MARKET PERFORMANCE WITH MULTIPRODUCT FIRMS[1]

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Abstract

We revisit the fundamental issue of market provision of variety associated with Chamberlin, Spence, and Dixit and Stiglitz when firms sell several products. Both products and firms are envisaged as differentiated. We propose a nested demand model where consumers decide upon a firm then which variant to buy, and use it to determine the market's biases when firms compete in product ranges and prices. The market system attracts too many firms with too few products per firm: firms restrain product ranges to relax price competition, but this exacerbates over-entry. The results extend to generalized nested CES models.

KEY WORDS: Multiproduct firms, excess variety, nested demand, product line competition

JEL Classification: L11, L13, D43

1 Introduction

The economic analysis of the market provision of variety goes back to Hotelling (1929) and Chamberlin (1933). Hotelling was concerned with product selection for duopoly, whereas Chamberlin was interested in free entry equilibrium. The Chamberlinian monopolistic competition model was later examined rigorously by Spence (1976) and by Dixit and Stiglitz (1977). These papers, along with almost all of the subsequent literature have assumed that each firm produces a single product. The intention of this paper is to broaden the discussion of market performance by allowing firms to produce several products and developing a tractable demand system for analyzing the problem.

The analysis of multiproduct firms introduces a further dimension to competition, that of the product range. When a firm brings in a further variant to its product line, it attracts more custom but at a cost of cannibalizing its existing products. The decision also has a strategic effect that the firm may prefer to mitigate. This is that rivals may price more aggressively in the face of tougher competition. These effects are not addressed under the standard assumption of single-product firms. The product range also contributes a further dimension to performance.

The reason why there has been little theoretical economic analysis of price competition with multiproduct firms is that the problem is intrinsically difficult.[1] To characterize profit-maximizing prices for a firm

[1] See Katz (1984), Champsaur and Rochet (1989), and Shaked and Sutton (1990) for previous analyses of multiproduct firms under price competition. Recent work by Johnson and M (2003) and Grossman (2003) treats multiproduct firms under

selling m products requires simultaneously solving m first-order conditions, each of which involves the derivatives of the demands for m products. Likewise, to find the profit-maximizing product range for a firm necessitates finding not only the direct effect on profit from an additional product, but also the equilibrium pricing response of all other firms for all other products. Finally, the free-entry equilibrium is determined from the condition that further entry be unprofitable.

To make the problem tractable, we set out a specific model and use symmetry assumptions liberally for the demand functions (in the tradition of monopolistic competition). This symmetry leads to a symmetric welfare benchmark. In our basic model, we parameterize differentiation at two different levels. These correspond to differentiation of products within the firm, and differentiation across the firms themselves. The demand for any particular variant sold by a firm then depends on the two sources of product variety. Corresponding to the two levels of differentiation, market performance can be gauged by two quantities: the number of products per firm and the total number of firms. These two measures are to be compared at the equilibrium to the corresponding magnitudes for the social optimum.

The demand model has considerable interest in its own right. Although we treat all the variants produced by any firm as equally good substitutes for each other, and we assume symmetry in the choice of which firm to buy from, the substitution pattern across variants pro-

Cournot competition.

duced by different firms can be rather complex. We consider a general nested demand structure that builds on the nested logit model first proposed in the transportation context by Ben-Akiva (1973) and rationalized by McFadden (1978). The nested logit model was subsequently used theoretically by Anderson and de Palma (1992) to study the performance of multiproduct firms and in several empirical studies in industrial organization.[2] The idea behind our nested demand model is that product selection can be split into a two-stage process. First, consumers choose a firm, then subsequently they choose a specific product to buy from that firm. When choosing a firm, consumers anticipate they will then optimally choose among the products available, although at this stage they do not know exactly what products are available. Think of restaurants: a consumer may not know exactly what is on the menu on a given day, but she knows that she will choose optimally once she gets there, and she anticipates her expected utility level.[3] The two levels of differentiation in the model correspond to the diversity across restaurants and the diversity within a restaurant's menu.

In the basic model, we assume that each consumer buys one unit of one product. This assumption makes it simple to carry out the welfare comparison because social surplus is independent of the price level,

[2] A sophisticated application of the nested logit model by Goldberg (1995) studies firm pricing in the car market.

[3] The two-stage process described here can readily be extended to three or more stages. For example, a consumer may choose for her vacation a country, then a resort, then a hotel.

and we can then directly compare market equilibrium with the first-best optimum solution. Later on we allow for downward sloping individual demand. For this case, we compare the second-best (zero profit constrained) optimum to the equilibrium. The extension of the basic analysis is fairly straightforward, but it broadens the scope considerably. This extension also encompasses the nested CES model. We also describe in this section a class of generalized nested demand models that have a consumer theoretic foundation with consumers making discrete choices of which product to buy.[4]

The performance analysis can be summarized quite succinctly. Firms hold back on product ranges in order to relax price competition.[5] Indeed, a broader product range makes the firm more attractive to consumers and so provokes a more competitive price response. Holding back elicits instead a more comfortable pricing environment. However, this also means that firm profitability is higher than it would be with more aggressive (larger) product ranges so that the market signal for firms to enter (i.e., profit) is stronger than the social signal (surplus contribution). This means that the market solution has too many firms, each one with too narrow a product range.[6]

The structure of the paper is as follows. Section 2 provides an

[4] Anderson, de Palma, and Thisse (1992) use a similar procedure to disaggregate the standard CES representative consumer model. Verboven (1996) does likewise for the nested logit and generalized CES models.

[5] This is the same reason that firms choose different qualities in models of vertical differentiation - see Shaked and Sutton (1982).

[6] Some alternative market structures are discussed in the conclusions.

overview of the analysis. In Section 3, we introduce the demand function and the nesting structure. In Section 4, we derive the social welfare function, establish symmetry, and characterize the first best optimum number of firms and the optimal variety offered by each firm. In Section 5, we compute the equilibrium game: firms decide first whether to enter the market or not, then how many products to offer, and finally how to price them. We then compare the market solution and the optimal solution, and show that the market induces over-entry of firms and under-provision of variety per firm. In section 6, we examine the case of variable individual consumption. Section 7 concludes with some further discussion.

2 Overview

Let there be n firms, indexed $i = 1...n$, and let Firm i produce m_i products, indexed $k = 1...m_i$. The demand for product ik (the kth product of Firm i) is given by

$$D_{ik} = N\mathbb{P}_i\mathbb{P}_{k|i}, \tag{1}$$

where N is the number of consumers in the market, $\mathbb{P}_i$ is the fraction of consumers buying from firm i, and $\mathbb{P}_{k|i}$ is the fraction of consumers who choose product ik given that they have selected Firm i. Costs per unit produced are constant at rate c, and the fixed costs for Firm i producing m_i products are $K(m_i) = k_0 + k_1 m_i$. The optimal allocation is symmetric, and at a symmetric allocation, $\mathbb{P}_i$ will equal $1/n$ while $\mathbb{P}_{k|i}$ will equal $1/m$.

The optimum values of n and m are determined from costs and the consumer benefit function that underlies the demand system. Specifically, the consumer benefit function may be written as a weighted sum of the benefits from variety at each level, the two levels being the product range and the firm. The relative importance of each level is described by weights σ_A and σ_B that reflect the heterogeneity of the two levels. The key component benefit functions are increasing and concave functions that are written as $A(m)$ and $B(n)$ respectively, and so depend on the amount of variety available at each level.

The market equilibrium is the outcome of a three-stage game involving entry, product ranges, and prices. The equilibrium number of firms is determined by a zero profit condition, so $\frac{N}{nm}(p-c) = K(m)$, where p is the price per unit. The equilibrium number of products per firm is determined from a marginal profit condition that accounts for both the direct effect of an extra product in the range and the strategic effect on other firms' prices. The latter effect is negative because further products provoke more price competition from rivals, which the firm wants to avoid. The former effect depends on the extra benefit to consumers from more product variety, and so is proportional to $A'(m)$: this link to the optimal problem is what enables us to find the direction of the bias in the market system.

The equilibrium mark-up is determined from the derivative of $\mathbb{P}_i$ when this is evaluated at a symmetric solution. This mark-up is inversely proportional to $\Omega(n)$, where $\Omega(n)$ is a third key component of the model. It, like $B(n)$, is determined by the tastes underlying the consumer demand function. Under symmetry, the marginal social benefit from a further firm is proportional to $B'(n)$ while the net revenue from an nth firm is proportional to $1/n\Omega(n)$ (the constant of proportionality being the same). The comparison of the equilibrium and optimum numbers of firms is then made possible by using an inequality proved in Anderson, de Palma, and Nesterov (1995); that $B'(n) < 1/n\Omega(n)$, implying roughly that the private incentive to enter exceeds the social one. In the sequel, we flesh out the details.

3 Nested demand

Our model of choice is inspired from the nested logit model used in many econometric applications (see e.g. Train, 2003). We model choice as a two-step procedure. First, a consumer selects a firm (firms are synonymous with nests), then she buys one unit of one of the variants that the selected firm sells.

Recall from (1) that the demand for product ik (sold by Firm i) is $D_{ik} = N\mathbb{P}_i\mathbb{P}_{k|i}$, which is written as the product of two fractions: the fraction of consumers buying from i and the (conditional) fraction of those buyers who then choose the particular variant. This latter fraction, $\mathbb{P}_{k|i}$, is determined from a discrete choice model in the following manner. Once a consumer has chosen a firm, she draws a vector of match values, $\epsilon_{1|i}...\epsilon_{m_i|i}$ (one for each of the firm's m_i variants), and chooses the variant for which the conditional utility

$$u_{k|i} = y - p_{ik} + \sigma_A \epsilon_{k|i}, \tag{2}$$

is greatest. Here p_{ik} is the price of Firm i's kth variant, $\sigma_A \geq 0$ parameterizes the degree of substitutability among i's variants (for a given distribution of $\epsilon_{1|i}...\epsilon_{m_i|i}$) and y is consumer income.

The N individuals are assumed to be statistically identical and independent (that is, their preferences are the realization of the same probability distribution). The $\epsilon_{k|i}$ are assumed to be i.i.d. random variables (across variants and individuals) with zero mean and unit variance, $i = 1...n$, $k = 1...m_i$. Their common density function, $f(.)$, is twice differentiable and log-concave over a convex support I_2 (that is, $\ln f(.)$ is concave).[7] Since the individuals are statistically identical, the expected fraction of consumers selecting product k is equal to the conditional probability that an individual, randomly chosen in the population (given her previous choice of Firm i) selects product k. Therefore (1) represents the expected demand for product ik. The conditional probability that an individual selects product k given she chooses nest i is the probability that product k gives her the highest utility among all alternatives in nest i. That is $\mathbb{P}_{k|i} = \text{Prob}\{u_{k|i} \geq u_{\ell|i},\ \ell = 1...m_i\}$ for $k = 1...m_i$ and $i = 1...n$, or:

[7] Most of the usual distributions used in economics (uniform, normal, Gumbell, log-normal, beta, gamma, etc.) are log-concave. Log-concavity plays an important role in showing existence of a Nash price equilibrium with differentiated products, as shown by Caplin and Nalebuff (1991).

$$\mathbb{P}_{k|i} \equiv \int_{\mathcal{B}(k|i)} \prod_{l=1}^{m_i} f\left(e_l\right) de_1 ... de_{m_i}. \tag{3}$$

Here the integral is taken over the set $\mathcal{B}\left(k|i\right)$ of realizations $\left(e_1, ..., e_{m_i}\right)$ for which choice k in nest i yields the largest utility:

$$\mathcal{B}\left(k|i\right) \equiv \left\{ e_k : y - p_{ik} + \sigma_A e_k = \max_{l=1...m_i} \left(y - p_{il} + \sigma_A e_l\right) \right\}.$$

This expression can also be written as a one-dimension integral:

$$\mathbb{P}_{k|i} = \int_{I_2} f(x) \prod_{\substack{\ell=1 \\ \ell \neq k}}^{m_i} F\left(\frac{p_{i\ell} - p_{ik}}{\sigma_A} + x\right) dx, \tag{4}$$

where $F(.)$ is the common cumulative distribution of $\epsilon_{k|i}$. To interpret this expression, notice that $F\left(\left(p_{i\ell} - p_{ik}\right)/\sigma_A + x\right)$ in (4) is simply the probability that product ik is preferred to product $i\ell$ when the match value for product ik is x. Given the i.i.d. assumption, the product term in (4) is the probability that ik is the most preferred of i's variants given a draw x. Integrating over all possible x then gives the probability that ik is bought, conditional on buying from i.

The choice of firm is determined in a similar manner using the attractiveness of the various firms. Let V_i denote the attractiveness of Firm i, measured as the expected consumer surplus that a consumer selecting Firm i should expect. Hence V_i is the expected value of the maximum of the conditional utilities $u_{k|i}$, $i = 1...m_i$, so we can write

$$V_i = y + \int_{I_2} ... \int_{I_2} \max_{k=1...m_i} \left(-p_{ik} + \sigma_A e_k\right) \prod_{l=1}^{m_i} f(e_l) de_1 ... de_{m_i}. \tag{5}$$

Moreover, since the function f is continuous, then the sufficient condition for differentiability under the integral sign holds, and we have $\partial V_i / \partial p_{ik} = -\mathbb{P}_{k|i}$ (see (3)), where the domain of integration for demand is the set of realizations $\left(e_1, ..., e_{m_i}\right)$ such that product ik is the most preferred - which is the domain $\mathcal{B}(k|i)$. In summary, we have:

Lemma 1. *The within-nest conditional choice probabilities are given by:*

$$\mathbb{P}_{k|i} = -\partial V_i / \partial p_{ik} \ . \tag{6}$$

Note that (5) has all the properties of a (conditional) indirect utility function (see Anderson et al., 1992), and that it is linear in income, y, so that the result in the Lemma is effectively Roy's Identity.

When all of i's variants are priced at the same price, p_i, then V_i reduces to[8]

$$\widehat{V_i} = y - p_i + \sigma_A A(m_i), \tag{7}$$

where

$$A(m_i) = m_i \int_{I_2} x f(x) F^{m_i - 1}(x) dx. \tag{8}$$

Since $A(m_i)$ is the expected value of the maximum of m_i i.i.d. random variables, it is an increasing and strictly concave function. This is because getting more draws raises the expected value of maximum but at

[8] Roy's identity also applies here insofar as it yields the conditional demand as 1, which is just the assumption that each consumer buys one unit.

a decreasing rate.[9] Thus $A'(m_i) > 0$ and $A''(m_i) < 0$. In what follows we shall use a condition on the elasticity of $A'(m_i)$:

ASSUMPTION A: *Marginal intra-nest surplus is inelastic with respect to the product range:*

$$\frac{mA''(m)}{A'(m)} \leq -1.$$

Note that this assumption holds for standard log-concave distributions such as the uniform and power functions, exponential, and the double exponential (in which case the elasticity is -1).

We can now describe the consumer's choice of firm. Like variants, firms are also differentiated. Brand name, firm location, waiting time, and quality of service all contribute to firm differentiation. Consumer utility from choosing Firm i is assumed to be given by

$$u_i = V_i + \sigma_B \varepsilon_i, \qquad i = 1...n, \tag{9}$$

where $\sigma_B \geq 0$ parameterizes the degree of substitutability across firms. We assume that the ε_i are i.i.d. random variables with zero mean and unit variance with twice differentiable density function $g(.)$, which is log-concave over its convex support I_1. Hence,

$$\mathbb{P}_i = \text{Prob}\{u_i \geq u_j,\ j = 1...n\} = \int_{I_1} g(x) \prod_{\substack{j=1 \\ j \neq i}}^{n} G\left(\frac{V_i - V_j}{\sigma_B} + x\right) dx, \tag{10}$$

[9] When the random terms are distributed according to the double exponential (also known as the Gumbel), i.e. $F(x) = \exp[-\exp(-x/\mu_2 - \gamma)]$, where γ is Euler's constant, then $A(m_i) = \ln m_i$, which is clearly increasing and strictly concave in m_i. In this case, the IIA property restricts the scope of the demand model.

where $G(.)$ is the common cumulative distribution of ε_i (cf. (4) and (5)). The special case where the random terms are double exponentially distributed at both levels corresponds to the nested logit model treated by Anderson and de Palma (1992). The framework considered here allows for a broad palette of possible demand patterns at each level.

It is important for what follows in the equilibrium analysis to find the derivative of (10) evaluated where all the V_j's are the same. This is given by

$$\frac{\partial \mathbb{P}_i}{\partial V_i} = \frac{n-1}{\sigma_B} \int_{I_1} g^2(x) G^{n-2}(x)\, dx = \frac{\Omega(n)}{\sigma_B n} \tag{11}$$

where we have thus defined

$$\Omega(n) \equiv n(n-1) \int_{I_1} g^2(e)\, G^{n-2}(e)\, de > 0. \tag{12}$$

We return to this key magnitude in the analysis of equilibrium.

3.1 Properties of the demand system

The nested demand system has some interesting properties that are worth pointing out before we proceed. As expected, the demand addressed to Firm i increases as V_i increases and decreases as V_j increases ($j \neq i$). The demand derivative for Firm i's product k with respect to Firm j's product h (recalling $D_{ik} = N\mathbb{P}_i\mathbb{P}_{k|i}$ and using Lemma 1) is:

$$\frac{\partial D_{ik}}{\partial p_{jh}} = \frac{\partial D_{ik}}{\partial V_j}\frac{\partial V_j}{\partial p_{jh}} = N\frac{\partial \mathbb{P}_i}{\partial V_j}\frac{\partial V_j}{\partial p_{jh}}\mathbb{P}_{k|i} = -N\frac{\partial \mathbb{P}_i}{\partial V_j}\mathbb{P}_{h|j}\mathbb{P}_{k|i} \geq 0, \quad i \neq j. \tag{13}$$

Thus variants produced by different firms are substitutes.

For variants produced by the same firm, we have

$$\frac{\partial D_{ik}}{\partial p_{i\ell}} = N\left[\mathbb{P}_i\frac{\partial \mathbb{P}_{k|i}}{\partial p_{i\ell}} - \frac{\partial \mathbb{P}_i}{\partial V_i}\mathbb{P}_{\ell|i}\mathbb{P}_{k|i}\right], \ \ell \neq k. \tag{14}$$

The first term is non-negative, whereas the second is non-positive.[10] Conditional on choosing Firm i, i's variants are substitutes (first term); however, when p_{il} rises, Firm i becomes less attractive. This decreases total demand for i's variants and hence cuts into product k's demand. If the latter effect outweighs the former, variants sold by Firm i are complements. Otherwise they are substitutes.

Complementarity can arise when the nest effect dominates, meaning that a price rise deteriorates consumers' evaluations of the firm so much as to offset within nest substitution into the other variants. If all goods are substitutes, then McFadden (1981) has shown that under certain regularity conditions (see Anderson et al., 1992, Ch. 3 for details) the demand system can be rationalized by a single-stage discrete choice random utility model with consumer taste heterogeneity described by a distribution of taste parameters across products and individuals. The present approach uses a discrete choice random utility model with two stages, the first for the firm and the second for the particular variant.

Thus, if all variants are always substitutes (i.e., if (14) is always positive), then the demand system does have a standard discrete choice representation.[11] Otherwise it does not. We shall show below that the demand system is consistent with a representative consumer regardless.

[10] The elasticity form of (14) is $\frac{p_{i\ell}}{D_{ik}}\frac{\partial D_{ik}}{\partial p_{i\ell}} = \frac{p_{i\ell}}{\mathbb{P}_{k|i}}\frac{\partial \mathbb{P}_{k|i}}{\partial p_{i\ell}} + \left[\frac{V_i}{\mathbb{P}_i}\frac{\partial \mathbb{P}_i}{\partial V_i}\right]\left[\frac{p_{il}}{V_l}\frac{\partial \mathbb{V}_i}{\partial p_i}\right]$. This shows that variants within the same firm are substitutes if intra-nest elasticity (first term) dominates the inter-nest elasticity.

[11] This condition holds, for example, for the nested logit model when $\sigma_B \geq \sigma_A$.

3.2 Consumer surplus

Just as V_i was interpreted as a conditional benefit function, the expected maximum of the u_i provides a utilitarian measure that we shall use as a consumer welfare measure. The consumer surplus for the population of N consumers is (cf. the construction of (5)):

$$CS = N \int_{I_1} ... \int_{I_1} \max_{i=1...n} [V_i + \sigma_B e_i] \prod_{j=1}^{n} g(e_j) de_1 ... de_n, \qquad (15)$$

with V_i given by (5). We show below how (15) enables us to recover the demand system. Using an argument parallel to that substantiating Lemma 1, we can establish a parallel property:

Lemma 2 *The demand addressed to Firm i is given by:*

$$D_i = N\mathbb{P}_i = \frac{\partial CS}{\partial V_i} \; i = 1...n$$

and the demand for product ik is

$$D_{ik} = -N\mathbb{P}_i\mathbb{P}_{k|i} = -\frac{\partial CS}{\partial p_{ik}}, \; k = 1...m_i, \; i = 1...n.$$

Indeed, the first relation is derived just as before, noting that the derivative of (15) with respect to V_i uncovers the mass of consumers who prefer i to the other nests. The second expression then follows from the chain rule and Lemma 1:

$$\frac{\partial CS}{\partial p_{ik}} = \frac{\partial CS}{\partial V_i}\frac{\partial V_i}{\partial p_{ik}} = -N\mathbb{P}_i\mathbb{P}_{k|i} = -D_{ik}$$

Once more, these demands are consistent with Roy's identity and the reason (as shown below in Proposition 1) is that (15) is a valid indirect utility function.

If Firm i sells all its m_i variants at the same price and if all the firms have the same attractivity (i.e. $\hat{V}_i = V$, $i = 1...n$), then, following the same procedure as we did for V, we have $CS = N[V + \sigma_A B(n)]$ where

$$B(n) = n \int_{I_1} x g(x) G^{n-1}(x) dx, \tag{16}$$

and $B(.)$ is increasing and concave in n (i.e., $B'(n) > 0$ and $B''(n) < 0$). Parallel to Assumption A, we now suppose:

ASSUMPTION B: *Marginal inter-nest surplus is inelastic with respect to the number of nests:*

$$\frac{nB''(n)}{B'(n)} \leq -1.$$

In the symmetric case (same prices and same number of variants per firm) the expression (15) reduces to (cf. the argument preceding (7)):

$$CS = N\left[y - p + \sigma_A A(m) + \sigma_B B(n)\right]. \tag{17}$$

One interpretation of the demand model uses the choice of restaurant meal as an example. The selection of a particular dish at a particular restaurant can be seen as the outcome of a two-stage process. The first stage is the choice of restaurant, and the second is that of a specific dish offered there. The consumer knows that when she gets to the restaurant, she will order the dish that pleases her most (as per (2)). However, before getting there she does not know precisely what is on the menu that day (but she does know her distribution of valuations of dishes). The valuation she attributes to a specific restaurant comprises an individual-specific match component ($\sigma_B \varepsilon_i$ in equation (9)) plus the expected value of choosing the best dish once gets there (V_i).

Another interpretation of the model is to treat the general form of (15) as the indirect utility function of a representative consumer:

Proposition 1 *The demand model (1) with (4) and (10) is consistent with the preferences of a representative consumer whose indirect utility function can be written as (15).*

Proof. We need to show that (15) is an indirect utility function. First note that (from (13)) $\partial D_{ik}/\partial p_{jh} = \partial D_{jh}/\partial p_{ik}$ since $\partial \mathbb{P}_i/\partial V_j = \partial \mathbb{P}_j/\partial V_i$ $(i \neq j)$ from the definition of $\mathbb{P}_i$ and $\mathbb{P}_j$; from (14), $\partial D_{ik}/\partial p_{il} = \partial D_{il}/\partial p_{ik}$ since $\partial \mathbb{P}_{k|i}/\partial p_{ij} = \partial \mathbb{P}_{l|i}/\partial p_{ik}$. Hence the matrix of cross-derivatives is symmetric. This property is equivalent to the symmetry of the Slutsky matrix for the representative consumer. We also require that the indirect utility function be quasi-convex in prices (see also McFadden, 1981). Indeed, here it is convex in prices since the maximum of linear functions is convex. The demand model is therefore consistent with the preferences of a representative consumer whose indirect utility function is given by (15). ■

The representative consumer approach provides an alternative theoretic underpinning to the demand model. Representative consumer models (with different structural assumptions) have been previously used by Spence (1976) and Dixit and Stiglitz (1977) to compare optimum with equilibrium product diversity when firms sell but one product each.

4 Welfare analysis

On the cost side, let $K(m_i) = k_0 + k_1 m_i$ denote the fixed costs of a firm with m_i variants, with k_0 therefore the fixed cost per firm. Average variable production costs for Firm i are constant and given by c per unit. These cost assumptions can correspond to a single production line which must be closed down (to alter specifications) to switch production to a different variant: the more often the line is closed down to switch, the bigger the cost.[12]

The welfare maximand is assumed to be the sum of consumer surplus and firm profits. The social surplus analysis is simplified using prices to decentralize the optimum: clearly marginal cost pricing does the trick. We show in Appendix 1 that optimality requires that each firm produces the same amount of each of its variants and that all product ranges must be the same size. Hence each firm produces the same quantity of each of m products. This renders the welfare function, W, quite simple, as the following result summarizes:

Proposition 2 *The social optimum entails each firm producing the same number of variants, m, and producing an equal quantity, N/mn, of each*

[12] An alternative cost assumption, we can consider Firm i as running m_i different production lines, each with its own fixed and variable costs. The two cost assumptions are formally equivalent when marginal production costs are constant. The model can readily be extended (but with additional notational heaviness) to convex production costs.

variant. The welfare function is

$$W(m,n) = N\left[y + \sigma_A A(m) + \sigma_B B(n)\right] - nK(m) - cN. \tag{18}$$

We can now determine the optimal values of m and n.

4.1 Optimum number of firms and variety

Given Proposition 2, the first-order condition for the optimal choice of m implicitly defines the locus $m^o(n)$ which is the optimal product range for a given number of firms. Thus m^o solves[13] $\partial W(m,n)/\partial m = 0$, or

$$N\sigma_A A'(m^o) - nk_1 = 0. \tag{19}$$

The slope of this locus is

$$\frac{dm^o}{dn} = \frac{k_1}{N\sigma_A A''(m^o)} < 0, \tag{20}$$

which is necessarily negative since $A'' < 0$. The larger the number of firms, the more narrow the desired product range of each one because more firms can substitute for range size.

Likewise, the first-order condition for the optimal choice of n implicitly defines the locus $n^o(m)$ which is the optimal number of firms for a given product range and solves $\partial W(m,n)/\partial n = 0$, or

$$N\sigma_B B'(n^o) - K(m) = 0. \tag{21}$$

[13]Note that this also corresponds to setting $\lambda = 0$ in (41) in Appendix 1 for m_j, since when M is optimally chosen in the maximization problem the marginal social benefit of an extra variant is identically zero.

The corresponding derivative is

$$\frac{dn^o}{dm} = \frac{k_1}{N\sigma_B B''(n^o)} < 0, \tag{22}$$

where the negative slope follows from the concavity of $B(.)$. The solution does not involve either the number of firms nor the product range size tending to infinity since the marginal benefit from each source of diversity goes to zero as n or m get large enough while marginal costs are strictly positive. The solution does not involve either value going to zero as long as the corresponding costs are low enough, which we assume.

The loci $m^o(n)$ and $n^o(m)$ (see equations (19) and (21)) are illustrated in Figure 1. The intersection of the two loci is the social optimum.

Insert here Figure 1: The optimal number of firms and product ranges.

In the Figure, we have drawn the curve $m^o(n)$ as more shallow than $n^o(m)$ around the intersection point. We now argue that this relation must hold under our assumptions. From (20) and (22), this slope condition is

$$\frac{k_1}{N\sigma_A A''(m^o)} > \frac{N\sigma_B B''(n^o)}{k_1}. \tag{23}$$

Now, this is also the condition that the determinant of the matrix of second derivatives of W be strictly negative. Since (19) and (21) are strictly decreasing in m^o and n^o, respectively, the Hessian of W is negative definite if the inequality above holds. Thus, if (23) holds at any intersection of the two loci, then since the loci are continuous functions, we know that they can only intersect once and that this unique intersection point must be a local maximum. The solution does not involve either the number of firms nor the product range size tending to infinity since the marginal benefit from each source of diversity goes to zero as n or m get large enough while marginal costs are strictly positive. The solution does not involve either value going to zero as long as the corresponding costs are low enough, which we assume.

Therefore there is an intersection of the two loci, it is unique, and constitutes a global maximum of $W(m, n)$ if (23) holds there. For (23) to hold at any intersection of the two loci, then it must be that (19) and (21) hold, so that we can use these relations to substitute out the μ's and write the desired condition as

$$\frac{m^o A''(m^o)}{A'(m^o)} < \left(\frac{m^o k_1}{K(m^o)} \frac{B'(n^o)}{n^o B''(n^o)} \right). \tag{24}$$

Defining $\eta_{A'}$ as the (absolute value of the) elasticity of A' and similarly for $\eta_{B'}$ and $\eta_K = \frac{k_1 m}{K(m)} < 1$, we can rewrite this inequality as:

$$\eta_{A'} > \frac{\eta_K}{\eta_{B'}}.$$

Assumptions A and B imply $\eta_{A'} \geq 1$ and $\eta_{B'} \geq 1$. The inequality then must hold since $\eta_K < 1$ (marginal cost for increasing the product range is lower than average cost).

Hence, (19) and (21) characterize the unique global maximum of (18), and via Proposition 2, of the social welfare. To summarize:

Proposition 3 *Under Assumptions A and B, the social optimum number of firms and the optimum variety are the unique positive solution of*

(19) and (21).

When σ_A rises, the $m^o(n)$ locus shifts up in Figure 1 while the $n^o(m)$ locus remains unchanged. Thus a greater preference for variety within the firm leads to larger product ranges which leads to fewer firms since the two dimensions of diversity are substitutes. Conversely, the case of single product firms arises for σ_A is small enough. A similar analysis implies that the optimal number of firms decreases as σ_B decreases but that range size rises. For σ_B low enough there is optimally a single firm on the market.

The comparative static properties with respect to market size, N, and cost parameters, also involve simple shifts of the loci in Figure 1. They are quite intuitive and are left to the reader.

5 Market equilibrium

We are interested in characterizing the symmetric equilibrium at which n^e firms each produce m^e products.[14] We proceed in two steps. First we consider the symmetric equilibrium choice of product ranges for a given number of firms, $m^e(n)$. Then we discuss the equilibrium number of firms, as determined by the zero profit condition, when all firms have the same size of product range. This gives the $n^e(m)$ locus. Throughout we ignore the integer constraint and treat both n and m as continuous

[14] We shall not be concerned here about showing that such an equilibrium exists, although we note that existence and symmetry was proved for the special case treated in Anderson and de Palma (1992), so we are not dealing with a vacuous problem.

variables (as in the previous section). The intersection of the $n^e(m)$ and $m^e(n)$ loci gives the equilibrium.

5.1 Equilibrium price

The equilibrium is that of a three-stage game. In the first stage, firms enter the market. In the second stage they choose product ranges, and in the third stage they choose prices, which are the same for all the products of any firm.[15] At each stage they internalize the effects of their decisions on the subsequent sub-game equilibria. In the last (price) stage, if all firms produce the same number of variants, m, and all other firms charge the same price for all their variants, then the inter-firm choice probabilities are independent of m, so that profit is

$$\pi_i = N\,(p_i - c)\,\mathbb{P}_i(\hat{V}_i, \hat{V}_{-i}) - K(m), \tag{25}$$

where the second argument in Firm i's choice probability function, $\hat{V}_{-i}$, denotes the vector of all other expected surpluses, given that each firm charges the same price for all its variants.

The candidate symmetric equilibrium price satisfies:

$$\left.\frac{\partial \pi_i}{\partial p_i}\right|_{Sym} = N\,(p_i - c)\,\frac{\partial \mathbb{P}_i(\hat{V}_i, \hat{V}_{-i})}{\partial \widehat{V}_i}\left.\frac{\partial \widehat{V}_i}{\partial p_i}\right|_{Sym} + \frac{N}{n} = 0.$$

Using (11) and recalling $\partial \widehat{V}_i\,/\partial p_i = -1$, the equilibrium price is given explicitly by:

$$p\,(n) = c + \frac{\sigma_B}{\Omega(n)}, \tag{26}$$

[15] It can be readily shown that each firm optimally sets the same price for each of its variants. This property follows from maximizing profit within the nest, subject to the constraint of providing a given expected surplus level, $\hat{V}_i$.

where $\Omega(n)$ is defined in (12).

Note that the equilibrium price (26) is independent of m since $A(m_i)$ is the same for all firms. This is because the product range effect cancels out in a cross-firm comparison of attractiveness. The equilibrium price is a simple mark-up that depends only on the degree of firm heterogeneity and the number of firms. Since $\Omega(n)$ is increasing under log-concavity of $g(.)$ (see Anderson et al., 1995), the price of each firm's product range falls the more competing firms there are.

The quantities $\Omega(n)$ and $B'(n)$ depend on the density function $g(.)$ and satisfy the following property.

Lemma 3 *(Anderson, de Palma, and Nesterov, 1995). If the density function $g(.)$ is log-concave, then $n\Omega(n)B'(n) < 1$.*

Anderson, de Palma, and Nesterov (1995) actually show that[16]

$$B(n) - B(n-1) = \int_{I_1} [1 - G(\varepsilon)] G^{n-1}(\varepsilon) \, d\varepsilon \leq \frac{1}{n\Omega(n)}.$$

Since $B(.)$ is strictly concave, the left-hand-side exceeds $B'(n)$ and so the inequality given in the Lemma above follows immediately.

5.2 Equilibrium versus optimum varieties

We first determine the equilibrium product range, for n fixed. Then, we consider the free entry equilibrium. In the product range stage, suppose

[16] This version would enable us to explicitly consider the issue that the number of firms should be an integer. The product line analysis is rather more cumbersome with explicit integer constraints though.

firm i produces m_i variants while all other produce $\overline{m}$ variants each. Firm i's profit is then

$$\pi_i = N\left(p_i - c\right)\mathbb{P}_i(m_i, \bar{m}; p_i, \bar{p}) - K(m_i), \tag{27}$$

where $K\left(m\right) = k_0 + k_1 m$. Taking the derivative with respect to m_i we have

$$\frac{d\pi_i}{dm_i} = N\left(p_i - c\right)\left(\frac{\partial \mathbb{P}_i}{\partial m_i} + \frac{\partial \mathbb{P}_i}{\partial \bar{p}}\frac{d\bar{p}}{dm_i}\right) - k_1, \tag{28}$$

where $d\bar{p}/dm_i$ denotes the change in the equilibrium price set by all other firms as Firm i increases its product range. Now, noting that $\frac{\partial \mathbb{P}_i}{\partial m_i} = -\sigma_A A'(m_i)\frac{\partial \mathbb{P}_i}{\partial p_i}$, and that $\frac{\partial \mathbb{P}_i}{\partial \bar{p}} = -\frac{\partial \mathbb{P}_i}{\partial p_i}$, so that:

$$\left(\frac{\partial \mathbb{P}_i}{\partial m_i} + \frac{\partial \mathbb{P}_i}{\partial \bar{p}}\frac{d\bar{p}}{dm_i}\right) = -\frac{\partial \mathbb{P}_i}{\partial p_i}\left(\sigma_A A'(m_i) + \frac{d\bar{p}}{dm_i}\right).$$

Using $\left(p_i - c\right)\frac{\partial \mathbb{P}_i}{\partial p_i} + \mathbb{P}_i = 0$ (i.e. the first-order condition for pricing) and evaluating equation (28) at a symmetric equilibrium for the equilibrium choice of variety, denoted by m^e yields:

$$N\sigma_A A'(m^e) + N\frac{d\bar{p}}{dm_i} - nk_1 = 0. \tag{29}$$

This equation characterizes the $m^e\left(n\right)$ locus. In comparison with equation (19), the only difference between the equilibrium and the optimum is the term $Nd\overline{p}/dm_i$ which can be interpreted as a strategic effect on equilibrium prices. It is shown in Appendix 2 that this term is negative.

Lemma 4. *Rivals' equilibrium prices fall as Firm i boosts its product range: $d\overline{p}/dm_i < 0$.*

The equilibrium product range for a fixed number of firms, $m^e(n)$, is therefore smaller than the optimum one. In terms of Figure 1, the $m^e(n)$ locus is below the $m^o(n)$ locus. As seen from the analysis above, the difference is completely attributable to a strategic effect that competing firms internalize. Adding a variant leads to more intense competition and lower prices of rivals' variants. At the margin, firms avoid too much provocation by holding back on their product ranges.

Now consider the equilibrium number of firms for given symmetric product ranges, m. This is determined by the zero profit condition:

$$\pi = \frac{N}{n}\left(p(n) - c\right) - K(m) = 0,$$

Using (26), the equilibrium number of firms, n^e, of firms satisfies:

$$\frac{N\sigma_B}{n^e\Omega(n^e)} - K(m) = 0. \tag{30}$$

This equation characterizes the $n^e\left(m\right)$ locus and is directly comparable with (21) for the optimal number of firms. The $n^e(m)$ locus lies right of the $n^o(m)$ locus if $n\Omega(n)B'(n) < 1$. This is precisely the condition in Lemma 3.

Insert here Figure 2: Equilibrium and optimum product variety

The consequent results are illustrated in Figure 2, where we see that the optimum number of firms is smaller than the equilibrium number and the optimum product variety is larger than the equilibrium one. Anderson, de Palma, and Nesterov (1995) established over-entry of single-product firms: the special assumptions made here allow us to establish this property more broadly.[17] We summarize our results in

Proposition 4 *Given unit demand by consumers, the market equilibrium involves too many firms and too few products per firm with respect to the optimum.*

Since firms hold back on product ranges to lessen price competition, prices stay excessively high so that profits exceed the social value of a firm and too many firms enter the market. In the next section we relax the assumption of unit demand.

6 Variable consumption

The analysis so far has treated unit demand by consumers insofar as each consumer has been assumed to buy one unit of the preferred good independently of the price level. In this section, we broaden the vista to allow the quantity demanded to depend in a decreasing fashion on price. We retain the discrete choice assumption at the level of choice of good to buy, but we allow the quantity of that good bought to decrease with price. We make extensive use of Roy's identity in the demand relations. Our extension allows us to pick up the classic case of CES preferences here extended to the nested CES.

[17] In the earlier analysis, the equilibrium and optimum coincide only if the taste density is log-linear. Here, even if this condition holds for $g(.)$ so that the $n^o(m)$ locus is coincident with the $n^e(m)$ locus, the divergence of the other loci suffices to encourage strict over-entry.

The basic demand structure is as above except that we write demand as

$$D_{ik} = Nq\left(p_{ik}\right)\mathbb{P}_i\mathbb{P}_{k|i},$$

where the function $q\left(.\right)$ is to be interpreted as a conditional demand function (conditional on choosing product ik) and the probability components are much as before.

The extension works as follows. Let the conditional (indirect) utility of consumer buying variant (ik) be $u_{ik} = y + v\left(p_{ik}\right) + \sigma_A\varepsilon_{ik}$, where $v\left(p_{ik}\right)$ is the conditional surplus function. This surplus function is increasing and convex. Applying Roy's identity yields the conditional demand as $q\left(p_{ik}\right) = -v'\left(p_{ik}\right)$.[18] Given that the consumer who selects Firm i will choose the variant ik that maximizes u_{ik}, the conditional probability of choosing good ik in nest i when all intra-nest prices are equal to p_i is just $\mathbb{P}_{k|i} = 1\,/m_i$ and the expected demand for the variants sold by Firm i is just $Nq\left(p_i\right)\mathbb{P}_i$. Here, $\mathbb{P}_i$ is determined by the attractivity of the various nests, so that $\mathbb{P}_i = \text{Prob}\{V_i + \sigma_B\varepsilon_i \geq V_j + \sigma_B\varepsilon_j,\ j = 1...n\}$, as before, or:

$$\mathbb{P}_i = \int_{I_1} g(x)\prod_{j\neq i} G\left(\frac{V_i - V_j}{\sigma_B} + x\right)dx,\ i = 1...n,$$

where $V_j = y + v\left(p_j\right) + \sigma_A A\left(m_j\right)$, when Firm j set the same price p_j for all of its variants. We return to these expressions below when we find

[18] In the analysis up to here we have assumed effectively that $v\left(p_{ik}\right) = -p_{ik}$; applying Roy's identity yields the conditional demand as unity, which is consistent with the unit demand assumption.

the market equilibrium.

Proposition 5 *The nested demand model with variable consumption is consistent with the preferences of a representative consumer whose indirect utility function is given by:*

$$CS = N \int_{I_1} \dots \int_{I_1} \max_{i=1\dots n} \left[V_i + \sigma_B e_i\right] \prod_{j=1}^{n} g(e_j) de_1 \dots de_n,$$

where

$$V_i = y + \int_{I_2} \dots \int_{I_2} \max_{k=1\dots m_i} \left(v\left(p_{ik}\right) + \sigma_A e_k\right) \prod_{l=1}^{m_i} f(e_l) de_1 \dots de_{m_i}.$$

Proof. Following the lines used in the proof of Proposition 1, we need to show that the matrix of cross-derivatives is symmetric, and that the indirect utility function is quasi-convex in prices. The first property follows since $\partial CS/\partial p_{jh} = N\mathbb{P}_j \partial V_j/\partial p_{jh}$ and $\partial V_j/\partial p_{jh} = -q\left(p_{jh}\right)\mathbb{P}_{h|j}$, so $\partial CS/\partial p_{jh} = -D_{jh}$. (Indeed, the cross-derivative is $\partial^2 CS/\partial p_{jh}\partial p_{ik} = q\left(p_{jh}\right) q\left(p_{ik}\right) \mathbb{P}_{h|j}\mathbb{P}_{k|i}\partial\mathbb{P}_i/\partial V_j$, from which symmetry is apparent since in discrete choice models $\partial\mathbb{P}_i/\partial V_j = \partial\mathbb{P}_j/\partial V_i$). The second argument follows since $v\left(.\right)$ is convex and therefore $V\left(.\right)$ is convex in prices (this is a property of the maximum operator). Moreover, the function $CS\left(.\right)$ is then convex in prices for the same reason. ■

We now find the optimum allocation. Under symmetry, all of the $V's$ are equal and the social surplus is given by

$$W = N\left[y + v\left(p\right) + \sigma_B B(n) + \sigma_A A(m)\right] + n\pi. \tag{31}$$

We look for a second-best optimum such that firms are constrained to make zero profits. This means that aggregate net revenues minus the total set-up cost is zero or

$$n\pi = N\left(p-c\right)q\left(p\right) - nK(m) = 0. \tag{32}$$

The corresponding Lagrangian $\mathcal{L}\left(m,n,p,\lambda\right)$ is:

$$\mathcal{L}{=}N\left[y+v\left(p\right)+\sigma_A A(m)+\sigma_B B(n)\right]{+}(1+\lambda)\left[N\left(p-c\right)q\left(p\right)-nK(m)\right],$$

where λ denotes the Lagrangian multiplier associated to the aggregate zero-profit constraint. The first-order condition for the locus $m^o\left(n\right)$ is given by $\partial\mathcal{L}\,/\partial m = 0$, or:

$$N\sigma_A A'\left(m^o\right) = \left(1+\lambda\right)nk_1. \tag{33}$$

The locus $n^o\left(m\right)$ is given by $\partial\mathcal{L}\,/\partial n = 0$, or

$$N\sigma_B B'(n^o) = \left(1+\lambda\right)K(m). \tag{34}$$

The pricing condition is given by $\partial\mathcal{L}\,/\partial p = 0$, or, recalling $q\left(p_{ik}\right) = -v'\left(p_{ik}\right)$,

$$\left(1+\lambda\right) = \frac{q\left(p\right)}{q\left(p\right)+\left(p-c\right)q'\left(p\right)}, \tag{35}$$

and the final first order condition is (32).

We now derive the analogous conditions for the equilibrium. The profit of Firm i is

$$\pi_i = N\left(p_i-c\right)q\left(p_i\right)\mathbb{P}_i - K\left(m_i\right).$$

The optimality condition for the number of products offered by Firm i is:

$$\frac{d\pi_i}{dm_i} = N\left(p_i-c\right)q\left(p_i\right)\left[\frac{\partial\mathbb{P}_i}{\partial m_i}+\frac{\partial\mathbb{P}_i}{\partial\overline{V}}\frac{\partial\overline{V}}{\partial m_i}\right]-k_1,$$

where $\overline{V}$ denotes the common attractivity of each other firm. Note that $\overline{V}$ incorporates the sub-game equilibrium prices ensuing from the product range game. Using an argument analogous to that in Appendix 2, $\frac{\partial \overline{V}}{\partial m_i}$ is positive: rival firms decrease their equilibrium prices (as so raise their attractivities) when Firm i increases its product range. Note too that the expression for $\frac{d\pi_i}{dm_i}$ also uses the envelope theorem in the fact that p_i is optimally chosen by Firm i in the pricing sub-game. Now, $\frac{\partial \mathbb{P}_i}{\partial m_i}$ may be decomposed as $\frac{\partial \mathbb{P}_i}{\partial m_i} = \frac{\partial \mathbb{P}_i}{\partial V_i}\frac{\partial V_i}{\partial m_i}$, while $\frac{\partial \mathbb{P}_i}{\partial \overline{V}} = -\frac{\partial \mathbb{P}_i}{\partial V_i}$. Substituting, we get:

$$\frac{d\pi_i}{dm_i} = N\,\Psi\left[\frac{\partial V_i}{\partial m_i} - \frac{\partial \overline{V}}{\partial m_i}\right] - k_1, \tag{36}$$

where $\Psi = (p_i - c)\,q\,(p_i)\,\frac{\partial \mathbb{P}_i}{\partial V_i}$.

We can use the first-order condition for the choice of p_i to rewrite Ψ. This pricing first-order condition ($\frac{d\pi_i}{dp_i} = 0$) is:

$$q\,(p_i)\,\mathbb{P}_i + (p_i - c)\,q'\,(p_i)\,\mathbb{P}_i + \Psi\,\frac{\partial V_i}{\partial p_i} = 0. \tag{37}$$

Substituting Roy's identity ($\frac{\partial V_i}{\partial p_i}/\frac{\partial V_i}{\partial y} = v'\,(p_i) = -q\,(p_i)$), we get:

$$\Psi = \frac{q\,(p_i) + (p_i - c)\,q'\,(p_i)}{q\,(p_i)}\mathbb{P}_i. \tag{38}$$

At a symmetric equilibrium, $\mathbb{P}_i = 1/n$, and noting that $\frac{\partial V_i}{\partial m_i} = \sigma_A A'\,(m)$, we get:

$$\frac{d\pi_i}{dm_i}|_{sym} = N\frac{q\,(p) + (p - c)\,q'\,(p)}{q\,(p)}\frac{1}{n}\left[\sigma_A A'\,(m) - \frac{\partial \overline{V}}{\partial m_i}\right] - k_1 = 0$$

or

$$N\left[\sigma_A A'(m) - \frac{\partial \overline{V}}{\partial m_i}\right] = \frac{q(p)}{q(p) + (p-c)\,q'(p)} n k_1. \tag{39}$$

Comparing this expression with the relation for the optimum, (33) with (35), for the same values of n and p, the value of m solving this expression is lower, so that $m^e(n) < m^o(n)$.

Similarly, the free entry condition is: $\pi = N(p-c)\,q(p)/n - K(m) = 0$. Recall that $\Psi = (p_i - c)\,q(p_i)\frac{\partial \mathbb{P}_i}{\partial V_i}$ and that $\frac{\partial \mathbb{P}_i}{\partial V_i} = \frac{\Omega(n)}{\sigma_B n}$ (by (11)) so that this zero profit condition becomes $N\Psi\sigma_B/\Omega(n) = K(m)$. Now from (38) we can write the equilibrium condition as:

$$\frac{N\sigma_B}{n\Omega(n)} = K(m)\frac{q(p)}{q(p) + (p-c)\,q'(p)}. \tag{40}$$

From (35), the LHS is simply $K(m)(1+\lambda)$ when the price is the same as at the optimum (i.e., when the zero-profit constraint holds). Comparing then (34) with (40) and using Lemma 3 ($n\Omega(n)\,B'(n) < 1$) shows that for the same values of m and p, the value of n solving (40) is higher. This means that $n^e(m) > n^o(m)$.

In summary, both relations hold just as in Figure 2 for the extension to variable (price-sensitive) individual demand. This implies that the conclusion of the previous section applies to this case, with the qualification that the welfare benchmark is the second best subject to a zero-profit constraint. In summary:

Proposition 6 *The market equilibrium involves too many firms and too few products per firm with respect to the zero-profit constrained second-best social optimum.*

7 Conclusions

We have emphasized in this paper that there is a systematic market bias towards over-entry of firms and too narrow product lines. The latter effect provokes and attenuates the former: because product line competition is strategically restricted to moderate price competition, profits are kept higher than is optimal. This in turn encourages and exacerbates the excess entry that is the hallmark of models on optimal and market variety for single product firms.

Our analysis follows the Chamberlin (1933) tradition in its interest in comparing equilibrium and optimal diversity, but there is another parallel that bears developing. Chamberlin looked at single-product firms and assumed a production cost structure that is familiar in standard perfectly competitive analysis, a U-shaped average cost function. He noted that his "tangency condition" of the perceived demand (dd) with average production cost implied that production is below minimum efficient scale, namely the "excess capacity" theorem. He then noted that this configuration may be close to the optimum because a preference for product variety implies that production efficiencies ought not be exhausted. Instead, production at a lower scale enables more varieties to be produced, albeit at a higher price per unit bought. We have concentrated on the product range of multiproduct firms, but in the text have assumed that production costs are constant as a function of both output per variety and the number of varieties. The more interesting of the two generalizations is to allow the cost function for varieties to be U-shaped as a function of m_i.

That is, suppose now that $K(m)/m$ has the classic U shape as a function of m (with $K'(m)$ passing through its minimum).[19] Notice first that the (zero profit constrained) optimum solution has the range size below the minimum average cost if consumers value products produced by different firms more than an extension in the range of a given firm at the margin.[20] The equilibrium relation then looks similar to a Chamberlinian tangency, although his demand curve is replaced by an average revenue curve per product. This slopes down because of the cannibalization effect and the property that a larger range toughens the competition. This tangency equilibrium is at a lower range level than the optimal range by the result we have emphasized that firms' keep their ranges too narrow.

Our equilibrium analysis also yields some predictions for empirical regularities. For example, larger markets (higher N) typically attract more firms in standard models of product differentiation (and in actual markets, comparing across cities or countries). This source of higher product diversity underscores a key source of gains from trade in the context of globalization. The endogenous product ranges in the current

[19] The elasticity form of the optimality condition corrsponding to (24) is now $\eta_{A'} + \eta_{K'} > \frac{\eta_K}{\eta_{B'}}$, where $\eta_{K'}$ is the elasticity of $K'(m)$.

[20] To see this, suppose that product ranges were above the minimum efficient scale. Then reducing product ranges and creating new firms at the same time (in order to keep the total number of products constant) would raise consumer benefits from variety. At the same time this would reduce average production costs per variety, so there is a distinct gain in shifting.

analysis provide a further source of potential gains from market expansion. Larger markets provide the incentive for firms to bring in broader product ranges (for given firm numbers) since the fixed costs of bringing in more products is spread over a broader consumer base. Larger markets also lead more firms to enter, for any given product range size. In terms of Figure 2, both curves shift out with N. Thus one would expect both wider product ranges and more firms in larger markets, so two types of increased variety.

Finally, the over-entry result bears comment. Our solution concept uses free-entry equilibrium with many firms driving profit to zero. In markets that are small relative to costs of firm and product introduction, there is room for more complex strategic behavior with respect to entry deterrence. In particular, it was noted in the text that broader product ranges give rise to more intense competition. For entry deterrence, this is a good thing (see also Schmalensee, 1979). Indeed, insofar as one might then expect fewer firms, and more products per firm than our current solution, this type of deterrence equilibrium may be closer to the social optimum than the free entry equilibrium we consider. The deterrence solution remains an open research question.

Appendix 1

Proof of Proposition 1. Assume that Firm i has product range m_i. Optimality requires that it charges the same price, denoted p_i, for all its variants. Under symmetry, V_i reduces to $\widehat{V}_i = y - p_i + \sigma_A A(m_i)$ (see (7)).

Using (15) we can write consumer surplus as $CS = CS\left[\widehat{V}_1 ... \widehat{V}_n\right]$ and we recall from Lemma 2 that $\partial CS / \partial \widehat{V}_i = -\partial CS / \partial p_i = D_i$. Suppose the total number of variants is fixed at $M = \sum_{i=1}^{n} m_i$. The choice of the number of variants per firm is given by the solution to the following Lagrangian:

$$\max_{\{m_1...m_n\}} CS\left[\widehat{V}_1 ... \widehat{V}_n\right] + \sum_{i=1}^{n} (p_i - c) D_i$$
$$- \sum_{i=1}^{n} K(m_i) + \mu \left[M - \sum_{i=1}^{n} m_i]\right]$$

Note first that the optimal choice of prices requires

$$\sum_{i=1}^{n} (p_i - c) \frac{\partial D_i}{\partial p_j} = 0, \; j = 1..n.$$

This is clearly satisfied by marginal cost pricing.[21] Now note that

$$\frac{\partial CS\left[\widehat{V}_1 ... \widehat{V}_n\right]}{\partial m_j} = \frac{\partial CS}{\partial \widehat{V}_j} \sigma_A A'(m_j) = \sigma_A A'(m_j) D_j.$$

Given that prices are optimally chosen, and treating the m_i as perfectly divisible, the first-order conditions to the maximization problem yield

$$\sigma_A A'(m_j) D_j + \sum_{i=1}^{n} (p_i - c) \frac{\partial D_i}{\partial m_j} - k_1 = \mu. \quad (41)$$

[21] It is also satisfied by choosing identical markups over marginal cost, since $\sum_{i=1}^{n} \partial D_i / \partial p_j = 0$, for all $j = 1...n$.

Since mark-ups are identical, the middle term on the LHS is zero, and thus $\sigma_A A'(m_j) D_j - k_1 = \mu$, $j = 1...n$. This implies that $m_j = m_i = m, i, j = 1...n$, since $A(.)$ is concave and D_j is increasing in m_j. Q.E.D.

Appendix 2

Proof of Lemma 4.

We show here that $d\bar{p}/dm_i < 0$, i.e. that competitors decrease their prices $\bar{p}$ as a deviant firm (Firm i) increases its product range, m_i. The first-order conditions defining the price sub-game are

$$(p_j - c)\frac{\partial \mathbb{P}_i}{\partial p_j} + \mathbb{P}_j = 0, \; j = 1...n. \tag{A1}$$

For the deviant firm we have

$$\mathbb{P}_i = \int_{I_1} f(x) F^{n-1}(\alpha + x) dx, \tag{A2}$$

where $\alpha \equiv [A(m_i) - A(\bar{m}) + \bar{p} - p_i]/\sigma_A$, the relative attractiveness of firm i. We henceforth set $\sigma_A = 1$ to ease clutter. Note also that

$$\frac{\partial \mathbb{P}_i}{\partial p_i} = -(n-1)\int_{I_1} f(x) f(\alpha + x) F^{n-2}(\alpha + x) dx. \tag{A3}$$

For the other firms, we must evaluate $\mathbb{P}_j$ and $\partial \mathbb{P}_j/\partial p_j$ at a symmetric common price, $\bar{p}$, so

$$\mathbb{P}_j = \overline{\mathbb{P}} = \int_{I_1} f(x) F^{n-2}(x) F(-\alpha + x) dx \tag{A4}$$

and

$$\frac{\partial \mathbb{P}_j}{\partial p_j}(\bar{p}) = -\int_{I_1} (n-2) f^2(x) F^{n-3}(x) F(x-\alpha) + f(x) F^{n-2}(x) f(x-\alpha) ds \tag{A5}$$

Note this is *not* the derivative of (A4) since $(p_i, \bar{p})$ should be the Nash equilibrium price sub-game stemming from $(m_i, \bar{m})$.

To find $d\bar{p}/dm_i$, we totally differentiate the two types of (A1) - for firm i and for a representative firm $k \neq i$. Define

$$h(p_i, \bar{p}, m_i) = (p_i - c)\frac{\partial \mathbb{P}_i}{\partial p_i} + \mathbb{P}_i = 0 \tag{A6}$$

and

$$g(p_i, \bar{p}, m_i) = (\bar{p} - c)\frac{\partial \mathbb{P}_k(\bar{p})}{\partial p_k} + \overline{\mathbb{P}} = 0, \tag{A7}$$

where all arguments are then to be evaluated at a symmetric solution, $m_i = \bar{m}$ and $p_i = \bar{p}$. From (A6) and (A7) we have

$$\frac{d\bar{p}}{dm_i} = \frac{\frac{\partial g}{\partial p_i}\frac{\partial h}{\partial m_i} - \frac{\partial g}{dm_i}\frac{\partial h}{\partial p_i}}{\frac{\partial h}{\partial p_i}\frac{\partial g}{\partial \bar{p}} - \frac{\partial g}{\partial p_i}\frac{\partial h}{\partial \bar{p}}}. \tag{A8}$$

The denominator is the product of own effects minus the product of cross effects, which we assume positive corresponding to the standard stability condition. Now, $\frac{\partial g}{\partial p_i} = -\frac{\partial g}{\partial \alpha}$ and $\frac{\partial g}{\partial m_i} = A'(m_i)\frac{\partial g}{\partial \alpha}$, so we wish to show that $\frac{\partial g}{\partial \alpha}(-A'(m_i)\frac{\partial h}{\partial p_i} - \frac{\partial h}{\partial m_i}) < 0$. From (A6), the term in brackets is simply $-A'(m_i)\frac{\partial \mathbb{P}_i}{\partial p_i} > 0$, so it suffices to show that $\frac{\partial g}{\partial \alpha} < 0$. From (A7) we have

$$\frac{\partial g}{\partial \alpha} = (\bar{p} - c)\frac{\partial \left(\frac{\partial \mathbb{P}_k(\bar{p})}{\partial p_k}\right)}{\partial \alpha} + \frac{\partial \overline{\mathbb{P}}}{\partial \alpha}. \tag{A9}$$

We can use the first order condition (A7) to simplify the remaining terms so that it suffices to show that

$$-\frac{\overline{\mathbb{P}}}{\frac{\partial \mathbb{P}_k(\bar{p})}{\partial p_k}} \frac{\partial\left(\frac{\partial \mathbb{P}_k(\bar{p})}{\partial p_k}\right)}{\partial \alpha}+\frac{\partial \overline{\mathbb{P}}}{\partial \alpha}<0. \tag{A10}$$

Now, evaluated at a symmetric equilibrium, ($p_i = \bar{p}$, $m_i = \bar{m}$), $\overline{\mathbb{P}} = 1/n$ and (see (A4) and (A5))

$$\frac{\partial \mathbb{P}_k(\bar{p})}{\partial p_k}=\frac{1}{(n-1)} \frac{\partial \overline{\mathbb{P}}}{\partial \alpha}=-(n-1) \int_{I_1} f^2(x) F^{n-2}(x) d x$$

Furthermore, from (A5) we have

$$\left.\frac{\partial\left(\frac{\partial \mathbb{P}_k(\bar{p})}{\partial p_k}\right)}{\partial \alpha}\right|_{\alpha=0}=\int_{I_1}(n-2) f^3(x) F^{n-3}+f^{\prime}(x) f(x) F^{n-2}(x) d x,$$

so (A10) becomes

$$\begin{gathered}\int_{I_1}\left[(n-2) f^3(x) F^{n-3}(x)+f^{\prime}(x) f(x) F^{n-2}(x)\right] d x< \\ n(n-1)\left[\int_{I_1} f^2(x) F^{n-2}(x) d x\right]^2\end{gathered} \tag{A11}$$

To prove (A11), recall that log-concavity of $f(\cdot)$ implies that $\mathbb{P}_i$ is log-concave. The latter condition implies that $\left[\partial \mathbb{P}_i / \partial \alpha\right] / \mathbb{P}_i$ is decreasing in α, or, using (A2) and (A3), this implies that the expression

$$\frac{(n-1) \int_{I_1} f(x) f(\alpha+x) F^{n-2}(\alpha+x) d x}{\int_{I_1} f(x) F^{n-1}(\alpha+x) d x}$$

is a decreasing function of α. Evaluating the derivative at $\alpha = 0$ means that

$$\frac{(n-1)}{n} \int_{I_1} \left[(n-2) f^3(x) F^{n-3}(x) + f'(x) f(x) F^{n-2}(x)\right] dx$$

$$-(n-1)^2 \left[\int_{I_1} f^2(x) F^{n-2}(x) dx\right] < 0.$$

This condition is equivalent to (A11). Q.E.D.

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Figure 1: The optimal number of firms and product ranges

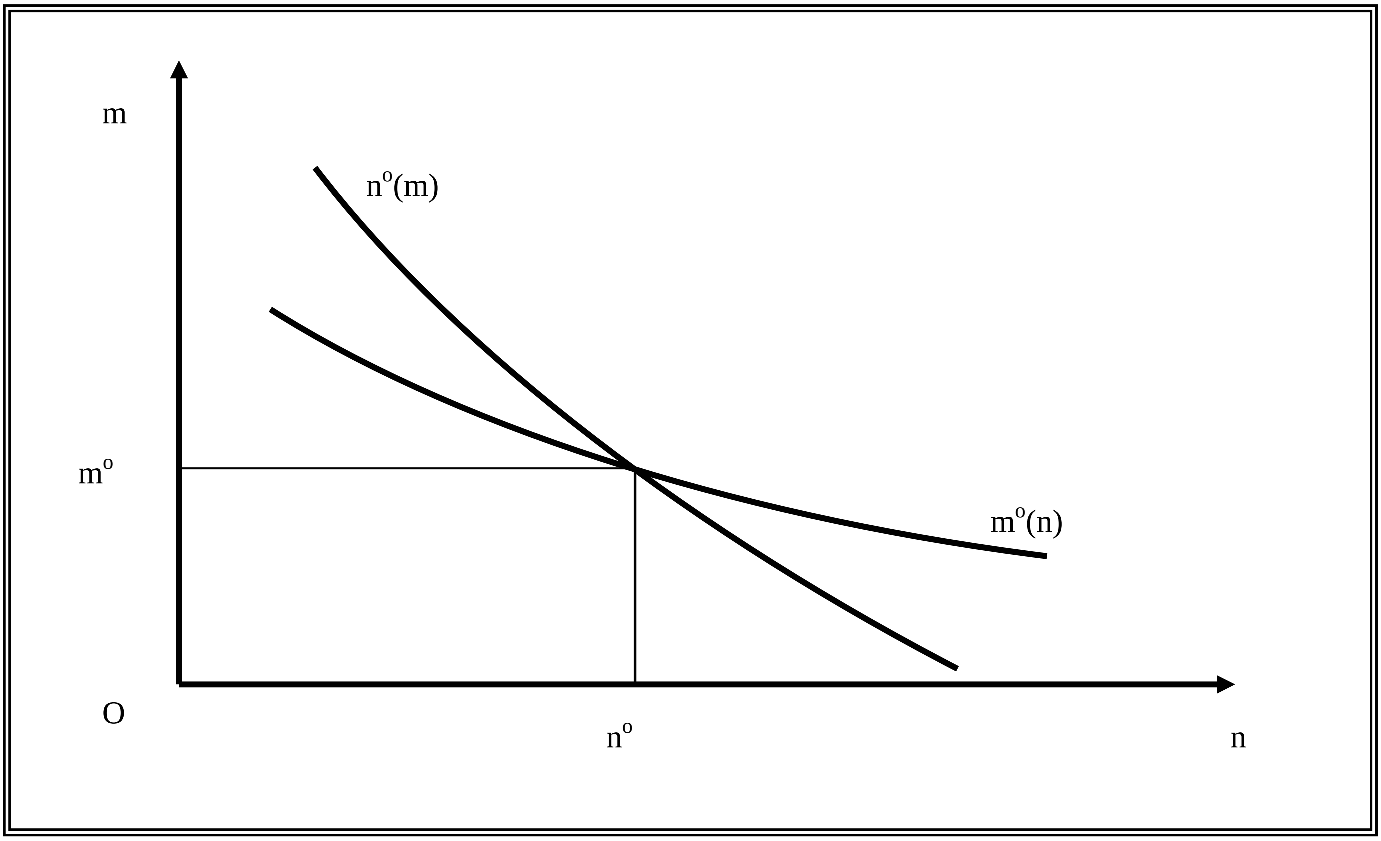

Figure 2: Equilibrium and optimum variety