The Efficiency of Indirect Taxes under Imperfect Competition

Simon P. Anderson

Department of Economics, University of Virginia

Charlottesville, VA 22903-3288, USA.

sa9w@virginia.edu

André de Palma

THEMA, Université de Cergy-Pontoise

33 Bd du Port, F-95011

Cergy-Pontoise Cedex, France.

depalma@u-cergy.fr

Brent Kreider

Department of Economics, University of Virginia

Charlottesville, VA 22903-3288, USA.

kreider@virginia.edu

February 2000

The Efficiency of Indirect Taxes under Imperfect Competition

Abstract

This paper considers the relative efficiency of ad valorem and unit taxes in imperfectly competitive markets. We provide a simple proof that ad valorem taxes are welfare-superior to unit taxes in the short run when production costs are identical across firms. The proof covers differentiated products and a wide range of market conduct. Cost asymmetries strengthen the case for ad valorem taxation under Cournot competition, but unit taxation may be welfare-superior under Bertrand competition with product differentiation. Ad valorem taxation is superior with free entry under Cournot competition, but not necessarily under price competition when consumers value variety.

Keywords: Excise tax, unit tax, specific tax, ad valorem tax, tax efficiency, product differentiation, imperfect competition.

JEL Classification #s: D43, H21, H22, L13

We thank Maxim Engers, Bill Johnson, Ed Olsen, John Pepper, Jim Poterba, and seminar participants at the University of Virginia Public Economics Workshop for helpful comments. An anonymous referee provided especially useful suggestions. The first author gratefully acknowledges financial support from the University of Virginia Bankard Fund and the National Science Foundation (grant # SBR-9617784); the second author thanks PREDIT (Ministère des Transports, France) for financial support.

1. Introduction

Tax authorities have raised revenue through indirect selective sales taxes (excise taxes) since the first known civilization began in Sumer, Babylonia some 6000 years ago. "Everything" in Sumer's city-state of Lagash was taxed,[1] from sheep-shearing (five shekels to the *ishakku*) to divorce (five shekels to the *ishakku* and another to the vizier) to perfume preparation (five shekels to the *ishakku*, a shekel to the vizier, and another to the palace steward). Wealth and property were "taxed to the limit." Even the dead could not be buried until an excise was paid. Historians emphasize the capaciousness of Lagash's tax policy, which is said to have ultimately lead to its invasion and destruction. More importantly, the form of the excises may have been inefficient as well.

Most modern governments also derive substantial revenue from excise taxes. Some governments impose very large excises. The Israeli government, for example, imposes a 95 percent ad valorem luxury excise on all private automobiles in addition to a 17 percent value added tax and 7 percent customs tax on certain imports (Fershtman, Gandal, and Markovich, 1997). Both unit (specific) and ad valorem (percentage) rates are applied at all levels of government, sometimes on the same classes of commodities.[2] The form of tax can have a substantial impact on social surplus. This paper provides a general approach for comparing efficiency across the two types of excise instruments.[3]

The relative efficiency of unit and ad valorem taxes is unambiguous for the polar cases of perfect competition and monopoly. Under perfect competition, the two tax types are equivalent; unit and ad valorem taxes constructed to raise the same amount of revenue lead to identical economic outcomes (both taxes shift the perceived demand curve down by the same amount at its intersection with supply). Under monopoly, an ad valorem tax is welfare superior to a unit tax that yields the same revenue because it renders the effective demand curve more elastic and so encourages production.

Most market structures fall between these two extreme cases – firms are neither so small as to effectively take the market price as given (with some exceptions, such as agricultural markets), nor are there many cases of private-sector firms without any competition. The predominant market form is oligopoly. Most of the work on taxation under imperfect competition has analyzed Cournot (quality) competition with a homogeneous product.[4] The main rival is the Bertrand (price) model with differentiated products. Products sold by firms are typically differentiated so that firms retain some

[1] Interpretation by Adams (1993) with additional information from Kramer (1959).

[2] For example, the federal government levies a \$3 per passenger excise on transportation if by water, 10 percent if by air; surface coal is taxed at the lesser of \$0.55 per ton or 4.4 percent of the sale price.

[3] For simplicity, we consider each tax type in isolation and do not allow for combinations (see Myles, 1996). We consider distributional issues in Anderson, de Palma, and Kreider (2000).

[4] See, e.g., Seade, 1987; Stern, 1987; Besley, 1989; Delipalla and Keen, 1992; Skeath and Trandel, 1994; and Keen's 1998 comprehensive survey paper.

market power even when there are many of them. Inappropriate assumptions about market structure and firm behavior can mislead policy analysis.

The standard argument for the superiority of ad valorem taxation in a monopoly setting, which uses marginal analysis, dates back to Wicksell (1896) and was further developed by Suits and Musgrave (1953).[5] Delipalla and Keen (1992) generalized the argument to show that ad valorem taxes welfare-dominate unit taxes for symmetric Cournot-Nash oligopolies.[6] They conclude that optimal policy from an efficiency perspective "requires maximum reliance on ad valorem taxation" (p.351).[7] Skeath and Trandel (1994) further strengthened the case for ad valorem taxes under monopoly by showing that for any given unit tax, there exists an ad valorem tax that yields higher consumer surplus, profits and tax revenue. For linear (and homogeneous) demand schedules, they show that this Pareto dominance also extends to Cournot oligopoly settings if revenue requirements are sufficiently large. They conclude that "welfare-maximization behavior on the part of the government would always imply that the ad valorem tax would be imposed" (p. 69).

We use a straightforward approach that provides a clean proof of the ad valorem efficiency claim. We extend this result to Bertrand price competition with differentiated products. Our analysis, which makes direct use of the properties of the profit function, is surprisingly general and applies to many oligopoly models, including Stackelberg leader/follower behavior. The approach underlying the method of our proof formally extends an idea first introduced by Suits and Musgrave (1953). We show how the general argument can be tailored to compare the relative efficiency of different taxation schemes under general assumptions about demand and market conditions.

We also demonstrate, however, that the relative efficiency of ad valorem taxation hinges critically on the assumptions of symmetric costs and a fixed number of firms. We show that unit taxation can be more efficient under Bertrand competition with differentiated products, even in the short run, if aggregate demand is sufficiently inelastic and firms produce at different costs. This result contrasts starkly with the Cournot case with homogeneous products (henceforth, the "Cournot model"), where we find that asymmetric cost structures always support ad valorem taxes. Although the superiority of ad valorem taxes for symmetric costs continues to hold in the Cournot model in the long run, we show that unit taxation can be welfare-superior in the long run when the market is characterized by Bertrand competition with differentiated products. When consumers value product variety (or tastes vary across consumers), a unit tax may be preferable on efficiency grounds because the unit tax leads to greater variety for given tax proceeds. We argue that the social desirability of one form of tax over the other depends on the elasticity of industry demand, the size of the government's revenue

[5] Cournot (1838) earlier recognized that the two taxes require separate analyses for monopoly.

[6] Hamilton (1999) extended their model to allow for output-dependent tax schedules.

[7] There is an analogous discussion of this issue in the trade literature, in a comparison of specific and ad valorem tariffs; see, e.g., Helpman and Krugman (1989).

requirement, and on the strength of tastes for variety relative to the magnitude of firm entry costs.

In the next section, we examine the short run efficiency differences between the two taxes with symmetric cost structures. Section 3 introduces cost asymmetries across firms into the analysis. Consequences for long run efficiency with free entry are then considered in Section 4. Section 5 concludes with a discussion of our results.

2. Short Run with Identical Costs

We first present our efficiency analysis for the case of an n-firm Cournot model, where each firm produces a homogeneous good at constant marginal cost c (and fixed costs are sunk in the short run). While the superiority of the ad valorem tax has previously been demonstrated for this case (see, e.g., Delipalla and Keen, 1992), we use this setting to showcase the methodology used below.[8]

Under a unit tax, t per unit sold, Firm i's profit when it produces output q_i is

$$\pi_i = [p(Q) - \widetilde{c}]\, q_i, \tag{2.1}$$

where $\widetilde{c} \equiv c + t$ is the effective marginal cost, $Q = \sum_{j=1}^{n} q_j$ is total output, and $p(Q)$ denotes the inverse demand curve. We make no concavity or differentiability assumptions: we only assume there exists a solution to the Cournot game (i.e., a Nash equilibrium in quantities).[9] Let $\underline{q}^* = (q_1^*, ..., q_n^*)$ denote this solution, with $Q^* = \sum_{j=1}^{n} q_j^*$, so that the tax revenue collected under the unit tax is $R_U = tQ^*$.

Under an ad valorem tax, the producer price is $(1-\tau)p(Q)$, where $\tau \in [0, 1]$ is the ad valorem tax rate.[10] Firm i's profit function is then $\pi_i = [(1-\tau)p(Q) - c]\, q_i$, or

$$\pi_i = (1-\tau)\, [p(Q) - \widetilde{c}]\, q_i, \tag{2.2}$$

where now $\widetilde{c} \equiv c/(1-\tau)$. The key observation is that this profit expression is the same as in (2.1) when taxes are set such that $c + t = c/(1-\tau)$, or, rewriting,

$$t = \frac{\tau}{1-\tau} c, \tag{2.3}$$

[8] A similar method was used in a proof by Delipalla and Keen (1992) to show that the optimal rate of unit taxation is nonpositive in the Cournot model with free entry (their Proposition 6).

[9] See McManus (1964) and Novshek (1985) for conditions that guarantee existence and uniqueness of a Cournot equilibrium. Given symmetric costs, these conditions are very mild.

[10] This definition of the ad valorem tax rate is simple to work with in what follows. Ad valorem tax rates are usually expressed as a percentage of the producer price, in which case the consumer price is $(1+s)$ times the producer price, where s is the ad valorem tax rate. The rate τ that we use is given by $s/(1+s)$: a sales tax of 10 percent $(s = 0.1)$ translates into $\tau = 1/11$.

except that the term $(1-\tau)$ premultiplies the expression in (2.2). Since (1-τ) is constant, it acts like a pure profit tax and is therefore neutral in the short run. Hence, whenever the other firms play their equilibrium strategies, q^*_{-i}, found for the unit tax, Firm i's best reply is to play q^*_i. Thus any equilibrium under the unit tax is also an equilibrium under the ad valorem tax (and vice versa), given (2.3).[11] The tax revenue collected under the ad valorem tax is $R_A = \tau p(Q^*)Q^*$.

We can now compare the tax revenues under the two different tax schemes given that (2.3) holds. The tax revenue difference is

$$\begin{aligned} R_A - R_U &= \tau p(Q^*)Q^* - tQ^* \\ &= [p(Q^*) - \widetilde{c}]\, \tau Q^*, \end{aligned} \tag{2.4}$$

where we have used (2.3). As long as $\tau > 0$, (corresponding to $t > 0$), and $\widetilde{c}$ is below the inverse demand curve intercept (so $Q^* > 0$), the common markup is positive $(p(Q^*) - \widetilde{c} > 0)$ and hence $R_A > R_U$.[12] (For a perfectly competitive market, price equals marginal cost so that $R_A - R_U = 0$, which is the standard tax equivalence result for perfect competition.) Hence for any given unit tax $t > 0$, there exists an ad valorem tax τ that yields the same social surplus with a higher tax revenue. As long as the equilibrium outcome is continuous in the ad valorem tax rate in the neighborhood of Q^* and a lower tax leads to a higher total output, then there is a slightly lower τ that leads to both higher social surplus (since surplus increases in Q^*) and a higher tax revenue (from the inequality that follows from (2.4)). This argument is the basis for defining one tax type to be more efficient if more tax revenue is raised for the same social surplus (consumer surplus plus profit plus tax revenue).[13]

Our criterion for relative tax efficiency is therefore a higher tax take for a given total output, which is equivalent to total output being larger for a given tax revenue under fairly mild conditions. For example, when marginal revenue is strictly decreasing, the tax revenue is a continuous function of the output induced by the tax rate (with zero tax revenue at both zero output and the free Cournot equilibrium output). Then for any revenue level attained under the unit tax, there exists an ad valorem tax rate that yields the same tax revenue at a higher output, and therefore higher profit plus consumer surplus. To see this for the monopoly case, it may help the reader to picture two Laffer curves, both anchored at zero output and the tax-free monopoly output, with

[11] Throughout this paper, we select the same equilibrium outcomes for the two tax types if there is more than one.

[12] The difference $R_A - R_U$ equals the difference of profit under the unit tax minus profit under the equivalent ad valorem tax, as is apparent from (2.1)-(2.3). This follows since Q^* is the same under both taxes. Therefore, the sum of profit plus tax revenue is equal under both regimes, and the higher tax revenue earned under the ad valorem tax is a transfer from profits.

[13] Another rationale for this definition follows if tax revenue is more highly weighted than profit in social welfare, which would be the case if it is socially desirable to raise tax revenue.

the one for the ad valorem tax above that for the unit tax. Lower tax rates correspond to higher outputs.[14] Our efficiency criterion is simply that the ad valorem Laffer curve is above the unit tax one; the alternative criterion is that the ad valorem tax one is to the right of the unit tax one at high enough outputs. Hence the two criteria are equivalent.

The argument for the superiority of the ad valorem tax also extends to Bertrand (price) competition with firms selling differentiated products. To see this, let $D_i(\underline{p})$ denote the demand addressed to Firm i when the prices set by firms are $\underline{p} \equiv (p_1, \ldots, p_n)$. Under a unit tax, Firm i's profit is then

$$\pi_i = (p_i - \widetilde{c})D_i(\underline{p}),$$

with $\widetilde{c} \equiv c + t$, while for an ad valorem tax it is

$$\pi_i = (1 - \tau)(p_i - \widetilde{c})D_i(\underline{p}),$$

with $\widetilde{c} \equiv c/(1 - \tau)$. If there exists a price equilibrium $\underline{p}^*$ under one tax, then the same consumer prices constitute an equilibrium for the other tax when (2.3) holds: if all other players choose their equilibrium prices, Firm i will too. Tax revenue collected from Firm i under the unit tax is $tD_i(\underline{p}^*)$; under the ad valorem tax it is $\tau p_i^* D_i(\underline{p}^*)$. Using (2.3), the latter exceeds the former as long as $\tau(p_i^* - \widetilde{c})D_i(\underline{p}^*) > 0$, which is true since price exceeds effective marginal cost (which is a generic property of Bertrand equilibria with differentiated products). Hence, tax revenues collected from each firm are higher under ad valorem taxation.[15]

We summarize the above results in the following proposition:

Proposition 1. *Consider a fixed number of firms with identical constant marginal costs. The ad valorem tax is more efficient than the unit tax under both the Cournot model and Bertrand competition with product differentiation provided there exists an equilibrium.*

The above efficiency argument readily extends to nonconstant continuous (but still identical across firms) marginal cost for symmetric equilibria. To see this for the Cournot case, it suffices to rewrite (2.1) for the unit tax case as

$$\pi_i = p(Q)q_i - C(q_i) - tq_i,$$

[14] A simple condition for total output to be inversely related to the tax is that the inverse demand function, $p(Q)$, be logconcave, meaning that $\ln p$ is concave in Q. This condition covers all concave demands as well as allowing demand functions that are not "too" convex. Our discussion paper provides details.

[15] The analogous argument is readily developed for Cournot competition with differentiated products; i.e., each firm's inverse demand function is given as $p_i(\underline{q})$.

where $C(q)$ is the total cost function. Denote the symmetric solution to the Cournot game under unit taxation as $Q^* = nq^*$. Likewise, (2.2) for the ad valorem case becomes

$$\pi_i = (1-\tau)\left[p(Q)q_i - \frac{C(q_i)}{1-\tau}\right].$$

The two tax regimes yield the same output per firm q^*, if

$$t = \frac{\tau}{1-\tau}C'(q^*).$$

(This relation reduces to (2.3) when marginal cost is constant.) Using the same argument as before,

$$R_A - R_U = \left[p(Q^*) - \frac{C'(q^*)}{1-\tau}\right]\tau Q^*,$$

which is strictly positive since price exceeds effective marginal cost at the oligopoly equilibrium. A parallel proof establishes the claim for the Bertrand case.

The efficiency result of Proposition 1 also extends to Stackelberg (leadership) structures (with equal and constant marginal cost), with either a sequential move structure with each firm choosing its output internalizing the reactions of subsequent firms in the order of moves (see Robson, 1990) or with m leaders and $n-m$ followers (see Daughety, 1990) as well as variants on these themes. Again, the structure of (2.1) and (2.2) together with (2.3) suffices to prove the result: the efficiency property does not rely on the market conduct. To illustrate, consider a sequential-move output choice game, with Firm 1 choosing output first, and Firm n producing last. Firm n's choice q_n^* is

$$\underset{q_n}{\operatorname{argmax}}\ \pi_n = \left[p\left(Q_{-n}+q_n\right) - (c+t)\right]q_n \tag{2.5}$$

under the unit tax, where Q_{-n} denotes the cumulative output of the preceding $n-1$ firms. Regardless of Q_{-n}, any solution to (2.5) is also a solution to

$$\underset{q_n}{\operatorname{argmax}}\ \pi_n = (1-\tau)\left[p\left(Q_{-n}+q_n\right) - \frac{c}{1-\tau}\right]q_n$$

when τ is set according to (2.3). Given this result, similar reasoning shows that the solution(s) to Firm $n-1$'s problem is also independent of the form of tax system given any cumulative output of the preceding $n-2$ firms, etc., so that any equilibrium solution under one tax system is also a solution under the other one. The superiority of the ad valorem tax then follows directly from the argument following equation (2.4). For any equilibrium under a unit tax, the same equilibrium exists under an appropriately chosen ad valorem tax (given by (2.3)) because the profit functions are identical in the two cases (apart from a neutral profits tax in the ad valorem case). Furthermore, tax revenue is higher under the ad valorem case, and by continuity there exists a slightly lower ad valorem rate that yields both more revenue and more output.

In the next sections we examine the robustness of this result by considering cost asymmetries across firms (Section 3) and the long run situation with free entry (Section 4).

3. Short Run with Asymmetric Costs

We now introduce cost differences across firms to see how such asymmetries can affect the efficiency result of Proposition 1. We consider first the Cournot model, and then the Bertrand differentiated products model. In both cases, there are n firms producing at constant marginal cost labeled so that $c_1 \leq c_2, ..., \leq c_n$.

3.1. Cournot analysis

As shown next in Proposition 2, the efficiency result in Proposition 1 extends to the case of asymmetric costs when market conduct is characterized by Cournot competition. In fact, the case for ad valorem taxation is strengthened with cost differences because this form of taxation leads to relatively less production from high-cost firms. Assume demand is differentiable with $p'(Q) < 0$ and that there exists a solution to the Cournot game under ad valorem taxation denoted by $q_1^*, ..., q_n^*$, with $Q^* = \sum_{j=1}^{n} q_j^*$ denoting the total equilibrium output. We then have:

Proposition 2. *The ad valorem tax is more efficient than the unit tax under the Cournot model with constant, but possibly different, marginal costs.*
Proof. Firms' first order conditions under the ad valorem tax are given by

$$p(Q) + p'(Q)q_i = c_i/(1-\tau), \; i = 1, ..., n. \tag{3.1}$$

Summing over these first order conditions implies

$$np(Q^*) + p'(Q^*)Q^* = \sum_{j=1}^{n} c_j/(1-\tau). \tag{3.2}$$

From (3.2), we find (since $p'(Q) < 0$) that the effective price exceeds the average marginal cost across firms:

$$(1-\tau)p(Q^*) = \overline{c} - \frac{1-\tau}{n} p'(Q^*)Q^* > \overline{c}, \tag{3.3}$$

where $\overline{c} \equiv \frac{1}{n} \sum_{j=1}^{n} c_j$.

The analogous first order conditions for the unit case are given by

$$p(Q) + p'(Q)q_i = c_i + t, \; i = 1, ..., n, \tag{3.4}$$

which lead to the aggregate condition

$$np(Q^*) + p'(Q^*)Q^* = \sum_{j=1}^{n} c_j + nt. \tag{3.5}$$

Equating the right-hand sides of (3.2) and (3.5), taxes t and τ yield the same total output Q^* if

$$t = \frac{\tau}{1-\tau}\overline{c}, \tag{3.6}$$

which reduces to (2.3) for the symmetric case. Total tax revenue is greater with the ad valorem tax if $\tau p(Q^*)Q^* > tQ^*$ when Q^* is attained under both taxes. When (3.6) holds, this revenue difference is positive if and only if $\frac{\tau}{1-\tau}\left[(1-\tau)p(Q^*) - \overline{c}\right] Q^* > 0$,which is true by (3.3). Thus, more revenue is raised under the ad valorem tax.

Since Q^* is fixed to be the same under the two taxes, $p(Q^*)$ and hence consumer surplus are the same. To establish the efficiency of the ad valorem tax, it remains to be shown that the total industry cost of producing Q^* is not lower under the unit tax. Under the unit tax, the total cost of producing Q^* is

$$TC_U = \frac{\sum_{j=1}^{n} c_j \left[p(Q^*) - c_j - t\right]}{-p'(Q^*)}, \tag{3.7}$$

where we have used the first order condition for the unit tax case. Similarly, the total cost of producing Q^* for the ad valorem case is

$$TC_A = \frac{\sum_{j=1}^{n} c_j \left[p(Q^*) - c_j/(1-\tau)\right]}{-p'(Q^*)}. \tag{3.8}$$

Hence, $TC_U > TC_A$ when $\sum_{j=1}^{n} c_j(c_j + t) < \sum_{j=1}^{n} c_j^2/(1-\tau)$, or, using (3.6), when $\frac{1}{n}\left(\sum_{j=1}^{n} c_j\right)^2 < \sum_{j=1}^{n} c_j^2$. This inequality follows directly from the Cauchy-Schwarz inequality. □

In essence, the proof follows the structure of that for Proposition 1 regarding the revenue comparison, with the extra element of showing that production is less efficient (more costly) under the unit tax. The intuition is that the ad valorem tax penalizes the high cost firms more than the low cost firms by adding a greater absolute burden, thus decreasing the output of the less efficient firms in the mix that comprises Q^*.

As we show next, cost asymmetries work very differently under Bertrand competition with product differentiation. Since we showed in Proposition 1 that the ad valorem tax is superior under symmetry, it will take a sufficient degree of asymmetry to overturn that result.

3.2. Bertrand analysis

To provide a simple Bertrand counterexample to the Cournot case, we consider a standard duopoly model with low-cost Firm 1 producing at constant marginal cost c_1 and high-cost Firm 2 producing at constant marginal cost c_2, with the production cost difference denoted $\Delta c \equiv c_2 - c_1 > 0$. The firms are located on opposite ends of a linear city which lies on a line segment of length 1. Consumers of measure 1 are located uniformly between the two firms, with a particular consumer's location indexed by $x \in [0, 1]$, the distance from Firm 1 (and thus the distance from Firm 2 is $1 - x$). Each consumer purchases one unit of the good from her most-preferred firm, yielding consumption benefits v. Product differentiation is captured by the presence of travel costs; consumers wish to minimize their total purchase costs, so for equal prices consumers prefer to purchase from the closer firm. Denoting p_1 the price charged by Firm 1, p_2 the price charged by Firm 2, and γ the constant disutility per unit of distance traveled, a consumer's total cost associated with buying from Firm 1 is $p_1 + \gamma x$ and total cost associated with buying from Firm 2 is $p_2 + \gamma(1 - x)$. Defining $\widehat{x}$ as the solution to $p_1 + \gamma \widehat{x} = p_2 + \gamma(1 - \widehat{x})$, consumers located in $[0, \widehat{x}]$ buy from Firm 1 at these arbitrary prices since $p_1 + \gamma x < p_2 + \gamma(1 - x)$, while the remaining consumers located in $(\widehat{x}, 1]$ buy from Firm 2. We assume that derived benefits of consumption v are sufficiently large relative to production and travel costs that all consumers wish to buy a unit of the good.[16] The demands for Firm 1's and Firm 2's products are given by $x_1(p_1, p_2) = \widehat{x}$ and $x_2(p_1, p_2) = (1 - \widehat{x})$, respectively, where $\widehat{x} = \frac{p_2 - p_1}{2\gamma} + \frac{1}{2}$. We assume that $\Delta c < \frac{3}{5}\gamma$ to ensure that the tax policies considered below do not drive the high-cost firm out of business.

Denoting $\widetilde{c} = c + t$ for the unit tax case and $\widetilde{c} = c/(1 - \tau)$ for the ad valorem tax case as before, Firm j chooses p_j to maximize $(p_j - \widetilde{c}_j)x_j(p_1, p_2)$, which yields the reaction functions

$$\begin{aligned} p_1 &= \tfrac{1}{2}\left(p_2 + \widetilde{c}_1 + \gamma\right) \\ p_2 &= \tfrac{1}{2}\left(p_1 + \widetilde{c}_2 + \gamma\right). \end{aligned}$$

We can then solve for the equilibrium prices

[16] For sufficiently large v, firms will not price anyone out of the entire market in equilibrium.

$$p_1^*(\widetilde{c}_1, \widetilde{c}_2, \gamma) = \tfrac{2}{3}\widetilde{c}_1 + \tfrac{1}{3}\widetilde{c}_2 + \gamma$$
$$p_2^*(\widetilde{c}_1, \widetilde{c}_2, \gamma) = \tfrac{2}{3}\widetilde{c}_2 + \tfrac{1}{3}\widetilde{c}_1 + \gamma.$$

Note that firms charge identical prices when production costs are the same: $p_1^* = p_2^* = \widetilde{c} + \gamma$.

With asymmetric costs, taxes alter the equilibrium price difference between firms, which in turn affects allocative efficiency. The equilibrium price difference is $p_2^* - p_1^* = \frac{1}{3}\Delta\widetilde{c}$, where $\Delta\widetilde{c} \equiv \widetilde{c}_2 - \widetilde{c}_1$. Equilibrium demand is then

$$x_1^*(\Delta\widetilde{c}, \gamma) = \frac{1}{2}\left(1 + \frac{\Delta\widetilde{c}}{3\gamma}\right) \tag{3.9}$$
$$x_2^*(\Delta\widetilde{c}, \gamma) = \frac{1}{2}\left(1 - \frac{\Delta\widetilde{c}}{3\gamma}\right).$$

For identical production costs, the two firms share the market equally with $x_1^* = x_2^* = \frac{1}{2}$. Even with asymmetric costs, the unit tax has no effect on the price difference or demands; in this case $\Delta\widetilde{c} = \Delta c$, which does not depend on t and so does not further distort the market outcome. The ad valorem tax does affect prices and the output mix since then $\Delta\widetilde{c} = \frac{1}{1-\tau}\Delta c$, which rises with τ.

To examine the consequences of taxation for efficiency, we consider the effects of the taxes on social surplus, measured as consumer surplus plus firm profits plus tax revenue. Writing out this expression, total welfare for the ad valorem case is

$$W_A = \int_0^{x_{1A}^*} [(v - p_{1A}^* - \gamma x_{1A}) + (1-\tau)(p_{1A}^* - \widetilde{c}_{1A}) + \tau p_{1A}^*]\, dx_{1A}$$
$$+ \int_{x_{1A}^*}^1 [(v - p_{2A}^* - \gamma x_{2A}) + (1-\tau)(p_{2A}^* - \widetilde{c}_{2A}) + \tau p_{2A}^*]\, dx_{1A},$$

which becomes

$$W_A = v - c_1 x_{1A}^* - c_2 x_{2A}^* - \frac{\gamma}{2}\left(x_{1A}^{*2} + x_{2A}^{*2}\right). \tag{3.10}$$

Similarly, welfare under the unit tax is given by

$$W_U = v - c_1 x_{1U}^* - c_2 x_{2U}^* - \frac{\gamma}{2}\left(x_{1U}^{*2} + x_{2U}^{*2}\right). \tag{3.11}$$

Since aggregate demand is fixed and each consumer receives utility v, maximizing social welfare is equivalent to minimizing total costs (production plus travel) of generating that utility. The average travel cost associated with x_1 customers purchasing from Firm 1 is $\frac{\gamma}{2}x_1$, implying a total travel cost of $\frac{\gamma}{2}x_1^2$ (and similarly for Firm 2).

After substituting the demand values from (3.9) into the welfare function, it is easily shown that the market shares that maximize social welfare are given by $x_1^o = \frac{1}{2} + \frac{1}{2}\frac{\Delta c}{\gamma}$ and $x_2^o = \frac{1}{2} - \frac{1}{2}\frac{\Delta c}{\gamma}$. Note from (3.9), however, that the equilibrium shares are given by $x_1^* = \frac{1}{2} + \frac{1}{6}\frac{\Delta \widetilde{c}}{\gamma}$ and $x_2^* = \frac{1}{2} - \frac{1}{6}\frac{\Delta \widetilde{c}}{\gamma}$ so that the high-cost firm serves an inefficiently large share of the market in the absence of taxation. This is because the low-cost firm, while pricing below the high-cost firm, nevertheless uses its cost advantage to overprice its product.

Since the allocation of resources is independent of the unit tax rate, the unit tax neither corrects nor exacerbates the duopoly demand distortion. The ad valorem tax, however, does affect equilibrium market share by increasing the effective cost differential between firms. Since $\Delta\widetilde{c} = \frac{\Delta c}{1-\tau}$, sufficiently low values of τ improve welfare by narrowing the gap between x_1^* and x_1^o. It is easily verified that the gap is eliminated when $\tau = 2/3$. For values of τ greater than $2/3$, the effective production costs for Firm 2 become so high relative to Firm 1 that Firm 1 actually *over*produces. For a sufficiently high value of τ, social welfare will fall below that associated with the unit tax case. This implies that unit taxation is socially preferable to ad valorem taxation for sufficiently high revenue requirements. Substituting the demand values in (3.9) into the welfare functions (3.10) and (3.11) yields (after some algebra) the welfare difference

$$W_A - W_U = \frac{\tau(1-\tau)^2\,(\Delta c)^2}{180\gamma}\left(\tfrac{4}{5} - \tau\right).$$

Hence, the unit tax dominates the ad valorem tax if the ad valorem rate exceeds 4/5. This result is summarized as Proposition 3:

Proposition 3. *The unit tax is more efficient than the ad valorem tax under Bertrand competition in the linear city model when marginal production costs differ and revenue requirements are sufficiently high.*

This simple counterexample involves perfectly inelastic individual demand. If demand is not perfectly inelastic, the relatively nondistortionary effect of ad valorem taxation on individual demand must be considered in addition to the effect on the production mix.

We have shown that cost asymmetries strengthen the desirability of ad valorem taxes under Cournot competition with homogeneous products, but unit taxes may be preferred under Bertrand competition with product differentiation. Although these are the prominent models in the literature, the analysis leaves open the question whether the mode of competition or product differentiation is responsible for unseating the ad valorem tax. If the product sold is perfectly homogeneous, Bertrand competition leads to the lowest cost producer serving the market at the cost, c_2, of the next lower cost producer (assuming that this cost is below the first firm's monopoly price).

For a unit tax, t, the equilibrium price is $c_2 + t$ with sales of $Q(c_2 + t)$. An ad valorem tax τ set so that $c_2 + t = c_2/(1-\tau)$ yields the same equilibrium price since the producer price under ad valorem taxation is $p(1-\tau) = c_2$. Sales are therefore the same and so is tax revenue: $tQ(c_2+t) = \frac{\tau}{1-\tau}c_2Q\left(\frac{c_2}{1-\tau}\right)$ when $c_2 + t = c_2/(1-\tau)$. Both taxes are thus equally efficient. However, even with a small amount of product differentiation ($\gamma > 0$ in the example above), the preceding analysis shows that either tax type may dominate depending on revenue requirements. Under the Cournot model, since ad valorem taxation strictly dominates unit taxation for all revenue requirements, adding a small enough amount of product differentiation will not upset that result (by continuity). This argument (weakly) suggests that it is primarily the mode of competition that is responsible for the result. We have found no counterexamples, and simulations for a linear-demand differentiated products system showed no flips in welfare superiority of ad valorem taxes under Cournot competition, while there were such flips for Bertrand competition (e.g., the result of Proposition 3).[17] Nevertheless, we have not been able to prove that the ad valorem tax is always preferred under Cournot competition with asymmetric costs and differentiated products

4. Long Run Equilibrium

The long run equilibrium analysis allows for the number of firms to depend on the tax policy and a fixed entry cost, K. Production costs are the same across firms. As with previous sections, we first extend the result of the superiority of the ad valorem tax for the Cournot case, and then we provide a counterexample to show that unit taxation can be welfare-superior for the Bertrand case. The state of the art in the literature for the long run Cournot case is that "the set of circumstances in which welfare is improved by specific taxation is a strict subset of that in which it is improved by ad valorem taxation" (Delipalla and Keen, 1992, p. 364). They show this by considering small taxes (in the neighborhood of the tax-free equilibrium). In contrast to this local result, our result is a global one that establishes the superiority of the ad valorem tax in attaining any given (feasible) revenue requirement. In the Bertrand case, Kay and Keen (1983) have suggested that unit taxation can dominate ad valorem taxes in the long run for high enough revenue requirements. We provide an example below in which unit taxation dominates for any revenue requirement.

4.1. Cournot analysis

The next proposition extends the efficiency result in Proposition 1 for Cournot to the long run:

Proposition 4. *In the long run equilibrium under the Cournot model, the ad valorem tax is more efficient than the unit tax if marginal costs are constant and equal across*

[17] Indeed, our simulations for the linear demand model under Cournot competition suggest that the welfare advantage of the ad valorem tax grows monotonically with the revenue requirement.

firms provided there exists a unique symmetric equilibrium.

Proof. From the zero profit condition under the two taxes, and at a symmetric equilibrium at which total output is the same under both taxes, we have

$$(p-c-t)\frac{Q}{n_U} = (1-\tau)\left(p - \frac{c}{1-\tau}\right)\frac{Q}{n_A} = K, \tag{4.1}$$

where n_U and n_A denote the numbers of firms under the unit and ad valorem taxes, respectively. Since output is the same in both cases, the first equality in (4.1) implies

$$(p-c-t)n_A = (1-\tau)\left(p - \frac{c}{1-\tau}\right)n_U, \tag{4.2}$$

from which we can substitute the first order conditions (3.4) and (3.1) to give

$$-p'q_U n_A = -p'q_A(1-\tau)n_U, \tag{4.3}$$

where $q_U = Q/n_U$ and $q_A = Q/n_A$, the outputs per firm under the two taxes. Using the expressions for q_U and q_A yields:

$$n_A^2 = (1-\tau)n_U^2, \tag{4.4}$$

which implies $n_A < n_U$. This inequality means that production of Q is necessarily less efficient under the unit tax because more firms are involved (since marginal cost is constant and each firm entails a fixed cost).

To establish the greater efficiency of the ad valorem tax, it therefore suffices to show that more revenue is raised. We wish to show $R_A > R_U$, or $\tau p Q > tQ$, given that τ and t are set such that Q is the equilibrium outcome under both taxes. The latter relation is given by (4.2). We substitute (4.4) into this equation to yield $(p-c-t) = \sqrt{1-\tau}\,[p - c/(1-\tau)]$, or

$$t = p - c - \sqrt{1-\tau}\left(p - \frac{c}{1-\tau}\right). \tag{4.5}$$

We therefore must show that the RHS of (4.5) is less than τp, or

$$\left(\frac{1}{\sqrt{1-\tau}} - 1\right)[(1-\tau)p - c] > 0. \tag{4.6}$$

This inequality necessarily holds since $0 < \tau < 1$ and $(1-\tau)p > c$ in the oligopoly equilibrium. □

Delipalla and Keen (1992) have shown that if small unit taxes improve social welfare then small ad valorem taxes are also welfare-improving. This conclusion stems from the

Mankiw and Whinston (1986) result that the free entry equilibrium under the Cournot model entails a socially excessive number of firms (in the absence of taxes), coupled with the property that ad valorem taxation is associated with relatively low short run profits compared with unit taxation. The ad valorem tax is particularly effective for discouraging entry and reducing aggregate fixed costs.[18] Our result is that for any unit tax, there is an ad valorem tax that leads to the same aggregate output, but fewer firms are involved in producing that output. Ad valorem taxation is more efficient since fixed costs are lower in producing a given output level.

4.2. Bertrand analysis

The previous subsection showed that ad valorem taxation continues to dominate unit taxation in the long run with free entry under the Cournot model. We will show that Bertrand competition and product differentiation can lead to the opposite conclusions about tax efficiency. We first provide an example in which the unit tax is always welfare-superior to the ad valorem tax. We then briefly discuss other models.

The counterexample uses a common discrete choice framework for consumer demand (see Anderson et al., 1992). Let the conditional indirect utility of an individual choosing good i from n alternatives be given by

$$U_i = y - p_i + \mu\varepsilon_i, \quad i = 1...n, \tag{4.7}$$

where y is the consumer's income, p_i is the price of good i, and $\mu\varepsilon_i$ is an idiosyncratic match value between the consumer and good i. Each consumer purchases one unit of the good for which her utility is greatest. The match term consists of a scale parameter $\mu > 0$, which measures the intensity of consumer preferences across the different goods, and a realization of random match variable ε_i. These variables are assumed to be identically and independently distributed (which implies that the goods are symmetric substitutes). The common distribution and density functions are given by $F(x)$ and $f(x)$, respectively. We further assume that the total mass of consumers is unity so that the number of consumers purchasing good i is given by $D_i = \Pr[U_i > U_j,\ j = 1...n, i \neq j] = \Pr\left[\varepsilon_j < \frac{p_j - p_i}{\mu} + \varepsilon_i,\ j = 1...n, i \neq j\right]$, which can be rewritten as

$$D_i = \int_{-\infty}^{b} f(x) \prod_{j \neq i}^{n} F\left(\frac{p_j - p_i}{\mu} + x\right) dx.$$

To provide the sharpest contrast between the Cournot model and Bertrand competition with differentiated products, we assume that the taste distribution is given by $F(x) = f(x) = \exp(x - b)$, $x \in (-\infty, b]$.

[18] Besley (1989) previously showed that unit taxes can be used to correct the overentry problem. He did not consider ad valorem taxes.

Under a unit tax, the Firm i's profit is

$$\pi_i = (p_i - \widetilde{c})D_i - K,$$

where $\widetilde{c} = c + t$. The first order condition for profit maximization is

$$\frac{\partial \pi_i}{\partial p_i} = D_i + (p_i - \widetilde{c})\frac{\partial D_i}{\partial p_i} = 0.$$

At a symmetric equilibrium, $D_i = \frac{1}{n}$ and

$$\left.\frac{\partial D_i}{\partial p_i}\right|_{sym} = -\frac{n-1}{\mu}\int_{-\infty}^{b} f^2(x)F^{(n-2)}(x)dx,$$

so $\frac{\partial D_i}{\partial p_i} = -\frac{n-1}{n\mu}$ at equal prices. Thus a firm's markup is given by

$$p - \widetilde{c} = \frac{\mu}{n-1}. \tag{4.8}$$

Firm profit as a function of n is then $\pi_i(n) = \frac{\mu}{n(n-1)} - K$, so that the long run equilibrium number of firms n^e satisfies the entry condition

$$\frac{\mu}{n^e(n^e+1)} - K < 0 \leq \frac{\mu}{n^e(n^e-1)} - K; \tag{4.9}$$

the last entrant earns nonnegative profits while an additional entrant would earn negative profits.

Social welfare for this model is given by

$$W = y + \mu\int_{-\infty}^{b} xg(x)dx - c - nK,$$

where $g(\cdot)$ is the p.d.f. of the maximum ε. The c.d.f. of the maximum ε is given by $G(x) = [F(x)]^n$, which implies $g(x) = n[F(x)]^{n-1} f(x) = n\exp[n(x-b)]$. Using integration by parts, social welfare is given by $W = y + \mu b - \frac{\mu}{n} - c - nK$,which implies that the welfare change associated with an additional entrant is $W(n+1) - W(n) = \frac{\mu}{n(n+1)} - K$.Social efficiency thus requires n^o firms such that

$$\frac{\mu}{n^o(n^o+1)} - K < 0 \leq \frac{\mu}{n^o(n^o-1)} - K.$$

But this is the same condition as (4.9): the long run equilibrium is characterized by the socially efficient number of firms in the absence of taxation (see also Anderson et al., 1995) and under the unit tax. Since the optimal number of firms enters the market in the absence of taxation, and the unit tax is nondistortionary (in the sense that the tax is fully passed on to consumers), short run profits and hence the long run number of firms are unchanged under the unit tax.

As shown next, however, the ad valorem tax is not fully passed on and so diminishes profits and the number of firms in the long run equilibrium. For the ad valorem case, profit is given by

$$\pi_i = (1-\tau)(p_i - \widetilde{c})D_i - K,$$

where now $\widetilde{c} = c/(1-\tau)$. As for the unit case, a firm's markup at a symmetric equilibrium is given by (4.8) so that profit per firm is given by $\pi_i(n) = \frac{(1-\tau)\mu}{n(n-1)} - K$. The long run equilibrium number of firms under the ad valorem tax satisfies the entry condition

$$\frac{(1-\tau)\mu}{n^e(n^e+1)} - K < 0 \leq \frac{(1-\tau)\mu}{n^e(n^e-1)} - K.$$

Since profits are lower for a given number of firms under the ad valorem tax, the equilibrium number of firms is (weakly) lower compared with the unit tax. As the ad valorem rate τ rises, the number of active firms declines and consumers lose surplus in the form of lost product variety. Since the unit tax equilibrium involves the efficient number of firms, it is always welfare-superior to the ad valorem tax.

Proposition 5. *For the discrete choice model with exponentially distributed match values under Bertrand competition with free entry, the unit tax is a more efficient revenue instrument than the ad valorem tax.*

For the logit demand specification (when the ε terms have a double exponential distribution), the relative efficiency of the two tax forms depends on the government's revenue requirement and the strength of tastes for variety relative to entry costs (i.e., μ/K). In the absence of taxes, the logit demand specification involves inefficiently many firms in the free entry equilibrium. For a sufficiently low revenue requirement, the ad valorem tax is a more efficient revenue instrument because it decreases profit and induces exit while the unit tax just transfers surplus from consumers to the government without affecting profit and hence without affecting the (excessive) number of firms. When either μ/K or revenue requirements are sufficiently large, the unit tax becomes more efficient because the ad valorem tax induces excessive firm exit.

This result extends to a large class of discrete choice models, of which the logit is the best known example. Indeed, Anderson et al. (1995) show that the market equilibrium involves overentry in the absence of taxes when density of the match values is logconcave.[19] Overentry is ameliorated by small ad valorem taxes because they reduce firm numbers by decreasing profit per firm, but high tax rates reduce profits so much as to take the equilibrium number of firms too far below the optimum.

[19] The exponential distribution considered above is a limit case since it is log-linear, hence the equivalence of optimum and equilibrium numbers.

The well-known circle model generalizes the linear city model discussed in Section 3.2 to allow for endogenous free entry. In that model, long run equilibrium is characterized by substantial overentry of firms (see Salop, 1979). The unit tax is again nondistortionary, and the ad valorem tax can improve social welfare by reducing profits and hence the long run number of firms. Travel costs rise as the number of firms declines, however, and ad valorem taxation induces excessive firm exit from the market for sufficiently high revenue requirements. In the Salop framework n firms are equispaced around a circle whose circumference is normalized to unity. Consumers are uniformly distributed around the circle, and each buys one unit from the firm with the lowest delivered price, transport costs being linear in distance at rate γ. The social cost associated with n firms is thus $nK + \gamma/4n$, where the first term is total fixed costs and the second term is aggregate transport costs (since the average distance traveled is $1/4n$). Under an ad valorem tax, the equilibrium after-tax markup is $\gamma(1-\tau)/n$ and each firm serves $1/n$th of the market so that the equilibrium number of firms is $\sqrt{\gamma(1-\tau)/K}$. Hence, the corresponding social cost is $\sqrt{\gamma K}\left(\sqrt{1-\tau}+\frac{1}{4}\frac{1}{\sqrt{1-\tau}}\right)$. A unit tax has no effect on the markup, so the equilibrium number of firms is $\sqrt{\gamma/K}$, at social cost $\frac{5}{4}\sqrt{\gamma/K}$. The social cost therefore at first decreases with τ as the number of firms falls toward the optimal level, with minimum at $\tau = \frac{3}{4}$ (see Kay and Keen, 1983), corresponding to a sales tax rate of $s = 3$ (and $\frac{1}{2}\sqrt{\gamma/K}$ firms). The social cost thereafter rises, and reaches the level attained by the unit tax at $\widehat{\tau} = \frac{15}{16}$, which corresponds to a tax rate of $s = 15$. Thus the unit tax is only superior for an ad valorem tax in excess of 1500%. This high tax rate reflects the massive overentry of firms in the circle model. In contrast, the discrete choice model with i.i.d. tastes (the exponential model and the Logit are examples) typically has a much lower degree of excess entry. Consequently, in this case the unit tax is more likely to be preferred since ad valorem taxes will take the equilibrium below the optimum (see also Kay and Keen, 1983, for an early discussion of the role for unit taxation and preferences for diversity).

We have discussed models with completely inelastic aggregate demand to provide simple counterexamples to the conclusion for the Cournot model. When industry output as well as product variety depends on the form of tax, the ad valorem tax has the often desirable quality that it has a smaller impact on total consumption compared with the unit tax. Under the CES demand specification, aggregate demand is sufficiently elastic that the ad valorem tax always welfare-dominates the unit tax, despite the detrimental effect of ad valorem taxation compared with unit taxation on product variety. This suggests that unit taxes might be the preferable form of excise taxation on relatively inelastic commodities like cigarettes to preserve product variety, while ad valorem taxes might be preferable for more elastic commodities where losses associated with the distortion to aggregate demand may outweigh those associated with lost product variety.

5. Conclusion

We have shown that ad valorem taxes are more efficient than unit taxes in the short run with symmetric costs across firms. Our straightforward approach, which exploits an insight in Suits and Musgrave (1953), can be applied quite generally. The result encompasses both traditional Cournot environments and Bertrand-Nash environments with differentiated products. We have also shown, however, that conclusions may change for asymmetric costs or in long run equilibrium. With cost asymmetries, ad valorem taxes exacerbate the absolute differences in marginal costs across firms. For low taxes, this is advantageous because high-cost firms tend to overprice relative to low-cost ones in oligopoly equilibrium, and the ad valorem tax redresses the production balance. When products are differentiated, the output of high-cost firms is not perfectly substitutable for that of low-cost firms so that high enough ad valorem taxes may lead to relative underproduction of high-cost goods. It is then possible that the unit tax is preferable, as we showed in an example with inelastic demand and Bertrand competition. Inelastic demand is important because it lessens the tax distortion and allows us to focus on relative output, even though unit taxation tends to be more distortionary because more of the tax is passed on to consumers than for ad valorem taxes (see Anderson, de Palma, and Kreider, 2000). This leaves open the question whether it is the mode of competition or the introduction of product differentiation that is primarily responsible for the difference in results. We provided weak evidence in favor of Bertrand competition.

The same question applies to the long-run analysis. The ad valorem tax reduces profits more than the unit tax by more efficient transfer from profit to tax revenue. When products are differentiated, the welfare loss from lost product variety may then overturn the short-run desirability of the ad valorem tax. Whether this is possible depends on whether the variety effect can dominate the static disadvantage of the unit tax. We showed that this can happen under Bertrand competition. It is not possible under Cournot competition with homogeneous products because there is no product variety. If products were differentiated and the mode of competition were Cournot, one might expect it to be less likely that the unit tax could dominate since Cournot competition typically yields greater profits for a given number of firms (it is less "competitive" than the Bertrand mode). This suggests that the Cournot equilibrium will typically entail greater overentry than its Bertrand counterpart and so the static inefficiency of unit taxation must be incurred for a greater extent before it could possibly overcome the variety effect. Thus it seems less likely that Cournot competition with differentiated products could render the unit tax more efficient (for some revenue requirement) although we have no formal proof. Insofar as the main competing models of oligopoly interaction are the Cournot mode with homogeneous products and the Bertrand mode with differentiated products, we have shown that the ad valorem tax dominates under the former with either cost asymmetries in the short run or symmetric costs in the long run, while the latter formulation can sometimes rationalize use of the unit tax.

While we have provided cases in which the unit tax may dominate the ad valorem tax, administrative and equity issues may justify use of the latter. For example, the percentage-based ad valorem tax automatically adjusts to inflation while the unit tax does not. There may also be practical difficulties associated with defining the units of a product, making it potentially difficult to impose unit taxes in some cases (see Keen, 1998, for a more complete discussion of such issues).

The framework in this paper is readily adapted to incorporate other effects of economic interest, through its roots in discrete choice theory, such as differences across firms in the objective quality of produced goods. For example, Anderson, et al. (1992, Ch. 7) have extended the differentiated products model to consider multiproduct firms, and tax issues could be addressed within that extended context. Our setting could be extended to international trade to study issues such as optimal tariffs, and the partial equilibrium setting would ideally be extended to the case of general equilibrium in order to study the taxation of goods in conjunction with the taxation of income and capital. Such issues, we hope, will be the focus of subsequent analyses.

References

[1] Adams, Charles, (1993). *For Good and Evil: The Impact of Taxes on the Course of Civilization*, Madison Books, New York, p. 2.

[2] Anderson, Simon P., André de Palma, and Brent Kreider. Tax Incidence in Differentiated Product Oligopoly. *Journal of Public Economics*, (forthcoming, this issue?).

[3] Anderson, Simon P., André de Palma, and Yuri Nesterov (1995). Oligopolistic Competition and the Optimal Provision of Products. *Econometrica*, 63, 1281-1301.

[4] Anderson, Simon P., André de Palma, and Jacques-François Thisse (1992). *Discrete Choice Theory of Product Differentiation.* MIT Press, Cambridge, MA.

[5] Besley, Timothy (1989). Commodity Taxation and Imperfect Competition: A Note on the Effects of Entry. *Journal of Public Economics*, 40, 359-367.

[6] Cournot, Augustin, (1838, translated 1960). *Researches into the Mathematical Principles of the Theory of Wealth.* Frank Cass & Co., London.

[7] Daughety, Andrew (1990). Beneficial Concentration. *American Economic Review*, 80, 1231-1237.

[8] Delipalla, Sophia and Michael Keen (1992). The Comparison between Ad Valorem and Specific Taxation under Imperfect Competition. *Journal of Public Economics*, 49, 351-361.

[9] Fershtman, Chaim, Neil Gandal, and Sarit Markovich (1997). Tax Incidence in Differentiated Good Oligopolistic Markets: The Cases of the Automobile Market in Israel. Mimeo, Tel-Aviv University.

[10] Hamilton, Stephen (1999). Tax Incidence Under Oligopoly: A Comparison of Policy Approaches. *Journal of Public Economics*, 71, 233-245.

[11] Helpman, Elhanan and Paul R. Krugman (1989). *Trade Policy and Market Structure* MIT Press, Cambridge Mass.

[12] Kay, John A. and Michael J. Keen (1983). How Should Commodities be Taxed? *European Economic Review*, 23, 339-358.

[13] Keen, Michael (1998), The Balance between Specific and Ad Valorem Taxation, *Fiscal Studies,* 19, 1-37.

[14] Kramer, Samuel (1959). *History Begins at Sumer*, Doubleday & Company, Garden City, NY, 47-48.

[15] Mankiw, Gregor and Michael Whinston (1986). Free Entry and Social Inefficiency. *Rand Journal of Economics*,17, 48-58.

[16] McManus, M. (1964). Equilibrium, Number and Size in Cournot Oligopoly. *Yorkshire Bulletin of Social and Economic Research*, 16, 68-75.

[17] Myles, Gareth (1996). Imperfect Competition and the Optimal Combination of Ad Valorem and Specific Taxation, *International Tax and Public Finance*, 3, 29-44.

[18] Novshek, William (1985). On the Existence of Cournot Equilibrium. *Review of Economic Studies*, 52, 85-98.

[19] Robson, Arthur (1990). Stackelberg and Marshall. *American Economic Review*, 80, 69-82.

[20] Salop, Steven C. (1979). Monopolistic Competition with Outside Goods. *Bell Journal of Economics*, 10, 141-156.

[21] Seade, Jesus (1987). Profitable Cost Increases and the Shifting of Taxation: Equilibrium Responses of Markets in Oligopoly. Mimeo.

[22] Skeath, Susan and Gregory Trandel (1994). A Pareto Comparison of Ad Valorem and Unit Taxes in Noncompetitive Environments. *Journal of Public Economics*, 53, 53-71.

[23] Stern, Nicholas (1987). The Effects of Taxation, Price Control, and Government Contracts in Oligopoly and Monopolistic Competition. *Journal of Public Economics*, 32, 133-158.

[24] Suits, Daniel B. and Richard A. Musgrave (1953). Ad Valorem and Unit Taxes Compared. *Quarterly Journal of Economics,* 67, 598-604.

[25] Wicksell, Knut (1896). Taxation in the Monopoly Case. Translation from the original in: Musgrave and Shoups, eds. (1959) *Readings in the Economics of Taxation,* Irwin, Homewood, Il.