Consumer Information and Firm Pricing:

Negative Externalities from Improved Information

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Abstract

We analyze the effect of consumer information on firm pricing in a model where consumers search for prices and matches with products. We consider two types of consumers. Uninformed consumers do not know in advance their match values with firms, whereas informed consumers do. Prices are lower the greater the proportion of uninformed consumers. Hence uninformed consumers exert a positive externality on the others, in contrast to standard results. This leads to socially excessive investment in gathering prior information when aggregate demand is price-sensitive.

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I - Introduction.

Economic intuition would suggest that informed consumers typically impart a positive externality on uninformed ones. Because firms compete more vigorously for informed consumers, the uninformed ones will benefit through lower prices and/or higher product qualities. Uninformed consumers can then free ride on the informed ones; this suggests that consumers will underinvest in information gathering, as was pointed out by Tirole (1988, p.108). Several results in the literature bear out this intuition (see details below). Our intention in this paper is to show that there are important cases where informed consumers cause market prices to rise, so that such consumers impart a *negative* externality on the informed ones. Under these circumstances, consumers will then *overinvest* in information gathering. Along the way, we also develop the theory of pricing in markets where consumers have imperfect information and must incur search costs to improve their knowledge.

Our starting point is a search model due to Wolinsky (1986).[1] The idea is that consumers care about more than just prices. They also care about the products sold by firms. Saturday afternoon shopping at the mall is about finding low prices, but it is also about finding products that suit consumer tastes. What one consumer likes, another consumer may not like so much (clothes, shoes, television sets, stereos, restaurant menus, red Cadillacs or blue Lincolns). That is, products are differentiated, and consumers value the products sold by firms differently. The goods we consider are "search" (or "inspection") goods, whose characteristics can be determined prior to purchase. If indeed we accept that consumer tastes differ, then

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[1] Wolinsky's paper was written to provide insight into monopolistic competition. See also Wolinsky (1984) and Fischer (1993) for related approaches using Salop's (1979) circle model to describe product differentiation.

a consumer who finds a product at the price she anticipated may nevertheless continue to search for a good that better matches her tastes. This brings firms into direct competition with each other, and gets around the Diamond (1971) paradox that, for homogeneous goods, the only equilibrium price is the monopoly one if consumers must incur a search cost to find out the price charged by a firm. It also means that consumers do actually search in equilibrium, so providing a search model with search (there is no search in equilibrium in Diamond).

This framework enables us to deal with differing degrees of information across consumers by allowing some consumers to be uninformed as to their product matches, while others know exactly their matches with particular firms.[2] As we show, the greater the proportion of the latter, the worse off are the former. Loosely, this is because the informed consumers reveal their tastes by always first checking out the price of the firm they prefer. This behavior renders firm demands more inelastic (an informed consumer will not check out another firm's product if the preferred firm's price exceeds the anticipated price by less than the search cost) and increases equilibrium prices. Hence the informed consumers impart a negative externality on the others. This means that consumers can overinvest in information if we allow for a prior choice of whether to become informed about products.

The result that informed consumers impart a negative externality on uninformed ones is in sharp opposition to what would happen if prior information were about prices rather than product characteristics. The literature on heterogeneous search costs illustrates the effect of prior information on prices. Heterogeneity in search costs is another way to avoid Diamond's (1971) monopoly pricing result and to

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[2] There is a small literature on imperfect consumer information on matches and firm advertising. See for example Grossman and Shapiro (1984) and Meurer and Stahl (1994).

have consumers searching in equilibrium in a market for a homogeneous good (rather than heterogeneity in tastes, as in Wolinsky, 1986), as in Rob (1985) and Stahl (1989, 1995).[3] In these papers, prices are kept down because there are enough shoppers whose search costs are zero or arbitrarily close to zero.[4] These authors find that equilibrium prices may be less than the monopoly price and tend smoothly towards marginal cost as the search cost distribution puts more weight in the neighborhood of zero. These models yield price dispersion, and those consumers who have low search costs keep searching if the prices quoted to them are too high. Hence, those with low search costs impart a positive externality on the others (a related story is the "Bargains and Ripoffs" model of Salop and Stiglitz (1977), that has been popularized as the "Tourists and Natives" model - see e.g., Carlton and Perloff (1994)).

Our result is also to be contrasted with models in which products are vertically differentiated and some consumers are imperfectly informed of product qualities (but know prices). Models in this vein include Wolinsky (1983) and the version by Tirole (1988, Ch.2.3.1.1), and Bagwell and Riordan (1991), where again a greater proportion of informed consumers benefits the uninformed ones.

In the next section we lay out the basic structure of the model and discuss the back-drop case of zero search costs. Section III describes the properties of equilibrium when all consumers face search costs and do not know their product matches. Section IV extends the analysis to allow for a fraction of consumers who do know their match values in advance, and presents the negative externality result. This

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[3] Other methods of circumventing the Diamond Paradox are described in Reinganum (1979), Burdett and Judd (1983), Bénabou (1989, 1992), Robert and Stahl (1993), and Stegeman (1989) (the last two build on Butters, 1977). See Stiglitz (1989) and Stegeman (1989) for surveys of the literature.

[4] Axell (1977) is a pioneering work in this area. Stahl (1995) gives a detailed comparison of papers that follow this route. Carlson and McAfee (1983) introduce heterogeneity in both search costs and production costs.

result is elaborated in Section V, where the model is extended to allow for price-sensitive demands and we show that the degree of investment in information is socially excessive. Section VI concludes and offers some further research directions.

II. Preliminaries.

The models that follow share a common structure that we can develop at the outset. There are two firms, 1 and 2. Each sells one variant of a horizontally differentiated product at price p_i, $i = 1,2$. Production entails constant marginal cost which is the same for both firms so that we can set it equal to zero without loss of generality.

Consumers incur a search (or sampling) cost c to check out a firm's product and price.[5] Once a firm has been sampled, recall is costless, meaning that no extra cost is incurred if a consumer buys from a firm sampled earlier. Thus search costs are 2c for a consumer who checks out both firms, regardless of which product is bought; a consumer who stops at the first firm sampled pays only c in search costs. Each consumer has income y and will buy at most one unit of the good. Any remaining income is spent on the numéraire. If a consumer does not buy, her utility is arbitrarily low, so she will always buy unless she cannot afford to (that is, if price exceeds y).[6] The total measure of consumers is one.

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[5] Our analysis is essentially static. In particular, we do not allow for consumers to have information from previous purchases, which would require a dynamic analysis. Our model is best thought of as applying to durable goods with purchases so far apart in time that any information from past purchases is irrelevant.

[6] We are implicitly assuming here that search costs are nonmonetary. The assumption that non-purchase yields an arbitrarily low utility effectively means that each consumer's reservation price exceeds y+c. (The analysis is not fundamentally changed if search costs have a monetary component - see below.) Note price will never exceed income in equilibrium. In some cases price will equal income: we refer to such

Consumers differ in their valuations of the goods sold by the two firms. Gross of any search costs, consumer ℝ's (indirect) utility conditional on purchasing good i at price p_i is:

$$u_{Ri}(p_i) = y - p_i + \varepsilon_{Ri} .$$ (1)

where $y-p_i$ is the amount of numeraire consumed and ε_{Ri} is consumer ℝ's match value with product i.[7] A fully-informed consumer will buy the product for which the realization of (1) is greatest. The match values, ε_{Ri}, are realizations of random variables that are identically and independently distributed across consumers and firms, with a common density function f whose support is an interval [a,b] of the extended real line. Let F be the corresponding distribution function. We shall also assume:

A1: The density f is twice continuously differentiable and logconcave on [a,b].

Logconcavity is a weaker requirement than concavity (but stronger than quasiconcavity), and is satisfied by most commonly used densities (including the normal, the exponential and the double-exponential; see Caplin and Nalebuff, 1991, for a more complete list). Logconcavity implies the monotone (increasing) hazard rate property (see footnote 12 below). Caplin and Nalebuff (1991) establish that

cases as the Diamond Paradox. Our primary interest is in the cases where the paradox does NOT hold. In our formulation the problem noted by Stiglitz (1979) - that the market may fail to exist if the first search is costly - does not arise. In the Stiglitz version, firms charge the consumer reservation price, so that consumers never buy because they would expect to pay a full price equal to the reservation price plus the cost of the first search, and so never enter the market. In our version, income caps the maximum amount consumers will pay.

[7] The numeraire is purchased under perfect information.

logconcavity of the joint density of match values guarantees the existence of a price equilibrium in the Perloff and Salop (1985) model of oligopoly with differentiated products (see also Anderson, de Palma, and Thisse, 1992). We refer below to this model (which does not entertain search costs) as the *benchmark model*.

The benchmark model is a limit case of ours, and it is useful for what follows to derive its symmetric equilibrium price. For two firms, and given the utility function (1) with no search costs, the demand facing Firm 1 if it charges p_1 while Firm 2 charges p^* is

$$D_1(p_1, p^*) = \int_a^b F(\epsilon + p^* - p_1) f(\epsilon) d\epsilon. \tag{2}$$

A symmetric equilibrium price $p^* < y$ must satisfy

$$D_1(p^*, p^*) + p^* \frac{\partial D_1}{\partial p_1}(p^*, p^*) = 0. \tag{3}$$

By symmetry, $D_1(p^*, p^*) = \frac{1}{2}$, and so the equilibrium price with no search costs, p_n^*, is:

$$p_n^* = \frac{1}{2 \int_a^b f(\epsilon)^2 d\epsilon}. \tag{4}$$

The following result is proved in Anderson, de Palma, and Thisse (1992), using the analysis of Caplin and Nalebuff (1991).

Proposition 1: *Under A1, if search costs are zero and y is above (resp. below) p_n^*, there exists a unique symmetric equilibrium price given by p_n^* (resp. y).*

Let us now return to the model with positive search costs. We seek equilibria at which both firms charge the same price.[8] Since a consumer expects no gain from price differences when she searches, her sampling behavior in equilibrium is entirely based on her information about her matches with products. If she is indifferent between sampling either firm, we assume that each is sampled with probability one half. We consider two alternative assumptions on consumer information. The first corresponds to Wolinsky (1986).

(a) *Uninformed consumers*. The consumer does not know her realization $_{,R1}$ ($_{,R2}$) unless she has sampled Firm 1 (Firm 2). She samples a first firm at random. If she gets a low draw she then samples the other firm if the expected utility gain exceeds the search cost. She returns to the first firm if she realizes an even worse utility value from the second firm.

(b) *Fully-informed consumers*. The consumer knows the realizations of both her match values in advance. She expects $p_1 = p_2$, so she first samples the firm for which $_{,Ri}$ is greater.

Section III considers the pure case (a). Section IV adds consumers of type (b), with a fraction k of the latter.

In keeping with much of the literature, the equilibria considered throughout the paper are Perfect Bayesian equilibria (see Fudenberg and Tirole, 1991). For firms, this simply means that each maximizes

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[8] There are also degenerate equilibria at which both firms set prices above y, so that no one buys. In such Nash equilibria, if consumers expect prices above y, they do not enter the market, so that firms have no incentive to change their prices. We ignore such equilibria in the sequel. We show in a later footnote that the only equilibrium (with prices no larger than y) is the symmetric one. The existence of a unique, symmetric, pure-strategy equilibrium should be contrasted with models of heterogeneous search costs in which there may be only asymmetric equilibria (e.g. Salop and Stiglitz, 1977, and Rob, 1985), or else only equilibria in mixed strategies (e.g. Stahl, 1989, 1995).

its profit given the anticipated price of the rival firm and consumer search behavior. A consumer's information set is characterized by the prices and match values already observed. Each consumer maximizes her expected utility at each information set, given her expectations of the unknown prices and match values. Price expectations are consistent with equilibrium strategies of firms and are unaffected by prices already observed. Hence if a consumer has sampled one firm and finds out that its price is different from the equilibrium price, she follows an optimal search rule, still expecting the other firm to charge the equilibrium price.

III. Uninformed Consumers.

In this section it is assumed that no consumer knows her match value with a product until she samples the firm selling it. The model is basically that of Wolinsky (1986) with two firms. Henceforth we drop the subscript R on valuations to ease notation.

In order to calculate the symmetric equilibrium, we must find Firm 1's demand (Firm 2's demand is given in a similar manner). We first need to characterize consumer behavior given the equilibrium price p^* is expected to prevail at each firm. Half the consumers search Firm 1 first, and actually find its price to be p_1 (which is not necessarily equal to p^*, since we must characterize consumer reactions to prices off the equilibrium path) and match ϵ_1. If a consumer who starts at Firm 1 then searches Firm 2 (where she correctly expects the equilibrium price p^* to prevail), she will return to Firm 1 if $-p_1 + \epsilon_1 > -p^* + \epsilon_2$, or $-\delta + \epsilon_1 > \epsilon_2$, where $\delta \equiv (p_1 - p^*)$ is the premium charged by Firm 1. Therefore the expected benefit from searching Firm 2 is:

$$\int_{\epsilon_1 - \Delta}^{b} \left(\epsilon - \epsilon_1 + \Delta \right) f(\epsilon) d\epsilon. \tag{5}$$

Define $\hat{x}$ as the critical value of the lower bound of the integral such that a consumer holding $\hat{x}$ is indifferent between searching again and sticking with Firm 1. Thus $\hat{x}$ is defined by:

$$\int_{\hat{x}}^{b} (\epsilon - \hat{x}) f(\epsilon) d\epsilon = c \tag{6}$$

(cf. Wolinsky, 1986, eq. 3).

It is readily verified that the left-hand side of (6) is continuous and decreasing in $\hat{x}$, is infinite at $\hat{x} = -4$, and is zero at $\hat{x} = b$, so that $\hat{x}$ is uniquely determined, and consumers for whom $(\epsilon_1 - \delta) > \hat{x}$ will not go on to search Firm 2 (for such consumers, (5) is less than c). Hence a fraction $[1 - F(\hat{x} + \delta)]$ of the consumers searching Firm 1 first stop there.

Since $\hat{x}$ is decreasing in c, fewer consumers search when the cost of search goes up. Note also that if $\hat{x} < a - \delta$, then no consumer will search again. Thus, if $\hat{x} < a$, no consumer would want to search beyond the first firm unless she anticipated a large enough price difference. In this case, the only equilibrium is that both prices equal consumer income, y, since at any lower common price, either firm could increase its price slightly without losing any customer.[9] This is the Diamond Paradox. We now study the model when $\hat{x} > a$.[10]

Firm 1 expects consumers who start at Firm 2 to find a price of p^*. When these consumers decide

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[9] If c were a monetary cost, the equilibrium price would simply be y-c. The same comment applies to the other models.

[10] The case $\hat{x} = a$ is described at the end of this Section.

whether to go on and sample Firm 1, they face a problem similar to that of consumers who start at Firm 1, which we just analyzed. Since they expect Firm 1 to play its equilibrium strategy, and thus to also charge p*, the expected fraction of them choosing to stay with Firm 2 without further search is $[1- F(\hat{x})]$.

For those who do search both firms, the indifferent consumer is defined by $u_1(p_1) = u_2(p_2)$, or

$$\varepsilon_1 = \varepsilon_2 + \Delta .$$ (7)

The partition of the $(\varepsilon_1, \varepsilon_2)$ space is given in Figure 1, and the firms' demands can be directly calculated from the Figure.

<center>∟INSERT FIGURE 1 ⌐</center>

The square $[a,b] \times [a,b]$ is divided into four rectangles. Each represents a different outcome for a consumer's search strategy. Area I ($\varepsilon_1 > \hat{x} + \Delta$) and $\varepsilon_2 > \hat{x}$) corresponds to consumers who stop at the first firm sampled. Those consumers who sample both firms no matter where they start are represented by Area III ($\varepsilon_1 < \hat{x} + \Delta$) and $\varepsilon_2 < \hat{x}$). Finally, Area II (Area IV) corresponds to consumers who sample both firms if and only if they start at Firm 2 (Firm 1).

In order to derive Firm 1's demand, we also draw the indifferent consumer relation (7). Consumers who have sampled both firms and whose match value pair is below that line buy from Firm 1. Thus Firm 1's demand comprises half the consumers in Area I, all consumers in Area II, and those in Area III who

$$D_1(p_1,p^*) = \frac{1}{2}\left[1 - F(\hat{x})\right]\left[1 - F(\hat{x} + \Delta)\right]$$
$$+ F(\hat{x})\left[1 - F(\hat{x} + \Delta)\right] + \int_{*}^{\hat{x}+\Delta} F(\epsilon - \Delta)f(\epsilon)d\epsilon, \tag{8}$$

are below the dashed line:

where we recall that $\Delta \equiv (p_1 - p^*)$, so that demand depends only on price differences. Integrating by parts and letting $u = \epsilon - \Delta$, (8) can be rewritten as

$$D_1(p_1,p^*) = \frac{1}{2}\left[1 - F(\hat{x})\right]\left[1 - F(\hat{x} + \Delta)\right] + F(\hat{x}) - \int_{*-\Delta}^{\hat{x}} F(u + \Delta)f(u)du. \tag{9}$$

(Note that these expressions hold no matter what the sign of Δ is.)

A symmetric equilibrium price for the uninformed consumer model, $p_u^* \equiv (0,y)$, must satisfy (3) with $p^* = p_u^*$ (the subscript u denotes that all consumers are uninformed), so that[11]

$$p_u^* = \frac{1}{\left[1 - F(\hat{x})\right]f(\hat{x}) + 2\int_{*}^{\hat{x}} f(\epsilon)^2 d\epsilon} \tag{10}$$

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[11] There are some conceptual difficulties in formulating what an asymmetric equilibrium would look like. In particular, it is not clear what a consumer should believe when confronted with a deviation - or even if she could detect a deviation if a firm deviates to its competitor's candidate equilibrium price. One way around the problem is to assume that a consumer finds out the identity of the firm only when she samples it (so that she is a priori indifferent between checking out either firm); she can then also tell if it has deviated. Under this assumption, the unique equilibrium is the symmetric one by the following argument. Suppose there were an asymmetric equilibrium. Then it would satisfy $D_1 + p_1 \partial D_1 / \partial p_1 = D_2 + p_2 \partial D_2 / \partial p_2 = 0$. Since $D_1 = 1 - D_2$, we have $\partial D_1 / \partial p_1 = - \partial D_2 / \partial p_1 = \partial D_2 / \partial p_2$, and therefore
$$(p_1 - p_2) \partial D_1 / \partial p_1 = D_2 - D_1.$$
Now, since $p_1 > p_2 \Rightarrow D_1 < D_2$, the only possible solution to the first order conditions has $p_1 = p_2$. Similar analysis rules out one firm setting a price equal to y with the other firm setting a lower price; a price of zero by either firm clearly cannot be an equilibrium.

(cf. Wolinsky, 1986, eq. 16: thanks to the assumption that all consumers buy, we find an explicit solution for the equilibrium price whereas Wolinsky's is in implicit form).

It is instructive to determine how the candidate equilibrium price evolves with the search cost. Wolinsky (1986) notes that if match values are iid uniform (the case for which he proves the existence of a price equilibrium) and under monopolistic competition, price is strictly increasing in the search cost. Intuition would suggest that this should be the case more generally, at least under reasonable conditions. As we now show, the "reasonable condition" here is the logconcavity of 1-F:

Proposition 2: *Assume f is differentiable on [a,b] and let $\hat{x} > a$. The price p_u^* is increasing in c if and only if 1 - F is logconcave on [a,b]. Furthermore,*

$$\lim_{c \to 0} p_u^* = \frac{1}{2\int_a^b f(\epsilon)^2 d\epsilon}. \tag{11}$$

which is the equilibrium price p_n^* of the benchmark (no search cost) model (see (4)).

Proof: The price p_u^* depends on the search cost c through $\hat{x}$ (which is monotonically decreasing in c). With f differentiable, $dp_u^*/d\hat{x}$ is well defined and has the sign of

$$- [1 - F(\hat{x})]f'(\hat{x}) - f(\hat{x})^2. \tag{12}$$

This expression is negative over the whole interval if and only if 1-F is logconcave. Since f is differentiable, it is continuous. As c $\to$ 0, $\hat{x} \to$ b, and (11) follows from (10). **Q.E.D.**

The condition shown in Proposition 2 regarding logconcavity of 1-F is related to our assumption A1 by the following:

Corollary 1: *Under A1, the price p_u^* is increasing in the search cost.*

Proof: Under A1 f is differentiable and logconcave on [a,b]. This guarantees 1 - F is logconcave,[12] and the result then follows from Proposition 2. **Q.E.D.**

The search cost in this model is the cost of getting more information. As this cost rises, consumers are more likely to stick with the first firm sampled. This gives firms more market power and they raise prices (Corollary 1). Clearly consumers are better off with lower search costs. They benefit from the direct effect of being more likely to get a better match, and the indirect effect of lower prices. Another type of cost of getting better information is introduced in the next Section. As we argue there, lower costs of *pre-search* information on product matches (as opposed to the lower costs of search considered here) may actually decrease consumer welfare.

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[12] Using the argument of Anderson, de Palma, and Nesterov (1995), first write

$$\frac{f(b) - f(x)}{1 - F(x)} = \frac{\int_x^b f'(u)\,du}{\int_x^b f(u)\,du} \leq \frac{f'(x)}{f(x)}$$

where the inequality follows from the logconcavity of f (i.e., f'/f is increasing). Hence $0 \leq f(b)f(x) \leq f'(x)[1 - F(x)] + f^2(x)$, so 1 - F(x) is logconcave (see also Bagnoli and Bergstrom, 1989, and Borell, 1975).

The intuition behind the second part of the Proposition is that almost all consumers search both firms when search costs are low, regardless of what they find first. Since they can then costlessly recall the preferred product, this is just as if there were no search costs (i.e., the benchmark model).

The second part of Proposition 2 is important to the analysis of equilibrium existence that follows because of the next result:

Corollary 2: *Under A1, p_u^* is greater than or equal to the symmetric equilibrium price, p_n^*, of the benchmark model of no search costs.*

Proof: By Corollary 1 and the observation that (11) is equal to (4). **Q.E.D.**

The following proposition gives sufficient conditions for p_u^* to be an equilibrium price.[13]

Proposition 3: *Suppose A1 holds, f is increasing, and that all consumers are uninformed. Then there exists a unique single-price equilibrium*:

(i) If $p_u^* < y$ and $\hat{x} > a$, both firms charge p_u^*.

(ii) If $p_u^* \$ y$ or $\hat{x} < a$, both firms charge y.

(iii) If $p_u^* < y$ and $\hat{x} = a$, both firms charge p if and only if $p \ O \ [1/f(a), y]$.

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[13] The condition that f be strictly increasing is stronger than is required for these results, but this condition renders tractable the analysis of existence of equilibrium for the (more complex) model of the next section.

The proof is in the Appendix. Let us briefly discuss the case of high search costs. As $\hat{x}$ tends to a, the right-hand side of (10) tends to 1/f(a). Thus, if f(a) = 0, p_u^* goes to infinity as c goes to (E, - a) (see (6)). This means that the equilibrium price hits the budget constraint of consumers for some $\hat{x} >$ a. In contrast, if f(a) is sufficiently large (greater than 1/y), then the equilibrium price is $p_u^* <$ y for c < (E, - a) (i.e. for $\hat{x}$ > a), and is equal to y for c > (E, - a). Finally, the equilibrium price takes any value in [1/f(a),y] for c = (E, - a). To illustrate, consider the increasing exponential distribution, f(,) = exp[,] for , 0 [-4, 0] (so a = -4 and b = 0). Here $\hat{x}$ satisfies exp($\hat{x}$) -$\hat{x}$ - 1 = c, and there are no restrictions on c. The equilibrium price is p_u^* = exp(-$\hat{x}$), as long as this is less than y, and is y otherwise.

IV. Informed and Uninformed Consumers.

We now consider what happens if the consumer population comprises a fraction 1-k of uninformed consumers (like those considered in the previous Section) and k fully-informed consumers. The informed consumers know both valuations in advance. The uninformed consumers know neither valuation until they have sampled the firms. Let $D_u(p_1,p^*)$ as given by (9) be the average demand for Firm 1 by uninformed consumers. Each of the informed consumers first checks out the firm where she knows she has a better match. Since consumers expect firms to set the same price, and by symmetry of the distribution of matches, half the informed consumers initially check out each firm. They will not shift if the actual price observed is less than c above what was expected. Hence for p_1 # p^* + c the demand addressed to Firm 1 is:

$$D_1(p_1,p^*) = (1 - k)D_u(p_1,p^*) + k/2, \tag{13}$$

and the candidate equilibrium price, if strictly below y, is given by $p^* = p_k^*$ where p_k^* is the common

equilibrium price in the model with k informed consumers, so

$$p_k^* = \frac{1}{(1-k)\left[1 - F(\hat{x})f(\hat{x}) + 2\int_a^{\hat{x}} f(\epsilon)^2 d\epsilon\right]}.$$

(14)

This is just $1/(1-k)$ times the price p_u^* as given in (10), so that if $k = 0$ we have the uninformed consumer

equilibrium, and as $k \to 1$, price goes to infinity so that the budget constraint eventually becomes binding

(we discuss this limit in more detail below). We now extend Proposition 3 to allow for informed

consumers.

Proposition 4: *Suppose A1 holds, f is increasing, and that a fraction k of consumers are informed*

of their match values. Then there exists a unique single-price equilibrium:

(i) If $p_k^* < y$ and $\hat{x} > a$, both firms charge p_k^*.

(ii) If $p_k^* \$ y$ or $\hat{x} < a$, both firms charge y.

(iii) If $p_k^* < y$ and $\hat{x} = a$, both firms charge p if and only if $p \in [1/(1-k)f(a), y]$.

Proof: As in the proof of Proposition 3, it suffices for (i) to show that p_k^* maximizes profit over the interval

$[h,+\infty)$, where $h \equiv \max\{0, p_k^* + a - \hat{x}\}$ (for $0 \# p_1 < p_k^* + a - \hat{x}$, the demand from informed and uninformed

consumers is constant and profit is increasing in p_1). In the previous proof, it is also shown that under the

present assumptions, D_u is concave in p_1 if p_1 satisfies $\hat{x} + \delta \# b$. Furthermore, for $p_1 \# p_k^* + c$ (i.e.

for $\delta < c$) the informed consumers' demand is constant.

We now show that $\hat{x}+c$ is always less than b, and thus, for $p_1 \# p_k^* + c$, D_1 is concave in p_1 since

it is the sum of two concave functions. First note that $\hat{x}+c = b$ for $c = 0$. Using (6), the derivative of $\hat{x}+c$

with respect to c is $1 + 1/[F(\hat{x})-1] \# 0$. Hence $\hat{x}+c \# b$ for all values of c. Since demand is concave for

$p_1 \# p_k^* + c$, profit is also concave and is maximized by p_k^* given by (14) for this range of prices. We

now show that profit is decreasing for $p_1 > p_k^* + c$ and therefore reaches a global maximum at p_k^*.

The price p_k^* defined by (14) is clearly larger than p_u^*. Thus, using Lemma A.1 in the Appendix

(if demand depends only on the price difference, and profit is decreasing for prices above the rival's price,

then profit is decreasing for prices above the rival's price when the rival sets a higher price), $p_1 D_u(p_1, p_k^*)$

is decreasing for $p_1 > p_k^* + c$. The demand from informed consumers is no longer constant: once they find

out that p_1 is larger than $p_k^* + c$, some may decide to go to Firm 2. For those who stay we must have $, _1 -$

$p_1 > , _2 - p_k^* - c$. Thus the informed consumers' demand is:

$$\int_a^b F(\epsilon - p_1 + p_k^* + a)\hat{f}(\epsilon)d\epsilon . \tag{15}$$

This is simply Firm 1's demand in the benchmark model with no search costs if Firm 2's price is $p_k^* + c$ (see

(2)). Since $p_k^* > p_u^*$ (for $k > 0$) and Corollary 2 shows that p_u^* exceeds the benchmark price (for $c > 0$),

then so does $p_k^* + c$ exceed the benchmark price p_n^* given in (4). We therefore know by Lemma A.1 that

Firm 1's profit from informed consumers must be decreasing in p_1 for $p_1 > p_k^* + c$. Thus overall profit is

decreasing since it is a convex combination of two decreasing functions.

The proofs for cases (ii) and (iii) are similar to those in Proposition 3. **Q.E.D.**

The sufficient conditions for existence are rather strong and they are clearly not necessary. In

particular, the assumption that f is increasing ensures that D_u is concave in own price, so that, since demand

from uninformed consumers is constant for prices less than c above the rival's price, overall demand is concave for this range of prices, which renders profit concave as well. To see what can go wrong for decreasing densities (even though A1 is satisfied) consider the decreasing exponential, $f(x) = e^{-x}$ for x 0 $[0,4)$. The candidate equilibrium price is then simple to calculate; from (14) we have $p_k^* = [1/(1 - k)]$ (note that this is independent of c since, by Proposition 2, p_u^* is independent of c, as 1 - F is loglinear). The corresponding candidate equilibrium profit is simply $1/[2(1 - k)]$. Now consider a deviation by one firm to $p_k^* + c$.[14] To calculate the deviation profit, first note that D_u, as given by (8) or (9), is $(1/2)e^{-)}$ with $)$ = c. This is the demand from uninformed consumers, while the demand from informed consumers is still 1/2 since price has been increased by less than c (and consumers find this out only when they sample the firm). The deviation profit is thus

$$\left(\frac{1}{1-k} + c\right)\left(\frac{k}{2} + \frac{1}{2}(1 - k)e^{-c}\right)$$

which exceeds the equilibrium profit if and only if

$$(1 + (1 - k)c)(k + (1 - k)e^{-c}) > 1.$$

It is readily verified that this holds for k = 1/2 if c is sufficiently close to 1, so that p_k^* is not an equilibrium price for this case, and indeed it is not an equilibrium price for k large enough. Furthermore, if $y > p_k^*$ (as in Proposition 4(i)), the profit derivative when both firms charge y, as given by (3) (for $p^* = y$), is strictly

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[14]We assume $c < 1$ so that, from (6), $\hat{x} = -\ln c > 0$. Otherwise we are in the Diamond paradox case covered by Proposition 4(ii).

negative so that y cannot be an equilibrium price either (a slight decrease in price would be profitable). This shows that no pure strategy symmetric equilibrium exists.

In order to guarantee that an equilibrium exists, demand from uninformed consumers must decrease sufficiently fast when a firm raises its price so that such a deviation is not profitable, in spite of the extra revenue extracted from the captive informed consumers. An increasing density guarantees this even if there is a large informed consumers population. This is a strong sufficient condition, especially if informed consumers are few. For instance, it can actually be shown that with only uninformed consumers ($k = 0$) and a decreasing exponential distribution of matches (as above), there is a symmetric pure strategy equilibrium as given by the candidate equilibrium expression (10), in which both firms charge 1.

Returning now (and henceforth) to the assumptions of Proposition 4, since $p_u^* > 0$, the equilibrium price (16) is clearly increasing in k:[15]

Proposition 5: *Under the assumptions of Proposition 4(i), the equilibrium price is strictly increasing in the fraction of informed consumers.*

The result of Proposition 5 implies that the informed consumers impart a negative externality on the uninformed ones. That is, more informed consumers lead to higher prices. The reason is that uninformed consumers are the source of elasticity in demand - the smaller the fraction of them, the fewer customers will be lost following a price hike.

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[15] The price p_k^* is also increasing in the search cost c under the assumptions of Proposition 2.

This result is in stark opposition to the usual reasoning about the effects of informed consumers. It is usually argued that there is too little investment in information gathering because the informed consumers positively affect the uninformed ones, so the latter free ride. Let us consider these arguments. The major contexts in which information differences across consumers have been considered are the quality choice model summarized in Tirole (1988) (which draws on Wolinsky, 1983), the quality-signalling model of Bagwell and Riordan (1991), and the price dispersion model of Stahl (1989) and others. In the quality choice model, the issue is whether firms will produce high or low quality, for a given price that is observed by all consumers. Some consumers are informed about quality, while others are not informed until they have purchased the good (so that quality is an experience characteristic for the uninformed consumers). For prices in a medium range, the greater the fraction of informed consumers, the greater the equilibrium probability that firms produce the high quality. In the quality-signalling model, quality is exogenous and observed by only a fraction of consumers, and the firm uses its price (observed by all consumers) to signal its quality. A low-quality firm also has low cost, and so loses more profit at a high price than a high-quality (and high-cost) firm. The distortion in prices from signalling a high quality is greatest when there are only uninformed consumers; the price for high quality falls as the fraction of informed consumers rises because the benefit to a low-quality firm from mimicking the high-quality price is less.

Prices are not observed in the Stahl (1989) model with heterogeneous search costs, but since some consumers have zero search costs they are effectively informed about prices. The more such consumers there are, the more firms will compete for them, as manifested in lower prices in the equilibrium mixed strategy. In our model, observing price is costly to all consumers, and information differences pertain only to matches. In contrast to Stahl, in our model informed consumers shop *less* than uninformed ones (not

at all in equilibrium). Fewer shoppers means higher prices in both models.

To see that there is an externality to consumers in the collection of pre-search information in our context, suppose that different consumers have different costs of gathering information. The fraction of consumers who will get fully informed is determined by the benefit of full information, which entails getting the best match for sure and obviating any further search costs. Although the information decision is privately optimal, because each consumer does not take into account the adverse effect of her improved information on prices, there will be *too many informed consumers from the perspective of consumers taken as a group*.[16]

We now consider the limit case of $k = 1$ in more detail. In this case, all consumers know their match values in advance, and they also know which firm sells which product. This information is the undoing of the consumers. To see why, suppose that the consumers anticipate prices p_1 and p_2, one of them being strictly less than y. Each consumer then chooses the firm offering her the highest utility. In equilibrium, she is not disappointed and buys from the first firm sampled. Thus a firm expects to be the best deal for a consumer who samples it. That consumer could either buy, or else incur the search cost and check out the other firm. Clearly, if the price of the first firm exceeds the anticipated price by some amount less than c, she will still buy from it as long as the actual price does not exceed y. Hence if the firm whose price is less than y is sampled, it can raise its price by some small amount above the anticipated price without losing any

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customers. Thus there is no equilibrium at which a firm sampled with positive probability has a price strictly below y. Since the firm with the lower anticipated price (below y) would always have some customers, the only equilibrium outcome is for both firms to price at y.

This discussion is summarized in:

Proposition 6: *If all consumers are informed, the unique equilibrium outcome has both firms charging y.*

This is the analogue to the Diamond result for homogeneous goods, and arises here even though products are heterogeneous. Firms know that consumers will first sample the firm they like best, and therefore that consumers would not switch if faced with a price slightly higher (but less than c higher) than expected. The firms can use this information to extract as much surplus as is possible given the budget constraint.

V. Overinvestment in information with price-sensitive demands.

There are two notable restrictions in the analysis above. The first is that the income level y has been used to bound consumer spending from above, so that the price that corresponds to the Diamond paradox is y. The second is that the total surplus is independent of the price level. This implies that total surplus per uninformed consumer (and per informed consumer) is unaffected by increases in the fraction of informed consumers, k, since the consumer surplus lost is exactly balanced by an increase in firm profits. In this Section we generalize the model to price-sensitive individual demands. As well as constituting a useful

broadening of the model, this extension effectively replaces the upper bound, y, with the monopoly price

on the individual demand curve. Second, once there is some elasticity in demand, an increase in k will be

seen to reduce social surplus per consumer for those whose prior information has not changed. Since

demand is totally inelastic in the model above, the socially optimal degree of information investment (that

which maximizes the sum of consumer surplus and firm profits) is attained in equilibrium: higher prices are

simply a transfer from consumers to firms.[17] When we extend the model to allow for price-sensitive

individual demands, each individual, once she chooses a product, buys an amount that depends (negatively)

on the price charged by the firm. Then the price rise that follows the information investment of an additional

consumer will cause total consumer benefits to fall by more than firm profits rise - there is a direct welfare

loss due to higher prices. Thus there will be *excessive investment in prior information from the social*

welfare perspective.

To introduce price-sensitivity, rewrite (1) as:

$$u_{f,i}(p_i) = y + v(p_i) + \epsilon_{f,i} \tag{18}$$

where $v(p_i)$ is the consumer's conditional surplus from buying good i. We assume that $v(.)$ is positive,

strictly decreasing, strictly convex and twice continuously differentiable on $[0, \bar{p}]$ with $v(p) = 0$ for all $p \geq \bar{p}$

(so that $\bar{p}$ is the demand intercept). This surplus is associated with a conditional individual demand of $q(p_i)$,

which, by Roy's identity, is equal to $-v'(p_i)$. That is, if a consumer buys good i, she buys a quantity $q = -$

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[17] The optimal degree of prior information investment involves equating the benefit to informing one
more consumer with the extra cost - this is exactly the private incentive too.

$v'(p_i)$ of it as long as $- p_i v'(p_i) < y$ (as assumed below). Let $\mathcal{O}(p_i)$ $(= q'(p_i)p_i/q(p_i) < 0)$ be the elasticity of the conditional demand function, with $\mathcal{O}$ strictly decreasing in price (as is standard for many demand functions, such as linear demand: see also Anderson, de Palma, and Nesterov, 1995). Furthermore, let there exist a finite price $p_m > 0$ such that $\mathcal{O}(p_m) = -1$: this is just the standard monopoly price and will be shown to be the "Diamond price" (which firms will charge if no consumer searches). We assume that consumer expenditure at price p_m is less than y (hence this is true at all relevant prices), so that income constraints are never binding.

The convenient property of this formulation is that the revenue per consumer associated to the (conditional) individual demand is concave in v_i $/$ $v(p_i)$. Indeed, let $R(v_i) = p_i q(p_i)$ denote this revenue (where the dependence of p_i on v_i has been suppressed: recall that $p_i(v_i) = v^{-1}(v_i)$). Hence $R' = -(\mathcal{O}+1)$, which is negative in the relevant price range below the monopoly price, and R' is strictly decreasing since $\mathcal{O}$ is strictly increasing in v_i. Now, it suffices to note that any demand function D_1 derived in the previous sections constitutes the fraction of consumers buying from Firm 1 when the price variables are now replaced by minus the conditional surpluses, i.e., consumers base their choices on surpluses. The previous analysis is a limit case of this transformation, with $v(p_i) = r - p_i$ and r $(> y)$ a positive constant. M o r e specifically, for the benchmark model with no search costs, a consumer will buy from Firm 1 if $v_1 + \iota_1$ exceeds $v^* + \iota_2$, where v^* is the equilibrium conditional surplus level at Firm 2, so that the fraction of consumers buying from Firm 1 is given by (2), with v^* replacing $- p^*$ and v_1 replacing $- p_1$. Likewise, the consumer search problem is analogous to that of Section 3, and the consumer will search if $\iota_1 - \mathbf{)} < \hat{x}$, where $\hat{x}$ is given by (6) and $\mathbf{)} = v^* - v_1$. Thus $\mathbf{)}$ is the expected conditional surplus premium from switching to Firm 2. The fraction of uninformed consumers buying from Firm 1 is then given by the right-

hand side of (8) (equivalently (9)).

If we write Firm i's profit as $B_i(v_i, v_j) = R(v_i)D_i(v_i, v_j)$, i,j = 1,2, i ... j, then we now seek a Nash equilibrium at which the v_i's are the strategic variables. A symmetric Nash equilibrium now entails a level of conditional surplus v* such that

$$R'(v^*)D_1(v^*, v^*) + R(v^*)\frac{\partial D_1}{\partial v_1}(v^*, v^*) = 0. \tag{19}$$

(cf. (3)) so that all previous equilibrium expressions are now replaced with -R/R' on the left-hand side (leaving the right-hand side unchanged). For the general case with a fraction k of informed consumers, the equilibrium surplus value is the unique solution to (see (14))

$$\frac{- R(v_k^*)}{R'(v_k^*)} = \frac{1}{(1-k)\left[1 - F(x)]f(x) + 2\int_a^x f(\epsilon)^2 d\epsilon\right]}. \tag{20}$$

Note that for the special case of totally inelastic demand, $R = p_i$ and $v(p_i) = r-p_i$, so that $R(v_i) = r-v_i$, and hence $-R/R' = p_i$, as in (14). Where demand has some elasticity, -R/R' is given by $p_i q(p_i)/(O(p_i)+1)$, which tends to infinity as p_i tends to p_m from below, so that price is always below the monopoly price. Furthermore, since R is concave, -R/R' is a decreasing function of v_i, and thus an increasing function of p_i. This means that the equilibrium price rises as the right-hand side of the relevant equilibrium expression increases. In particular, the equilibrium price rises with c if and only if 1-F is logconcave (see Proposition 2), and price is strictly increasing in the number of informed consumers (see Propositions 5 and 8). Furthermore, the counterpart to Proposition 6 is that both firms charge p_m when all consumers are informed about their matches. The following equilibrium existence result is proved in the Appendix.

Proposition 7: *Suppose A1 holds, f is increasing, a fraction k of consumers are informed of their match values, and individual conditional surplus, v, has the properties given above. Then there exists a unique single-price equilibrium*:

(i) If $\hat{x} > a$, both firms charge the price $p(v_k^*) = v^{-1}(v_k^*)$, where v_k^* is given by (18).

(ii) If $\hat{x} < a$, both firms charge p_m .

(iii) If $\hat{x} = a$, both firms charge $p(v)$ if and only if $v \in [v(p_m), v_k^*]$.

In comparison to Proposition 4, the assumption of a downward sloping demand curve obviates the need for the income constraint y. If $\hat{x} < a$, no consumer searches and we have the Diamond paradox that both firms charge the monopoly price p_m. In order to determine the welfare properties of the equilibrium, it is useful to define the conditional total surplus, S, as the sum of conditional consumer surplus, v_k^*, plus both firms' equilibrium profits. (The full expression for total surplus also accounts for search costs, expected match values, plus any cost of obtaining prior information as introduced below.)

Proposition 8. *If $\hat{x} > a$, the equilibrium price p_k^*. is increasing in the fraction of informed consumers, k. Hence conditional total surplus, S, is decreasing in k.*

Proof: The right-hand side of (18) is increasing in k, so that v_k^* is decreasing since -R/R' is a decreasing function by the concavity of R in v. Hence p_k^* is increasing since v is decreasing in p. The next step is to show that S is decreasing in k, i.e. $v(p_k^*) - p_k^* v'(p_k^*)$ is decreasing in p_k^* (where the term $-p_k^* v'(p_k^*)$ denotes firm revenues, which equal total profits given the equilibrium price is symmetric and the assumption of zero

production costs). The derivative of S with respect to p_k^* is simply $-p_k^* v''(p_k^*)$, which is negative under the assumption that v is strictly convex. **Q.E.D.**

The latter result is simply that deadweight loss increases in price when demand slopes down. Here this increase in deadweight loss implies that consumers who gather prior information exert a negative externality. With this result in hand, we can now address the suboptimality of prior investment in information. To this end, suppose there is a continuum of consumers and let us rank consumers in order of increasing costs of gathering prior information. Let the cost for consumer k be $N(k)$, which we assume for simplicity to be continuous and strictly increasing in k. Each consumer weighs her cost of obtaining prior information against her private benefit b(c). This benefit is positive and comprises the expected incremental match value achieved plus lower expected search costs (since the consumer immediately goes to the firm where she knows her match value is higher). It is important to note that the benefit b(c) is independent of k: although p_k^* is increasing in k, any individual consumer's choice has a negligible effect on the equilibrium price. Hence the equilibrium value of k, denoted k*, is zero if $N(0) \$ b(c)$, unity if $N(1) \# b(c)$, and satisfies $N(k^*) = b(c)$ otherwise. We are now in a position to prove:

Proposition 9: *If k* 0 (0,1), there is excessive investment in prior information.*

Proof. The equilibrium condition is that $N(k^*) = b(c)$. Since N is increasing, for any k $\$$ k* we have $N(k)$ $\$$ b(c). However, the marginal social benefit is always strictly below b(c) by Proposition 8 that total conditional social surplus is strictly decreasing in k. Hence the social optimum must involve a level of k

strictly below k*. **Q.E.D.**

The equilibrium condition for investment in prior information is that the cost of getting the information equals the expected benefit for the marginal consumer. This consumer takes into account neither the effect on other consumers of her decision, nor the effect on firms' profits. The other consumers are worse off by the consequent price increase while the firms are better off. However, since the equilibrium is symmetric, there is no change in the matching process (except for the marginal consumer, whose decision is privately optimal and can thus be ignored). Since demand is price-sensitive, the loss in consumer surplus from the price increase exceeds the gain in firms' profit from moving closer to the monopoly level.

VI. Conclusion.

We have shown that consumers may overinvest in information about product characteristics because each consumer does not account for the higher price that ensues from firms taking greater advantage of the informed consumers. An example of such an investment in information is the purchase of travel guides. In such guides, price information is often vague or unreliable (for instance most restaurant guides do not provide information about the price of wine). If travelers use such guides in order to pick among various products of comparable quality (restaurants, hotels, etc.), when they actually start searching, they are in a position comparable to that of the informed consumers of our model. The model suggests that the availability of such guides should lead to higher prices - the cheaper the guides, the higher the price. In the same vein, tourist information bureaux can also be a way for localities to relax price competition among local businesses, so firms would encourage such offices. However these results depend on the

existence of a symmetric pure strategy equilibrium which is proved under the rather strong sufficient condition that the density of tastes is non decreasing.

In this paper we have considered consumers who are uninformed about prices and matches, and consumers who are uninformed about prices only, with the result that the latter impart a negative externality on the former. One could also formulate models in which some consumers may be informed about prices, in which case one would expect there to be positive externalities from informed consumers (see Stahl, 1989, for example). Such differential information across consumers leads one to think about advertising. Indeed, our result that profits rise with the number of informed consumers suggests that firms would want to engage in product advertising. Product advertising would benefit firms if they could commit not to advertise prices at the same time. However, firms are unlikely to be able to thus commit themselves, and price advertising may end up destroying the advantages that product specification advertising creates. In this context, one may ask whether product advertising or price advertising, or both, are used in equilibrium, and whether advertising is excessive or insufficient in these dimensions. These are topics of our ongoing research.

Appendix 1

Lemma A.1: *Suppose $B_a(p_1) = p_1 D(p_1 - p_a)$ is decreasing in p_1 for $p_1 \$ p_a \$ 0$, with $D' \# 0$ whenever D is positive. Then for $p_b \$ p_a$, $B_b(p_1) = p_1 D(p_1 - p_b)$ is decreasing in p_1 for $p_1 \$ p_b$.*

Proof: By hypothesis,

$$p_1 D'(\ )\ ) + D(\ )\ ) \# 0 \ \text{ or } \ )\ D'(\ )\ ) + D(\ )\ ) + p_a D'(\ )\ ) \# 0$$

for $p_1 > p_a$ with $)\ = p_1 - p_a$. Since $p_b > p_a$ and $D' < 0$, it follows that

$$)\ D'(\ )\ ) + D(\ )\ ) + p_b D'(\ )\ ) < 0, \ \text{ or } \ p_1 D'(\ )\ ) + D(\ )\ ) < 0 \text{ with })\ = p_1 - p_b,$$

i.e., profit B_b is strictly decreasing for $p_1 > p_b$. **Q.E.D.**

Proof of Proposition 3: First consider case (i). If Firm 1's price is $p_1 \# p_u + a - b$, its demand is $1 - \frac{1}{2}[1 - F(\hat{x})]$ (Firm 1 retains all consumers coming to it first as well as all those coming from Firm 2: $\hat{x}$ is the highest match value with Firm 2 in the latter group). Thus, for this range of prices Firm 1's profit is linear and increasing in p_1. Let $h /\ \max\{0, p_u^* + a - \hat{x}\}$. To show that p_u^* maximizes profit on $Ú_+$ it now suffices to show that it maximizes profit on $[h, +4)$. Taking the second derivative of (9) with respect to p_1 we have:

$$\frac{\partial^2 D_1}{\partial p_1^2}(p_1, p_u^*) = -\frac{1}{2}[1 - F(\hat{x})]f'(\hat{x} + \Delta)$$
$$- f(a - \Delta)f(a) - \int_{a-\Delta}^{\hat{x}} f'(u + \Delta)f(u)du \tag{A3}$$

Thus if f' is nonnegative on $[a,b]$, D_1 is concave in p_1 for prices at least equal to h, such that $\hat{x} +)\ \# b$. Thus p_u^* maximizes profit for this range of prices.

For $\hat{x} +)\ > b$, (8) can be rewritten as:

$$D_1(p_1, p_u^*) = \int_a^b F(\epsilon - \Delta)f(\epsilon)d\epsilon$$

which is the same as (2), i.e., Firm 1's demand in the benchmark model with no search costs when Firm 2's price is p_u^* (for this range of prices, no consumer will stay at Firm 1 without searching Firm 2). As

shown by Caplin and Nalebuff (1991), D_1 as given by (A4) is logconcave under assumption A1. Furthermore, we know that in the benchmark model, the equilibrium price p_n^* is less than p_u^*. By logconcavity of D_1, profit is logconcave and thus decreasing for all $p_1 > p_n^*$ when the rival sets price p_n^*. Letting $Ð()\,) = D_1(p_1,p_u^*)$, it follows from Lemma A.1 that

$$) Ð'()\,) + Ð()\,) + p_u^* Ð'()\,) < 0,$$

i.e., profit is strictly decreasing for $p_1 > p_u^*$ when the rival sets p_u^*. Hence profit is decreasing for all p_1 such that $\hat{x} +) > b$, which is the parameter range under consideration.

There are two sub-cases to case (ii). First consider $\hat{x} < a$, meaning that no consumer will search if the observed price at the first firm searched is equal to the price expected at the other firm. Moreover, if the putative equilibrium price, p^*, were below y, then each firm's demand would be perfectly inelastic in a neighborhood of p^* so that profits could be increased by unilaterally raising price. For $p^* = y$, Firm 1's demand is zero for a higher price and constant for a lower price, so the equilibrium is $p^* = y$. Now consider the subcase $p_u^* > y$. If p^* were below y, then Firm 1's profit derivative at p^* as given by the left-hand side of (3) would be positive (see (3) and recall $p^* < . p_u^*$, while D_1 and D_1' are independent of p^*), so again Firm 1 can gain by raising its price above p^*. If $p^* = y$ such an upward deviation yields zero profit; y is an equilibrium price since profit increases in price up to y by the concavity of profit established for $) < 0$ in part (i) above, in conjunction with the positive derivative of (3) at $p^* = y$.

Finally case (iii) arises when $c = (E, - a)$. Then, if Firm 2 charges an equilibrium price $p^* \# y$ (correctly anticipated by consumers), Firm 1's profit kinks at $p_1 = p^*$. While the right derivative is still given by (3), the left derivative is always 1/2 which is larger than what it would be for lower values of c (i.e., $\hat{x} > a$). This is because a decrease in price does not lead to a higher demand from those consumers sampling Firm 1 first (since Firm 1 gets all of them at $p_1 = p^*$ anyway) and those consumers (of measure 1/2) represent all of Firm 1's demand (since those sampling Firm 2 first do not engage in search either). Since the arguments in the proof for quasiconcavity of profit hold here for $p_1 \$ p^*$, a necessary and sufficient condition for p^* to be an equilibrium price is that the left-hand side of (3) (which is the right derivative at $p_1 = p^*$) be negative, which translates into $p^* \$ 1/f(a)$. **Q.E.D.**

Proof of Proposition 7: Here we prove the existence of equilibrium for price-sensitive demand under the assumption that the (per consumer) demand elasticity, $\complement$, is decreasing in price (or, equivalently, increasing in conditional surplus).

For Case (i), the proof parallels the development of Propositions 3 and 4: we first prove equilibrium existence for $k = 0$ and then extend the result to $k > 0$. Thus in Step 1 we show that $R(v_1)D_u(v_1, v_u^*)$ is quasiconcave and maximized at v_u^*; in Step 2 we show that profit is quasiconcave in v_1 and maximized at $v_1 = v_k^*$ for $v_2 = v_k^*$. For reference to the preceding cases, note that now $\big) = v^* - v_1$ (which is just the generalization of the inelastic demand case).

Step 1. First note that the fraction of consumers buying from Firm 1 is $1 - \tfrac{1}{2}[1 - F(\hat{x})]$ when its surplus satisfies $\big) \# h = \max\{v_u^* - v(0), a - \hat{x}\}$ (all consumers who check out Firm 1 buy from it). Over this range, Firm 1's profit is decreasing in v_1 since revenue per consumer, R, is decreasing in v_1 (equivalently, revenue increases with price since demand is inelastic for prices below the price corresponding to v_u^*).

For values of v_1 such that $\big) \complement [a - \hat{x}, b - \hat{x}]$, the second derivative of D_1 with respect to v_1 is given by the left-hand side of (A3), so that D_1 is concave in v_1 over this range for $f\!\!N$ nonnegative on $[a,b]$. Since too R is concave in v_1 (and also positive), profit is logconcave (as the product of logconcave functions) and $v_1 = v_u^*$ maximizes profit against v_u^* for $\big) \# b - \hat{x}$. It remains to be shown that profit is increasing in v_1 for $\big) > b - \hat{x}$. Over this range, the fraction of consumers buying from Firm 1 is given by the left-hand side of (A4). Moreover, v_u^* is decreasing in the search cost, and so is lower than when the search cost is zero. let v_n^* denote the equilibrium level of surplus when search costs are zero. Since profit is logconcave for no search costs (D_1 is logconcave, R is concave and positive and hence logconcave, and the product of logconcave functions is logconcave), profit is increasing with v_1 for $v_1 < v_n^*$. It thus suffices to show that the latter property implies that profit is increasing with v_1 for $v_1 < v_u^*$ when the rival charges v_u^*. That is, we must show that when $R(v_1)\mathcal{D}(\big))$ is increasing for $v_1 < v_n^*$ with $\big) = v_n^* - v_1$, then it is increasing for $v_1 < v_u^*$ with $\big) = v_u^* - v_1$, given that $v_u^* \# v_n^*$ (see also Lemma A.1). Since the condition for profit to be increasing in v_1 is $R'/R - \mathcal{D}'/\tilde{\mathcal{D}} > 0$, the desired result follows since R is concave in v_1 (and hence R'/R is decreasing in v_1), and $\mathcal{D}'/\mathcal{D}$ is increasing in v_1 since $\mathcal{D}$ is logconcave.

Step 2. As in Step 1, profit is increasing for v_1 sufficiently large. Here it is increasing for $\gamma \leq h = \max\{v_k^* - v(0), a - \hat{x}\}$.

Now consider $\gamma \in [a - \hat{x}, c]$. We then have $D_1 = (1 - k)D_u + k/2$. Arguments similar to those used in the proof of Proposition 4 show that this is a logconcave function of v_1. Then the logconcavity of profit follows from an argument analogous to that used in Step 1 for $\gamma \in [a - \hat{x}, b - \hat{x}]$. We now show that profit is rising in v_1 for higher γ on both informed and uninformed consumers thus establishing that profit is quasiconcave.

As in Proposition 4, for $\gamma > c$ the proportion of informed consumers staying with Firm 1 starts falling. The argument used in Step 1 (where all consumers are uninformed) may be applied again here to show that profit from informed consumers is then increasing in v_1. To see this note that here the fraction of informed consumers staying with Firm 1 is $D(\gamma - c)$, where D has the same functional form as there (it is the fraction buying from Firm 1 when search costs are 0 and conditional surpluses are v_1 at Firm 1 and $v_k^* - c$ at Firm 2).

It is now shown that profit from uninformed consumers is increasing in v_1 for $\gamma > c$, Since for $\gamma = 0$, overall profit has a zero derivative with respect to v_1 and, since $v_k^* > v(p_m)$, profit from informed consumers is decreasing in v_1. Thus, profit from uninformed consumers must be increasing at $\gamma = 0$. Since it was shown in Step 1 that profit from uninformed consumers is logconcave in v_1 for $\gamma \leq b - \hat{x}$, it must be increasing for $\gamma \in [c, b - \hat{x}]$. For $\gamma > b - \hat{x}$, profit is increasing in v_1 using an argument similar to that used at the end of Step 1. This concludes the proof for case (i) since overall profit is increasing for $\gamma \geq c$.

Arguments for cases (ii) and (iii) are analogous to those for Proposition 3. **Q.E.D.**

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