# discussion papers 

FS IV 99-37

Demand for Customized Products, Production Flexibility, and Price Competition

William Novshek
Lynda Thoman
Purdue University

December 1999

ISSN Nr. 0722-6748

Forschungsschwerpunkt
Marktprozeß und Unter-
nehmensentwicklung

## Research Area

Market Processes and Corporate Development

William Novshek, Lynda Thoman, Demand for Customized Products, Production Flexibility, and Price Competition, Discussion Paper FS IV 99-37, Wissenschaftszentrum Berlin, 1999.

Wissenschaftszentrum Berlin für Sozialforschung gGmbH, Reichpietschufer 50, 10785 Berlin, Tel. (030) 25491 - 0

# Demand for Customized Products, Production Flexibility, and Price Competition 

by William Novshek and Lynda Thoman *

We examine markets where, in addition to production of standard (core) products, mass customization is technologically feasible. We compare a setting where a monopolist provides both standard and custom goods to a setting where an entrant joins the custom market, and find customers' tastes affect the social desirability of entry. The entrant is unconcerned about the impact of his custom production on the incumbent's core product market and in some cases may supply more custom products than is socially desirable. In contrast to previous literature, we show that increased variability of demand may lead to a reduction in investment in flexible production technologies.

## ZUSAMMENFASSUNG

## Nachfrage nach Spezialprodukten, Fertigungsflexibilität und Preiswettbewerb

In diesem Beitrag werden Märkte untersucht, auf denen zusätzlich zur Produktion von Standardprodukten die Herstellung von Spezialprodukten in großen Stückzahlen (,,mass customization") technologisch möglich ist. Es wird eine Situation verglichen, in der ein Monopolist sowohl Standard- als auch Spezialprodukte anbietet mit einer Situation, in der der eintretende Konkurrent den Markt für Spezialprodukte bedient. Der Vergleich zeigt, daß der Kundengeschmack die soziale Wünschbarkeit des Eintritts beeinflußt. Für den Eintretenden ist es unerheblich wie sich seine Spezialprodukte auf den Hauptproduktmarkt des eingesessenen Unternehmens auswirken und in manchen Fällen kann es dazu kommen, daß er mehr Spezialprodukte anbietet als gesellschaftlich wünschbar. Im Gegensatz zur bisherigen Literatur wird gezeigt, daß zunehmende Variabilität der Nachfrage zu einer Verringerung von Investitionen in flexible Fertigungstechnologien führen kann.

[^0]
## 1. Introduction

Firms realize they can often add value for their customers by tailoring or customizing their products to meet the special needs of individual buyers. For many products the standard (or uncustomized) market is becoming more competitive, squeezing profit margins. This creates additional incentives to find methods to generate added value for customers, part of which can be extracted as profit for the firm. At the same time, in many industries, the additional cost necessary to customize a product has decreased, making product customization a viable alternative. ${ }^{1}$ Improved technologies such as programmable robots, the advanced training of workers to make them more adaptable and responsive, and the redesigned of the production process to reflect innovations such as just-in-time inventory management have all contributed to making customization possible at reasonable costs.

When Siemens found profit margins shrinking in its standard electric motor market, it responded by emphasizing custom motors for individual purchasers. The custom motors quickly became a substantial fraction of overall sales. The market for personal computers, where standard setups are essentially a homogeneous commodity, has become extremely competitive, with little room for profit. Producers such as Dell now custom build their machines to order, with literally thousands of combinations of features possible. Deere \& Co., faced with continued declines in industry demand, has introduced a new strategy whereby it customizes farm equipment to meet its customers' requirements. Mass customization has made inroads into the clothing industry; Levi Strauss \& Co will make a pair of Levi's to any individual's specific measurements.

The ability to customize a product is an aspect of production flexibility. Many meanings for the term production flexibility have been discussed in the literature. Often the desire for flexibility arises out of a need to respond to uncertainty about the quantity that will be demanded in a single commodity context, or about the mix of quantities that will be demanded in a multiproduct context. In this literature, if the set of produced products is not fixed, the set of alternatives is very limited (for example, to producing one or both of two potential products).

[^1]In contrast, the notion of production flexibility we have been discussing is motivated by the dispersion in taste among potential purchasers. It cannot be modeled adequately with a small number of alternatives. At the same time, uncertainty about demand is unnecessary. Instead, our notion of flexible production allows the firm to create value for its customers by tailoring the products to fit different individuals' needs or desires, and then to capture some of that added value through appropriate pricing.

We model this notion of flexibility and examine its consequences. Our model is simple, to retain tractability, but captures key features of this idea. There is a market for a single core product, but within that market, value is added by customizing a product for an individual consumer. The problem is that there is a dispersion of desired customizations, and only a consumer's particular desired customization adds value for her. A "wrong" customization does not destroy value for her, but is merely a useless (for her) but harmless change.

We start from a point at which a single firm, called the incumbent, produces a single variant of the core product. ${ }^{2}$ Our game begins when customization becomes feasible. We consider both the monopoly problem (incumbent alone) and a duopoly problem (incumbent plus entrant). There are three strategic stages. First, firms invest in flexibility, to lower the cost of being able to produce a larger range of customized products. Next, firms choose the range of customized products they will produce. Finally, firms set prices for each of their products, including the incumbent's original product, which we call the standard product.

We explicitly model the consumer sector used to generate demands, and we use cost functions to model the technological possibilities for firms. To capture the idea that, in the duopoly case, customization increases the competitive pressure for the standard product, we assume per unit production costs for customized products are not too much larger than per unit production costs for the incumbent's standard product. Since individuals view "wrongly" customized products as no worse than the incumbent's standard product, the entrant can compete with the incumbent for the standard product market (individuals without an appropriate customized product or for whom the customized product is relatively too expensive). Our assumption means that, in equilibrium, the per unit production cost of a customized product is a binding upper bound on the price the incumbent charges for the standard product. This is our version

[^2]of the standard product market being more competitive after the introduction of customization (and an entrant).

We first examine monopoly and duopoly models without uncertainty. We consider two distributions of consumers tastes, a "positive" case" and a "negative case." In the positive case, consumers with high valuations for the standard product also have high value added (and thus valuations) for customization. In this case, when the cost of flexibility is low, in both the monopoly and duopoly outcomes the full range of custom products is produced, but the duopoly outcome has higher total surplus. When the cost of flexibility is higher, but not too high, in the duopoly outcome the full range of custom products is produced but the monopolist offers only a standard product. Total surplus is again higher with the duopoly. The entrant has more incentive to offer custom products than the incumbent/monopolist whose custom products compete with its own standard product.

In the negative case, consumers with high valuations for the standard product have low value added and relatively low valuations for customization. In this case the monopolist is willing to offer a full range of custom products even at flexibility costs which are high enough to prevent the entrant from doing so. At flexibility costs low enough that the full range of custom products is offered in both the monopoly and duopoly outcomes, total surplus is larger in the monopoly solution than with duopoly. This is because the monopolist is able to adjust both standard and custom product prices to sell to all consumers; those with high valuations for the standard (and relatively low valuations for the custom) product buy the standard product while those with high valuations for the custom (and low valuations for the standard) product buy the custom product. The entrant must battle with the incumbent's standard product to sell more custom products, and must lower price too much to attract as many buyers. Here the monopolist does a better job of selling custom products to those consumers who should be getting them (because their value added exceeds the extra cost of producing the custom product).

The differences between the positive and negative cases suggest that the nature of the distribution of consumer tastes will play an important role in determining the extent to which the market for customized products will be served by a single firm, and whether additional entry should be encouraged.

We also examine a monopoly problem with random demand. Here we show that increased variability of demand may lead to reduced investment in flexibility (and
reduced product range), contrary to the results for other types of production flexibility. This highlights the difference between the alternative notions of flexibility.

Of the large variety of issues addressed in the literature under the general heading of flexibility in production, we will mention the three most relevant to our paper. ${ }^{3}$ The first topic concerns uncertainty driven desire for flexibility in the single firm case. Some input levels must be chosen before uncertainty (about the prices for outputs and other inputs) is resolved while other inputs are chosen after some or all uncertainty is resolved. Turnovsky (1973) and Epstein (1978) address such problems for a competitive firm. Jones and Ostroy (1984) examine a more general version of this type of question, and emphasize the relationship between the level of flexibility embodied in the initial choices and the amount of information that will be received later. Greater flexibility preserves more options for the future use of the information that will arrive. A conclusion of this strand of literature that is relevant to our paper is the observation that increased variability of, for example, demand for the output good, increases the level of flexibility chosen initially. This conclusion does not hold in our context, as we show in Section 4.

Strategic choice of flexibility is introduced in Dixon $(1985,1986)$. Firms choose flexibility in the first stage of a model in which they act as competitive price takers in the second stage. In Dixon (1985), the strategic choice is the level of capital, where increased capital lowers the marginal cost curve for the price-taking stage. Underinvestment in capital reduces flexibility in the second stage, in the sense of Stigler (1939), and increases profit. Dixon (1986) extends the analysis to allow a commitment to the levels of one or both of the two inputs to production, or to neither input, in the first stage

The third strand of literature involves multiple product firms. It is related to an extensive literature in Operations Management concerning flexible manufacturing systems. There are two product markets, and firms must initially decide whether to use dedicated machines, each of which produce just one of the products or to use a flexible machine, which can produce either product. He and Pindyck (1992) analyze a firm that is a monopolist in both markets. The firm initially decides which type of machines to employ, then continually invests in the chosen technology. Demands are random, with negative correlations between markets. Even though the flexible machine is more expensive, it may be more profitable, and an increase in the variability of demand (with

[^3]the negative correlation between markets) increases the relative advantage of flexible over dedicated machines.

Roeller and Tombak (1990) analyze a game in which two firms engage in quantity competition in the two markets in the second stage, thus introducing strategic interaction to the second as well as first stage. In the first stage, firms choose either a flexible production technology, which allows a firm to compete in both product markets, or a dedicated technology, which forces a firm to choose a single market in which to compete. They identify the conditions under which firms would choose the flexible technology.

Our paper differs substantially from the existing literature because the desire for flexibility is driven by the possibility to customize products for individual customers. With a continuum of potential customizations, the question is not whether to use a flexible machine, but rather, how flexible to make the machine, and what range of customizations to offer.

The remainder of the paper is organized as follows. Section 2 contains the model and some initial observations on the nature of the equilibria in the duopoly case. Section 3 compares monopoly and duopoly outcomes when there is no uncertainty. Section 4 shows that for a monopolist facing uncertainty about demand, increased variability may lead to a decrease, rather than an increase in the desired level of flexibility. Concluding comments are contained in Section 6.

## 2. Model

This section introduces the model and makes some basic observations about the nature of the duopoly equilibrium. These observations simplify the discussion of the duopoly model in Section 3.

The set of potential variants of the standard product is a continuum, represented by the unit interval $[0,1]$. Each type $t \in[0,1]$ represents a different customization of the standard product.

Individual consumers are competitive price takers who will purchase either nothing, or one unit of one of the available types of the product. Each consumer is characterized by a triple, $\left(v_{0}, v_{1}, t\right)$, where $v_{0} \geq 0, v_{1} \geq v_{0}$, and $t \in[0,1]$. The type, $t$, is
the only customization of the product that the consumer views as having added value. A consumer whose desired customization is type $t$ will also be called a type $t$ consumer. Variants of the product with types other than $t$ are perfect substitutes for one another, and the consumer's reservation value for any type other than $t$ is $v_{0}$. The consumer's reservation value for type $t$ is $v_{1}$. Hence, for customer $t$, the value added by the customization $t$ is $v_{1}-v_{0} \geq 0$; no other customization creates additional value. It is important to keep in mind that there is no notion of types being similar in preference terms. Either the type is the desired one, or it is not. Thus, though the structure may look similar to many spatial or differentiated product models, it is in fact quite different.

The demand behavior for a consumer characterized by ( $v_{0}, v_{1}, t$ ) is simple. If type $t$ is the only one available, and has price $p_{1}$, then the consumer purchases one unit of type $t$ when $v_{1}-p_{1} \geq 0$. If type $t$ is not available, all available types are perfect substitutes, and the effective price for the consumer is the lowest price for any available type. If this lowest price is $p_{0}$, then the consumer purchases one unit of one of the lowest priced types when $v_{0}-p_{0} \geq 0$. If type $t$ is available at price $p_{1}$, and some other types are available, with lowest price $p_{0}$, then the consumer compares $v_{1}-p_{1}, v_{0}-p_{0}$, and 0 to determine whether to purchase one unit of type $t$, one unit of one of the lowest priced other types, or nothing. This is illustrated in Figure 1. Note that those who purchase the customized type $t$ good have relatively high value added for that type, or $v_{1}-v_{0} \geq p_{1}-p_{0}$.

We will consider three different distributions of consumer characteristics, $\left(v_{0}, v_{1}, t\right)$, in each of which there is a continuum of consumers. In each case, the conditional distribution on $\left(v_{0}, v_{1}\right)$ given $t$ is independent of $t$ as long as type $t$ is desired by some consumers. This greatly simplifies our analysis since each " $t$ market" will have similar demand properties.

The first two distributions are for the nonrandom cases in Section 3. For each of these the mass of consumers is one, and every potential type is desired by some consumers. Given $S>0$, the first distribution is uniform on $\left\{\left(v_{0}, v_{1}, t\right) \mid 0 \leq v_{0} \leq 1, v_{1}=(1+S) v_{0}, 0 \leq t \leq 1\right\}$. Note $v_{0}$ and $v_{1}$ are perfectly positively correlated, as are $v_{0}$ and the value added, $v_{1}-v_{0}$. This will be referred to as the positive

Figure 1.
Purchase decisions for a consumer of type $t$.

case. Given $T>1$, the second distribution is uniform on $\left\{\left(v_{0}, v_{1}, t\right) \mid 0 \leq v_{0} \leq 1, v_{1}=T-v_{0}(T-1), 0 \leq t \leq 1\right\}$. Note $v_{0}$ and $v_{1}$ are perfectly negatively correlated, as are $v_{0}$ and the value added, $v_{1}-v_{0}$. This will be referred to as the negative case. These distributions are used to investigate how the correlations matter for the effect of increased competition on the equilibrium range of customized products. ${ }^{4}$

The third distribution is for the random case in Section 4. The distribution is uniform on $\left\{\left(v_{0}{ }^{*}, v_{1}{ }^{*}, t\right) \mid 0 \leq t \leq \tilde{m}\right\}$, where $0<v_{0}{ }^{*}<v_{1}{ }^{*}$, and $\tilde{m}$ is random taking values $m_{1}$ and $m_{2}$ each with probability $1 / 2$, where $1>m_{2}>m_{1}>0$. Not every potential product type is desired by some consumers, and the set of product types that are desired is random. For simplicity, in this case the conditional distribution on $\left(v_{0}, v_{1}\right)$ given $t \leq \tilde{m}$ is degenerate, with all weight on $\left(v_{0}{ }^{*}, v_{1}{ }^{*}\right)$.

At this point it will be useful to return to Figure 1, assume type $t$ is available, introduce each of the distributions in turn, and consider which of the $\left(v_{0}, v_{1}\right)$ would purchase $t$, purchase another good, or purchase nothing, given the prices in the figure. For the positive case, the possible $\left(v_{0}, v_{1}\right)$ combinations lie on a line with slope greater than one, beginning at $(0,0)$. Those with low $v_{0}$ and $v_{1}$ purchase nothing; those with intermediate $v_{0}$ and $v_{1}$ purchase the cheapest type instead of type $t$; while those with high $v_{0}$ and $v_{1}$ purchase type $t .{ }^{5}$ For the negative case, the possible $\left(v_{0}, v_{1}\right)$ combinations lie on a line with negative slope beginning at $(0, T)$. Those with low $v_{0}$ and high $v_{1}$ purchase type $t$; those with intermediate $v_{0}$ and $v_{1}$ purchase nothing, while those with high $v_{0}$ and low $v_{1}$ purchase the cheapest type instead of type $t$. For the random case, only $\left(v_{0}{ }^{*}, v_{1}{ }^{*}\right)$ is possible, so all make the same purchase decision.

We now turn to the production technology. There are two types of machines: dedicated machines and flexible machines. A dedicated machine can produce only a single type of good. The machine has a strictly positive fixed cost and unlimited production capacity with constant per-unit production cost. For notational convenience, all values $v_{0}$ and $v_{1}$ are measured as net of the per-unit production cost, and that cost can be treated as zero. Since a dedicated machine has a strictly positive machine cost, it

[^4]is only profitable if it is used to sell to a positive mass of consumers. None of the three distributions we consider has any lump of mass at any type $t$, so the relevant demand must come from sales as a standard, rather than customized, product (i.e., from sales to consumers who desire a different type but purchase this instead as the cheapest of the non-ideal types). Hence no firm would ever purchase more than one dedicated machine, and the actual type produced by the dedicated machine is irrelevant. For notational convenience we will not specify the type produced by any dedicated machine, but will refer to it as the standard product.

The second type of machine available is a flexible machine, which is able to produce more than one type. There are three kinds of costs associated with a flexible machine: machine cost, product line cost, and unit production cost. The machine cost for flexibility level $K>0$ is $r K$, where $r>0$. The product line cost is determined by the range of types of goods the machine will be used to produce; it does not depend on total production or the production levels of any type produced. For any range of types the product line cost is lower the more flexible the machine. Because of our distributional assumptions, the number of different types that can be produced matters, but the specific types do not. The product line choice is $y$, the length of the range of producible types. Given flexibility $K$, the product line cost for a range of products of length $y$ is $y^{2} / K$. For any of the types in the range of types chosen, there is unlimited production capacity with constant per-unit production cost, which is the same for all types in the range, and is independent of the flexibility and product range choices. Since the per unit production cost for a dedicated machine was normalized to zero, we assume the per unit production cost for a flexible machine is $c>0$.

The machine cost plus product line cost, $r K+y^{2} / K$, is such that there are "constant returns to scale" in $y$ when $K$ and $y$ are coordinated. That is, if $K$ is chosen to minimize the machine cost plus product line cost for $y$, the optimal $K$ is $K^{*}(y)=y / \sqrt{r}$, with machine plus product line cost $r y / \sqrt{r}+y^{2} /(y / \sqrt{r})=(2 \sqrt{r}) y$. For a monopoly with no uncertainty, this observation will simplify the analysis, by allowing us to combine the $K$ and $y$ choice stages.

We will consider both the problem for a single firm and for a duopoly. Firms are risk neutral, expected profit maximizers. We start from a position in which one firm already has a dedicated machine, and the machine costs are sunk. As noted earlier, this firm would never purchase a second dedicated machine. Also since we will use price setting by firms in the final strategic stage, if the second firm had a dedicated machine, the resulting prices would equal the unit production cost, and the second firm could not
cover the dedicated machine cost. Thus, the second firm would not purchase a dedicated machine. For these reasons, we simplify our game by removing the possibility of the first firm purchasing a second dedicated machine and the possibility of the second firm purchasing a dedicated machine. Only the choice of a flexible machine is considered.

Noting that stages 1 and 3 are relevant only for the random case in Section 4, our time line is as follows:

1. [For the random case] Nature picks a realization of the consumer distribution parameter $\tilde{m}$.
2. Firms choose their investment levels in flexibility, $K_{i}$, simultaneously and independently.
3. [For the random case] The realized value of $\tilde{m}$ is observed.
4. With the flexibility choices common knowledge, firms choose their product ranges, $y_{i}$, simultaneously and independently
5. With the product range choices common knowledge, firms choose prices for each of the types they produce (including the type produced by the dedicated machine), simultaneously and independently.
6. Consumers decide which types, if any, to purchase; firms produce to meet demand for each type, and profits are realized.

Our solution concept is pure strategy, subgame perfect, Nash equilibrium.
Let $a$ and $b$ denote the first and second firms, respectively. Let $p$ denote the first firm's price for the type produced by its dedicated machine. For $i=a, b$, for each type $t$ in firm $i$ 's range of products, let $p_{i}(t)$ denote firm $i$ 's price for type $t$.

For a consumer of type $t$, the effective prices for the type t good and the standard good, $p_{1}(t)$ and $p_{0}$, respectively, are the lowest prices for the type $t$ product and the lowest prices for all other types combined. If type $t$ is produced by firm $i$ alone, then $p_{1}(t)=p_{i}(t)$. If type $t$ is produced by both firms, then $p_{1}(t)$ is the minimum of $p_{a}(t)$
and $p_{b}(t) .{ }^{6}$ The price for the standard product is the lowest price overall, $p_{0}=\min \left\{p, \min \left\{p_{1}\left(t^{\prime}\right) \mid 0 \leq t^{\prime} \leq 1\right\}\right\} .{ }^{7}$

When consumers are indifferent between purchasing and not purchasing, or between purchasing different types, we need to impose tie-breaking rules. We assume consumers purchase rather than not purchase when indifferent between the two. This matters for the random case but not for the others. When consumers are indifferent between multiple products, with two exceptions, we assume they are split equally. The equal sharing assumption has no impact on the results. The first exception to an equal split is standard for multiproduct monopoly: if consumers are indifferent, then their demand allocation is that which the monopolist prefers. The second exception to an equal split is the standard one for Bertrand competition with different unit costs: if consumers are indifferent between products, one of which is offered at a price exceeding its unit cost of production while the others are offered at prices equal to their unit cost of production, then all demand goes to the product with price above unit cost. This assumption will have an effect in all duopoly equilibria since we impose parameter restrictions: $c<S /(2+4 S)$ for the positive case and $c<T /(1+2 T)$ for the negative case. By assuming that the unit costs of production for a flexible machine are not too much larger than the unit costs of production for a dedicated machine, we guarantee that in all duopoly equilibria, the price for the standard product produced by the dedicated machine will equal the unit production cost for the flexible machine as discussed below.

In the remainder of this section we make some basic observations about the nature of equilibrium in the duopoly problem. These observations will allow us to carry out the analysis in Section 3 in terms of just three prices: the incumbent's (monopolist's) standard product price and a single custom price for each firm.

If $y_{b}=0$, then the pricing stage is just a monopoly optimization problem, so consider an equilibrium in which $y_{b}>0$. What must be true about any duopoly equilibrium in which $y_{b}>0$ ?

First note that in any duopoly equilibrium, the standard product of the incumbent has positive sales. This follows from its cost advantage, $c>0$, and the fact that for each

[^5]type, there are some consumers of that type who have value added, $v_{1}-v_{0}$, which is less than the cost advantage.

If the standard product had positive sales at a price exceeding $c$, the entrant could use just one of the types in its product range to undercut the standard product price, for a discrete increase in profit serving the standard market with negligible impact on the entrant's customized product sales. Thus the standard product price cannot exceed $c$. As mentioned earlier, we impose parameter restrictions such that, in duopoly equilibrium, this is a binding constraint, so the incumbent will always end up selling its standard product at price $p=c$.

If both firms produce type t , then in equilibrium, $p_{a}(t)=p_{b}(t)=c$. This is because the standard product is already priced at $c$, so pricing for type $t$ cannot affect the standard product market (recall the tie-breaking rule for allocation of demand leaves all sales with the incumbent's standard product) and it does not affect any other $t^{\prime}$ market. Thus type $t$ has a classic homogenous good Bertrand market.

Given that overlapping custom markets lead to marginal cost pricing, firms cannot cover the cost of product range expansion into these markets (and for the incumbent, pricing type $t$ at marginal cost can only hurt its standard product sales). Thus, in equilibrium $y_{a}+y_{b} \leq 1$, with no overlap of the types produced

The assumption that the distribution of ( $v_{0}, v_{1}$ ) among consumers is independent of $t$, means that every $t$ market in which a firm operates has the same demand function, and there is a single standard product price, so the incumbent's custom product price will be the same for every $t$ in its product range, and the entrant's custom product price will be the same for every $t$ in its product range. ${ }^{8}$ Thus the remaining analysis can be carried out in terms of three prices: the incumbent's standard product price, $p$, the incumbent's custom product price, $p_{a}$, and the entrant's custom product price. $p_{b}$. This last observation applies to the monopoly problem as well; the monopoly problem can be carried out in terms of a standard price, $p$, and a custom price $p_{a}$.

[^6]
## 3. Monopoly and Duopoly Flexibility Choice

In this section we examine and compare the monopoly and duopoly solutions for both the positive and negative cases. We start by considering a single type $t$ of consumer for whom one of the firms produces the custom product type $t$. Let $p_{1}$ be the price for the type $t$ good and let $p_{0}$ be the lowest other price. Figure 2 indicates the relevant demand regions for the positive case. If $\left(p_{0}, p_{1}\right)$ is in Region A1, then both the standard and type $t$ good will have positive demand. If $\left(p_{0}, p_{1}\right)$ is above region A1, demand is zero for type $t$ but positive for the standard good. In region B1, demand is zero for the standard good and positive for type $t$. Let $d_{0}$ and $d_{1}$ denote the demands by type $t$ consumers for the standard and type $t$ products, respectively. Then for the positive case,


Insert Figure 2 here

Figure 3 indicates the relevant demand regions for the negative case. In both regions A2 and B2 both the standard and type $t$ products have positive demands. In A2, but not in B2, all consumers purchase something. For the negative case, the demands by type $t$ consumers are:

$$
\left(d_{0}\left(p_{0}, p_{1}\right), d_{1}\left(p_{0}, p_{1}\right)\right)=\left\{\begin{array}{cl}
\left(\frac{p_{1}-p_{0}}{T}, \frac{T-p_{1}+p_{0}}{T}\right) & \text { in A2 } \\
\left(1-p_{0}, \frac{T-p_{1}}{T-1}\right) & \text { in B2 }
\end{array}\right.
$$

Insert Figure 3 here

Figure 2
Demand regions for type $t$ in positive case.


Figure 3
Demand regions for type $t$ in negative case.


Aggregate demands are obtained by combining the $t$ market demands just discussed with the demand by consumer types for which no custom product is available. For each of these types, $d_{0}\left(p_{0}\right)=1-p_{0}$. Integrating over all the types, we obtain the aggregate demand for the standard product, $D_{0}$, the aggregate demand for the incumbent's custom products, $D_{a}$, and the aggregate demand for the entrant's custom products, $D_{b}$. For $y_{a}+y_{b} \leq 1$, the aggregate demands, in terms of the incumbent's prices, $p$ and $p_{a}$, and the entrant's price, $p_{b}$, are:

$$
\begin{aligned}
& D_{0}\left(p, p_{a}, p_{b}\right)=\left(1-y_{a}-y_{b}\right)(1-p)+y_{a} d_{0}\left(p, p_{a}\right)+y_{b} d_{0}\left(p, p_{b}\right), \\
& D_{a}\left(p, p_{a}\right)=y_{a} d_{1}\left(p, p_{a}\right) \text { and } \\
& D_{b}\left(p, p_{b}\right)=y_{b} d_{1}\left(p, p_{b}\right) .
\end{aligned}
$$

The corresponding profits, given $y_{a}, y_{b}, K_{a}$, and $K_{b}$, are ${ }^{9}$

$$
\begin{aligned}
& \pi_{a}\left(p, p_{a}, p_{b}\right)=p D_{0}\left(p, p_{a}, p_{b}\right)+\left(p_{a}-c\right) D_{a}\left(p, p_{a}\right)-y_{a}^{2} / K_{a}-r K_{a} \text { and } \\
& \pi_{b}\left(p, p_{b}\right)=\left(p_{b}-c\right) D_{b}\left(p, p_{b}\right)-y_{b}^{2} / K_{b}-r K_{b} .
\end{aligned}
$$

First consider the monopoly problem ( $K_{b} \equiv y_{b} \equiv 0$ ), starting with the choice of prices $p$ and $p_{a}$ given $y_{a}$ and $K_{a}$. In the positive case, given our parameter restriction $c<S /(2+4 S)$, the optimal ( $p, p_{a}$ ) always lies in the interior of region A1. The optimal prices are:

$$
\left(p^{m}, p_{a}^{m}\right)=\left(\frac{1}{2}, \frac{1+c+S}{2}\right) .
$$

In the negative case, given our parameter restriction $c<T /(1+2 T)$, the optimal ( $p, p_{a}$ ) always lies on the boundary between the A 2 and B 2 regions:

$$
\left(p^{m}, p_{a}^{m}\right)=\left(\frac{1+y(T-c)}{2[1+y(T-1)]}, \frac{1+T+y(T-1)(c+T)}{2[1+y(T-1)]}\right) .
$$

[^7]As noted earlier, for the monopoly problem without uncertainty, the $y_{a}$ and $K_{a}$ choice stages can be combined into a single choice of $y_{a}$ at cost $(2 \sqrt{r}) y$, where the corresponding $K_{a}$ is $y / \sqrt{r}$. Substituting this and the optimal $p^{m}$ and $p_{a}^{m}$ as functions of $y_{a}$ into the profit function, $\pi_{a}^{m}$, we observe that profit is linear in $y_{a}$. Comparing profit when $y_{a}=0$ to profit when $y_{a}=1$, we obtain the overall monopoly solution. For the positive and negative cases, the values for $K_{a}^{m}, y_{a}^{m}, p^{m}, p_{a}^{m}$ (where relevant), $\pi_{a}^{m}$ and total surplus (producer's plus consumers' surplus) are listed in Table 1. The term $r_{m}^{\max }$ is used to denote the largest value of $r$ for which the monopolist is willing to invest in flexibility.

Insert Table 1 here

We now turn to the duopoly problem starting with the price setting stage given $y_{a}, y_{b}, K_{a}$, and $K_{b}$. If $y_{b}=0$, this coincides with the monopoly case, so assume $y_{b}>0$. Recall we have made parameter restrictions $(c<S /(2+4 S)$ in the positive case, $c<T /(1+2 T)$ in the negative case) that guarantee the incumbent's equilibrium price for the standard product is $p=c$. Thus the duopoly pricing game can be carried out entirely in terms of $p_{a}$ and $p_{b}$, given $p=c$.

In the positive case, there are now two versions of Figure 2, one for $p=c$ and $p_{a}$ and the other for $p=c$ and $p_{b}$. Each firm has a dominant "strategy" for its custom price, since the custom markets do not interact directly, and the pair of dominant "strategies" forms the pricing equilibrium: $p=c, p_{a}=(3 c+S) / 2$, and $p_{b}=(2 c+S) / 2$. The negative case is similar, with dominant "strategies" and pricing equilibrium: $p=c$, $p_{a}=(3 c+T) / 2$, and $p_{b}=(2 c+T) / 2$.

It is no longer possible to combine the $K$ and $y$ choice stages into a single step since the choice of $K$ may have strategic implications in the $y$ choice stage. First consider the $y$ choice stage, given the $K$ choices. There may be multiple equilibria, since the firms do not want to have overlapping product ranges. Again there is no direct interaction between $y_{a}$ and $y_{b}$, unless $y_{a}+y_{b} \geq 1$. To find all the equilibria at this stage, we first determine the maximum product range each firm would desire, as a function of its investment in flexibility, K. As long as the other firm's product range is small enough that, when added to the maximum range, the total is no more than 1 , then the firm's best response is its maximum range. Otherwise, the best response makes the sum of product ranges equal to 1 .

Table 1. Monopoly Equilibrium

| Case | Parameter restriction | $K_{a}^{m}$ | $y_{a}^{m}$ | $p^{m}$ | $p_{a}^{m}$ | $\pi_{a}^{m}$ | Total Surplus |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Positive | $\begin{aligned} & r>r_{m}^{\max } \\ & =\frac{(S-c)^{4}}{64 S^{2}} \end{aligned}$ | 0 | 0 | $\frac{1}{2}$ | - | $\frac{1}{4}$ | $\frac{3}{8}$ |
|  | $\begin{aligned} & r<r_{m}^{\max } \\ & =\frac{(S-c)^{4}}{64 S^{2}} \end{aligned}$ | $\frac{1}{\sqrt{r}}$ | 1 | $\frac{1}{2}$ | $\frac{1+c+S}{2}$ | $\begin{gathered} \frac{1}{4}+\frac{(S-c)^{2}}{4 S} \\ -2 \sqrt{r} \end{gathered}$ | $\begin{gathered} \frac{3\left((S-c)^{2}+S\right)}{8 S} \\ -2 \sqrt{r} \end{gathered}$ |
| Negative | $\begin{aligned} & r>r_{m}^{\max } \\ & =\frac{\left((T-c)^{2}+1+T-2 c\right)^{2}}{64 T^{2}} \end{aligned}$ | 0 | 0 | $\frac{1}{2}$ | - | $\frac{1}{4}$ | $\frac{3}{8}$ |
|  | $\begin{aligned} & r<r_{m}^{\max } \\ & =\frac{\left((T-c)^{2}+1+T-2 c\right)^{2}}{64 T^{2}} \end{aligned}$ | $\frac{1}{\sqrt{r}}$ | 1 | $\frac{T+1-c}{2 T}$ | $\frac{T^{2}+1+c(T-1)}{2 T}$ | $\begin{gathered} \frac{1}{4}+\frac{(T-c)^{2}+1+T-2 c}{4 T} \\ -2 \sqrt{r} \end{gathered}$ | $\begin{gathered} \frac{3(T-c)^{2}+6 T-2 c-1}{8 T} \\ -2 \sqrt{r} \end{gathered}$ |

For the positive case, the maximum product ranges are:

$$
\begin{aligned}
& y_{a}\left(K_{a}\right)= \begin{cases}\frac{K_{a}(S-c)^{2}}{8 S} & \text { if } K_{a}<\frac{8 S}{(S-c)^{2}} \\
1 & \text { if } K_{a} \geq \frac{8 S}{(S-c)^{2}}\end{cases} \\
& y_{b}\left(K_{b}\right)= \begin{cases}\frac{K_{b} S}{8} & \text { if } K_{b}<\frac{8}{S} \\
1 & \text { if } K_{b} \geq \frac{8}{S}\end{cases}
\end{aligned}
$$

If $y_{a}\left(K_{a}\right)+y_{b}\left(K_{b}\right) \leq 1$, then these maximum ranges form the unique equilibrium. If the sum exceeds 1 , then any pair $\left(y_{a}, y_{b}\right)$ such that $y_{a}+y_{b}=1$ and $y_{i} \leq y_{i}\left(K_{i}\right)$ for $i=a, b$, forms an equilibrium. The results for the negative case are similar, with maximum product ranges

$$
\begin{aligned}
& y_{a}\left(K_{a}\right)=\left\{\begin{array}{ll}
\frac{K_{a}\left[-2 c+c^{2}\left(4+\frac{1}{T}\right)+T\right]}{8} & \text { if } K_{a}<\frac{8}{-2 c+c^{2}\left(4+\frac{1}{T}\right)+T} \\
y_{b}\left(K_{b}\right)= \begin{cases}\frac{K_{b} T}{8} & \text { if } K_{b}<\frac{8}{T} \\
1 & \text { if } K_{a} \geq \frac{8}{T}\end{cases}
\end{array} . \begin{array}{l}
-2 c+c^{2}\left(4+\frac{1}{T}\right)+T
\end{array}\right. \\
& y_{b}^{1}
\end{aligned}
$$

and

For both the positive and the negative cases, note that $y_{a}(K) \leq y_{b}(K)$, with strict inequality if $0<y_{a}(K)<1$. This should not be surprising since by increasing its product range, firm a increases its range of competition with its own standard commodity.

Turning now to the K choice stage, it is clear the outcome depends on the equilibrium anticipated for the $y$ choice stage. Multiple equilibria are possible at this stage as well. The easiest way to find all equilibria at this stage is to start with the firm anticipating that the next stage equilibrium will be the one in which its product range will be $y_{i}\left(K_{i}\right)$ if it chooses $K_{i}, i=a, b$. Substituting this anticipated product range equilibrium and the resulting pricing stage equilibrium into the firm's profit function, we obtain profit as a function of $K_{i}$. When differentiating this with respect to $K_{i}$, by the Envelope Theorem, since $y_{i}\left(K_{i}\right)$ is optimal given $K_{i}$, the indirect effect of $K_{i}$ through $y_{i}$ is zero, and only the direct cost reducing effect, $\left(y_{i}\left(K_{i}\right) / K_{i}\right)^{2}-r$, remains. Note from the previous formulas, in each case, $y_{i}\left(K_{i}\right) / K_{i}, i=a, b$, is constant as long as $y_{i}\left(K_{i}\right)<1$. For $K_{i}$ such that $y_{i}\left(K_{i}\right)=1$, the profit derivative is $\left(1 / K_{i}\right)^{2}-r$, which is decreasing in $K_{i}$.

For each firm $i$, let $h_{i}$ denote the fixed ratio $y_{i}\left(K_{i}\right) / K_{i}$ when $y_{i}\left(K_{i}\right)<1$. Then the optimal $K_{i}$ is easy to describe. If $r<\left(h_{i}\right)^{2}$, then profit is increasing in $K_{i}$ until $y_{i}\left(K_{i}\right)=1$, and beyond, until $\left(1 / K_{i}\right)^{2}=r$. In this case the firm invests enough to pick product range 1 , and invests enough, $K_{i}=1 / \sqrt{r}$, to make product range 1 as inexpensive as possible. If $r>\left(h_{i}\right)^{2}$, then the firm would invest nothing. At $r=\left(h_{i}\right)^{2}$, the firm is indifferent among any investment level between zero and $1 / \sqrt{r}$.

Multiple equilibria arise for $r$ such that $r \leq\left(h_{i}\right)^{2}, i=a, b$. Both firms would like to be the sole producer of all custom types. For such $r$, any positive $K_{a}$ and $K_{b}$ such that $K_{a}+K_{b}=1 / \sqrt{r}$ can be sustained as an equilibrium in which $y_{a}+y_{b}=1$ and each firm is productively efficient in the sense that its $K_{i}$ is the investment level that minimizes the cost of product range $y_{i}$ (i.e., $K_{i}=y_{i} / \sqrt{r}$ ). ${ }^{10}$

Let $r_{i}^{\text {max }}, i=a, b$, be the largest r for which firm i is willing to invest. For the positive and negative cases these are listed in Table 2. Note that in each case, $r_{b}^{\max }>r_{a}^{\max }$, so for r between the two values there is a unique equilibrium. For these $r$ values, for the positive and negative cases, the unique equilibrium values for $K_{i}, y_{i}, p_{i}$, $\pi_{i}, i=a, b$, and for $p$ and for total surplus are listed in Table 2.

[^8]| Table 2. Unique duopoly equilibrium when $\boldsymbol{r}_{b}^{\text {max }}>\boldsymbol{r}>\boldsymbol{r}_{a}^{\text {max }}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Firm | $\boldsymbol{r}_{i}^{\text {max }}$ | $K_{i}$ | $y_{i}$ | $p$ | $p_{i}$ | $\pi_{i}$ | Total Surplus |
| Positive Case: <br> Firm a | $\frac{(S-c)^{4}}{64 S^{2}}$ | 0 | 0 |  | - | $c\left(\frac{1}{2}-c\right)$ |  |
| Firm b | $\frac{S^{2}}{64}$ | $\frac{1}{\sqrt{r}}$ | 1 | c | $\frac{2 c+S}{2}$ | $\frac{S}{4}-2 \sqrt{r}$ | 8 - $2 \sqrt{r}$ |
| Negative Case: <br> Firm a | $\frac{\left((T-c)^{2}+4 c^{2} T\right)^{2}}{64 T^{2}}$ | 0 | 0 |  | - | $\frac{c}{2}$ |  |
| Firm b | $\frac{T^{2}}{64}$ | $\frac{1}{\sqrt{r}}$ | 1 | c | $\frac{2 c+T}{2}$ | $\frac{T}{4}-2 \sqrt{r}$ | $\frac{3 T-4 c+4}{8}-2 \sqrt{r}$ |

For $r<r_{a}^{\max }$, the equilibrium is not unique, but the values listed in Table 2 describe the equilibrium that is most profitable for the entrant. At the other extreme is the equilibrium in which $K_{b}=0$ and $K_{a}=1 / \sqrt{r}$. Since $K_{b}=0$ this reverts to the monopoly problem for subsequent stages. (Note the monopolist is willing to invest $K=1 / \sqrt{r}$ whenever the incumbent would do so in the duopoly problem.) Thus the previous monopoly solution may be used.

We now turn to a comparison of the different equilibria for fixed parameter values. First, for $r \leq r_{a}^{\max }\left(<r_{b}^{\max }\right)$ consider the continuum of productively efficient equilibria in which the two firms share the custom market. Each of these equilibria is specified by a value for $y_{b}$, with $0<y_{b}<1, y_{a}=1-y_{b}$, and, for productive efficiency, $K_{i}=y_{i} / \sqrt{r}$ for $i=a, b$. Since there are no income effects, total surplus, the sum of consumers' plus producers' surplus, is a reasonable welfare measure.

Proposition 1: Assume $r \leq r_{a}^{\max }$. Then among the productively efficient duopoly equilibria in which the entrant has a positive share of the custom market, $0<y_{b}<1$, total surplus is increasing in $y_{b}$, the product range of the entrant.

Proof: For the positive case, for each of these equilibria, for each type $t$, the fraction of consumers who purchase something is $1-c$, independent of whether the type is produced by the incumbent or entrant. Also the total investment plus product range cost is $2 \sqrt{r}$ for each of these equilibria. Thus in total surplus terms, the equilibria differ only in the individuals who receive a custom rather than standard product. In each $t$ market served by the incumbent, consumers with $v_{0} \geq 1 / 2+c / 2 S$ purchase the custom good, while in those $t$ markets served by the entrant, consumers with $v_{0} \geq 1 / 2$ purchase the custom good. To maximize total surplus, every consumer with $v_{1}=(1+S) v_{0}>v_{0}+c$ should receive the custom rather than standard good, so every consumer with $v_{0}>c / S$ should receive a custom good. Since $c / S<1 / 2$, neither firm sells custom goods to all the consumers who should receive them, but the entrant does better than the incumbent. Thus total surplus is increasing in $y_{b}$.

The negative case is similar except that in every one of the productively efficient duopoly equilibria, all consumers purchase something. In the $t$ markets served by the incumbent, consumers with $v_{0} \leq 1 / 2-c / 2 T$ receive the custom good, while in $t$ markets served by the entrant, consumers with $v_{0} \leq 1 / 2$ receive the custom good. To maximize total surplus, every consumer with $v_{1}=T-v_{0}(T-1)>v_{0}+c$, or $v_{0}<(T-c) / T$, should receive the custom rather than standard good. Since $(T-c) / T>1 / 2$, neither firm sells custom goods to all consumers who should receive
them, but the entrant does better than the incumbent. Thus total surplus is increasing in $y_{b}$. Q.E.D.

Proposition 1 proves that if there are multiple duopoly equilibria, then among those in which the entrant has positive product range, the best in terms of total surplus is the productively efficient equilibrium in which $y_{b}=1$. It does not prove that the equilibrium with $y_{b}=1$ is better than that with $y_{b}=0$. This is because of the discontinuity in equilibrium pricing. When $y_{b}=0$, the problem reverts to that of the monopolist, where the standard product price is no longer constrained to equal $c$, the unit production cost for the custom good. Here we need to compare the monopoly solution to the duopoly equilibrium with $y_{b}=1$, which exists if $r \leq r_{b}^{\max }$.

Proposition 2: Assume $r \leq r_{b}^{\max }$. In the positive case, total surplus is larger in the duopoly equilibrium with $y_{b}=1$ than in the monopoly solution. In the negative case, total surplus is smaller in the duopoly equilibrium with $y_{b}=1$ than in the monopoly solution.

Proof: For the positive case, note that $r_{b}^{\max }>r_{m}^{\max }$, so for $r \leq r_{m}^{\max }$, the monopoly solution has $y_{m}=1$ while for $r_{m}^{\max }<r \leq r_{b}^{\max }$ the monopoly solution has $y_{m}=0$. For $0<r \leq r_{m}^{\max }$, both monopoly and duopoly have product range 1 , so what matters are the fraction of consumers who get any product and the fraction that get the custom product. In the monopoly solution, all consumers with $v_{0}$ between $1 / 2$ and $1 / 2+c / 2 S$ receive the standard product, and those with $v_{0}$ between $1 / 2+c / 2 S$ and 1 receive the custom product. In the duopoly equilibrium more of those who should receive the custom product, do receive it (those with $v_{0}$ between $1 / 2$ and 1 ), and additional consumers receive the standard product (those with $v_{0}$ between $c$ and $1 / 2$ ). Thus total surplus is larger with the duopoly. Here the monopoly solution is worse than any of the productively efficient duopoly equilibria. In any $t$ market served by the incumbent in duopoly, the same set of consumers get the custom good as would in monopoly, and all who would get the standard product with a monopoly, plus additional consumers ( $c \leq v_{0}<1 / 2$ ), get it with the duopoly.

For $r_{m}^{\text {max }}<r \leq r_{b}^{\text {max }}$, the monopoly solution includes no custom products. All consumers with $v_{0}$ between $1 / 2$ and 1 receive the standard product. The duopoly equilibrium adds surplus for all these individuals, since they now receive a custom product whose value, $(S+1) v_{0}$, exceeds the standard product value $v_{0}$ plus the extra production cost, $c$. Consumers with $v_{0}$ between $c$ and $1 / 2$ now receive the standard product, with value $v_{0}$. Because the duopoly has $y_{b}=1$ while the monopoly has no
custom products, there is one additional cost for which to account--the total investment plus product range cost of $2 \sqrt{r}$. The change in total surplus moving from monopoly to duopoly is $\left(3 S+1-4 c-4 c^{2}\right) / 8-2 \sqrt{r}$ which is strictly positive for every $S>0$, $c<S /(2+4 S)$, and $r<r_{b}^{\max }$.

For the negative case, first note that $r_{m}^{\max }>r_{b}^{\max }$, so the comparison is always between monopoly with full product range and duopoly with full product range. Using the results in Tables 1 and 2 , monopoly minus duopoly total surplus is $(1-c)(2 T-1-3 c) / 8 T$, which is strictly positive since $c<T /(1+2 T)$. Q.E.D.

The result for the negative case may seem surprising, so it is worth examining carefully. Because of the negative correlation between $v_{0}$ and $v_{1}$, every consumer has either a relatively large $v_{0}$ or a relatively large $v_{1}$. Even with the monopoly solution, prices are such that every consumer purchases either its custom good or the standard product. The same is true in the duopoly equilibrium, and every product type is available in each equilibrium. The prices charged affect the allocation of surplus between consumers and firms, but in terms of total surplus, the only difference between monopoly and duopoly is which consumers get custom goods. In monopoly, all consumers with $v_{0}<(T+1-c) / 2 T$ get custom goods, while in duopoly, those with $v_{0}<1 / 2<(T+1-c) / 2 T$ do. More consumers receive the custom good in the monopoly solution than in the duopoly equilibrium. Because the monopolist will sell to every consumer, it can internalize the benefits and costs of changing the consumer who is on the margin between buying the standard product and the custom product, by changing both prices in such a way as to keep prices as high as possible while still selling to all consumers and attaining the target marginal consumer. In the duopoly equilibrium, the standard product price is fixed at $c$, so that a change in the custom price has less of an effect on the marginal consumer. Lowering the custom price to serve more of the consumers does not have as large a benefit as in the monopoly case (where the standard product price adjusts to accommodate the change), so the duopolist serves too small a fraction of the market with custom goods.

It is interesting to note that, for some parameter values, the monopolist sells custom products to a higher fraction of consumers than would receive them in a total surplus maximizing outcome. As seen in the proof of Proposition 1, to maximize total surplus, all consumers with $v_{0}<(T-c) / T$ should receive the custom good. For $T<(1+\sqrt{3}) / 2$ and $c$ between $T-1$ and its upper bound of $T /(1+2 T)$, $(T+1-c) / 2 T>(T-c) / T$, and more consumers get the custom good in the monopoly solution than in the social optimum. Because the monopolist charges only a single price
for the standard good, and a single price for the custom good of each type, its incentives do not match social incentives. It may not want to lower its standard product price (and correspondingly raise its custom price) when social incentives would call for it to do so.

## 4. Uncertainty and the choice of flexibility

A general conclusion of papers in the first strand of production flexibility literature discussed in the introduction, is that with uncertainty in demand, for a single firm problem, increased variability in demand leads to an increased desire for flexibility. This result need not hold in our context.

Consider a monopolist in the random case, where for each type $t$, all consumers of type $t$ have the same valuation, $\left(v_{0}{ }^{*}, v_{1}{ }^{*}\right)$; assume $V=v_{1}{ }^{*}-v_{0}{ }^{*}-c>0$. The range of consumer types is $[0, \tilde{m}]$, where $\tilde{m}$ is $m_{1}=m^{*}-d$ with probability $1 / 2$ and $m_{2}=m^{*}+d$ with probability $1 / 2$, where $0<m^{*}<1$ and $0<d<\min \left\{m^{*}, 1-m^{*}\right\}$. Given any $K$ and $y$, the pricing stage is trivial: the monopolist always chooses price $v_{0} *$ for the standard product and price $v_{1} *$ for the custom good. (Recall by our tie-breaking rule, since $v_{1} *-c>v_{0} *$, all consumers whose type is available will purchase the custom product; consumers are indifferent while the monopolist earns higher profit with that allocation.)

Given $K$, at the $y$ choice stage, the realization $m_{i}$ of the random parameter $\tilde{m}$ has been observed, and, for $0 \leq y \leq m_{i}$, profit is $\left(m_{i}-y\right) v_{o} *+y\left(v_{1} *-c\right)-y^{2} / K-r K$. The optimal $y$ is

$$
y_{m}\left(K, m_{i}\right)=\left\{\begin{array}{cl}
m_{i} & \text { if } m_{i} \leq K V / 2 \\
K V / 2 & \text { if } m_{i}>K V / 2
\end{array}\right.
$$

If $K \geq 2 m_{i} / V$, then the resulting profit is $m_{i}\left(v_{1}^{*}-c\right)-m_{i}^{2} / K-r K$; while if $K<2 m_{i} / V$, the resulting profit is $m_{i} v_{o}^{*}+K V^{2} / 4-r K$.

The $K$ choice occurs before the realization of $\tilde{m}$ is observed. The form of expected profit depends on the value of $K$ relative to $2 m_{1} / V$ and $2 m_{2} / V$, since those values determine whether the $y$ choice will be $m_{i}$ or $K V / 2$. Expected profit, $\Pi$, as a function of $K$ is

$$
\Pi(K)=\left\{\begin{array}{cc}
m * v_{0}+\frac{K V^{2}}{4}-r K & \text { if } K \leq \frac{2\left(m^{*}-d\right)}{V} \\
\frac{m *\left(v_{1} *+v_{0} *-c\right)}{2}-\frac{d V}{2}+\frac{K V^{2}}{8}-\frac{\left(m^{*}-d\right)^{2}}{2 K}-r K & \text { if } \frac{2\left(m^{*}-d\right)}{V}<K \leq \frac{2\left(m^{*}+d\right)}{V} \\
\left(v_{1}^{*}-c\right) m *-\frac{m^{* 2}+d^{2}}{K}-r K & \text { if } \frac{2\left(m^{*}+d\right)}{V}<K
\end{array}\right.
$$

For some parameter values, an increase in the variability of demand (i.e., an increase in $d$ ) leads to an increase in the optimal investment in flexibility, $K$, as in the previous literature. For example if $r<V^{2}\left(m^{*}+d^{2}\right) /\left(4\left(m^{*}+d\right)^{2}\right)$, then the optimal $K$, is $\sqrt{\left(m *^{2}+d^{2}\right) / r}$, which is increasing in $d$. Given this optimal $K$, for each realization of $\tilde{m}$, the optimal $y$ is equal to $\tilde{m}$, so the expected product range is constant at $\mathrm{m}^{*}$. However, the optimal investment in flexibility may be decreasing in the level of variability of demand, as shown by the following result.

Proposition 3: If $V^{2}\left(m^{* 2}+d^{2}\right) /\left(4\left(m^{*}+d\right)^{2}\right)<r<V^{2} / 4$, then both the optimal investment in flexibility and the resulting expected product range are decreasing in the amount of variability of demand.

Proof: For these parameter values the optimal $K$ is $2\left(m^{*}-d\right) / \sqrt{8 r-V^{2}} ; y=m^{*}-d$ when $\tilde{m}=m_{1}$, but $y=K V / 2=\left(m^{*}-d\right) V / \sqrt{8 r-V^{2}}$ when $\tilde{m}=m_{2}$. The optimal $K$ and each of the product ranges are decreasing in $d$. Q.E.D.

## 5. Conclusion

Our analysis of the desire for production flexibility that is driven by profit opportunities in product customization has derived some unexpected conclusions. As shown in Section 4, when flexibility is motivated by production customization, even in the single firm case, where there are no strategic effects, increased demand variability may lead to reduced investment in flexibility. This is in sharp contrast to other papers examining the desire for flexibility based on other motivations.

The results in Section 3 show the significant differences that arise when valuations have negative rather than positive correlations. With positive correlations, the duopoly equilibrium has higher total surplus than the monopoly outcome, and the entrant would produce the full range of custom product types even when the monopolist would not. With negative correlations, the monopoly outcome has higher total surplus than the duopoly equilibrium and the monopolist would produce the full range of custom product types even when the entrant in duopoly would not. This suggests that the extent to which the market for customized products will be served depends upon not only the number of firms with access to the market, but also the nature of the distribution in valuations $\left(v_{0}, v_{1}\right)$.

Our model is quite simple, but some of the conclusions seem robust. For example, the nonoverlap of product ranges in duopoly equilibrium should be true in general with price setting. A more realistic model would include multiple "dimensions" in which customization might occur, where "dimensions" are determined by the technology of flexibility. Perhaps the set of machines used or the organization of production must change to produce products in different "dimensions." To avoid overlap, and the resulting fierce price competition that would result, firms might specialize in the "dimensions" of customization they offer. As long as the firms each retain some exclusive types of customization, they will be able to create added value for consumers, and retain some market power.

The importance of customization will depend on both the distributions of valuations, as well as the degree to which the costs of customization can be lowered to approximate those of standard goods. If we remove the assumption of equal density of consumers across types, then the product range question takes on additional realism and complexity. For which custom types is the potential market largest, and can those different types be produced together at reasonable cost? Firms face a complex and interesting challenge in identifying the best mix of custom product types, both in terms of demand and cost.

## References

Beckman, Sara. "Manufacturing flexibility: The next source of competitive advantage." in Strategic Manufacturing, Patricia Moody (Ed), Business One Irwin (1990).

Dixon, Huw. "The Cournot and Bertrand outcomes as equilibria in a strategic metagame." Economic Journal. Vol. 96, Supplement (1986), pp 59-70.
---- "Strategic investment in an industry with a competitive product market." The Journal of Industrial Economics, Vol. 33, No. 4 (June 1985), pp 483-499.

Epstein, Larry. "Production flexibility and the behaviour of the competitive firm under price uncertainty." Review of Economic Studies. Vol. 45, No. 2 (June 1978), pp 251-261.

Fleischer, Manfred. The inefficiency trap: Strategy failure in the German machine tool industry. Berlin: Edition Sigma (1997).

He, Hua and Robert Pindyck. "Investments in flexible production capacity." Journal of Economic Dynamics and Control. Vol. 16 (1992), pp 575-599.

Jones, Robert and Joseph Ostroy. "Flexibility and uncertainty." Review of Economic Studies. Vol. 51 (1984), pp 13-32.

Roeller, Lars-Hendrik and Mihkel Tombak. "Strategic choice of flexible production technologies and welfare implications." The Journal of Industrial Economics. Vol. 38, No. 4 (June 1990), pp. 417-431.

Stigler, George. "Production and distribution in the short run." Journal of Political Economy. Vol. 47 (1939), pp 305-328.

Turnovsky, Stephen. "Production flexibility, price uncertainty and the behavior of the competitive firm." International Economic Review. Vol. 14, No. 2 (June 1973), pp. 395-413.

# Bücher des Forschungsschwerpunkts Marktprozeß und Unternehmensentwicklung Books of the Research Area Market Processes and Corporate Development <br> (nur im Buchhandel erhältlich/available through bookstores) 

Horst Albach, Ulrike Görtzen, Rita Zobel (Hg.)
Information Processing as a Competitive
Advantage of Japanese Firms
1999, edition sigma
Dieter Köster
Wettbewerb in Netzproduktmärkten
1999, Deutscher Universitäts-Verlag/Gabler Verlag
Christian Wey
Marktorganisation durch Standardisierung: Ein
Beitrag zur Neuen Institutionenökonomik des
Marktes
1999, edition sigma
Horst Albach, Meinolf Dierkes, Ariane Berthoin Antal, Kristina Vaillant (Hg.)
Organisationslernen - institutionelle und kulturelle Dimensionen
1998, edition sigma
Lars Bergman, Chris Doyle, Jordi Gual, Lars Hultkrantz, Damien Neven, Lars-Hendrik Röller, Leonard Waverman
Europe's Network Industries: Conflicting
Priorities - Telecommunications
Monitoring European Deregulation 1
1998, Centre for Economic Policy Research
Manfred Fleischer
The Inefficiency Trap
Strategy Failure in the
German Machine Tool Industry
1997, edition sigma

Christian Göseke
Information Gathering and Dissemination
The Contribution of JETRO to
Japanese Competitiveness
1997, Deutscher Universitäts-Verlag

Andreas Schmidt
Flugzeughersteller zwischen globalem Wettbewerb und internationaler Kooperation Der Einfluß von Organisationsstrukturen auf die Wettbewerbsfähigkeit von Hochtechnologie-Unternehmen 1997, edition sigma

Horst Albach, Jim Y. Jin, Christoph Schenk (eds.) Collusion through Information Sharing? New Trends in Competition Policy 1996, edition sigma

Stefan O. Georg
Die Leistungsfähigkeit japanischer Banken
Eine Strukturanalyse des Bankensystems in Japan
1996, edition sigma

Stephanie Rosenkranz
Cooperation for Product Innovation
1996, edition sigma
Horst Albach, Stephanie Rosenkranz (eds.) Intellectual Property Rights and Global Competition - Towards a New Synthesis 1995, edition sigma.

David B. Audretsch
Innovation and Industry Evolution
1995, The MIT Press.
Julie Ann Elston
US Tax Reform and Investment: Reality and Rhetoric in the 1980s
1995, Avebury
Horst Albach
The Transformation of Firms and Markets:
A Network Approach to Economic
Transformation Processes in East Germany
Acta Universitatis Upsaliensis, Studia Oeconomiae
Negotiorum, Vol. 34
1994, Almqvist \& Wiksell International
(Stockholm).
Horst Albach
"Culture and Technical Innovation: A CrossCultural Analysis and Policy
Recommendations"
Akademie der Wissenschaften zu Berlin (Hg.)
Forschungsbericht 9, S. 1-597
1994, Walter de Gruyter.

Horst Albach
Zerissene Netze. Eine Netzwerkanalyse des ostdeutschen Transformationsprozesses 1993, edition sigma.

Zoltan J. Acs/David B. Audretsch (eds)
Small Firms and Entrepreneurship: An EastWest Perspective 1993, Cambridge University Press.

Anette Boom
Nationale Regulierungen bei internationalen Pharma-Unternehmen: Eine theoretische Analyse der Marktwirkungen 1993, Nomos Verlagsgesellschaft.

| Horst Albach | Unternehmensgründungen in Deutschland Potentiale und Lücken | FS IV 98-1 |
| :---: | :---: | :---: |
| Dietmar Harhoff | Vertical Organization, Technology Flows and R\&D Incentives - An Exploratory Analysis | FS IV 98-2 |
| Karel Cool Lars-Hendrik Röller Benoit Leleux | Der Einfluß des tatsächlichen und des potentiellen Wettbewerbs auf die Rentabilität von Unternehmen der pharmazeutischen Industrie | FS IV 98-3 |
| Horst Albach | Blühende Landschaften? <br> Ein Beitrag zur Transformationsforschung | FS IV 98-4 |
| Shino Futagami Tomoki Waragai Thomas Westphal | Shukko in Japanese Companies and its Economic and Managerial Effects | FS IV 98-5 |
| Dietmar Harhoff Timm Körting | Lending Relationships in Germany: Empricial Results from Survey Data | FS IV 98-6 |
| Johan Lagerlöf | Are We Better Off if Our Politicians Know How the Economy Works? | FS IV 98-7 |
| Justus Haucap Christian Wey Jens Barmbold | Location Costs, Product Quality, and Implicit Franchise Contracts | FS IV 98-8 |
| Manfred Fleischer | Patenting and Industrial Performance: The Case of the Machine Tool Industry | FS IV 98-9 |
| Dieter Köster | Was sind Netzprodukte? - Eigenschaften, Definition und Systematisierung von Netzprodukten | FS IV 98-10 |
| Andreas Blume | Coordination and Learning with a Partial Language | FS IV 98-11 |
| Andreas Blume Uri Gneezy | An Experimental Investigation of Optimal Learning in Coordination Games | FS IV 98-12 |
| Andreas Blume Douglas V. DeJong George R. Neumann Nathan E. Savin | Learning in Sender-Receiver Games | FS IV 98-13 |
| Hans Mewis | The Stability of Information Cascades: How Herd Behavior Breaks Down | FS IV 98-14 |
| Lars-Hendrik Röller Minkel M. Tombak Ralph Siebert | The Incentives to Form Research Joint Ventures: Theory and Evidence | FS IV 98-15 |
| Christine Zulehner | Econometric Analysis of Cattle Auctions | FS IV 98-16 |
| Catherine Matraves | Market Structure, R\&D and Advertising in the Pharmaceutical Industry | FS IV 98-17 |


| Suchan Chae Paul Heidhues | Bargaining Power of a Coalition in Parallel Bargaining: Advantage of Multiple Cable System Operators | FS IV 99-1 |
| :---: | :---: | :---: |
| Christian Wey | Compatibility Investments in Duopoly with Demand Side Spillovers under Different Degrees of Cooperation | FS IV 99-2 |
| Horst Albach | Des paysages florissants? Une contribution à la recherche sur la transformation | FS IV 99-3 |
| Jeremy Lever | The Development of British Competition Law: A Complete Overhaul and Harmonization | FS IV 99-4 |
| Damien J. Neven Lars-Hendrik Röller Zhentang Zhang | Union Power and Product Market Competition: Evidence from the Airline Industry | FS IV 99-5 |
| Justus Haucap Uwe Pauly Christian Wey | The Incentives of Employers' Associations to Raise Rivals' Costs in the Presence of Collective Bargaining | FS IV 99-6 |
| Jianbo Zhang Zhentang Zhang | Asymptotic Efficiency in Stackelberg Markets with Incomplete Information | FS IV 99-7 |
| Justus Haucap Christian Wey | Standortwahl als Franchisingproblem | FS IV 99-8 |
| Yasar Barut Dan Kovenock Charles Noussair | A Comparison of Multiple-Unit All-Pay and Winner-Pay Auctions Under Incomplete Information | FS IV 99-9 |
| Jim Y. Jin | Collusion with Private and Aggregate Information | FS IV 99-10 |
| Jos Jansen | Strategic Information Revelation and Revenue Sharing in an R\&D Race with Learning Labs | FS IV 99-11 |
| Johan Lagerlöf | Incomplete Information in the Samaritan's Dilemma: The Dilemma (Almost) Vanishes | FS IV 99-12 |
| Catherine Matraves | Market Integration and Market Structure in the European Soft Drinks Industry: Always Coca-Cola? | FS IV 99-13 |
| Pinelopi Koujianou Goldberg Frank Verboven | The Evolution of Price Discrimination in the European Car Market | FS IV 99-14 |
| Olivier Cadot Lars-Hendrik Röller Andreas Stephan | A Political Economy Model of Infrastructure Allocation: An Empirical Assessment | FS IV 99-15 |
| Holger Derlien Tobias Faupel Christian Nieters | Industriestandort mit Vorbildfunktion? Das ostdeutsche Chemiedreieck | FS IV 99-16 |

Versandstelle - WZB
Reichpietschufer 50
D-10785 Berlin

## BESTELLSCHEIN / ORDERFORM

Bitte schicken Sie mir aus der Liste der Institutsveröffentlichungen folgende Papiere zu:

Bitte schicken Sie bei Ihren Bestellungen von WZB-Papers unbedingt eine 1-DM-Briefmarke pro paper und einen an Sie adressierten Aufkleber mit. Danke.

For each paper you order please send a "CouponRéponse International" (international money order) plus a self-addressed adhesive label. Thank You.

Please send me the following papers from your Publication List:

## Paper Nr./No.

Autor/Author + Kurztitel/Short Title


[^0]:    * Most of the work on this paper was completed while the authors were visiting Wissenschaftszentrum Berlin (WZB). We wish to thank WZB for financial support.

[^1]:    ${ }^{1}$ This is in contrast to an earlier period such as that discussed by Fleischer (1997) for the machine tool industry in Germany, in which producers of specialized products suffered serious market erosion due to drastically reduced prices for standard products made possible by major cost reductions for production of standard products. Those same cost reducing techniques are now being applied to customization.

[^2]:    2 As discussed in Section 2, with two such producers, this market would lead to losses for each firm.

[^3]:    ${ }^{3}$ Beckman (1990) discusses a wide range of types of flexibility in manufacturing.

[^4]:    ${ }^{4}$ We have also examined a distribution similar to the negative case, but with $0<T<1$, in which $v_{0}$ and $v_{1}$ are perfectly positively correlated while $v_{0}$ and the value added $v_{1}-v_{0}$, are perfectly negatively correlated. Not unexpectedly, the results, not reported in this paper, form a transition zone between the results for the positive and negative cases.

    5 Depending on the prices and parameters, not all three purchase decisions necessarily occur. This discussion indicates the "order" in which the decisions occur, if they do occur. These comments apply to negative case as well.

[^5]:    ${ }^{6}$ We ignore the fact that the dedicated machine is used to produce one of the types, since that single type has no effect on the aggregate demand faced by either firm.
    ${ }^{7} \quad$ This should exclude $p_{1}(t)$ itself, but the change has no effect on aggregate demand faced by either firm.

[^6]:    ${ }^{8}$ For one product in its product range, the entrant must set $p_{b}(t)=c$ to maintain the equilibrium standard product price for the incumbent. The single type is negligible in terms of the entrant's profit.

[^7]:    ${ }^{9} \quad$ Here and throughout the rest of the paper, we ignore the sunk cost of the dedicated machine when listing the profit for the incumbent/monopolist or the surplus.

[^8]:    ${ }^{10}$ If the anticipated product range equilibrium selection is "perverse," equilibria in which the firms are not productively efficient can also be sustained.

