the firm must have received so that it does not want to exit. Given the BUS the public state is always positive. By definition in the period before a RD, a firm knows of as many positive as negative signals. Hence, it takes exactly 1 positive signal to survive the upcoming RD. In other words the minimal number of positive signals needed is one. Therefore, "no reversion" reveals one positive signal. ■

The action of young firms conveys no information since they do not reverse no matter what signals they have received. Only actions of experienced players are of interest to the public. No attention is paid to the decision of newcomers.

The next two lemmas establish that after a RD mature firms are identical with respect to their information.

Lemma 7 After a RD all firms have knowledge of the same number of positive signals.

Proof. Since the number of positive signals is the smallest number which guarantees no reversion, all these positive signals are public. ■

Lemma 8 Given the BUS at every RD mature firms have the same information ratio $(a + 1)/a$, where a is the number of negative signals.

Proof. After a RD all …rms have knowledge of the same number of positive signals (Lemma 7). Therefore at the following RD, all …rms have the same information ratio, because under the BUS a …rm receives only negative signals until the …rm has the same number of positive and negative signals, i.e. until the next RD is just one period ahead. ■

From Lemma 8 it follows that any RD is uniquely de…ned by the number of negative signals a. Furthermore it follows that any …rm that might have left at the previous RD might leave at the current RD. Moreover, according to lemma 8 after a RD all mature …rms are information-wise identical. Therefore, it does not matter which mature …rm reverses …rst. The revealed information is the same. This leads to the following proposition.

Proposition 9 The information revealed by a …rst reversion does not depend on the age of the …rm but only on the date of the …rst reversion.

Proof. Follows from lemma 8 and the following observation. Since all positive signals are public and under the BUS all mature …rms have the same information ratio $(a + 1)=a$, in every information cascade which has not experienced any reversions yet, all mature …rms have the same number of unknown signals $f_{t;e}$. Therefore, the revealed signals are independent of the …rm's age. ■

The population of …rms is subdivided into two groups: mature and young …rms. By de…nition only mature …rms reveal information through their action. The …rm's age, that is its identity is not important. An action's information content does not so much depend on who acts but rather at what point in time the action takes place.

Lemma 10 shows how the group of mature …rms grows in the course of the cascade.

Lemma 10 Each RD is for exactly one …rm, which entered within the information cascade, its …rst RD.

Proof. Suppose there were two …rms which have their …rst RD at some RD $(a + 1)=a$. This means that they both have received only negative signals and one positive signal in the current period. Hence, both have received $a+1$ private signals and a revealed signals, therefore, both entered $a+1$ period ago. However, that is not possible since only one …rm may enter per period. Therefore, at most one …rm can have its …rst RD at RD $(a + 1)=a$. By the same argument and given that the information cascade is running, it is not possible that no …rm has its …rst RD at RD $(a + 1)=a$. Therefore, at any RD exactly one …rm has its …rst RD. ■

The …rst RD is at period 4, than follows 6, 10, 15, 21, 28, 36 and so on. The distance grows by one period. At period 6 two …rms have their …rst RD, which are …rms 2 and 3. Yet only …rm 3 entered within the information cascade. Firm 3 has not revealed its initial signal, while …rm 2's initial signal is public. Hence, the two …rms have the same number of unrevealed signals. That is the reason for the irregularity at the beginning of the series of RDs.

From Lemmas 8 and 10 it follows that at each RD the number of mature …rms grows by exactly 1 …rm.

Lemma 11 The time span between two RDs equals $n + 1$ where n is the number of mature …rms at the previous RD.

Proof. At any RD the ratio of positive to negative signals is given by $(a + 1)=a$. There are, say, n mature …rms, then n positive signals are revealed at the end of the RD. The information ratio is than $(a + 1 + n \text{ ¡ } 1)=a = (a + n)=a$ for any of the n mature …rms. Note that …rms do not double count their own public signals. It takes at least $a + n + 1$ to outweigh the $a + n$ positive signals. Hence, the next RD is $n + 1$ periods ahead. ■

Now we can state the main result of this section.

Proposition 12 The longer the information cascade lasts and no reversion occurs the less likely a reversion is.

Proof. From Lemmas 8, 10, and 11 follows that the time span between RD grows as the information cascade continues. Under the BUS during this time all …rm receive negative signals until the next RD is reached. However, for any information cascade, which has not seen any reversions yet, it is true that a …rm that exits at a RD has received only negative signals since the last RD. Since the distance between RD grows it becomes ever less likely that such a sequence of negative signals will occur. ■

Note, that this proposition generalizes to all information cascade processes where no reversion occurs or up to the period of the …rst reversion.

The proposition shows that information cascades gain in stability as they grow older. Note that the proposition looks at the necessary condition for breakdown. Yet, reversion by one …rm is not su¢cient for the breakdown of a cascade. In other words the proposition states that the necessary condition for breakdown becomes ever less likely, therefore the breakdown itself, too.

However, the later the …rst reversion occurs the more information it conveys. The intuition is that an unexpected incident reveals more information than the expected.

3 Simulation

The main focus of this paper is the question of whether or under what circumstances an information cascade breaks down. Therefore, we assume that an "enter"- information cascade goes o¤. That means we assume that the …rst …rm gets a positive signal (+1) in the …rst period and that the second …rm gets a positive signal in the second period. Those two signals are …xed. Firm three then enters for sure. It does not follow his own signal anymore. Decision makers ignore their private information and follow the crowd. The cascade breaks down if all …rms have exited[6].

The process consists of the decision rules and the signal generating stochastic process. The signals are binary (-1,1) and independent across time and …rms, and are identically distributed.

We have run simulations assuming either a good or a bad state of the world for several signal qualities. For each scenario we have run 5000 simulations. We have limited each simulation to 200 periods. First results suggested that almost all breakdowns occur during the …rst 50 periods. This is in line with the results from section 2.5. The probability of a breakdown after 50 periods, given no reversion has occurred so far, should be rather small[7]. The

[6] Actually the information cascade breaks down if some manager follows his own signal again. That is if the publicly known positive and negative signals cancel out. Call this the strict de…nition of a break down. Yet, to make sure that the process comes to an end and the cascade does not restart we look at the case where all …rms have left the market and pstate is negative. pstate<0 rules a restart out.

[7] The next RD is at 55. Public state has reached $+45$ by then, given no exit has occurred so far. Mature …rms have 46 unknown signals. It takes a sequence of 46 negative signals to make the youngest mature …rm leave. Even at a probability to get a negative signal of 0.8, that is almost zero. For the older …rms the probability will not be much higher. Certainly not all unknown private signals have to be negative but the few positive signals have all to be in the right place such that the …rm did not want to leave at some earlier RD. This reduces the admissable permutations. Note that at 0.9 no cascade has made it to period 55.

probability of a breakdown is even smaller. In fact in all 40.000 simulation rounds, only once a breakdown occurred after the 50th period. Table I summarizes the results.

The simulation leads to the following conclusions. In the case of a correct "enter"-cascade and a signal quality of $p = 0.9$ all cascades were fully revealing and for $p = 0.8$ only 2 out of 5000 were non-fully revealing. At signal quality $p = 0.7$ still only 18 out of 5000 were non-fully revealing. That is still well below 0.5%. Therefore, in these cases the probability to have a fully revealing cascade is almost 1. At a very poor signal quality of $p = 0.6$ the probability to have a fully revealing cascade is still just below 95%. Hence, in the case of a good cascade the information gathering process is rather efficient. The probability to have a fully revealing cascade is simply the counter probability of a breakdown after 200 periods. These are listed in the last column of Table I.

Table I

Relative Frequency of Breakdown						
$\mu = 1$						
p	after 5	after 7	after 10	after 20	after 50	after 200
0.9	0.6716	0.9612	0.9888	1	1	1
0.8	0.3920	0.7240	0.9044	0.9894	0.9962	0.9964
0.7	0.1834	0.4380	0.7186	0.8788	0.8952	0.8952
0.6	0.0756	0.2218	0.4436	0.5672	0.5754	0.5754
$\mu = -1$						
p	after 5	after 7	after 10	after 20	after 50	after 200
0.9	0	0	0	0	0	0
0.8	0	0.0002	0.0004	0.0004	0.0004	0.0004
0.7	0.0004	0.0030	0.0038	0.0038	0.0038	0.0038
0.6	0.0046	0.0304	0.0506	0.0530	0.0530	0.0530

Number of Simulation per Case: 5000

In the case of a wrong "enter"-cascade the estimated probabilities of having a fully revealing cascade are the relative frequencies of a breakdown after 200 periods in table I. The information gathering process is again rather efficient for signal qualities above $p = 0.7$. For poorer signal qualities the efficiency rapidly deteriorates. At $p = 0.6$ the probability of a fully revealing cascade is down to 0.58. Note that these are not asymptotic results. The values do not change already after 20 periods.

Observation 1: The information cascade process is better at detecting true cascades than wrong cascades.

We also calculated an empirical hazard rate ξ_t for each sample.

Definition 13 The empirical hazard rate ξ_t is the relative frequency that an information cascade breaks down in t, given that it has reached period $t-1$:

$$\xi_t := \frac{BD_t}{N_{t-1}},$$

where BD_t is the number of breakdowns in period t, and N_{t-1} is the number of running information cascades in the previous period.

Table II shows the results (see below). Breakdowns occur only after one or more firms have reversed their decisions. As we have seen in section 2.5 exit is only possible at RDs given that no exit has occurred so far. Therefore, one would expect to see the highest empirical hazard rates one and two periods after such a RD. After that ξ_t should decrease to almost zero until the next RD. Especially in later periods, as the population is larger, the effect of exit should take at least two periods to come through. This is exactly what we found. Moreover, as the cascade goes on, without any reversions, optimism should pick up and hazard rates should be much lower even after RDs. This can be seen in Table II as well. In fact ξ_t is zero from period 40 onwards. ξ_t is largest in wrong cascade with high signal quality at early stages. Note that in the case of signal quality $p = 0.9$ and a bad state the last cascades breaks down after 16 periods. That is why the empirical hazard rate is 1 here.

Observation 2: Empirical hazard rates are big after an RD. Moreover, the later the RD the smaller the empirical hazard rate.

Clearly, the process is better at "detecting" a true cascade than a wrong cascade. Why is that the case? As long as no reversion occurs only positive signals are revealed and thereby their effect is multiplied. This leads to the described spread of optimism among the population. An "enter"-cascade tends to reinforce itself simply by surviving. There is no equivalent for the development of an pessimistic attitude. In fact, negative news multiply rapidly. Therefore, the breakdown of a cascade is always a matter of a few periods. The breakdown itself is also a kind of cascade. The revealed negative signals outweigh the private signals. Yet, the speed of that cascade renders the term information avalanche more adequate[8]. Table III presents

[8] The term information avalanche was first proposed by Lee (1993).

the average time an “exit” information avalanche lasts. The duration of an information avalanche is de…ned as the time between the …rst exit and the breakdown of the information cascade. If the information cascade does not break down there is no exit information avalanche. The mean duration of an “exit” information avalanche is between 1 and 2.5 periods. Hence, breakdowns occur at a considerable speed. Given a bad state as the signal quality decreases the duration of an information avalanche increases. This is due to the fact that positive signals are more likely and therefore, there is a larger amount of positive signals that work in the opposite direction. Since the limited sample size of breakdowns in the case of a good state and high signal quality an interpretation of these results is not possible.

Table III

True State	Signal Qual.	Number of Breakdowns	Mean Duration of Avalanche*	
bad	0.9	5,000	1.084	(0.294)
	0.8	4,982	1.378	(0.715)
	0.7	4,476	1.862	(1.448)
	0.6	2,877	2.452	(2.149)
good	0.9	0	-	(-)
	0.8	2	2	(0)
	0.7	19	2.095	(0.625)
	0.6	265	2.479	(1.491)

Number of Simulation: 5000

* in periods, Standard Diviation in Brackets

Observation 3: If an “entry” cascade breaks down, the breakdown is very quick.

4 Conclusion

In this paper we have adapted the standard information cascade model to situations where agents have the choice between switching to some new alternative or sticking to the old one. The opportunity of a new market or the availability of some promising new technology are good examples. We have enriched the model by two new features. First if agents opt for “switching” they continue to receive private signals each period. Thereby we take into account that from the point of view of the agent after switching to the new alternative the game is not over. The agent gains experience handling the new alternative and might finally learn whether it has any value. Here the second new feature comes into play. If the agent is convinced that switching was a mistake he may change his decision. We focused on how "enter"- information cascades evolve and how information is passed on within such an information cascade.

The inclusion of these two new features changes the learning process completely. We have identified rules which describe the learning process within an "enter"-cascade. Moreover, we have shown that an "enter"-cascade gains in stability as it goes on. To investigate the properties of this modified information cascade model we ran simulations for 8 scenarios. These differ in the underlying state of the world and the signal quality. For each of the scenarios we ran 5000 simulations.

The simulations reproduced what we have found analytically: Empirical hazard rates for an information cascade approached zero as time increased. Already after 50 periods all but 1 out of 17,621 breakdowns had occurred. We found that good "enter"-cascades with a relatively high signal quality were fully revealing. Even with a very poor signal quality 95% of the cascades were fully revealing. The information gathering process was less efficient in the case of a bad "enter"-cascade. Nevertheless, with a poor signal quality, still 58% of the cascade were fully revealing. This improved rather quickly to over 99% as the signal quality increased.

The results of this more realistic set up contradicts the results of the existing literature in two important points. First, the information gathering process seems to be more efficient than the standard information cascade suggested by earlier work (see e.g. Bikhchandani et al. (1992)). And second, the information cascades we analyzed are not as fragile. Information cascades gain in stability the longer they last.

Therefore, the attempt to correct a wrong "enter"-cascade by some public information release has to be timed quite carefully. On the one hand the problem might go away without any interference. On the other hand as the information cascade lasts it gains stability and therefore, a late information release must be quite powerful to overcome the optimistic attitude in the cascade. Hence, to much hesitation might be expensive.

Tabel II: Empirical Hazard Rate

State	signal quality	*at 4*	at 5	*at 6*	at 7	at 8	at 9	*at 10*	at 11	at 12	at 13	at 14	*at 15*	at 16	at 17	at 18	at 19
		Empirical Hazard Rate															
bad	0.9	0,00	0,67	0,16	0,86	0,71	0,02	0,00	0,01	0,89	0,00	0,00	0,00	1,00	-	-	-
	0.8	0,00	0,39	0,17	0,45	0,63	0,06	0,01	0,35	0,60	0,02	0,00	0,00	0,25	0,42	0,00	0,00
	0.7	0,00	0,18	0,14	0,20	0,44	0,08	0,02	0,09	0,34	0,03	0,02	0,01	0,03	0,20	0,01	0,00
	0.6	0,00	0,08	0,10	0,06	0,21	0,08	0,02	0,03	0,11	0,03	0,01	0,01	0,01	0,03	0,01	0,00
good	0.9	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
	0.8	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
	0.7	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
	0.6	0,00	0,00	0,02	0,01	0,01	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

State	signal quality	at 20	*at 21*	at 22	at 23	at 24	at 25	at 26	at 27	*at 28*	at 29	at 30	at 31	at 32	at 33	at 34	at 35
		Empirical Hazard Rate															
bad	0.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	0.8	0,00	0,00	0,08	0,37	0,00	0,00	0,00	0,03	0,00	0,00	0,33	0,00	0,00	0,00	0,00	0,00
	0.7	0,00	0,00	0,00	0,07	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,00	0,00	0,00	0,00	0,00
	0.6	0,00	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
good	0.9	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
	0.8	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
	0.7	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
	0.6	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

State	signal quality	*at 36*	at 37	at 38	at 39	at 40	at 41	at 42	at 43	at 44	*at 45*	at 46	at 47	at 48	at 49	at 50
		Empirical Hazard Rate														
bad	0.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	0.8	0,00	0,00	0,05	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
	0.7	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
	0.6	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
good	0.9	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
	0.8	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
	0.7	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
	0.6	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

Colunmheads in italics indicate reversion dates (RD). Periods at which breakdowns should be most likely are shaded.

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