Sectoral Reallocation, Growth and Labor Income Inequality*

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November 12th, 2003

Abstract

This paper focuses on the transitory relationship between output level and Income inequality. As a result of either permanent or transitory sectoral technological shocks the economy will adjust to a new steady state equilibrium, but during the transition the dynamics of wages and workers will generate departures from the steady state level of income inequality.

* I thank Rody Manuelli and Ananth Seshadri for the guidance, comments and useful discussions
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1. Introduction

There is a broad literature both theoretical and empirical trying to understand the relation between income inequality and growth. Unfortunately the evidence in favor of the different theories is not conclusive, suggesting that we may need new explanations to this relationship. Based on frictions in the labor market this paper develops transitory departures from a steady state relationship between output level and income inequality as responses to sectoral specific technological shocks.

As Benabou (1996) reports theories relating growth and income inequality have followed three main avenues.

The first one is the one based on the political system as the pivotal mechanism trough which inequality affects growth. This part of the literature is characterized by models of intertemporal utility maximizers agents where there exists a government that redistributes income across households, and for that matter has to levy a tax on everybody's income. The endogenous determination of the tax, implies that the poorer the median voter (or the worst the income distribution), the higher the tax rate and therefore the lower the incentives to invest in this economy. In that sense, income inequality and economic growth are negatively related. Examples of work in this area are: Persson and Tabellini (1994, 1999), Alessina and Rodrik (1994), Perotti (1993), Krusell, Quadrini and Rios-Rull (1994), Krusell and Rios-Rull (1997), Wright (1996) and Boldrin and Rustichini (2000).

The second group of theories where a relationship between inequality and growth arises is based on imperfections in the asset markets. The basic idea is that the existence of imperfections such as borrowing constraints in any form (i.e. collateral constraints), prevent agents from undertaking efficient investment level and therefore growth or level of output is negatively affected. Work in this area include Galor and Zeira (1993), Banerjee and Newman (1993), Aghion and Bolton (1997) and Piketty (1997).

The last group bases the link between inequality and growth in the social conflict. That is, the higher the inequality, the more likely the possibility that the property rights will not be protected and therefore the higher the possibility of being confiscated, which in tern affects the investment decision negatively. References on this area include Grossman and Kim (1996) and Benhabib and Rustichini (1996).

All of these theories encounter severe difficulties when empirically tested. Though lightened by the work of Deininger and Squire (1996), the lack of reliable and internationally comparable data remains is a major problem, but also the evidence suggests that there may be something else going on. Benabou (1996) reports the results for 23 empirical studies on this area. The first group of theories suggests that the poorer the median voter the higher the transfer from rich to poor, which is something that is not verified. The theories based on asset market imperfections, has an even greater data reliability problem and therefore studies on that area have been even more limited. And finally the very notion of instability and the way it is measured puts another question mark in the studies trying to verify the third group of theories. In this case the idea of a two way relation between growth and inequality seems to be very plausible and this identification problem sets a limit in the ability of the data to conclusively support the theory.

So far the theories exposed above have focused themselves in the long run effects of inequality on growth, without paying attention to transitory effects. That is, they mainly focused on the steady state equilibrium of the economy and the relationship between inequality and growth there.

Findings by both Benabou (1996) and Banerjee and Duflo (2000) suggest that there may be a convergence in the level of inequality and that differences across countries may arise due to different stages in the convergence process. In both studies they find a negative correlation between changes in inequality and past inequality suggesting the existence of mean reversion in inequality and a convergence to a long run equilibrium level of inequality.

On this line of reasoning this paper rests entirely on transitional adjustments to a new equilibrium as a result of a sectoral technology shock. Previous work where the transition to a new steady state involves changes in the distribution of income as a result of a technological change include Greendwood and Yorokoglu (1997) and Manuelli (2001). Greenwood and Yorokoglu introduce human capital which will be used more intensively the newer the technology, therefore the introduction of new technologies creates a departure from the steady state level of inequality since the decision of human capital accumulation is previous to the technology improvement. The story behind Greendwood and Yorokoglu (1997) is one of differences across wages due to human capital and the fact that the productivity improvements make a relative intensive use of human capital. In their model there is no unemployment, whereas in this model unemployment plays a central role in allowing workers to change across sectors in the economy. Manuelli (2001) introduces a friction in the labor market and explores the effects of the introduction of a new technology since its discovery to its practical adoption.

Based on Phelan and Trejos (2000), the economy will be composed by two sectors, one of them producing capital and durable goods and the other non-durable goods and services, where both sectors will be exposed to sectoral specific technology shocks, which can be transitory or permanent. As a result of sectoral shocks, total factor productivity across sectors will differ and labor will have to be reallocated. The reallocation mechanism will not be instantaneous, in the sense that in order to transfer workers from one sector to another, they will have to stay unemployed for at least one period. Therefore any shock that induces reallocation of workers will induce a movement in the unemployment rate and during the transition to the new equilibrium wages across sectors will be different, which will induce to a different distribution of income during the transition to a new steady state equilibrium than the one that will prevail in equilibrium.

As noted before, the changes in income inequality will only appear in the transition, since unemployment will be the same across steady states and wages will be equal once in a steady state equilibrium.

The model will generate similar short term responses to permanent and transitory shocks, in particular with respect to inequality, with very different implications for growth or output level in the long run. Clearly permanent technological shocks will have an effect in the level of output in the future steady state, but transitory will have no effect in the new steady state once it is reached, but the initial response may be similar. In other words, the relationship between inequality and growth is more complex than in previous work in the sense that we may have countries undergoing transitory shocks and permanent shocks, with effects on inequality but the first ones will experience no long run growth whereas the second will show effects on the level of gdp. Therefore the interpretation of a simple panel regression where we mix temporary and permanent effects may be misleading.

The paper is organized as follows: In section 2 the dynamic model is presented, where the economy will be divided into two sectors and there will be a friction in the labor market that will generate transitional differences in wages across sectors and therefore transitional increases in the level of labor income inequality. In section 3, the model is calibrated mainly using data from micro evidence or evidence from specific studies. In section 4 three experiments are run. First transitional increases in productivities, then permanent ones and finally data from the post war US economy are introduced into the model and dynamics calculated. Finally in section 5 the conclusion is presented.

2. The model

The economy will be populated by a continuoum of infinitely lived individuals with mass equal to one. Time is discrete. The instantaneous utility function has the following form:

$$u(c_x, c_y) = \frac{\left[\left(\alpha c_x^{\rho} + (1-\alpha)\, c_y^{\rho}\right)^{1/\rho}\right]^{1-\xi}}{1-\xi}$$

where c_x denotes the flow of services from the durable goods and c_y is the consumption of non durable goods and services. And $0 \leq \alpha \leq 1$, $-\infty \leq \rho \leq 1$.

There will be two sectors in the economy, x and y, where x will be the capital and durable goods producing sector and y the non-durable goods an services sector. The production function of each sector is represented by

$$\begin{aligned} x &= a_x \left(L_x(1-m_x)\right)^{\gamma} K_x^{1-\gamma} \\ y &= a_y \left(L_y(1-m_y)\right)^{\theta} K_x^{1-\theta} \end{aligned}$$

where x and y are the total outputs in each sector, $K_x$and K_y is the amount of capital devoted to production in each sector, $(1 - m_x)$ and $(1 - m_y)$ are the hours per worker directly involved in the production process, since m_x and m_y will be the "per worker recruiting effort", L_x and L_y are the fraction of population working for sector x and y, and a_x and a_y are productivity parameters. And $0 \leq \gamma \leq 1$, $0 \leq \theta \leq 1$.

Workers can be either in sector x, y or unemployed, and since population is normalized to one, we have

$$1 = L_x + L_y + u$$

The law of motion for workers in each sector, taken from Phelan and Trejos (2000) is given by

$$L_j' = L_j(1 - \phi) + \psi L_j u^\eta m_j^{1-\eta}; j = x, y$$

where a fraction ϕ is exogenously separated from their jobs and new workers come into each sector according to the matching function $\psi L_j u^\eta m_j^{1-\eta}; j = x, y$, that is, the number of new matches is increasing in the number of people unemployed u and on the average searching effort in each sector $L_j m_j; j = x, y$. And $0 \leq \eta \leq 1$, $\psi \geq 0$.

Note that the law of motion for workers in each sector takes the amount of workers in period t and the unemployment rate in period t as state variables. So far I am not letting the workers quit their jobs in any moment. Only those exogenously separated from their jobs may be matched next period in one of the sectors. This restriction has no implication in the steady state, but makes the dynamics slower than in the case when workers are allowed to quit their jobs as a response to a technological change.

Capital in each sector follow a standard law of motion

$$K_j' = K_j(1-\delta) + I_j; j = x, y$$

where δ is the depreciation rate, $0 \leq \delta \leq 1$, and $I_j; j = x, y$ are the investment levels in each of the sectors.

We will assume that capital will only be produced in sector x, that is, sector x will produce capital goods which will be used in the production of new capital goods, and non durable goods as well, appart from renting the services of the durable goods to the household. So we have the following constraints

$$\begin{aligned} c_x + I_x + I_y &= x \\ c_y &= y \\ I_x + I_y &\geq 0 \end{aligned}$$

Let β be the discount rate.

So the planner's problem can be written as:

$$V(L_x, L_y, K_x, K_y) = \max_{m_j, I_j, L'_j, K'_j} u(c_x, c_y) + \beta V(L'_x, L'_y, K'_x, K'_y) \; ; \; j = x, y$$

Subject to

$$\begin{aligned}
u(c_x, c_y) &= \frac{\left[\left(\alpha c_x^{\rho} + (1-\alpha)\, c_y^{\rho}\right)^{1/\rho}\right]^{1-\xi}}{1-\xi} \\
x &= a_x \left(L_x (1 - m_x)\right)^{\gamma} K_x^{1-\gamma} \\
y &= a_y \left(L_y (1 - m_y)\right)^{\theta} K_x^{1-\theta} \\
1 &= L_x + L_y + u \\
K'_j &= K_j (1 - \delta) + I_j ; j = x, y \\
c_x + I_x + I_y &= x \\
c_y &= y \\
L'_j &= L_j (1 - \phi) + \psi L_j u^{\eta} m_j^{1-\eta} ; j = x, y \\
& (L_x, L_y, K_x, K_y)_0 \text{ given} \\
I_x + I_y &\geq 0
\end{aligned}$$

For the solution of the planner's problem, see the appendix.

2.1. Competitive Equilibrium

If we think of a representative household that has to allocate a continuum of individuals working in sector x, working in sector y, be unemployed and engage in the recruit effort for both sectors. That is, the household should engage in both unemployment and recruiting effort in order to supply sectoral specific labor to the market. The household takes as given his/her unemployment level (it is a state variable) and the average recruitment effort in the rest of the economy M_j $j = x, y$, the return on sectoral capital $r_j, j = x, y$, wages in each sector $w_j, j = x, y$ and the relative price of non-durable goods with respect to durable goods p. The consumer's problem can be written as follows:

$$V_C(l_{xi}, l_{yi}, k_{xi}, k_{yi}) = \max_{c_{ji}, l'_{ji}, k'_{ji}} u(c_{xi}, c_{yi}) + \beta V_C(l'_{xi}, l'_{yi}, k'_{xi}, k'_{yi}) \ ; \ j = x, y$$

subject to

$$\begin{aligned}
u(c_{xi}, c_{yi}) &= \frac{\left[\left(\alpha c_{xi}^{\rho} + (1-\alpha)\, c_{yi}^{\rho}\right)^{1/\rho}\right]^{1-\xi}}{1-\xi} \\
c_{xi} + i_{xi} + i_{yi} + pc_{yi} &= w_x(1-m_{ix})l_{xi} + w_y(1-m_{iy})l_{yi} + r_x k_{xi} + r_y k_{yi} \\
l'_{ji} &= l_{ji}(1-\phi) + l_{ij} u_i^{\eta} m_{ij}^{1-\eta-\zeta} P(M_j); j = x, y \\
P(M_j) &= \sigma \psi M_j^{\zeta} \\
\sigma &= \left(\frac{1-\eta-\xi}{1-\eta}\right)^{1-\eta-\zeta} \\
k'_{ji} &= k_{ji}(1-\delta) + i_{ji}; j = x, y \\
i_{xi} + i_{yi} &\geq 0
\end{aligned}$$

Where lower case letter indexed by i represent individual level variables. c_{xi} represents the flow of services from durable goods that the household rents from the durable goods producer sector x and c_{yi} represents the consumption by household i of non durable goods and services. l_{ji} represents the time consumer i spends working for sector j, l'_{ji} represent the time consumer i will spend working for sector j next period, k_{ji} is the amount of capital consumer i owns which is an input in the production function of sector j, k'_{ji} is the amount of capital consumer i will own next period which will be an input in the production function of sector j. c_{ji} is the amount consumed by consumer i from goods produced in sector j. i_{ji} is the amount invested in sector j's capital goods by consumer i. u_i is the amount of time unemployed. m_{ij} is the amount of time consumer i spends recruiting for sector j . w_j and r_j are the wage rate and the interest rate respectively paid by sector j. p is the relative price of goods produced in sector y taking goods produced in sector x as numeraire.

Note that using the law of motion for hours worked in sector j, we can obtain m_{ij} and replace it into the budget constraint, and then replace c_x into the utility function.

So, the problem of the household can be solved by choosing l'_{ji}, k'_{ji} and c_{yi} for $j = x, y$, given u_i, M_j, w_j, r_j and p for $j = x, y$.

The representative firm in sector x solves the following static problem

$$\pi_x = \max_{l_x, k_x} \; a_x \left(l_x(1 - m_x)\right)^{\gamma} k_x^{1-\gamma} - w_x(1 - m_x)l_x - r_x k_x$$

whereas the representative firm in sector y solves the following static problem

$$\pi_y = \max_{l'_y, k_y} \; p a_y \left(l_y(1 - m_y)\right)^{\theta} k_y^{1-\theta} - w_y(1 - m_y)l_y - r_y k_y$$

Where l_x and l_y are the employment levels in the representative firm in sector x and y respectively at the beginning of each period. k_x and k_y are the capital used in each sector.

From the firm's problem wages and interest rates are determined as follows:

$$
\begin{aligned}
w_x &= \gamma a_x \left(L_x(1-m_x)\right)^{\gamma-1} K_x^{1-\gamma} \\
w_y &= p\theta a_y \left(L_y(1-m_y)\right)^{\theta-1} K_y^{1-\theta} \\
r_x &= (1-\gamma)a_x \left(L_x(1-m_x)\right)^{\gamma} K_x^{-\gamma} \\
r_y &= p(1-\theta)a_y \left(L_y(1-m_y)\right)^{\theta} K_y^{-\theta}
\end{aligned}
$$

For the solution of the competitive equilibrium, see the appendix.

The model will deliver an endogenous labor income inequality. That is, it will endogenously determine unemployment, and labor shares in the sector x and y, together with wages in sector x and y. Assuming that the fraction of unemployed workers has no income, it determines a three point Lorenz curve determining a gini coefficient for the labor income.

3. Calibration

The sectors in the model will be matched to the sectors in the US economy from 1950 to 2000. Sector x, the capital and durable goods producer will be matched to the Manufacturing in durable goods sector plus the Construction sector, and sector y, the non durable goods sector will be matched to the rest of the private US economy, that is non-durable manufacturing, services and agriculture.

The model has parameters for preferences, technology and the matching function. The preferences parameters are estimated by Ogaki and Reinhart (1998). There they estimate the exact same utility function and obtain parameters $\alpha = 0.2709$, $\rho = 0.143$, $\xi = 0.23$. In addition to those the discount factor β is set equal to 0.96 which is a common value in the literature.

The matching function parameters are calibrated as follows. ϕ, the exogenous match destruction rate is equal to 0.055 as in Cole and Rogerson (1999)[1]. η, the elasticity of the matching function with respect to unemployment is set equal to 0.4 as in Blanchard and Diamond (1989), which is a value frequently used in this kind of literature. Other papers like Andolfatto (1996), Merz (1995) and Phelan and Trejos (2000) use the same estimate. Finally the constant in front of the matching function ψ is calibrated so that the model delivers a steady state level of unemployment of 5.5%.

The technology parameters are the ones that are calibrated to match moments of the pre WWII US economy. a_y is set equal to 1 and a_x, γ and θ are calibrated to deliver a share of employment in the durable goods sector equal to the average from 1929 to 1940 of 18.53%, the average of Investment to GDP[2] from 1929 to 1940 of 19.05% and a

[1] They use 0.055 as the quarterly match destruction rate, and argue that it may be over estimating the actual match destruction rate, so, I'll use 0.055 as a yearly match destruction rate in order to account for this over estimation

[2] Note that since in the model the household rents the capital services from sector x. The value of investment that is analogous to the model is the sum of consumption of durables and private investment from NIPA tables

labor share of GDP of 70%. Finally the depreciation rate is set equal to 10%, which is a common value in the literature.

4. Dynamics

The model will be solved as follows. First calculate an initial steady state and a final steady state (which will differ from the previous only in the final values of productivity parameters a_x and a_y. With those calculated, now I feed to the model a vector of parameters a_{xt} and a_{yt} and set a sufficiently large time frame for the model to converge to the final steady state. The model has four Euler equations per period, so the system will have the number of equations and unknowns equal to four times the number of periods that takes the system to converge to a new equilibrium, which will deliver the vectors for sectoral employment and sectoral capital. Once I have the system of equations, I solve it nonlinearly. Once solved for L_j', K_j' $j = x, y$, the wages, interest rate, relative prices, total output, unemployment and labor income distribution are calculated using the sectoral employment and capital as inputs.

I will run three experiment in order to learn about the relationship between the labor income inequality and Output. First it will be a permanent increase in the productivity levels a_x and a_y separately, second a temporary increase in the productivity levels a_x and a_y separately and finally I will feed the vectors of a_x and a_y as reported by the Bureau of

Labor Statistics for the values of Multifactor productivity in the corresponding sectors for the US economy for the period 1950-2000.

4.1. Permanent increases in productivity

When feeding to the model a vector of productivity a_x, where productivity increases 10% once and for all, the results in terms of labor income inequality, total output, and other relevant quantities and prices are described by figures 1, 2, 3 and 4.. On the other hand, if the same change in productivity takes place in the non durable goods and services sector as depicted by figure 3, the results in terms of labor income inequality total output and other relevant quantities and prices are shown in figures 5, 6, 7 and 8.

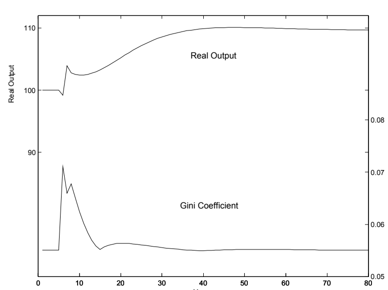


Figure 1

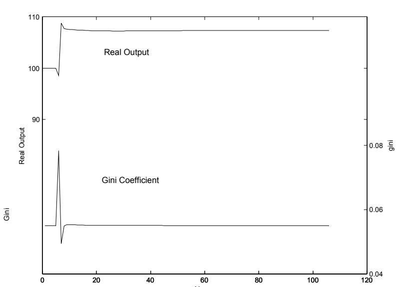


Figure 5

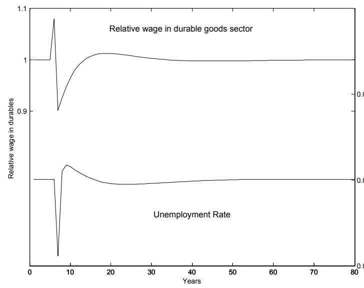


Figure 2

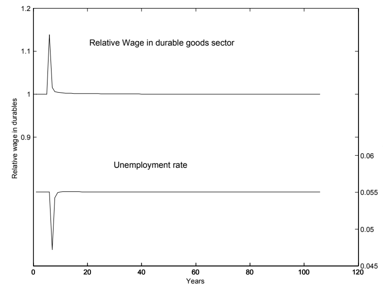


Figure 6

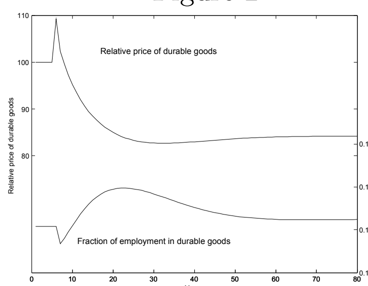


Figure 3

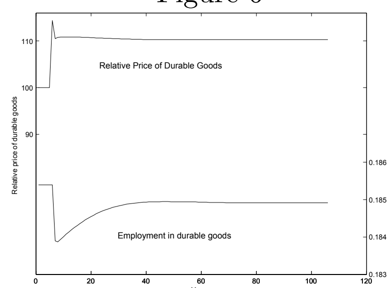


Figure 7

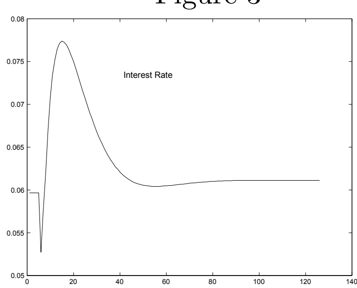


Figure 4

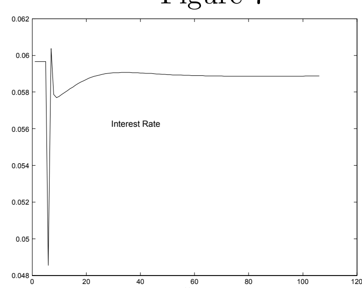


Figure 8

We can see from the results shown above that the evolution of relative prices, interest rate, and relative wages display a slower adjustment to a new equilibrium in the case where the jump in productivity occurs in the durable goods sector. In both cases unemployment rate falls initially, which is driven by changes as a response of the shock of the search effort. The evolution of relative wages is responsible for the different reaction in the gini coefficient. The relative price of durable goods in the case where the increase happens in its sector, shows an initial increase and then a long monotonic decreasing convergence to a lower equilibrium. The initial reaction of the relative price of durable goods is similar in the case where the increase in productivity took place in the non durable goods and services sector, but instead of decreasing afterwards, it stays at a higher level. The employment also displays a similar initial pattern in the two cases, but is different in its convergence to the new long run equilibrium. In the case of the increase in the durable goods sector, the employment in that sector, decreases, but then overshoots its equilibrium and converges finally, whereas in the other case, it converges monotonically to the equilibrium level after the initial drop.

4.2. Temporary increases in productivity

If instead of feeding a permanent increase in either of the productivities, I feed a temporary one , that jumps 10% in one period, stays 10% above its long run level for 5 periods and returns to the long run level in the 6th period, to stay there forever, the results are as follows. When the temporary increase of 10% occurs in the durable goods sector, the relationship between labor income inequality, total output and other relevant prices and quantities are shown in figures 9 through 12..

On the other hand, when the temporary increase in productivity happens in the non durable goods sector the results are reported by figures 13 through 16.

As reported in figures 9 and 13 the initial reaction of real output, measured with relative prices constant at the initial level, in both cases it drops. That reaction is induced by the fact that the total search effort increases, so there are less workers devoted to the production process. In the case where the temporary increase occurs in the durable goods sector, the total search effort increases from 7% of total labor force to 9.7%, with a dramatic increase in the search effort in the non durable goods and services sector and a fall in the durable goods sector search effort.

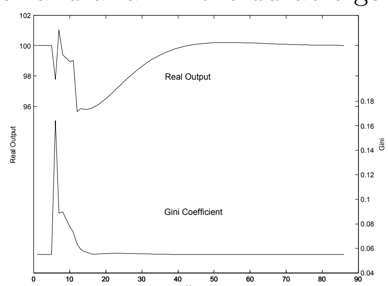


Figure 9

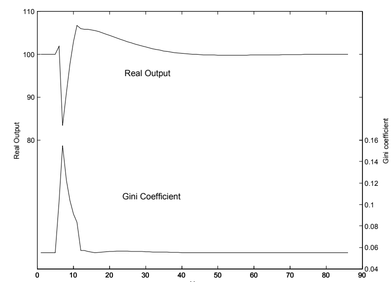


Figure 13

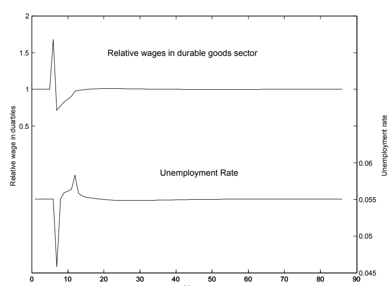


Figure 10

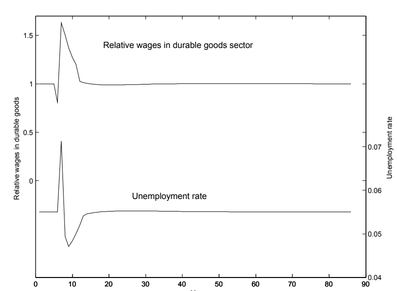
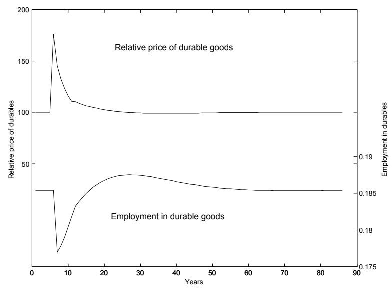


Figure 14



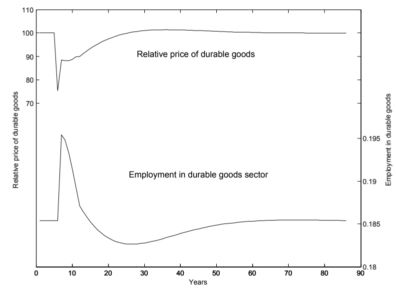
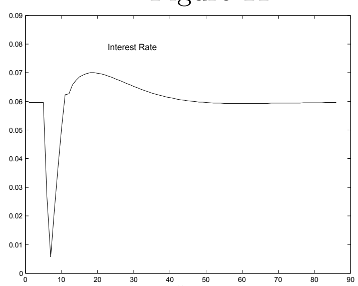


Figure 11

Figure 15



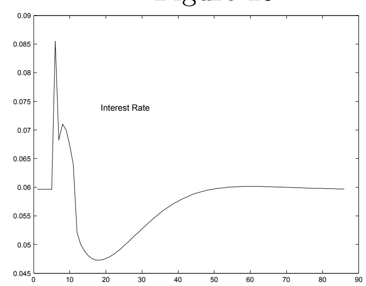


Figure 12

Figure 16

The agents are exploiting the temporary increase in the durable goods sector by devoting as many workers as possible to the production process and therefore temporarily abandoning the search effort, which explains why employment in the sector experiencing the temporary increase in productivity suffers a decrease in its share of labor force as reported in figures 11 and 15. In the case where the temporary increase in productivity occurs in the non durable goods sector we see an increase and a later decrease in unemployment, which is entirely determined by the evolution of the search effort. On aggregate it falls from 7% to 4.5% initially.

The evolution of unemployment together with the path displayed by relative wages determine the evolution of the gini coefficient. The relative wages follow an expected pattern, with an initial increase in the wage of the sector that underwent the temporary productivity increase. Prices and interest rates also follow opposite behaviors according to where the productivity increase took place.

4.3. Simulation for the post war US economy

In order to feed the actual vectors of productivities of the US economy for the period 1950-2000, I will assume that the economy was in a balanced growth path from the distant past to 1950, and from 1950 to 2000, I will feed the deviations from the aggregate multifactor productivity vector for each of the sectors as shown in figure 17. That is, I will assume that the planner knew in advance the evolution of aggregate multifactor productivity in the economy and that the initial belief was that the sectoral multifactor productivities would display the same evolution as the aggregate one. Then, in 1950 the agents are surprised by new paths of sectoral multifactor productivities as shown. The actual values for sectoral multifactor productivities where taken from BLS for the US sectors manufacturing in durable goods and total private multifactor productivity. With those I calculated the path of multifactor productivity in the non durable and service sector by calculating the percentage deviation of the sectoral multifactor productivity to the aggregate multifactor productivity assuming that the three series start in 1950 with the value of 1.

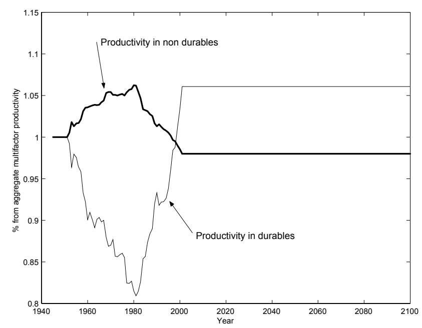


Figure 17

The results of the experiment of feeding the series as in Figure 17, are shown in figures 18 through 21.

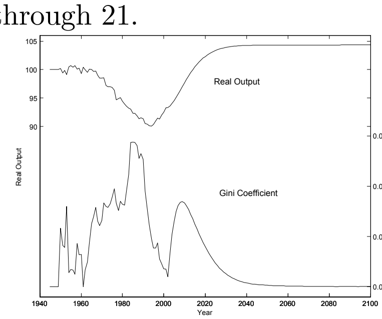


Figure 18

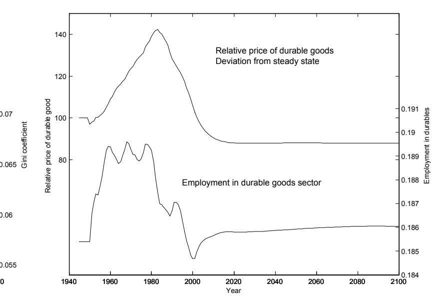


Figure 20

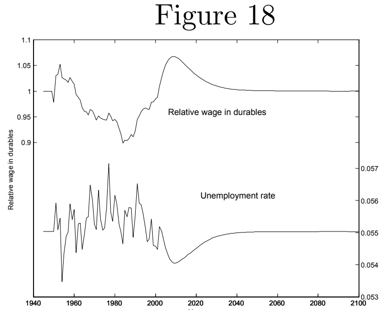


Figure 19

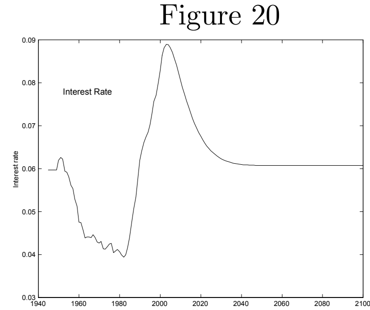


Figure 21

As shown the real output stays below its balanced growth path and the relative price of capital stays above its path. Reallocation is taking place as we can see from the path of employment in the durable goods sector and also in the relative wage. Note that both employment and wages in the durable goods sector display a similar patter with an initial jump and a subsequent decrease to the equilibrium. The unemployment rate is driven by the search effort, and here the search effort in the durable goods sector in particular is increasing. The gini coefficient stays above its equilibrium level during the convergence to the new steady state, which is driven by the optimal timing in the allocation of workers. That is, the reallocation across sectors is permanent till the new equilibrium is reached and that we can see from the evolution of relative wages in figure 19. There is an initial increase in the relative wage in durable goods sector, but then around 1960 the model predicts that the wage in the non durable goods sector be higher than that in the durable goods sector.

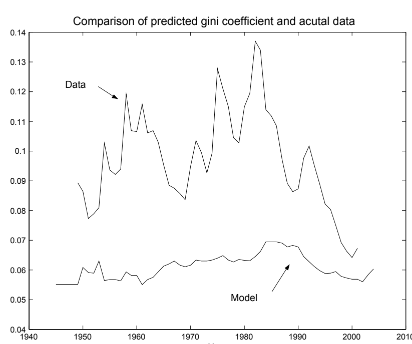


Figure 22

If I calculate the very same gini coefficient from the actual data for the post WWII US economy, the relation between the data and the model is depicted by figure 22. The correlation coefficient between the model and the data is 0.52. Comparing the two series we can see that there is a difference in the ranges of labor income inequality, the model predicts it to be between 0.055 to 0.07 and in the data it is between 0.08 and 0.14. This difference in scale may be driven by differences in human capital required across sectors which would induce an equilibrium inequality higher than that induced solely by unemployment as in the model. We also see that the behavior of the actual gini coefficient is much more volatile than that suggested by the model, which could be explained by the assumption of perfect foresight in the model. That is, in the model, the agents know in 1950 the path of productivities from then on.

5. Conclusion

So, in essence there are two effects determining the relationship between output and inequality. First there is a physical rigidity due to the fact that the planner finds optimal to reallocate workers across sectors following a cyclical pattern as seen in figure 20. That pattern generates increases in inequality which are captured in figure 18 from 1950 to 2000. Then there is an information effect. The timing of information and the perception of wheter the changes in productivities are permanent or transitory generate different reactions of the economy

As it can be seen from the results of the experiments undertaken above, the relationship between inequality and growth depends not only on the relative productivities across sectors during a transition to a new steady state, but also on the structure of information that the planner has. That is whether he/she believes that some change in productivities is permanent or transitory and how in advance of the actual change in productivities the news is known. The introduction of uncertainty and a more complex informational structure would be the future steps in research to try to reproduce the actual path of inequality.

These results are in line with the findings of Benabou (1996) and Banerjee and Duflo (2000) in the sense that inequality displays mean reversion, but raise a question mark on panel data studies trying to relate growth and inequality in the sense that the structure of information should be taken into account and there may be countries experiencing transitory changes in productivities whereas others permanent ones with different implications for the relationship inequality - growth.

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7. Appendix

The solution of the planner's problem is the following:

$$V(L_x, L_y, K_x, K_y) = \max_{m_j, I_j, L'_j, K'_j} u(c_x, c_y) + \beta V(L'_x, L'_y, K'_x, K'_y) \text{ ; } j = x, y$$

Subject to

$$u(c_x, c_y) = \frac{\left[\left(\alpha c_x^{\rho} + (1-\alpha)\, c_y^{\rho} \right)^{1/\rho} \right]^{1-\xi}}{1-\xi}$$

$$
\begin{aligned}
x &= a_x \left(L_x(1-m_x)\right)^{\gamma} K_x^{1-\gamma} \\
y &= a_y \left(L_y(1-m_y)\right)^{\theta} K_x^{1-\theta} \\
1 &= L_x + L_y + u \\
K'_j &= K_j(1-\delta) + I_j; j = x, y \\
c_x + I_x + I_y &= x \\
c_y &= y \\
L'_j &= L_j(1-\phi) + \psi L_j u^{\eta} m_j^{1-\eta}; j = x, y \\
& (L_x, L_y, K_x, K_y)_0 \text{ given} \\
I_x + I_y &\geq 0
\end{aligned}
$$

Note that from the law of motion for workers in each sector we can obtain m_x and m_y as

$$
\begin{aligned}
m_x(L'_x, L_x, L_y) &= \left[\left(\frac{L'_x}{L_x} - 1 + \phi\right) \frac{1}{\psi\left(1 - L_x - L_y\right)^{\eta}}\right]^{\frac{1}{1-\eta}} \\
m_y(L'_y, L_y, L_x) &= \left[\left(\frac{L'_y}{L_y} - 1 + \phi\right) \frac{1}{\psi\left(1 - L_x - L_y\right)^{\eta}}\right]^{\frac{1}{1-\eta}}
\end{aligned}
$$

Therefore the consumption functions in are only functions of L_j, L'_j, K_j and K'_j for $j = x, y$, so, the planner's problem can be reduced to the choice of L'_j and K'_j given the state variables L_j and K_j for $j = x, y$.

The FOC are given by:

$$
\begin{aligned}
\left(\frac{\partial u}{\partial c_x}\right)_t \left(\frac{\partial c_x}{\partial m_x}\right)_t \left(\frac{\partial m_x}{\partial L'_x}\right)_t + \beta \frac{\partial V(L'_x, L'_y, K'_x, K'_y)}{\partial L'_x} &= 0 \\
\left(\frac{\partial u}{\partial c_y}\right)_t \left(\frac{\partial c_y}{\partial m_y}\right)_t \left(\frac{\partial m_y}{\partial L'_y}\right)_t + \beta \frac{\partial V(L'_x, L'_y, K'_x, K'_y)}{\partial L'_y} &= 0 \\
\left(\frac{\partial u}{\partial c_x}\right)_t \left(\frac{\partial c_x}{\partial K'_x}\right)_t + \beta \frac{\partial V(L'_x, L'_y, K'_x, K'_y)}{\partial K'_x} &= 0 \\
\left(\frac{\partial u}{\partial c_x}\right)_t \left(\frac{\partial c_x}{\partial K'_y}\right)_t + \beta \frac{\partial V(L'_x, L'_y, K'_x, K'_y)}{\partial K'_y} &= 0
\end{aligned}
$$

The Envelope conditions are given by:

$$
\begin{aligned}
\frac{\partial V(L_x, L_y, K_x, K_y)}{\partial L_x} &= \left(\frac{\partial u}{\partial c_x}\right)_t \left[\left(\frac{\partial c_x}{\partial L_x}\right)_t + \left(\frac{\partial c_x}{\partial m_x}\right)_t \left(\frac{\partial m_x}{\partial L_x}\right)_t\right] \\
&\quad + \left(\frac{\partial u}{\partial c_y}\right)_t \left(\frac{\partial c_y}{\partial m_y}\right)_t \left(\frac{\partial m_y}{\partial L_x}\right)_t \\
\frac{\partial V(L_x, L_y, K_x, K_y)}{\partial L_y} &= \left(\frac{\partial u}{\partial c_y}\right)_t \left[\left(\frac{\partial c_y}{\partial L_y}\right)_t + \left(\frac{\partial c_y}{\partial m_y}\right)_t \left(\frac{\partial m_y}{\partial L_y}\right)_t\right] \\
&\quad + \left(\frac{\partial u}{\partial c_x}\right)_t \left(\frac{\partial c_x}{\partial m_x}\right)_t \left(\frac{\partial m_x}{\partial L_y}\right)_t \\
\frac{\partial V(L_x, L_y, K_x, K_y)}{\partial K_x} &= \left(\frac{\partial u}{\partial c_x}\right)_t \left(\frac{\partial c_x}{\partial K_x}\right)_t \\
\frac{\partial V(L_x, L_y, K_x, K_y)}{\partial K_y} &= \left(\frac{\partial u}{\partial c_x}\right)_t \left(\frac{\partial c_x}{\partial K_y}\right)_t + \left(\frac{\partial u}{\partial c_y}\right)_t \left(\frac{\partial c_y}{\partial K_y}\right)_t
\end{aligned}
$$

The Foc and the Envelope conditions properly updated once determine the four Euler equations governing the dynamics of the system. Therefore the system has 4 Euler equations per period determining L'_j, K'_j $j = x, y$.

If T= number of periods in the system, I solve 4T equations and unknonwns nonlinearly

The Competitive equilibrium solution coincides with the planner's problem as follows:

From the firm's problem wages and interest rates are determined as follows:

$$
\begin{aligned}
w_x &= \gamma a_x \left(L_x(1-m_x)\right)^{\gamma-1} K_x^{1-\gamma} \\
w_y &= p\theta a_y \left(L_y(1-m_y)\right)^{\theta-1} K_y^{1-\theta} \\
r_x &= (1-\gamma) a_x \left(L_x(1-m_x)\right)^{\gamma} K_x^{-\gamma} \\
r_y &= p(1-\theta) a_y \left(L_y(1-m_y)\right)^{\theta} K_y^{-\theta}
\end{aligned}
$$

For the household problem, define

$$
\begin{aligned}
\widetilde{c_x} &= w_x(1-m_{ix}(l'_{xi}, l_{xi}, l_{yi}))l_{xi} + w_y(1-m_{iy}(l'_{yi}, l_{yi}, l_{xi}))l_{yi} + \\
&\quad +(1-\delta+r_x)k_{xi} + (1-\delta+r_y)k_{yi} - k'_{xi} - k'_{yi} - pc_{yi}
\end{aligned}
$$

Then

$$
\frac{\partial u}{\partial c_y} - p\frac{\partial u}{\partial c_x} = 0
$$

$$\left(\frac{\partial u}{\partial \widetilde{c_x}}\right)_t\left(\frac{\partial \widetilde{c_x}}{\partial m_{ix}}\right)_t\left(\frac{\partial m_{ix}}{\partial l'_{xi}}\right)_t+\beta\frac{\partial V_C(l'_{xi},l'_{yi},k'_{xi},k'_{yi})}{\partial l'_{xi}} = 0$$

$$\left(\frac{\partial u}{\partial \widetilde{c_x}}\right)_t\left(\frac{\partial \widetilde{c_x}}{\partial m_{iy}}\right)_t\left(\frac{\partial m_{iy}}{\partial l'_{yi}}\right)_t+\beta\frac{\partial V_C(l'_{xi},l'_{yi},k'_{xi},k'_{yi})}{\partial l'_{yi}} = 0$$

$$\left(\frac{\partial u}{\partial \widetilde{c_x}}\right)_t\left(\frac{\partial \widetilde{c_x}}{\partial k'_{ix}}\right)_t+\beta\frac{\partial V_C(l'_{xi},l'_{yi},k'_{xi},k'_{yi})}{\partial k'_{xi}} = 0$$

$$\left(\frac{\partial u}{\partial \widetilde{c_x}}\right)_t\left(\frac{\partial \widetilde{c_x}}{\partial k'_{iy}}\right)_t+\beta\frac{\partial V_C(l'_{xi},l'_{yi},k'_{xi},k'_{yi})}{\partial k'_{yi}} = 0$$

And the envelope conditions:

$$\begin{aligned}
\frac{\partial V_C(l_{xi},l_{yi},k_{xi},k_{yi})}{\partial l_{xi}} &= \left(\frac{\partial u}{\partial \widetilde{c_x}}\right)_t\left(\frac{\partial \widetilde{c_x}}{\partial l_{ix}}\right)_t+\left(\frac{\partial u}{\partial \widetilde{c_x}}\right)_t\left(\frac{\partial \widetilde{c_x}}{\partial m_{ix}}\right)_t\left(\frac{\partial m_{ix}}{\partial l_{xi}}\right)_t \\
&\quad+\left(\frac{\partial u}{\partial \widetilde{c_x}}\right)_t\left(\frac{\partial \widetilde{c_x}}{\partial m_{iy}}\right)_t\left(\frac{\partial m_{iy}}{\partial l_{xi}}\right)_t \\
\frac{\partial V_C(l_{xi},l_{yi},k_{xi},k_{yi})}{\partial l_{yi}} &= \left(\frac{\partial u}{\partial \widetilde{c_x}}\right)_t\left(\frac{\partial \widetilde{c_x}}{\partial l_{iy}}\right)_t+\left(\frac{\partial u}{\partial \widetilde{c_x}}\right)_t\left(\frac{\partial \widetilde{c_x}}{\partial m_{ix}}\right)_t\left(\frac{\partial m_{ix}}{\partial l_{yi}}\right)_t \\
&\quad+\left(\frac{\partial u}{\partial \widetilde{c_x}}\right)_t\left(\frac{\partial \widetilde{c_x}}{\partial m_{iy}}\right)_t\left(\frac{\partial m_{iy}}{\partial l_{yi}}\right)_t \\
\frac{\partial V_C(l_{xi},l_{yi},k_{xi},k_{yi})}{\partial k_{xi}} &= \left(\frac{\partial u}{\partial \widetilde{c_x}}\right)_t(1+r_x-\delta) \\
\frac{\partial V_C(l_{xi},l_{yi},k_{xi},k_{yi})}{\partial k_{yi}} &= \left(\frac{\partial u}{\partial \widetilde{c_x}}\right)_t(1+r_y-\delta)
\end{aligned}$$

After applying the equilibrium prices and equilibrium quantities ($m_{ij} = M_j$) in the expressions above we get the exact same four Euler equations as in the planner's problem.