Ten Little Treasures of Game Theory and Ten Intuitive Contradictions

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ABSTRACT

This paper reports laboratory data for a series of two-person games that are played only once. These games span the standard categories: static and dynamic games with complete and incomplete information. For each game, the treasure is a treatment for which behavior conforms quite nicely to the predictions of the Nash equilibrium or relevant refinement. In each case we change a key payoff parameter in a manner that does not alter the equilibrium predictions, but this theoretically neutral payoff change has a major (often dramatic) effect on observed behavior. These contradictions are generally consistent with simple economic intuition and with a model of iterated noisy introspection for one-shot games.

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I. INTRODUCTION

The Nash equilibrium has been the centerpiece of game theory since its introduction about fifty years ago. Along with supply and demand, the Nash equilibrium is one of the most commonly used theoretical constructs in economics, and it is increasingly being applied in other social sciences. Indeed, game theory has finally gained the central role envisioned by von Neumann and Morgenstern, and in some areas of economics (e.g., industrial organization) virtually all recent theoretical developments are applications of game theory. The impression one gets from recent surveys and game theory textbooks is that the field has reached a comfortable maturity, with neat classifications of games and successively stronger (more "refined") versions of the basic approach being appropriate for more complex categories of games: Nash equilibrium for static games with complete information, Bayesian Nash for static games with incomplete information, subgame perfectness for dynamic games with complete information, and some refinement of the sequential Nash equilibrium for dynamic games with incomplete information (e.g. Gibbons, 1997). The rationality assumptions that underlie this analysis are often preceded by persuasive adjectives like "perfect," "intuitive," and "divine." If any noise in decision making is admitted, it is eliminated in the limit in a process of "purification." It is hard not to notice parallels with theology, and the highly mathematical nature of the developments makes this work about as inaccessible to mainstream economists as medieval treatises on theology would have been to the general public.

The discordant note in this view of game theory has come primarily from laboratory experiments, but the prevailing opinion among game theorists seems to be that behavior will eventually converge to Nash predictions under the right conditions.[1] This paper presents a much more unsettled perspective of the current state of game theory. In each of the major types of

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[1] For example, Mailath's (1998) survey of evolutionary models cites the failure of backward induction as the main cause of behavioral deviations from Nash predictions.

games, we present one or more examples for which the relevant version of the Nash equilibrium predicts remarkably well. These "treasures" are observed in games played only once by financially motivated subjects who have had prior experience in other, similar, strategic situations. In each of these games, however, we show that a change in the payoff structure can produce a large inconsistency between theoretical prediction(s) and human behavior. For example, a payoff change that does not alter the unique Nash equilibrium may move the data to the opposite side of the range of feasible decisions. Alternatively, a payoff change may cause a major change in the game-theoretic predictions and have no noticeable effect on actual behavior. The observed contradictions are typically quite intuitive, even though they are not explained by standard theory. In a simultaneous effort-choice coordination game, for example, an increase in the cost of players' "effort" decisions is shown to cause a dramatic decrease in effort, despite the fact that *any* common effort is a Nash equilibrium for a range of effort costs. In some of these games, it seems like the Nash equilibrium works only by coincidence, e.g. in symmetric cases where the costs of errors in each direction are balanced. In other games, the Nash equilibrium has considerable drawing power, but economically significant deviations remain to be explained.

The notion that game theory should be tested with laboratory experiments is as old as the notion of a Nash equilibrium, and indeed, the classic prisoner's dilemma paradigm was inspired by a laboratory experiment conducted at the RAND Corporation in 1950. Some of the strategic analysts at RAND were dissatisfied with the received theory of cooperative and zero-sum games in von Neumann and Morgenstern's *Theory of Games and Economic Behavior*. In particular, nuclear conflict was not thought of as a zero-sum game because both parties may lose. Nasar (1998) describes the interest at RAND when word spread that a graduate student at Princeton had generalized von Neumann's existence proof for zero-sum games to the class of all games with finite numbers of strategies. Two mathematicians, Dresher and Flood, had been running some game experiments with their colleagues, and they were skeptical that human behavior would be consistent with Nash's notion of equilibrium. In fact, they designed an experiment that was run on the same day they heard about Nash's proof. Each player in this game had a dominant strategy to defect, but both would earn more if they both used the cooperative strategy. The game was repeated 100 times with the same two players, and a fair amount of cooperation was observed. One of Nash's professors, Al Tucker, saw the payoffs for this game written on a blackboard, and he invented the prisoner's dilemma story that was later used in a lecture on game theory that he gave in the Psychology Department at Stanford.

Interestingly, Nash's response to the Dresher and Flood's repeated prisoner's dilemma experiment is contained in a note to the authors that was published as a footnote to their paper: "The flaw in the experiment as a test of equilibrium point theory is that the experiment really amounts to having the players play one large multi-move game. One cannot just as well think of the thing as a sequence of independent games as one can in zero-sum cases. There is just too much interaction..." (quoted from Nasar, 1998, p. 119). In contrast, the experiments that we report in this paper involved games that were played *only once*, although related results for repeated games with random matching will be cited where appropriate. The categories of games considered are based on the usual distinctions: static versus dynamic and complete versus incomplete information. Section II describes the experiments based on static games with complete information: social dilemma, matching pennies, and coordination games. Section III contains results from dynamic games with complete information: bargaining games and games with threats that are not credible. The games reported in sections IV and V have incomplete information about other players' payoffs: in static settings (auctions) and two-stage settings (signaling games).

It is well known that psychological factors, such as framing, aspiration levels, social distance, and heuristics, can affect behavior in decision making and games (e.g., Kahneman, Slovic, and Tversky, 1982; Eckel and Wilson, 1999). In this paper we try to hold psychological factors constant and focus on payoff changes that are primarily *economic* in nature. As noted below, economic theories can and are being devised to explain the resulting anomalies. For example, the rational-choice assumption underlying the notion of a Nash equilibrium eliminates all errors, but if the cost of "overshooting" an optimal decision are much lower than the costs of "undershooting," one might expect an upward bias in decisions. In an interactive game, the endogenous effects of such biases may be reinforcing in a way that creates a "snowball" effect that moves decisions well away from a Nash prediction. Models that introduce (possibly small) amounts of noise into the decision making process can produce predictions that are quite far from any Nash equilibrium (McKelvey and Palfrey, 1995; Capra et al. 1999). A second type of rationality assumption that is built into the Nash equilibrium is that beliefs are consistent with actual decisions. Beliefs are not likely to be confirmed out of equilibrium, and learning will presumably occur in such cases. There is a large recent literature on incorporating learning into models of adjustment in games that are played repeatedly with different partners. Learning from experience is not possible in games that are only played once, and beliefs must be formed from introspective thought processes, which may be subject to considerable noise. Without noise, a model of iterated best responses will converge to a Nash equilibrium, if it converges at all. Our approach to explaining systematic deviations from Nash decisions is based on a model that injects increasing amounts of noise into higher levels of iterated beliefs. The predictions derived from this approach, discussed in section VI, are generally consistent with conformity to Nash predictions in the treasure treatments and systematic, intuitive deviations in the contradiction treatments. Some conclusions are offered in section VII.

II. Static Games with Complete Information

In this section we consider a series of two-player, simultaneous-move games, for which the Nash equilibria show an increasing degree of complexity. The first game is a "social dilemma" in which the pure-strategy Nash equilibrium coincides with the unique rationalizable outcome. Next, we consider a matching pennies game with a unique Nash equilibrium in mixed strategies. Finally, we discuss coordination games that have multiple Nash equilibria, some of which are better for all players than others.

In all of the games reported here and in subsequent sections, we used cohorts of student subjects recruited from undergraduate economics classes at the University of Virginia. Each cohort consisted of 10 students who were paid $6 for arriving on time, plus all cash that they earned in the games played. Earnings for the two-hour sessions ranged from $15 to $60, with an average of about $35.[2] Each one-shot game began with the distribution and reading of the instructions for that game (see http://www.people.virginia.edu/~cah2k/datapage.html). These instructions contained assurances that all money earned would be paid and that the game would

[2] These one-shot games followed an initial "part A" in which the subjects played the same two-person game for 10 periods with new pairings made randomly in each period. The part A for some of the sessions only lasted 9 periods, and for these sessions pairings were deterministic so that each person interacted with each of the others exactly once. Random pairings in part A of the other sessions were made by draws of numbered ping pong balls.

be followed by "another, quite different, decision-making experiment."[3]

The One-Shot Traveler's Dilemma Game

The Nash equilibrium concept is based on the twin assumptions of perfect error-free decision making and the consistency of actions and beliefs. The latter requirement may seem especially strong in the presence of multiple equilibria when there is no obvious way for players to coordinate. More compelling arguments can be given for the Nash equilibrium when it predicts the play of the unique justifiable, or *rationalizable*, action (Bernheim, 1984; Pierce, 1984). Rationalizability is based on the idea that players should eliminate those strategies that are never a best response for any possible beliefs, and realize that other (rational) players will do the same.[4]

To illustrate this procedure, consider the game in which two players independently and simultaneously choose integer numbers between (and including) 180 and 300. Both players are paid the *lower* of the two numbers, and, in addition, an amount $R > 1$ is transferred from the player with the higher number to the player with the lower number. For instance, if one person chooses 210 and the other chooses 250, they receive payoffs of $210 + R$ and $210 - R$ respectively. Since $R > 1$, the best response is to undercut the other's decision by 1 (if that decision were known), and therefore, the upper bound 300 is never a best response to any possible beliefs that one could have. Consequently, a rational person must assign a probability of zero to a choice of 300, and hence 299 cannot be a best response to any possible beliefs that rule out choices of 300, etc. Only the lower bound 180 survives this iterated deletion process and is thus the unique rationalizable action, and hence the unique Nash equilibrium.[5] This game was introduced by

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[3] We only had time to run about 6 one-shot games in each session, so the data are obtained from a large number of sessions where part A involved a wide range of repeated games, including public goods, coordination, price competition, and auction games that are reported in other papers. The one-shot games never followed a repeated game of the same type.

[4] A well-known example for which this iterated deletion process results in a unique outcome is a Cournot duopoly game with linear demand (Fudenberg and Tirole, 1993, pp. 47-48).

[5] In other games, rationalizability may allow outcomes that are not Nash equilibria, so it is a weaker concept than that of a Nash equilibrium, allowing a wider range of possible behavior. It is in this sense that Nash is more persuasive when it corresponds to the unique rationalizable outcome.

Basu (1994) who coined it the "traveler's dilemma" game.[6]

Although the Nash equilibrium for this game can be motivated by successively dropping those strategies that are never a best response (to any beliefs about strategies that have not yet been eliminated from consideration), this deletion process may be too lengthy for human subjects with limited cognitive abilities. When the cost of having the higher number is small, i.e. for small values of R, one might expect more errors in the direction of high claims, well away from the unique equilibrium at 180, and indeed this is the intuition behind the dilemma. In contrast, with a large penalty for having the higher of the two claims, players are likely to end up with claims that are near the unique Nash prediction, 180.

To test these hypotheses we asked 50 subjects (25 pairs) to make choices in a treatment with $R = 180$, and again in a matched treatment with $R = 5$. All subjects made decisions in each treatment, and the two games were separated by a number of other one-shot games. The ordering of the two treatments was alternated. The instructions asked the participants to devise their own numerical examples to be sure that they understood the payoff structure.

Figure 1 shows the frequencies for each 10-cent category centered around the claim label on the horizontal axis. The lighter bars pertain to the high-R "treasure" treatment, where close to 80 percent of all the subjects chose the Nash equilibrium strategy, with an average decision of 201. However, roughly the same fraction chose the *highest* possible number in the low-R treatment, for which the average was 280, as shown by the darker bars. Notice that the data in the contradiction treatment are clustered at the opposite end of the set of feasible decisions from the unique (rationalizable) Nash equilibrium.[7]

One might wonder whether the "anomalous" result for the low-R treatment disappears

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[6] The associated story is that two travelers purchase identical antiques while on a tropical vacation. Their luggage is lost on the return trip, and the airline asks them to make independent claims for compensation. In anticipation of excessive claims, the airline representative announces: "We know that the bags have identical contents, and we will entertain any claim between $180 and $300, but you will each be reimbursed at an amount that equals the *minimum* of the two claims submitted. If the two claims differ, we will also pay a reward R to the person making the smaller claim and we will deduct a penalty R from the reimbursement to the person making the larger claim."

[7] This result is statistically significant at all conventional levels, given the strong treatment and the relatively large number of independent observations (two paired observations for each of the 50 subjects). We will not report specific non-parametric tests for cases that are so clearly significant. The individual choice data are provided in the Data Appendix for this paper on: http://www.people.virginia.edu/~cah2k/datapage.html.

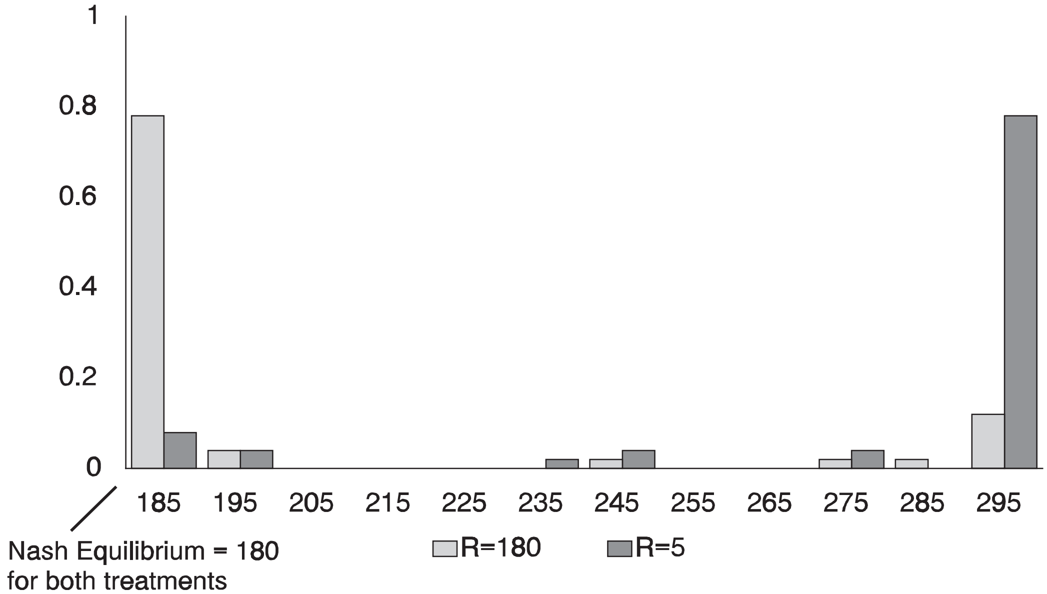

Figure 1. Claim Frequencies in a Traveler's Dilemma
for R = 180 (light bars) and R = 5 (dark bars)

when subjects play the game repeatedly and have the opportunity to learn. In Capra, *et al*. (1999) we discuss the results of a repeated version of this game (with random matching) and show that the opposite is true. Subjects chose numbers in the range [80, 200] with R = 5. The average claim *rose* from approximately 180 in the first period to 196 in period 5, and the average remained above 190 in later periods. Different cohorts played this game with different values of R, and successive increases R resulted in successive reductions in the average claims.[8] None of these treatment changes alter the unique Nash prediction of 80, so standard game theory simply cannot explain the most salient feature of the data, i.e. the effect of the penalty/reward parameter on average claims, a feature that is consistent with simple economic intuition.

[8] With a penalty/reward parameter of 5, 10, 20, 25, 50, and 80 the average claims in the final three periods were 195, 186, 119, 138, 85, and 81 respectively. Even though there is one treatment reversal, the effect of the penalty/reward parameter on average claims is significant at the 1 percent level. Capra, *et al*. (1999) show that the patterns of adjustment are well explained by a naive Bayesian learning model with decision error, and that the claim distributions for the final five periods are close to the distributions predicted by a logit equilibrium in the sense of McKelvey and Palfrey (1995, 1996).

A Matching Pennies Game

Consider a symmetric matching pennies game in which the row player chooses between *Top* and *Bottom* and the column player simultaneously chooses between *Left* and *Right*, as shown in top part of Table 1. The payoff for the row player is \$0.80 when the outcome is (*Top*, *Left*) or (*Bottom*, *Right*) and \$0.40 otherwise. The motivations for the two players are exactly opposite: column earns \$0.80 when row earns \$0.40, and vice versa. Since the players have opposite interests there is no equilibrium in pure strategies. Moreover, in order not to be exploited by the opponent, neither player should favor one of their strategies, and the mixed-strategy Nash equilibrium involves randomizing over both alternatives with equal probabilities. As before, we obtained decisions from 50 subjects in a one-shot version of this game (5 cohorts of 10 subjects, who were randomly matched and assigned row or column roles). The choice percentages are shown in parentheses next to the decision labels in the top part of Table 1. Note that the choice percentages are essentially "fifty-fifty," or as close as possible given that there was an odd number of players in each role.

Now consider what happens if the row player's payoff of \$0.80 in the (*Top*, *Left*) box is increased to \$3.20, as shown in the asymmetric matching pennies game in the middle part of Table 1. In a mixed-strategy equilibrium, a player's *own* decision probabilities should be such that the *other* player is made indifferent between the two alternatives. Since the column player's payoffs are unchanged, the mixed-strategy Nash equilibrium predicts that *row's* decision probabilities do not change either. In other words, the row player should ignore the unusually high payoff of \$3.20 and still choose *Top* or *Bottom* with probabilities of one-half. (Since column's payoffs are either 40 or 80 for playing *Left* and either 80 or 40 for playing *Right*, row's decision probabilities must equal 1/2 to keep column indifferent between *Left* and *Right*, and hence willing to randomize.)[9] This counter-intuitive prediction is dramatically rejected by the data, with 96% of the row players choosing the Top decision that gives a chance of the high \$3.20 payoff. Interestingly, the column players seemed to have anticipated this, and they played

[9] The predicted equilibrium probabilities for the row player are not affected if we relax the assumption of risk neutrality. Since there are only two possible payoff levels for column so without loss of generality columns' utilities for payoffs of 40 and 80 can be normalized to 0 and 1. Hence even a risk-averse column player will only be indifferent when row uses choice probabilities of 1/2.

Table 1. Three One-Shot Matching Pennies Games
(with choice percentages)

		Left (48%)	*Right* (52%)
Symmetric Matching Pennies	*Top* **(48%)**	80, 40	40, 80
	Bottom **(52%)**	40, 80	80, 40
		Left (16%)	*Right* (84%)
Asymmetric Matching Pennies	*Top* **(96%)**	320, 40	40, 80
	Bottom **(4%)**	40, 80	80, 40
		Left (80%)	*Right* (20%)
Reversed Asymmetry	*Top* **(8%)**	44, 40	40, 80
	Bottom **(92%)**	40, 80	80, 40

Right 84% of the time, which is quite close their equilibrium mixed-strategy of 7/8. Next, we lowered the row player's (*Top*, *Left*) payoff to $0.44, which again should leave the row player's own choice probabilities unaffected in a mixed-strategy Nash equilibrium. Again the effect is dramatic, with 92% of the choices being *Down*, as shown in the bottom part of Table 1. As before, the column players seemed to have anticipated this reaction, playing *Left* 80% of the time. To summarize, the unique Nash prediction is for the bolded row-choice percentages to be unchanged at 50% for all three treatments. This prediction is violated in an intuitive manner, with row players' choices responding to their own payoffs. In this context, *the Nash mixed-*

strategy prediction seems to work only by coincidence, when the payoffs are symmetric.[10]

A Coordination Game with a Secure Outside Option

Games with multiple Nash equilibria pose interesting new problems for predicting behavior, especially when some equilibria produce higher payoffs for all players. The problem of coordinating on the high-payoff equilibrium may be complicated by the possible gains and losses associated with payoffs that are not part of any equilibrium outcome. Consider a coordination game in which players receive $1.80 if they coordinate on the high-payoff equilibrium (H,H), $0.90 if they coordinate on the low-payoff equilibrium (L,L), and they receive nothing if they fail to coordinate (i.e. when one player chooses H and the other L). In addition, the column player has a secure option S that yields $0.40 for column and results in a zero payoff for the row player. This game is given in Table 2 when $x = 0$. To analyze the Nash equilibria of this game, notice that for the column player a fifty-fifty combination of L and H dominates S, and a rational player should therefore avoid the secure option. Eliminating S turns the game into a standard two-by-two coordination game that has three Nash equilibria: both players choosing L, both choosing H, and a mixed-strategy equilibrium in which both players choose L with probability 2/3.

[10] This anomaly is persistent when subjects play the game repeatedly. Ochs (1995) investigates a matching pennies game with an asymmetry similar to that of the middle game in Table 1, and the row player's continue to select *Top* considerably more than one-half of the time, even after as many as fifty rounds. These results are replicated in McKelvey, Palfrey, and Weber (1997). We have also conducted some repeated matching pennies games that exactly match those in Table 1, using 10 repetitions with the same partner. The results are qualitatively similar but less dramatic than those in Table 1. One possible explanation for the weaker own-payoff effect in the repeated game is that there may be strategic "teaching" whereby a player may choose a particular strategy several times in a row in order to alter the other's beliefs and decisions.

Table 2. An Extended Coordination Game

	L	H	S
L	90, 90	0, 0	x, 40
H	0, 0	180, 180	0, 40

The Nash equilibria are independent of x, which is the payoff to the row player when (L,S) is the outcome, since the argument for eliminating S is based solely on column's payoffs. However, the magnitude of x may affect the coordination process: for $x = 0$, row is indifferent between L and H when column selects S, and row is likely to prefer H when column does not select S (since then L and H have the same number of zero payoffs for row, but the potential positive payoff is higher with a choice of H). Row is thus more likely to choose H, which is then also the optimal action for the column player. However, when x is large, say 400, the column player may anticipate that row will select L in which case column should avoid H.

This intuition is borne out by the experimental data: in the treasure treatment with $x = 0$, 96 percent of the row players and 84 percent of the column players chose the high-payoff action H, while in the contradiction treatment with $x = 400$ only 64 percent of the row players and 76 percent of the column players chose H. The percentages of outcomes that were coordinated on the high-payoff equilibrium were 80 for the treasure treatment versus 32 for the contradiction treatment. In the latter treatment, an additional 16 percent of the outcomes were coordinated on the low-payoff equilibrium, but more than half of all the outcomes were uncoordinated, non-Nash outcomes.

A Minimum-Effort Coordination Game

The next game we consider is also a coordination game with multiple equilibria, but in this case the focus is on the effect of payoff asymmetries that determine the risks of deviating in the upward and downward directions. The two players in this game choose "effort" levels simultaneously, and the cost of effort determines the risk of deviation. The joint product is of the fixed-coefficients variety, so that each person's payoff is the minimum of the two efforts, minus the product of the player's own effort and a constant cost factor, *c*. In the experiment, we let efforts be any integer in the range from 110 to 170. If $c < 1$, any common effort in this range is a Nash equilibrium, because a unilateral one-unit increase in effort above a common starting point will not change the minimum but will reduce one's payoff by *c*. Similarly, a one-unit decrease in effort will reduce one's payoff by $1 - c$, i.e. the reduction in the minimum is less than the savings in effort costs when $c < 1$. Obviously, a higher effort cost increases the risk of raising effort and reduces the risk of lowering effort. Thus simple economic intuition suggests that effort levels will be inversely related to effort costs, despite the fact that any common effort level is a Nash equilibrium.

We ran one treatment with a low effort cost of 0.1, and the data for 50 randomly matched subjects in this treatment are shown by the dark bars in Figure 2. Notice that behavior is quite concentrated at the highest effort level of 170; subjects coordinate on the Pareto-dominant outcome. The high effort cost treatment ($c = 0.9$), however, produced a preponderance of efforts at the lowest possible level, as can be seen by the lighter bars in the figure. Clearly, the extent of this "coordination failure" is affected by the key economic variable in this model in a manner that is quite intuitive, even though Nash theory is silent.[11]

The Kreps Game

The previous examples demonstrate how the cold logic of game theory can be at odds with intuitive notions about human behavior. This tension has not gone unnoticed by some game theorists. For instance, Kreps (1995) discusses a variant of the game in the top part of Table 3 (where we have scaled back the payoffs to levels that are appropriate for the laboratory). The pure-strategy equilibrium outcomes of this game are (*Top*, *Left*) and (*Bottom*, *Right*). In addition,

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[11] The standard analysis of equilibrium selection in coordination is based on the Harsanyi-Selten notion of risk dominance, which allows a formal analysis of the tradeoff between risk and payoff dominance. There is no agreement on how to generalize risk dominance beyond 2×2 games, but see Anderson, Goeree, and Holt (1998c) for a proposed generalization based on the "stochastic potential." Experiments with repeated plays of coordination games have shown that behavior may begin near the Pareto-dominant equilibrium, but later converge to the equilibrium that is worst for all concerned (van Huyck, Battalio, and Beil, 1990). Moreover, the equilibrium that is selected may be affected by the payoff structure for dominated strategies (Cooper *et al*., 1992). See Goeree and Holt (1998) for results of a repeated coordination game with random matching. They show that the dynamic patterns of effort decisions are well explained by a simple evolutionary model of noisy adjustment toward higher payoffs.

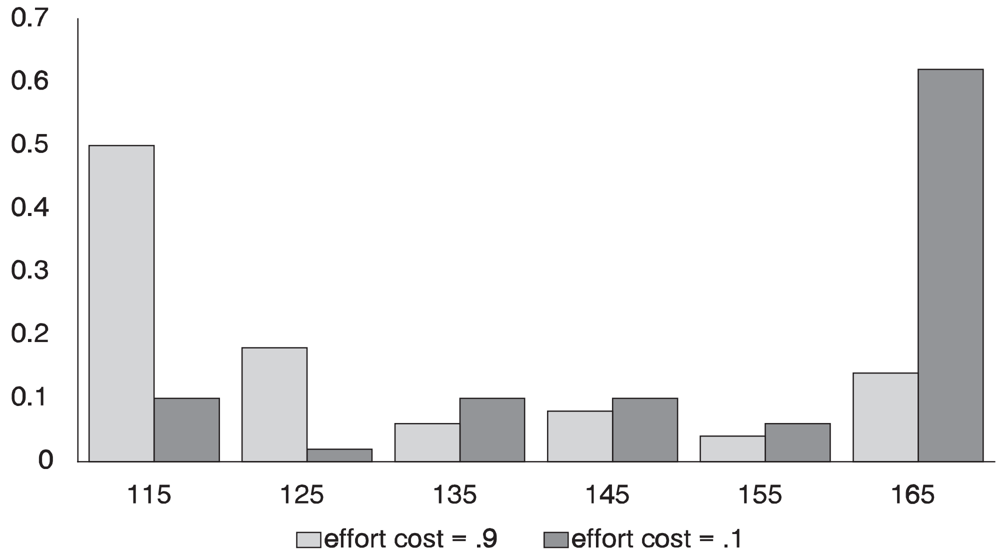

Figure 2. Effort Choice Frequencies for a Minimum Effort Coordination Game With High Effort Cost (Light Bars) and Low Effort Cost (Dark Bars)

there is a mixed-strategy equilibrium in which row randomizes between *Top* and *Bottom* and column randomizes between *Left* and *Middle*. The only column strategy that is not part of any Nash equilibrium is labeled *Non-Nash*. Kreps argues that column players will tend to choose *Non-Nash* because the other options yield at best a slightly higher payoff (i.e. 10, 15, or 20 cents higher) but could lead to substantial losses of $1 or $2.50. Notice that this intuition is based on payoff magnitudes out of equilibrium, in contrast to Nash calculations based only on signs of payoff differences.

Kreps did try the high-hypothetical-payoff version of this game on several graduate students, but lets consider what happens with financially motivated subjects in an anonymous laboratory situation. As before, we randomly paired 50 subjects and let them make a single choice. Subjects were told that losses would be subtracted from prior earnings, which were quite substantial by that point. As seen from the percentages in parentheses in the top part of the table, the *Non-Nash* decision was selected by approximately two-thirds of the column players. Of

Table 3. Two Versions of the Kreps Game
(with choice percentages)

		Left (26%)	*Middle* (8%)	*Non-Nash* (68%)	*Right* (0%)
Basic Game	*Top* (68%)	200, 50	0, 45	10, 30	20, -250
	Bottom (32%)	0, -250	10, -100	30, 30	50, 40

		Left (24%)	*Middle* (12%)	*Non-Nash* (64%)	*Right* (0%)
Positive Payoff Frame	*Top* (84%)	500, 350	300, 345	310, 330	320, 50
	Bottom (16%)	300, 50	310, 200	330, 330	350, 340

course, it is possible that this result is simply a consequence of "loss-aversion," i.e. the disutility of losing some amount of money is greater than the utility associated with winning the same amount (Kahneman, Knetsch, and Thaler, 1991). Since all the other columns contain negative payoffs, loss-averse subjects would thus be naturally inclined to choose *Non-Nash*. Therefore, we ran another 50 subjects through the same game, but with 300 cents added to payoffs to avoid losses, as shown in the bottom part of Table 3. The choice percentages shown in parentheses indicate very little change, with close to two-thirds of column players choosing *Non-Nash* as before. Finally, we ran 50 new subjects through the original version in the top part of the table, with the (*Bottom*, *Right*) payoffs of (50, 40) being replaced by (350, 400), which (again) does not alter the equilibrium structure of the game. With this admittedly heavy-handed enhancement of the equilibrium in that cell, we observed 96% *Bottom* choices and 84% *Right* choices, with 16% *Non-Nash* persisting in this, the "treasure" treatment.

III. Dynamic Games with Complete Information

As game theory became more widely used in fields like industrial organization, the complexity of the applications increased to accommodate dynamics and asymmetric information.

One of the major developments coming out of these applications was the use of backward induction rationality to eliminate equilibria with threats that are not "credible" (Selten, 1975). Backward induction was also used to develop solutions to alternating-offer bargaining games (Rubinstein, 1982), which was the first major advance on this historically perplexing topic since Nash's axiomatic approach. However, there have been persistent doubts that people will be able to figure out complicated, multi-stage backward induction arguments. Rosenthal (1981) quickly proposed a game, later dubbed the "centipede game," in which backward induction over a large number of stages (e.g. 100 stages) was thought to be particularly problematic (e.g. McKelvey and Palfrey, 1992). Many of the games in this section are inspired by Rosenthal's (1981) doubts and Beard and Beil's (1994) experimental results. Indeed, the anomalies in this section are better known than those in other sections, but we focus on very simple games with two or three stages, using parallel procedures and subjects who have previously made a number of strategic decisions in different one-shot games.

Should You Trust the Others to be Rational?

The power of backward induction is illustrated in the top game in Figure 3. The first player begins by choosing between a safe decision, *S*, and a risky decision, *R*. If *R* is chosen, the second player must choose between a decision *P* that punishes both of them and a decision *N* that leads to a Nash equilibrium that is also a joint-payoff maximum. There is, however, a second Nash equilibrium where the first player chooses *S* and the second chooses *P*. The second player has no incentive to deviate from this equilibrium because the self-inflicted punishment occurs off of the equilibrium path. Subgame perfectness rules out this equilibrium by requiring equilibrium behavior in each subgame, i.e. that the second player behave optimally in the event that the second stage subgame is reached.

Again, we used 50 randomly paired subjects who played this game only once. The data for this treasure treatment are quite consistent with the subgame perfect equilibrium; the preponderance of first players trust the other's rationality enough to choose *R*, and there are no irrational *P* decisions that follow. The game shown in the bottom part of Figure 3 is identical, except that the second player only forgoes 2 cents by choosing *P*. This change does not alter the

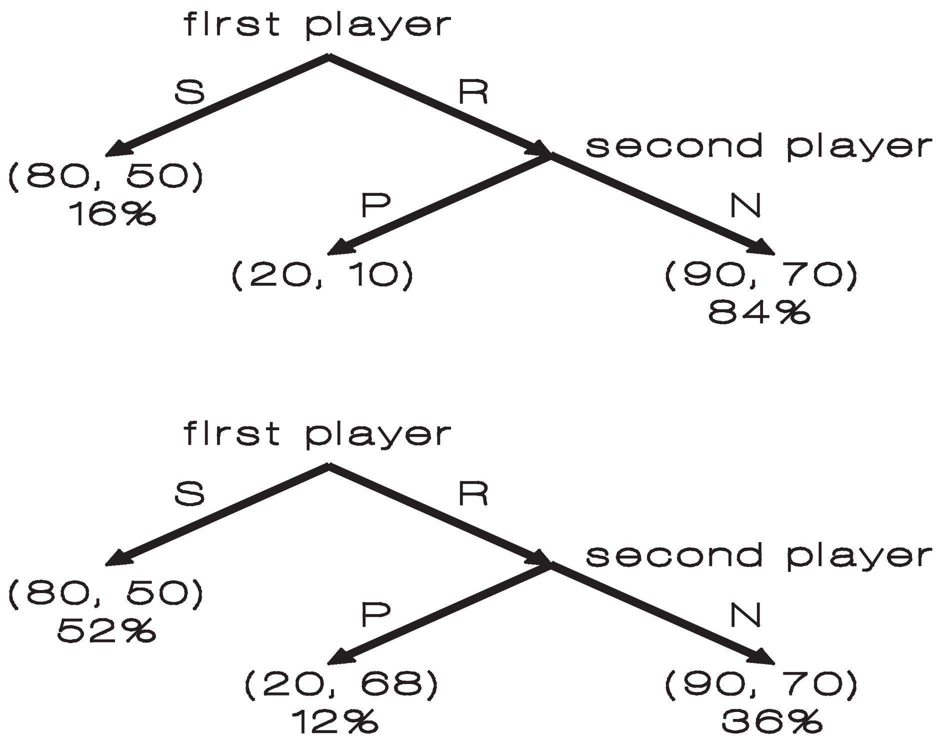

Figure 3. Should You Trust the Rationality of Others?

fact that there are two Nash equilibria, one of which is ruled out by subgame perfectness. The choice percentages for 50 subjects indicate that a majority of the first players did not trust others to be perfectly rational when the cost of irrationality is so small. Only about a third of the outcomes matched the subgame perfect equilibrium in this game.[12] We did a third treatment (not shown) in which we multiplied all payoffs by a factor of 5, except that the *P* decision led to (100, 348) instead of (100, 340). This large increase in payoffs produced an even more dramatic result; only 16% of the outcomes were subgame perfect, and 80% of the outcomes were at the Nash equilibrium that is not subgame perfect.

Are "Credible" Threats Really Credible?

The game just considered is a little unusual in that the second player has no reason to

[12] See Beard and Beil (1994) for similar results in a two-stage game played only once.

punish, since the first player's *R* decision also benefits the second player. This is not the case for the game in Figure 4, where an *R* decision by the first player will lower the second player's payoff. As before, there are two Nash equilibria, with the *R*, *P* equilibrium ruled out by subgame perfectness. In addition to not being credible, the threat to play *P* is a relatively costly punishment for the second player to administer (40 cents).

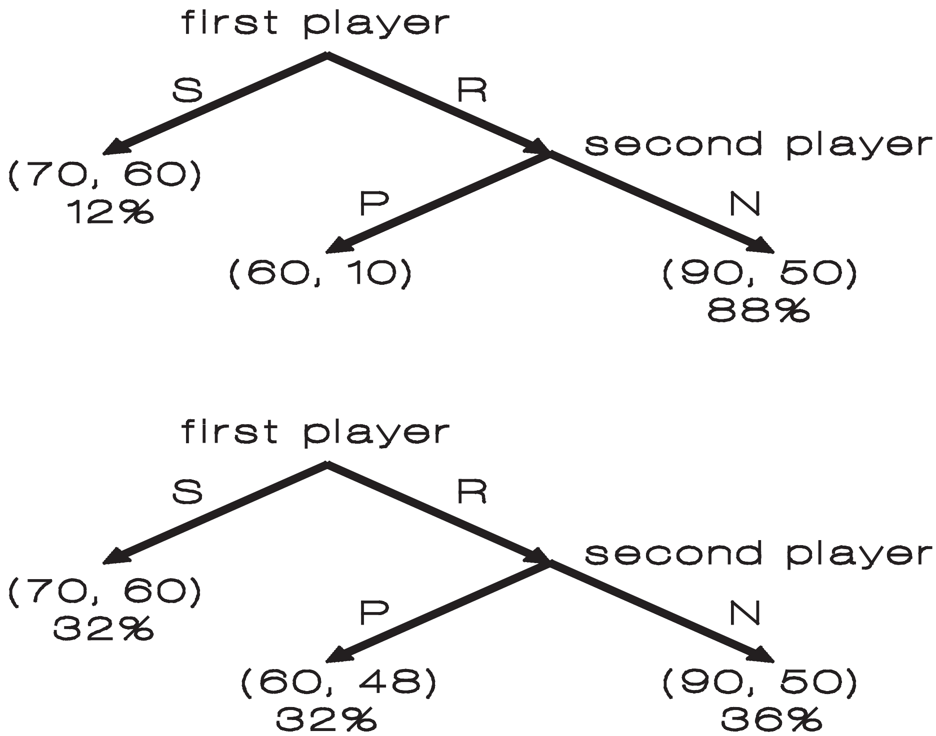

Figure 4. Should You Believe a Threat That Is Not Credible?

The threat to play *P* in the top part of Figure 4 is evidently not believed, and 88% of the first players choose the *R* strategy, with impunity. The threat is cheap (2 cents) for the game in the bottom part of the figure, and outcomes for 25 subject pairs are about evenly divided between the subgame imperfect outcome, the incredible threat outcome, and the subgame perfect outcome. Cheap threats are often not (and apparently should not) be believed. Again we see that payoff magnitudes and off-equilibrium-path risks matter.

These results would not come as any surprise to Reinhard Selten, the originator of the notion of subgame perfectness. His attitude toward game theory is that there is a sharp contrast between standard theory and behavior. For a long time he essentially wore different hats when he did theory and ran experiments, although his 1995 Nobel prize was clearly for his contributions in theory. This schizophrenic stance may seem inconsistent, but it may prevent unnecessary anxiety, and some of Selten's recent theoretical work is based on models of boundedly rational (directional) learning (Selten and Buchta, 1994). In contrast, John Nash was reportedly discouraged by the predictive failures of game theory and gave up on both experimentation and game theory (Nasar, 1998, p.150).

Two-Stage Bargaining Games

Bargaining has long been considered a central part of economic analysis, and at the same time, one of the most difficult problems for economic theory. One promising approach is to model unstructured bargaining situations "as if" the parties take turns making offers, with the costs of delayed agreement reflected in a shrinking size of the pie to be divided. This problem is particularly easy to analyze when the number of alternating offers is fixed and small.

Consider a bargaining game in which each player gets to make a single proposal for how to split a pie, but the amount of money to be divided falls from $5 in the first stage to $2 in the second. The first player proposes a split of $5 that is either accepted (and implemented) or rejected, in which case the second player proposes a split of $2 that is either accepted or rejected by the first player. This final rejection results in payoffs of zero for both players, so the second player can (in theory) successfully demand $1.99 in the second stage if the first player prefers a penny to nothing. Knowing this, the first player should demand $3 and offer $2 to the other in the first stage. In a subgame perfect equilibrium, the first player receives the amount by which the pie shrinks, so a larger cost of delay confers a greater advantage to the player making the initial demand, which seems reasonable. For example, a similar argument shows that if the pie shrinks by $4.50, from $5 to $0.50, then the first player should make an initial demand of $4.50.

We used 60 subjects (6 cohorts of 10 subjects each), who were randomly paired for each of the two treatments described above (alternating in order and separated by other one-shot games). The average demand for the first player was $2.83 for the $5/$2 treatment, with a standard deviation of $0.29. This is quite close to the predicted $3.00 demand, and 14 of the 30 initial demands were exactly equal to $3.00 in this treasure treatment. But the average demand only increased to $3.38 for the other treatment with a $4.50 prediction, and 28 of the 30 demands were below the prediction of $4.50. Rejections were quite common in this treatment with higher demands and correspondingly lower offers to the second player, which is not surprising given the smaller costs of rejecting "stingy" offers.

These results fit into a larger pattern surveyed in Davis and Holt (1993, chapter 5) and Roth (1995); initial demands in two-stage bargaining games tend to be "too low" relative to theoretical predictions when the equilibrium demand is high, say more than 80% of the pie as in our $5.00/$0.50 treatment, and initial demands tend to be close to predictions when the equilibrium demand is 50-75% of the pie (as in our $5.00/$2.00 treatment). Interestingly, initial demands are "too high" when the equilibrium demand is less than half of the pie. Here is an example of why theoretical explanations of behavior should not be based on experiments in only one part of the parameter space, and why theorists should have more than just a casual, second-hand knowledge of the experimental economics literature.[13] Many of the diverse theoretical explanations for anomalous behavior in bargaining games hinge on models of preferences in which a person's utility depends on the payoffs of both players, i.e. distribution matters (Bolton, 1998, Bolton and Ockenfels, 1999, and Fehr and Schmidt, 1997). The role of fairness is illustrated dramatically in the experiment reported in Goeree and Holt (2000a), who obtained even larger deviations from subgame perfect Nash predictions than those reported here by giving subjects asymmetric money endowments that raise fairness issues without altering the subgame perfect Nash prediction.

IV. Static Games with Incomplete Information

Vickrey's models of auctions with incomplete information constitute one of the most widely used applications of game theory. If private values are drawn from a uniform distribution, the Bayesian Nash equilibrium predicts that bids will be proportional to value, which is generally consistent with laboratory evidence. The main deviation from theoretical predictions is the tendency of human subjects to "overbid," which is commonly rationalized in terms of risk

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[13] Another example is the development of theories of generalized expected utility to explain "fanning out" preferences in Allais paradox situations, when later experiments in other parts of the probability triangle found "fanning in."

aversion, an explanation that has lead to some controversy. Harrison (1989), for instance, argues that deviations from the Nash equilibrium may well be caused by a lack of monetary incentives since the costs of such deviations are rather small: the "flat maximum critique." Our approach here is to specify two auction games with identical Nash equilibria, but with differing incentives not to overbid.

First, consider a game in which each of two bidders receives a private value for a prize to be auctioned in a first-price, sealed bid auction. In other words, the prize goes to the highest bidder for a price equals that bidder's own bid. Each bidder's value for the prize is equally likely to be $0, $2, or $5. Bids are constrained to be integer dollar amounts, with ties decided by the flip of a coin.

Table 4. Equilibrium Expected Payoffs for the (0,2,5) Treatment
(Optimal Bids Are Denoted by an Asterisk *)

	bid = 0	bid = 1	bid = 2	bid = 3	bid = 4	bid = 5
value = $0	0*	-.5	-1.66	- 3	- 4	- 5
value = $2	.33	.5*	0	-1	-2	- 3
value = $5	.83	2	2.5*	2	1	0

The relevant Nash equilibrium in this game with incomplete information about others' preferences is the Bayesian Nash equilibrium, which specifies an equilibrium bid for each possible realization of a bidder's value. It is straightforward but tedious to verify that the Nash equilibrium bids are $0, $1, and $2 for a value of $0, $2, and $5 respectively, as can be seen from the equilibrium expected payoffs in Table 4. For example, consider a bidder with a private value of $5 (in the bottom row) who faces a rival that bids according to the proposed Nash solution. A bid of 0 has a 1/2 chance of winning (decided by a coin flip) if the rival's value, and hence the rival's bid, is zero, which happens with probability 1/3. Therefore, the expected payoff of a zero bid with a value of $5 equals 1/2*1/3*($5 - $0) = $5/6 = .83. If the bid is raised to $1, the probability of winning becomes 1/2 (1/3 when the rival's value is $0 plus 1/6 when the rival's value is $2). Hence, the expected payoff of a $1 bid is 1/2*($5 - $1) = $2. The other numbers in Table 4 are derived in a similar way. The maximum expected payoff in each row coincides with the equilibrium bid, as indicated by an asterisk (*). Note that the equilibrium involves bidding about one half of the value.[14]

Table 5. Equilibrium Expected Payoffs for the (0,3,6) Treatment
(Optimal Bids Are Denoted by an Asterisk *)

	bid = 0	bid = 1	bid = 2	bid = 3	bid = 4	bid = 5
value = $0	0*	-.5	-1.66	- 3	- 4	- 5
value = $3	.5	1*	.83	0	-1	- 2
value = $6	1	2.5	3.33*	3	2	1

Table 5 shows the analogous calculations for the second treatment, with equally likely private values of $0, $3, or $6. Interestingly, this increase in values does not alter the equilibrium bids in the unique Bayesian Nash equilibrium, as indicated by the location of optimal bids for each value. Even though the equilibria are the same, we expected more of an upward bias in bids in the second (0, 3, 6) treatment. The intuition can be seen by looking at payoff losses associated with deviations from the Nash equilibrium. Consider, for instance, the middle-value bidder with expected payoffs shown in the second rows of Tables 4 and 5. In the (0, 3, 6) treatment, the cost of bidding $1 above the equilibrium bid is $1 - $0.83 = $0.17, which is less than the cost of bidding $1 below the equilibrium bid: $1 - $0.50 = $0.50. In the (0, 2, 5) treatment, the cost of an upward deviation from the equilibrium bid is greater than the cost of a downward deviation, see the middle row of Table 4. A similar argument applies to the high-value bidders, while deviation costs are the same in both treatments for the low-value bidder. Hence we expected more overbidding for the (0, 3, 6) treatment.

This intuition is borne out by bid data for the 50 subjects who participated in a single auction under each condition. Eighty percent of the bids in the (0, 2, 5) treatment matched the equilibrium: the average bids for low, medium, and high value bidders were $0, $1.06, and $2.64

[14] The bids would be exactly half of value if the highest value were $4 instead of $5, but we had to raise the highest value to eliminate multiple Nash equilibria.

Table 6. Bid Frequencies (Equilibrium Bids Marked with an Asterisk *)

	(0, 2, 5) treatment			(0, 3, 6) treatment	
	bid	frequency		bid	frequency
value = 0	0*	20	value = 0	0*	17
value = 2	1*	15	value = 3	1*	5
	2	1		2	11
	3	0		3	2
value = 5	1	1	value = 6	1	0
	2*	5		2*	3
	3	6		3	4
	4	2		4	6
	5	0		5	1
	6	0		6	1

respectively. In contrast, the average bids for the (0, 3, 6) treatment were \$0, \$1.82, and \$3.40 for the three value levels. The bid frequencies for each value are shown in Table 6. As in previous games, deviations from Nash behavior in these private value auctions seem to be sensitive to the costs of deviation. Of course, this does not rule out the possibility that risk aversion or some other factor may also have some role in explaining the overbidding observed here, especially the slight overbidding for the high value in the (0, 2, 5) treatment.[15]

V. Dynamic Games with Incomplete Information: Signaling

Signaling games are complex and interesting because the two-stage structure allows an

[15] Goeree, Holt, and Palfrey (1999) report a first-price auction experiment with 6 possible values, under repeated random matching for ten periods. An econometric model that includes both decision error and risk aversion provides a good fit of the data. Several plausible alternatives to risk aversion, like nonlinear cumulative probability weighting or a "joy of winning," are considered. Lucking-Reiley (1999) mentions risk aversion as a possible explanation for overbidding in a variety of auction experiments.

opportunity for players to make inferences and change others' inferences about private information. This complexity often generates multiple equilibria that, in turn, have stimulated a sequence of increasingly complex refinements of the Nash equilibrium condition. Although it is unlikely that introspective thinking about the game will produce equilibrium behavior in a single play of a game this complex (except by coincidence), the one-shot play reveals useful information about subjects' cognitive processes.

Table 7. Signaling with a Separating Equilibrium
(sender's payoff, responder's payoff)

	response to Left signal				response to Right signal		
	C	D	E		C	D	E
type A sends Left	300, 300	0, 0	500, 300	type A sends Right	450, 900	150, 150	1000, 300
type B sends Left	500, 500	300, 450	300, 0	type B sends Right	450, 0	0, 300	0, 150

In the experiment, half of the subjects were designated as "senders" and half as "responders." After reading the instructions, we began by throwing a die for each sender to determine whether the sender was of type A or B. Everybody knew that the *ex ante* probability of a type A sender was 1/2. The sender, knowing his/her own type would choose a signal, *Left* or *Right*. This signal determined whether the payoffs on the right or left side of Table would be used. (The instructions used letters to identify the signals, but we will use words here to facilitate the explanations.) This signal would be communicated to the responder that was matched with that sender. The responder would see the sender's signal, *Left* or *Right*, but not the sender's type, and then choose a response, *C*, *D*, or *E*. The payoffs were determined by Table 7, with the sender's payoff to the left in each cell.

First, consider the problem facing a type A sender, for whom the possible payoffs from sending a *Left* signal (300, 0, 500) seem, in some loose sense, is less attractive than those for sending a *Right* signal (450, 150, 1000). (For example, if each response is thought to be equally likely, then the *Right* signal has a higher expected payoff.) Consequently, the *Right* row has been shaded in the top right part of the table. Similarly, a type B sender looking at the payoffs in the bottom row of the table might be more attracted by the *Left* signal, with payoffs of (500, 300, 300) as compared with (450, 0, 0).[16] Therefore, the payoffs for type B sending the *Left* signal have been shaded. In fact, all of the type B subjects did send the *Left* signal, and 7 of the 10 type A subjects sent the *Right* signal. All responses in this game were *C*, so all but three of the outcomes were in the two boxes that have thick outlines. Notice that this is an equilibrium, since neither type of sender would benefit from sending the other signal, and the respondent cannot do any better than the maximum payoff received in the boxes. This is a separating Nash equilibrium; the signal reveals the sender's type.

The payoff structure for this game becomes a little clearer if you think of the responses as one of three answers to a request: Concede, Deny, or Evade. Evade is sufficiently unattractive to respondents that it is never selected, so consider the other two responses and note that a sender always prefers that the responder choose Concede instead of Deny. In the separating equilibrium, the signals reveal the senders' types, the responder always Concedes, and all players are satisfied. There is, however, a second equilibrium for the game in Table 7 in which the responder Concedes to *Left* and Denies *Right*, and therefore both sender types send *Left* to avoid being Denyed. To check that the responder has no incentive to deviate, note that Concede is a best response to a *Left* regardless of the sender's type, and that Deny is a best response to a deviant Right signal if the responder believes that it was sent by a type B. Backward induction rationality (of the sequential Nash equilibrium) does not rule out these beliefs, since a deviation does not occur in equilibrium, and the respondent is making a best response to the beliefs. What is unintuitive about these beliefs (that a deviant *Right* signal comes from a type B) is that the type B is earning 500 in this (Left, Concede) equilibrium outcome, and no deviation could conceivably increase this payoff, whereas, the type A is earning 300 in the *Left* side pooling equilibrium, and this type could possibly earn more (450 or even 1000), depending on the

[16] These are not dominance arguments, since the responder can respond differently to each signal, and the lowest payoff from sending one signal is not higher than the highest payoff that can be obtained from sending the other signal.

response to a deviation. The Cho and Kreps (1987) *intuitive criterion* rules out these beliefs, and therefore, selects the separating equilibrium that was observed in the treasure treatment.

Table 8. Signaling with a Pooling Equilibrium At *Right* That Is Not Intuitive
(sender's payoff, responder's payoff)

	response to Left signal				response to Right signal		
	C	D	E		C	D	E
type A sends Left	300, 300	0, 0	500, 300	type A sends Right	450, 900	150, 150	1000, 300
type B sends Left	300, 300	300, 450	300, 0	type B sends Right	450, 0	0, 300	0, 150

The game in Table 8 is a minor variation on the previous game, with the only change being that the (500, 500) in the bottom left part of Table 7 is replaced by a (300, 300) payoff. Again a casual consideration of the sender's payoffs may lead one to expect that type A senders will choose *Right* and that type *B* senders will choose *Left*, as indicated by the shading. In the experiment, all but 3 of the 13 type A senders did choose *Right*, and all but 2 of the 11 type B senders did choose *Left*. But the separation observed in this contradiction treatment is not a Nash equilibrium, since the respondents would prefer to Concede to a *Right* signal thought to be sent by a type A and to Deny a *Left* signal thought to be sent by a type B. In fact, half of the *Left* signals received the negative *D* response, whereas only 2 of the 12 *Right* signals were Denied.[17]

In an experiment (unlike ours) with repeated plays of the signaling game, this Deny response to *Left* signals should cause the type B senders to switch to the *Right* signals. This is precisely the pattern of adjustment reported in Brandts and Holt (1993), where the *Right* signal became predominant for both types in later matchings (with different partners for each matching). This "pooling" in which both types send *Right* is a Nash equilibrium; neither would want to

[17] Unlike the paired treatments considered previously, the payoff change for these signaling games does alter the set of Nash equilibria.

switch if they anticipated receiving a *D* response to the *Left* signal. Of course, the *D* response to *Left* is only appropriate if the respondent thinks the deviant *Left* signal comes from a type B sender. But these beliefs about the deviant are ruled out by the intuitive criterion argument given above: the type B sender gets 450 in the pooling *Right* equilibrium, and there is no payoff on the left side of the bottom row of table 8 that would be better for this sender type. In contrast, the type A sender who also gets 450 in equilibrium, could possibly get a 500 payoff (from an E response) to a deviation on the top row, left side of the table. There is a second Nash equilibrium for the game in Table 8: both types send *Left* and respondent Concedes to *Left* and Denies *Right*. The beliefs that support these responses, that a deviant Right signal is sent by a type B, are not ruled out by the intuitive criterion, since the type B sender could possibly obtain a payoff increase (from 300 to 450) from such a deviation. There is no support for this "intuitive" equilibrium, either in the one-shot games or in repeated-matching design of Brandts and Holt.[18]

The important lesson from this contradiction treatment is that restrictions on what beliefs are "reasonable" must be derived from an analysis of the process of adjustment to equilibrium. Once people get to an equilibrium (e.g. with both types sending signal *Right*), when the respondent sees a deviant signal, the inference about the type that deviated will be based on the type that tended to send that signal in the past, before behavior converged to the other signal. In repeated signaling games, beliefs "off the equilibrium path" are determined by the out-of-equilibrium behavior actually observed earlier during the process of adjustment. This learning-based analysis of beliefs can be quite different from the deductive analysis used in the refinements literature, which begins by looking at what payoffs players *do* get in equilibrium and then looking at which type of sender could conceivably do better by deviating. In addition, a better understanding of behavior in one-shot games, however complex, will help us predict the initial type/signal correlation that may have a big influence on respondents' beliefs even after one

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[18] This is essentially the refinement recommended by Gibbons (1997) for this type of game, and any equilibrium ruled out by the intuitive criterion in a two-stage signaling game will be ruled out by stronger refinements like "divinity" or "strategic stability." Cho and Kreps (1987) show that, in the context of these two-stage signaling games, any equilibrium ruled out by the intuitive criterion will also be ruled out by stronger refinements like "divinity" or "strategic stability," so the Brandts and Holt (1993) experiment is inconsistent with these refinements as well.

of the signals is no longer being used very often in later periods of a repeated game.

VI. Noisy Decision Making and Introspection in One-Shot Games

A Nash equilibrium can be found by considering only the *signs* of payoff differences. The games considered thusfar indicate that payoff *magnitudes* may matter, and that the directions of deviation from Nash may be affected by payoff asymmetries. Biases in behavior can be self-reinforcing in a dramatic manner in situations with a lot of payoff interdependence, as in a traveler's dilemma. In all of the games considered, however, there are payoff conditions where Nash or the relevant refinement yields accurate predictions. This section describes a model of noisy introspection that is intended to explain both the treasures and the contradictions for these one-shot games. Recall that the Nash equilibrium requires perfect payoff maximization (no errors) and consistency of actions and beliefs (no surprises). Our approach is to allow decision errors that are sensitive to the magnitudes of payoff differences, as in a logit probabilistic choice model. Play in many types of one-shot games is likely to contain surprises, no matter how carefully players think about the payoffs before deciding. Therefore, we also relax the second Nash assumption, that of consistency of actions and beliefs, by introducing a process of iterated stochastic conjectures.

A payoff-sensitive choice rule can be based on an assumption that decision probabilities are positively related to expected payoffs. The logit rule, for example, specifies that the choice probabilities, p_i, for options $i = 1,...,m$, are proportional to an exponential function of the associated expected payoffs π_i^e:

$$p_i = \frac{\exp(\pi_i^e/\mu)}{\sum_{j=1,..,m} \exp(\pi_j^e/\mu)}, \tag{1}$$

where the sum in the denominator ensures that the probabilities sum to one. The expected payoffs in (1) are determined by a player's beliefs about the rival. Let q_i, $i = 1,...,m$, denote players' "belief probabilities." For instance, if a player thinks that all options are equally likely to be chosen by the opponent, then q_i is simply $1/m$ for all i. The logit choice rule (1) maps belief probabilities to choice probabilities, i.e. $p = \phi_\mu(q)$, where ϕ_μ represents the map on the right side of (1). The "error parameter," μ, determines how sensitive choice probabilities are to payoff differences. As μ goes to infinity, the arguments of the exponential expressions go to zero, and the probabilities go to $1/m$, regardless of expected payoffs. Thus a high μ represents noisy decision making that makes choices essentially random. In contrast, dividing expected payoffs by a low value of μ means that payoff differences are blown up, making choice probabilities sensitive to payoff differences. Hence the "noisy best response" rule in (1) includes perfectly rational behavior and completely random behavior as limiting cases.

We model the process of pre-play introspection as being stochastic, since it is very likely that speculation about others' decisions is highly subjective. Consider a symmetric, two-player game in normal form with m decisions for each player. The simplest way to proceed is to begin introspection by assuming that all possible decisions made by the other player are equally likely, and use the $1/m$ probabilities to calculate the expected payoffs, π_i^e, associated with each decision. These expected payoffs can be used in (1) to generate a new set of conjectured probabilities, those that are logit probabilistic responses to the initial conjectures. Iterating in this manner, the thought process will produce more refined conjectures.[19] However, this iterative procedure becomes increasingly complex, and we will assume that the error rate grows (geometrically) with each further iteration. With a vector of initial belief probabilities, p_0, the vector of noisy best response probabilities is given by the logit functions in (1): $p = \phi_\mu(p_0)$, where μ is an error rate associated with the decision. There is likely to be more error associated with beliefs about the other player's responses, so we increase the error parameter for one iteration to $t\,\mu$, where $t > 1$. Thus $p = \phi_\mu(\phi_{t\mu}(p_0))$ represents the noisy (μ) responses to an even noisier (tμ) response to p_0. This process can be iterated backwards, with the "telescoping" parameter $t > 1$ determining how fast the error rate blows up with further iterations; the error rate for the nth iteration is given by

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[19] For an alternative approach, see Capra (1998). In her model, beliefs are represented by degenerate distributions that put all probability mass at a single point. The location of the belief points is, *ex ante*, stochastic. A deterministic model of introspection in 2×2 games is presented in Olcina and Urbano (1994). This model uses an axiomatic approach to select a prior distribution, which is revised by a simulated learning process. This latter process is essentially a partial adjustment from current beliefs to best responses to current beliefs. The model has the attractive property that it selects the risk-dominant Nash equilibrium. Since the simulated learning process has no noise, it will converge to the unique Nash equilibrium in mixed strategies in the asymmetric matching pennies games, which is an undesirable feature of the model in light of the one-shot data reported here. Our conjecture is that it will not track data patterns in the traveler's dilemma game either, since the (noise-free) best-responses in this game are unaffected by the magnitude of the R parameter that has such a large impact on the observed decisions.

$t^{n-1}\mu$. We are interested in the choice probabilities in the limit as the number of iterations goes to infinity:

$$p = \lim_{n \to \infty} \phi_{\mu}(\phi_{t\mu}(\cdots \phi_{t^n\mu}(p_0))). \qquad (2)$$

One issue is whether the limit in (2) converges. In Goeree and Holt (2000b) we use continuity arguments to show that this limit is well defined for $t > 1$. A second issue is what to do about the seemingly arbitrary initial probability vector. Note that, since ϕ_{∞} maps the whole probability simplex to a single point, the process is independent of the initial belief vector p_0. The geometrically increasing error rate in (2) captures the idea that it becomes more and more complex to think further back. For a t value between 2 and 4, say, the process converges quickly and the iterated probabilities remain more or less the same after several steps.[20] Finally, the limit case $t = 1$ is of special interest. For some games (e.g. matching pennies) the process will not converge when $t = 1$, but when it does, the limit probabilities, p^*, must be invariant under the logit map: $\phi_{\mu}(p^*) = p^*$. A fixed point of this type constitutes a "logit equilibrium," which is a special case of the quantal response equilibrium defined in McKelvey and Palfrey (1995, 1996). It is in this sense that the logit equilibrium arises as a limit case of the noisy introspective process defined in (2). When $t > 1$, the choice probabilities on the left side of (2) generally do not match the belief probabilities at any stage of the iterative process on the right. In other words, the introspective process allows for surprises, which are likely to occur in oneshot games.

For given values of the telescope and error parameters, the predicted distribution of decisions can be found by iterating the stochastic best responses to any initial distribution,

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[20] The convergence proof in Goeree and Holt (2000b) allows the telescope parameter to be person specific and to differ for different levels of introspection. The only restriction is that the telescope parameters be strictly positive and that there be more noise at higher levels of iteration. Instead of increases in error parameters from μ to $t\mu$ to $t^2\mu$ ..., for example, the increase can be from μ to $t_1\mu$ to $t_2\mu$, where $1 < t_1 < t_2$. This formulation is flexible and allows many special cases. If t_1 is very large, the model essentially generates a μ stochastic best response to a uniform distribution over all of the other player's decisions, which is the way we start the simulations in Capra, *et al.* (1999). Similarly, if $\mu = \infty$, the choice probabilities on the left side of (2) are uniform as is the case for Stahl and Wilson's (1995) "level-0 rationality." If μ goes to zero and t goes to infinity, we have a (non-stochastic) best response to a uniform distribution, which corresponds to Stahl and Wilson's "level 1 rationality." Higher levels can be generated similarly. Roughly speaking, low values of t in our model correspond to higher rationality levels in their formulation, in the sense that the precision of the thought process is relatively insensitive to the number of iterations. Rather than assuming a fixed number of iterations, (2) allows parsimonious representation of a wide range of rationality levels.

starting with $t^{n-1}\mu$ and reducing n until reaching 1 for the final (μ) stochastic response. The outcome of this process is typically independent of the starting value of n, as long as it is larger than 5 or 6, but we use higher values just to be safe. With asymmetries, there is a noisy best response function for each player, and these functions alternate in the iteration process in (2).

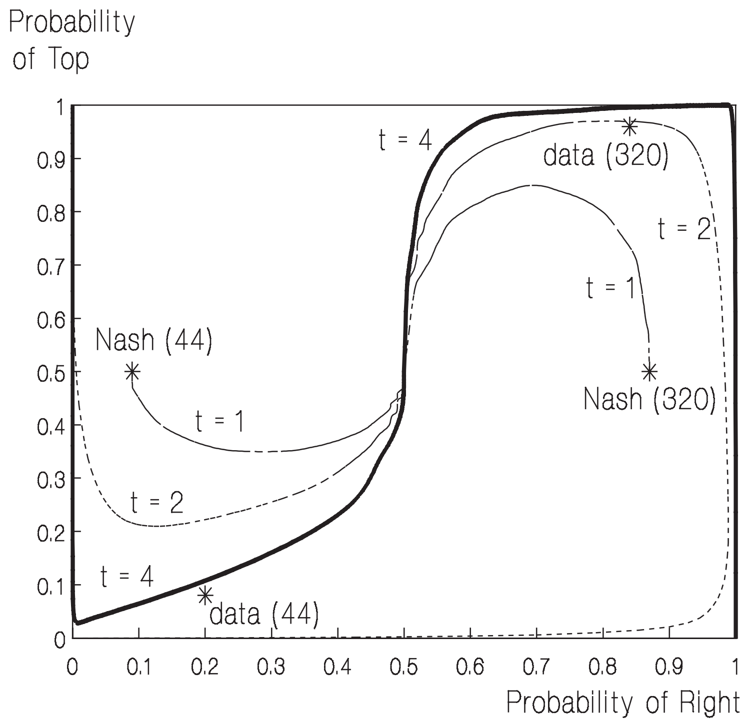

Figure 5. Loci of Introspection Predictions for the Asymmetric Matching Pennies Games For t = 4, t = 2, and t = 1 (Logit Equilibria) and for $2 < \mu < \infty$

For the matching pennies games in Table 1, the iterations produce a probability of *Top* for the Row player and a probability of *Right* for the Column player. In the symmetric version, these probabilities are 0.5, consistent with the Nash prediction and the data for the treasure treatment. Next consider the asymmetric treatment for the case where Row's payoff from a (*Top*, *Left*) outcome in the middle part of Table 1 is 320. The curves in the upper right quadrant of Figure 5 trace out the solutions for a range of μ values and for three values of t (1, 2, and 4), with the probabilities of *Top* and *Right* on the vertical and horizontal axes, respectively. The uppermost (dark) curve, which represents the t = 4 case, begins in the lower right side for μ = 2, and circles up along the right side of the graph in a counterclockwise direction, ending at the center point (0.5, 0.5) as μ goes to infinity. Notice that this line passes near the asterisk at (0.84, 0.96) which represents the actual choice proportions for the "320" contradiction treatment. The curve in the lower left part of the figure is the analogous locus for the "44" contradiction treatment (bottom part of Table 1), and again this line passes close to the asterisk representing the data proportions. The dashed curves in upper-right and lower-left regions represent the loci of solutions with t values of 1 and 2. The $t = 1$ case traces the logit equilibria, starting at the Nash mixed equilibria for the case of no error, $\mu = 0$, and again ending up in the center as μ goes to infinity.[21] Unlike the one-shot game case, many of the deviations from Nash predictions in games with repeated random matching are well approximated by a logit equilibrium (McKelvey and Palfrey, 1995; Goeree and Holt, 1999; Reynolds, 1999).

Space constraints prevent a detailed analysis of each of the ten treasure/contradiction games presented here, but in the remainder of this section we will sketch how the introspective model explains the qualitative features of conformity in the treasure treatments and divergence from Nash predictions in the contradiction treatments. To derive point predictions we need to fix values for the error and telescope parameters: we use out-of-sample estimates of $\mu = 6$ (from Capra *et al*., 2000) and $t = 4$ (from Goeree and Holt, 2000b). The introspection model tracks the data in the traveler's dilemma quite closely, producing predicted claim averages of 180 for $R = 180$ and 280 for $R = 5$, while the actual data averages were 201 and 280 respectively. The predictions of the introspective model also conform nicely with the data in the coordination game that has a range of Nash equilibria for each integer on [110, 170]: the prediction and data averages are 155 and 156 respectively for the low-effort-cost treatment and 125 and 130 for the high-effort-cost treatment. The introspection model predicts the general pattern of behavior for the matrix coordination game in Table 2 and the prevalence of Non-Nash decisions by Column in Table 3, but the effects of the treatment change are "over-predicted" in each case.[22]

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[21] It may not be a coincidence that the $t = 4$ lines fit reasonably well. We used maximum-likelihood techniques on data from 32 (unrelated) one-shot 2×2 matrix games to estimate $t = 4.1$ (.4), with the standard error in parentheses. The data were for a series of variations of prisoner's dilemma, chicken, and matching pennies games, both symmetric and asymmetric, as reported in Goeree and Holt (2000b).

[22] For Table 2, the predicted rate of coordination on the (H, H) outcome when $x = 0$ is 100%, whereas the subjects only achieved an 80% coordination rate. With $x = 400$, the predicted coordination rate on the (L, L) outcome is 83%, as compared to only 16% in the data. A similar pattern is observed for the Kreps game in Table 3: the introspection

Table 9. Two-Stage Trust and Threat Games: Data Averages (Introspection Predictions)

	Treatment	Safe Outcome (not subgame perfect) (S)	Punishment Outcome (R, P)	Risky Outcome (subgame perfect nash) (R, N)
Trust Game (Figure 3)	Treasure	16% (36%)	0% (0%)	84% (64%)
	Contradiction	52% (98%)	12% (1%)	36% (1%)
Threat Game (Figure 4)	Treasure	12% (8%)	0% (0%)	88% (92%)
	Contradiction	32% (28%)	32% (30%)	36% (42%)

Table 9 shows data for the two-stage Trust Game in Figure 3 and the Threat Game in Figure 4. The introspection predictions are shown in parentheses to the right of the data averages. These predictions generally track the data shifts away from the "risky" subgame-perfect Nash decision when the cost of punishment is reduced to 2 cents, although this shift is strongly over-predicted for the Trust Game treatments in the top two rows. The introspection model also predicts the failure of the proposers to fully capitalize on their strategic advantage in the two-stage bargaining games. For the treasure treatment with a Nash demand of 300, the data and introspection predictions are 283 and 270 respectively, whereas with a Nash demand of 450 the data and introspection predictions are 338 and 371 respectively. For the signaling game, the introspection model predicts the strong separation observed in the data, both when it is a Nash equilibrium (treasure) and when it is not (contradiction). In the private value auction, the introspection correctly predicts higher bids in the (0, 3, 6) treatment with low upside risk, as observed in the data. But the slight overbidding for the treasure treatment is not predicted, which may be due to risk aversion or some other omitted factor.

The analysis of introspection is a relatively understudied topic in game theory, as compared with equilibrium refinements and learning, for example. Our model of noisy iterated introspection does a fairly good job of organizing the qualitative patterns of conformity and

model (with $\mu = 6$ and $t = 4$) predicts that 100% of the column decisions will be on the Non-Nash decision, as compared to 68% in the data, and the heavy-handed treasure treatment that raises the (*Bottom, Right*) payoffs is predicted to draw about 99% of Row and Column decisions, whereas the data averages are 96% and 84%.

deviation from the predictions of standard theory, but there are obvious discrepancies. We hope that this paper will stimulate further theoretical work on models of behavior in one-shot games. One potentially useful approach may be to elicit beliefs directly as the games are played (Schotter and Narkov, 1998; Offerman, 1997).

VII. Conclusion

Games played only once are interesting because many games are in fact only played once; single play is especially relevant in applications of game theory in other fields, e.g. international conflicts, election campaigns, and legal disputes. The decision makers in these contexts, like the subjects in our experiments, typically have experience in similar games with other people. One-shot games are also appealing because they allow us to abstract away from issues of learning and attempts to manipulate others' beliefs, behavior, or preferences (e.g. cooperativeness). This paper reports the results of ten pairs of games that are played only once by subjects who have experience with other one-shot and repeated games. The Nash equilibrium (or relevant refinement) provides accurate predictions for standard versions of these games. In each case, however, there is a matched game for which the Nash prediction clearly fails, although it fails in a way that is consistent with economic intuition. The results for these experienced subjects show:

1) Behavior may diverge sharply from the unique rationalizable (Nash) equilibrium in a social (traveler's) dilemma. In these games, the Nash equilibrium is located on one side of the range of feasible decisions, and data for the contradiction treatment have a mode on the *opposite* side of this range. The most salient feature of the data is the extreme sensitivity to a parameter that has no effect on the Nash outcome.

2) Students suffering through game theory classes may have good reasons when they have trouble understanding why a change in one player's payoffs only affects the *other* player's decision probabilities in a mixed-strategy Nash equilibrium. The data from matching pennies experiments show strong "own-payoff" effects that are not predicted by the unique (mixed-strategy) Nash equilibrium. The Nash analysis seems to work only by coincidence, when the payoff structure is symmetric and deviation risks are balanced.

3) Effort choices are strongly influenced by the cost of effort in coordination games, an

intuitive result that is not explained by standard theory, since *any* common effort is a Nash equilibrium in such games. Moreover, as Kreps conjectured, it is possible to design coordination games where the majority of one player's decisions correspond to the only action that is not part of any Nash equilibrium.

4) Subjects often do not trust others to be rational when irrationality is relatively costless. Moreover, "threats" that are *not* credible in a technical sense may nevertheless alter behavior in simple two-stage games when carrying out these threats is not costly.

5) Deviations from Nash predictions in alternating-offer bargaining games and in private-value auctions are inversely related to the costs of such deviations. The effects of these biases can be quite large in the games considered.

6) It is possible to set up a simple signaling game in which the decisions reveal the signaler's type (separation), even though the equilibrium involves pooling. Moreover, this separation coincides with respondent's beliefs (about signal/type correlation) that are ruled out by *all* standard refinements of the sequential Nash equilibrium.

So what should be done? Reinhard Selten, one of the three game theorists to share the 1995 Nobel Prize, has said: "Game theory is for proving theorems, not for playing games."[23] Indeed, the internal elegance of traditional game theory is appealing, and it has been defended as being a normative theory about how perfectly rational people should play games with each other, rather than a positive theory that predicts actual behavior. It is natural to separate normative and positive studies of individual decision making, which allows one to compare actual and optimal decision making. This normative-based defense is not convincing for interactive games, however, since the best way for one to play a game depends on how others *actually* play, not on how some theory dictates that rational people *should* play. John Nash, one of the other Nobel recipients, saw no way around this dilemma, and when his experiments were not providing support to theory, he lost whatever confidence he had in the relevance of game theory and focused his later research in more purely mathematical topics (Nasar, 1998).

Nash seems to have undersold the importance of his insight, and we will be the first to admit that we begin the analysis of a new strategic problem by considering the equilibria derived

[23] Selten reiterated this point of view in a personal communication to the authors.

from standard game theory, before considering the effects of payoff and risk asymmetries on incentives to deviate. But in an interactive game, biases can have a reinforcing effect that drives behavior well away from Nash predictions, and economists are starting to explain such deviations using computer simulations and theoretical analyses of learning and decision error processes. There has been relatively little theoretical analysis of games played once, where learning is impossible, and the model of noisy iterated introspection presented here offers some promise in that it explains the qualitative features of the deviations from Nash predictions enumerated above. Taken together, these new approaches to a stochastic game theory enhance the behavioral relevance of standard game theory. And looking at laboratory data is less stressful than before.

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